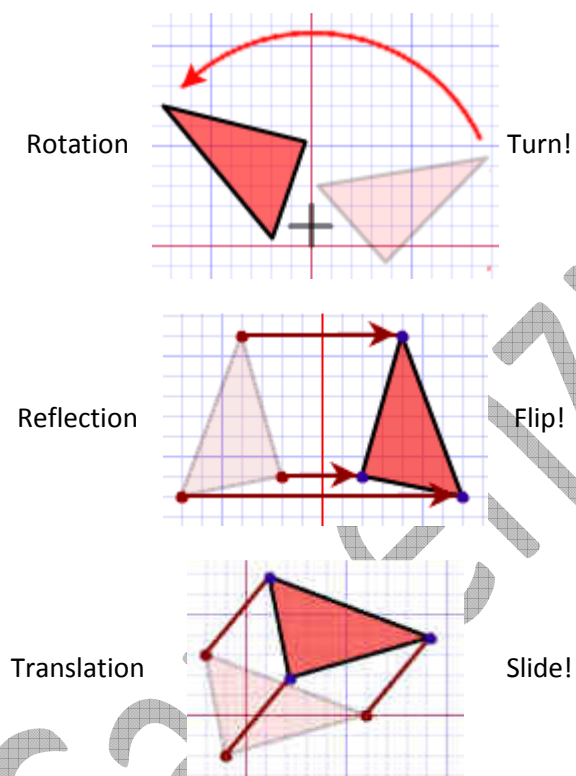


CHAPTER 9: TRANSFORMATIONS

A **transformation** changes the position or size of a shape.

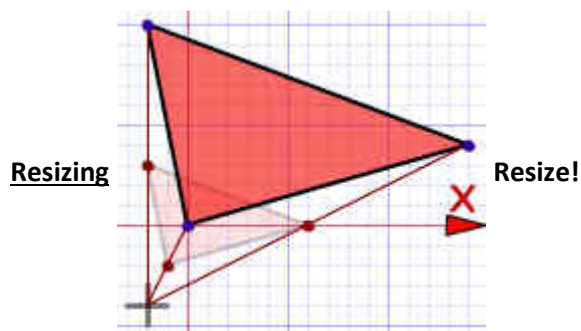
The three main transformations are:



After any of these transformations the shape still has the **same shape, size and side lengths**.

Enlargements

The other important Transformation is **Resizing** (also called *dilation*, *contraction*, *compression*, *enlargement* or even *expansion*). The shape becomes bigger or smaller:



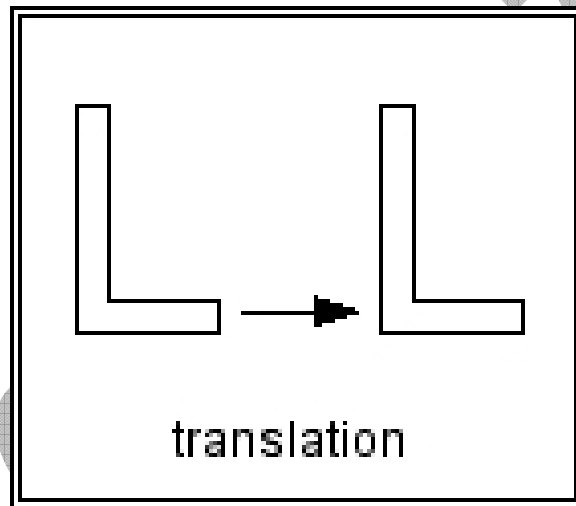
If you have to use Resizing to make one shape become another then the shapes are not Congruent, they are **Similar**.

The starting shape is called the **object**.

The final shape is called the **image**.

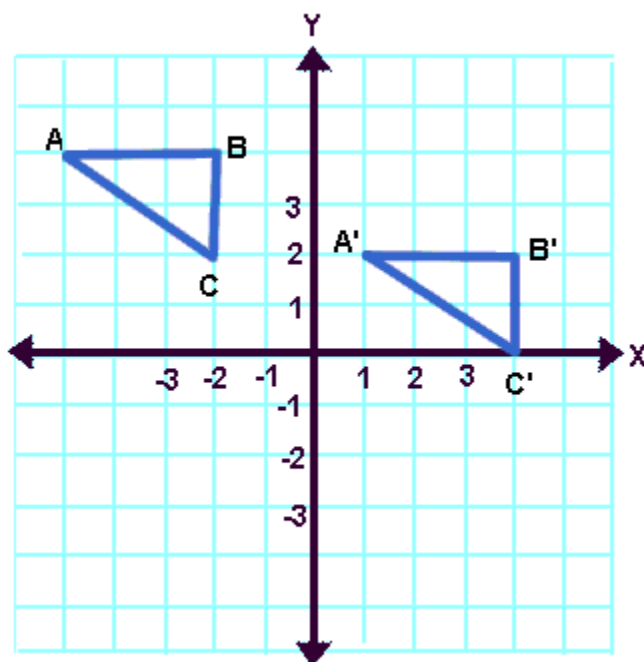
The object **maps onto** the image.

SECTION 9.1 TRANSLATIONS



A translation is like **sliding** the object over a certain distance.

The original object and its translation have the **same shape** and **size**, and they **face in the same direction**.

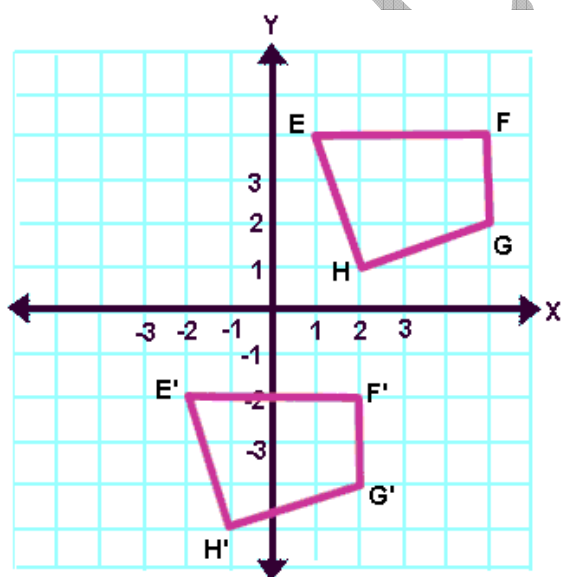


This graph illustrates a translation of a vector of $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$

This is because:

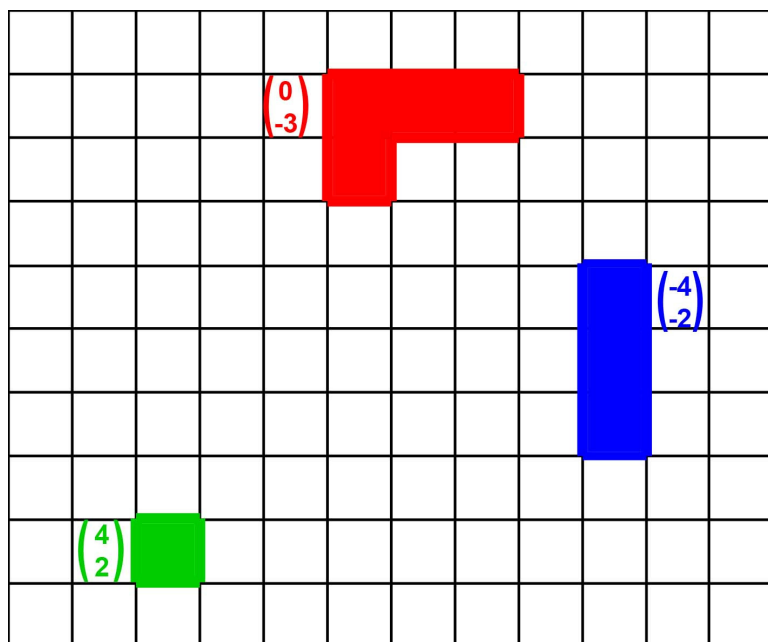
The original diagram was **moved 6 units** on the **x-axis** and **-2** on the **y-axis**.

