Chapter 9

Chapter 9 Maintaining Mathematical Proficiency (p. 477)

1. $x^2 + 10x + 25 = x^2 + 2(x)(5) + 5^2$
$= (x + 5)^2$

2. $x^2 - 20x + 100 = x^2 - 2(x)(10) + 10^2$
$= (x - 10)^2$

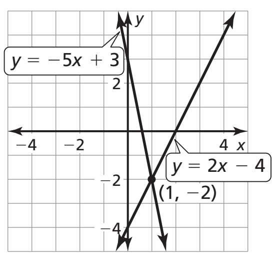
3. $x^2 + 12x + 36 = x^2 + 2(x)(6) + 6^2$
$= (x + 6)^2$

4. $x^2 - 18x + 81 = x^2 - 2(x)(9) + 9^2$
$= (x - 9)^2$

5. $x^2 + 16x + 64 = x^2 + 2(x)(8) + 8^2$
$= (x + 8)^2$

6. $x^2 - 30x + 225 = x^2 - 2(x)(15) + 15^2$
$= (x - 15)^2$

7. $y = -5x + 3$
$y = 2x - 4$



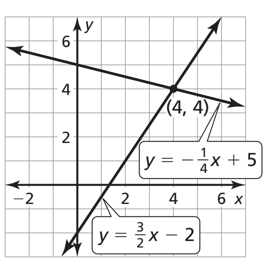
The lines appear to intersect at $(1, -2)$.

Check

$y = -5x + 3$	$y = 2x - 4$
$-2 \stackrel{?}{=} -5(1) + 3$	$-2 \stackrel{?}{=} 2(1) - 4$
$-2 \stackrel{?}{=} -5 + 3$	$-2 \stackrel{?}{=} 2 - 4$
$-2 = -2$ ✓	$-2 = -2$ ✓

The solution is $(1, -2)$.

8. $y = \frac{3}{2}x - 2$
$y = -\frac{1}{4}x + 5$



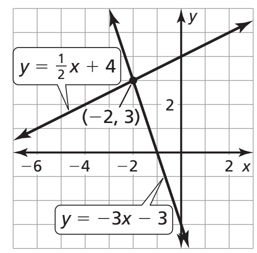
The lines appear to intersect at $(4, 4)$.

Check

$y = \frac{3}{2}x - 2$	$y = -\frac{1}{4}x + 5$
$4 \stackrel{?}{=} \frac{3}{2}(4) - 2$	$4 \stackrel{?}{=} -\frac{1}{4}(4) + 5$
$4 \stackrel{?}{=} 6 - 2$	$4 \stackrel{?}{=} -1 + 5$
$4 = 4$ ✓	$4 = 4$ ✓

The solution is $(4, 4)$.

9. $y = \frac{1}{2}x + 4$
$y = -3x - 3$



The lines appear to intersect at $(-2, 3)$.

Check

$y = \frac{1}{2}x + 4$	$y = -3x - 3$
$3 \stackrel{?}{=} \frac{1}{2}(-2) + 4$	$3 \stackrel{?}{=} -3(-2) - 3$
$3 \stackrel{?}{=} -1 + 4$	$3 \stackrel{?}{=} 6 - 3$
$3 = 3$ ✓	$3 = 3$ ✓

The solution is $(-2, 3)$.

10. A polynomial of the form $x^2 + bx + c$ is a perfect square trinomial when b is twice the square root of c. So, the value of c must be $\left(\frac{b}{2}\right)^2$.

Chapter 9 Mathematical Practices (p. 478)

1. *Sample answer:*

Guess	Check	How to Revise
-1.6	$(-1.6)^2 - 1.6 - 1 = -0.04$	Decrease guess.
-1.65	$(-1.65)^2 - 1.65 - 1 = 0.0725$	Increase guess.
-1.62	$(-1.62)^2 - 1.62 - 1 = 0.0044$	Increase guess.
-1.615	$(-1.615)^2 - 1.615 - 1 = -0.006775$	Decrease guess.
-1.618	$(-1.618)^2 - 1.618 - 1 = -0.000076$	Decrease guess.
-1.6181	$(-1.6181)^2 - 1.6181 - 1 \approx 0.00015$	The solution is between -1.618 and -1.6181.

So, to the nearest thousandth, the negative solution of the equation is $x \approx -1.618$.

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2. *Sample answer:*

Guess	Check	How to Revise
1.3	$(1.3)^2 + 1.3 - 3 = -0.01$	Increase guess.
1.31	$(1.31)^2 + 1.31 - 3 = 0.0261$	Decrease guess.
1.305	$(1.305)^2 + 1.305 - 3 = 0.008025$	Decrease guess.
1.302	$(1.302)^2 + 1.302 - 3 = -0.002796$	Increase guess.
1.3025	$(1.3025)^2 + 1.3025 - 3 \approx -0.00099$	Increase guess.
1.303	$(1.303)^2 + 1.303 - 3 = 0.000806$	The solution is between 1.3025 and 1.303.

So, to the nearest thousandth, the positive solution of the equation is $x \approx 1.303$.

Guess	Check	How to Revise
-2.3	$(-2.3)^2 - 2.3 - 3 = -0.01$	Decrease guess.
-2.31	$(-2.31)^2 - 2.31 - 3 = 0.0261$	Increase guess.
-2.305	$(-2.305)^2 - 2.305 - 3 = 0.008025$	Increase guess.
-2.302	$(-2.302)^2 - 2.302 - 3 = -0.002796$	Decrease guess.
-2.3025	$(-2.3025)^2 - 2.3025 - 3 = -0.00099$	Decrease guess.
-2.303	$(-2.303)^2 - 2.303 - 3 = 0.000809$	The solution is between -2.303 and -2.3025.

So, to the nearest thousandth, the negative solution of the equation is $x \approx -2.303$.

9.1 Explorations (p. 479)

1. a. $\sqrt{36} + \sqrt{64} = 6 + 8 = 14$ and $\sqrt{36 + 64} = \sqrt{100} = 10$. Because $14 \neq 10$, $\sqrt{36} + \sqrt{64}$ does *not* equal $\sqrt{36 + 64}$. So, the general expressions $\sqrt{a} + \sqrt{b}$ and $\sqrt{a + b}$ are *not* equal.

b. $\sqrt{4} \bullet \sqrt{9} = 2 \bullet 3 = 6$ and $\sqrt{4 \bullet 9} = \sqrt{36} = 6$. Because $6 = 6$, $\sqrt{4} \bullet \sqrt{9} = \sqrt{4 \bullet 9}$ is true. Also, $\sqrt{a} \bullet \sqrt{b} = a^{1/2} \bullet b^{1/2}$, and by the Power of a Product Property, $a^{1/2} \bullet b^{1/2} = (a \bullet b)^{1/2}$. Also, $(a \bullet b)^{1/2} = \sqrt{a \bullet b}$. So, the general expressions $\sqrt{a} \bullet \sqrt{b}$ and $\sqrt{a \bullet b}$ are equal.

c. $\sqrt{64} - \sqrt{36} = 8 - 6 = 2$ and $\sqrt{64 - 36} = \sqrt{28}$. Because $2 \neq \sqrt{28}$, $\sqrt{64} - \sqrt{36}$ does *not* equal $\sqrt{64 - 36}$. So, the general expressions $\sqrt{a} - \sqrt{b}$ and $\sqrt{a - b}$ are *not* equal.

d. $\dfrac{\sqrt{100}}{\sqrt{4}} = \dfrac{10}{2} = 5$ and $\sqrt{\dfrac{100}{4}} = \sqrt{25} = 5$. Because $5 = 5$, $\dfrac{\sqrt{100}}{\sqrt{4}} = \sqrt{\dfrac{100}{4}}$ is true. Also, $\dfrac{\sqrt{a}}{\sqrt{b}} = \dfrac{a^{1/2}}{b^{1/2}}$, and by the Power of a Product Property, $\dfrac{a^{1/2}}{b^{1/2}} = \left(\dfrac{a}{b}\right)^{1/2}$. Also, $\left(\dfrac{a}{b}\right)^{1/2} = \sqrt{\dfrac{a}{b}}$. So, the general expressions $\dfrac{\sqrt{a}}{\sqrt{b}}$ and $\sqrt{\dfrac{a}{b}}$ are equal.

2. *Sample answer:* A counterexample for adding square roots is $\sqrt{9} + \sqrt{16} \neq \sqrt{25}$, and a counterexample for subtracting square roots is $\sqrt{16} - \sqrt{9} \neq \sqrt{7}$.

3. Multiply or divide the numbers inside the square root symbols and take the square root of the product or quotient.

4. *Sample answer:* An example of multiplying square roots is $\sqrt{9} \bullet \sqrt{16} = \sqrt{9 \bullet 16} = \sqrt{144} = 12$. An example of dividing square roots is $\dfrac{\sqrt{16}}{\sqrt{4}} = \sqrt{\dfrac{16}{4}} = \sqrt{4} = 2$.

5. a. Because $\sqrt{a} \bullet \sqrt{b}$ and $\sqrt{a \bullet b}$ are equal, an algebraic rule for the product of square roots is $\sqrt{a} \bullet \sqrt{b} = \sqrt{a \bullet b}$.

b. Because $\dfrac{\sqrt{a}}{\sqrt{b}}$ and $\sqrt{\dfrac{a}{b}}$ are equal, an algebraic rule for the quotient of square roots is $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$.

9.1 Monitoring Progress (pp. 480–484)

1. $\sqrt{24} = \sqrt{4 \bullet 6}$
$= \sqrt{4} \bullet \sqrt{6}$
$= 2\sqrt{6}$

2. $-\sqrt{80} = -\sqrt{16 \bullet 5}$
$= -\sqrt{16} \bullet \sqrt{5}$
$= -4\sqrt{5}$

3. $\sqrt{49x^3} = \sqrt{49 \bullet x^2 \bullet x}$
$= \sqrt{49} \bullet \sqrt{x^2} \bullet \sqrt{x}$
$= 7x\sqrt{x}$

4. $\sqrt{75n^5} = \sqrt{25 \bullet 3 \bullet n^4 \bullet n}$
$= \sqrt{25} \bullet \sqrt{3} \bullet \sqrt{n^4} \bullet \sqrt{n}$
$= 5 \bullet \sqrt{3} \bullet n^2 \bullet \sqrt{n}$
$= 5 \bullet n^2 \bullet \sqrt{3} \bullet \sqrt{n}$
$= 5n^2\sqrt{3n}$

5. $\sqrt{\dfrac{23}{9}} = \dfrac{\sqrt{23}}{\sqrt{9}}$
$= \dfrac{\sqrt{23}}{3}$

6. $-\sqrt{\dfrac{17}{100}} = -\dfrac{\sqrt{17}}{\sqrt{100}}$
$= -\dfrac{\sqrt{17}}{10}$

7. $\sqrt{\dfrac{36}{z^2}} = \dfrac{\sqrt{36}}{\sqrt{z^2}}$
$= \dfrac{6}{z}$

8. $\sqrt{\dfrac{4x^2}{64}} = \dfrac{\sqrt{4x^2}}{\sqrt{64}}$
$= \dfrac{\sqrt{4} \bullet \sqrt{x^2}}{8}$
$= \dfrac{2 \bullet x}{8}$
$= \dfrac{x}{4}$

9. $\sqrt[3]{54} = \sqrt[3]{27 \cdot 2}$
$= \sqrt[3]{27} \cdot \sqrt[3]{2}$
$= 3\sqrt[3]{2}$

10. $\sqrt[3]{16x^4} = \sqrt[3]{8 \cdot 2 \cdot x^3 \cdot x}$
$= \sqrt[3]{8} \cdot \sqrt[3]{2} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x}$
$= 2 \cdot \sqrt[3]{2} \cdot x \cdot \sqrt[3]{x}$
$= 2 \cdot x \cdot \sqrt[3]{2} \cdot \sqrt[3]{x}$
$= 2x\sqrt[3]{2x}$

11. $\sqrt[3]{\dfrac{a}{-27}} = \dfrac{\sqrt[3]{a}}{\sqrt[3]{-27}}$
$= \dfrac{\sqrt[3]{a}}{-3}$
$= -\dfrac{\sqrt[3]{a}}{3}$

12. $\sqrt[3]{\dfrac{25c^7d^3}{64}} = \dfrac{\sqrt[3]{25c^7d^3}}{\sqrt[3]{64}}$
$= \dfrac{\sqrt[3]{25} \cdot \sqrt[3]{c^7} \cdot \sqrt[3]{d^3}}{4}$
$= \dfrac{\sqrt[3]{25} \cdot \sqrt[3]{c^6} \cdot \sqrt[3]{c} \cdot \sqrt[3]{d^3}}{4}$
$= \dfrac{\sqrt[3]{25} \cdot c^2 \cdot \sqrt[3]{c} \cdot d}{4}$
$= \dfrac{c^2d\sqrt[3]{25} \cdot \sqrt[3]{c}}{4}$
$= \dfrac{c^2d\sqrt[3]{25c}}{4}$

13. $\dfrac{1}{\sqrt{5}} = \dfrac{1}{\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}}$
$= \dfrac{\sqrt{5}}{\sqrt{25}}$
$= \dfrac{\sqrt{5}}{5}$

14. $\dfrac{\sqrt{10}}{\sqrt{3}} = \dfrac{\sqrt{10}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$
$= \dfrac{\sqrt{30}}{\sqrt{9}}$
$= \dfrac{\sqrt{30}}{3}$

15. $\dfrac{7}{\sqrt{2x}} = \dfrac{7}{\sqrt{2x}} \cdot \dfrac{\sqrt{2x}}{\sqrt{2x}}$
$= \dfrac{7\sqrt{2x}}{\sqrt{4x^2}}$
$= \dfrac{7\sqrt{2x}}{\sqrt{4} \cdot \sqrt{x^2}}$
$= \dfrac{7\sqrt{2x}}{2x}$

16. $\sqrt{\dfrac{2y^2}{3}} = \dfrac{\sqrt{2y^2}}{\sqrt{3}}$
$= \dfrac{\sqrt{2} \cdot \sqrt{y^2}}{\sqrt{3}} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$
$= \dfrac{\sqrt{2} \cdot y \cdot \sqrt{3}}{3}$
$= \dfrac{y\sqrt{2} \cdot \sqrt{3}}{3}$
$= \dfrac{y\sqrt{6}}{3}$

17. $\dfrac{5}{\sqrt[3]{32}} = \dfrac{5}{\sqrt[3]{8} \cdot \sqrt[3]{4}} \cdot \dfrac{\sqrt[3]{2}}{\sqrt[3]{2}}$
$= \dfrac{5 \cdot \sqrt[3]{2}}{2 \cdot \sqrt[3]{8}}$
$= \dfrac{5\sqrt[3]{2}}{2 \cdot 2}$
$= \dfrac{5\sqrt[3]{2}}{4}$

18. $\dfrac{8}{1 + \sqrt{3}} = \dfrac{8}{1 + \sqrt{3}} \cdot \dfrac{1 - \sqrt{3}}{1 - \sqrt{3}}$
$= \dfrac{8\left(1 - \sqrt{3}\right)}{1^2 - \left(\sqrt{3}\right)^2}$
$= \dfrac{8(1) - 8\sqrt{3}}{1 - 3}$
$= \dfrac{8 - 8\sqrt{3}}{-2}$
$= \dfrac{8}{-2} - \dfrac{8\sqrt{3}}{-2}$
$= -4 + 4\sqrt{3}$

19. $\dfrac{\sqrt{13}}{\sqrt{5} - 2} = \dfrac{\sqrt{13}}{\sqrt{5} - 2} \cdot \dfrac{\sqrt{5} + 2}{\sqrt{5} + 2}$
$= \dfrac{\sqrt{13}\left(\sqrt{5} + 2\right)}{\left(\sqrt{5}\right)^2 - 2^2}$
$= \dfrac{\sqrt{13} \cdot \sqrt{5} + \sqrt{13}(2)}{5 - 4}$
$= \dfrac{\sqrt{65} + 2\sqrt{13}}{1}$
$= \sqrt{65} + 2\sqrt{13}$

20. $\dfrac{12}{\sqrt{2} + \sqrt{7}} = \dfrac{12}{\sqrt{2} + \sqrt{7}} \cdot \dfrac{\sqrt{2} - \sqrt{7}}{\sqrt{2} - \sqrt{7}}$
$= \dfrac{12\left(\sqrt{2} - \sqrt{7}\right)}{\left(\sqrt{2}\right)^2 - \left(\sqrt{7}\right)^2}$
$= \dfrac{12\sqrt{2} - 12\sqrt{7}}{2 - 7}$
$= \dfrac{12\sqrt{2} - 12\sqrt{7}}{-5}$
$= -\dfrac{12\sqrt{2} - 12\sqrt{7}}{5}$

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21. $d = \sqrt{\frac{3h}{2}}$
$= \sqrt{\frac{3(35)}{2}}$
$= \frac{\sqrt{105}}{\sqrt{2}}$
$= \frac{\sqrt{105}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{\sqrt{210}}{\sqrt{4}}$
$= \frac{\sqrt{210}}{2}$

You can see $\frac{\sqrt{210}}{2}$, or about 7.25 miles.

22. Let ℓ be the length of the longer side.

$\frac{1+\sqrt{5}}{2} = \frac{\ell}{50}$
$2\ell = 50(1+\sqrt{5})$
$\frac{2\ell}{2} = \frac{50(1+\sqrt{5})}{2}$
$\ell = 25(1+\sqrt{5})$
$\ell = 25(1) + 25\sqrt{5}$
$\ell = 25 + 25\sqrt{5}$
$\ell \approx 80.90$

The length of the longer side is about 81 feet.

23. $3\sqrt{2} - \sqrt{6} + 10\sqrt{2} = 3\sqrt{2} + 10\sqrt{2} - \sqrt{6}$
$= (3 + 10)\sqrt{2} - \sqrt{6}$
$= 13\sqrt{2} - \sqrt{6}$

24. $4\sqrt{7} - 6\sqrt{63} = 4\sqrt{7} - 6\sqrt{9 \cdot 7}$
$= 4\sqrt{7} - 6\sqrt{9} \cdot \sqrt{7}$
$= 4\sqrt{7} - 6 \cdot 3 \cdot \sqrt{7}$
$= 4\sqrt{7} - 18\sqrt{7}$
$= (4 - 18)\sqrt{7}$
$= -14\sqrt{7}$

25. $4\sqrt[3]{5x} - 11\sqrt[3]{5x} = (4 - 11)\sqrt[3]{5x}$
$= -7\sqrt[3]{5x}$

26. $\sqrt{3}(8\sqrt{2} + 7\sqrt{32}) = \sqrt{3}(8\sqrt{2} + 7\sqrt{16 \cdot 2})$
$= \sqrt{3}(8\sqrt{2} + 7\sqrt{16} \cdot \sqrt{2})$
$= \sqrt{3}(8\sqrt{2} + 7 \cdot 4 \cdot \sqrt{2})$
$= \sqrt{3}(8\sqrt{2} + 28\sqrt{2})$
$= \sqrt{3}[(8 + 28)\sqrt{2}]$
$= \sqrt{3}(36\sqrt{2})$
$= 36 \cdot \sqrt{3} \cdot \sqrt{2}$
$= 36\sqrt{6}$

27. $(2\sqrt{5} - 4)^2 = (2\sqrt{5})^2 - 2(2\sqrt{5})(4) + 4^2$
$= 2^2(\sqrt{5})^2 - 2 \cdot 2 \cdot 4 \cdot \sqrt{5} + 16$
$= 4 \cdot 5 - 16\sqrt{5} + 16$
$= 20 - 16\sqrt{5} + 16$
$= 20 + 16 - 16\sqrt{5}$
$= 36 - 16\sqrt{5}$

28. $\sqrt[3]{-4}(\sqrt[3]{2} - \sqrt[3]{16}) = \sqrt[3]{-4} \cdot \sqrt[3]{2} - \sqrt[3]{-4} \cdot \sqrt[3]{16}$
$= \sqrt[3]{-8} - \sqrt[3]{-64}$
$= -2 - (-4)$
$= -2 + 4$
$= 2$

9.1 Exercises (pp. 485–488)

Vocabulary and Core Concept Check

1. The process of eliminating a radical from the denominator of a radical expression is called rationalizing the denominator.

2. The conjugate is $\sqrt{6} - 4$.

3. First, rewrite $\sqrt{\frac{2x}{9}}$ as $\sqrt{\frac{1}{9} \cdot 2x}$. Then, by the Product Property of Square Roots, $\sqrt{\frac{1}{9} \cdot 2x} = \sqrt{\frac{1}{9}} \cdot \sqrt{2x}$. Also, $\sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3}$. So, $\frac{1}{3}\sqrt{2x}$ and $\sqrt{\frac{2x}{9}}$ are equivalent.

4. The expression that does not belong is $-\frac{1}{3}\sqrt{6}$. The other three expressions have like radicals of $\sqrt{3}$.

Monitoring Progress and Modeling with Mathematics

5. The expression $\sqrt{19}$ is in simplest form.

6. The expression $\sqrt{\frac{1}{7}}$ is not in simplest form because the radicand is a fraction.

7. The expression $\sqrt{48}$ is not in simplest form because the radicand has a perfect square factor of 16.

8. The expression $\sqrt{34}$ is in simplest form.

9. The expression $\frac{5}{\sqrt{2}}$ is not in simplest form because a radical appears in the denominator of the fraction.

10. The expression $\frac{3\sqrt{10}}{4}$ is in simplest form.

11. The expression $\frac{1}{2 + \sqrt[3]{2}}$ is not in simplest form because a radical appears in the denominator of the fraction.

12. The expression $6 - \sqrt[3]{54}$ is not in simplest form because the radicand has a perfect cube factor of 27.

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13. $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$

14. $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

15. $\sqrt{128} = \sqrt{64 \cdot 2} = \sqrt{64} \cdot \sqrt{2} = 8\sqrt{2}$

16. $-\sqrt{72} = -\sqrt{36 \cdot 2} = -\sqrt{36} \cdot \sqrt{2} = -6\sqrt{2}$

17. $\sqrt{125b} = \sqrt{25 \cdot 5b} = \sqrt{25} \cdot \sqrt{5b} = 5\sqrt{5b}$

18. $\sqrt{4x^2} = \sqrt{4 \cdot x^2} = \sqrt{4} \cdot \sqrt{x^2} = 2x$

19. $-\sqrt{81m^3} = -\sqrt{81 \cdot m^2 \cdot m} = -\sqrt{81} \cdot \sqrt{m^2} \cdot \sqrt{m} = -9m\sqrt{m}$

20. $\sqrt{48n^5} = \sqrt{16 \cdot 3 \cdot n^4 \cdot n} = \sqrt{16} \cdot \sqrt{3} \cdot \sqrt{n^4} \cdot \sqrt{n} = 4 \cdot \sqrt{3} \cdot n^2 \cdot \sqrt{n} = 4 \cdot n^2 \cdot \sqrt{3} \cdot \sqrt{n} = 4n^2\sqrt{3n}$

21. $\sqrt{\frac{4}{49}} = \frac{\sqrt{4}}{\sqrt{49}} = \frac{2}{7}$

22. $-\sqrt{\frac{7}{81}} = -\frac{\sqrt{7}}{\sqrt{81}} = -\frac{\sqrt{7}}{9}$

23. $-\sqrt{\frac{23}{64}} = -\frac{\sqrt{23}}{\sqrt{64}} = -\frac{\sqrt{23}}{8}$

24. $\sqrt{\frac{65}{121}} = \frac{\sqrt{65}}{\sqrt{121}} = \frac{\sqrt{65}}{11}$

25. $\sqrt{\frac{a^3}{49}} = \frac{\sqrt{a^3}}{\sqrt{49}} = \frac{\sqrt{a^2 \cdot a}}{7} = \frac{\sqrt{a^2} \cdot \sqrt{a}}{7} = \frac{a\sqrt{a}}{7}$

26. $\sqrt{\frac{144}{k^2}} = \frac{\sqrt{144}}{\sqrt{k^2}} = \frac{12}{k}$

27. $\sqrt{\frac{100}{4x^2}} = \frac{\sqrt{100}}{\sqrt{4 \cdot x^2}} = \frac{10}{\sqrt{4} \cdot \sqrt{x^2}} = \frac{10}{2x} = \frac{5}{x}$

28. $\sqrt{\frac{25v^2}{36}} = \frac{\sqrt{25 \cdot v^2}}{\sqrt{36}} = \frac{\sqrt{25} \cdot \sqrt{v^2}}{6} = \frac{5v}{6}$

29. $\sqrt[3]{16} = \sqrt[3]{8 \cdot 2} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$

30. $\sqrt[3]{-108} = \sqrt[3]{-27 \cdot 4} = \sqrt[3]{-27} \cdot \sqrt[3]{4} = -3\sqrt[3]{4}$

31. $\sqrt[3]{-64x^5} = \sqrt[3]{-64 \cdot x^3 \cdot x^2} = \sqrt[3]{-64} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = -4x\sqrt[3]{x^2}$

32. $-\sqrt[3]{343n^2} = -\sqrt[3]{343 \cdot n^2} = -\sqrt[3]{343} \cdot \sqrt[3]{n^2} = -7\sqrt[3]{n^2}$

33. $\sqrt[3]{\frac{6c}{-125}} = \frac{\sqrt[3]{6c}}{\sqrt[3]{-125}} = \frac{\sqrt[3]{6c}}{-5} = -\frac{\sqrt[3]{6c}}{5}$

34. $\sqrt[3]{\frac{8h^4}{27}} = \frac{\sqrt[3]{8h^4}}{\sqrt[3]{27}} = \frac{\sqrt[3]{8 \cdot h^3 \cdot h}}{3} = \frac{\sqrt[3]{8} \cdot \sqrt[3]{h^3} \cdot \sqrt[3]{h}}{3} = \frac{2h\sqrt[3]{h}}{3}$

35. $-\sqrt[3]{\frac{81y^2}{1000x^3}} = -\frac{\sqrt[3]{81y^2}}{\sqrt[3]{1000x^3}} = -\frac{\sqrt[3]{27 \cdot 3y^2}}{\sqrt[3]{1000 \cdot x^3}} = -\frac{\sqrt[3]{27} \cdot \sqrt[3]{3y^2}}{\sqrt[3]{1000} \cdot \sqrt[3]{x^3}} = -\frac{3\sqrt[3]{3y^2}}{10x}$

36. $\sqrt[3]{\frac{21}{-64a^3b^6}} = \frac{\sqrt[3]{21}}{\sqrt[3]{-64a^3b^6}} = \frac{\sqrt[3]{21}}{\sqrt[3]{-64 \cdot a^3 \cdot b^6}} = \frac{\sqrt[3]{21}}{\sqrt[3]{-64} \cdot \sqrt[3]{a^3} \cdot \sqrt[3]{b^6}} = \frac{\sqrt[3]{21}}{-4ab^2} = -\frac{\sqrt[3]{21}}{4ab^2}$

37. The radicand 18 has a perfect square factor of 9. So, it is not in simplest form.

$\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$

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38. The denominator should be $\sqrt[3]{125}$.

$$\sqrt[3]{\frac{128y^3}{125}} = \frac{\sqrt[3]{128y^3}}{\sqrt[3]{125}} = \frac{\sqrt[3]{64 \cdot 2 \cdot y^3}}{5} = \frac{\sqrt[3]{64} \cdot \sqrt[3]{2} \cdot \sqrt[3]{y^3}}{5} = \frac{4 \cdot \sqrt[3]{2} \cdot y}{5} = \frac{4y\sqrt[3]{2}}{5}$$

39. To rationalize the denominator of the expression $\frac{4}{\sqrt{6}}$, multiply by a factor of $\frac{\sqrt{6}}{\sqrt{6}}$.

40. To rationalize the denominator of the expression $\frac{1}{\sqrt{13z}}$, multiply by a factor of $\frac{\sqrt{13z}}{\sqrt{13z}}$.

41. To rationalize the denominator of the expression $\frac{2}{\sqrt[3]{x^2}}$, multiply by a factor of $\frac{\sqrt[3]{x^2}}{\sqrt[3]{x^2}}$.

42. To rationalize the denominator of the expression $\frac{3m}{\sqrt[3]{4}}$, multiply by a factor of $\frac{\sqrt[3]{2}}{\sqrt[3]{2}}$.

43. To rationalize the denominator of the expression $\frac{\sqrt{2}}{\sqrt{5} - 8}$, multiply by a factor of $\frac{\sqrt{5} + 8}{\sqrt{5} + 8}$.

44. To rationalize the denominator of the expression $\frac{5}{\sqrt{3} + \sqrt{7}}$, multiply by a factor of $\frac{\sqrt{3} - \sqrt{7}}{\sqrt{3} - \sqrt{7}}$.

45. $$\frac{2}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

46. $$\frac{4}{\sqrt{3}} = \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{9}} = \frac{4\sqrt{3}}{3}$$

47. $$\frac{\sqrt{5}}{\sqrt{48}} = \frac{\sqrt{5}}{\sqrt{16 \cdot 3}} = \frac{\sqrt{5}}{\sqrt{16} \cdot \sqrt{3}} = \frac{\sqrt{5}}{4 \cdot \sqrt{3}} = \frac{\sqrt{5}}{4 \cdot \sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{4 \cdot \sqrt{9}} = \frac{\sqrt{15}}{4 \cdot 3} = \frac{\sqrt{15}}{12}$$

48. $$\sqrt{\frac{4}{52}} = \sqrt{\frac{1}{13}} = \frac{\sqrt{1}}{\sqrt{13}} = \frac{1}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}}{\sqrt{169}} = \frac{\sqrt{13}}{13}$$

49. $$\frac{3}{\sqrt{a}} = \frac{3}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{3\sqrt{a}}{\sqrt{a^2}} = \frac{3\sqrt{a}}{a}$$

50. $$\frac{1}{\sqrt{2x}} = \frac{1}{\sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}} = \frac{\sqrt{2x}}{\sqrt{4x^2}} = \frac{\sqrt{2x}}{\sqrt{4 \cdot x^2}} = \frac{\sqrt{2x}}{\sqrt{4} \cdot \sqrt{x^2}} = \frac{\sqrt{2x}}{2x}$$

51. $$\sqrt{\frac{3d^2}{5}} = \frac{\sqrt{3d^2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{15d^2}}{\sqrt{25}} = \frac{\sqrt{15 \cdot d^2}}{5} = \frac{\sqrt{15} \cdot \sqrt{d^2}}{5} = \frac{d\sqrt{15}}{5}$$

52. $$\frac{\sqrt{8}}{\sqrt{3n^3}} = \frac{\sqrt{4 \cdot 2}}{\sqrt{3 \cdot n^2 \cdot n}} = \frac{\sqrt{4} \cdot \sqrt{2}}{\sqrt{3} \cdot \sqrt{n^2} \cdot \sqrt{n}} = \frac{2\sqrt{2}}{n\sqrt{3n}} \cdot \frac{\sqrt{3n}}{\sqrt{3n}} = \frac{2\sqrt{6n}}{n\sqrt{9n^2}} = \frac{2\sqrt{6n}}{n \cdot \sqrt{9} \cdot \sqrt{n^2}} = \frac{2\sqrt{6n}}{n \cdot 3 \cdot n} = \frac{2\sqrt{6n}}{3n^2}$$

53. $$\frac{4}{\sqrt[3]{25}} = \frac{4}{\sqrt[3]{25}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{4\sqrt[3]{5}}{\sqrt[3]{125}} = \frac{4\sqrt[3]{5}}{5}$$

Chapter 9

54. $\sqrt[3]{\dfrac{1}{108y^2}} = \dfrac{\sqrt[3]{1}}{\sqrt[3]{108y^2}}$

$= \dfrac{1}{\sqrt[3]{27 \cdot 4y^2}}$

$= \dfrac{1}{\sqrt[3]{27} \cdot \sqrt[3]{4y^2}}$

$= \dfrac{1}{3\sqrt[3]{4y^2}} \cdot \dfrac{\sqrt[3]{2y}}{\sqrt[3]{2y}}$

$= \dfrac{\sqrt[3]{2y}}{3\sqrt[3]{8y^3}}$

$= \dfrac{\sqrt[3]{2y}}{3 \cdot \sqrt[3]{8} \cdot \sqrt[3]{y^3}}$

$= \dfrac{\sqrt[3]{2y}}{3 \cdot 2 \cdot y}$

$= \dfrac{\sqrt[3]{2y}}{6y}$

55. $\dfrac{1}{\sqrt{7}+1} = \dfrac{1}{\sqrt{7}+1} \cdot \dfrac{\sqrt{7}-1}{\sqrt{7}-1}$

$= \dfrac{1(\sqrt{7}-1)}{(\sqrt{7})^2 - 1^2}$

$= \dfrac{\sqrt{7}-1}{7-1}$

$= \dfrac{\sqrt{7}-1}{6}$

56. $\dfrac{2}{5-\sqrt{3}} = \dfrac{2}{5-\sqrt{3}} \cdot \dfrac{5+\sqrt{3}}{5+\sqrt{3}}$

$= \dfrac{2(5+\sqrt{3})}{5^2 - (\sqrt{3})^2}$

$= \dfrac{2(5+\sqrt{3})}{25-3}$

$= \dfrac{2(5+\sqrt{3})}{22}$

$= \dfrac{5+\sqrt{3}}{11}$

57. $\dfrac{\sqrt{10}}{7-\sqrt{2}} = \dfrac{\sqrt{10}}{7-\sqrt{2}} \cdot \dfrac{7+\sqrt{2}}{7+\sqrt{2}}$

$= \dfrac{\sqrt{10}(7+\sqrt{2})}{7^2 - (\sqrt{2})^2}$

$= \dfrac{\sqrt{10} \cdot 7 + \sqrt{10} \cdot \sqrt{2}}{49-2}$

$= \dfrac{7\sqrt{10} + \sqrt{20}}{47}$

$= \dfrac{7\sqrt{10} + \sqrt{4} \cdot \sqrt{5}}{47}$

$= \dfrac{7\sqrt{10} + 2\sqrt{5}}{47}$

58. $\dfrac{\sqrt{5}}{6+\sqrt{5}} = \dfrac{\sqrt{5}}{6+\sqrt{5}} \cdot \dfrac{6-\sqrt{5}}{6-\sqrt{5}}$

$= \dfrac{\sqrt{5}(6-\sqrt{5})}{6^2 - (\sqrt{5})^2}$

$= \dfrac{\sqrt{5} \cdot 6 - \sqrt{5} \cdot \sqrt{5}}{36-5}$

$= \dfrac{6\sqrt{5} - \sqrt{25}}{31}$

$= \dfrac{6\sqrt{5} - 5}{31}$

59. $\dfrac{3}{\sqrt{5}-\sqrt{2}} = \dfrac{3}{\sqrt{5}-\sqrt{2}} \cdot \dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}$

$= \dfrac{3(\sqrt{5}+\sqrt{2})}{(\sqrt{5})^2 - (\sqrt{2})^2}$

$= \dfrac{3(\sqrt{5}+\sqrt{2})}{5-2}$

$= \dfrac{3(\sqrt{5}+\sqrt{2})}{3}$

$= \sqrt{5}+\sqrt{2}$

60. $\dfrac{\sqrt{3}}{\sqrt{7}+\sqrt{3}} = \dfrac{\sqrt{3}}{\sqrt{7}+\sqrt{3}} \cdot \dfrac{\sqrt{7}-\sqrt{3}}{\sqrt{7}-\sqrt{3}}$

$= \dfrac{\sqrt{3}(\sqrt{7}-\sqrt{3})}{(\sqrt{7})^2 - (\sqrt{3})^2}$

$= \dfrac{\sqrt{3} \cdot \sqrt{7} - \sqrt{3} \cdot \sqrt{3}}{7-3}$

$= \dfrac{\sqrt{21} - \sqrt{9}}{4}$

$= \dfrac{\sqrt{21} - 3}{4}$

61. a. $t = \sqrt{\dfrac{h}{16}} = \sqrt{\dfrac{55}{16}} = \dfrac{\sqrt{55}}{\sqrt{16}} = \dfrac{\sqrt{55}}{4}$

It takes $\dfrac{\sqrt{55}}{4}$, or about 1.85 seconds for the earring to hit the ground.

b. $h = 55 - 22 = 33$

$t = \sqrt{\dfrac{h}{16}} = \sqrt{\dfrac{33}{16}} = \dfrac{\sqrt{33}}{\sqrt{16}} = \dfrac{\sqrt{33}}{4} \approx 1.44$

The earring hits the ground about $1.85 - 1.44 = 0.41$ second sooner when it is dropped from two stories below the roof.