Try the following:

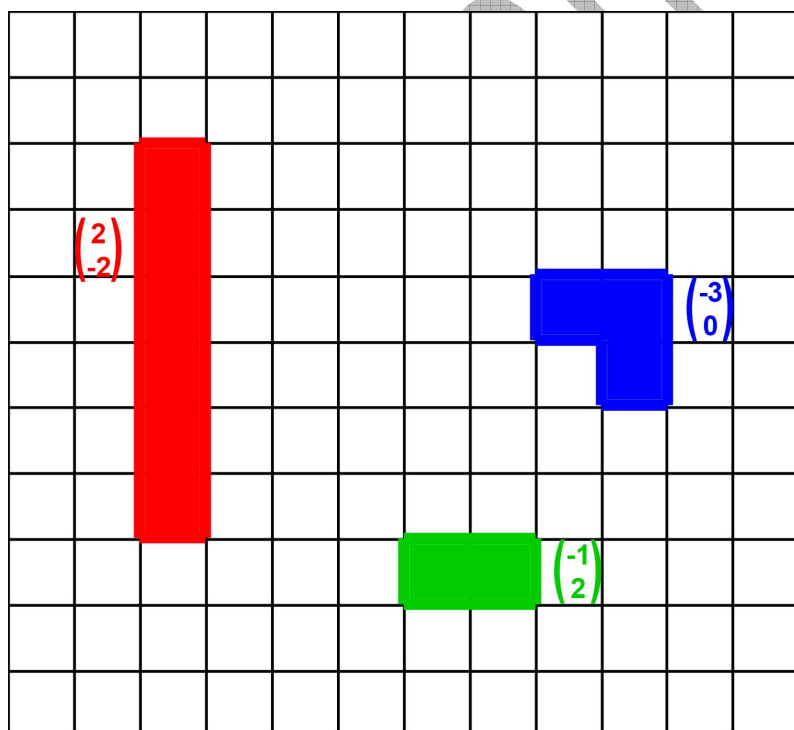


The figure is translated by vector

Example 1: Translate the shapes with the given vectors and find the resulting letter



Example 2: Translate the shapes with the given vectors and find the resulting letter

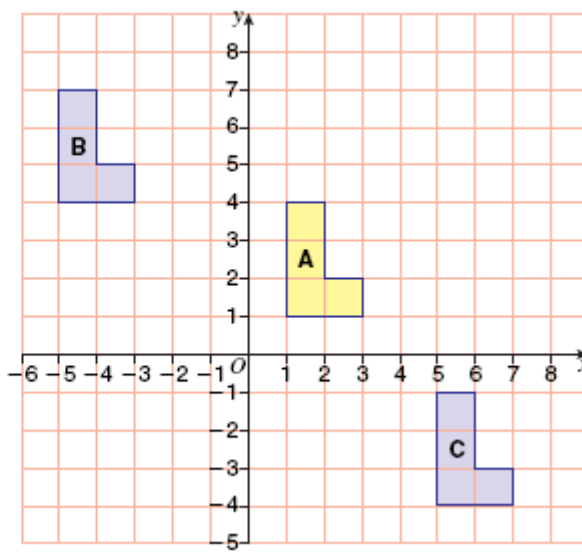
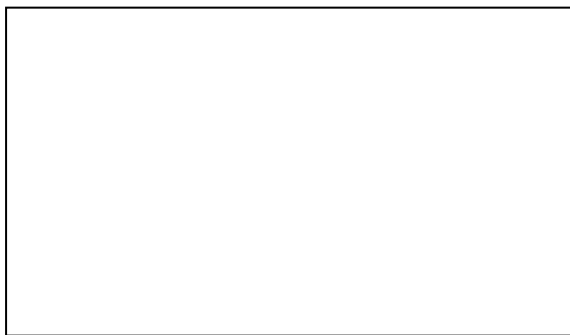


Example 3

a Describe the translation that maps shape A onto **i** shape B, **ii** shape C.

b Translate shape A by the vector $\begin{pmatrix} -3 \\ -5 \end{pmatrix}$.

Label this new shape D.

**Consolidation**

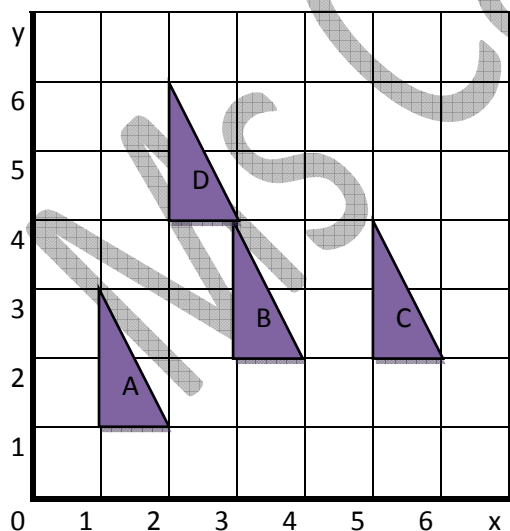
Use vectors to describe the translations of the following triangles.

a. A to B

b. B to C

c. C to D

d. D to A



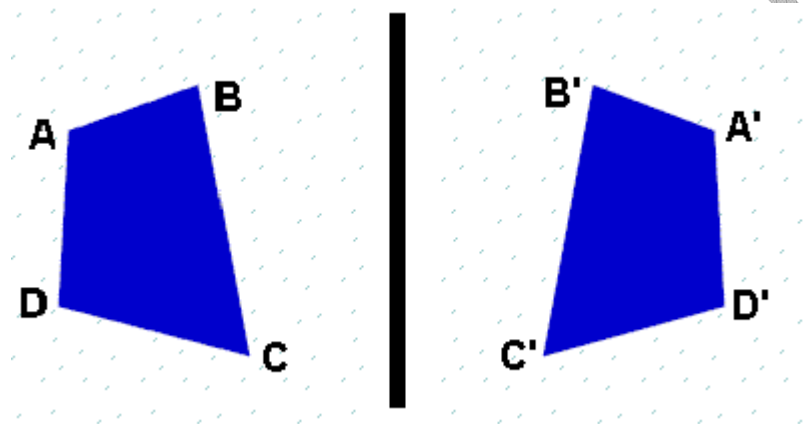
Support Exercise Pg 217 Ex 14A Nos 1 - 4

Handout

SECTION 9.2 REFLECTION

A **reflection** can be seen in water, in a mirror, in glass, or in a shiny surface. An object and its reflection have the **same shape and size**, but the **figures face in opposite directions**. In a mirror, for example, right and left are switched.

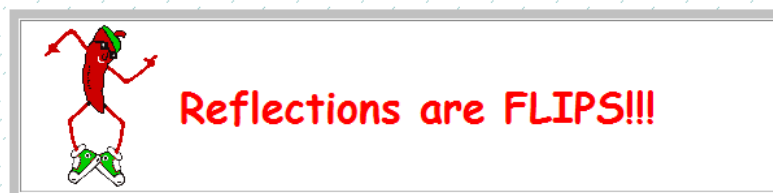
In mathematics, the reflection of an object is called its **image**. If the original object was labeled with letters, such as polygon ABCD, the image may be labeled with the same letters followed by a **prime** symbol, $A'B'C'D'$.



The line (where a mirror may be placed) is called the **line of reflection**. The distance from a point to the line of reflection is the same as the distance from the point's image to the line of reflection.

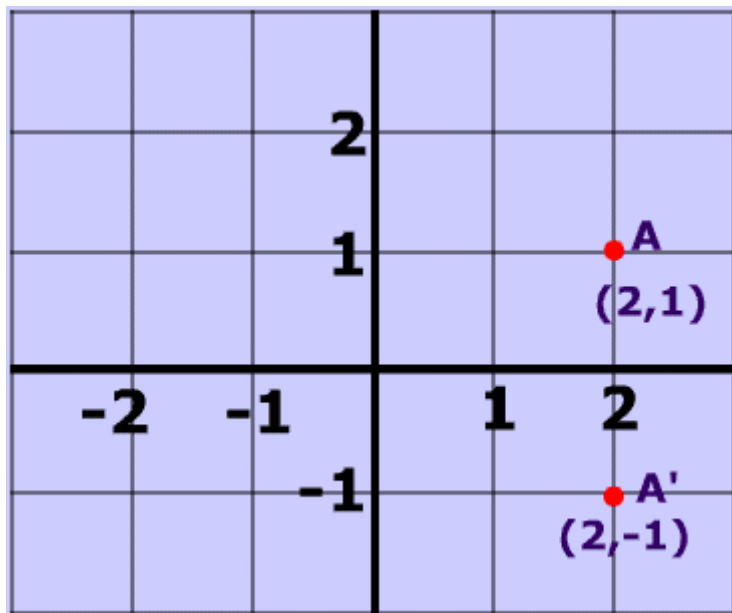
A reflection can be thought of as a "**flipping**" of an object over the line of reflection

Remember:

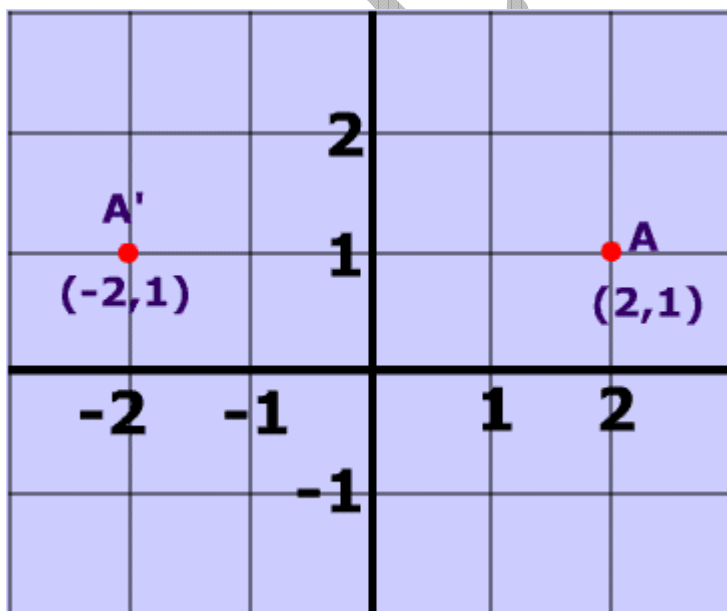


Reflection in x-axis

A reflection in the x-axis can be seen in the picture below in which A is reflected to its image A'. The general rule for a reflection in the x-axis: $(A,B) \rightarrow (A, -B)$

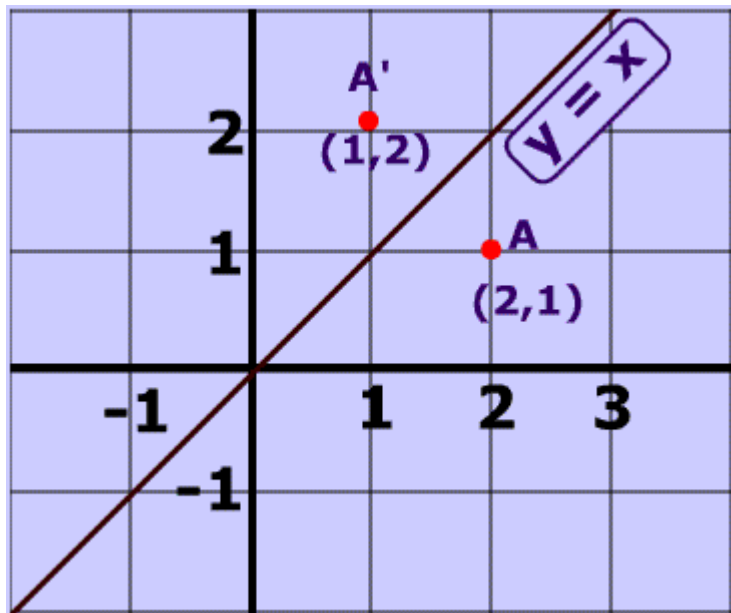
**Reflection in the y-axis**

A reflection in the y-axis can be seen in the picture below in which A is reflected to its image A'. The general rule for a reflection in the y-axis: $(A,B) \rightarrow (-A, B)$



Reflection in the line $y=x$

A reflection in the line $y = x$ can be seen in the picture below in which A is reflected to its image A'. The general rule for a reflection in the y -axis: $(A, B) \rightarrow (B, A)$

**Method 1**

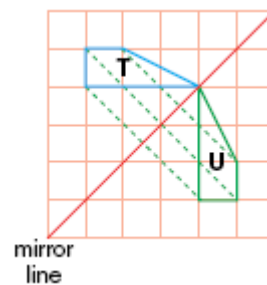
Reflect each corner in the mirror line so that its image is the same distance behind the mirror line as the corner is in front

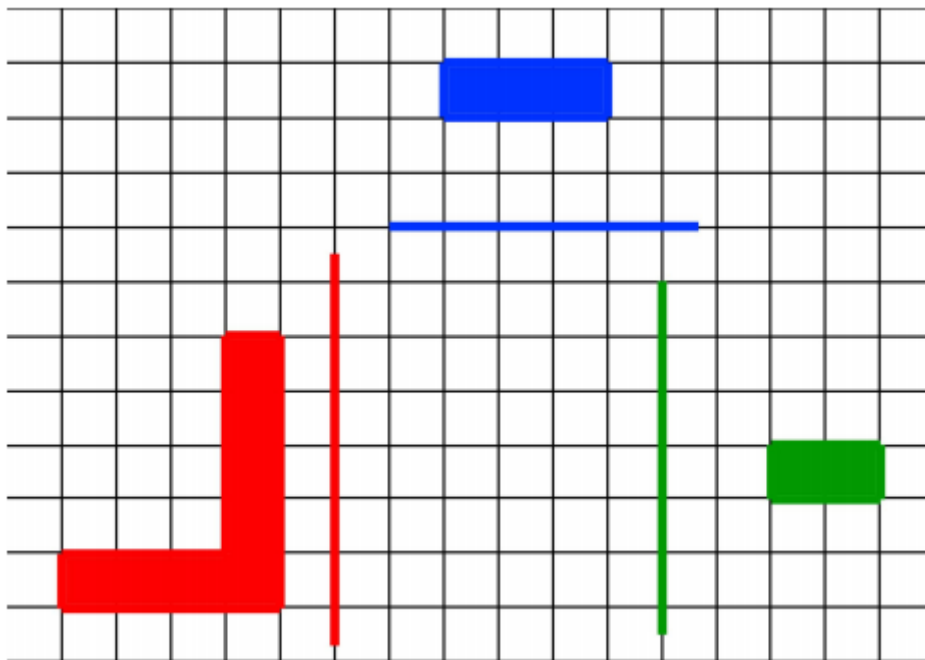
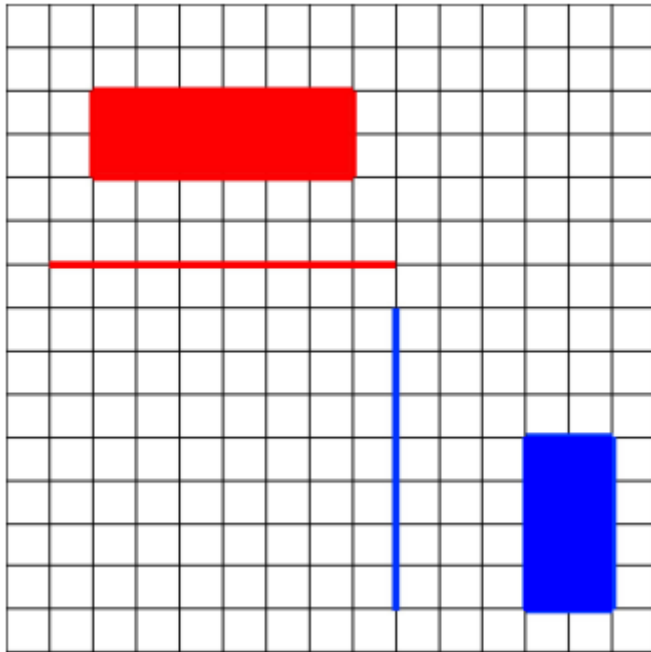
Notice that:

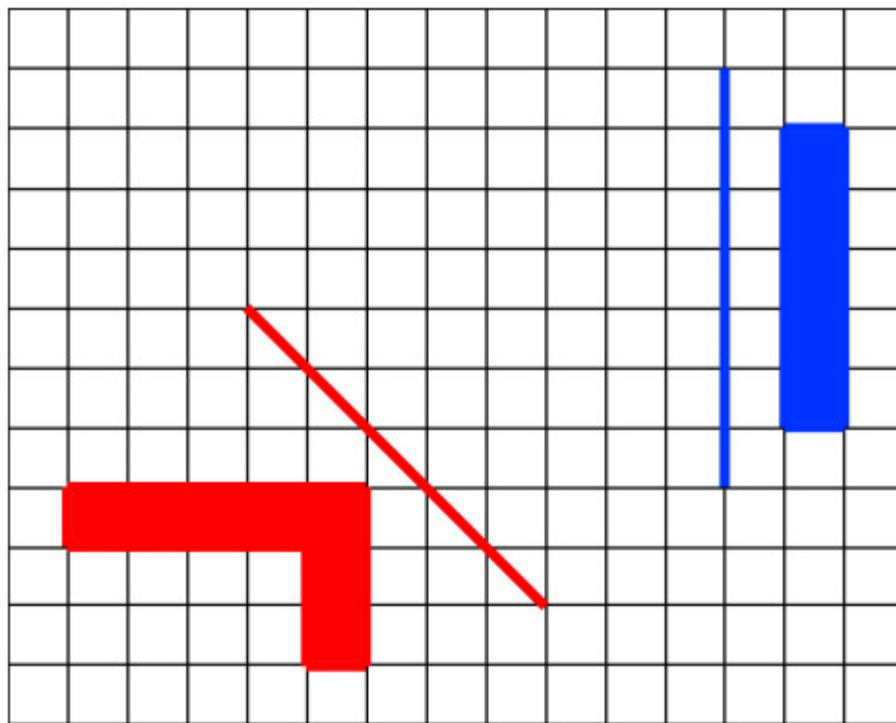
- the line joining each corner to its image is perpendicular to the mirror line.
- the image of the corner which is on the mirror line is also on the mirror line

Method 2

- Put the edge of a sheet of tracing paper (piece of plastic) on the mirror line and make a tracing of the trapezium.
- Turn the tracing paper over and put the edge of the tracing paper back on the mirror line
- Mark the images of the corners with a pencil or compass point.