62. a. $P = \sqrt{d^3}$

$= \sqrt{d^2 \cdot d}$

$= \sqrt{d^2} \cdot \sqrt{d}$

$= d\sqrt{d}$

So, the formula for a planet's orbital period is $P = d\sqrt{d}$.

b. $P = d\sqrt{d}$

$= 5.2\sqrt{5.2}$

$\approx 5.2(2.2804)$

≈ 11.86 Earth years

Chapter 9

63. $I = \sqrt{\frac{P}{R}}$

$= \sqrt{\frac{147}{5}}$

$= \frac{\sqrt{147}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{\sqrt{735}}{\sqrt{25}}$

$= \frac{\sqrt{49 \cdot 15}}{5}$

$= \frac{\sqrt{49} \cdot \sqrt{15}}{5}$

$= \frac{7\sqrt{15}}{5}$

The current the appliance uses is $\frac{7\sqrt{15}}{5}$, or about 5.42 amperes.

64. Account 1: $r = \sqrt{\frac{v_2}{v_0}} - 1 = \sqrt{\frac{293}{275}} - 1 \approx 0.0322 \approx 3.2\%$

Account 2: $r = \sqrt{\frac{v_2}{v_0}} - 1 = \sqrt{\frac{382}{361}} - 1 \approx 0.0287 \approx 2.9\%$

Account 3: $r = \sqrt{\frac{v_2}{v_0}} - 1 = \sqrt{\frac{214}{199}} - 1 \approx 0.0370 \approx 3.7\%$

Account 4: $r = \sqrt{\frac{v_2}{v_0}} - 1 = \sqrt{\frac{272}{254}} - 1 \approx 0.0348 \approx 3.5\%$

Account 5: $r = \sqrt{\frac{v_2}{v_0}} - 1 = \sqrt{\frac{406}{386}} - 1 \approx 0.0256 \approx 2.6\%$

Invest money in Account 3 because it has the greatest interest rate of about 3.7%.

65. $h(x) = \sqrt{5x}$

$h(10) = \sqrt{5(10)}$

$= \sqrt{50}$

$= \sqrt{25 \cdot 2}$

$= \sqrt{25} \cdot \sqrt{2}$

$= 5\sqrt{2}$

So, $h(10) = 5\sqrt{2}$, or about 7.07.

66. $g(x) = \sqrt{3x}$

$g(60) = \sqrt{3(60)}$

$= \sqrt{180}$

$= \sqrt{36 \cdot 5}$

$= \sqrt{36} \cdot \sqrt{5}$

$= 6\sqrt{5}$

So, $g(60) = 6\sqrt{5}$, or about 13.42.

67. $r(x) = \sqrt{\frac{3x}{3x^2 + 6}}$

$r(4) = \sqrt{\frac{3(4)}{3(4)^2 + 6}}$

$= \sqrt{\frac{12}{3(16) + 6}}$

$= \sqrt{\frac{12}{48 + 6}}$

$= \sqrt{\frac{12}{54}}$

$= \sqrt{\frac{2}{9}}$

$= \frac{\sqrt{2}}{\sqrt{9}}$

$= \frac{\sqrt{2}}{3}$

So, $r(4) = \frac{\sqrt{2}}{3}$, or about 0.47.

68. $p(x) = \sqrt{\frac{x - 1}{5x}}$

$p(8) = \sqrt{\frac{8 - 1}{5(8)}}$

$= \sqrt{\frac{7}{40}}$

$= \frac{\sqrt{7}}{\sqrt{40}}$

$= \frac{\sqrt{7}}{\sqrt{4} \cdot \sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}}$

$= \frac{\sqrt{70}}{2 \cdot \sqrt{100}}$

$= \frac{\sqrt{70}}{2 \cdot 10}$

$= \frac{\sqrt{70}}{20}$

So, $r(8) = \frac{\sqrt{70}}{20}$, or about 0.42.

69. $\sqrt{a^2 + bc} = \sqrt{(-2)^2 + (8)\left(\frac{1}{2}\right)}$

$= \sqrt{4 + 4}$

$= \sqrt{8}$

$= \sqrt{4 \cdot 2}$

$= \sqrt{4} \cdot \sqrt{2}$

$= 2\sqrt{2}$, or about 2.83

70. $-\sqrt{4c - 6ab} = -\sqrt{4\left(\frac{1}{2}\right) - 6(-2)(8)}$

$= -\sqrt{2 + 96}$

$= -\sqrt{98}$

$= -\sqrt{49 \cdot 2}$

$= -\sqrt{49} \cdot \sqrt{2}$

$= -7\sqrt{2}$, or about -9.90

71. $-\sqrt{2a^2 + b^2} = -\sqrt{2(-2)^2 + 8^2}$

$= -\sqrt{2(4) + 64}$

$= -\sqrt{8 + 64}$

$= -\sqrt{72}$

$= -\sqrt{36 \cdot 2}$

$= -\sqrt{36} \cdot \sqrt{2}$

$= -6\sqrt{2}$, or about -8.49

72. $\sqrt{b^2 - 4ac} = \sqrt{8^2 - 4(-2)\left(\frac{1}{2}\right)}$

$= \sqrt{64 + 4}$

$= \sqrt{68}$

$= \sqrt{4 \cdot 17}$

$= \sqrt{4} \cdot \sqrt{17}$

$= 2\sqrt{17}$, or about 8.25

73. $$\begin{aligned} \frac{1+\sqrt{5}}{2} &= \frac{6}{w} \\ w\left(1+\sqrt{5}\right) &= 12 \\ \frac{w\left(1+\sqrt{5}\right)}{\left(1+\sqrt{5}\right)} &= \frac{12}{\left(1+\sqrt{5}\right)} \\ w &= \frac{12}{1+\sqrt{5}} \\ &= \frac{12}{1+\sqrt{5}} \bullet \frac{1-\sqrt{5}}{1-\sqrt{5}} \\ &= \frac{12\left(1-\sqrt{5}\right)}{1^2-\left(\sqrt{5}\right)^2} \\ &= \frac{12 \bullet 1 - 12 \bullet \sqrt{5}}{1-5} \\ &= \frac{12-12\sqrt{5}}{-4} \\ &= \frac{-4\left(-3+3\sqrt{5}\right)}{-4} \\ &= -3+3\sqrt{5} \approx 3.71 \end{aligned}$$

The width of the text is about 3.71 inches.

74. $$\begin{aligned} \frac{1+\sqrt{5}}{2} &= \frac{42}{w} \\ w\left(1+\sqrt{5}\right) &= 84 \\ \frac{w\left(1+\sqrt{5}\right)}{\left(1+\sqrt{5}\right)} &= \frac{84}{\left(1+\sqrt{5}\right)} \\ w &= \frac{84}{1+\sqrt{5}} \\ &= \frac{84}{1+\sqrt{5}} \bullet \frac{1-\sqrt{5}}{1-\sqrt{5}} \\ &= \frac{84\left(1-\sqrt{5}\right)}{1^2-\left(\sqrt{5}\right)^2} \\ &= \frac{84 \bullet 1 - 84 \bullet \sqrt{5}}{1-5} \\ &= \frac{84-84\sqrt{5}}{-4} \\ &= \frac{-4\left(-21+21\sqrt{5}\right)}{-4} \\ &= -21+21\sqrt{5} \approx 25.96 \end{aligned}$$

The width of the flag is about 25.96 inches.

75. $$\begin{aligned} \sqrt{3} - 2\sqrt{2} + 6\sqrt{2} &= \sqrt{3} + (-2+6)\sqrt{2} \\ &= \sqrt{3} + 4\sqrt{2} \end{aligned}$$

76. $$\begin{aligned} \sqrt{5} - 5\sqrt{13} - 8\sqrt{5} &= \sqrt{5} - 8\sqrt{5} - 5\sqrt{13} \\ &= (1-8)\sqrt{5} - 5\sqrt{13} \\ &= -7\sqrt{5} - 5\sqrt{13} \end{aligned}$$

77. $$\begin{aligned} 2\sqrt{6} - 5\sqrt{54} &= 2\sqrt{6} - 5\sqrt{9 \bullet 6} \\ &= 2\sqrt{6} - 5 \bullet \sqrt{9} \bullet \sqrt{6} \\ &= 2\sqrt{6} - 5 \bullet 3 \bullet \sqrt{6} \\ &= 2\sqrt{6} - 15\sqrt{6} \\ &= (2-15)\sqrt{6} \\ &= -13\sqrt{6} \end{aligned}$$

78. $$\begin{aligned} 9\sqrt{32} + \sqrt{2} &= 9\sqrt{16 \bullet 2} + \sqrt{2} \\ &= 9 \bullet \sqrt{16} \bullet \sqrt{2} + \sqrt{2} \\ &= 9 \bullet 4 \bullet \sqrt{2} + \sqrt{2} \\ &= 36\sqrt{2} + \sqrt{2} \\ &= (36+1)\sqrt{2} \\ &= 37\sqrt{2} \end{aligned}$$

79. $$\begin{aligned} \sqrt{12} + 6\sqrt{3} + 2\sqrt{6} &= \sqrt{4 \bullet 3} + 6\sqrt{3} + 2\sqrt{6} \\ &= \sqrt{4} \bullet \sqrt{3} + 6\sqrt{3} + 2\sqrt{6} \\ &= 2\sqrt{3} + 6\sqrt{3} + 2\sqrt{6} \\ &= (2+6)\sqrt{3} + 2\sqrt{6} \\ &= 8\sqrt{3} + 2\sqrt{6} \end{aligned}$$

80. $$\begin{aligned} 3\sqrt{7} - 5\sqrt{14} + 2\sqrt{28} &= 3\sqrt{7} - 5\sqrt{14} + 2\sqrt{4 \bullet 7} \\ &= 3\sqrt{7} - 5\sqrt{14} + 2 \bullet \sqrt{4} \bullet \sqrt{7} \\ &= 3\sqrt{7} - 5\sqrt{14} + 2 \bullet 2 \bullet \sqrt{7} \\ &= 3\sqrt{7} - 5\sqrt{14} + 4\sqrt{7} \\ &= 3\sqrt{7} + 4\sqrt{7} - 5\sqrt{14} \\ &= (3+4)\sqrt{7} - 5\sqrt{14} \\ &= 7\sqrt{7} - 5\sqrt{14} \end{aligned}$$

81. $$\begin{aligned} \sqrt[3]{-81} + 4\sqrt[3]{3} &= \sqrt[3]{-27 \bullet 3} + 4\sqrt[3]{3} \\ &= \sqrt[3]{-27} \bullet \sqrt[3]{3} + 4\sqrt[3]{3} \\ &= -3\sqrt[3]{3} + 4\sqrt[3]{3} \\ &= (-3+4)\sqrt[3]{3} \\ &= 1\sqrt[3]{3} \\ &= \sqrt[3]{3} \end{aligned}$$

82. $$\begin{aligned} 6\sqrt[3]{128t} - 2\sqrt[3]{2t} &= 6\sqrt[3]{64 \bullet 2t} - 2\sqrt[3]{2t} \\ &= 6 \bullet \sqrt[3]{64} \bullet \sqrt[3]{2t} - 2\sqrt[3]{2t} \\ &= 6 \bullet 4 \bullet \sqrt[3]{2t} - 2\sqrt[3]{2t} \\ &= 24\sqrt[3]{2t} - 2\sqrt[3]{2t} \\ &= (24-2)\sqrt[3]{2t} \\ &= 22\sqrt[3]{2t} \end{aligned}$$

83. $$\begin{aligned} \sqrt{2}\left(\sqrt{45} + \sqrt{5}\right) &= \sqrt{2} \bullet \sqrt{45} + \sqrt{2} \bullet \sqrt{5} \\ &= \sqrt{90} + \sqrt{10} \\ &= \sqrt{9 \bullet 10} + \sqrt{10} \\ &= \sqrt{9} \bullet \sqrt{10} + \sqrt{10} \\ &= 3\sqrt{10} + \sqrt{10} \\ &= (3+1)\sqrt{10} \\ &= 4\sqrt{10} \end{aligned}$$

Chapter 9

84. $\sqrt{3}\left(\sqrt{72} - 3\sqrt{2}\right) = \sqrt{3} \cdot \sqrt{72} - \sqrt{3} \cdot 3\sqrt{2}$
$= \sqrt{216} - 3 \cdot \sqrt{3} \cdot \sqrt{2}$
$= \sqrt{36 \cdot 6} - 3\sqrt{6}$
$= \sqrt{36} \cdot \sqrt{6} - 3\sqrt{6}$
$= 6\sqrt{6} - 3\sqrt{6}$
$= (6 - 3)\sqrt{6}$
$= 3\sqrt{6}$

85. $\sqrt{5}\left(2\sqrt{6x} - \sqrt{96x}\right) = \sqrt{5} \cdot 2\sqrt{6x} - \sqrt{5} \cdot \sqrt{96x}$
$= 2 \cdot \sqrt{5} \cdot \sqrt{6x} - \sqrt{480x}$
$= 2\sqrt{30x} - \sqrt{16 \cdot 30x}$
$= 2\sqrt{30x} - \sqrt{16} \cdot \sqrt{30x}$
$= 2\sqrt{30x} - 4\sqrt{30x}$
$= (2 - 4)\sqrt{30x}$
$= -2\sqrt{30x}$

86. $\sqrt{7y}\left(\sqrt{27y} + 5\sqrt{12y}\right) = \sqrt{7y} \cdot \sqrt{27y} + \sqrt{7y} \cdot 5\sqrt{12y}$
$= \sqrt{189y^2} + 5\sqrt{84y^2}$
$= \sqrt{9 \cdot 21 \cdot y^2} + 5\sqrt{4 \cdot 21 \cdot y^2}$
$= \sqrt{9} \cdot \sqrt{21} \cdot \sqrt{y^2} + 5 \cdot \sqrt{4} \cdot \sqrt{21} \cdot \sqrt{y^2}$
$= 3 \cdot \sqrt{21} \cdot y + 5 \cdot 2 \cdot \sqrt{21} \cdot y$
$= 3y\sqrt{21} + 10y\sqrt{21}$
$= (3y + 10y)\sqrt{21}$
$= 13y\sqrt{21}$

87. $\left(4\sqrt{2} - \sqrt{98}\right)^2 = \left(4\sqrt{2}\right)^2 - 2\left(4\sqrt{2}\right)\left(\sqrt{98}\right) + \left(\sqrt{98}\right)^2$
$= 4^2\left(\sqrt{2}\right)^2 - 8\sqrt{196} + 98$
$= 16 \cdot 2 - 8 \cdot 14 + 98$
$= 32 - 112 + 98$
$= -80 + 98$
$= 18$

88. $\left(\sqrt{3} + \sqrt{48}\right)\left(\sqrt{20} - \sqrt{5}\right)$
$= \sqrt{3} \cdot \sqrt{20} - \sqrt{3} \cdot \sqrt{5} + \sqrt{48} \cdot \sqrt{20} - \sqrt{48} \cdot \sqrt{5}$
$= \sqrt{60} - \sqrt{15} + \sqrt{960} - \sqrt{240}$
$= \sqrt{4 \cdot 15} - \sqrt{15} + \sqrt{64 \cdot 15} - \sqrt{16 \cdot 15}$
$= \sqrt{4} \cdot \sqrt{15} - \sqrt{15} + \sqrt{64} \cdot \sqrt{15} - \sqrt{16} \cdot \sqrt{15}$
$= 2\sqrt{15} - \sqrt{15} + 8\sqrt{15} - 4\sqrt{15}$
$= (2 - 1 + 8 - 4)\sqrt{15}$
$= 5\sqrt{15}$

89. $\sqrt[3]{3}\left(\sqrt[3]{4} + \sqrt[3]{32}\right) = \sqrt[3]{3} \cdot \sqrt[3]{4} + \sqrt[3]{3} \cdot \sqrt[3]{32}$
$= \sqrt[3]{12} + \sqrt[3]{96}$
$= \sqrt[3]{12} + \sqrt[3]{8 \cdot 12}$
$= \sqrt[3]{12} + \sqrt[3]{8} \cdot \sqrt[3]{12}$
$= \sqrt[3]{12} + 2\sqrt[3]{12}$
$= (1 + 2)\sqrt[3]{12}$
$= 3\sqrt[3]{12}$

90. $\sqrt[3]{2}\left(\sqrt[3]{135} - 4\sqrt[3]{5}\right) = \sqrt[3]{2} \cdot \sqrt[3]{135} - \sqrt[3]{2} \cdot 4 \cdot \sqrt[3]{5}$
$= \sqrt[3]{270} - 4\sqrt[3]{10}$
$= \sqrt[3]{27 \cdot 10} - 4\sqrt[3]{10}$
$= \sqrt[3]{27} \cdot \sqrt[3]{10} - 4\sqrt[3]{10}$
$= 3\sqrt[3]{10} - 4\sqrt[3]{10}$
$= (3 - 4)\sqrt[3]{10}$
$= -1\sqrt[3]{10}$
$= -\sqrt[3]{10}$

91. $C \approx 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$
$\approx 2\pi\sqrt{\dfrac{20^2 + 16^2}{2}}$
$\approx 2\pi\sqrt{\dfrac{400 + 256}{2}}$
$\approx 2\pi\sqrt{\dfrac{656}{2}}$
$\approx 2\pi\sqrt{328}$
$\approx 2\pi\sqrt{4 \cdot 82}$
$\approx 2\pi\sqrt{4} \cdot \sqrt{82}$
$\approx 2\pi \cdot 2 \cdot \sqrt{82}$
$\approx 4\pi\sqrt{82}$
≈ 113.79

The circumference of the room is about 114 square feet.

92. a. The expression $4 + \sqrt{6}$ represents an irrational number because 6 is not a perfect square.

b. The expression $\dfrac{\sqrt{48}}{\sqrt{3}}$ represents a rational number. By the Quotient Property of Square Roots, $\dfrac{\sqrt{48}}{\sqrt{3}} = \sqrt{\dfrac{48}{3}}$. $\sqrt{\dfrac{48}{3}}$ is equal to $\sqrt{16}$, and 16 is a perfect square. So, $\dfrac{\sqrt{48}}{\sqrt{3}} = \sqrt{16} = 4$, and 4 is a rational number.

c. The expression $\dfrac{8}{\sqrt{12}}$ represents an irrational number because 12 is not a perfect square.

d. The expression $\sqrt{3} + \sqrt{7}$ represents an irrational number because 3 and 7 are not pefect squares.

e. The expression $\dfrac{a}{\sqrt{10} - \sqrt{2}}$ represents an irrational number because 2 and 10 are not perfect squares.

f. The expression $\dfrac{2 + \sqrt{5}}{2b + \sqrt{5b^2}}$ represents a rational number.

By the Distributive Property,

$$\frac{2 + \sqrt{5}}{2b + \sqrt{5b^2}} = \frac{2 + \sqrt{5}}{b\left(2 + \sqrt{5}\right)},$$

and when you simplify the expression, you get $\dfrac{1}{b}$, which is a rational number when b is a positive integer.

93. $\sqrt[5]{\dfrac{13}{5x^5}} = \dfrac{\sqrt[5]{13}}{\sqrt[5]{5x^5}}$

$= \dfrac{\sqrt[5]{13}}{\sqrt[5]{5} \cdot \sqrt[5]{x^5}}$

$= \dfrac{\sqrt[5]{13}}{\sqrt[5]{5} \cdot x}$

$= \dfrac{\sqrt[5]{13}}{x\sqrt[5]{5}} \cdot \dfrac{\sqrt[5]{625}}{\sqrt[5]{625}}$

$= \dfrac{\sqrt[5]{8125}}{x\sqrt[5]{3125}}$

$= \dfrac{\sqrt[5]{8125}}{x \cdot 5}$

$= \dfrac{\sqrt[5]{8125}}{5x}$

94. $\sqrt[4]{\dfrac{10}{81}} = \dfrac{\sqrt[4]{10}}{\sqrt[4]{81}} = \dfrac{\sqrt[4]{10}}{3}$

95. $\sqrt[4]{256y} = \sqrt[4]{256} \cdot \sqrt[4]{y}$

$= 4\sqrt[4]{y}$

96. $\sqrt[5]{160x^6} = \sqrt[5]{32 \cdot 5 \cdot x^5 \cdot x}$

$= \sqrt[5]{32} \cdot \sqrt[5]{5} \cdot \sqrt[5]{x^5} \cdot \sqrt[5]{x}$

$= 2 \cdot \sqrt[5]{5} \cdot x \cdot \sqrt[5]{x}$

$= 2 \cdot x \cdot \sqrt[5]{5} \cdot \sqrt[5]{x}$

$= 2x\sqrt[5]{5x}$

97. $6\sqrt[4]{9} - \sqrt[5]{9} + 3\sqrt[4]{9} = 6\sqrt[4]{9} + 3\sqrt[4]{9} - \sqrt[5]{9}$

$= (6 + 3)\sqrt[4]{9} - \sqrt[5]{9}$

$= 9\sqrt[4]{9} - \sqrt[5]{9}$

98. $\sqrt[5]{2}\left(\sqrt[4]{7} + \sqrt[5]{16}\right) = \sqrt[5]{2} \cdot \sqrt[4]{7} + \sqrt[5]{2} \cdot \sqrt[5]{16}$

$= \sqrt[5]{2}\sqrt[4]{7} + \sqrt[5]{32}$

$= \sqrt[5]{2}\sqrt[4]{7} + 2$

99. a.

	2	$\frac{1}{4}$	0	$\sqrt{3}$	$-\sqrt{3}$	π
2	4	$2\frac{1}{4}$	2	$2 + \sqrt{3}$	$2 - \sqrt{3}$	$2 + \pi$
$\frac{1}{4}$	$2\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4} + \sqrt{3}$	$\frac{1}{4} - \sqrt{3}$	$\frac{1}{4} + \pi$
0	2	$\frac{1}{4}$	0	$\sqrt{3}$	$-\sqrt{3}$	π
$\sqrt{3}$	$2 + \sqrt{3}$	$\frac{1}{4} + \sqrt{3}$	$\sqrt{3}$	$2\sqrt{3}$	0	$\pi + \sqrt{3}$
$-\sqrt{3}$	$2 - \sqrt{3}$	$\frac{1}{4} - \sqrt{3}$	$-\sqrt{3}$	0	$-2\sqrt{3}$	$\pi - \sqrt{3}$
π	$2 + \pi$	$\frac{1}{4} + \pi$	π	$\pi + \sqrt{3}$	$\pi - \sqrt{3}$	2π

b.

	2	$\frac{1}{4}$	0	$\sqrt{3}$	$-\sqrt{3}$	π
2	4	$\frac{1}{2}$	0	$2\sqrt{3}$	$-2\sqrt{3}$	2π
$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{16}$	0	$\frac{\sqrt{3}}{4}$	$-\frac{\sqrt{3}}{4}$	$\frac{\pi}{4}$
0	0	0	0	0	0	0
$\sqrt{3}$	$2\sqrt{3}$	$\frac{\sqrt{3}}{4}$	0	3	-3	$\pi\sqrt{3}$
$-\sqrt{3}$	$-2\sqrt{3}$	$-\frac{\sqrt{3}}{4}$	0	-3	3	$-\pi\sqrt{3}$
π	2π	$\frac{\pi}{4}$	0	$\pi\sqrt{3}$	$-\pi\sqrt{3}$	π^2

100. a. The sum of a rational number and a rational number is always rational because the sum of two fractions can always be written as a fraction.

b. The sum of a rational number and an irrational number is always irrational because if one of the factors is a nonrepeating decimal, then the sum cannot be written as the ratio of two integers.

c. The sum of an irrational number and an irrational number is sometimes irrational. The sum is either 0, or it is irrational. For example, $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$, which is irrational. However, $\sqrt{3} + \left(-\sqrt{3}\right) = 0$, and zero is a rational number because it can be written as a ratio of two integers, such as $\frac{0}{1} = 0$.

d. The product of a rational number and a rational number is always rational because the product of two fractions can always be written as a fraction.

e. The product of a nonzero rational number and an irrational number is always irrational because if one of the factors is a nonrepeating decimal, then the product cannot be written as the ratio of two integers.

f. The product of an irrational number and an irrational number is sometimes irrational. An example of a product that is irrational is $\sqrt{3} \cdot \pi = \pi\sqrt{3}$, but an example of a product that is rational is $\sqrt{3} \cdot \sqrt{3} = 3$.

101. The simplified form of the expression $\sqrt{2^m}$ contains a radical when m is odd, because 2 to an odd power is not a perfect square. The simplified form of the expression $\sqrt{2^m}$ does not contain a radical when m is even, because 2 to an even power is a perfect square.

102. *Sample answer:* If $s = \sqrt[3]{2}$, then the side length, $\sqrt[3]{2}$, is an irrational number, the surface area is $6\left[\left(\sqrt[3]{2}\right)^2\right]$, which is an irrational number, but the volume is $\left(\sqrt[3]{2}\right)^3 = 2$, which is a rational number.

103. When $a < b$, if you multiply each side of the inequality by a, you get $a^2 < ab$. Similarly, when $a < b$, if you multiply each side of the inequality by b, you get $ab < b^2$. So, putting these two inequalities together, you get $a^2 < ab < b^2$. When you take the square root of each part of this inequality, you get $a < \sqrt{ab} < b$. So, it must be that $\sqrt{ab}$ lies between a and b on a number line.

Chapter 9

104. Your friend is incorrect. Using the sum and difference pattern to simplify the product of the denominator $4 + \sqrt[3]{5}$ and $4 - \sqrt[3]{5}$, you get $4^2 - \left(\sqrt[3]{5}\right)^2 = 16 - \sqrt[3]{5} \bullet \sqrt[3]{5} = 16 - \sqrt[3]{25}$, which means the denominator will still contain a radical.

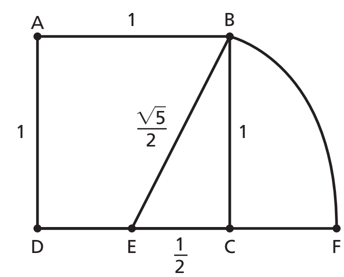
105.
$$\begin{aligned}
\frac{1+\sqrt{5}}{2} &= \frac{610}{x} \\
x\left(1+\sqrt{5}\right) &= 1220 \\
\frac{x\left(1+\sqrt{5}\right)}{\left(1+\sqrt{5}\right)} &= \frac{1220}{\left(1+\sqrt{5}\right)} \\
x &= \frac{1220}{1+\sqrt{5}} \\
&= \frac{1220}{1+\sqrt{5}} \bullet \frac{1-\sqrt{5}}{1-\sqrt{5}} \\
&= \frac{1220\left(1-\sqrt{5}\right)}{1^2 - \left(\sqrt{5}\right)^2} \\
&= \frac{1220\left(1-\sqrt{5}\right)}{1-5} \\
&= \frac{1220\left(1-\sqrt{5}\right)}{-4} \\
&= -305\left(1-\sqrt{5}\right) \\
&= -305 \bullet 1 - 305\left(-\sqrt{5}\right) \\
&= -305 + 305\sqrt{5} \\
&\approx 377
\end{aligned}$$

So, the preceding term is 377.

106. a.
$$\begin{aligned}
x^2 - x - 1 &= 0 \\
\left(\frac{1+\sqrt{5}}{2}\right)^2 - \left(\frac{1+\sqrt{5}}{2}\right) - 1 &\stackrel{?}{=} 0 \\
\frac{1^2 + 2(1)\left(\sqrt{5}\right) + \left(\sqrt{5}\right)^2}{2^2} + \frac{-\left(1+\sqrt{5}\right)}{2} - 1 &\stackrel{?}{=} 0 \\
\frac{1 + 2\sqrt{5} + 5}{4} + \frac{-1-\sqrt{5}}{2} - 1 &\stackrel{?}{=} 0 \\
\frac{6 + 2\sqrt{5}}{4} + \frac{-1-\sqrt{5}}{2} - 1 &\stackrel{?}{=} 0 \\
4 \bullet \frac{6+2\sqrt{5}}{4} + 4 \bullet \frac{\left(-1-\sqrt{5}\right)}{2} - 4 \bullet 1 &\stackrel{?}{=} 4 \bullet 0 \\
6 + 2\sqrt{5} + 2\left(-1-\sqrt{5}\right) - 4 &\stackrel{?}{=} 0 \\
6 + 2\sqrt{5} + 2(-1) - 2 \bullet \sqrt{5} - 4 &\stackrel{?}{=} 0 \\
6 + 2\sqrt{5} - 2 - 2\sqrt{5} - 4 &\stackrel{?}{=} 0 \\
(6 - 2 - 4) + \left(2\sqrt{5} - 2\sqrt{5}\right) &\stackrel{?}{=} 0 \\
0 + 0 &\stackrel{?}{=} 0 \\
0 &= 0 \checkmark
\end{aligned}$$

$$\begin{aligned}
x^2 - x - 1 &= 0 \\
\left(\frac{1-\sqrt{5}}{2}\right)^2 - \left(\frac{1-\sqrt{5}}{2}\right) - 1 &\stackrel{?}{=} 0 \\
\frac{1^2 - 2(1)\left(\sqrt{5}\right) + \left(\sqrt{5}\right)^2}{2^2} + \frac{-\left(1-\sqrt{5}\right)}{2} - 1 &\stackrel{?}{=} 0 \\
\frac{1 - 2\sqrt{5} + 5}{4} + \frac{-\left(1+\sqrt{5}\right)}{2} - 1 &\stackrel{?}{=} 0 \\
\frac{6 - 2\sqrt{5}}{4} + \frac{-1+\sqrt{5}}{2} - 1 &\stackrel{?}{=} 0 \\
4 \bullet \frac{6-2\sqrt{5}}{4} + 4 \bullet \frac{\left(-1+\sqrt{5}\right)}{2} - 4 \bullet 1 &\stackrel{?}{=} 4 \bullet 0 \\
6 - 2\sqrt{5} + 2\left(-1+\sqrt{5}\right) - 4 &\stackrel{?}{=} 0 \\
6 - 2\sqrt{5} + 2(-1) + 2 \bullet \sqrt{5} - 4 &\stackrel{?}{=} 0 \\
6 - 2\sqrt{5} - 2 + 2\sqrt{5} - 4 &\stackrel{?}{=} 0 \\
(6 - 2 - 4) + \left(-2\sqrt{5} + 2\sqrt{5}\right) &\stackrel{?}{=} 0 \\
0 + 0 &\stackrel{?}{=} 0 \\
0 &= 0 \checkmark
\end{aligned}$$

b. *Sample answer:* $DF = \dfrac{1+\sqrt{5}}{2}$



107. In order to rationalize the denominator of $\dfrac{2}{\sqrt[3]{x}+1}$, let $a = \sqrt[3]{x}$ and let $b = 1$ and multiply the numerator and denominator by

$a^2 - ab + b^2 = \left(\sqrt[3]{x}\right)^2 - \sqrt[3]{x}(1) + 1^2 = \sqrt[3]{x^2} - \sqrt[3]{x} + 1.$

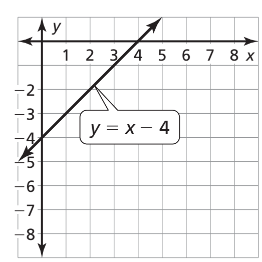
$$\begin{aligned}
\frac{2}{\sqrt[3]{x}+1} \bullet \frac{\sqrt[3]{x^2} - \sqrt[3]{x} + 1}{\sqrt[3]{x^2} - \sqrt[3]{x} + 1} &= \frac{2\left(\sqrt[3]{x^2} - \sqrt[3]{x} + 1\right)}{\left(\sqrt[3]{x}\right)^3 + 1^3} \\
&= \frac{2\sqrt[3]{x^2} - 2\sqrt[3]{x} + 2}{x+1}
\end{aligned}$$

So, $\dfrac{2}{\sqrt[3]{x}+1} = \dfrac{2\sqrt[3]{x^2} - 2\sqrt[3]{x} + 2}{x+1}$.

Maintaining Mathematical Proficiency

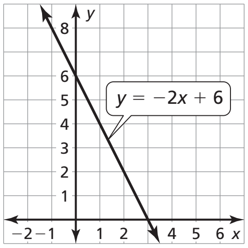
108. To graph $y = x - 4$, use slope $m = 1$ and y-intercept $b = -4$.

The graph crosses the x-axis at (4, 0). So, the x-intercept is 4.



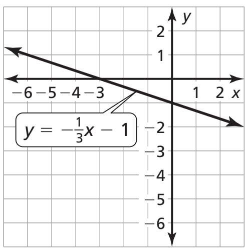
Chapter 9

109. To graph $y = -2x + 6$, use slope $m = -2$ and y-intercept $b = 6$.



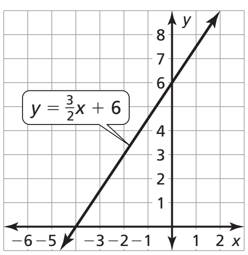
The graph crosses the x-axis at $(3, 0)$. So, the x-intercept is 3.

110. To graph $y = -\frac{1}{3}x - 1$, use slope $m = -\frac{1}{3}$ and y-intercept $b = -1$.



The graph crosses the x-axis at $(-3, 0)$. So, the x-intercept is -3.

111. To graph $y = \frac{3}{2}x + 6$, use slope $m = \frac{3}{2}$ and y-intercept $b = 6$.



The graph crosses the x-axis at $(-4, 0)$. So, the x-intercept is -4.

112.
$$\begin{aligned} 32 &= 2^x \\ 2^5 &= 2^x \\ 5 &= x \end{aligned}$$

Check
$$\begin{aligned} 32 &= 2^x \\ 32 &\stackrel{?}{=} 2^5 \\ 32 &= 32 \checkmark \end{aligned}$$

The solution is $x = 5$.

113.
$$\begin{aligned} 27^x &= 3^{x-6} \\ (3^3)^x &= 3^{x-6} \\ 3^{3x} &= 3^{x-6} \\ 3x &= x - 6 \\ \underline{-x} &\quad \underline{-x} \\ 2x &= -6 \\ \frac{2x}{2} &= \frac{-6}{2} \\ x &= -3 \end{aligned}$$

Check
$$\begin{aligned} 27^x &= 3^{x-6} \\ 27^{-3} &\stackrel{?}{=} 3^{-3-6} \\ \frac{1}{27^3} &\stackrel{?}{=} 3^{-9} \\ \frac{1}{19{,}683} &\stackrel{?}{=} \frac{1}{3^9} \\ \frac{1}{19{,}683} &= \frac{1}{19{,}683} \checkmark \end{aligned}$$

The solution is $x = -3$.

114.
$$\begin{aligned} \left(\frac{1}{6}\right)^{2x} &= 216^{1-x} \\ (6^{-1})^{2x} &= (6^3)^{1-x} \\ 6^{-2x} &= 6^{3(1-x)} \\ -2x &= 3(1 - x) \\ -2x &= 3(1) - 3(x) \\ -2x &= 3 - 3x \\ \underline{+3x} &\quad \underline{+3x} \\ x &= 3 \end{aligned}$$

Check
$$\begin{aligned} \left(\frac{1}{6}\right)^{2x} &= 216^{1-x} \\ \left(\frac{1}{6}\right)^{2 \cdot 3} &\stackrel{?}{=} 216^{1-3} \\ \left(\frac{1}{6}\right)^{6} &\stackrel{?}{=} 216^{-2} \\ \frac{1^6}{6^6} &\stackrel{?}{=} \frac{1}{216^2} \\ \frac{1}{46{,}656} &= \frac{1}{46{,}656} \checkmark \end{aligned}$$

The solution is $x = 3$.