Example 1



Support Exercise Pg 223 Ex 14C No 1 – 6
Handout

SECTION 12.3 ROTATION

A **rotation** is a transformation that turns a figure about a fixed point called the center of rotation. An object and its rotation are the **same shape and size**, but the **figures may be turned in different directions**. A rotation transforms a shape to a new position by turning it about a fixed point called the **centre of rotation**.

Note:

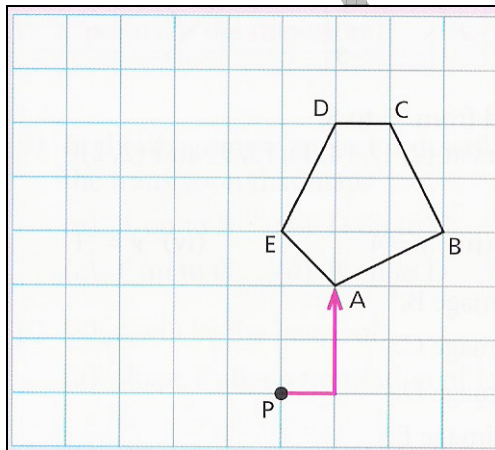
- The direction of turn or the angle of rotation is expressed as clockwise or anticlockwise.
- The position of the centre of rotation is always specified.
- The rotations 180° clockwise and 180° anticlockwise are the same.

The L MethodRotating on Grid with Centre of Rotation

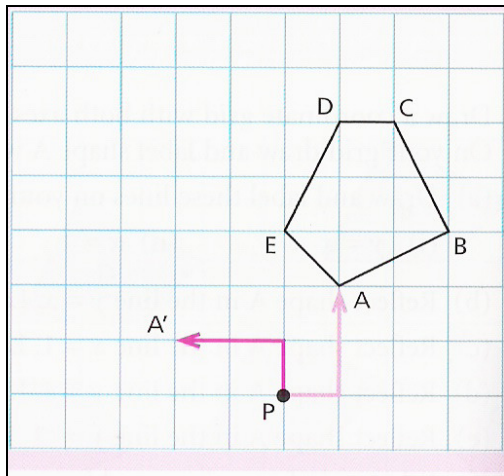
When rotating a shape on a grid it would be very helpful if you rotate each corner with the help of an 'L'

Below is an example of how we use the 'L' to rotate a shape through 90° anticlockwise.

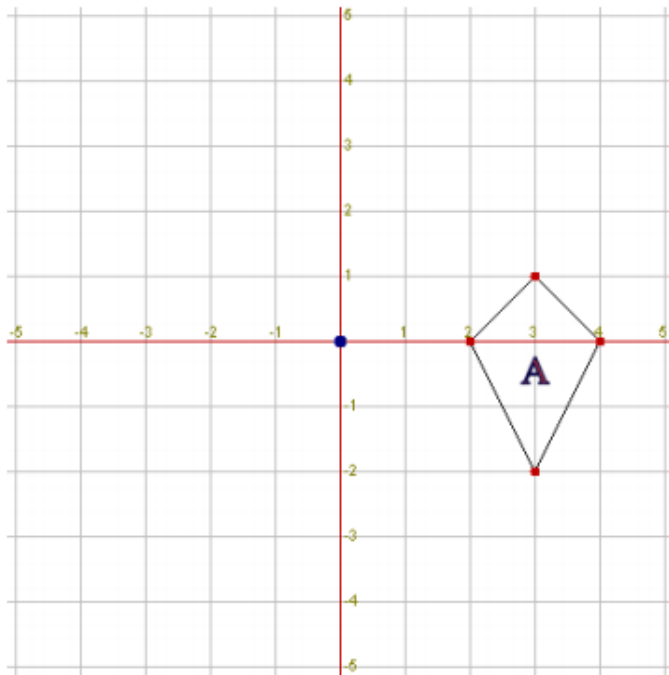
Draw an 'L' from the centre of rotation to the corner of the object.



Rotate the L 90° anticlockwise. The end of the 'L' will mark the new position of the corner.

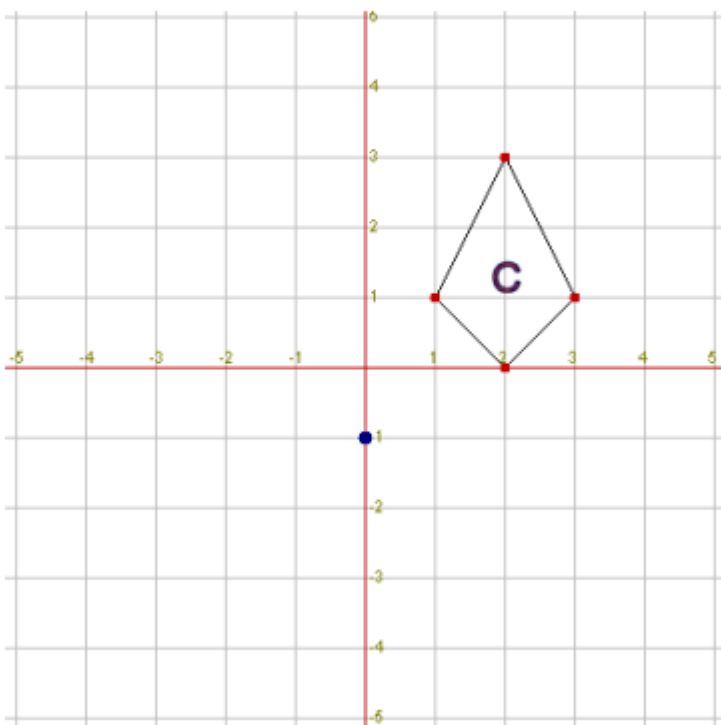
**Example 1**

Rotate shape A through 90° , anti-clockwise, about (0,0). Label it B.



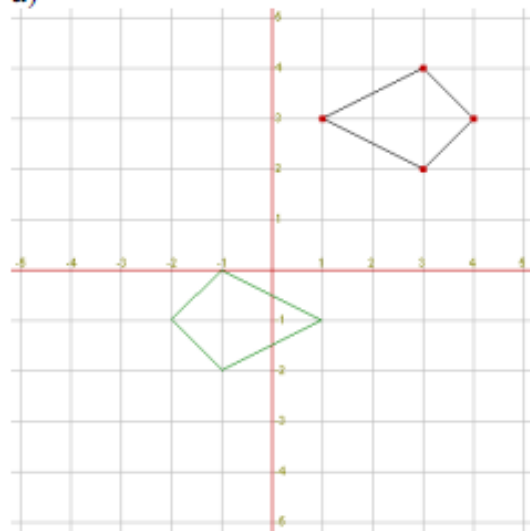
Example 2

Rotate shape C through 180° , centre $(0, -1)$. Label it D.

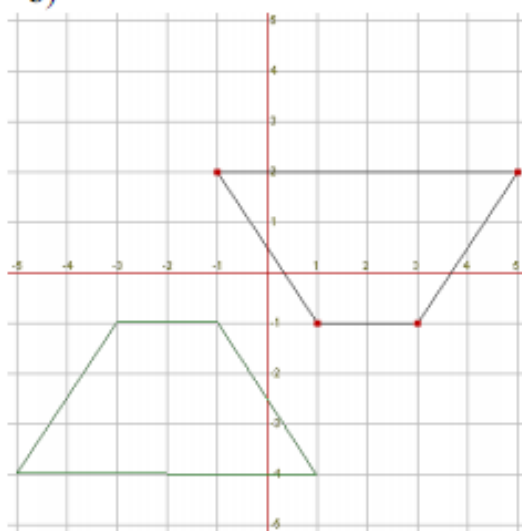
**Example 3**

3 Find the coordinates of the centres of these 180° rotations.

a)