115.
$$\begin{aligned} 625^x &= \left(\frac{1}{25}\right)^{x+2} \\ (5^4)^x &= \left(\frac{1}{5^2}\right)^{x+2} \\ 5^{4x} &= (5^{-2})^{x+2} \\ 5^{4x} &= 5^{-2(x+2)} \\ 4x &= -2(x + 2) \\ 4x &= -2(x) - 2(2) \\ 4x &= -2x - 4 \\ \underline{+2x} &\quad \underline{+2x} \\ 6x &= -4 \\ \frac{6x}{6} &= \frac{-4}{6} \\ x &= -\frac{2}{3} \end{aligned}$$

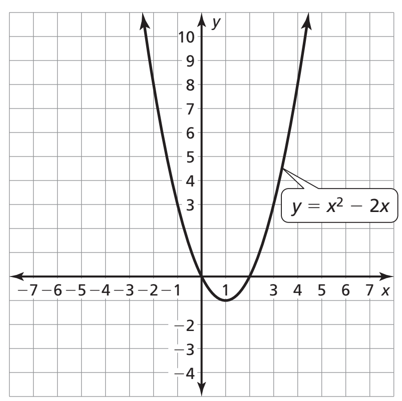
Check
$$\begin{aligned} 625^x &= \left(\frac{1}{25}\right)^{x+2} \\ 625^{-2/3} &\stackrel{?}{=} \left(\frac{1}{25}\right)^{-2/3+2} \\ (625^{-2})^{1/3} &\stackrel{?}{=} \left(\frac{1}{25}\right)^{4/3} \\ \left(\frac{1}{625^2}\right)^{1/3} &\stackrel{?}{=} \left[\left(\frac{1}{25}\right)^4\right]^{1/3} \\ \left(\frac{1}{390{,}625}\right)^{1/3} &\stackrel{?}{=} \left(\frac{1^4}{25^4}\right)^{1/3} \\ \left(\frac{1}{390{,}625}\right)^{1/3} &= \left(\frac{1}{390{,}625}\right)^{1/3} \checkmark \end{aligned}$$

The solution is $x = -\frac{2}{3}$.

9.2 Explorations (p. 489)

1. a.

x	-1	0	1	2	3
y	3	0	-1	0	3



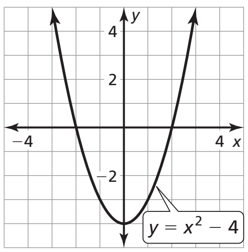
b. An x-intercept of a graph is the x-coordinate of a point where the graph crosses the x-axis. This graph crosses the x-axis at two points. So, it has two x-intercepts. They are 0 and 2.

c. The solution of an equation in x is the value of x that makes the equation true. The equation $x^2 - 2x = 0$ has two solutions because there are two points on the graph that have a y-value of 0. The solutions are $x = 0$ and $x = 2$.

d. You can verify the solutions from part (c) by substituting the solutions into the equation and then simplifying the equation.

2. a. Graph $y = x^2 - 4$.

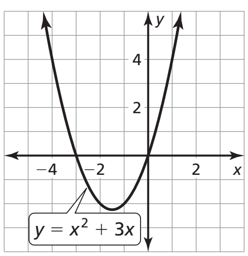
x	-2	-1	0	1	2
y	0	-3	-4	-3	0



The graph crosses the x-axis at $(-2, 0)$ and $(2, 0)$. So, the solutions are $x = -2$ and $x = 2$.

b. Graph $y = x^2 + 3x$.

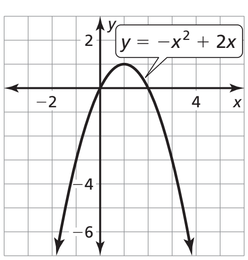
x	-4	-3	-2	-1	0	1
y	4	0	-2	-2	0	4



The graph crosses the x-axis at $(-3, 0)$ and $(0, 0)$. So, the solutions are $x = -3$ and $x = 0$.

c. Graph $y = -x^2 + 2x$.

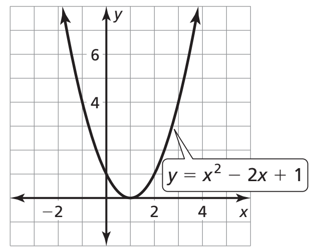
x	-1	0	1	2	3
y	-3	0	1	0	-3



The graph crosses the x-axis at $(0, 0)$ and $(2, 0)$. So, the solutions are $x = 0$ and $x = 2$.

d. Graph $y = x^2 - 2x + 1$

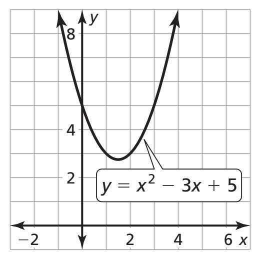
x	-1	0	1	2	3
y	4	1	0	1	4



The graph touches the x-axis at $(1, 0)$. So, the solution is $x = 1$.

e. Graph $y = x^2 - 3x + 5$

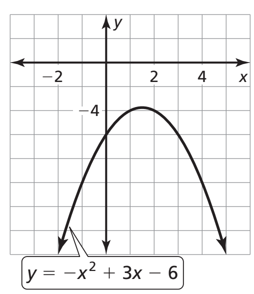
x	-1	0	1	2	3	4
y	9	5	3	3	5	9



The graph does not cross the x-axis. So, the equation has no real solutions.

f. Graph $y = -x^2 + 3x - 6$

x	-1	0	1	2	3	4
y	-10	-6	-4	-4	-6	-10



The graph does not cross the x-axis. So, the equation has no real solutions.

3. In order to solve a quadratic equation by graphing, first write the equation in standard form $ax^2 + bx + c = 0$. Then graph the related function $y = ax^2 + bx + c$, and find the x-intercepts.

Chapter 9

4. In order to check a solution algebraically, substitute the solution into the equation and verify that the value of the variable makes the equation true.

a.
$$x^2 - 4 = 0 \qquad x^2 - 4 = 0$$
$$(-2)^2 - 4 \stackrel{?}{=} 0 \qquad 2^2 - 4 \stackrel{?}{=} 0$$
$$4 - 4 \stackrel{?}{=} 0 \qquad 4 - 4 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

b.
$$x^2 + 3x = 0 \qquad x^2 + 3x = 0$$
$$(-3)^2 + 3(-3) \stackrel{?}{=} 0 \qquad 0^2 + 3(0) \stackrel{?}{=} 0$$
$$9 - 9 \stackrel{?}{=} 0 \qquad 0 + 0 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

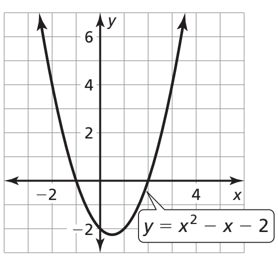
c.
$$-x^2 + 2x = 0 \qquad -x^2 + 2x = 0$$
$$-0^2 + 2(0) \stackrel{?}{=} 0 \qquad -(2)^2 + 2(2) \stackrel{?}{=} 0$$
$$0 + 0 \stackrel{?}{=} 0 \qquad -4 + 4 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

d.
$$x^2 - 2x + 1 = 0$$
$$1^2 - 2(1) + 1 \stackrel{?}{=} 0$$
$$1 - 2 + 1 \stackrel{?}{=} 0$$
$$-1 + 1 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark$$

5. A quadratic equation has no solution if the related graph has no x-intercepts.

9.2 Monitoring Progress (pp. 490–493)

1. Graph $y = x^2 - x - 2$.



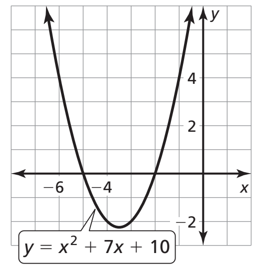
The x-intercepts are -1 and 2.

Check
$$x^2 - x - 2 = 0 \qquad x^2 - x - 2 = 0$$
$$(-1)^2 - (-1) - 2 \stackrel{?}{=} 0 \qquad 2^2 - 2 - 2 \stackrel{?}{=} 0$$
$$1 + 1 - 2 \stackrel{?}{=} 0 \qquad 4 - 2 - 2 \stackrel{?}{=} 0$$
$$2 - 2 \stackrel{?}{=} 0 \qquad 2 - 2 \stackrel{?}{=} 0$$
$$0 = 0 \checkmark \qquad 0 = 0 \checkmark$$

So, the solutions are $x = -1$ and $x = 2$.

2.
$$x^2 + 7x = -10$$
$$x^2 + 7x + 10 = -10 + 10$$
$$x^2 + 7x + 10 = 0$$

Graph $y = x^2 + 7x + 10$.



The x-intercepts are -5 and -2.

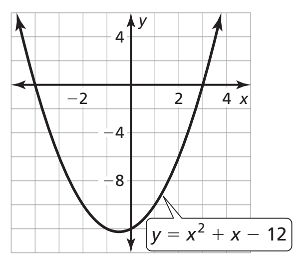
Check
$$x^2 + 7x = -10$$
$$(-5)^2 + 7(-5) \stackrel{?}{=} -10$$
$$25 - 35 \stackrel{?}{=} -10$$
$$-10 = -10 \checkmark$$

$$x^2 + 7x = -10$$
$$(-2)^2 + 7(-2) \stackrel{?}{=} -10$$
$$4 - 14 \stackrel{?}{=} -10$$
$$-10 = -10 \checkmark$$

The solutions are $x = -5$ and $x = -2$.

3.
$$x^2 + x = 12$$
$$x^2 + x - 12 = 12 - 12$$
$$x^2 + x - 12 = 0$$

Graph $y = x^2 + x - 12$.



The x-intercepts are -4 and 3.

Check
$$x^2 + x = 12 \qquad x^2 + x = 12$$
$$(-4)^2 - 4 \stackrel{?}{=} 12 \qquad 3^2 + 3 \stackrel{?}{=} 12$$
$$16 - 4 \stackrel{?}{=} 12 \qquad 9 + 3 \stackrel{?}{=} 12$$
$$12 = 12 \checkmark \qquad 12 = 12 \checkmark$$

The solutions are $x = -4$ and $x = 3$.

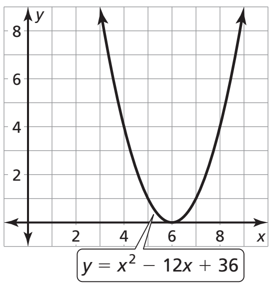
Chapter 9

4. $x^2 + 36 = 12x$

$x^2 + 36 - 12x = 12x - 12x$

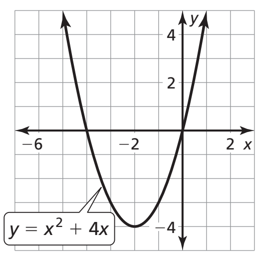
$x^2 - 12x + 36 = 0$

Graph $y = x^2 - 12x + 36$.



The only x-intercept is at the vertex, (6, 0). So, the solution is $x = 6$.

5. Graph $y = x^2 + 4x$.



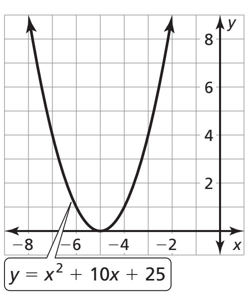
The x-intercepts are -4 and 0. So, the solutions are $x = -4$ and $x = 0$.

6. $x^2 + 10x = -25$

$x^2 + 10x + 25 = -25 + 25$

$x^2 + 10x + 25 = 0$

Graph $y = x^2 + 10x + 25$.



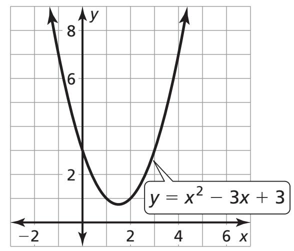
The only x-intercept is at the vertex, $(-5, 0)$. So, the solution is $x = -5$.

7. $x^2 = 3x - 3$

$x^2 - 3x + 3 = 3x - 3x - 3 + 3$

$x^2 - 3x + 3 = 0$

Graph $y = x^2 - 3x + 3$.



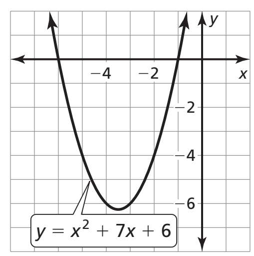
There are no x-intercepts. So, $x^2 = 3x - 3$ has no real solutions.

8. $x^2 + 7x = -6$

$x^2 + 7x + 6 = -6 + 6$

$x^2 + 7x + 6 = 0$

Graph $y = x^2 + 7x + 6$.



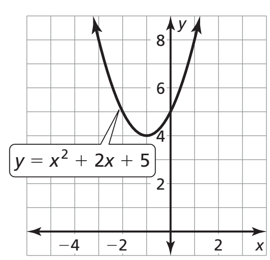
The x-intercepts are -6 and -1. So, the solutions are $x = -6$ and $x = -1$.

9. $2x + 5 = -x^2$

$2x + 5 + x^2 = -x^2 + x^2$

$x^2 + 2x + 5 = 0$

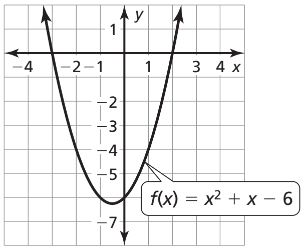
Graph $y = x^2 + 2x + 5$.



There are no x-intercepts. So, $2x + 5 = -x^2$ has no real solutions.

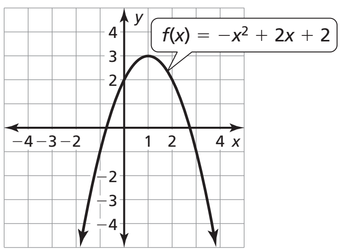
Chapter 9

10. Graph $f(x) = x^2 + x - 6$.



The x-intercepts are -3 and 2. So, the zeros of f are -3 and 2.

11. Graph $f(x) = -x^2 + 2x + 2$



x	-0.9	-0.8	-0.7	-0.6	-0.5
$f(x)$	-0.61	-0.24	0.11	0.44	0.75

change in signs

x	-0.4	-0.3	-0.2	-0.1
$f(x)$	1.04	1.31	1.56	1.79

x	2.1	2.2	2.3	2.4	2.5
$f(x)$	1.79	1.56	1.31	1.04	0.75

x	2.6	2.7	2.8	2.9
$f(x)$	0.44	0.11	-0.24	-0.61

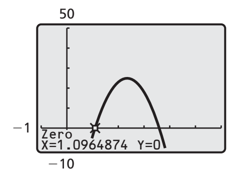
change in signs

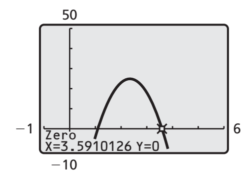
In each table, the function value closest to 0 is 0.11. So, the zeros of f are about -0.7 and 2.7.

12. From the table it is reasonable to estimate that the height of the football is 65 feet slightly more than 1 second and about 3.5 seconds after it is kicked.

$$-16t^2 + 75t + 2 = 65$$
$$-16t^2 + 75t + 2 - 65 = 65 - 65$$
$$-16t^2 + 75t - 63 = 0$$

Graph $h(t) = -16t^2 + 75t - 63$.





The football is 65 feet above the ground after about 1.1 seconds and about 3.6 seconds, which supports the estimates from the table.

9.2 Exercises (pp. 494–496)

Vocabulary and Core Concept Check

1. A quadratic equation is an equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

2. The equation that does not belong is $x^2 + x - 4 = 0$, because it is the only equation written in standard form.

3. Graph the quadratic equation and count the number of times the graph crosses the x-axis. The number of x-intercepts is the number of real solutions.

4. Solutions, roots, x-intercepts, and zeros are all basically the same. Equations have solutions or roots. Graphs have x-intercepts. Functions have zeros.

Monitoring Progress and Modeling with Mathematics

5. The graph crosses the x-axis at $(-1, 0)$ and $(3, 0)$. So, the solutions are $x = -1$ and $x = 3$.

6. The graph crosses the x-axis at $(2, 0)$ and $(4, 0)$. So, the solutions are $x = 2$ and $x = 4$.

7. The only x-intercept is at the vertex, $(-4, 0)$. So, the solution is $x = -4$.

8. The graph does not have any x-intercepts. So, the equation $-x^2 - 4x - 6 = 0$ has no real solutions.

9.
$$4x^2 = 12$$
$$4x^2 - 12 = 12 - 12$$
$$4x^2 - 12 = 0$$

The rewritten equation is $4x^2 - 12 = 0$, or $-4x^2 + 12 = 0$.

10.
$$-x^2 = 15$$
$$-x^2 + x^2 = 15 + x^2$$
$$0 = x^2 + 15$$

The rewritten equation is $x^2 + 15 = 0$, or $-x^2 - 15 = 0$.

Chapter 9

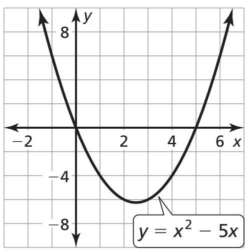
11. $$2x - x^2 = 1$$
$$2x - 2x - x^2 + x^2 = 1 - 2x + x^2$$
$$0 = x^2 - 2x + 1$$

The rewritten equation is $x^2 - 2x + 1 = 0$, or $-x^2 + 2x - 1 = 0$.

12. $$5 + x = 3x^2$$
$$5 - 5 + x - x = 3x^2 - 5 - x$$
$$0 = 3x^2 - x - 5$$

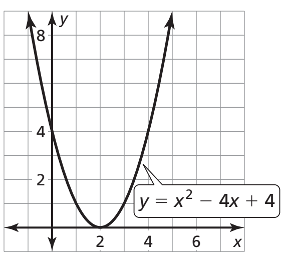
The rewritten equation is $3x^2 - x - 5 = 0$, or $-3x^2 + x + 5 = 0$.

13. Graph $y = x^2 - 5x$.



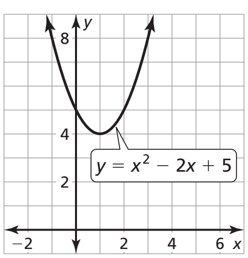
The x-intercepts are 0 and 5. So, the solutions are $x = 0$ and $x = 5$.

14. Graph $y = x^2 - 4x + 4$.



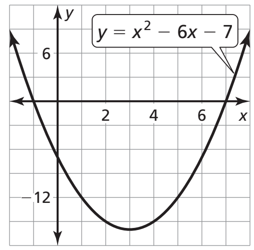
The only x-intercept is at the vertex, (2, 0). So, the solution is $x = 2$.

15. Graph $y = x^2 - 2x + 5$.



There are no x-intercepts. So, $x^2 - 2x + 5 = 0$ has no real solutions.

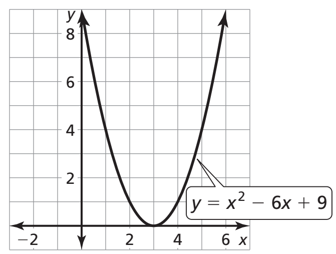
16. Graph $y = x^2 - 6x - 7$.



The x-intercepts are -1 and 7. So, the solutions are $x = -1$ and $x = 7$.

17. $$x^2 = 6x - 9$$
$$x^2 - 6x + 9 = 6x - 6x - 9 + 9$$
$$x^2 - 6x + 9 = 0$$

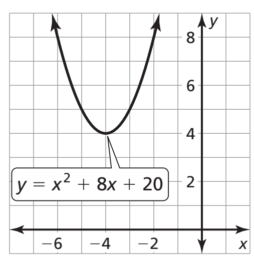
Graph $y = x^2 - 6x + 9$.



The only x-intercept is at the vertex, (3, 0). So, the solution is $x = 3$.

18. $$-x^2 = 8x + 20$$
$$-x^2 + x^2 = 8x + 20 + x^2$$
$$0 = x^2 + 8x + 20$$

Graph $y = x^2 + 8x + 20$.



There are no x-intercepts. So, $-x^2 = 8x + 20$ has no real solutions.

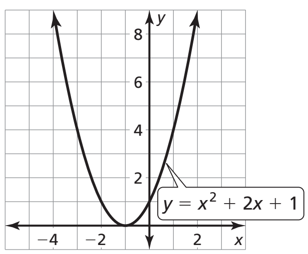
Chapter 9

19. $x^2 = -1 - 2x$

$x^2 + 1 + 2x = -1 + 1 - 2x + 2x$

$x^2 + 2x + 1 = 0$

Graph $y = x^2 + 2x + 1$.



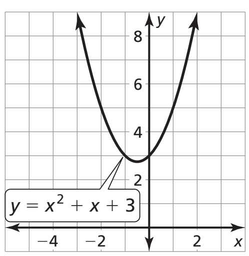
The only x-intercept is at the vertex, $(-1, 0)$. So, the solution is $x = -1$.

20. $x^2 = -x - 3$

$x^2 + x + 3 = -x + x - 3 + 3$

$x^2 + x + 3 = 0$

Graph $y = x^2 + x + 3$.



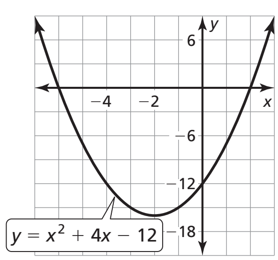
There are no x-intercepts. So, $x^2 = -x - 3$ has no real solutions.

21. $4x - 12 = -x^2$

$4x - 12 + x^2 = -x^2 + x^2$

$x^2 + 4x - 12 = 0$

Graph $y = x^2 + 4x - 12$.



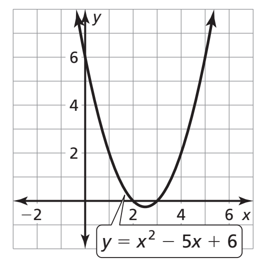
The x-intercepts are -6 and 2. So, the solutions are $x = -6$ and $x = 2$.

22. $5x - 6 = x^2$

$5x - 5x - 6 + 6 = x^2 - 5x + 6$

$0 = x^2 - 5x + 6$

Graph $y = x^2 - 5x + 6$.



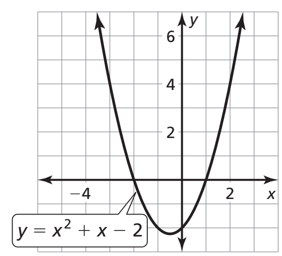
The x-intercepts are 2 and 3. So, the solutions are $x = 2$ and $x = 3$.

23. $x^2 - 2 = -x$

$x^2 - 2 + x = -x + x$

$x^2 + x - 2 = 0$

Graph $y = x^2 + x - 2$.



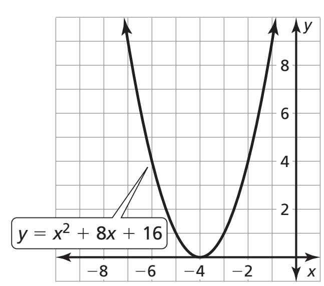
The x-intercepts are -2 and 1. So, the solutions are $x = -2$ and $x = 1$.

24. $16 + x^2 = -8x$

$16 + x^2 + 8x = -8x + 8x$

$x^2 + 8x + 16 = 0$

Graph $y = x^2 + 8x + 16$.



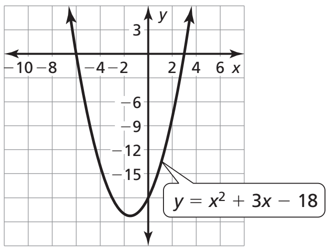
The only x-intercept is at the vertex, $(-4, 0)$. So, the solution is $x = -4$.

Chapter 9

25. The equation needs to be rewritten in standard form.

$$x^2 + 3x = 18$$
$$x^2 + 3x - 18 = 18 - 18$$
$$x^2 + 3x - 18 = 0$$

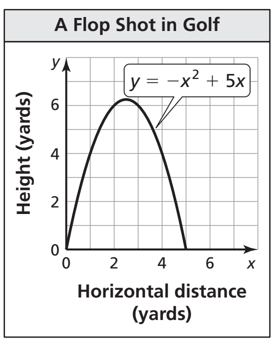
Graph $y = x^2 + 3x - 18$.



The x-intercepts are -6 and 3. So, the solutions of the equation $x^2 + 3x = 18$ are $x = -6$ and $x = 3$.

26. The solution is the x-intercept, not the y-intercept. The only x-intercept of the graph of $y = x^2 + 6x + 9$ is at the vertex, $(-3, 0)$. So, the solution of the equation $x^2 + 6x + 9 = 0$ is $x = -3$.

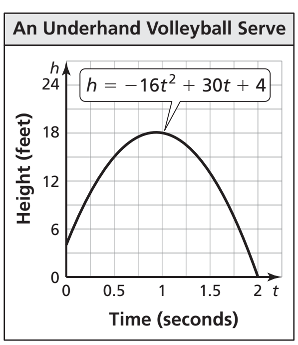
27. Graph $y = -x^2 + 5x$.



a. The x-intercepts represent when the golf ball is on the ground, where the height is 0 yards.

b. The x-intercepts are 0 and 5. So, the ball lands $5 - 0 = 5$ yards from where it is hit.

28. Graph $h = -16t^2 + 30t + 4$.



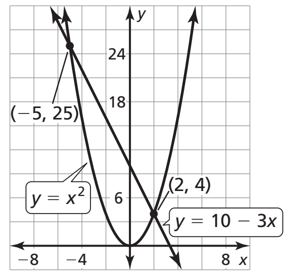
a. One of the t-intercepts is negative, but a negative time does not make sense in this context. So, the ball must have been served from a height above $h = 0$, where the ball landed.

b. The only valid t-intercept occurs at $(2, 0)$. So, the ball hit the ground after 2 seconds.

29. Graph each side of the equation $x^2 = 10 - 3x$.

$y = x^2$

$y = 10 - 3x$

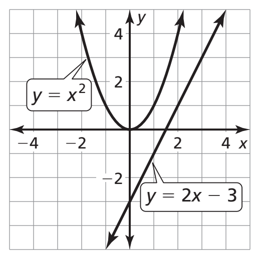


The graphs intersect at $(-5, 25)$ and $(2, 4)$. So, the solutions of the equation $x^2 = 10 - 3x$ are $x = -5$ and $x = 2$.

30. Graph each side of the equation $2x - 3 = x^2$.

$y = 2x - 3$

$y = x^2$

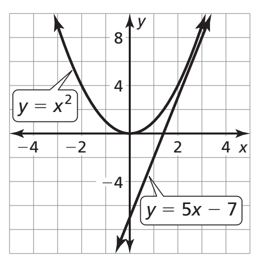


The graphs do not intersect. So, the equation $2x - 3 = x^2$ has no real solutions.

31. Graph each side of the equation $5x - 7 = x^2$.

$y = 5x - 7$

$y = x^2$



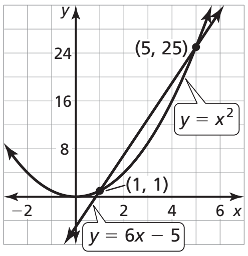
The graphs do not intersect. So, the equation $5x - 7 = x^2$ has no real solutions.

Chapter 9

32. Graph each side of the equation $x^2 = 6x - 5$.

$y = x^2$

$y = 6x - 5$

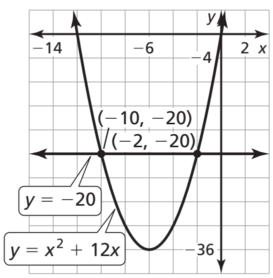


The graphs intersect at (1, 1) and (5, 25). So, the solutions of $x^2 = 6x - 5$ are $x = 1$ and $x = 5$.

33. Graph each side of the equation $x^2 + 12x = -20$.

$y = x^2 + 12x$

$y = -20$

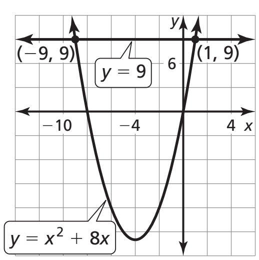


The graphs intersect at $(-10, -20)$ and $(-2, -20)$. So, the solutions of $x^2 + 12x = -20$ are $x = -10$ and $x = -2$.

34. Graph each side of the equation $x^2 + 8x = 9$.

$y = x^2 + 8x$

$y = 9$

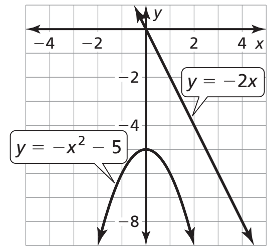


The graphs intersect at $(-9, 9)$ and $(1, 9)$. So, the solutions of $x^2 + 8x = 9$ are $x = -9$ and $x = 1$.

35. Graph each side of the equation $-x^2 - 5 = -2x$.

$y = -x^2 - 5$

$y = -2x$

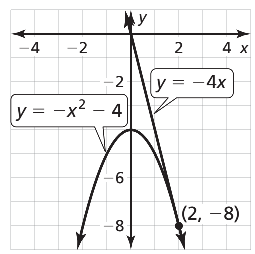


The graphs do not intersect. So, the equation $-x^2 - 5 = -2x$ has no real solutions.

36. Graph each side of the equation $-x^2 - 4 = -4x$.

$y = -x^2 - 4$

$y = -4x$



The graphs intersect at point $(2, -8)$. So, the solution of the equation $-x^2 - 4 = -4x$ is $x = 2$.

37. The x-intercepts are -1, 0, and 2.

Check $f(x) = (x - 2)(x^2 + x)$

$$0 \stackrel{?}{=} (-1 - 2)\left[(-1)^2 + (-1)\right]$$
$$0 \stackrel{?}{=} (-3)(1 - 1)$$
$$0 \stackrel{?}{=} (-3)(0)$$
$$0 = 0 \checkmark$$

$f(x) = (x - 2)(x^2 + x)$

$$0 \stackrel{?}{=} (0 - 2)(0^2 + 0)$$
$$0 \stackrel{?}{=} (-2)(0 + 0)$$
$$0 \stackrel{?}{=} (-2)(0)$$
$$0 = 0 \checkmark$$

$f(x) = (x - 2)(x^2 + x)$

$$0 \stackrel{?}{=} (2 - 2)(2^2 + 2)$$
$$0 \stackrel{?}{=} (0)(4 + 2)$$
$$0 \stackrel{?}{=} (0)(6)$$
$$0 = 0 \checkmark$$

So, the zeros are -1, 0, and 2.

Chapter 9

38. The x-intercepts are -4, -2, and -1.

Check $f(x) = (x+1)(x^2+6x+8)$

$$0 \stackrel{?}{=} (-4+1)\left[(-4)^2+6(-4)+8\right]$$
$$0 \stackrel{?}{=} (-3)(16-24+18)$$
$$0 \stackrel{?}{=} (-3)(-8+8)$$
$$0 \stackrel{?}{=} (-3)(0)$$
$$0 = 0 \checkmark$$

$f(x) = (x+1)(x^2+6x+8)$

$$0 \stackrel{?}{=} (-2+1)\left[(-2)^2+6(-2)+8\right]$$
$$0 \stackrel{?}{=} (-1)(4-12+8)$$
$$0 \stackrel{?}{=} (-1)(-8+8)$$
$$0 \stackrel{?}{=} (-1)(0)$$
$$0 = 0 \checkmark$$

$f(x) = (x+1)(x^2+6x+8)$

$$0 \stackrel{?}{=} (-1+1)\left[(-1)^2+6(-1)+8\right]$$
$$0 \stackrel{?}{=} (0)(1-6+8)$$
$$0 \stackrel{?}{=} (0)(-5+8)$$
$$0 \stackrel{?}{=} (0)(3)$$
$$0 = 0 \checkmark$$

So, the zeros are -4, -2, and -1.

39. The x-intercepts are -3 and 1.

Check $f(x) = (x+3)(-x^2+2x-1)$

$$0 \stackrel{?}{=} (-3+3)\left[-(-3)^2+2(-3)-1\right]$$
$$0 \stackrel{?}{=} (0)(-9-6-1)$$
$$0 \stackrel{?}{=} (0)(-15-1)$$
$$0 \stackrel{?}{=} (0)(-16)$$
$$0 = 0 \checkmark$$

$f(x) = (x+3)(-x^2+2x-1)$

$$0 \stackrel{?}{=} (1+3)\left(-(1)^2+2(1)-1\right)$$
$$0 \stackrel{?}{=} (4)(-1+2-1)$$
$$0 \stackrel{?}{=} (4)(1-1)$$
$$0 \stackrel{?}{=} (4)(0)$$
$$0 = 0 \checkmark$$

The zeros are -3 and 1.

40. The only x-intercept is 5.

Check $f(x) = (x-5)(-x^2+3x-3)$

$$0 \stackrel{?}{=} (5-5)\left(-(5)^2+3(5)-3\right)$$
$$0 \stackrel{?}{=} (0)(-25+15-3)$$
$$0 \stackrel{?}{=} (0)(-10-3)$$
$$0 \stackrel{?}{=} (0)(-13)$$
$$0 = 0 \checkmark$$

The zero is 5.

41. The x-intercepts are -3, -2, 1, and 2.

Check $f(x) = (x^2-4)(x^2+2x-3)$

$$0 \stackrel{?}{=} \left[(-3)^2-4\right]\left[(-3)^2+2(-3)-3\right]$$
$$0 \stackrel{?}{=} (9-4)(9-6-3)$$
$$0 \stackrel{?}{=} (5)(3-3)$$
$$0 \stackrel{?}{=} 5(0)$$
$$0 = 0 \checkmark$$

$f(x) = (x^2-4)(x^2+2x-3)$

$$0 \stackrel{?}{=} (1^2-4)(1^2+2(1)-3)$$
$$0 \stackrel{?}{=} (1-4)(1+2-3)$$
$$0 \stackrel{?}{=} (-3)(3-3)$$
$$0 \stackrel{?}{=} (-3)(0)$$
$$0 = 0 \checkmark$$

$f(x) = (x^2-4)(x^2+2x-3)$

$$0 \stackrel{?}{=} \left[(-2)^2-4\right]\left[(-2)^2+2(-2)-3\right]$$
$$0 \stackrel{?}{=} (4-4)(4-4-3)$$
$$0 \stackrel{?}{=} (0)(0-3)$$
$$0 \stackrel{?}{=} (0)(-3)$$
$$0 = 0 \checkmark$$

$f(x) = (x^2-4)(x^2+2x-3)$

$$0 \stackrel{?}{=} (2^2-4)\left[2^2+2(2)-3\right]$$
$$0 \stackrel{?}{=} (4-4)(4+4-3)$$
$$0 \stackrel{?}{=} (0)(8-3)$$
$$0 \stackrel{?}{=} (0)(5)$$
$$0 = 0 \checkmark$$

The zeros are -3, -2, 1, and 2.

42. The x-intercepts are -1 and 2.

Check $f(x) = (x^2+1)(x^2-x-2)$

$$0 \stackrel{?}{=} \left[(-1)^2+1\right]\left[(-1)^2-(-1)-2\right]$$
$$0 \stackrel{?}{=} (1+1)(1+1-2)$$
$$0 \stackrel{?}{=} (2)(2-2)$$
$$0 \stackrel{?}{=} (2)(0)$$
$$0 = 0 \checkmark$$

$f(x) = (x^2+1)(x^2-x-2)$

$$0 \stackrel{?}{=} (2^2+1)(2^2-2-2)$$
$$0 \stackrel{?}{=} (4+1)(4-2-2)$$
$$0 \stackrel{?}{=} (5)(2-2)$$
$$0 \stackrel{?}{=} (5)(0)$$
$$0 = 0 \checkmark$$

The zeros are -1 and 2.

43.

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	2.51	2.04	1.59	1.16	0.75

x	0.6	0.7	0.8	0.9
$f(x)$	0.36	−0.01	−0.36	−0.69

change in signs

x	4.1	4.2	4.3	4.4	4.5
$f(x)$	−0.69	−0.36	−0.01	0.36	0.75

change in signs

x	4.6	4.7	4.8	4.9
$f(x)$	1.16	1.59	2.04	2.51

In each table, the function value closest to 0 is −0.01. So, the zeros of f are about 0.7 and 4.3.

44.

x	−3.9	−3.8	−3.7	−3.6	−3.5
$f(x)$	2.51	2.04	1.59	1.16	0.75

x	−3.4	−3.3	−3.2	−3.1
$f(x)$	0.36	−0.01	−0.36	−0.69

change in signs

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	−0.69	−0.36	−0.01	0.36	0.75

change in signs

x	0.6	0.7	0.8	0.9
$f(x)$	1.16	1.59	2.04	2.51

In each table, the function value closest to 0 is −0.01. So, the zeros of f are about −3.3 and 0.3.

45.

x	−0.9	−0.8	−0.7	−0.6
$f(x)$	−1.61	−1.24	−0.89	−0.56

x	−0.5	−0.4	−0.3	−0.2	−0.1
$f(x)$	−0.25	0.04	0.31	0.56	0.79

change in signs

x	2.1	2.2	2.3	2.4	2.5
$f(x)$	0.79	0.56	0.31	0.04	−0.25

change in signs

x	2.6	2.7	2.8	2.9
$f(x)$	−0.56	−0.89	−1.24	−1.61

In each table, the function value closest to 0 is 0.04. So, the zeros of f are about −0.4 and 2.4.

46.

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	−1.41	−0.84	−0.29	0.24	0.75

change in signs

x	0.6	0.7	0.8	0.9
$f(x)$	1.24	1.71	2.16	2.59

x	5.1	5.2	5.3	5.4	5.5
$f(x)$	2.59	2.16	1.71	1.24	0.75

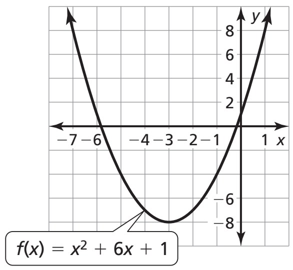
x	5.6	5.7	5.8	5.9
$f(x)$	0.24	−0.29	−0.84	−1.41

change in signs

In each table, the function value closest to 0 is 0.24. So, the zeros of f are about 0.4 and 5.6.

Chapter 9

47. Graph $f(x) = x^2 + 6x + 1$.



x	−5.9	−5.8	−5.7	−5.6	−5.5
$f(x)$	0.41	−0.16	−0.71	−1.24	−1.75

change in signs

x	−5.4	−5.3	−5.2	−5.1
$f(x)$	−2.24	−2.71	−3.16	−3.59

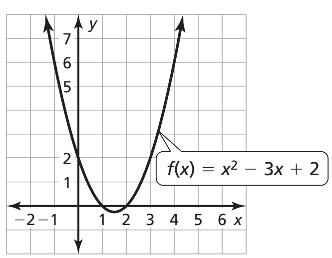
x	−0.9	−0.8	−0.7	−0.6	−0.5
$f(x)$	−3.59	−3.16	−2.71	−2.24	−1.75

x	−0.4	−0.3	−0.2	−0.1
$f(x)$	−1.24	−0.71	−0.16	0.41

change in signs

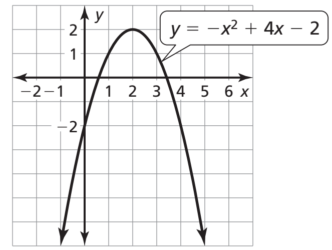
In each table, the function value closest to 0 is −0.16. So, the zeros of f are about −5.8 and about −0.2.

48. Graph $f(x) = x^2 - 3x + 2$.



The x-intercepts are 1 and 2. So, the zeros of f are 1 and 2.

49. Graph $y = -x^2 + 4x - 2$.



x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	−1.61	−1.24	−0.89	−0.56	−0.25	0.04

change in signs

x	0.7	0.8	0.9
$f(x)$	0.31	0.56	0.79

x	3.1	3.2	3.3	3.4	3.5
$f(x)$	0.79	0.56	0.31	0.04	−0.25

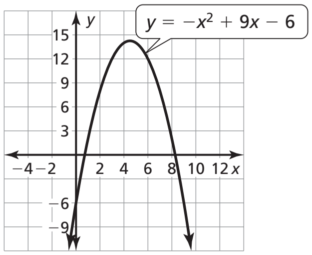
change in signs

x	3.6	3.7	3.8	3.9
$f(x)$	−0.56	−0.89	−1.24	−1.61

In each table, the function value closest to 0 is 0.04. So, the zeros of f are about 0.6 and about 3.4.

Chapter 9

50. Graph $y = -x^2 + 9x - 6$.



x	0.1	0.2	0.3	0.4	0.5
$f(x)$	−5.11	−4.24	−3.39	−2.56	−1.75

x	0.6	0.7	0.8	0.9
$f(x)$	−0.96	−0.19	0.56	1.29

change in signs

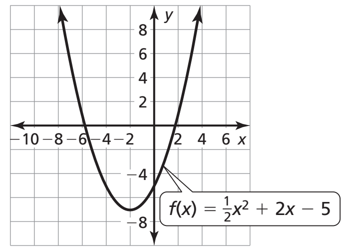
x	8.1	8.2	8.3	8.4	8.5
$f(x)$	1.29	0.56	−0.19	−0.96	−1.75

change in signs

x	8.6	8.7	8.8	8.9
$f(x)$	−2.56	−3.39	−4.24	−5.11

In each table, the function value closest to 0 is -0.19. So, the zeros of f are about 0.7 and about 8.3.

51. Graph $f(x) = \frac{1}{2}x^2 + 2x - 5$.



x	−5.9	−5.8	−5.7	−5.6	−5.5
$f(x)$	0.61	0.22	−0.16	−0.52	−0.88

change in signs

x	−5.4	−5.3	−5.2	−5.1
$f(x)$	−1.22	−1.56	−1.88	−2.20

x	1.1	1.2	1.3	1.4	1.5
$f(x)$	−2.20	−1.88	−1.56	−1.22	−0.88

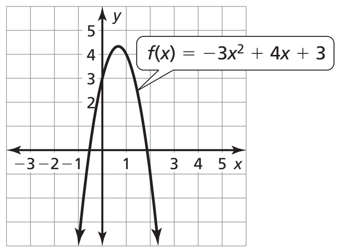
x	1.6	1.7	1.8	1.9
$f(x)$	−0.52	−0.16	0.22	0.61

change in signs

In each table, the function value closest to 0 is -0.16. So, the zeros of f are about -5.7 and about 1.7.

Chapter 9

52. Graph $f(x) = -3x^2 + 4x + 3$.



x	-0.9	-0.8	-0.7	-0.6	-0.5
$f(x)$	-3.03	-2.12	-1.27	-0.48	0.25

change in signs

x	-0.4	-0.3	-0.2	-0.1
$f(x)$	0.92	1.53	2.08	2.57

x	1.1	1.2	1.3	1.4	1.5
$f(x)$	3.77	3.48	3.13	2.72	2.25

x	1.6	1.7	1.8	1.9
$f(x)$	1.72	1.13	0.48	-0.23

change in signs

In the first table, the function value closest to 0 is 0.25. In the second table, the function value closest to 0 is -0.23. So, the zeros of f are about -0.5 and about 1.9.

53. a.

Seconds, t	0	1	2	3	4	5
Height, h	6	118	198	246	262	246

Seconds, t	6	7	8	9
Height, h	198	118	6	-138

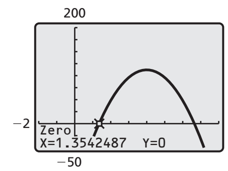
The height of the cannonball is 118 feet after 1 second, 198 feet after 2 seconds, 246 feet after 3 seconds, 262 feet after 4 seconds, 246 feet after 5 seconds, 198 feet after 6 seconds, 118 feet after 7 seconds, and 6 feet after 8 seconds.

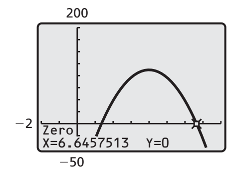
b. Based on the function values, it is reasonable to estimate that the height of the cannonball is 150 feet about 1.5 seconds and about 6.5 seconds after it is fired.

c.

$$-16t^2 + 128t + 6 = 150$$
$$-16t^2 + 128t + 6 - 150 = 150 - 150$$
$$-16t^2 + 128t - 144 = 0$$

So, graph $h = -16t^2 + 128t - 144$.





The cannonball is 150 feet above the ground after about 1.4 seconds and about 6.6 seconds.

54. a.

Seconds, t	0	1	2	3
Height, h	5	29	21	-19

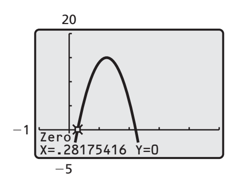
The height of the softball is 29 feet after 1 second and 21 feet after 2 seconds.

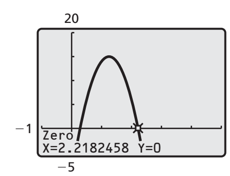
b. Based on the function values, it is reasonable to estimate that the height of the softball is 15 feet less than 0.5 second and slightly more than 2 seconds after it is thrown.

c.

$$-16t^2 + 40t + 5 = 15$$
$$-16t^2 + 40t + 5 - 15 = 15 - 15$$
$$-16t^2 + 40t - 10 = 0$$

So, graph $h = -16t^2 + 40t - 10$.



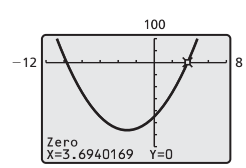


The softball is 15 feet above the ground after about 0.3 second and about 2.2 seconds.

55.

$$S = 2\pi r^2 + 2\pi rh$$
$$225 = 2\pi r^2 + 2\pi r(6)$$
$$225 = 2\pi r^2 + 12\pi r$$
$$225 - 225 = 2\pi r^2 + 12\pi r - 225$$
$$0 = 2\pi r^2 + 12\pi r - 225$$

Graph $y = 2\pi r^2 + 12\pi r - 225$.



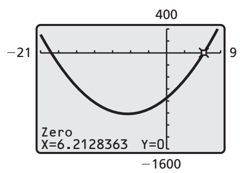
The length of the radius must be positive. The graph's only positive x-intercept is about 3.7. So, the radius is about 3.7 feet.

Chapter 9

56.

$$
\begin{aligned}
S &= 2\pi r^2 + 2\pi rh \\
750 &= 2\pi r^2 + 2\pi r(13) \\
750 &= 2\pi r^2 + 26\pi r \\
750 - 750 &= 2\pi r^2 + 26\pi r - 750 \\
0 &= 2\pi r^2 + 26\pi r - 750
\end{aligned}
$$

Graph $y = 2\pi r^2 + 26\pi r - 750$.



The length of the radius must be positive. The graph's only positive x-intercept is about 6.2. So, the radius is about 6.2 meters.

57. Graph the function to determine which integers the zeros are between. Then make tables using x-values between the integers with an interval of 0.1. Look for a change of sign in the function values. Of these two function values, pick the one that is closest to zero. The x-value that corresponds with this y-value is an approximate zero of the function.

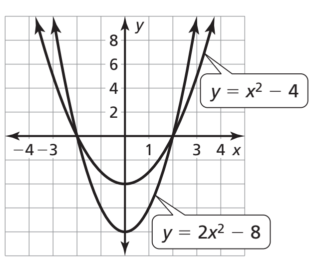
58. a. The graphs intersect in two places. So, the quadratic equation $x^2 = -3x + 4$ has two solutions.

b. The graph of $y = x^2 + 3x - 4$ intersects the x-axis in two places, where $x = -4$ and $x = 1$.

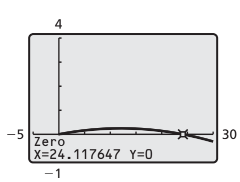
59. *Sample answer:* Method 1 is preferred because you only have to graph one equation in order to find the solutions.

60. There are infinitely many parabolas that have -2 and 2 as x-intercepts.

Sample answer:



61. Use a graphing calculator to graph $y = -0.0017x^2 + 0.041x$.



The x-intercepts of the graph are 0 and about 24.1. So, the width of the road is about $24.1 - 0 = 24.1$ feet.

62. $y = -0.003x^2 + 0.58x + 3$

$$
\begin{aligned}
y &= -0.003(57)^2 + 0.58(57) + 3 \\
&= -0.003(3249) + 33.06 + 3 \\
&= -9.747 + 33.06 + 3 \\
&= 26.313
\end{aligned}
$$

When $x = 57$, the value of y is 26.313, which means the water is reaching higher than 26 feet on the building when the firefighter is standing 57 feet away. So, the water will pass through the window that is 26 feet above the ground.

63. An example of an equation with a negative a value that has two x-intercepts is $y = -2x^2 + 1$. An example of an equation with a negative a value that has no x-intercepts is $y = -2x^2 + (-1)$. So, the graph of $y = ax^2 + c$ sometimes has two x-intercepts when a is negative.

64. If a and c have the same sign, then the sign of y is the same over the entire graph. Because there is no sign change, the graph does not cross the x-axis and has no x-intercepts. So, the graph of $y = ax^2 + c$ always has no x-intercepts when a and c have the same sign.

65. Quadratic equations have at most two x-intercepts. So, the graph of $y = ax^2 + bx + c$ never has more than two x-intercepts when $a \neq 0$.

Maintaining Mathematical Proficiency

66.

$+1 \quad +1 \quad +1$

x	-1	0	1	2
y	18	3	$\frac{1}{2}$	$\frac{1}{12}$

$\times\frac{1}{6} \quad \times\frac{1}{6} \quad \times\frac{1}{6}$

As x increases by 1, y is multiplied by $\frac{1}{6}$. Because $0 < \frac{1}{6} < 1$, the table represents an exponential decay function.

67.

$+1 \quad +1 \quad +1$

x	0	1	2	3
y	2	8	32	128

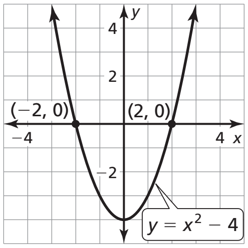
$\times 4 \quad \times 4 \quad \times 4$

As x increases by 1, y is multiplied by 4. Because $4 > 1$, the table represents an exponential growth function.

Chapter 9

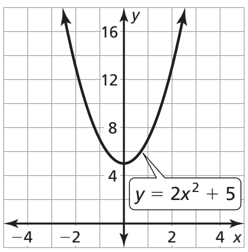
9.3 Explorations (p. 497)

1. a. Graph $y = x^2 - 4$.



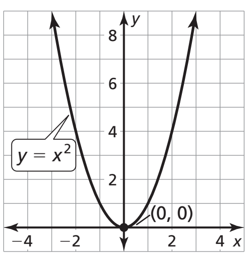
The x-intercepts are -2 and 2. So, the solutions are -2 and 2.

b. Graph $y = 2x^2 + 5$.



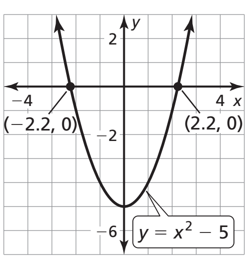
The graph does not cross the x-axis. So, $2x^2 + 5 = 0$ has no real solutions.

c. Graph $y = x^2$.



The graph has one x-intercept, and it is at the vertex, (0, 0). So, the solution is $x = 0$.

d. Graph $y = x^2 - 5$.



The x-intercepts are about -2.2 and 2.2. So, the solutions are $x \approx -2.2$ and $x \approx 2.2$.

The number of solutions is equal to the number of x-intercepts in the related graph.

2. a.

x	$x^2 - 5$
2.21	-0.1159
2.22	-0.0716
2.23	-0.0271
2.24	0.0176
2.25	0.0625
2.26	0.1076

b.

x	$x^2 - 5$
-2.21	-0.1159
-2.22	-0.0716
-2.23	-0.0271
-2.24	0.0176
-2.25	0.0625
-2.26	0.1076

In each table, the function value closest to 0 is 0.0176. So, the solutions are $x \approx 2.24$ and $x \approx -2.24$.

3. a. Adding 5 to each side of $x^2 - 5 = 0$ gives $x^2 = 5$. So, the equations are equivalent.

b. Using a calculator to find the positive square root of 5 and the negative square root of 5, you get $x = \sqrt{5} \approx 2.236$ or $x = -\sqrt{5} \approx -2.236$. The estimates in Explorations 2 were accurate to the nearest hundredths.

c. The exact solutions of $x^2 - 5 = 0$ are $x = \sqrt{5}$ and $x = -\sqrt{5}$.

4. Graph the related equation $y = ax^2 + c$. The number of solutions will be the same as the number of x-intercepts.

5. a. $x^2 - 2 = 0$

$$\begin{aligned} \underline{+\,2} \quad & \underline{+\,2} \\ x^2 &= 2 \\ \sqrt{x^2} &= \sqrt{2} \\ x &= \pm\sqrt{2} \\ x &\approx \pm 1.41 \end{aligned}$$

The exact solutions are $x = \sqrt{2}$ and $x = -\sqrt{2}$. The estimated solutions are $x \approx 1.41$ and $x \approx -1.41$.

b. $3x^2 - 18 = 0$

$$\begin{aligned} \underline{+\,18} \quad & \underline{+\,18} \\ 3x^2 &= 18 \\ \frac{3x^2}{3} &= \frac{18}{3} \\ x^2 &= 6 \\ \sqrt{x^2} &= \sqrt{6} \\ x &= \pm\sqrt{6} \\ x &\approx \pm 2.45 \end{aligned}$$

The exact solutions are $x = \sqrt{6}$ and $x = -\sqrt{6}$. The estimated solutions are $x \approx 2.45$ and $x \approx -2.45$.

c. $x^2 = 8$

$$\begin{aligned} \sqrt{x^2} &= \sqrt{8} \\ x &= \pm\sqrt{4 \cdot 2} \\ x &= \pm\sqrt{4} \cdot \sqrt{2} \\ x &= \pm 2\sqrt{2} \\ x &\approx \pm 2.83 \end{aligned}$$

The exact solutions are $x = 2\sqrt{2}$ and $x = -2\sqrt{2}$. The estimated solutions are $x \approx 2.83$ and $x \approx -2.83$.

Chapter 9

9.3 Monitoring Progress (pp. 499–500)

1. $-3x^2 = -75$

$$\frac{-3x^2}{-3} = \frac{-75}{-3}$$
$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$

The solutions are $x = -5$ and $x = 5$.

2. $x^2 + 12 = 10$

$$\underline{-12} \quad \underline{-12}$$
$$x^2 = -12$$

The square of a real number cannot be negative. So, the equation has no real solutions.

3. $4x^2 - 15 = -15$

$$\underline{+15} \quad \underline{+15}$$
$$4x^2 = 0$$
$$\frac{4x^2}{4} = \frac{0}{4}$$
$$\sqrt{x^2} = \sqrt{0}$$
$$x = 0$$

The only solution is $x = 0$.

4. $(x + 7)^2 = 0$

$$\sqrt{(x+7)^2} = \sqrt{0}$$
$$x + 7 = 0$$
$$\underline{-7} \quad \underline{-7}$$
$$x = -7$$

The only solution is $x = -7$.

5. $4(x - 3)^2 = 9$

$$\frac{4(x-3)^2}{4} = \frac{9}{4}$$
$$(x - 3)^2 = \tfrac{9}{4}$$
$$\sqrt{(x-3)^2} = \sqrt{\tfrac{9}{4}}$$
$$x - 3 = \pm \tfrac{3}{2}$$
$$\underline{+3} \quad \underline{+3}$$
$$x = 3 \pm \tfrac{3}{2}$$

So, the solutions are $x = 3 + \frac{3}{2} = \frac{9}{2}$ and $x = 3 - \frac{3}{2} = \frac{3}{2}$.

6. $(2x + 1)^2 = 36$

$$\sqrt{(2x+1)^2} = \sqrt{36}$$
$$2x + 1 = \pm 6$$
$$\underline{-1} \quad \underline{-1}$$
$$2x = -1 \pm 6$$
$$\frac{2x}{2} = \frac{-1 \pm 6}{2}$$
$$x = \frac{-1 \pm 6}{2}$$

The solutions are $x = \dfrac{-1 + 6}{2} = \dfrac{5}{2}$ and $x = \dfrac{-1 - 6}{2} = -\dfrac{7}{2}$.

7. $x^2 + 8 = 19$

$$\underline{-8} \quad \underline{-8}$$
$$x^2 = 11$$
$$\sqrt{x^2} = \sqrt{11}$$
$$x = \pm\sqrt{11}$$
$$x \approx \pm 3.32$$

The solutions are $x \approx 3.32$ and $x \approx -3.32$.

8. $5x^2 - 2 = 0$

$$\underline{+2} \quad \underline{+2}$$
$$5x^2 = 2$$
$$\frac{5x^2}{5} = \frac{2}{5}$$
$$x^2 = \frac{2}{5}$$
$$\sqrt{x^2} = \sqrt{\frac{2}{5}}$$
$$x = \pm\sqrt{\frac{2}{5}}$$
$$x = \pm\frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$
$$x = \pm\frac{\sqrt{10}}{5}$$
$$x \approx \pm 0.63$$

The solutions are $x \approx 0.63$ and $x \approx -0.63$.

Chapter 9

9. $3x^2 - 30 = 4$

$$\begin{aligned} 3x^2 - 30 &= 4 \\ +30 \quad & \quad +30 \\ 3x^2 &= 34 \\ \frac{3x^2}{3} &= \frac{34}{3} \\ x^2 &= \frac{34}{3} \\ \sqrt{x^2} &= \sqrt{\frac{34}{3}} \\ x &= \pm\sqrt{\frac{34}{3}} \\ x &= \pm\frac{\sqrt{34}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \\ x &= \pm\frac{\sqrt{102}}{3} \\ x &\approx \pm 3.37 \end{aligned}$$

The solutions are $x \approx 3.37$ and $x \approx -3.37$.

10.

$$\begin{aligned} V &= \ell wh \\ 315 &= 3w(w)(3) \\ 315 &= 9w^2 \\ \frac{315}{9} &= \frac{9w^2}{9} \\ 35 &= w^2 \\ \sqrt{35} &= \sqrt{w^2} \\ \pm\sqrt{35} &= w \end{aligned}$$

The solutions are $\sqrt{35}$ and $-\sqrt{35}$. Use the positive solution. So, the width is $\sqrt{35} \approx 5.9$ feet, and the length is $3\sqrt{35} \approx 17.7$ feet.

11.

$$\begin{aligned} S &= 4\pi r^2 \\ \frac{S}{4\pi} &= \frac{4\pi r^2}{4\pi} \\ \frac{S}{4\pi} &= r^2 \\ \sqrt{\frac{S}{4\pi}} &= \sqrt{r^2} \\ \sqrt{\frac{S}{4\pi}} &= r \end{aligned}$$

$$r = \sqrt{\frac{S}{4\pi}} = \sqrt{\frac{804}{4\pi}} = \sqrt{\frac{201}{\pi}} \approx 8.0$$

The radius of the globe is about 8 inches.

9.3 Exercises (pp. 501–502)

Vocabulary and Core Concept Check

1. The equation $x^2 = d$ has two real solutions when $d > 0$.

2. The one that is different is "Solve $x^2 + 146 = 2$ using square roots."

$$\begin{aligned} x^2 + 146 &= 2 \\ -146 \quad & \quad -146 \\ x^2 &= -144 \end{aligned}$$

The square of a real number cannot be negative. So, the equation has no real solutions. The other three are equivalent to $x^2 = 144$.

$$\begin{aligned} x^2 &= 144 \\ \sqrt{x^2} &= \sqrt{144} \\ x &= \pm 12 \end{aligned}$$

The solutions are $x = 12$ and $x = -12$.

Monitoring Progress and Modeling with Mathematics

3. Because $d = 25 > 0$, $x^2 = 25$ has two real solutions.

$$\begin{aligned} x^2 &= 25 \\ \sqrt{x^2} &= \sqrt{25} \\ x &= \pm 5 \end{aligned}$$

The solutions are $x = 5$ and $x = -5$.

4. Because $d = -36 < 0$, $x^2 = -36$ has no real solutions.

5. Because $d = -21 < 0$, $x^2 = -21$ has no real solutions.

6. Because $d = 400 > 0$, $x^2 = 400$ has two real solutions.

$$\begin{aligned} x^2 &= 400 \\ \sqrt{x^2} &= \sqrt{400} \\ x &= \pm 20 \end{aligned}$$

The solutions are $x = 20$ and $x = -20$.

7. Because $d = 0$, $x^2 = 0$ has one real solution.

$$\begin{aligned} x^2 &= 0 \\ \sqrt{x^2} &= \sqrt{0} \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

8. Because $d = 169 > 0$, $x^2 = 169$ has two real solutions.

$$\begin{aligned} x^2 &= 169 \\ \sqrt{x^2} &= \sqrt{169} \\ x &= \pm 13 \end{aligned}$$

The solutions are $x = 13$ and $x = -13$.

9. $x^2 - 16 = 0$

$$\begin{aligned} x^2 - 16 &= 0 \\ +16 \quad & \quad +16 \\ x^2 &= 16 \\ \sqrt{x^2} &= \sqrt{16} \\ x &= \pm 4 \end{aligned}$$

The solutions are $x = 4$ and $x = -4$.

Chapter 9

10.
$$\begin{aligned} x^2 + 6 &= 0 \\ \underline{-6} \quad & \underline{-6} \\ x^2 &= -6 \end{aligned}$$

The square of a real number cannot be negative. So, the equation has no real solutions.

11.
$$\begin{aligned} 3x^2 + 12 &= 0 \\ \underline{-12} \quad & \underline{-12} \\ 3x^2 &= -12 \\ \frac{3x^2}{3} &= \frac{-12}{3} \\ x^2 &= -4 \end{aligned}$$

The square of a real number cannot be negative. So, the equation has no real solutions.

12.
$$\begin{aligned} x^2 - 55 &= 26 \\ \underline{+55} \quad & \underline{+55} \\ x^2 &= 81 \\ \sqrt{x^2} &= \sqrt{81} \\ x &= \pm 9 \end{aligned}$$

The solutions are $x = 9$ and $x = -9$.

13.
$$\begin{aligned} 2x^2 - 98 &= 0 \\ \underline{+98} \quad & \underline{+98} \\ 2x^2 &= 98 \\ \frac{2x^2}{2} &= \frac{98}{2} \\ x^2 &= 49 \\ \sqrt{x^2} &= \sqrt{49} \\ x &= \pm 7 \end{aligned}$$

The solutions are $x = 7$ and $x = -7$.

14.
$$\begin{aligned} -x^2 + 9 &= 9 \\ \underline{-9} \quad & \underline{-9} \\ -x^2 &= 0 \\ \frac{-x^2}{-1} &= \frac{0}{-1} \\ x^2 &= 0 \\ \sqrt{x^2} &= \sqrt{0} \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

15.
$$\begin{aligned} -3x^2 - 5 &= -5 \\ \underline{+5} \quad & \underline{+5} \\ -3x^2 &= 0 \\ \frac{-3x^2}{-3} &= \frac{0}{-3} \\ x^2 &= 0 \\ \sqrt{x^2} &= \sqrt{0} \\ x &= 0 \end{aligned}$$

The only solution is $x = 0$.

16.
$$\begin{aligned} 4x^2 - 371 &= 29 \\ \underline{+371} \quad & \underline{+371} \\ 4x^2 &= 400 \\ \frac{4x^2}{4} &= \frac{400}{4} \\ x^2 &= 100 \\ \sqrt{x^2} &= \sqrt{100} \\ x &= \pm 10 \end{aligned}$$

The solutions are $x = 10$ and $x = -10$.

17.
$$\begin{aligned} 4x^2 + 10 &= 11 \\ \underline{-10} \quad & \underline{-10} \\ 4x^2 &= 1 \\ \frac{4x^2}{4} &= \frac{1}{4} \\ x^2 &= \frac{1}{4} \\ \sqrt{x^2} &= \sqrt{\frac{1}{4}} \\ x &= \pm \frac{1}{2} \end{aligned}$$

The solutions are $x = \frac{1}{2}$ and $x = -\frac{1}{2}$.

18.
$$\begin{aligned} 9x^2 - 35 &= 14 \\ \underline{+35} \quad & \underline{+35} \\ 9x^2 &= 49 \\ \frac{9x^2}{9} &= \frac{49}{9} \\ x^2 &= \frac{49}{9} \\ \sqrt{x^2} &= \sqrt{\frac{49}{9}} \\ x &= \pm \frac{7}{3} \end{aligned}$$

The solutions are $x = \frac{7}{3}$ and $x = -\frac{7}{3}$.

19.
$$\begin{aligned} (x + 3)^2 &= 0 \\ \sqrt{(x + 3)^2} &= \sqrt{0} \\ x + 3 &= 0 \\ \underline{-3} \quad & \underline{-3} \\ x &= -3 \end{aligned}$$

The only solution is. $x = -3$.

20.
$$\begin{aligned} (x - 1)^2 &= 4 \\ \sqrt{(x - 1)^2} &= \sqrt{4} \\ x - 1 &= \pm 2 \\ \underline{+1} \quad & \underline{+1} \\ x &= 1 \pm 2 \end{aligned}$$

The solutions are $x = 1 + 2 = 3$ and $x = 1 - 2 = -1$.

21.
$$\begin{aligned} (2x - 1)^2 &= 81 \\ \sqrt{(2x - 1)^2} &= \sqrt{81} \\ 2x - 1 &= \pm 9 \\ \underline{+1} \quad & \underline{+1} \\ 2x &= 1 \pm 9 \\ \frac{2x}{2} &= \frac{1 \pm 9}{2} \\ x &= \frac{1 \pm 9}{2} \end{aligned}$$

The solutions are $x = \frac{1 + 9}{2} = \frac{10}{2} = 5$ and $x = \frac{1 - 9}{2} = \frac{-8}{2} = -4$.

Chapter 9

22. $(4x + 5)^2 = 9$

$$\sqrt{(4x + 5)^2} = \sqrt{9}$$
$$4x + 5 = \pm 3$$
$$\underline{-5 \quad -5}$$
$$4x = -5 \pm 3$$
$$\frac{4x}{4} = \frac{-5 \pm 3}{4}$$
$$x = \frac{-5 \pm 3}{4}$$

The solutions are $x = \frac{-5 + 3}{4} = \frac{-2}{4} = -\frac{1}{2}$ and $x = \frac{-5 - 3}{4} = \frac{-8}{4} = -2$.

23. $9(x + 1)^2 = 16$

$$\frac{9(x + 1)^2}{9} = \frac{16}{9}$$
$$(x + 1)^2 = \frac{16}{9}$$
$$\sqrt{(x + 1)^2} = \sqrt{\frac{16}{9}}$$
$$x + 1 = \pm \frac{4}{3}$$
$$\underline{-1 \quad -1}$$
$$x = -1 \pm \frac{4}{3}$$

The solutions are $x = -1 \pm \frac{4}{3} = \frac{1}{3}$ and $x = -1 - \frac{4}{3} = -\frac{7}{3}$.

24. $4(x - 2)^2 = 25$

$$\frac{4(x - 2)^2}{4} = \frac{25}{4}$$
$$(x - 2)^2 = \frac{25}{4}$$
$$\sqrt{(x - 2)^2} = \sqrt{\frac{25}{4}}$$
$$x - 2 = \pm \frac{5}{2}$$
$$\underline{+2 \quad +2}$$
$$x = 2 \pm \frac{5}{2}$$

The solutions are $x = 2 + \frac{5}{2} = \frac{9}{2}$ and $x = 2 - \frac{5}{2} = -\frac{1}{2}$.

25. $x^2 + 6 = 13$

$$\underline{-6 \quad -6}$$
$$x^2 = 7$$
$$\sqrt{x^2} = \sqrt{7}$$
$$x = \pm \sqrt{7}$$
$$x \approx \pm 2.65$$

The solutions are $x \approx 2.65$ and $x \approx -2.65$.

26. $x^2 + 11 = 24$

$$\underline{-11 \quad -11}$$
$$x^2 = 13$$
$$\sqrt{x^2} = \sqrt{13}$$
$$x = \pm \sqrt{13}$$
$$x \approx \pm 3.61$$

The solutions are $x \approx 3.61$ and $x \approx -3.61$.

27. $2x^2 - 9 = 11$

$$\underline{+9 \quad +9}$$
$$2x^2 = 20$$
$$\frac{2x^2}{2} = \frac{20}{2}$$
$$x^2 = 10$$
$$\sqrt{x^2} = \sqrt{10}$$
$$x = \pm \sqrt{10}$$
$$x \approx \pm 3.16$$

The solutions are $x \approx 3.16$ and $x \approx -3.16$.

28. $5x^2 + 2 = 6$

$$\underline{-2 \quad -2}$$
$$5x^2 = 4$$
$$\frac{5x^2}{5} = \frac{4}{5}$$
$$x^2 = \frac{4}{5}$$
$$\sqrt{x^2} = \sqrt{\frac{4}{5}}$$
$$x = \pm\frac{2}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}}$$
$$x = \pm\frac{2\sqrt{5}}{5}$$
$$x \approx \pm 0.89$$

The solutions are $x \approx 0.89$ and $x \approx -0.89$.

29. $-21 = 15 - 2x^2$

$$\underline{-15 \quad -15}$$
$$-36 = -2x^2$$
$$\frac{-36}{-2} = \frac{-2x^2}{-2}$$
$$18 = x^2$$
$$\sqrt{18} = \sqrt{x^2}$$
$$\pm\sqrt{9 \bullet 2} = x$$
$$\pm\sqrt{9} \bullet \sqrt{2} = x$$
$$\pm 3\sqrt{2} = x$$
$$\pm 4.24 \approx x$$

The solutions are $x \approx 4.24$ and $x \approx -4.24$.

30. $2 = 4x^2 - 5$

$$\underline{+5 \quad +5}$$
$$7 = 4x^2$$
$$\frac{7}{4} = \frac{4x^2}{4}$$
$$\frac{7}{4} = x^2$$
$$\sqrt{\frac{7}{4}} = \sqrt{x^2}$$
$$\pm\frac{\sqrt{7}}{2} = x$$
$$\pm 1.32 \approx x$$

The solutions are $x \approx 1.32$ and $x \approx -1.32$.

Chapter 9

31. The number 36 has both a positive and negative square root.

$$2x^2 - 33 = 39$$
$$\underline{+33 \quad +33}$$
$$2x^2 = 72$$
$$\frac{2x^2}{2} = \frac{72}{2}$$
$$x^2 = 36$$
$$\sqrt{x^2} = \sqrt{36}$$
$$x = \pm 6$$

The solutions are $x = 6$ and $x = -6$.

32. Let w be the width and $2w$ be the length of the pond.

$$V = \ell wh$$
$$72{,}000 = 2w(w)(24)$$
$$72{,}000 = 48w^2$$
$$\frac{72{,}000}{48} = \frac{48w^2}{48}$$
$$1500 = w^2$$
$$\sqrt{1500} = \sqrt{w^2}$$
$$\pm\sqrt{1500} = w$$
$$\pm 10\sqrt{15} = w$$

The solutions are $10\sqrt{15}$ and $-10\sqrt{15}$. Use the positive solutions. So, the width is $10\sqrt{15} \approx 38.7$ inches and the length is $2 \bullet 10\sqrt{15} \approx 77.5$ inches.

33.

$$h = -16x^2 + 24$$
$$0 = -16x^2 + 24$$
$$\underline{-24 \qquad\qquad -24}$$
$$-24 = -16x^2$$
$$\frac{-24}{-16} = \frac{-16x^2}{-16}$$
$$\frac{3}{2} = x^2$$
$$\sqrt{\frac{3}{2}} = \sqrt{x^2}$$
$$\pm\frac{\sqrt{3}}{2} \bullet \frac{\sqrt{2}}{\sqrt{2}} = x$$
$$\pm\frac{\sqrt{6}}{2} = x$$

The solutions are $\frac{\sqrt{6}}{2}$ and $-\frac{\sqrt{6}}{2}$. Use the positive solution. So, it takes $\frac{\sqrt{6}}{2}$, or about 1.2 seconds for the sunglasses to hit the ground.

34. Subtract 4 from each side.

$$x^2 + 4 = 0$$
$$\underline{-4 \quad -4}$$
$$x^2 = -4$$

The square of a real number cannot be negative. So, the equation has no real solutions, and your cousin is correct.

35. Area of inner rug = 25% of Total area

$$x^2 = 0.25 \bullet 6^2$$
$$x^2 = 9$$
$$\sqrt{x^2} = \sqrt{9}$$
$$x = \pm 3$$

Use the positive solution. The inner square should have a side length of 3 feet.

36. a. $A = \pi r^2$

$$\frac{A}{\pi} = \frac{\pi r^2}{\pi}$$
$$\frac{A}{\pi} = r^2$$
$$\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$
$$\sqrt{\frac{A}{\pi}} = r$$

The formula for r is $r = \sqrt{\frac{A}{\pi}}$.

b. $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{113}{\pi}} \approx 6.0$

The radius of the first circle is about 6 feet.

$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{1810}{\pi}} \approx 24.0$

The radius of the second circle is about 24 inches.

$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{531}{\pi}} \approx 13.0$

The radius of the third circle is about 13 meters.

c. If you solve the formula for r, then it takes fewer steps to find the radius because the steps for solving only need to be completed once.

37. *Sample answer:* Isolate the variable term. Then use a calculator to find the approximate value of the variable.

38. $ax^2 + c = 0$

$$\underline{-c \quad -c}$$
$$ax^2 = -c$$
$$\frac{ax^2}{a} = \frac{-c}{a}$$
$$x^2 = -\frac{c}{a}$$

a. The equation $ax^2 + c = 0$ has two real solutions when $-\frac{c}{a} > 0$, or when a and c have opposite signs.

b. The equation $ax^2 + c = 0$ has one real solution when $-\frac{c}{a} = 0$, or when $a \neq 0$ and $c = 0$.

c. The equation $ax^2 + c = 0$ has no real solutions when $-\frac{c}{a} < 0$, or when a and c have the same sign.

Chapter 9

39. The graphs of $y = x^2$ and $y = 9$ intersect at the points $(-3, 9)$ and $(3, 9)$ because when $y = x^2 = 9$, $x = \pm\sqrt{9} = \pm 3$.