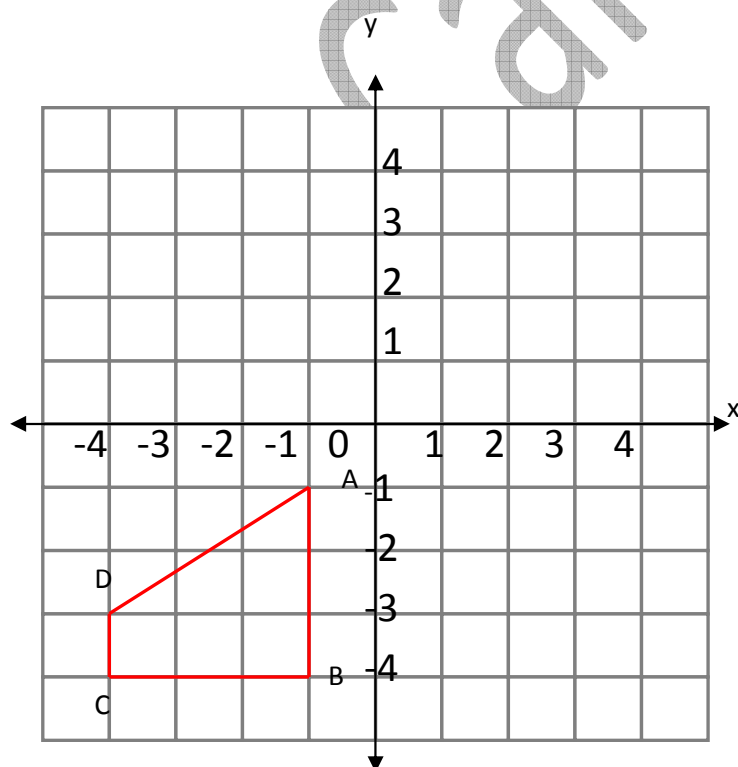
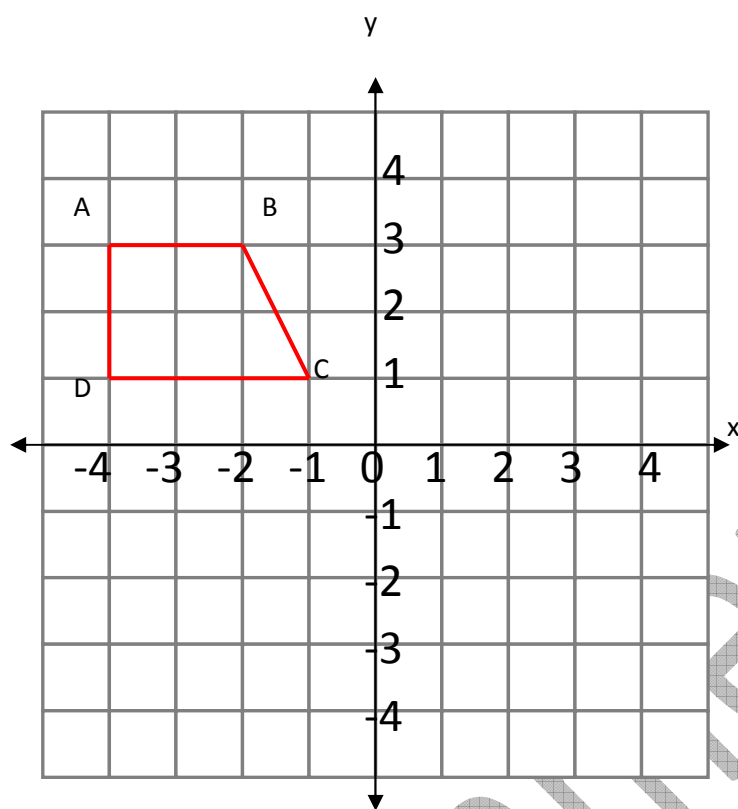


b)



Example 4

Rotate the following shapes by 90° clockwise at about $(0,0)$

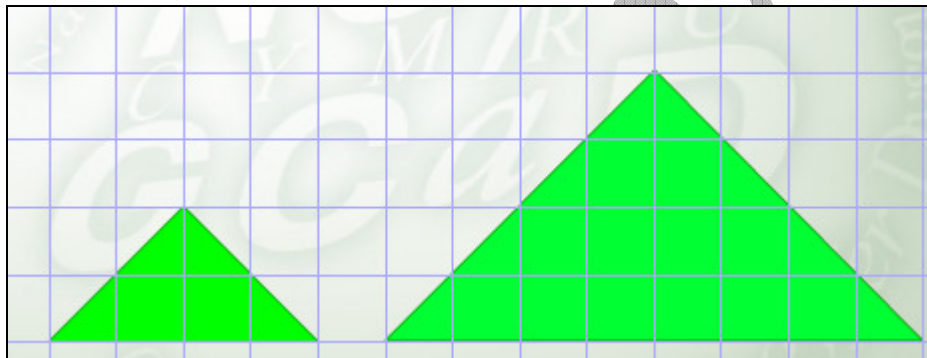


SECTION 12.4 ENLARGEMENTS

Enlargement is a transformation involving an increase or decrease in the length and sides of a shape with a constant factor. It always has a **center of enlargement** and a **scale factor**.

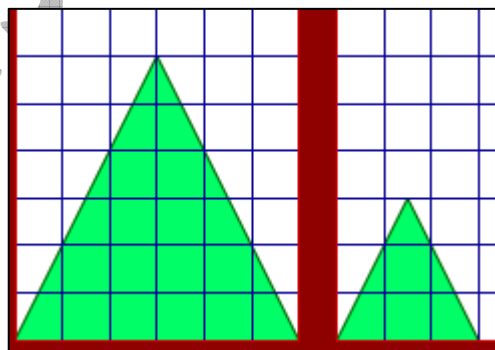
The **scale factor** of enlargement is the number that all the original lengths are multiplied by to produce the image.

If the scale factor, k , is **greater than 1**, the image is an enlargement (a stretch).



Scale Factor 2

If the scale factor is between 0 and 1, the image is a reduction (a shrink).

Scale Factor $\frac{1}{2}$ (0.5)

How can the scale factor be calculated?

The scale factor can be calculated by measuring a pair of corresponding sides in the original shape and in the image and calculating the ratio

$$\frac{\text{Enlarged Length}}{\text{Original Length}}$$

You can check your calculation by choosing a different pair of corresponding sides and finding their ratio. It should be the same.

Every length on the enlarged shape will be:

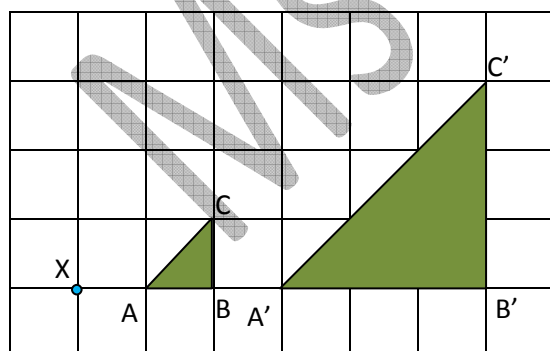
$$\text{original length} \times \text{scale factor}$$

The distance of each image point on the enlargement from the centre of enlargement will be:

$$\text{distance of original point from centre of enlargement} \times \text{scale factor}$$

Example 1

The diagram shows the enlargement of triangle ABC by scale factor 3 about the centre of enlargement X.



Note:

- Each length on the enlargement $A'B'C'$ is three times the corresponding length on the original shape.

This means that the corresponding sides are in the same ratio:

$$AB : A'B' = AC : A'C' = BC : B'C' = 1 : 3$$

- The distance of any point on the enlargement from the centre of enlargement is three times the distance from the corresponding point on the original shape to the centre of enlargement.

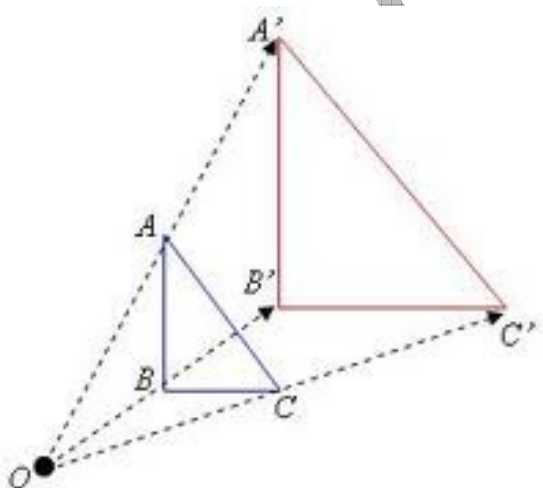
There are two distinct ways to enlarge a shape: the **ray method** and the **coordinate method**.

Ray Method

This is the only way to construct an enlargement when the diagram is not on a grid.

Example 2

Enlarge triangle ABC by scale factor 2 about the centre of enlargement O.



Notice that the rays have been drawn from the centre of enlargement to each vertex and beyond.

The distance from O to each vertex on triangle ABC is measured and multiplied by 2 to give the distance from O to vertex A' , B' and C' for the enlarged triangle $A'B'C'$.