40. The graph of $f(x) = (x - 1)^2$ has one x-intercept. So, the equation $(x - 1)^2 = 0$ has one solution.

41. Because the square of 12 is 144, you can conclude that $1.2^2 = 1.44$.

$$\begin{aligned} x^2 &= 1.44 \\ \sqrt{x^2} &= \sqrt{1.44} \\ x &= \pm 1.2. \end{aligned}$$

So, the solutions are $x = 1.2$ and $x = -1.2$.

42.
$$\begin{aligned} \left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ \sqrt{\left(x + \frac{b}{2a}\right)^2} &= \sqrt{\frac{b^2 - 4ac}{4a^2}} \\ x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{\sqrt{4a^2}} \\ x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \\ -\frac{b}{2a} \quad & \quad -\frac{b}{2a} \\ x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

The solutions of the equation $ax^2 + bx + c = 0$ are $x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$.

43.
$$\begin{aligned} y &= \tfrac{1}{2}(x - 2)^2 + 1 \\ 9 &= \tfrac{1}{2}(x - 2)^2 + 1 \\ -1 \quad & \qquad\qquad\quad -1 \\ 8 &= \tfrac{1}{2}(x - 2)^2 \\ 2 \cdot 8 &= 2 \cdot \tfrac{1}{2}(x - 2)^2 \\ 16 &= (x - 2)^2 \\ \sqrt{16} &= \sqrt{(x - 2)^2} \\ \pm 4 &= x - 2 \\ +2 \quad & \quad +2 \\ 2 \pm 4 &= x \end{aligned}$$

The x-coordinates are $x = 2 + 4 = 6$ and $x = 2 - 4 = -2$.

44. a.
$$\begin{aligned} x^2 - 12x + 36 &= 64 \\ x^2 - 2(x)(6) + 6^2 &= 64 \\ (x - 6)^2 &= 64 \\ \sqrt{(x - 6)^2} &= \sqrt{64} \\ x - 6 &= \pm 8 \\ +6 \quad & \quad +6 \\ x &= 6 + 8 \end{aligned}$$

The solutions are $x = 6 + 8 = 14$ and $x = 6 - 8 = -2$.

b.
$$\begin{aligned} x^2 + 14x + 49 &= 16 \\ x^2 + 2(x)(7) + 7^2 &= 16 \\ (x + 7)^2 &= 16 \\ \sqrt{(x + 7)^2} &= \sqrt{16} \\ x + 7 &= \pm 4 \\ -7 \quad & \quad -7 \\ x &= -7 \pm 4 \end{aligned}$$

The solutions are $x = -7 + 4 = -3$ and $x = -7 - 4 = -11$.

Maintaining Mathematical Proficiency

45. $x^2 + 8x + 16 = x^2 + 2(x)(4) + 4^2$
$= (x + 4)^2$

46. $x^2 - 4x + 4 = x^2 - 2(x)(2) + 2^2$
$= (x - 2)^2$

47. $x^2 - 14x + 49 = x^2 - 2(x)(7) + 7^2$
$= (x - 7)^2$

48. $x^2 + 18x + 81 = x^2 + 2(x)(9) + 9^2$
$= (x + 9)^2$

49. $x^2 + 12x + 36 = x^2 + 2(x)(6) + 6^2$
$= (x + 6)^2$

50. $x^2 - 22x + 121 = x^2 - 2(x)(11) + 11^2$
$= (x - 11)^2$

9.1–9.3 What Did You Learn? (p. 503)

1. For part (c), examples that make the statement true are $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$, $\sqrt{3} + \pi = \pi + \sqrt{3}$, $-\sqrt{3} + \left(-\sqrt{3}\right) = -2\sqrt{3}$, $-\sqrt{3} + \pi = \pi - \sqrt{3}$, $\pi + \sqrt{3} = \pi + \sqrt{3}$, $\pi + \left(-\sqrt{3}\right) = \pi - \sqrt{3}$, and $\pi + \pi = 2\pi$. For part (c), counterexamples (that make the statement false) are $\sqrt{3} + \left(-\sqrt{3}\right) = 0$ and $-\sqrt{3} + \sqrt{3} = 0$.

For part (f), examples that make the statement true are $\sqrt{3} \cdot \pi = \pi\sqrt{3}$, $-\sqrt{3} \cdot \pi = -\pi\sqrt{3}$, $\pi \cdot \sqrt{3} = \pi\sqrt{3}$, $\pi \cdot \left(-\sqrt{3}\right) = -\pi\sqrt{3}$, and $\pi \cdot \pi = \pi^2$. For part (f), counterexamples (that make the statement false) are $\sqrt{3} \cdot \sqrt{3} = 3$, $\sqrt{3} \cdot \left(-\sqrt{3}\right) = -3$, $-\sqrt{3} \cdot \sqrt{3} = -3$, and $-\sqrt{3} \cdot \left(-\sqrt{3}\right) = 3$.

Chapter 9

2. *Sample answer:* Exercise 54 on page 496 is most similar to Example 6 on page 493. The situations described are similar, and each problem has three parts with the same type of questions. You can also refer to Example 5 on page 492 because both involve making a table of values and looking for a change in sides.

3. *Sample answer:* Solving the simpler equation $x^2 = 144$ helps because the solution of the equation $x^2 = 1.44$ can be found by moving the decimal point in the solution of $x^2 = 144$.

9.1–9.3 Quiz (p. 504)

1.
$$\begin{aligned}\sqrt{112x^3} &= \sqrt{16 \cdot 7 \cdot x^2 \cdot x}\\ &= \sqrt{16} \cdot \sqrt{7} \cdot \sqrt{x^2} \cdot \sqrt{x}\\ &= 4 \cdot \sqrt{7} \cdot x \cdot \sqrt{x}\\ &= 4 \cdot x \cdot \sqrt{7} \cdot \sqrt{x}\\ &= 4x\sqrt{7x}\end{aligned}$$

2.
$$\begin{aligned}\sqrt{\frac{18}{81}} &= \sqrt{\frac{2}{9}}\\ &= \frac{\sqrt{2}}{\sqrt{9}}\\ &= \frac{\sqrt{2}}{3}\end{aligned}$$

3.
$$\begin{aligned}\sqrt[3]{-625} &= \sqrt[3]{-125 \cdot 5}\\ &= \sqrt[3]{-125} \cdot \sqrt[3]{5}\\ &= -5\sqrt[3]{5}\end{aligned}$$

4.
$$\begin{aligned}\frac{12}{\sqrt{32}} &= \frac{12}{\sqrt{16 \cdot 2}}\\ &= \frac{12}{\sqrt{16} \cdot \sqrt{2}}\\ &= \frac{12}{4\sqrt{2}}\\ &= \frac{3}{\sqrt{2}}\\ &= \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}\\ &= \frac{3\sqrt{2}}{\sqrt{4}}\\ &= \frac{3\sqrt{2}}{2}\end{aligned}$$

5.
$$\begin{aligned}\frac{4}{\sqrt{11}} &= \frac{4}{\sqrt{11}} \cdot \frac{\sqrt{11}}{\sqrt{11}}\\ &= \frac{4\sqrt{11}}{\sqrt{121}}\\ &= \frac{4\sqrt{11}}{11}\end{aligned}$$

6.
$$\begin{aligned}\sqrt{\frac{144}{13}} &= \frac{\sqrt{144}}{\sqrt{13}}\\ &= \frac{12}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}}\\ &= \frac{12\sqrt{13}}{\sqrt{169}}\\ &= \frac{12\sqrt{13}}{13}\end{aligned}$$

7.
$$\begin{aligned}\sqrt[3]{\frac{54x^4}{343y^6}} &= \frac{\sqrt[3]{54x^4}}{\sqrt[3]{343y^6}}\\ &= \frac{\sqrt[3]{27 \cdot 2 \cdot x^3 \cdot x}}{\sqrt[3]{343 \cdot y^6}}\\ &= \frac{\sqrt[3]{27} \cdot \sqrt[3]{2} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x}}{\sqrt[3]{343} \cdot \sqrt[3]{y^6}}\\ &= \frac{3 \cdot \sqrt[3]{2} \cdot x \cdot \sqrt[3]{x}}{7 \cdot y^2}\\ &= \frac{3 \cdot x \cdot \sqrt[3]{2} \cdot \sqrt[3]{x}}{7y^2}\\ &= \frac{3x\sqrt[3]{2x}}{7y^2}\end{aligned}$$

8.
$$\begin{aligned}\sqrt{\frac{4x^2}{28y^4z^5}} &= \frac{\sqrt{4x^2}}{\sqrt{28y^4z^5}}\\ &= \frac{\sqrt{4 \cdot x^2}}{\sqrt{4 \cdot 7 \cdot y^4 \cdot z^4 \cdot z}}\\ &= \frac{\sqrt{4} \cdot \sqrt{x^2}}{\sqrt{4} \cdot \sqrt{7} \cdot \sqrt{y^4} \cdot \sqrt{z^4} \cdot \sqrt{z}}\\ &= \frac{2 \cdot x}{2 \cdot \sqrt{7} \cdot y^2 \cdot z^2 \cdot \sqrt{z}}\\ &= \frac{2x}{2 \cdot y^2 \cdot z^2 \cdot \sqrt{7} \cdot \sqrt{z}}\\ &= \frac{x}{y^2z^2\sqrt{7z}}\\ &= \frac{x}{y^2z^2\sqrt{7z}} \cdot \frac{\sqrt{7z}}{\sqrt{7z}}\\ &= \frac{x\sqrt{7z}}{y^2z^2\sqrt{49z^2}}\\ &= \frac{x\sqrt{7z}}{y^2 \cdot z^2 \cdot \sqrt{49} \cdot \sqrt{z^2}}\\ &= \frac{x\sqrt{7z}}{y^2 \cdot z^2 \cdot 7 \cdot z}\\ &= \frac{x\sqrt{7z}}{7y^2z^3}\end{aligned}$$

Chapter 9

9. $\dfrac{6}{5+\sqrt{3}} = \dfrac{6}{5+\sqrt{3}} \cdot \dfrac{5-\sqrt{3}}{5-\sqrt{3}}$

$= \dfrac{6(5-\sqrt{3})}{5^2-(\sqrt{3})^2}$

$= \dfrac{6(5)-6(\sqrt{3})}{25-3}$

$= \dfrac{30-6\sqrt{3}}{22}$

$= \dfrac{2(15-3\sqrt{3})}{22}$

$= \dfrac{15-3\sqrt{3}}{11}$

10. $2\sqrt{5}+7\sqrt{10}-3\sqrt{20} = 2\sqrt{5}+7\sqrt{10}-3 \cdot \sqrt{4 \cdot 5}$

$= 2\sqrt{5}+7\sqrt{10}-3 \cdot \sqrt{4} \cdot \sqrt{5}$

$= 2\sqrt{5}+7\sqrt{10}-3 \cdot 2 \cdot \sqrt{5}$

$= 2\sqrt{5}+7\sqrt{10}-6\sqrt{5}$

$= 2\sqrt{5}-6\sqrt{5}+7\sqrt{10}$

$= (2-6)\sqrt{5}+7\sqrt{10}$

$= -4\sqrt{5}+7\sqrt{10}$

11. $\dfrac{10}{\sqrt{8}-\sqrt{10}} = \dfrac{10}{\sqrt{8}-\sqrt{10}} \cdot \dfrac{\sqrt{8}+\sqrt{10}}{\sqrt{8}+\sqrt{10}}$

$= \dfrac{10(\sqrt{8}+\sqrt{10})}{(\sqrt{8})^2-(\sqrt{10})^2}$

$= \dfrac{10(\sqrt{8}+\sqrt{10})}{8-10}$

$= \dfrac{10(\sqrt{8}+\sqrt{10})}{-2}$

$= -5(\sqrt{8}+\sqrt{10})$

$= -5\sqrt{8}-5\sqrt{10}$

$= -5\sqrt{4 \cdot 2}-5\sqrt{10}$

$= -5 \cdot \sqrt{4} \cdot \sqrt{2}-5\sqrt{10}$

$= -5 \cdot 2 \cdot \sqrt{2}-5\sqrt{10}$

$= -10\sqrt{2}-5\sqrt{10}$

12. $\sqrt{6}(7\sqrt{12}-4\sqrt{3}) = \sqrt{6}(7\sqrt{12})-\sqrt{6}(4\sqrt{3})$

$= 7\sqrt{6 \cdot 12}-4\sqrt{6 \cdot 3}$

$= 7\sqrt{72}-4\sqrt{18}$

$= 7\sqrt{36 \cdot 2}-4\sqrt{9 \cdot 2}$

$= 7 \cdot \sqrt{36} \cdot \sqrt{2}-4 \cdot \sqrt{9} \cdot \sqrt{2}$

$= 7 \cdot 6 \cdot \sqrt{2}-4 \cdot 3 \cdot \sqrt{2}$

$= 42\sqrt{2}-12\sqrt{2}$

$= (42-12)\sqrt{2}$

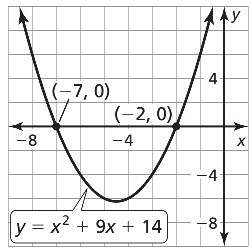
$= 30\sqrt{2}$

13. The x-intercepts of the graph are -1 and 3. So, the solutions are $x=-1$ and $x=3$.

14. The graph of $y = x^2 - 2x + 3$ does not cross the x-axis. So, $x^2 - 2x + 3 = 0$ has no real solutions.

15. The only x-intercept is at the vertex, $(-5, 0)$. So, the solution is $x = -5$.

16. Graph $y = x^2 + 9x + 14$.



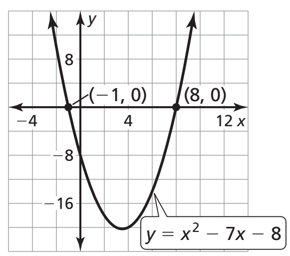
The x-intercepts are -7 and -2. So, the solutions are $x = -7$ and $x = -2$.

17. $x^2 - 7x = 8$

$x^2 - 7x - 8 = 8 - 8$

$x^2 - 7x - 8 = 0$

Graph $y = x^2 - 7x - 8$.



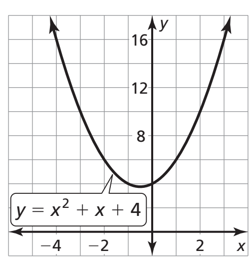
The x-intercepts are -1 and 8. So, the solutions are $x = -1$ and $x = 8$.

18. $x + 4 = -x^2$

$x + 4 + x^2 = -x^2 + x^2$

$x^2 + x + 4 = 0$

Graph $y = x^2 + x + 4$.



The graph of $y = x^2 + x + 4$ does not cross the x-axis. So, $x + 4 = -x^2$ has no real solution.

Chapter 9

19. $4x^2 = 64$

$$\frac{4x^2}{4} = \frac{64}{4}$$
$$x^2 = 16$$
$$\sqrt{x^2} = \sqrt{16}$$
$$x = \pm 4$$

The solutions are $x = -4$ and $x = 4$.

20. $-3x^2 + 6 = 10$

$$\underline{-\ 6} \quad \underline{-\ 6}$$
$$-3x^2 = 4$$
$$\frac{-3x^2}{-3} = \frac{4}{-3}$$
$$x^2 = -\frac{4}{3}$$

The square of a real number cannot be negative. So, $-3x^2 + 6 = 10$ has no real solutions.

21. $(x - 8)^2 = 1$

$$\sqrt{(x-8)^2} = \sqrt{1}$$
$$x - 8 = \pm 1$$
$$\underline{+\ 8} \quad \underline{+\ 8}$$
$$x = 8 \pm 1$$

So, the solutions are $x = 8 + 1 = 9$ and $x = 8 - 1 = 7$.

22. Because $d = 100 > 0$, the equation $x^2 = 100$ has two real solutions.

23. Let w be the width, and let $4w$ be the length of the rectangular prism.

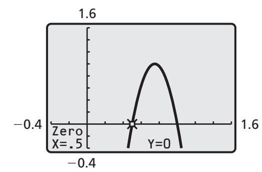
$$V = \ell wh$$
$$380 = 4w(w)(5)$$
$$380 = 20w^2$$
$$\frac{380}{20} = \frac{20w^2}{20}$$
$$19 = w^2$$
$$\sqrt{19} = \sqrt{w^2}$$
$$\pm\sqrt{19} = w$$

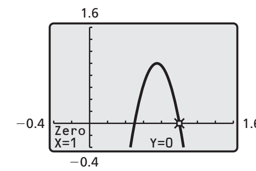
Use the positive solution. So, the width of the rectangular prism is $\sqrt{19}$, or about 4.4 meters, and the length is $4\sqrt{19}$, or about 17.4 meters.

24. a.

$$h = -16t^2 + 24t + 4$$
$$12 = -16t^2 + 24t + 4$$
$$12 - 12 = -16t^2 + 24t + 4 - 12$$
$$0 = -16t^2 + 24t - 8$$

Graph $y = -16t^2 + 24t - 8$.



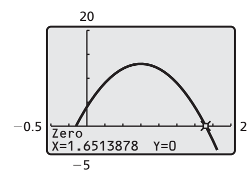


The zeros are 0.5 and 1. So, the fishing lure reaches a height of 12 feet after 0.5 second and again after 1 second.

b. $h = -16t^2 + 24t + 4$

$0 = -16t^2 + 24t + 4$

Graph $y = -16t^2 + 24t + 4$.



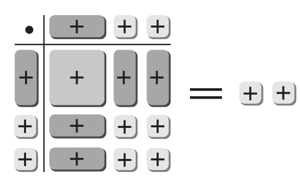
The fishing lure hits the water after about 1.65 seconds.

9.4 Explorations (p. 505)

1. a. The left side has one positive x-squared block and four positive x blocks, and the right has two negative unit blocks. So, the equation modeled by the algebra tiles is $x^2 + 4x = -2$.

b. By the Addition Property of Equality, when the same amount is added to each side of an equation, the new equation is equivalent.

c.



d. $x^2 + 4x + 4 = 2$

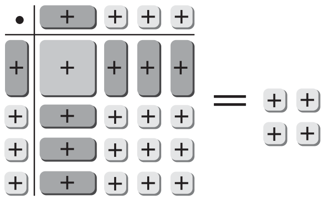
$$(x + 2)^2 = 2$$
$$\sqrt{(x+2)^2} = \sqrt{2}$$
$$x + 2 = \pm\sqrt{2}$$
$$\underline{-2} \quad \underline{-2}$$
$$x = -2 \pm \sqrt{2}$$

The solutions are $x = -2 + \sqrt{2} \approx -0.59$ and $x = -2 - \sqrt{2} \approx 3.41$.

Chapter 9

2. a. The equation is $x^2 + 6x = -5$.

b.



c.
$$\begin{aligned} x^2 + 6x + 9 &= 4 \\ (x+3)^2 &= 4 \\ \sqrt{(x+3)^2} &= \sqrt{4} \\ x + 3 &= \pm 2 \\ \underline{-3} \quad & \quad \underline{-3} \\ x &= -3 \pm 2 \end{aligned}$$

The solutions are $x = -3 + 2 = -1$ and $x = -3 - 2 = -5$.

d. Check

$$\begin{aligned} x^2 + 6x &= -5 \\ (-1)^2 + 6(-1) &\stackrel{?}{=} -5 \\ 1 - 6 &\stackrel{?}{=} -5 \\ -5 &= -5 \checkmark \end{aligned} \qquad \begin{aligned} x^2 + 6x &= -5 \\ (-5)^2 + 6(-5) &\stackrel{?}{=} -5 \\ 25 - 30 &\stackrel{?}{=} -5 \\ -5 &= -5 \checkmark \end{aligned}$$

3. Write the equation in the form $x^2 + bx = d$. Add $\left(\frac{b}{2}\right)^2$ to each side of the equation. Factor the resulting expression on the left side as the square of a binomial. Solve the resulting equation using square roots.

4. a.
$$\begin{aligned} x^2 - 2x &= 1 \\ x^2 - 2x + 1^2 &= 1 + 1^2 \\ x^2 - 2x + 1 &= 1 + 1 \\ (x-1)^2 &= 2 \\ \sqrt{(x-1)^2} &= \sqrt{2} \\ x - 1 &= \pm\sqrt{2} \\ \underline{+1} \quad & \quad \underline{+1} \\ x &= 1 \pm \sqrt{2} \end{aligned}$$

The solutions are $x = 1 + \sqrt{2} \approx 2.41$ and $x = 1 - \sqrt{2} \approx -0.41$.

b.
$$\begin{aligned} x^2 - 4x &= -1 \\ x^2 - 4x + 2^2 &= -1 + 2^2 \\ x^2 - 4x + 4 &= -1 + 4 \\ (x-2)^2 &= 3 \\ \sqrt{(x-2)^2} &= \sqrt{3} \\ x - 2 &= \pm\sqrt{3} \\ \underline{+2} \quad & \quad \underline{+2} \\ x &= 2 \pm \sqrt{3} \end{aligned}$$

The solutions are $x = 2 + \sqrt{3} \approx 3.73$ and $x = 2 - \sqrt{3} \approx 0.27$.

c.
$$\begin{aligned} x^2 + 4x &= -3 \\ x^2 + 4x + 2^2 &= -3 + 2^2 \\ x^2 + 4x + 4 &= -3 + 4 \\ (x+2)^2 &= 1 \\ \sqrt{(x+2)^2} &= \sqrt{1} \\ x + 2 &= \pm 1 \\ \underline{-2} \quad & \quad \underline{-2} \\ x &= -2 \pm 1 \end{aligned}$$

The solutions are $x = -2 + 1 = -1$ and $x = -2 - 1 = -3$.

9.4 Monitoring Progress (pp. 506–510)

1. $x^2 + 10x$

Step 1 $\frac{b}{2} = \frac{10}{2} = 5$

Step 2 $5^2 = 25$

Step 3 $x^2 + 10x + 25$

So, $x^2 + 10x + 25 = (x + 5)^2$.

2. $x^2 - 4x$

Step 1 $\frac{b}{2} = \frac{-4}{2} = -2$

Step 2 $(-2)^2 = 4$

Step 3 $x^2 - 4x + 4$

So, $x^2 - 4x + 4 = (x - 2)^2$.

3. $x^2 + 7x$

Step 1 $\frac{b}{2} = \frac{7}{2}$

Step 2 $\left(\frac{7}{2}\right)^2 = \frac{49}{4}$

Step 3 $x^2 + 7x + \frac{49}{4}$

So, $x^2 + 7x + \frac{49}{4} = \left(x + \frac{7}{2}\right)^2$.

4.
$$\begin{aligned} x^2 - 2x &= 3 \\ x^2 - 2x + (-1)^2 &= 3 + (-1)^2 \\ x^2 - 2x + 1 &= 3 + 1 \\ (x-1)^2 &= 4 \\ \sqrt{(x-1)^2} &= \sqrt{4} \\ x - 1 &= \pm 2 \\ \underline{+1} \quad & \quad \underline{+1} \\ x &= 1 \pm 2 \end{aligned}$$

The solutions are $x = 1 + 2 = 3$ and $x = 1 - 2 = -1$.

Chapter 9

5.
$$\begin{aligned} m^2 + 12m &= -8 \\ m^2 + 12m + 6^2 &= -8 + 6^2 \\ m^2 + 12m + 36 &= -8 + 36 \\ (m + 6)^2 &= 28 \\ \sqrt{(m + 6)^2} &= \sqrt{28} \\ m + 6 &= \pm\sqrt{28} \\ \underline{-6} \quad & \underline{-6} \\ m &= -6 \pm\sqrt{28} \end{aligned}$$

The solutions are $m = -6 + \sqrt{28} \approx -0.71$ and $m = -6 - \sqrt{28} \approx -11.29$.

6.
$$\begin{aligned} 3g^2 - 24g + 27 &= 0 \\ \underline{-27} \quad & \underline{-27} \\ 3g^2 - 24g &= -27 \\ 3(g^2 - 8g) &= -27 \\ \frac{3(g^2 - 8g)}{3} &= \frac{-27}{3} \\ g^2 - 8g &= -9 \\ g^2 - 8g + 4^2 &= -9 + 4^2 \\ g^2 - 8g + 16 &= -9 + 16 \\ (g - 4)^2 &= 7 \\ \sqrt{(g - 4)^2} &= \sqrt{7} \\ g - 4 &= \pm\sqrt{7} \\ \underline{+4} \quad & \underline{+4} \\ g &= 4 \pm \sqrt{7} \end{aligned}$$

The solutions are $g = 4 + \sqrt{7} \approx 6.65$ and $g = 4 - \sqrt{7} \approx 1.35$.

7.
$$\begin{aligned} y &= -x^2 - 4x + 4 \\ y - 4 &= -x^2 - 4x + 4 - 4 \\ y - 4 &= -x^2 - 4x \\ y - 4 &= -(x^2 + 4x) \\ y - 4 - 4 &= -(x^2 + 4x + 4) \\ y - 8 &= -(x + 2)^2 \\ \underline{+8} \quad & \underline{+8} \\ y &= -(x + 2)^2 + 8 \end{aligned}$$

The vertex is $(-2, 8)$. Because a is negative $(a = -1)$, the parabola opens down and the y-coordinate of the vertex is the maximum value. So, the function has a maximum value of 8.

8.
$$\begin{aligned} y &= x^2 + 12x + 40 \\ y - 40 &= x^2 + 12x + 40 - 40 \\ y - 40 &= x^2 + 12x \\ y - 40 + 36 &= x^2 + 12x + 36 \\ y - 4 &= (x + 6)^2 \\ \underline{+4} \quad & \underline{+4} \\ y &= (x + 6)^2 + 4 \end{aligned}$$

The vertex is $(-6, 4)$. Because a is positive $(a = 1)$, the parabola opens up and the y-coordinate of the vertex is a minimum value. So, the function has a minimum value of 4.

9.
$$\begin{aligned} y &= x^2 - 2x - 2 \\ y + 2 &= x^2 - 2x - 2 + 2 \\ y + 2 &= x^2 - 2x \\ y + 2 + 1 &= x^2 - 2x + 1 \\ y + 3 &= (x - 1)^2 \\ \underline{-3} \quad & \underline{-3} \\ y &= (x - 1)^2 - 3 \end{aligned}$$

The vertex is $(1, -3)$. Because a is positive $(a = 1)$, the parabola opens up and the y-coordinate of the vertex is a minimum value. So, the function has a minimum value of -3.

10. The graph of h opens up because $a = 1$ and $a > 0$. So, the function could not be represented by the graph.

11. The graph of n has two positive x-intercepts, and its graph opens down because $a = -2$ and $-2 < 0$. This means that n has a maximum value, and the vertex must be in the first quadrant. So, the graph could represent n.

12. a.
$$\begin{aligned} y &= -16x^2 + 128x \\ y &= -16(x^2 - 8x) \\ y - 16 \cdot 16 &= -16(x^2 - 8x + 16) \\ y - 256 &= -16(x - 4)^2 \\ \underline{+256} \quad & \underline{+256} \\ y &= -16(x - 4)^2 + 256 \end{aligned}$$

Because the maximum value is 256, the model rocket reaches a maximum height of 256 feet.

b. The vertex is $(4, 256)$. So, the axis of symmetry is $x = 4$. On the left side of $x = 4$, the height increases as time increases. On the right side of $x = 4$, the height decreases as time increases.

Chapter 9

13.

Area of chalkboard (square feet)	=	Length of chalkboard (feet)	•	Width of chalkboard (feet)
4	=	$(7 - 2x)$	•	$(3 - 2x)$

$$4 = (7 - 2x)(3 - 2x)$$
$$4 = 7(3) + 7(-2x) - 2x(3) - 2x(-2x)$$
$$4 = 21 - 14x - 6x + 4x^2$$
$$4 = 4x^2 - 20x + 21$$
$$4 - 21 = 4x^2 - 20x + 21 - 21$$
$$-17 = 4x^2 - 20x$$
$$-\frac{17}{4} = \frac{4x^2 - 20x}{4}$$
$$-\frac{17}{4} = x^2 - 5x$$
$$-\frac{17}{4} + \frac{25}{4} = x^2 - 5x + \frac{25}{4}$$
$$\frac{8}{4} = x^2 - 5x + \frac{25}{4}$$
$$2 = \left(x - \frac{5}{2}\right)^2$$
$$\sqrt{2} = \sqrt{\left(x - \frac{5}{2}\right)^2}$$
$$\pm\sqrt{2} = x - \frac{5}{2}$$
$$+\frac{5}{2} \qquad +\frac{5}{2}$$
$$\frac{5}{2} \pm \sqrt{2} = x$$

The solutions of the equation are $x = \frac{5}{2} + \sqrt{2} \approx 3.91$ and $x = \frac{5}{2} - \sqrt{2} \approx 1.09$. It is not possible for the width of the border to be 3.91 feet because the width of the door is only 3 feet. So, use 1.09 feet.

$$1.09 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = 13.08 \text{ in.}$$

The width of the border should be about 13 inches.

9.4 Exercises (pp. 511–514)

Vocabulary and Core Concept Check

1. The process of adding a constant c to the expression $x^2 + bx$ so that $x^2 + bx + c$ is a perfect square trinomial is called completing the square.

2. To complete the square for an expression of the form $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$ to $x^2 + bx$.

3. When b is even, $\frac{b}{2}$ is an integer. So, it is more convenient to complete the square for $x^2 + bx$ when b is even.

4. In order to find the maximum or minimum value of a quadratic function, first use completing the square to write the function in vertex form. Then the maximum or minimum is the y-coordinate of the vertex.

Monitoring Progress and Modeling with Mathematics

5. $x^2 - 8x + c$

$c = \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$

6. $x^2 - 2x + c$

$c = \left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$

7. $x^2 + 4x + c$

$c = \left(\frac{4}{2}\right)^2 = 2^2 = 4$

8. $x^2 + 12x + c$

$c = \left(\frac{12}{2}\right)^2 = 6^2 = 36$

9. $x^2 - 15x + c$

$c = \left(\frac{-15}{2}\right)^2 = \frac{225}{4}$

10. $x^2 + 9x + c$

$c = \left(\frac{9}{2}\right)^2 = \frac{81}{4}$

11. $x^2 - 10x$

Step 1 $\frac{b}{2} = \frac{-10}{2} = -5$

Step 2 $(-5)^2 = 25$

Step 3 $x^2 - 10x + 25$

So, $x^2 - 10x + 25 = (x - 5)^2$.

12. $x^2 - 40x$

Step 1 $\frac{b}{2} = \frac{-40}{2} = -20$

Step 2 $(-20)^2 = 400$

Step 3 $x^2 - 40x + 400$

So, $x^2 - 40x + 400 = (x - 20)^2$.

13. $x^2 + 16x$

Step 1 $\frac{b}{2} = \frac{16}{2} = 8$

Step 2 $8^2 = 64$

Step 3 $x^2 + 16x + 64$

So, $x^2 + 16x + 64 = (x + 8)^2$.

14. $x^2 + 22x$

Step 1 $\frac{b}{2} = \frac{22}{2} = 11$

Step 2 $11^2 = 121$

Step 3 $x^2 + 22x + 121$

So, $x^2 + 22x + 121 = (x + 11)^2$.

15. $x^2 + 5x$

Step 1 $\frac{b}{2} = \frac{5}{2}$

Step 2 $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

Step 3 $x^2 + 5x + \frac{25}{4}$

So, $x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$.

Chapter 9

16. $x^2 - 3x$

Step 1 $\frac{b}{2} = \frac{-3}{2} = -\frac{3}{2}$

Step 2 $\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$

Step 3 $x^2 - 3x + \frac{9}{4}$

So, $x^2 - 3x + \frac{9}{4} = \left(x - \frac{3}{2}\right)^2$.

17.
$$\begin{aligned} x^2 + 14x &= 15 \\ x^2 + 14x + 7^2 &= 15 + 7^2 \\ (x+7)^2 &= 15 + 49 \\ (x+7)^2 &= 64 \\ \sqrt{(x+7)^2} &= \sqrt{64} \\ x + 7 &= \pm 8 \\ \underline{-7} \quad & \quad \underline{-7} \\ x &= -7 \pm 8 \end{aligned}$$

The solutions are $x = -7 + 8 = 1$ and $x = -7 - 8 = -15$.

18.
$$\begin{aligned} x^2 - 6x &= 16 \\ x^2 - 6x + (-3)^2 &= 16 + (-3)^2 \\ (x-3)^2 &= 16 + 9 \\ (x-3)^2 &= 25 \\ \sqrt{(x-3)^2} &= \sqrt{25} \\ x + 3 &= \pm 5 \\ \underline{+3} \quad & \quad \underline{+3} \\ x &= 3 \pm 5 \end{aligned}$$

The solutions are $x = 3 + 5 = 8$ and $x = 3 - 5 = -2$.

19.
$$\begin{aligned} x^2 - 4x &= -2 \\ x^2 - 4x + (-2)^2 &= -2 + (-2)^2 \\ (x-2)^2 &= -2 + 4 \\ (x-2)^2 &= 2 \\ \sqrt{(x-2)^2} &= \sqrt{2} \\ x - 2 &= \pm\sqrt{2} \\ \underline{+2} \quad & \quad \underline{+2} \\ x &= 2 \pm \sqrt{2} \end{aligned}$$

The solutions are $x = 2 + \sqrt{2} \approx 3.41$ and $x = 2 - \sqrt{2} \approx 0.59$.

20.
$$\begin{aligned} x^2 + 2x &= 5 \\ x^2 + 2x + 1^2 &= 5 + 1^2 \\ (x+1)^2 &= 5 + 1 \\ (x+1)^2 &= 6 \\ \sqrt{(x+1)^2} &= \sqrt{6} \\ x + 1 &= \pm\sqrt{6} \\ \underline{-1} \quad & \quad \underline{-1} \\ x &= -1 \pm \sqrt{6} \end{aligned}$$

The solutions are $x = -1 + \sqrt{6} \approx 1.45$ and $x = -1 - \sqrt{6} \approx -3.45$.

21.
$$\begin{aligned} x^2 - 5x &= 8 \\ x^2 - 5x + \left(-\frac{5}{2}\right)^2 &= 8 + \left(-\frac{5}{2}\right)^2 \\ x - \left(\frac{5}{2}\right)^2 &= 8 + \frac{25}{4} \\ \left(x - \frac{5}{2}\right)^2 &= \frac{57}{4} \\ \sqrt{\left(x - \frac{5}{2}\right)^2} &= \sqrt{\frac{57}{4}} \\ x - \frac{5}{2} &= \pm\sqrt{\frac{57}{4}} \\ \underline{+\frac{7}{2}} \quad & \quad \underline{+\frac{7}{2}} \\ x &= \frac{5}{2} \pm \sqrt{\frac{57}{4}} \\ x &= \frac{5}{2} \pm \frac{\sqrt{57}}{2}, \text{ or } \frac{5 \pm \sqrt{27}}{2} \end{aligned}$$

The solutions are $x = \frac{5 + \sqrt{27}}{2} \approx 6.27$ and $x = \frac{5 - \sqrt{27}}{2} \approx -1.27$.

22.
$$\begin{aligned} x^2 + 11x &= -10 \\ x^2 + 11x + \left(\frac{11}{2}\right)^2 &= -10 + \left(\frac{11}{2}\right)^2 \\ \left(x + \frac{11}{2}\right)^2 &= \frac{81}{4} \\ \sqrt{\left(x + \frac{11}{2}\right)^2} &= \sqrt{\frac{81}{4}} \\ x + \frac{11}{2} &= \pm\frac{9}{2} \\ \underline{-\frac{11}{2}} \quad & \quad \underline{-\frac{11}{2}} \\ x &= -\frac{11}{2} \pm \frac{9}{2}, \text{ or } \frac{-11 \pm 9}{2} \end{aligned}$$

The solutions are $x = \frac{-11 + 9}{2} = \frac{-2}{2} = -1$ and $x = \frac{-11 - 9}{2} = \frac{-20}{2} = -10$.

23. a. Area = length • width

$216 = (x + 6) \cdot x$

$216 = x(x + 6)$

$216 = x(x) + x(6)$

$216 = x^2 + 6x$

An equation that represents the area of the patio is $x^2 + 6x = 216$.

b.
$$x^2 + 6x = 216$$
$$x^2 + 6x + 3^2 = 216 + 3^2$$
$$(x + 3)^2 = 216 + 9$$
$$(x + 3)^2 = 225$$
$$\sqrt{(x + 3)^2} = \sqrt{225}$$
$$x + 3 = \pm 15$$
$$\underline{-3} \quad \underline{-3}$$
$$x = -3 \pm 15$$

The solutions of the equation are $x = -3 + 15 = 12$ and $x = -3 - 15 = -18$. Disregard the negative value because a negative side length does not make sense. So, the width of the patio is 12 feet and the length is $12 + 6 = 18$ feet.

24. a. Volume = length • width • height
$$768 = x \cdot 1 \cdot (x - 8)$$
$$768 = x(x - 8)$$
$$768 = x(x) - x(8)$$
$$768 = x^2 - 8x$$

An equation that represents the volume of the glass case is $x^2 - 8x = 768$.

b.
$$x^2 - 8x = 768$$
$$x^2 - 8x + (-4)^2 = 768 + (-4)^2$$
$$(x - 4)^2 = 768 + 16$$
$$(x - 4)^2 = 784$$
$$\sqrt{(x - 4)^2} = \sqrt{784}$$
$$x - 4 = \pm 28$$
$$\underline{+4} \quad \underline{+4}$$
$$x = 4 \pm 28$$

The solutions of the equation are $x = 4 + 28 = 32$ and $x = 4 - 28 = -24$. Disregard the negative solution because a negative side length does not make sense. So, the length of the glass case is 32 centimeters and the height is $32 - 8 = 24$ centimeters.

25.
$$x^2 - 8x + 15 = 0$$
$$\underline{-15} \quad \underline{-15}$$
$$x^2 - 8x = -15$$
$$x^2 - 8x + (-4)^2 = -15 + (-4)^2$$
$$(x - 4)^2 = -15 + 16$$
$$(x - 4)^2 = 1$$
$$\sqrt{(x - 4)^2} = \sqrt{1}$$
$$x - 4 = \pm 1$$
$$\underline{+4} \quad \underline{+4}$$
$$x = 4 \pm 1$$

The solutions are $x = 4 + 1 = 5$ and $x = 4 - 1 = 3$.

26.
$$x^2 + 4x - 21 = 0$$
$$\underline{+21} \quad \underline{+21}$$
$$x^2 + 4x = 21$$
$$x^2 + 4x + 2^2 = 21 + 2^2$$
$$(x + 2)^2 = 21 + 4$$
$$(x + 2)^2 = 25$$
$$\sqrt{(x + 2)^2} = \sqrt{25}$$
$$x + 2 = \pm 5$$
$$\underline{-2} \quad \underline{-2}$$
$$x = -2 \pm 5$$

The solutions are $x = -2 + 5 = 3$ and $x = -2 - 5 = -7$.

27.
$$2x^2 + 20x + 44 = 0$$
$$\frac{2x^2 + 20x + 44}{2} = \frac{0}{2}$$
$$x^2 + 10x + 22 = 0$$
$$\underline{-22} \quad \underline{-22}$$
$$x^2 + 10x = -22$$
$$x^2 + 10x + 5^2 = -22 + 5^2$$
$$(x + 5)^2 = -22 + 25$$
$$(x + 5)^2 = 3$$
$$\sqrt{(x + 5)^2} = \sqrt{3}$$
$$x + 5 = \pm\sqrt{3}$$
$$\underline{-5} \quad \underline{-5}$$
$$x = -5 \pm \sqrt{3}$$

The solutions are $x = -5 + \sqrt{3} \approx -3.27$ and $x = -5 - \sqrt{3} \approx -6.73$.

28.
$$3x^2 - 18x + 12 = 0$$
$$\frac{3x^2 - 18x + 12}{3} = \frac{0}{3}$$
$$x^2 - 6x + 4 = 0$$
$$\underline{-4} \quad \underline{-4}$$
$$x^2 - 6x = -4$$
$$x^2 - 6x + (-3)^2 = -4 + (-3)^2$$
$$(x - 3)^2 = -4 + 9$$
$$(x - 3)^2 = 5$$
$$\sqrt{(x - 3)^2} = \sqrt{5}$$
$$x - 3 = \pm\sqrt{5}$$
$$\underline{+3} \quad \underline{+3}$$
$$x = 3 \pm \sqrt{5}$$

The solutions are $x = 3 + \sqrt{5} \approx 5.24$ and $x = 3 - \sqrt{5} \approx 0.76$.

Chapter 9

29. $-3x^2 - 24x + 17 = -40$

$$\begin{aligned}
-3x^2 - 24x + 17 &= -40 \\
\underline{-\ 17} \quad & \underline{-\ 17} \\
-3x^2 - 24x &= -57 \\
\frac{-3x^2 - 24x}{-3} &= \frac{-57}{-3} \\
x^2 + 8x &= 19 \\
x^2 + 8x + 4^2 &= 19 + 4^2 \\
(x+4)^2 &= 19 + 16 \\
(x+4)^2 &= 35 \\
\sqrt{(x+4)^2} &= \sqrt{35} \\
x + 4 &= \pm\sqrt{35} \\
\underline{-\ 4} \quad & \underline{-\ 4} \\
x &= -4 \pm \sqrt{35}
\end{aligned}$$

The solutions are $x = -4 + \sqrt{35} \approx 1.92$ and $x = -4 - \sqrt{35} \approx -9.92$

30. $-5x^2 - 20x + 35 = 30$

$$\begin{aligned}
-5x^2 - 20x + 35 &= 30 \\
\underline{-\ 35} \quad & \underline{-\ 35} \\
-5x^2 - 20x &= -5 \\
\frac{-5x^2 - 20x}{-5} &= \frac{-5}{-5} \\
x^2 + 4x &= 1 \\
x^2 + 4x + 2^2 &= 1 + 2^2 \\
(x+2)^2 &= 1 + 4 \\
(x+2)^2 &= 5 \\
\sqrt{(x+2)^2} &= \sqrt{5} \\
x + 2 &= \pm\sqrt{5} \\
\underline{-\ 2} \quad & \underline{-\ 2} \\
x &= -2 \pm \sqrt{5}
\end{aligned}$$

The solutions are $x = -2 + \sqrt{5} \approx 0.24$ and $x = -2 - \sqrt{5} \approx -4.24$.

31. $2x^2 - 14x + 10 = 26$

$$\begin{aligned}
2x^2 - 14x + 10 &= 26 \\
\underline{-\ 10} \quad & \underline{-\ 10} \\
2x^2 - 14x &= 16 \\
\frac{2x^2 - 14x}{2} &= \frac{16}{2} \\
x^2 - 7x &= 8 \\
x^2 - 7x + \left(-\frac{7}{2}\right)^2 &= 8 + \left(-\frac{7}{2}\right)^2 \\
\left(x - \frac{7}{2}\right)^2 &= 8 + \frac{49}{4} \\
\left(x - \frac{7}{2}\right)^2 &= \frac{81}{4} \\
\sqrt{\left(x - \frac{7}{2}\right)^2} &= \sqrt{\frac{81}{4}} \\
x - \frac{7}{2} &= \pm\frac{9}{2} \\
\underline{+\frac{7}{2}} \quad & \underline{+\frac{7}{2}} \\
x &= \frac{7}{2} \pm \frac{9}{2}, \text{ or } \frac{7 \pm 9}{2}
\end{aligned}$$

The solutions are $x = \frac{7 + 9}{2} = \frac{16}{2} = 8$ and $x = \frac{7 - 9}{2} = \frac{-2}{2} = -1$.

32. $4x^2 + 12x - 15 = 5$

$$\begin{aligned}
4x^2 + 12x - 15 &= 5 \\
\underline{+\ 15} \quad & \underline{+\ 15} \\
4x^2 + 12x &= 20 \\
\frac{4x^2 + 12x}{4} &= \frac{20}{4} \\
x^2 + 3x &= 5 \\
x^2 + 3x + \left(\frac{3}{2}\right)^2 &= 5 + \left(\frac{3}{2}\right)^2 \\
\left(x + \frac{3}{2}\right)^2 &= 5 + \frac{9}{4} \\
\left(x + \frac{3}{2}\right)^2 &= \frac{29}{4} \\
\sqrt{\left(x + \frac{3}{2}\right)^2} &= \sqrt{\frac{29}{4}} \\
x + \frac{3}{2} &= \pm\sqrt{\frac{29}{2}} \\
\underline{-\frac{3}{2}} \quad & \underline{-\frac{3}{2}} \\
x &= -\frac{3}{2} \pm \frac{\sqrt{29}}{2} \text{ or } \frac{-3 \pm \sqrt{29}}{2}
\end{aligned}$$

The solutions are $x = \frac{-3 + \sqrt{29}}{2} \approx 1.19$ and $x = \frac{-3 - \sqrt{29}}{2} \approx -4.19$.

Chapter 9

33. The number $4^2 = 16$ should be added to each side of the equation.

$$\begin{aligned} x^2 + 8x &= 10 \\ x^2 + 8x + 4^2 &= 10 + 4^2 \\ (x+4)^2 &= 10 + 16 \\ (x+4)^2 &= 26 \\ \sqrt{(x+4)^2} &= \sqrt{26} \\ x + 4 &= \pm\sqrt{26} \\ \underline{-4} \quad & \quad \underline{-4} \\ x &= -4 \pm \sqrt{26} \end{aligned}$$

The solutions are $x = -4 + \sqrt{26} \approx 1.1$ and $x = -4 - \sqrt{26} \approx -9.1$.

34. The leading coefficient should be 1 before completing the square.

$$\begin{aligned} 2x^2 - 2x - 4 &= 0 \\ \underline{+4} \quad & \quad \underline{+4} \\ 2x^2 - 2x &= 4 \\ \frac{2x^2 - 2x}{2} &= \frac{4}{2} \\ x^2 - x &= 2 \\ x^2 - x + \left(-\frac{1}{2}\right)^2 &= 2 + \left(-\frac{1}{2}\right)^2 \\ \left(x - \frac{1}{2}\right)^2 &= 2 + \frac{1}{4} \\ \left(x - \frac{1}{2}\right)^2 &= \frac{9}{4} \end{aligned}$$

35. In a perfect square trinomial, $c = \left(\frac{b}{2}\right)^2$.

$$\begin{aligned} \text{So, } c &= \left(\frac{b}{2}\right)^2 \\ 25 &= \left(\frac{b}{2}\right)^2 \\ \sqrt{25} &= \sqrt{\left(\frac{b}{2}\right)^2} \\ \pm 5 &= \frac{b}{2} \\ 2 \cdot (\pm 5) &= 2 \cdot \frac{b}{2} \\ \pm 10 &= b \end{aligned}$$

The values of b for which $x^2 + bx + 25$ is a perfect square trinomial are $b = 10$ and $b = -10$.

36. The first step is to divide each side of the equation by 3.

37. D;

$$\begin{aligned} y &= x^2 + 6x + 3 \\ y - 3 &= x^2 + 6x + 3 - 3 \\ y - 3 &= x^2 + 6x \\ y - 3 + 3^2 &= x^2 + 6x + 3^2 \\ y - 3 + 9 &= (x+3)^2 \\ y + 6 &= (x+3)^2 \\ y + 6 - 6 &= (x+3)^2 - 6 \\ y &= (x+3)^2 - 6 \end{aligned}$$

The graph of the function has a vertex of $(-3, -6)$ and opens up. So, it matches graph D.

38. A;

$$\begin{aligned} y &= -x^2 + 8x - 12 \\ y + 12 &= -x^2 + 8x - 12 + 12 \\ y + 12 &= -x^2 + 8x \\ y + 12 &= -(x^2 - 8x) \\ y + 12 - (-4)^2 &= -[x^2 - 8x + (-4)^2] \\ y + 12 - 16 &= -(x-4)^2 \\ y - 4 &= -(x-4)^2 \\ y - 4 + 4 &= -(x-4)^2 + 4 \\ y &= -(x-4)^2 + 4 \end{aligned}$$

The graph of the function has a vertex of $(4, 4)$ and opens down. So, it matches graph A.

39. B;

$$\begin{aligned} y &= -x^2 - 4x - 2 \\ y + 2 &= -x^2 - 4x - 2 + 2 \\ y + 2 &= -x^2 - 4x \\ y + 2 &= -(x^2 + 4x) \\ y + 2 - 2^2 &= -(x^2 + 4x + 2^2) \\ y + 2 - 4 &= -(x+2)^2 \\ y - 2 &= -(x+2)^2 \\ y - 2 + 2 &= -(x+2)^2 + 2 \\ y &= -(x+2)^2 + 2 \end{aligned}$$

The graph of the function has a vertex of $(-2, 2)$ and opens down. So, it matches graph B.

40. C;

$$\begin{aligned} y &= x^2 - 2x + 4 \\ y - 4 &= x^2 - 2x + 4 - 4 \\ y - 4 &= x^2 - 2x \\ y - 4 + (-1)^2 &= x^2 - 2x + (-1)^2 \\ y - 4 + 1 &= (x-1)^2 \\ y - 3 &= (x-1)^2 \\ y - 3 + 3 &= (x-1)^2 + 3 \\ y &= (x-1)^2 + 3 \end{aligned}$$

The graph of the function has a vertex of $(1, 3)$ and opens up. So, it matches graph C.

41.

$$
\begin{aligned}
y &= x^2 - 4x - 2\\
y + 2 &= x^2 - 4x - 2 + 2\\
y + 2 &= x^2 - 4x\\
y + 2 + (-2)^2 &= x^2 - 4x + (-2)^2\\
y + 2 + 4 &= (x - 2)^2\\
y + 6 &= (x - 2)^2\\
y + 6 - 6 &= (x - 2)^2 - 6\\
y &= (x - 2)^2 - 6
\end{aligned}
$$

The vertex is $(2, -6)$. Because a is positive $(a = 1)$, the parabola opens up and the y-coordinate of the vertex is a minimum value. So, the function has a minimum value of -6.

42.

$$
\begin{aligned}
y &= x^2 + 6x + 10\\
y - 10 &= x^2 + 6x + 10 - 10\\
y - 10 &= x^2 + 6x\\
y - 10 + 3^2 &= x^2 + 6x + 3^2\\
y - 10 + 9 &= (x + 3)^2\\
y - 1 &= (x + 3)^2\\
y - 1 + 1 &= (x + 3)^2 + 1\\
y &= (x + 3)^2 + 1
\end{aligned}
$$

The vertex is $(-3, 1)$. Because a is positive $(a = 1)$, the parabola opens up and the y-coordinate of the vertex is a minimum value. So, the function has a minimum value of 1.

43.

$$
\begin{aligned}
y &= -x^2 - 10x - 30\\
y + 30 &= -x^2 - 10x - 30 + 30\\
y + 30 &= -x^2 - 10x\\
y + 30 &= -(x^2 + 10x)\\
y + 30 - 5^2 &= -(x^2 + 10x + 5^2)\\
y + 30 - 25 &= -(x + 5)^2\\
y + 5 &= -(x + 5)^2\\
y + 5 - 5 &= -(x + 5)^2 - 5\\
y &= -(x + 5)^2 - 5
\end{aligned}
$$

The vertex is $(-5, -5)$. Because a is negative $(a = -1)$, the parabola opens down and the y-coordinate of the vertex is a maximum value. So, the function has a maximum value of -5.

44.

$$
\begin{aligned}
y &= -x^2 + 14x - 34\\
y + 34 &= -x^2 + 14x - 34 + 34\\
y + 34 &= -x^2 + 14x\\
y + 34 &= -(x^2 - 14x)\\
y + 34 - (-7)^2 &= -[x^2 - 14x + (-7)^2]\\
y + 34 - 49 &= -(x - 7)^2\\
y - 15 &= -(x - 7)^2\\
y - 15 + 15 &= -(x - 7)^2 + 15\\
y &= -(x - 7)^2 + 15
\end{aligned}
$$

The vertex is $(7, 15)$. Because a is negative $(a = -1)$, the parabola opens down and the y-coordinate of the vertex is a maximum value. So, the function has a maximum value of 15.

45.

$$
\begin{aligned}
f(x) &= -3x^2 - 6x - 9\\
f(x) + 9 &= -3x^2 - 6x - 9 + 9\\
f(x) + 9 &= -3x^2 - 6x\\
f(x) + 9 &= -3(x^2 + 2x)\\
f(x) + 9 - 3 \cdot 1^2 &= -3(x^2 + 2x + 1^2)\\
f(x) + 9 - 3 &= -3(x + 1)^2\\
f(x) + 6 &= -3(x + 1)^2\\
f(x) + 6 - 6 &= -3(x + 1)^2 - 6\\
f(x) &= -3(x + 1)^2 - 6
\end{aligned}
$$

The vertex is $(-1, -6)$. Because a is negative $(a = -3)$, the parabola opens down and the y-coordinate of the vertex is a maximum value. So, the function has a maximum value of -6.

46.

$$
\begin{aligned}
f(x) &= 4x^2 - 28x + 32\\
f(x) - 32 &= 4x^2 - 28x + 32 - 32\\
f(x) - 32 &= 4x^2 - 28x\\
f(x) - 32 &= 4(x^2 - 7x)\\
f(x) - 32 + 4 \cdot \left(-\tfrac{7}{2}\right)^2 &= 4\left[x^2 - 7x + \left(-\tfrac{7}{2}\right)^2\right]\\
f(x) - 32 + 49 &= 4\left(x - \tfrac{7}{2}\right)^2\\
f(x) + 17 &= 4\left(x - \tfrac{7}{2}\right)^2\\
f(x) + 17 - 17 &= 4\left(x - \tfrac{7}{2}\right)^2 - 17\\
f(x) &= 4\left(x - \tfrac{7}{2}\right)^2 - 17
\end{aligned}
$$

The vertex is $\left(\frac{7}{2}, -17\right)$. Because a is positive $(a = 1)$, the parabola opens up and the y-coordinate of the vertex is a minimum value. So, the function has a minimum value of -17.

47. The graph of the function $y = -(x + 8)(x + 3)$ has two negative x-intercepts and opens down because $a < 0$. This means that the function has a maximum value and the vertex must be in the second quadrant. So, the graph could represent the function.

48. The graph of the function $y = (x - 5)^2$ has an x-intercept of 5, but the graph shown has a negative x-intercept. So, the graph does not represent the function.

49. The graph of the function $y = \frac{1}{4}(x + 2)^2 - 4$ opens up because $a > 0$, which means the function has a minimum value. However, the vertex $(-2, -4)$ of the graph of the function is in the third quadrant. So, the graph does not represent the function.

50. The graph of the function $y = -2(x - 1)(x + 2)$ has one positive x-intercept and one negative x-intercept and opens down because $a < 0$. This means that the function has a maximum value, and the vertex could be in the first or second quadrant. So, the graph could represent the function.

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51. The graph of h opens up because $a > 0$, which means h has a minimum value. However, the vertex $(-2, 3)$ of the graph of h is in the second quadrant. So, the graph does not represent h.

The graph of f opens up because $a > 0$, which means f has a minimum value. The vertex $(-3, -2)$ of the graph of f is in the third quadrant.

$$\begin{aligned}
0 &= 2(x+3)^2 - 2 \\
0 + 2 &= 2(x+3)^2 - 2 + 2 \\
2 &= 2(x+3)^2 \\
\frac{2}{2} &= \frac{2(x+3)^2}{2} \\
1 &= (x+3)^2 \\
\sqrt{1} &= \sqrt{(x+3)^2} \\
\pm 1 &= x + 3 \\
\pm 1 - 3 &= x + 3 - 3 \\
-3 \pm 1 &= x
\end{aligned}$$

By solving $0 = 2(x + 3)^2 - 2$, you see that the x-intercepts of the graph of f are $-3 + 1 = -2$ and $-3 - 1 = -4$. So, the graph could represent f.

The graph of g has two positive x-intercepts and opens down because $a < 0$. So, the graph does not represent g.

The graph of m has two negative x-intercepts and opens up because $a > 0$. This means that m has a minimum value and the vertex must be in the third quadrant. So, the graph could represent m.

So, the graph could represent function f or function m.

52. The graph of r has one positive x-intercept and one negative x-intercept and opens down because $a < 0$. This means that r has a maximum value and the vertex must be in the first or second quadrant. So, the graph could represent r.

The graph of p opens down because $a < 0$, which means p has a maximum value. However, the graph of p has two positive x-intercepts. So, the graph does not represent p.

The graph of q opens up because $a > 0$, which means n has a minimum value, and the vertex $(-1, 4)$ of the graph of q is in the second quadrant. So, the graph does not represent q.

The graph of n opens down because $a < 0$, which means n has a maximum value. The vertex $(2, 9)$ of the graph of n is in the first quadrant.

$$\begin{aligned}
0 &= -(x-2)^2 + 9 \\
0 - 9 &= -(x-2)^2 + 9 - 9 \\
-9 &= -(x-2)^2 \\
\frac{-9}{-1} &= \frac{-(x-2)^2}{-1} \\
9 &= (x-2)^2 \\
\sqrt{9} &= \sqrt{(x-2)^2} \\
\pm 3 &= x - 2 \\
\pm 3 + 2 &= x - 2 + 2 \\
2 \pm 3 &= x
\end{aligned}$$

By solving $0 = -(x - 2)^2 + 4$, you see that the x-intercepts of the graph of n are $2 + 3 = 5$ and $2 - 3 = -1$. So, the graph could represent n.

So, the graph could represent function r or function n.

53. a.

$$\begin{aligned}
h &= -16t^2 + 48t \\
h &= -16(t^2 - 3t) \\
h - 16\left(-\tfrac{3}{2}\right)^2 &= -16\left[t^2 - 3t + \left(-\tfrac{3}{2}\right)^2\right] \\
h - 16\left(\tfrac{9}{4}\right) &= -16\left(t - \tfrac{3}{2}\right)^2 \\
h - 36 &= -16\left(t - \tfrac{3}{2}\right)^2 \\
h - 36 + 36 &= -16\left(t - \tfrac{3}{2}\right)^2 + 36 \\
h &= -16\left(t - \tfrac{3}{2}\right)^2 + 36
\end{aligned}$$

Because the maximum value is 36, the kickball reaches a maximum height of 36 feet.

b. The vertex is $\left(\frac{3}{2}, 36\right)$. So, the axis of symmetry is $x = \frac{3}{2}$. On the left side of $x = \frac{3}{2}$, the height increases as time increases. On the right side of $x = \frac{3}{2}$, the height decreases as time increases.

54. a.

$$\begin{aligned}
h &= -16t^2 + 32t + 16 \\
h - 16 &= -16t^2 + 32t + 16 - 16 \\
h - 16 &= -16t^2 + 32t \\
h - 16 &= -16(t^2 - 2t) \\
h - 16 - 16(-1)^2 &= -16(t^2 - 2t + (-1)^2) \\
h - 16 - 16 &= -16(t-1)^2 \\
h - 32 &= -16(t-1)^2 \\
h - 32 + 32 &= -16(t-1)^2 + 32 \\
h &= -16(t-1)^2 + 32
\end{aligned}$$

Because the maximum value is 32, the stone reaches a maximum height of 32 feet.

b. The vertex is $(1, 32)$. So, the axis of symmetry is $x = 1$. On the left side of $x = 1$, the height increases as time increases. On the right side of $x = 1$, the height decreases as time increases.

55.

$$\boxed{\begin{array}{c}\text{Area of brick}\\\text{patio}\\\text{(square feet)}\end{array}} = \boxed{\begin{array}{c}\text{Length}\\\text{of brick}\\\text{patio(feet)}\end{array}} \cdot \boxed{\begin{array}{c}\text{Width of}\\\text{brick patio}\\\text{(feet)}\end{array}}$$

$$\begin{aligned}
140 &= (20 - 2x) \cdot (16 - 2x)\\
140 &= (20 - 2x)(16 - 2x)\\
140 &= 20(16) + 20(-2x) - 2x(16) - 2x(-2x)\\
140 &= 320 - 40x - 32x + 4x^2\\
140 &= 4x^2 - 72x + 320\\
140 - 320 &= 4x^2 - 72x + 320 - 320\\
-180 &= 4x^2 - 72x\\
\frac{-180}{4} &= \frac{4x^2 - 72x}{4}\\
-45 &= x^2 - 18x\\
-45 + (-9)^2 &= x^2 - 18x + (-9)^2\\
-45 + 81 &= (x - 9)^2\\
36 &= (x - 9)^2\\
\sqrt{36} &= \sqrt{(x - 9)^2}\\
\pm 6 &= x - 9\\
\underline{+9} & \quad \underline{+9}\\
9 \pm 6 &= x
\end{aligned}$$

The solutions of the equation are $x = 9 + 6 = 15$ and $x = 9 - 6 = 3$. It is not possible for the width of the crushed stone border to be 15 feet because the width of the whole patio area is 16 feet. So, the width of the crushed stone border is 3 feet.

56.

$$\boxed{\begin{array}{c}\text{Total area}\\\text{of the poster}\\\text{(square inches)}\end{array}} = \boxed{\begin{array}{c}\text{Length of}\\\text{poster with}\\\text{border (inches)}\end{array}} \cdot \boxed{\begin{array}{c}\text{Width of}\\\text{poster with}\\\text{border (inches)}\end{array}}$$

$$\begin{aligned}
722 &= (28 + 2x) \cdot (22 + 2x)\\
722 &= (28 + 2x)(22 + 2x)\\
722 &= 28(22) + 28(2x) + 2x(22) + 2x(2x)\\
722 &= 616 + 56x + 44x + 4x^2\\
722 &= 4x^2 + 100x + 616\\
722 - 616 &= 4x^2 + 100x + 616 - 616\\
106 &= 4x^2 + 100x\\
\frac{106}{4} &= \frac{4x^2 + 100x}{4}\\
\frac{53}{2} &= x^2 + 25x\\
\frac{53}{2} + \left(\frac{25}{2}\right)^2 &= x^2 + 25x + \left(\frac{25}{2}\right)^2\\
\frac{53}{2} + \frac{625}{4} &= \left(x + \frac{25}{2}\right)^2\\
\frac{731}{4} &= \left(x + \frac{25}{2}\right)^2\\
\sqrt{\frac{731}{4}} &= \sqrt{\left(x + \frac{25}{2}\right)^2}\\
\frac{\pm\sqrt{731}}{2} &= x + \frac{25}{2}\\
\underline{-\frac{25}{2}} & \quad \underline{-\frac{25}{2}}\\
-\frac{25}{2} \pm \frac{\sqrt{731}}{2} &= x\\
\frac{-25 \pm \sqrt{731}}{2} &= x
\end{aligned}$$

The solutions of the equation are $x = \frac{-25 + \sqrt{731}}{2} \approx 1.02$ and $x = \frac{-25 - \sqrt{731}}{2} \approx -26.02$. It is not possible for the width of the border to be negative. So, the width of the border is about 1 inch.

Chapter 9

57.

$$\begin{aligned}
\text{Area} &= \tfrac{1}{2} \cdot \text{base} \cdot \text{height} \\
108 &= \tfrac{1}{2} \cdot (x + 6) \cdot x \\
108 &= \tfrac{1}{2}x(x + 6) \\
2 \cdot 108 &= 2 \cdot \tfrac{1}{2}x(x + 6) \\
216 &= x(x + 6) \\
216 &= x(x) + x(6) \\
216 &= x^2 + 6x \\
216 + 3^2 &= x^2 + 6x + 3^2 \\
216 + 9 &= (x + 3)^2 \\
225 &= (x + 3)^2 \\
\sqrt{225} &= \sqrt{(x + 3)^2} \\
\pm 15 &= x + 3 \\
\underline{-3} & \quad \underline{-3} \\
-3 \pm 15 &= x
\end{aligned}$$

The solutions of the equation are $x = -3 + 15 = 12$ and $x = -3 - 15 = -18$. Disregard the negative solution, because a negative height does not make sense in this situation. So, the value of x is 12.

58.

$$\begin{aligned}
\text{Area} &= \text{length} \cdot \text{width} \\
288 &= (2x + 10) \cdot (3x) \\
288 &= 3x(2x + 10) \\
288 &= 3x(2x) + 3x(10) \\
288 &= 6x^2 + 30x \\
\frac{288}{6} &= \frac{6x^2 + 30x}{6} \\
48 &= x^2 + 5x \\
48 + \left(\frac{5}{2}\right)^2 &= x^2 + 5x + \left(\frac{5}{2}\right)^2 \\
48 + \frac{25}{4} &= \left(x + \frac{5}{2}\right)^2 \\
\frac{217}{4} &= \left(x + \frac{5}{2}\right)^2 \\
\sqrt{\frac{217}{4}} &= \sqrt{\left(x + \frac{5}{2}\right)^2} \\
\frac{\pm\sqrt{217}}{2} &= x + \frac{5}{2} \\
\underline{-\frac{5}{2}} & \quad \underline{-\frac{5}{2}} \\
-\frac{5}{2} \pm \frac{\sqrt{217}}{2} &= x \\
\frac{-5 \pm \sqrt{217}}{2} &= x
\end{aligned}$$

The solutions of the equation are $x = \frac{-5 + \sqrt{217}}{2} \approx 4.87$ and $x = \frac{-5 - \sqrt{217}}{2} \approx -9.87$. Disregard the negative solution because $3(-9.87) \approx -29.6$ does not make sense as the width of the rectangle. So, the value of x is about 4.87.

59.

$$\begin{aligned}
0.5x^2 + x - 2 &= 0 \\
0.5x^2 + x - 2 + 2 &= 0 + 2 \\
0.5x^2 + x &= 2 \\
\frac{0.5x^2 + x}{0.5} &= \frac{2}{0.5} \\
x^2 + 2x &= 4 \\
x^2 + 2x + 1^2 &= 4 + 1^2 \\
(x + 1)^2 &= 4 + 1 \\
(x + 1)^2 &= 5 \\
\sqrt{(x + 1)^2} &= \sqrt{5} \\
x + 1 &= \pm\sqrt{5} \\
\underline{-1} & \quad \underline{-1} \\
x &= -1 \pm \sqrt{5}
\end{aligned}$$

The solutions are $x = -1 + \sqrt{5} \approx 1.24$ and $x = -1 - \sqrt{5} \approx -3.24$.

60.

$$\begin{aligned}
0.75x^2 + 1.5x &= 4 \\
\frac{0.75x^2 + 1.5x}{0.75} &= \frac{4}{0.75} \\
x^2 + 2x &= \frac{16}{3} \\
x^2 + 2x + 1^2 &= \frac{16}{3} + 1^2 \\
(x + 1)^2 &= \frac{19}{3} \\
\sqrt{(x + 1)^2} &= \sqrt{\frac{19}{3}} \\
x + 1 &= \pm\sqrt{\frac{19}{3}} \\
\underline{-1} & \quad \underline{-1} \\
x &= -1 \pm \sqrt{\frac{19}{3}}
\end{aligned}$$

The solutions are $x = -1 + \sqrt{\frac{19}{3}} \approx 1.52$ and $x = -1 - \sqrt{\frac{19}{3}} \approx -3.52$.

Chapter 9

61.
$$\frac{8}{3}x - \frac{2}{3}x^2 = -\frac{5}{6}$$
$$-\frac{3}{2}\left(\frac{8}{3}x\right) - \frac{3}{2}\left(-\frac{2}{3}x^2\right) = -\frac{3}{2}\left(-\frac{5}{6}\right)$$
$$-4x + x^2 = \frac{5}{4}$$
$$x^2 - 4x = \frac{5}{4}$$
$$x^2 - 4x + (-2)^2 = \frac{5}{4} + (-2)^2$$
$$(x-2)^2 = \frac{5}{4} + 4$$
$$(x-2)^2 = \frac{21}{4}$$
$$\sqrt{(x-2)^2} = \sqrt{\frac{21}{4}}$$
$$x - 2 = \pm\frac{\sqrt{21}}{2}$$
$$\underline{+2} \quad \underline{+2}$$
$$x = 2 \pm \frac{\sqrt{21}}{2}$$

The solutions are $x = 2 + \frac{\sqrt{21}}{2} \approx 4.29$ and $x = 2 - \frac{\sqrt{21}}{2} \approx -0.29$.

62.
$$\tfrac{1}{4}x^2 + \tfrac{1}{2}x - \tfrac{5}{4} = 0$$
$$4\left(\tfrac{1}{4}x^2\right) + 4\left(\tfrac{1}{2}x\right) - 4\left(\tfrac{5}{4}\right) = 4(0)$$
$$x^2 + 2x - 5 = 0$$
$$x^2 + 2x - 5 + 5 = 0 + 5$$
$$x^2 + 2x = 5$$
$$x^2 + 2x + 1^2 = 5 + 1^2$$
$$(x+1)^2 = 5 + 1$$
$$(x+1)^2 = 6$$
$$\sqrt{(x+1)^2} = \sqrt{6}$$
$$x + 1 = \pm\sqrt{6}$$
$$\underline{-1} \quad \underline{-1}$$
$$x = -1 \pm \sqrt{6}$$

The solutions are $x = -1 + \sqrt{6} \approx 1.45$ and $x = -1 - \sqrt{6} \approx -3.45$.

63.
$$d = 0.05s^2 + 2.2s$$
$$168 = 0.05s^2 + 2.2s$$
$$\frac{168}{0.05} = \frac{0.05s^2 + 2.2s}{0.05}$$
$$3360 = s^2 + 44s$$
$$3360 + 22^2 = s^2 + 44s + 22^2$$
$$3360 + 484 = (s+22)^2$$
$$3844 = (s+22)^2$$
$$\sqrt{3844} = \sqrt{(s+22)^2}$$
$$\pm 62 = s + 22$$
$$\underline{-22} \quad \underline{-22}$$
$$-22 \pm 62 = s$$

The solutions of the equation are $s = -22 + 62 = 40$ and $s = -22 - 62 = -84$. Disregard the negative solution, because a negative speed does not make sense in this situation. So, the maximum speed at which the car can travel is 40 miles per hour.

64.
$$h = -16t^2 + 24t + 16.4$$
$$16.4 - 3.2 = -16t^2 + 24t + 16.4$$
$$13.2 = -16t^2 + 24t + 16.4$$
$$13.2 - 16.4 = -16t^2 + 24t + 16.4 - 16.4$$
$$-3.2 = -16t^2 + 24t$$
$$\frac{-3.2}{-16} = \frac{-16t^2 + 24t}{-16}$$
$$\frac{1}{5} = t^2 - \frac{3}{2}t$$
$$\frac{1}{5} + \left(-\frac{3}{4}\right)^2 = t^2 - \frac{3}{2}t + \left(-\frac{3}{4}\right)^2$$
$$\frac{1}{5} + \frac{9}{16} = \left(t - \frac{3}{4}\right)^2$$
$$\frac{61}{80} = \left(t - \frac{3}{4}\right)^2$$
$$\sqrt{\frac{61}{80}} = \sqrt{\left(t - \frac{3}{4}\right)^2}$$
$$\pm\sqrt{\frac{61}{80}} = t - \frac{3}{4}$$
$$\underline{+\frac{3}{4}} \quad \underline{+\frac{3}{4}}$$
$$\frac{3}{4} \pm \sqrt{\frac{61}{80}} = t$$

The solutions of the equation are $t = \frac{3}{4} + \sqrt{\frac{61}{80}} \approx 1.62$ and $t = \frac{3}{4} - \sqrt{\frac{61}{80}} \approx -0.12$. Disregard the negative solution because a negative time does not make sense in this situation. So, the snowboarder is in the air for about 1.6 seconds.

Chapter 9

65. a. $\boxed{\begin{matrix}\text{Amount of}\\ \text{fencing (feet)}\end{matrix}} = \boxed{\begin{matrix}\text{Length}\\ \text{(feet)}\end{matrix}} + 2 \bullet \boxed{\begin{matrix}\text{Width}\\ \text{(feet)}\end{matrix}}$

$$80 \qquad = \qquad \ell \qquad + 2 \bullet \qquad w$$

$$80 = \ell + 2w$$

$\boxed{\begin{matrix}\text{Enclosed area}\\ \text{(square feet)}\end{matrix}} = \boxed{\begin{matrix}\text{Length}\\ \text{(feet)}\end{matrix}} \bullet \boxed{\begin{matrix}\text{Width}\\ \text{(feet)}\end{matrix}}$

$$750 \qquad = \qquad \ell \qquad \bullet \qquad w$$

$$750 = \ell w$$

An equation for the amount of fencing to be used is $80 = \ell + 2w$, and an equation for the area enclosed by the fencing is $750 = \ell w$.

b.

$$\begin{aligned} 80 &= \ell + 2w \\ 80 - 2w &= \ell + 2w - 2w \\ 80 - 2w &= \ell \end{aligned}$$

$$\begin{aligned} 750 &= \ell w \\ 750 &= (80 - 2w)w \\ 750 &= 80(w) - 2w(w) \\ 750 &= 80w - 2w^2 \\ 750 &= -2w^2 + 80w \\ \frac{750}{-2} &= \frac{-2w^2 + 80w}{-2} \\ -375 &= w^2 - 40w \\ -375 + (-20)^2 &= w^2 - 40w + (-20)^2 \\ -375 + 400 &= (w - 20)^2 \\ 25 &= (w - 20)^2 \\ \sqrt{25} &= \sqrt{(w - 20)^2} \\ \pm 5 &= w - 20 \\ \underline{+20} &\quad \underline{+20} \\ 20 \pm 5 &= w \end{aligned}$$

The solutions are $w = 20 + 5 = 25$ and $w = 20 - 5 = 15$. So, the pasture can have a width of 25 feet and a length of $80 - 2(25) = 30$ feet, or it can have a width of 15 feet and a length of $80 - 2(15) = 50$ feet.