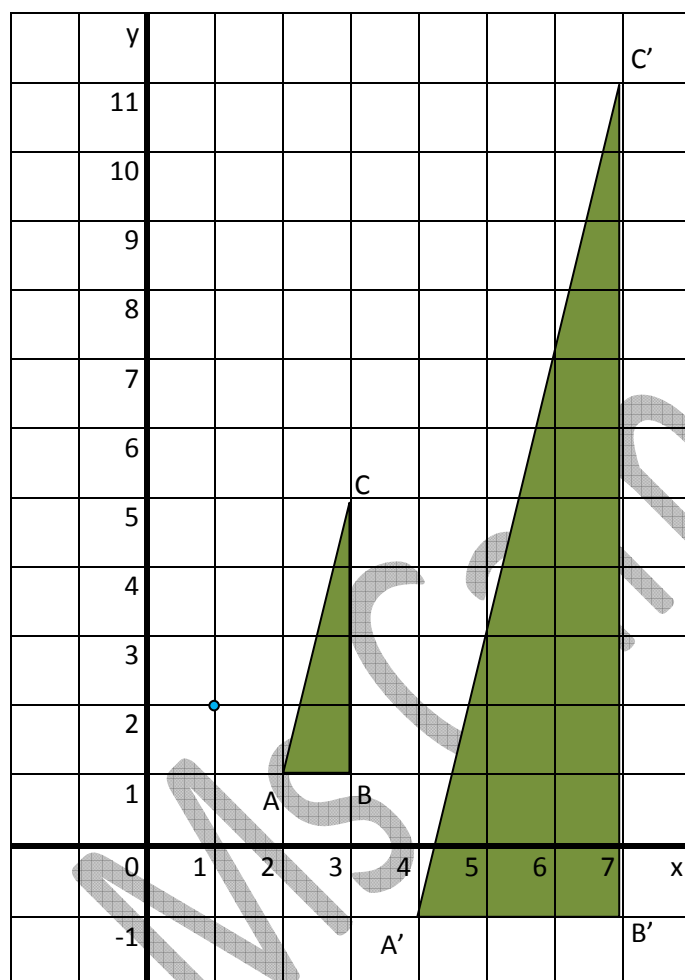
Once each image vertex has been found, the whole enlarged shape can be drawn.

Notice again that the length of each side on the enlarged triangle is two times the length of the corresponding side on the original triangle.

Coordinate Method

Example 3

Enlarge the triangle ABC by scale factor 3 from the centre of enlargement (1,2).



To find the coordinates of each image vertex, first work out the horizontal and vertical distances from each original vertex to the centre of enlargement.

Then multiply each of these distances by 3 to find the position of each image vertex.

For example, to find the coordinates of C' work out the distance from the centre of enlargement (1,2) to the point C (3,5).

Horizontal distance = 2

Vertical distance = 3

Make these 3 times longer to give:

New horizontal distance = $2 \times 3 = 6$

New vertical distance = $3 \times 3 = 9$

So the coordinates of C' are $(1 + 6, 2 + 9) = (7, 11)$

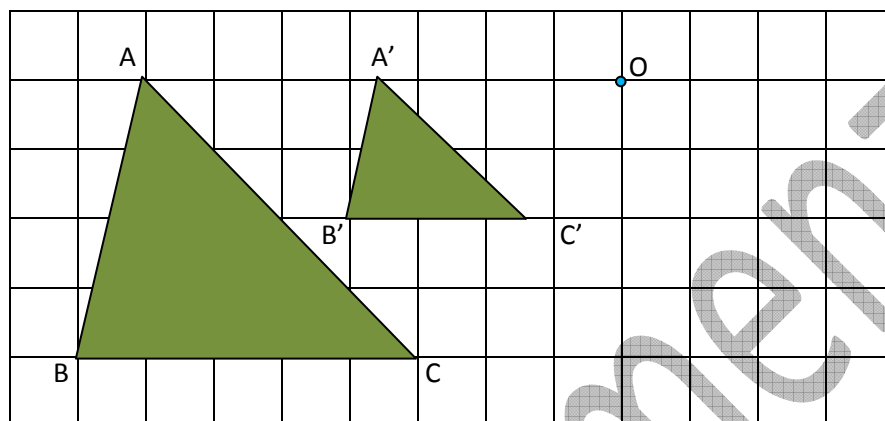
Notice again that the length of each side is three times as long in the enlargement.

Fractional Enlargement

Strange but true... you can have an enlargement in mathematics that is actually smaller than the original shape!

Example 4

Triangle ABC has been enlarged by a scale factor of $\frac{1}{2}$ about the centre of enlargement O to give triangle A'B'C'.



Negative Enlargement (Paper A)

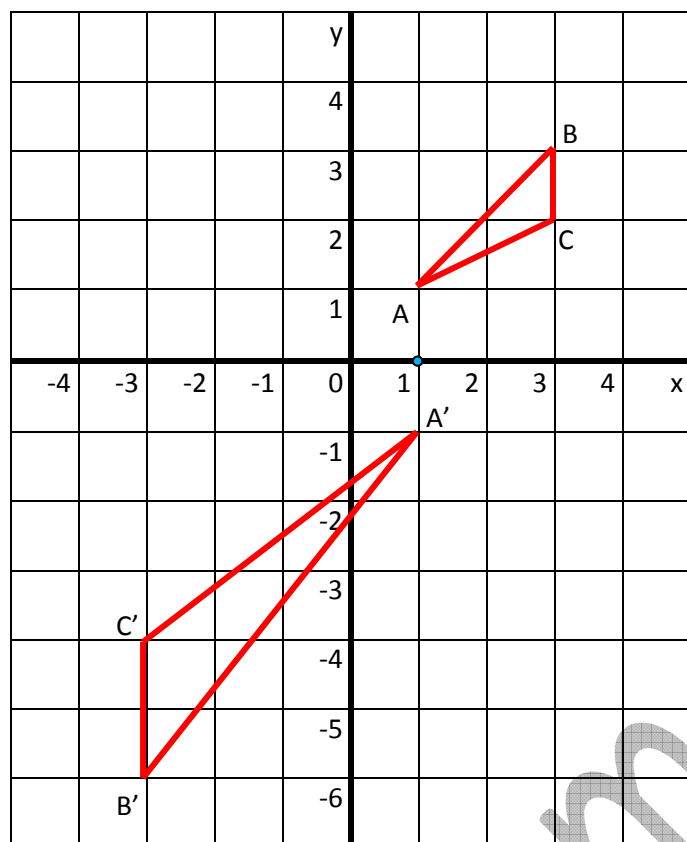
A negative enlargement produces an image shape on the opposite side of the centre of enlargement to the original shape.

Example 5

Triangle ABC has been enlarged by scale factor -2, with centre of enlargement at (1,0).

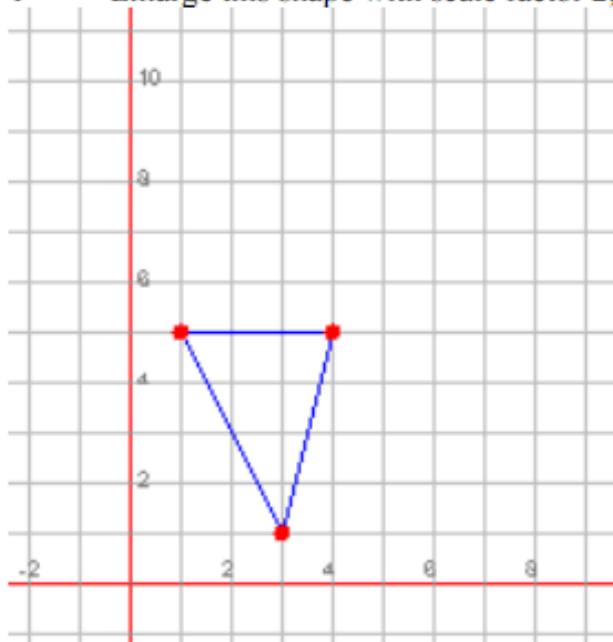
You can enlarge triangle ABC to give triangle A'B'C' by either the ray method or the coordinate method. You calculate the new lengths on the opposite side of the centre of enlargement to the original shape.

Notice how a negative scale factor also inverts the original shape.



Consolidation

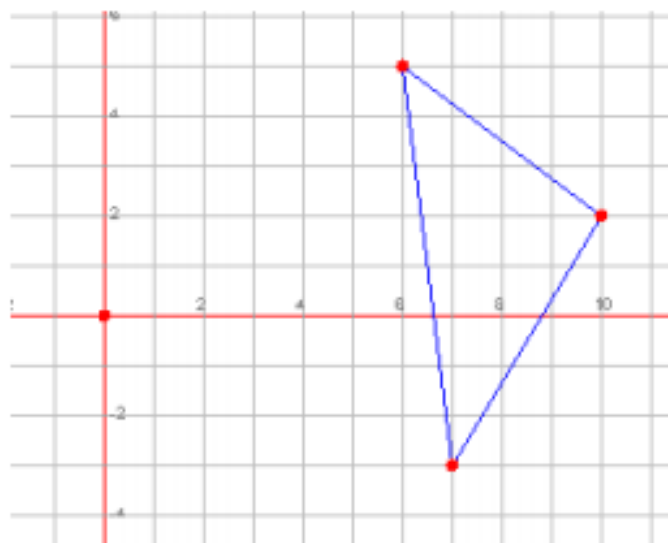
1 Enlarge this shape with scale factor 2, centre (0,0)



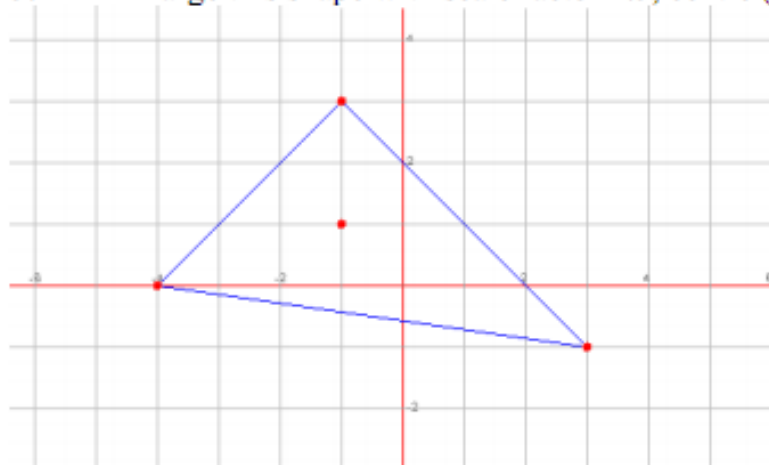
2. Enlarge this shape with scale factor -2 , centre $(0,0)$.