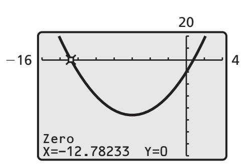
66. a. The x-values for which $y = 3$ are $x \approx 1$ and $x \approx 3$.

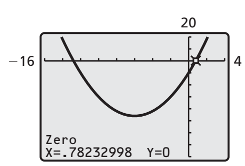
b. You can check your estimates in part (a) by substituting 3 for y in the equation and then solving the equation by completing the square.

67. a.

$$\begin{aligned} x^2 + 12x + 2 &= 12 \\ x^2 + 12x + 2 - 12 &= 12 - 12 \\ x^2 + 12x - 10 &= 0 \end{aligned}$$

Graph $y = x^2 + 12x - 10$.





The solutions are $x \approx -12.8$ and $x \approx 0.8$.

b.

$$\begin{aligned} x^2 + 12x + 2 &= 12 \\ x^2 + 12x + 2 - 2 &= 12 - 2 \\ x^2 + 12x &= 10 \\ x + 12x + 6^2 &= 10 + 6^2 \\ (x + 6)^2 &= 10 + 36 \\ (x + 6)^2 &= 46 \\ \sqrt{(x + 6)^2} &= \sqrt{46} \\ x + 6 &= \pm\sqrt{46} \\ \underline{-6} &\quad \underline{-6} \\ x &= -6 \pm \sqrt{46} \end{aligned}$$

The solutions are $x = -6 + \sqrt{46} \approx 0.78$ and $x = -6 - \sqrt{46} \approx -12.78$.

c. *Sample answer:* Completing the square is preferred because it gives an exact value as well as an approximate value. Graphing can be cumbersome, and unless the solutions are integers, it only gives an approximate value.

68. Let $x = 0$.

$$\begin{aligned} x^2 - 2xy + y^2 - x - y &= 0 \\ 0^2 - 2(0)y + y^2 - 0 - y &= 0 \\ y^2 - y &= 0 \\ y^2 - y + \left(-\tfrac{1}{2}\right)^2 &= 0 + \left(-\tfrac{1}{2}\right)^2 \\ \left(y - \tfrac{1}{2}\right)^2 &= \tfrac{1}{4} \\ \sqrt{\left(y - \tfrac{1}{2}\right)^2} &= \sqrt{\tfrac{1}{4}} \\ y - \tfrac{1}{2} &= \pm\tfrac{1}{2} \\ \underline{+\tfrac{1}{2}} &\quad \underline{+\tfrac{1}{2}} \\ y &= \tfrac{1}{2} \pm \tfrac{1}{2} \end{aligned}$$

$y = \frac{1}{2} + \frac{1}{2} = 1$ and $y = \frac{1}{2} - \frac{1}{2} = 0$

Let $x = 1$.

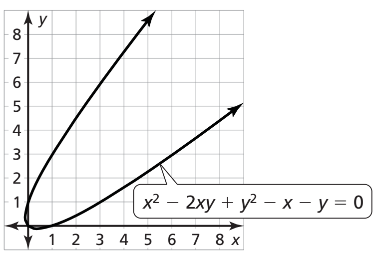
$$\begin{aligned} x^2 - 2xy + y^2 - x - y &= 0 \\ 1^2 - 2(1)y + y^2 - 1 - y &= 0 \\ 1 - 2y + y^2 - 1 - y &= 0 \\ y^2 - 2y - y + 1 - 1 &= 0 \\ y^2 - 3y &= 0 \\ y^2 - 3y + \left(-\tfrac{3}{2}\right)^2 &= 0 + \left(-\tfrac{3}{2}\right)^2 \\ \left(y - \tfrac{3}{2}\right)^2 &= \tfrac{9}{4} \\ \sqrt{\left(y - \tfrac{3}{2}\right)^2} &= \sqrt{\tfrac{9}{4}} \\ y - \tfrac{3}{2} &= \pm\tfrac{3}{2} \\ \underline{+\tfrac{3}{2}} &\quad \underline{+\tfrac{3}{2}} \\ y &= \tfrac{3}{2} \pm \tfrac{3}{2} \end{aligned}$$

$y = \frac{3}{2} + \frac{3}{2} = 3$ and $y = \frac{3}{2} - \frac{3}{2} = 0$

Let $x = 3$.

$$\begin{aligned}
x^2 - 2xy + y^2 - x - y &= 0\\
3^2 - 2(3)y + y^2 - 3 - y &= 0\\
9 - 6y + y^2 - 3 - y &= 0\\
y^2 - 6y - y + 9 - 3 &= 0\\
y^2 - 7y + 6 &= 0\\
y^2 - 7y + 6 - 6 &= 0 - 6\\
y^2 - 7y &= -6\\
y^2 - 7y + 6 - 6 &= 6\\
y^2 - 7y + \left(-\tfrac{7}{2}\right)^2 &= -6 + \left(-\tfrac{7}{2}\right)^2\\
\left(y - \tfrac{7}{2}\right)^2 &= -6 + \tfrac{49}{4}\\
\left(y - \tfrac{7}{2}\right)^2 &= \tfrac{25}{4}\\
\sqrt{\left(y - \tfrac{7}{2}\right)^2} &= \sqrt{\tfrac{25}{4}}\\
y - \tfrac{7}{2} &= \pm \tfrac{5}{2}\\
+\tfrac{7}{2} \quad & \quad +\tfrac{7}{2}\\
y &= \tfrac{7}{2} \pm \tfrac{5}{2}
\end{aligned}$$

$y = \dfrac{7 + 5}{2} = 6$ and $y = \dfrac{7 - 5}{2} = 1$



The graph of $x^2 - 2xy + y^2 - x - y = 0$ is a parabola at an angle.

69. Let n and $n + 2$ be two consecutive even integers.

$$\begin{aligned}
n(n + 2) &= 48\\
n(n) + n(2) &= 48\\
n^2 + 2n &= 48\\
n^2 + 2n + 1^2 &= 48 + 1^2\\
(n + 1)^2 &= 48 + 1\\
(n + 1)^2 &= 49\\
\sqrt{(n + 1)^2} &= \sqrt{49}\\
n + 1 &= \pm 7\\
-1 \quad & \quad -1\\
n &= -1 \pm 7
\end{aligned}$$

The solutions of the equation are $n = -1 + 7 = 6$ and $n = -1 - 7 = -8$. Disregard the negative value because the question asks for positive integers. So, the integers are 6 and $6 + 2 = 8$.

70. Let n and $n + 2$ be two consecutive odd integers.

$$\begin{aligned}
n(n + 2) &= 195\\
n(n) + n(2) &= 195\\
n^2 + 2n &= 195\\
n^2 + 2n + 1^2 &= 195 + 1^2\\
(n + 1)^2 &= 195 + 1\\
(n + 1)^2 &= 196\\
\sqrt{(n + 1)^2} &= \sqrt{196}\\
n + 1 &= \pm 14\\
-1 \quad & \quad -1\\
n &= -1 \pm 14
\end{aligned}$$

The solutions of the equation are $n = -1 + 14 = 13$ and $n = -1 - 14 = -15$. Disregard the positive solution because the question asks for negative integers. So, the integers are -15 and $-15 + 2 = -13$.

71. Substitute 23.50 for y in the model.

$$\begin{aligned}
y &= -0.025x^2 + x + 16\\
23.5 &= -0.025x^2 + x + 16\\
23.5 - 16 &= -0.025x^2 + x + 16 - 16\\
7.5 &= -0.025x^2 + x\\
\frac{7.5}{-0.025} &= \frac{-0.025x^2 + x}{-0.025}\\
-300 &= x^2 - 40x\\
-300 + (-20)^2 &= x^2 - 40x + (-20)^2\\
-300 + 400 &= (x - 20)^2\\
100 &= (x - 20)^2\\
\sqrt{100} &= \sqrt{(x - 20)^2}\\
\pm 10 &= x - 20\\
+20 \quad & \quad +20\\
20 \pm 10 &= x
\end{aligned}$$

The solutions are $x = 20 + 10 = 30$ and $x = 20 - 10 = 20$. So, the stock is worth \$23.50 ten days and 30 days after it is purchased. So, you could have sold the stock earlier for \$23.50 per share.

72. Factoring does not work for this equation, and graphing does not produce an exact solution.

Chapter 9

73. Let x be how much (in inches) you increase the width of the scarf.

$$\boxed{\begin{array}{c}\text{Area of scarf}\\\text{(square inches)}\end{array}} = \boxed{\begin{array}{c}\text{Length of scarf}\\\text{(inches)}\end{array}} \cdot \boxed{\begin{array}{c}\text{Width of}\\\text{scarf (inches)}\end{array}}$$

$$396 = (60 + 3x) \cdot (4 + x)$$

$$\begin{aligned}
396 &= (60 + 3x)(4 + x)\\
396 &= 60(4) + 60(x) + 3x(4) + 3x(x)\\
396 &= 240 + 60x + 12x + 3x^2\\
396 &= 3x^2 + 72x + 240\\
396 - 240 &= 3x^2 + 72x + 240 - 240\\
156 &= 3x^2 + 72x\\
\frac{156}{3} &= \frac{3x^2 + 72x}{3}\\
52 &= x^2 + 24x\\
52 + 12^2 &= x^2 + 24x + 12^2\\
52 + 144 &= (x + 12)^2\\
196 &= (x + 12)^2\\
\sqrt{196} &= \sqrt{(x + 12)^2}\\
\pm 14 &= x + 12\\
\underline{-12} &\quad \underline{-12}\\
-12 \pm 14 &= x
\end{aligned}$$

The solutions of the equation are $x = -12 + 14 = 2$ and $x = -12 - 14 = -26$. Disregard $x = -26$, because the scarf was 4 inches wide to begin with, and it does not make sense to increase the width by -26 inches. So, your scarf is $4 + 2 = 6$ inches wide and $60 + 3 \cdot 2 = 66$ inches long.

74. If $c < -\left(\frac{b}{2}\right)^2$, then when you add $\left(\frac{b}{2}\right)^2$ to each side of the inequality, you get $\left(\frac{b}{2}\right)^2 + c < 0$. Adding $\left(\frac{b}{2}\right)^2$ to each side of the equation $x^2 + bx = c$ gives

$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$, or $\left(x + \frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$. Because $\left(\frac{b}{2}\right)^2 + c < 0$, $\left(x + \frac{b}{2}\right)^2 < 0$. The square of a real number cannot be negative.

So, the equation has no real solutions.

Maintaining Mathematical Proficiency

75. The first term of the sequence is $a = 10$, and each term is 5 more than the previous term. So, a recursive rule for the sequence is $a_1 = 10$, $a_n = a_{n-1} + 5$.

76. The first term of the sequence is $a_1 = 3$, and each term is twice the previous term. So, a recursive rule for the sequence is $a_1 = 3$, $a_n = 2a_{n-1}$.

77. The first term of the sequence is $a_1 = -20$, and each term is 4 more than the previous term. So, a recursive rule for the sequence is $a_1 = -20$, $a_n = a_{n-1} + 4$.

78. $$\begin{aligned}\sqrt{b^2 - 4ac} &= \sqrt{(-6)^2 - 4(3)(2)}\\ &= \sqrt{36 - 24}\\ &= \sqrt{12}\\ &= \sqrt{4 \cdot 3}\\ &= \sqrt{4} \cdot \sqrt{3}\\ &= 2\sqrt{3}\end{aligned}$$

79. $$\begin{aligned}\sqrt{b^2 - 4ac} &= \sqrt{4^2 - 4(-2)(7)}\\ &= \sqrt{16 + 56}\\ &= \sqrt{72}\\ &= \sqrt{36 \cdot 2}\\ &= \sqrt{36} \cdot \sqrt{2}\\ &= 6\sqrt{2}\end{aligned}$$

80. $$\begin{aligned}\sqrt{b^2 - 4ac} &= \sqrt{6^2 - 4(1)(4)}\\ &= \sqrt{36 - 16}\\ &= \sqrt{20}\\ &= \sqrt{4 \cdot 5}\\ &= \sqrt{4} \cdot \sqrt{5}\\ &= 2\sqrt{5}\end{aligned}$$

9.5 Explorations (p. 515)

1. 2. Multiply each side by $4a$.

3. Add b^2 to each side.

4. Subtract $4ac$ from each side. Now the left side is a perfect square trinomial.

5. Write the left side in factored form as the square of a binomial.

6. Take the square root of each side.

7. Subtract b from each side.

8. Divide each side by $2a$.

2. a.

$$ax^2 + bx + c = 0$$
$$ax^2 + bx + c - c = 0 - c$$
$$ax^2 + bx = -c$$
$$\frac{ax^2 + bx}{a} = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$
$$x^2 + \frac{b}{a}x + \left(\frac{h}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$
$$\left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$$
$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$
$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{2a}}$$
$$-\frac{b}{2a} \qquad -\frac{b}{2a}$$
$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{2a}}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

b. Each side was multiplied by $4a$, and b^2 was added to each side so that the left side of the equation would be a perfect square trinomial.

3. In order to derive a formula that can be used to write the solutions of any quadratic equation, start with the general form of the quadratic equation, $ax^2 + bx + c = 0$, and solve for x by completing the square.

4. a. $x^2 + 2x - 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$
$$x = \frac{-2 \pm \sqrt{4 + 12}}{2}$$
$$x = \frac{-2 \pm \sqrt{16}}{2}$$
$$x = \frac{-2 \pm 4}{2}$$

The solutions are $x = \frac{-2 + 4}{2} = \frac{2}{2} = 1$ and $x = \frac{-2 - 4}{2} = \frac{-6}{2} = -3$.

b. $x^2 - 4x + 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(4)}}{2(1)}$$
$$= \frac{4 \pm \sqrt{16 - 16}}{2}$$
$$= \frac{4 \pm \sqrt{0}}{2}$$
$$= \frac{4 \pm 0}{2}$$
$$= \frac{4}{2}$$
$$= 2$$

The solution is $x = 2$.

c. $x^2 + 4x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$
$$= \frac{-4 \pm \sqrt{-4}}{2}$$

Because you cannot take the square root of a negative number, the equation has no real solutions.

5. *Sample answer:* The imaginary number, i, is $\sqrt{-1}$. No real number multiplied by itself produces a negative number. So, it is not possible to take the square root of a negative number using the real numbers only. Quadratic equations with no real solution have complex solutions, which include an imaginary part.

9.5 Monitoring Progress (pp. 516–520)

1. $x^2 - 6x + 5 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$
$$= \frac{6 \pm \sqrt{36 - 20}}{2}$$
$$= \frac{6 \pm \sqrt{16}}{2}$$
$$= \frac{6 \pm 4}{2}$$

So, the solutions are $x = \frac{6 + 4}{2} = \frac{10}{2} = 5$ and $x = \frac{6 - 4}{2} = \frac{2}{2} = 1$.

Chapter 9

2. $\frac{1}{2}x^2 + x - 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4\left(\frac{1}{2}\right)(-10)}}{2\left(\frac{1}{2}\right)}$$
$$= \frac{-1 \pm \sqrt{1 + 20}}{1}$$
$$= -1 \pm \sqrt{21}$$

So, the solutions are $x = -1 + \sqrt{21} \approx 3.6$ and $x = -1 - \sqrt{21} \approx -5.6$.

3. $-3x^2 + 2x + 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(-3)(7)}}{2(-3)}$$
$$= \frac{-2 \pm \sqrt{4 + 84}}{-6}$$
$$= \frac{-2 \pm \sqrt{88}}{-6}$$
$$= \frac{-1 \pm \sqrt{22}}{-3}$$

So, the solutions are $x = \frac{-1 + \sqrt{22}}{-3} \approx -1.2$ and $x = \frac{-1 - \sqrt{22}}{-3} \approx 1.9$.

4.
$$4x^2 - 4x = -1$$
$$4x^2 - 4x + 1 = -1 + 1$$
$$4x^2 - 4x + 1 = 0$$
$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$$
$$= \frac{4 \pm \sqrt{16 - 16}}{8}$$
$$= \frac{4 \pm \sqrt{0}}{8}$$
$$= \frac{4 \pm 0}{8}$$
$$= \frac{4}{8}$$
$$= \frac{1}{2}$$

So, the solution is $x = \frac{1}{2}$.

5.
$$y = 0.20x^2 + 1.8x - 3$$
$$60 = 0.20x^2 + 1.8x - 3$$
$$60 - 60 = 0.20x^2 + 1.8x - 3 - 60$$
$$0 = 0.20x^2 + 1.8x - 63$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1.8 \pm \sqrt{1.8^2 - 4(0.20)(-63)}}{2(0.20)}$$
$$= \frac{-1.8 \pm \sqrt{3.24 + 50.4}}{0.4}$$
$$= \frac{-1.8 \pm \sqrt{53.64}}{0.4}$$

The solutions are $x = \frac{-1.8 + \sqrt{53.64}}{0.4} \approx 13.8$ and $x = \frac{-1.8 - \sqrt{53.64}}{0.4} \approx -22.8$. Because x represents the number of years since 1990, x is greater than or equal to 0. So, there were about 60 breeding pairs 14 years after 1990, in 2004.

6. a.
$$y = 0.34x^2 + 13.1x + 51$$
$$160 = 0.34x^2 + 13.1x + 51$$
$$160 - 160 = 0.34x^2 + 13.1x + 51 - 160$$
$$0 = 0.34x^2 + 13.1x - 109$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-13.1 \pm \sqrt{13.1^2 - 4(0.34)(-109)}}{2(0.34)}$$
$$= \frac{-13.1 \pm \sqrt{171.61 + 148.24}}{0.68}$$
$$= \frac{-13.1 \pm \sqrt{319.85}}{0.68}$$

The solutions are $x = \frac{-13.1 + \sqrt{319.85}}{0.68} \approx 7.0$ and $x = \frac{-13.1 - \sqrt{319.85}}{0.68} \approx -45.6$. Because x represents the number of years since 2000, x is greater than or equal to 0. So, there were about 160 nesting pairs 7 years after 2000, in 2007.

b.
$$y = 0.34x^2 + 13.1x + 51$$
$$= 0.34(0)^2 + 13.1(0) + 51$$
$$= 0.34(0) + 0 + 51$$
$$= 0 + 51$$
$$= 51$$

In the year 2000, there were 51 bald eagle nesting pairs.

7.
$$b^2 - 4ac = 4^2 - 4(-1)(-4)$$
$$= 16 - 16$$
$$= 0$$

The discriminant is 0. So, the equation has one real solution.

8.
$$6x^2 + 2x = -1$$
$$6x^2 + 2x + 1 = -1 + 1$$
$$6x^2 + 2x + 1 = 0$$
$$b^2 - 4ac = 2^2 - 4(6)(1)$$
$$= 4 - 24$$
$$= -20$$

The discriminant is less than 0. So, the equation has no real solutions.

9.
$$\tfrac{1}{2}x^2 = 7x - 1$$
$$\tfrac{1}{2}x^2 - 7x + 1 = 7x - 7x - 1 + 1$$
$$\tfrac{1}{2}x^2 - 7x + 1 = 0$$
$$b^2 - 4ac = (-7)^2 - 4\left(\tfrac{1}{2}\right)(1)$$
$$= 49 - 2$$
$$= 47$$

The discriminant is greater than 0. So, the equation has two real solutions.

10.
$$y = -x^2 + x - 6$$
$$b^2 - 4ac = 1^2 - 4(-1)(-6)$$
$$= 1 - 24$$
$$= -23$$

Because the discriminant is less than 0, the equation $-x^2 + x - 6 = 0$ has no real solutions. So, the graph of $y = -x^2 + x - 6$ has no x-intercepts.

11.
$$y = x^2 - x$$
$$b^2 - 4ac = (-1)^2 - 4(1)(0)$$
$$= 1 - 0$$
$$= 1$$

Because the discriminant is greater than 0, the equation $x^2 - x = 0$ has two real solutions. So, the graph of $y = x^2 - x$ has two x-intercepts.

12.
$$f(x) = x^2 + 12x + 36$$
$$b^2 - 4ac = 12^2 - 4(1)(36)$$
$$= 144 - 144$$
$$= 0$$

Because the discriminant is 0, the equation $0 = x^2 + 12x + 36$ has one real solution. So, the graph of $f(x) = x^2 + 12x + 36$ has one x-intercept.

13. *Sample answer:* The equation is easily factorable. So, solve by factoring.
$$x^2 + 11x - 12 = 0$$
$$(x + 12)(x - 1) = 0$$
$$x + 12 = 0 \quad \text{or} \quad x - 1 = 0$$
$$\underline{-12} \quad \underline{-12} \qquad\qquad \underline{+1} \quad \underline{+1}$$
$$x = -12 \qquad\qquad x = 1$$

The solutions are $x = -12$ and $x = 1$.

14. *Sample answer:* The equation can be written in the form $x^2 = d$. So, solve using square roots.
$$9x^2 - 5 = 4$$
$$9x^2 - 5 + 5 = 4 + 5$$
$$9x^2 = 9$$
$$\frac{9x^2}{9} = \frac{9}{9}$$
$$x^2 = 1$$
$$\sqrt{x^2} = \sqrt{1}$$
$$x = \pm 1$$

The solutions are $x = 1$ and $x = -1$.

15. *Sample answer:* The equation is not factorable and the coefficient of the x^2-term is not 1. So, solve using the Quadratic Formula.
$$5x^2 - x - 1 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(5)(-1)}}{2(5)}$$
$$= \frac{1 \pm \sqrt{1 + 20}}{10}$$
$$= \frac{1 \pm \sqrt{21}}{10}$$

The solutions are $x = \frac{1 + \sqrt{21}}{10} \approx 0.56$ and $x = \frac{1 - \sqrt{21}}{10} \approx -0.36$.

16. *Sample answer:* The coefficient of the x^2-term is 1 and b is even. So, solve by completing the square.
$$x^2 = 2x - 5$$
$$x^2 - 2x = 2x - 2x - 5$$
$$x^2 - 2x = -5$$
$$x^2 - 2x + (-1)^2 = -5 + (-1)^2$$
$$(x - 1)^2 = -5 + 1$$
$$(x - 1)^2 = -4$$

No real number multiplied by itself gives a negative value. So, the equation has no real solutions.

9.5 Exercises (pp. 521–524)

Vocabulary and Core Concept Check

1. The formula you can use to solve any quadratic equation is the Quadratic Formula, which is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

2. The discriminant is the part of the Quadratic Formula that is inside the square root: $b^2 - 4ac$. The sign of this value can be used to determine the number of real solutions of a quadratic solution.

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Monitoring Progress and Modeling with Mathematics

3. $x^2 = 7x$

$x^2 - 7x = 7x - 7x$

$x^2 - 7x = 0$

So, $a = 1$, $b = -7$, and $c = 0$.

or

$x^2 = 7x$

$x^2 - x^2 = 7x - x^2$

$0 = -x^2 + 7x$

So, $a = -1$, $b = 7$, and $c = 0$.

4. $x^2 - 4x = -12$

$x^2 - 4x + 12 = -12 + 12$

$x^2 - 4x + 12 = 0$

So, $a = 1$, $b = -4$, and $c = 12$.

or

$x^2 - 4x = -12$

$x^2 - x^2 - 4x + 4x = -12 - x^2 + 4x$

$0 = -x^2 + 4x - 12$

So, $a = -1$, $b = 4$, and $c = -12$.

5. $-2x^2 + 1 = 5x$

$-2x^2 + 1 - 5x = 5x - 5x$

$-2x^2 - 5x + 1 = 0$

So, $a = -2$, $b = -5$, and $c = 1$.

or

$-2x^2 + 1 = 5x$

$-2x^2 + 2x^2 + 1 - 1 = 5x + 2x^2 - 1$

$0 = 2x^2 + 5x - 1$

So, $a = 2$, $b = 5$, and $c = -1$.

6. $3x + 2 = 4x^2$

$3x + 2 - 4x^2 = 4x^2 - 4x^2$

$-4x^2 + 3x + 2 = 0$

So, $a = -4$, $b = 3$, and $c = 2$.

or

$3x + 2 = 4x^2$

$3x - 3x + 2 - 2 = 4x^2 - 3x - 2$

$0 = 4x^2 - 3x - 2$

So, $a = 4$, $b = -3$, and $c = -2$.

7. $4 - 3x = -x^2 + 3x$

$4 - 3x + x^2 - 3x = -x^2 + x^2 + 3x - 3x$

$x^2 - 6x + 4 = 0$

So, $a = 1$, $b = -6$, and $c = 4$.

or

$4 - 3x = -x^2 + 3x$

$4 - 4 - 3x + 3x = -x^2 + 3x - 4 + 3x$

$0 = -x^2 + 6x - 4$

So, $a = -1$, $b = 6$, and $c = -4$.

8. $-8x - 1 = 3x^2 + 2$

$-8x - 1 - 3x^2 - 2 = 3x^2 - 3x^2 + 2 - 2$

$-3x^2 - 8x - 3 = 0$

So, $a = -3$, $b = -8$, and $c = -3$.

or

$-8x - 1 = 3x^2 + 2$

$-8x + 8x - 1 + 1 = 3x^2 + 2 + 8x + 1$

$0 = 3x^2 + 8x + 3$

So, $a = 3$, $b = 8$, and $c = 3$.

9. $x^2 - 12x + 36 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-12) \pm \sqrt{(-12)^2 - 4(1)(36)}}{2(1)}$$
$$= \frac{12 \pm \sqrt{144 - 144}}{2}$$
$$= \frac{12 \pm \sqrt{0}}{2}$$
$$= \frac{12}{2}$$
$$= 6$$

The solution is $x = 6$.

10. $x^2 + 7x + 16 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-7 \pm \sqrt{7^2 - 4(1)(16)}}{2(1)}$$
$$= \frac{-7 \pm \sqrt{49 - 64}}{2}$$
$$= \frac{-7 \pm \sqrt{-15}}{2}$$

The discriminant is less than 0. So, the equation has no real solutions.

11. $x^2 - 10x - 11 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(-11)}}{2(1)}$$
$$= \frac{10 \pm \sqrt{100 + 44}}{2}$$
$$= \frac{10 \pm \sqrt{144}}{2}$$
$$= \frac{10 \pm 12}{2}$$

The solutions are $x = \frac{10 + 12}{2} = \frac{22}{2} = 11$ and

$x = \frac{10 - 12}{2} = \frac{-2}{2} = -1$.

12. $2x^2 - x - 1 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-1)}}{2(2)} \\ &= \frac{1 \pm \sqrt{1 + 8}}{4} \\ &= \frac{1 \pm \sqrt{9}}{4} \\ &= \frac{1 \pm 3}{4} \end{aligned}$$

The solutions are $x = \frac{1 + 3}{4} = \frac{4}{4} = 1$ and $x = \frac{1 - 3}{4} = -\frac{2}{4} = -\frac{1}{2}$.

13. $2x^2 - 6x + 5 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(5)}}{2(2)} \\ &= \frac{6 \pm \sqrt{36 - 40}}{4} \\ &= \frac{6 \pm \sqrt{-4}}{4} \end{aligned}$$

The discriminant is less than 0. So, the equation has no real solutions.

14. $9x^2 - 6x + 1 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(9)(1)}}{2(9)} \\ &= \frac{6 \pm \sqrt{36 - 36}}{18} \\ &= \frac{6 \pm \sqrt{0}}{18} \\ &= \frac{6}{18} \\ &= \frac{1}{3} \end{aligned}$$

The solution is $x = \frac{1}{3}$.

15.

$$\begin{aligned} 6x^2 - 13x &= -6 \\ 6x^2 - 13x + 6 &= -6 + 6 \\ 6x^2 - 13x + 6 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-13) \pm \sqrt{(-13)^2 - 4(6)(6)}}{2(6)} \\ &= \frac{13 \pm \sqrt{169 - 144}}{12} \\ &= \frac{13 \pm \sqrt{25}}{12} \\ &= \frac{13 \pm 5}{12} \end{aligned}$$

The solutions are $x = \frac{13 + 5}{12} = \frac{18}{12} = \frac{3}{2}$ and $x = \frac{13 - 5}{12} = \frac{8}{12} = \frac{2}{3}$.

16.

$$\begin{aligned} -13x^2 + 6x &= 4 \\ -13x^2 + 6x - 4 &= 4 - 4 \\ -13x^2 + 6x - 4 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 \pm \sqrt{6^2 - 4(-13)(-4)}}{2(-13)} \\ &= \frac{-6 \pm \sqrt{36 - 208}}{-26} \\ &= \frac{-6 \pm \sqrt{-172}}{-26} \end{aligned}$$

The discriminant is less than 0. So, the equation has no real solutions.

17.

$$\begin{aligned} 1 - 8x &= -16x^2 \\ 1 - 8x + 16x^2 &= -16x^2 + 16x^2 \\ 16x^2 - 8x + 1 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(16)(1)}}{2(16)} \\ &= \frac{8 \pm \sqrt{64 - 64}}{32} \\ &= \frac{8 \pm \sqrt{0}}{32} \\ &= \frac{8}{32} \\ &= \frac{1}{4} \end{aligned}$$

The solution is $x = \frac{1}{4}$.

18. $x^2 - 5x + 3 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(3)}}{2(1)}$$
$$= \frac{5 \pm \sqrt{25 - 12}}{2}$$
$$= \frac{5 \pm \sqrt{13}}{2}$$

The solutions are $x = \frac{5 + \sqrt{13}}{2} \approx 4.3$ and $x = \frac{5 - \sqrt{13}}{2} \approx 0.7$.

19.
$$x^2 + 2x = 9$$
$$x^2 + 2x - 9 = 9 - 9$$
$$x^2 + 2x - 9 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{2^2 - 4(1)(-9)}}{2(1)}$$
$$= \frac{-2 \pm \sqrt{4 + 36}}{2}$$
$$= \frac{-2 \pm \sqrt{40}}{2}$$

The solutions are $x = \frac{-2 + \sqrt{40}}{2} \approx 2.2$ and $x = \frac{-2 - \sqrt{40}}{2} \approx -4.2$.

20.
$$5x^2 - 2 = 4x$$
$$5x^2 - 2 - 4x = 4x - 4x$$
$$5x^2 - 4x - 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(5)(-2)}}{2(5)}$$
$$= \frac{4 \pm \sqrt{16 + 40}}{10}$$
$$= \frac{4 \pm \sqrt{56}}{10}$$

The solutions are $x = \frac{4 + \sqrt{56}}{10} \approx 1.1$ and $x = \frac{4 - \sqrt{56}}{10} \approx -0.3$.

21.
$$2x^2 + 9x + 7 = 3$$
$$2x^2 + 9x + 7 - 3 = 3 - 3$$
$$2x^2 + 4x + 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(2)(4)}}{2(2)}$$
$$= \frac{-9 \pm \sqrt{81 - 32}}{4}$$
$$= \frac{-9 \pm \sqrt{49}}{4}$$
$$= \frac{-9 \pm 7}{4}$$

The solutions are $x = \frac{-9 + 7}{4} = \frac{-2}{4} = -\frac{1}{2}$ and $x = \frac{-9 - 7}{4} = \frac{-16}{4} = -4$.

22.
$$8x^2 + 8 = 6 - 9x$$
$$8x^2 + 8 - 6 + 9x = 6 - 6 - 9x + 9x$$
$$8x^2 + 9x + 2 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(8)(2)}}{2(8)}$$
$$= \frac{-9 \pm \sqrt{81 - 64}}{16}$$
$$= \frac{-9 \pm \sqrt{17}}{16}$$

The solutions are $x = \frac{-9 + \sqrt{17}}{16} \approx -0.3$ and $x = \frac{-9 - \sqrt{17}}{16} \approx -0.8$.

23.
$$h = -16t^2 + 26t$$
$$5 = -16t^2 + 26t$$
$$5 - 5 = -16t^2 + 26t - 5$$
$$0 = -16t^2 + 26t - 5$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-26 \pm \sqrt{26^2 - 4(-16)(-5)}}{2(-16)}$$
$$= \frac{-26 \pm \sqrt{676 - 320}}{-32}$$
$$= \frac{-26 \pm \sqrt{356}}{-32}$$

The solutions are $t = \frac{-26 + \sqrt{356}}{-32} \approx 0.2$ and $t = \frac{-26 - \sqrt{356}}{-32} \approx 1.4$. So, the dolphin is at a height of 5 feet after about 0.2 second and again after about 1.4 seconds.

24. a.

$$\begin{aligned} y &= -0.08x^2 + 1.6x + 10 \\ 15 &= -0.08x^2 + 1.6x + 10 \\ 15 - 15 &= -0.08x^2 + 1.6x + 10 - 15 \\ 0 &= -0.08x^2 + 1.6x - 5 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1.6 \pm \sqrt{1.6^2 - 4(-0.08)(-5)}}{2(-0.08)} \\ &= \frac{-1.6 \pm \sqrt{2.56 - 1.6}}{-0.16} \\ &= \frac{-1.6 \pm \sqrt{0.96}}{-0.16} \end{aligned}$$

The solutions are $x = \frac{-1.6 + \sqrt{0.96}}{-0.16} \approx 3.9$ and $x = \frac{-1.6 - \sqrt{0.96}}{-0.16} \approx 16.1$. So, about 15 tons of trout were caught in the lake after 4 years, in 1999, and again after 16 years, in 2011.

b. According to the model, the amount of trout caught after 2020 is negative. So, this model does not provide reasonable predictions for the amounts of trout caught in future years.

25. $x^2 - 6x + 10 = 0$

$$\begin{aligned} b^2 - 4ac &= (-6)^2 - 4(1)(10) \\ &= 36 - 40 \\ &= -4 \end{aligned}$$

The discriminant is less than 0. So, the equation has no real solutions.

26. $x^2 - 5x - 3 = 0$

$$\begin{aligned} b^2 - 4ac &= (-5)^2 - 4(1)(-3) \\ &= 25 + 12 \\ &= 37 \end{aligned}$$

The discriminant is greater than 0. So, the equation has two real solutions.

27.

$$\begin{aligned} 2x^2 - 12x &= -18 \\ 2x^2 - 12x + 18 &= -18 + 18 \\ 2x^2 - 12x + 18 &= 0 \\ b^2 - 4ac &= (-12)^2 - 4(2)(18) \\ &= 144 - 144 \\ &= 0 \end{aligned}$$

The discriminant is 0. So, the equation has one real solution.

28.

$$\begin{aligned} 4x^2 &= 4x - 1 \\ 4x^2 - 4x + 1 &= 4x - 4x - 1 + 1 \\ 4x^2 - 4x + 1 &= 0 \\ b^2 - 4ac &= (-4)^2 - 4(4)(1) \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

The discriminant is 0. So, the equation has one real solution.

29.

$$\begin{aligned} -\tfrac{1}{4}x^2 + 4x &= -2 \\ -\tfrac{1}{4}x^2 + 4x + 2 &= -2 + 2 \\ -\tfrac{1}{4}x^2 + 4x + 2 &= 0 \\ b^2 - 4ac &= 4^2 - 4\left(-\tfrac{1}{4}\right)(2) \\ &= 16 + 2 \\ &= 18 \end{aligned}$$

The discriminant is greater than 0. So, the equation has two real solutions.

30.

$$\begin{aligned} -5x^2 + 8x &= 9 \\ -5x^2 + 8x - 9 &= 9 - 9 \\ -5x^2 + 8x - 9 &= 0 \\ b^2 - 4ac &= 8^2 - 4(-5)(-9) \\ &= 64 - 180 \\ &= -116 \end{aligned}$$

The discriminant is less than 0. So, the equation has no real solutions.

31.

$$\begin{aligned} y &= x^2 + 5x - 1 \\ b^2 - 4ac &= 5^2 - 4(1)(-1) \\ &= 25 + 4 \\ &= 29 \end{aligned}$$

Because the discriminant is greater than 0, the equation $0 = x^2 + 5x - 1$ has two real solutions. So, the graph of $y = x^2 + 5x - 1$ has two x-intercepts.

32.

$$\begin{aligned} y &= 4x^2 + 4x + 1 \\ b^2 - 4ac &= 4^2 - 4(4)(1) \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

Because the discriminant is 0, the equation $0 = 4x^2 + 4x + 1$ has one real solution. So, the graph of $y = 4x^2 + 4x + 1$ has one x-intercept.