3 Enlarge this shape, scale factor 0.5 , centre $(0,0)$



5. Enlarge this shape with scale factor 1.5, centre $(-1,1)$



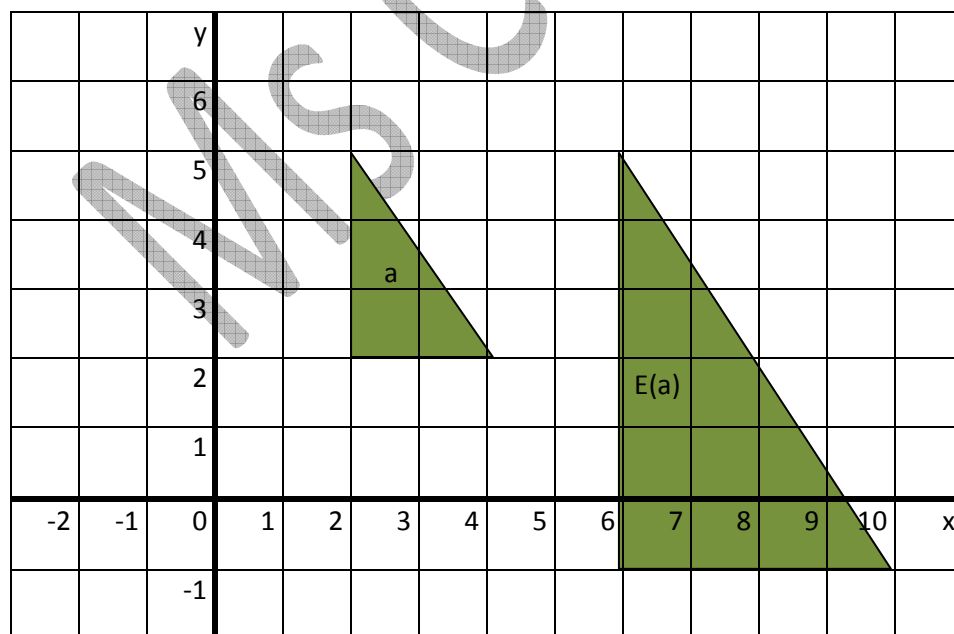
Support Exercise Pg226 Ex 14D No 1 – 7

Handout

SECTION 12.5 FURTHER ENLARGEMENTS

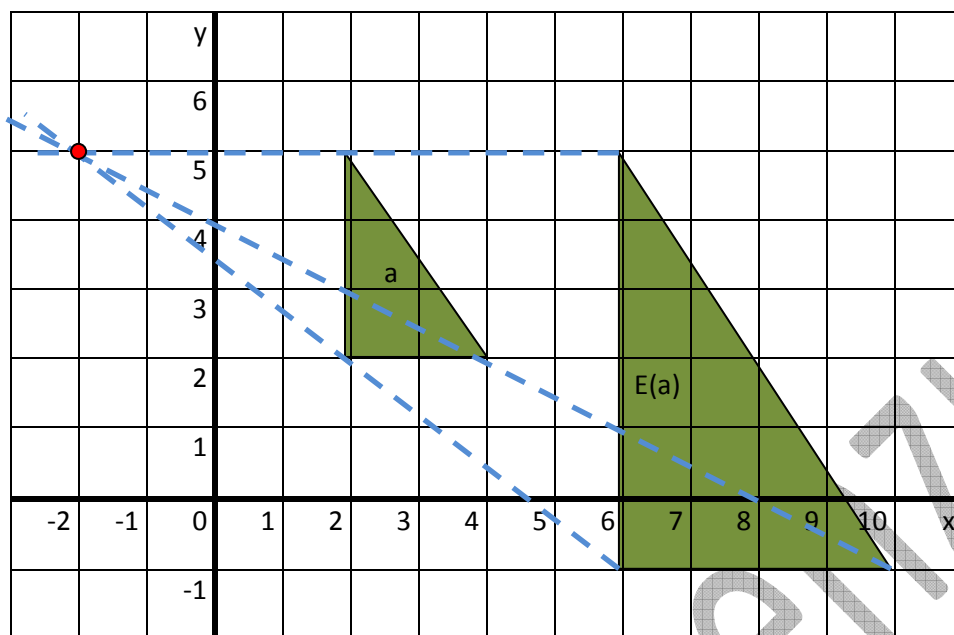
We can find the centre of an enlargement by drawing rays

Example 1



$E(a)$ is an enlargement of shape a .

Find the centre and the scale factor of the enlargement.



Draw rays through corresponding points. They should all meet at one point. This is the centre of enlargement.

In this case the centre is $(-2, 5)$.

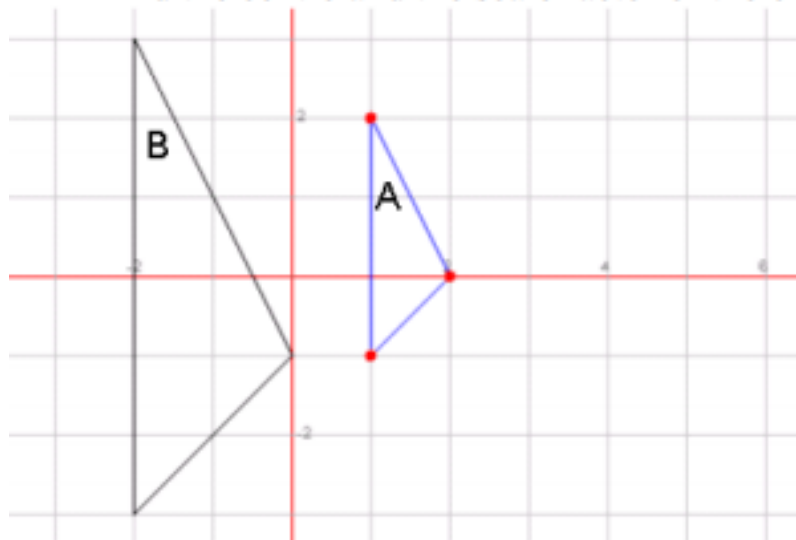
To find the scale factor, find the ratio of corresponding sides.

$$\text{Scale Factor} = \frac{\text{Height of } E(a)}{\text{Height of } a} = \frac{6}{3} = 2$$

Consolidation

1.

Find the centre and the scale factor of the enlargement from A to B.



2. Triangle A has vertices with coordinates (2,1), (4,1) and (4,4).

Triangle B has vertices with coordinates (-5, 1), (-5, 7) and (-1, 7).

Describe fully the single transformation that maps triangle A onto triangle B.

Support Exercise Pg 230 Ex 14e No 1 – 6

Handout

SECTION 12.6 DESCRIBING TRANSFORMATIONS

Questions will require you to use more than one **transformation**.

To describe what is happening in a transformation we must not forget to describe:

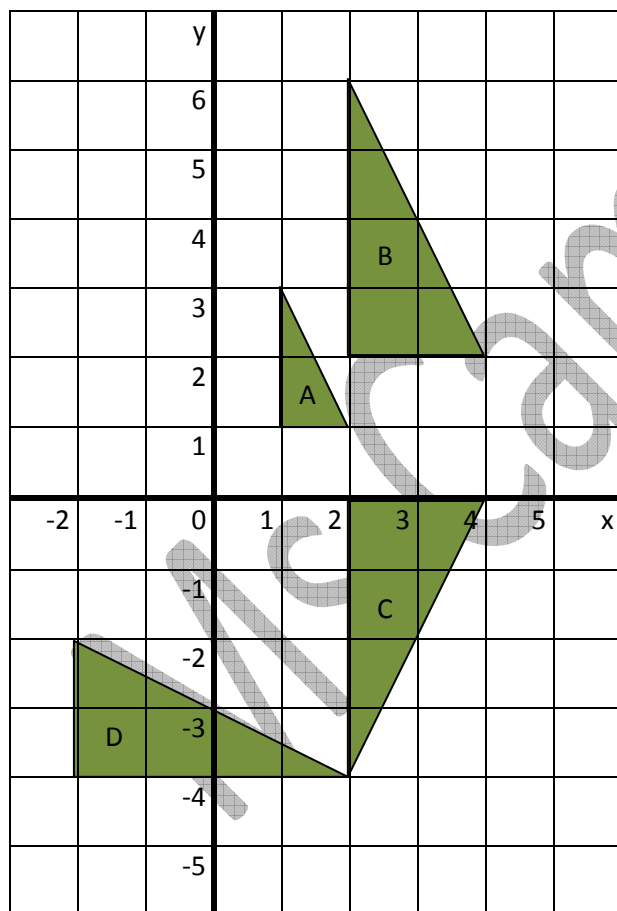
- a **translation**, you need to use a vector
- a **reflection**, you need to use a mirror line

- a **rotation**, you need a centre of rotation, an angle of rotation and a direction of turn
- an **enlargement**, you need a centre of enlargement and a scale factor

Consolidation

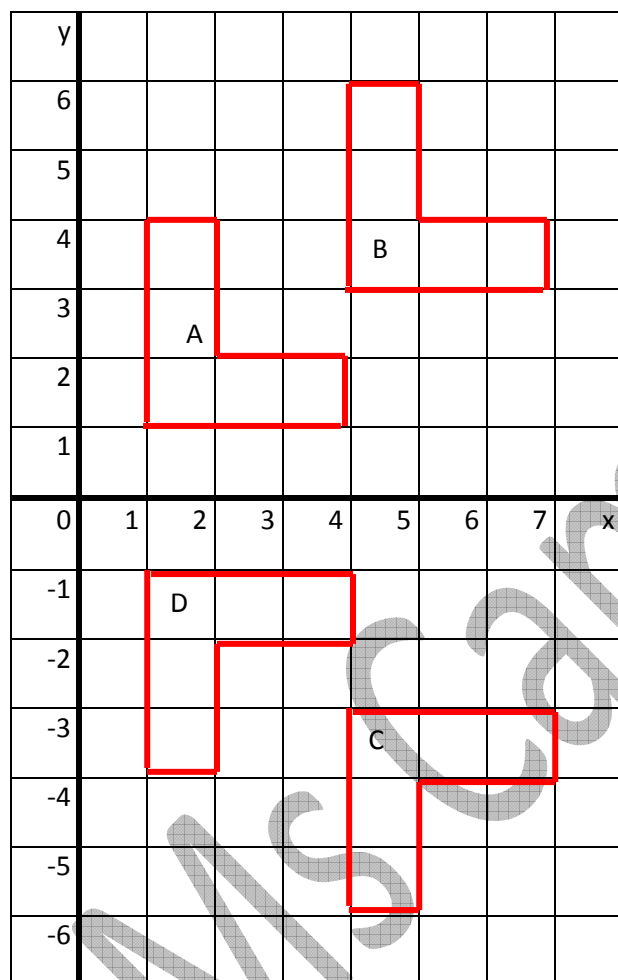
1. Describe fully the following transformations:

- B is the image of A
- C is the image of B
- D is the image of C



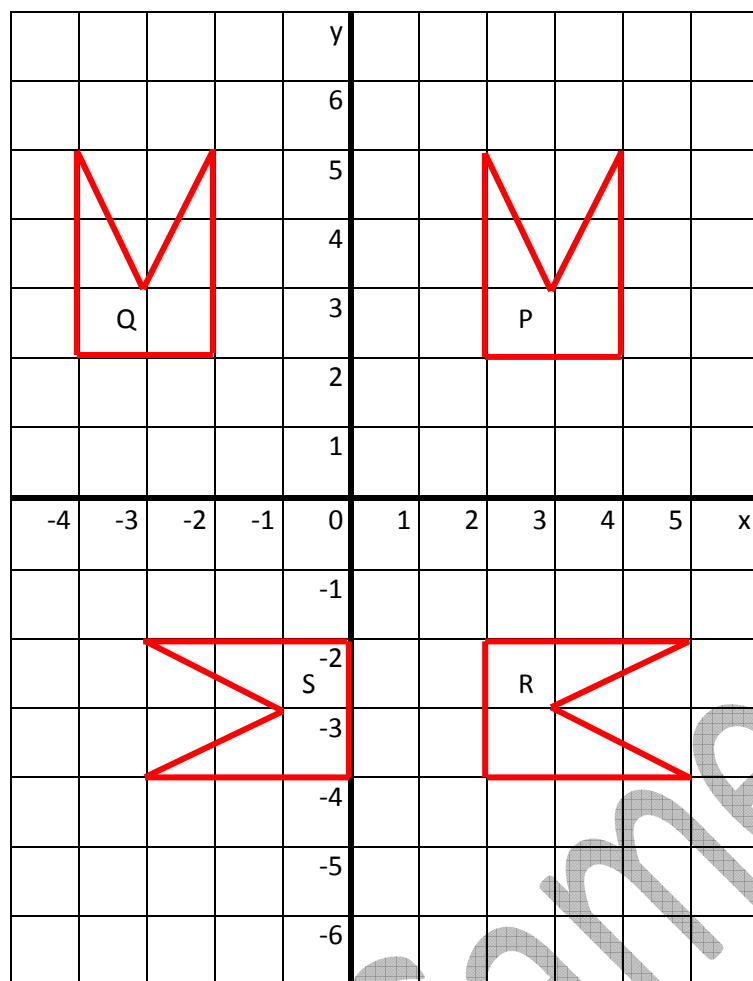
2. Describe fully the following transformations:

- a. A is mapped onto B
- b. B is mapped onto C
- c. C is mapped onto D
- d. D is mapped onto A



3. Describe fully the following transformations:

- a. P is mapped onto Q
- b. Q is mapped onto R
- c. R is mapped onto S
- d. P is mapped onto R

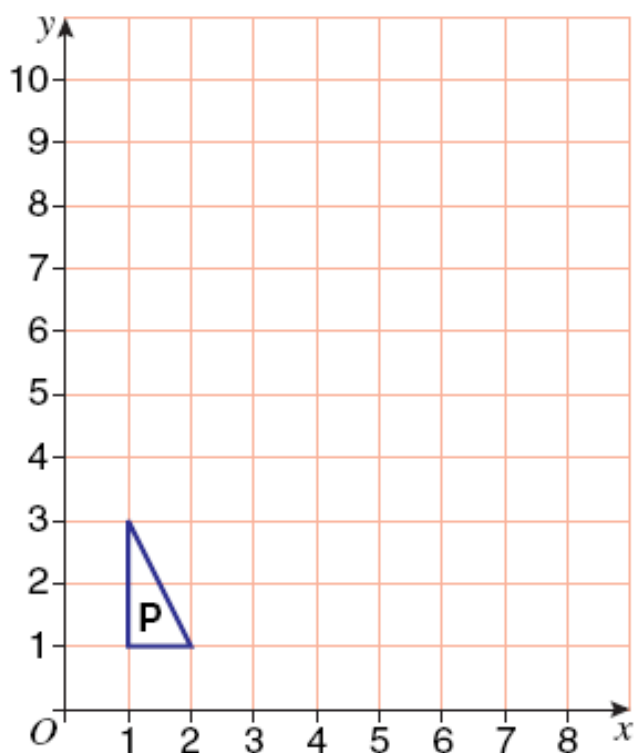
**Support Exercise Handout**

SECTION 12.7 COMBINED TRANSFORMATIONS (PAPER A)

It is sometimes possible to find a single transformation which has the same effect as a combination of transformations.

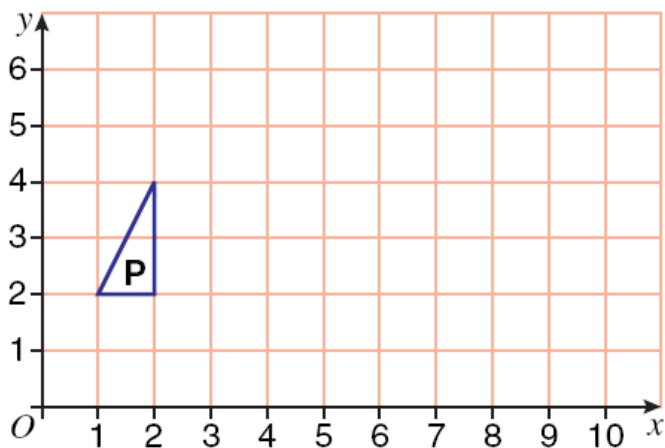
Example 1

- Reflect triangle P in the line $x = 3$. Label the new triangle Q.
- Reflect triangle Q in the line $y = 5$. Label the new image R.
- Describe fully the *single* transformation which maps triangle P onto triangle R.



Example 2

- Rotate triangle P 90° anticlockwise about (3,5). Label the new triangle Q.
- Rotate triangle Q 90° clockwise about (6, 2). Label the new triangle R.
- Describe fully the *single* transformation that maps triangle P onto triangle R.



Support Exercise Pg 234 Ex 14F No 1 – 6