33.

$$\begin{aligned} y &= -6x^2 + 3x - 4 \\ b^2 - 4ac &= 3^2 - 4(-6)(-4) \\ &= 9 - 96 \\ &= -87 \end{aligned}$$

Because the discriminant is less than 0, the equation $0 = -6x^2 + 3x - 4$ has no real solutions. So, the graph of $y = -6x^2 + 3x - 4$ has no x-intercepts.

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34. $$\begin{aligned} y &= -x^2 + 5x + 13 \\ b^2 - 4ac &= 5^2 - 4(-1)(13) \\ &= 25 + 52 \\ &= 77 \end{aligned}$$

Because the discriminant is greater than 0, the equation $0 = -x^2 + 5x + 13$ has two real solutions. So, the graph of $y = -x^2 + 5x + 13$ has two x-intercepts.

35. $$\begin{aligned} f(x) &= 4x^2 + 3x - 6 \\ b^2 - 4ac &= 3^2 - 4(4)(-6) \\ &= 9 + 96 \\ &= 105 \end{aligned}$$

Because the discriminant is greater than 0, the equation $0 = 4x^2 + 3x - 6$ has two real solutions. So, the graph of $y = 4x^2 + 3x - 6$ has two x-intercepts.

36. $$\begin{aligned} f(x) &= 2x^2 + 8x + 8 \\ b^2 - 4ac &= 8^2 - 4(2)(8) \\ &= 64 - 64 \\ &= 0 \end{aligned}$$

Because the discriminant is 0, the equation $0 = 2x^2 + 8x + 8$ has one real solution. So, the graph of $y = 2x^2 + 8x + 8$ has one x-intercept.

37. *Sample answer:* The equation is not easily factorable and $a \neq 1$. So, solve using the Quadratic Formula.

$$\begin{aligned} -10x^2 + 13x &= 4 \\ -10x^2 + 13x - 4 &= 4 - 4 \\ -10x^2 + 13x - 4 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-13 \pm \sqrt{13^2 - 4(-10)(-4)}}{2(-10)} \\ &= \frac{-13 \pm \sqrt{169 - 160}}{-20} \\ &= \frac{-13 \pm \sqrt{9}}{-20} \\ &= \frac{-13 \pm 3}{-20} \end{aligned}$$

The solutions are $x = \frac{-13 + 3}{-20} = \frac{-10}{-20} = \frac{1}{2}$ and

$x = \frac{-13 - 3}{-20} = \frac{-16}{-20} = \frac{4}{5}$.

38. *Sample answer:* The equation is easily factorable. So, solve by factoring.

$$x^2 - 3x - 40 = 0$$

$$(x - 8)(x + 5) = 0$$

$$\begin{array}{rclcrcl} x - 8 &=& 0 & \quad\text{or}\quad & x + 5 &=& 0 \\ \underline{+\,8} && \underline{+\,8} && \underline{-\,5} && \underline{-\,5} \\ x &=& 8 && x &=& -5 \end{array}$$

The solutions are $x = 8$ and $x = -5$.

39. *Sample answer:* The equation is not factorable, but $a = 1$ and b is even. So, solve by completing the square.

$$\begin{aligned} x^2 + 6x &= 5 \\ x^2 + 6x + 3^2 &= 5 + 3^2 \\ (x + 3)^2 &= 5 + 9 \\ (x + 3)^2 &= 14 \\ \sqrt{(x + 3)^2} &= \sqrt{14} \\ x + 3 &= \pm\sqrt{14} \\ \underline{-\,3} \quad & \quad \underline{-\,3} \\ x &= -3 \pm \sqrt{14} \end{aligned}$$

The solutions are $x = -3 + \sqrt{14} \approx 0.74$ and $x = -3 - \sqrt{14} \approx -6.74$.

40. *Sample answer:* The equation can be written in the form $x^2 = d$. So, solve using square roots.

$$\begin{aligned} -5x^2 &= -25 \\ \frac{-5x^2}{-5} &= \frac{-25}{-5} \\ x^2 &= 5 \\ \sqrt{x^2} &= \sqrt{5} \\ x &= \pm\sqrt{5} \end{aligned}$$

The solutions are $x = \sqrt{5} \approx 2.24$ and $x = -\sqrt{5} \approx -2.24$.

41. *Sample answer:* The equation is easily factorable. So, solve by factoring.

$$x^2 + x - 12 = 0$$

$$(x + 4)(x - 3) = 0$$

$$\begin{array}{rclcrcl} x + 4 &=& 0 & \quad\text{or}\quad & x - 3 &=& 0 \\ \underline{-\,4} && \underline{-\,4} && \underline{+\,3} && \underline{+\,3} \\ x &=& -4 && x &=& 3 \end{array}$$

The solutions are $x = -4$ and $x = 3$.

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42. *Sample answer:* The equation is not factorable, but $a = 1$ and b is even. So, solve by completing the square.

$$x^2 - 4x + 1 = 0$$
$$x^2 - 4x + 1 - 1 = 0 - 1$$
$$x^2 - 4x = -1$$
$$x^2 - 4x + (-2)^2 = -1 + (-2)^2$$
$$(x - 2)^2 = -1 + 4$$
$$(x - 2)^2 = 3$$
$$\sqrt{(x - 2)^2} = \sqrt{3}$$
$$x - 2 = \pm\sqrt{3}$$
$$\underline{+2} \quad \underline{+2}$$
$$x = 2 \pm \sqrt{3}$$

The solutions are $x = 2 + \sqrt{3} \approx 3.73$ and $x = 2 - \sqrt{3} \approx 0.27$.

43. *Sample answer:* The equation cannot be factored and $a \neq 1$. So, solve using the Quadratic Formula.

$$4x^2 - x = 17$$
$$4x^2 - x - 17 = 17 - 17$$
$$4x^2 - x - 17 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(4)(-17)}}{2(4)}$$
$$= \frac{1 \pm \sqrt{1 + 272}}{8}$$
$$= \frac{1 \pm \sqrt{273}}{8}$$

The solutions are $x = \frac{1 + \sqrt{273}}{8} \approx 2.19$ and $x = \frac{1 - \sqrt{273}}{8} \approx -1.94$.

44. *Sample answer:* The left side is a perfect square trinomial. So, solve using square roots.

$$x^2 + 6x + 9 = 16$$
$$x^2 + 2(x)(3) + 3^2 = 16$$
$$(x + 3)^2 = 16$$
$$\sqrt{(x + 3)^2} = \sqrt{16}$$
$$x + 3 = \pm 4$$
$$\underline{-3} \quad \underline{-3}$$
$$x = -3 \pm 4$$

The solutions are $x = -3 + 4 = 1$ and $x = -3 - 4 = -7$.

45. In the numerator of the fraction, $-b$ should be $-(-7) = 7$, not -7.

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-6)}}{2(3)}$$
$$= \frac{7 \pm \sqrt{49 + 72}}{6}$$
$$= \frac{7 \pm \sqrt{121}}{6}$$
$$= \frac{7 \pm 11}{6}$$

The solutions are $x = \frac{7 + 11}{6} = \frac{18}{6} = 3$ and $x = \frac{7 - 11}{6} = \frac{-4}{6} = -\frac{2}{3}$.

46. The equation needs to be in standard form, $ax^2 + bx + c = 0$. So, $c = -4$, not 4.

$$-2x^2 + 9x = 4$$
$$-2x^2 + 9x - 4 = 4 - 4$$
$$-2x^2 + 9x - 4 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-9 \pm \sqrt{9^2 - 4(-2)(-4)}}{2(-2)}$$
$$= \frac{-9 \pm \sqrt{81 - 32}}{-4}$$
$$= \frac{-9 \pm \sqrt{49}}{-4}$$
$$= \frac{-9 \pm 7}{-4}$$

The solutions are $x = \frac{-9 + 7}{-4} = \frac{-2}{-4} = \frac{1}{2}$ and $x = \frac{-9 - 7}{-4} = \frac{-16}{-4} = 4$.

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47.

$$\begin{aligned} y &= -0.006x^2 + 1.2x + 10 \\ 50 &= -0.006x^2 + 1.2x + 10 \\ 50 - 50 &= -0.006x^2 + 1.2x + 10 - 50 \\ 0 &= -0.006x^2 + 1.2x - 40 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1.2 \pm \sqrt{1.2^2 - 4(-0.006)(-40)}}{2(-0.006)} \\ &= \frac{-1.2 \pm \sqrt{1.44 - 0.96}}{-0.012} \\ &= \frac{-1.2 \pm \sqrt{0.48}}{-0.012} \end{aligned}$$

The solutions are $x = \frac{-1.2 + \sqrt{0.48}}{-0.012} \approx 42.3$ and $x = \frac{-1.2 - \sqrt{0.48}}{-0.012} \approx 157.7$. So, the water arc is 50 feet above the water when it is about 42 feet from the north shore and again when it is about 158 feet from the north shore.

48. a.

$$\begin{aligned} y &= -0.00046x^2 + 0.076x + 13 \\ 17 &= -0.00046x^2 + 0.076x + 13 \\ 17 - 17 &= -0.00046x^2 + 0.076x + 13 - 17 \\ 0 &= -0.00046x^2 + 0.076x - 4 \\ b^2 - 4ac &= 0.076^2 - 4(-0.00046)(-4) \\ &= 0.005776 - 0.00736 \\ &= -0.001584 \end{aligned}$$

Because the discriminant is less than 0, the equation has no real solutions. So, none of the days between April and September in Seattle have 17 hours of daylight.

b.

$$\begin{aligned} y &= -0.00046x^2 + 0.076x + 13 \\ 14 &= -0.00046x^2 + 0.076x + 13 \\ 14 - 14 &= -0.00046x^2 + 0.076x + 13 - 14 \\ 0 &= -0.00046x^2 + 0.076x - 1 \\ b^2 - 4ac &= 0.076^2 - 4(-0.00046)(-1) \\ &= 0.005776 - 0.00184 \\ &= 0.003936 \end{aligned}$$

Because the discriminant is greater than 0, the equation has two solutions. So, two of the days between April and September in Seattle have 14 hours of daylight.

49.

$$\begin{aligned} 2x^2 - 5x - 2 &= -11 \\ 2x^2 - 5x - 2 + 11 &= -11 + 11 \\ 2x^2 - 5x + 9 &= 0 \\ b^2 - 4ac &= (-5)^2 - 4(2)(9) \\ &= 25 - 72 \\ &= -47 \end{aligned}$$

Your friend is incorrect. You must first rewrite the equation in standard form, and the value of the discriminant is -47. Because the discriminant is less than 0, the equation has no solutions.

50. Substitute 4 for y in the model.

$$\begin{aligned} y &= -0.18x^2 + 1.6x \\ 4 &= -0.18x^2 + 1.6x \\ 4 - 4 &= -0.18x^2 + 1.6x - 4 \\ 0 &= -0.18x^2 + 1.6x - 4 \end{aligned}$$

Find the value of the discriminant.

$$\begin{aligned} b^2 - 4ac &= 1.6^2 - 4(-0.18)(-4) \\ &= 2.56 - 2.88 \\ &= -0.32 \end{aligned}$$

Because the discriminant is less than 0, the equation has no solutions when $y = 4$. So, a child who is 4 feet tall cannot walk under either one of the arches without having to bend over.

51. Area = length • width

$$\begin{aligned} 91 &= (2x + 3)(x + 2) \\ 91 &= 2x(x) + 2x(2) + 3(x) + 3(2) \\ 91 &= 2x^2 + 4x + 3x + 6 \\ 91 &= 2x^2 + 7x + 6 \\ 91 - 91 &= 2x^2 + 7x + 6 - 91 \\ 0 &= 2x^2 + 7x - 85 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-7 \pm \sqrt{7^2 - 4(2)(-85)}}{2(2)} \\ &= \frac{-7 \pm \sqrt{49 + 680}}{4} \\ &= \frac{-7 \pm \sqrt{729}}{4} \\ &= \frac{-7 \pm 27}{4} \end{aligned}$$

The solutions are $x = \frac{-7 + 27}{4} = \frac{20}{4} = 5$ and

$x = \frac{-7 - 27}{4} = \frac{-34}{4} = -8.5$. Disregard the solution $x = -8.5$ because $x + 2 = -8.5 + 2 = -6.5$ does not make sense as the width of a rectangle. So, the value of x is 5, the length of the rectangle is $2(5) + 3 = 13$ meters, and the width is $5 + 2 = 7$ meters.

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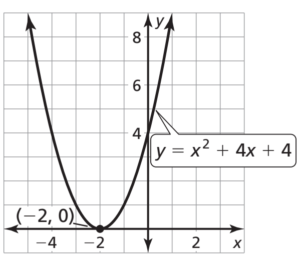
52.
$$\begin{aligned}
\text{Area} &= \text{length} \cdot \text{width}\\
209 &= (4x + 3)(4x - 5)\\
209 &= 4x(4x) + 4x(-5) + 3(4x) + 3(-5)\\
209 &= 16x^2 - 20x + 12x - 15\\
209 &= 16x^2 - 8x - 15\\
209 - 209 &= 16x^2 - 8x - 15 - 209\\
0 &= 16x^2 - 8x - 224\\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-(-8) \pm \sqrt{(-8)^2 - 4(16)(-224)}}{2(16)}\\
&= \frac{8 \pm \sqrt{64 + 14{,}336}}{32}\\
&= \frac{8 \pm \sqrt{14{,}400}}{32}\\
&= \frac{8 \pm 120}{32}
\end{aligned}$$

The solutions are $x = \frac{8 + 120}{32} = \frac{128}{32} = 4$ and $x = \frac{8 - 120}{32} = \frac{-112}{32} = -3.5$. Disregard the solution $x = -3.5$ because $4x - 5 = 4(-3.5) - 5 = -16$ does not make sense as the width of a rectangle. So, the value of x is 4, the length of the rectangle is $4(4) + 3 = 19$ feet, and the width is $4(4) - 5 = 11$ feet.

53. a. Graph $y = x^2 + 4x + 4$.



The x-intercept is -2. So, the solution is $x = -2$.

b.
$$\begin{aligned}
x^2 + 4x + 4 &= 0\\
x^2 + 2(x)(2) + 2^2 &= 0\\
(x + 2)^2 &= 0\\
x + 2 &= 0\\
\underline{-2} \quad & \underline{-2}\\
x &= -2
\end{aligned}$$

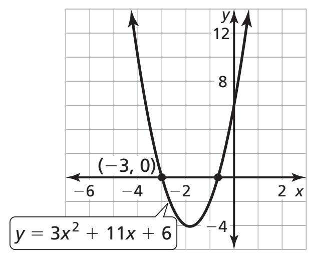
The solution is $x = -2$.

c. $x^2 + 4x + 4 = 0$
$$\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\\
&= \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}\\
&= \frac{-4 \pm \sqrt{16 - 16}}{2}\\
&= \frac{-4 \pm \sqrt{0}}{2}\\
&= -\frac{4}{2}\\
&= -2
\end{aligned}$$

The solution is $x = -2$.

Sample answer: For this equation, solve by factoring because the equation is easily factorable.

54. a. Graph $y = 3x^2 + 11x + 6$.



One x-intercept is $(-3, 0)$. Use a table to approximate the other.

x	-0.9	-0.8	-0.7	-0.6	-0.5
y	-1.47	-0.88	-0.23	0.48	1.25

change in signs

x	-0.4	-0.3	-0.2	-0.1
y	2.08	2.97	3.92	4.93

In the table, the function value closest to 0 is -0.23. So, the zeros of f are -3 and about -0.7, which means the solutions are $x = -3$ and $x \approx -0.7$.

b. $3x^2 + 11x + 6 = 0$

$(3x + 2)(x + 3) = 0$

$$\begin{aligned}
3x + 2 &= 0 & \text{or} \qquad x + 3 &= 0\\
\underline{-2} \quad & \underline{-2} & \underline{-3} \quad & \underline{-3}\\
3x &= -2 & x &= -3\\
\frac{3x}{3} &= -\frac{2}{3} & &\\
x &= -\frac{2}{3} & &
\end{aligned}$$

The solutions are $x = -\frac{2}{3}$ and $x = -3$.

c. $3x^2 + 11x + 6 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-11 \pm \sqrt{11^2 - 4(3)(6)}}{2(3)} \\ &= \frac{-11 \pm \sqrt{121 - 72}}{6} \\ &= \frac{-11 \pm \sqrt{49}}{6} \\ &= \frac{-11 \pm 7}{6} \end{aligned}$$

The solutions are $x = \frac{-11 + 7}{6} = \frac{-4}{6} = -\frac{2}{3}$ and $x = \frac{-11 - 7}{6} = \frac{-18}{6} = -3$.

Sample answer: For this equation, use the Quadratic Formula. The equation is not easily factorable, and graphing does not yield an exact solution because one of the x-intercepts is not an integer.

55. When a and c have different signs, their product, ac, is always negative. So, $-4ac$ is positive. Also, b^2 is always positive. So, when a and c have different signs, the discriminant will always be positive and the equation will have two real solutions.

56. When the discriminant is a perfect square, the square root of the discriminant is an integer. So, the solutions are rational.

57. a. *Sample answer:* $b^2 - 4ac > 0$

$$\begin{aligned} (-2)^2 - 4(1)c &= 8 \\ 4 - 4c &= 8 \\ \underline{-\ 4} \qquad & \quad \underline{-\ 4} \\ -4c &= 4 \\ \frac{-4c}{-4} &= \frac{4}{-4} \\ c &= -1 \end{aligned}$$

So, when $c = -1$, the discriminant is 8, which is greater than 0, and the equation $x^2 - 2x + c = 0$ has two real solutions.

b.
$$\begin{aligned} b^2 - 4ac &= 0 \\ (-2)^2 - 4(1)c &= 0 \\ 4 - 4c &= 0 \\ \underline{-\ 4} \qquad & \quad \underline{-\ 4} \\ -4c &= -4 \\ \frac{-4c}{-4} &= \frac{-4}{-4} \\ c &= 1 \end{aligned}$$

So, when $c = 1$, the discriminant is 0, and the equation $x^2 - 2x + c = 0$ has one real solution.

c. *Sample answer:* $b^2 - 4ac < 0$

$$\begin{aligned} (-2)^2 - 4(1)c &= -4 \\ 4 - 4c &= -4 \\ \underline{-\ 4} \qquad & \quad \underline{-\ 4} \\ -4c &= -8 \\ \frac{-4c}{-4} &= \frac{-8}{-4} \\ c &= 2 \end{aligned}$$

So, when $c = 2$, the discriminant is -4, which is less than 0, and the equation $x^2 - 2x + c = 0$ has no real solutions.

58. a. *Sample answer:* $b^2 - 4ac > 0$

$$\begin{aligned} (-8)^2 - 4(1)c &= 16 \\ 64 - 4c &= 16 \\ \underline{-\ 64} \qquad & \quad \underline{-\ 64} \\ -4c &= -48 \\ \frac{-4c}{-4} &= \frac{-48}{-4} \\ c &= 12 \end{aligned}$$

So, when $c = 12$, the discriminant is 16, which is greater than 0, and the equation $x^2 - 8x + c = 0$ has two real solutions.

b.
$$\begin{aligned} b^2 - 4ac &= 0 \\ (-8)^2 - 4(1)c &= 0 \\ 64 - 4c &= 0 \\ \underline{-\ 64} \qquad & \quad \underline{-\ 64} \\ -4c &= -64 \\ \frac{-4c}{-4} &= \frac{-64}{-4} \\ c &= 16 \end{aligned}$$

So, when $c = 16$, the discriminant is 0, and the equation $x^2 - 8x + c = 0$ has one real solution.

c. *Sample answer:* $b^2 - 4ac < 0$

$$\begin{aligned} (-8)^2 - 4(1)c &= -8 \\ 64 - 4c &= -8 \\ \underline{-\ 64} \qquad & \quad \underline{-\ 64} \\ -4c &= -72 \\ \frac{-4c}{-4} &= \frac{-72}{-4} \\ c &= 18 \end{aligned}$$

So, when $c = 18$, the discriminant is -8, which is less than 0, and the equation $4x^2 + 12x + c = 0$ has no real solutions.

59. a. *Sample answer:* $b^2 - 4ac > 0$

$$\begin{aligned} 12^2 - 4(4)c &= 48 \\ 144 - 16c &= 48 \\ \underline{-144} \qquad & \quad \underline{-144} \\ -16c &= -96 \\ \frac{-16c}{-16} &= \frac{-96}{-16} \\ c &= 6 \end{aligned}$$

So, when $c = 6$, the discriminant is 48, which is greater than 0, and the equation $4x^2 + 12x + c = 0$ has two real solutions.

b.

$$\begin{aligned} b^2 - 4ac &= 0 \\ 12^2 - 4(4)c &= 0 \\ 144 - 16c &= 0 \\ \underline{-144} \qquad & \quad \underline{-144} \\ -16c &= -144 \\ \frac{-16c}{-16} &= \frac{-144}{-16} \\ c &= 9 \end{aligned}$$

So, when $c = 9$, the discriminant is 0, and the equation $4x^2 + 12x + c = 0$ has one real solution.

c. *Sample answer:* $b^2 - 4ac < 0$

$$\begin{aligned} 12^2 - 4(4)c &= -16 \\ 144 - 16c &= -16 \\ \underline{-144} \qquad & \quad \underline{-144} \\ -16c &= -160 \\ \frac{-16c}{-16} &= \frac{-160}{-16} \\ c &= 10 \end{aligned}$$

So, when $c = 10$, the discriminant is -16, which is less than 0, and the equation $4x^2 + 12x + c = 0$ has no real solutions.

60. a. If the solutions are integers, then the equation can be written in intercept form, $y = (x - p)(x - q)$, where p and q are integers. So, the equation is factorable, and you can use factoring to solve the equation.

b. If the solutions are fractions, then the equation can be written in intercept form, $y = (x - p)(x - q)$, where p and q are fractions. So, the equation is factorable, and you can use factoring to solve the equation.

c. If a quadratic equation has rational solutions, then it can be solved by factoring.

61.

$$\begin{aligned} y &= -0.013x^2 + 1.25x + 5.6 \\ 32 &= -0.013x^2 + 1.25x + 5.6 \\ 32 - 32 &= -0.013x^2 + 1.25x + 5.6 - 32 \\ 0 &= -0.013x^2 + 1.25x - 26.4 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1.25 \pm \sqrt{1.25^2 - 4(-0.013)(-26.4)}}{2(-0.013)} \\ &= \frac{-1.25 \pm \sqrt{1.5625 - 1.3728}}{-0.026} \\ &= \frac{-1.25 \pm \sqrt{0.1897}}{-0.026} \end{aligned}$$

The solutions are $x = \dfrac{-1.25 + \sqrt{0.1897}}{-0.026} \approx 31.3$ and $x = \dfrac{-1.25 - \sqrt{0.1897}}{-0.026} \approx 64.8$. So, you can travel at about 31 miles per hour or about 65 miles per hour and have a fuel economy of 32 miles per gallon.

62.

$$\begin{aligned} d &= -0.25t^2 + 1.7t + 3.5 \\ 6 &= -0.25t^2 + 1.7t + 3.5 \\ 6 - 6 &= -0.25t^2 + 1.7t + 3.5 - 6 \\ 0 &= -0.25t^2 + 1.7t - 2.5 \\ t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1.7 \pm \sqrt{1.7^2 - 4(-0.25)(-2.5)}}{2(-0.25)} \\ &= \frac{-1.7 \pm \sqrt{2.89 - 2.5}}{-0.5} \\ &= \frac{-1.7 \pm \sqrt{0.39}}{-0.5} \end{aligned}$$

The solutions are $t = \dfrac{-1.7 + \sqrt{0.39}}{-0.5} \approx 2.2$ and $t = \dfrac{-1.7 - \sqrt{0.39}}{-0.5} \approx 4.6$. So, the river is 6 feet deep after 2.2 hours and again after 4.6 hours.

63.

$$\begin{aligned} y &= x^2 - 3x + 2 \\ b^2 - 4ac &= (-3)^2 - 4(1)(2) \\ &= 9 - 8 \\ &= 1 \end{aligned}$$

Because the discriminant is positive, the graph of y has two x-intercepts. Also, because $a > 0$, the graph opens up and has a minimum value. So, the vertex of the graph lies below the x-axis.

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64.

$$\begin{aligned} y &= 3x^2 - 6x + 3 \\ b^2 - 4ac &= (-6)^2 - 4(3)(3) \\ &= 36 - 36 \\ &= 0 \end{aligned}$$

Because the discriminant is 0, the vertex lies on the x-axis.

65.

$$\begin{aligned} y &= 6x^2 - 2x + 4 \\ b^2 - 4ac &= (-2)^2 - 4(6)(4) \\ &= 4 - 96 \\ &= -92 \end{aligned}$$

Because the discriminant is negative, the graph of y has no x-intercepts. Also, because $a > 0$, the graph opens up and has a minimum value. So, the vertex of the graph lies above the x-axis.

66.

$$\begin{aligned} y &= -15x^2 + 10x - 25 \\ b^2 - 4ac &= 10^2 - 4(-15)(-25) \\ &= 100 - 1500 \\ &= -1400 \end{aligned}$$

Because the discriminant is negative, the graph of y has no x-intercepts. Also, because $a < 0$, the graph opens down and has a maximum value. So, the vertex of the graph lies below the x-axis.

67.

$$\begin{aligned} f(x) &= -3x^2 - 4x + 8 \\ b^2 - 4ac &= (-4)^2 - 4(-3)(8) \\ &= 16 + 96 \\ &= 112 \end{aligned}$$

Because the discriminant is positive, the graph of y has two x-intercepts. Also, because $a < 0$, the graph opens down and has a maximum value. So, the vertex of the graph lies above the x-axis.

68.

$$\begin{aligned} f(x) &= 9x^2 - 24x + 16 \\ b^2 - 4ac &= (-24)^2 - 4(9)(16) \\ &= 576 - 576 \\ &= 0 \end{aligned}$$

Because the discriminant is 0, the vertex lies on the x-axis.

69.

$$\begin{aligned} h &= -11t^2 + 700t + 21{,}000 \\ 30{,}800 &= -11t^2 + 700t + 21{,}000 \\ 30{,}800 - 30{,}800 &= -11t^2 + 700t + 21{,}000 - 30{,}800 \\ 0 &= -11t^2 + 700t - 9800 \\ t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-700 \pm \sqrt{700^2 - 4(-11)(-9800)}}{2(-11)} \\ &= \frac{-700 \pm \sqrt{490{,}000 - 431{,}200}}{-22} \\ &= \frac{-700 \pm \sqrt{58{,}800}}{-22} \end{aligned}$$

The solutions are $t = \dfrac{-700 + \sqrt{58{,}800}}{-22} \approx 20.8$ and

$t = \dfrac{-700 - \sqrt{58{,}800}}{-22} \approx 42.8$. So, the height is 30,800 feet

after about 20.8 seconds and again after about 42.8 seconds. The passengers experience weightlessness for about $42.8 - 20.8 = 22$ seconds.

70.

$$\begin{aligned} (x + 1)\left(x + \tfrac{1}{4}\right) &= 0 \\ x(x) + x\left(\tfrac{1}{4}\right) + 1(x) + 1\left(\tfrac{1}{4}\right) &= 0 \\ x^2 + \tfrac{1}{4}x + 1x + \tfrac{1}{4} &= 0 \\ 4 \bullet x^2 + 4 \bullet \tfrac{1}{4}x + 4 \bullet 1x + 4 \bullet \tfrac{1}{4} &= 4 \bullet 0 \\ 4x^2 + x + 4x + 1 &= 0 \\ 4x^2 + 5x + 1 &= 0 \end{aligned}$$

So, $a = 4$, $b = 5$, and $c = 1$.

or

$$\begin{aligned} -4 \bullet x^2 - 4 \bullet \tfrac{1}{4}x - 4 \bullet 1x - 4 \bullet \tfrac{1}{4} &= -4 \bullet 0 \\ -4x^2 - x - 4x - 1 &= 0 \\ -4x^2 - 5x - 1 &= 0 \end{aligned}$$

So, $a = -4$, $b = -5$, and $c = -1$.

71. a.

$$\begin{aligned} \text{Perimeter} &= 4x + 3y \\ 1050 &= 4x + 3y \\ 1050 - 4x &= 4x - 4x + 3y \\ 1050 - 4x &= 3y \\ \frac{1050 - 4x}{3} &= \frac{3y}{3} \\ 350 - \frac{4}{3}x &= y \end{aligned}$$

So, $y = 350 - \dfrac{4}{3}x$.

b.

$$\text{Area} = \text{length} \bullet \text{width}$$
$$15{,}000 = x \bullet y$$
$$15{,}000 = xy$$
$$15{,}000 = x\left(350 - \frac{4}{3}x\right)$$
$$15{,}000 = x(350) - x\left(\frac{4}{3}x\right)$$
$$15{,}000 = 350x - \frac{4}{3}x^2$$
$$15{,}000 - 350x + \frac{4}{3}x^2 = 350x - 350 - \frac{4}{3}x^2 + \frac{4}{3}x^2$$
$$\frac{4}{3}x^2 - 350x + 15{,}000 = 0$$
$$\frac{3}{2} \bullet \frac{4}{3}x^2 - \frac{3}{2} \bullet 350x + \frac{3}{2} \bullet 15{,}000 = \frac{3}{2} \bullet 0$$
$$2x^2 - 525x + 22{,}500 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-(-525) \pm \sqrt{(-525)^2 - 4(2)(22{,}500)}}{2(2)}$$
$$= \frac{525 \pm \sqrt{275{,}625 - 180{,}000}}{4}$$
$$= \frac{525 \pm \sqrt{95{,}625}}{4}$$
$$= \frac{525 + \sqrt{95{,}625}}{4} \approx 208.6$$
$$y \approx 350 - \frac{4}{3}(208.6)$$
$$\approx 350 - 278.1$$
$$\approx 71.9$$

or

$$x = \frac{525 - \sqrt{95{,}625}}{4} \approx 53.9$$
$$y \approx 350 - \frac{4}{3}(53.9)$$
$$\approx 350 - 71.9$$
$$\approx 278.1$$

So, each pasture could have a length of about 209 feet and a width of about 72 feet, or each one could have a length of about 54 feet and a width of about 278 feet.

72. a. $h = -16t^2 + v_0t + s_0$

$h = -16t^2 + 45t + 2.5$

An equation that models this situation is $h = -16t^2 + 45t + 2.5$.

b.

$$h = -16t^2 + 45t + 2.5$$
$$5.5 = -16t^2 + 45t + 2.5$$
$$5.5 - 5.5 = -16t^2 + 45t + 2.5 - 5.5$$
$$0 = -16t^2 + 45t - 3$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-45 \pm \sqrt{45^2 - 4(-16)(-3)}}{2(-16)}$$
$$= \frac{-45 \pm \sqrt{2025 - 192}}{-32}$$
$$= \frac{-45 \pm \sqrt{1833}}{-32}$$

The solutions are $t = \frac{-45 + \sqrt{1833}}{-32} \approx 0.07$ and $t = \frac{-45 - \sqrt{1833}}{-32} \approx 2.74$. Choose the larger value because the ball is caught on its way back down. So, the football is in the air for about 2.74 seconds.

73.

$$\frac{1}{2}\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{1}{2}\left(\frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{1}{2}\left(\frac{-2b}{2a}\right)$$
$$= \frac{1}{2}\left(\frac{-b}{a}\right)$$
$$= -\frac{b}{2a}$$

The mean of the solutions is $-\frac{b}{2a}$, which is the x-coordinate of the vertex. The mean of the solutions is equal to the graph's axis of symmetry, which is where the vertex lies.

74. a. C; When the discriminant of a function is positive, the graph of the function has two x-intercepts.

b. A; When the discriminant of a function is 0, the graph of the function has one x-intercept.

c. B; When the discriminant of a function is negative, the graph of the function has no x-intercepts.

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75. $h = -16t^2 + v_0t + s_0$

$15 = -16t^2 + v_0t + 5.5$

$15 - 15 = -16t^2 + v_0t + 5.5 - 15$

$0 = -16t^2 + v_0t - 9.5$

In order for the vertex to occur at $h = 15$, the discriminant of $0 = -16t^2 + v_0t - 9.5$ must be 0.

$$b^2 - 4ac = 0$$
$$v_0^2 - 4(-16)(-9.5) = 0$$
$$v_0^2 - 608 = 0$$
$$\underline{+\,608} \quad \underline{+\,608}$$
$$v_0^2 = 608$$
$$\sqrt{v_0^2} = \sqrt{608}$$
$$v_0 = \pm\sqrt{608}$$

Use the positive solution. So, the minimum initial vertical velocity needed to reach the branch is $\sqrt{608}$, or about 24.7 feet per second.

76. $x = -\dfrac{b}{2a}$ is the axis of symmetry, and $\dfrac{\sqrt{b^2 - 4ac}}{2a}$ is the horizontal distance from the axis of symmetry to each x-intercept.

77. $$\frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2b}{2a} = -\frac{b}{a}$$

The sum is $-\dfrac{b}{a}$.

$$\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(\frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)$$
$$= \frac{(-b)^2 - \left(\sqrt{b^2 - 4ac}\right)^2}{(2a)(2a)}$$
$$= \frac{b^2 - \left(b^2 - 4ac\right)}{4a^2}$$
$$= \frac{b^2 - b^2 + 4ac}{4a^2}$$
$$= \frac{4ac}{4a^2} = \frac{c}{a}$$

The product is $\dfrac{c}{a}$.

Sample answer: $-\dfrac{b}{a} = 2$ and $\dfrac{c}{a} = \dfrac{1}{2}$

Let $a = 2$.

Then, $-\dfrac{b}{2} = 2$ and $\dfrac{c}{2} = \dfrac{1}{2}$

$-2 \cdot \left(-\dfrac{b}{2}\right) = -2 \cdot 2$ $\qquad$ $2 \cdot \dfrac{c}{2} = 2 \cdot \dfrac{1}{2}$

$b = -4$ $\qquad$ $c = 1$

So, one possible quadratic equation is $0 = 2x^2 - 4x + 1$.

78. $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1 - 4ac}}{2a}$$

A formula that can be used to find solutions of equations that have the form $ax^2 + x + c = 0$ is $x = \dfrac{-1 \pm \sqrt{1 - 4ac}}{2a}$.

For, $-2x^2 + x + 8 = 0$,

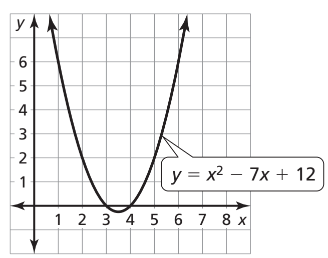
$$x = \frac{-1 \pm \sqrt{1 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1 - 4(-2)(8)}}{2(-2)}$$
$$= \frac{-1 \pm \sqrt{1 + 64}}{-4}$$
$$= \frac{-1 \pm \sqrt{65}}{-4}$$

The solutions are $x = \dfrac{-1 + \sqrt{65}}{-4} \approx -1.77$ and $x = \dfrac{-1 - \sqrt{65}}{-4} \approx 2.27$.

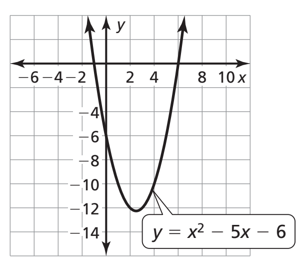
79. a.

Solutions	Factors	Quadratic equation
3, 4	$(x - 3), (x - 4)$	$x^2 - 7x + 12 = 0$
$-1, 6$	$(x + 1), (x - 6)$	$x^2 - 5x - 6 = 0$
0, 2	$x, (x - 2)$	$x^2 - 2x = 0$
$-\frac{1}{2}, 5$	$\left(x + \frac{1}{2}\right), (x - 5)$	$x^2 - \frac{9}{2}x - \frac{5}{2} = 0$

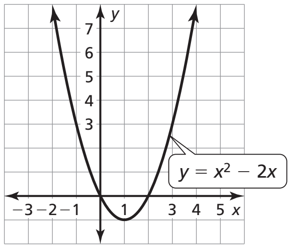
b.



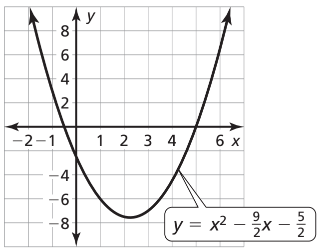
The zeros of the function are 3 and 4.



The zeros of the function are -1 and 6.



The zeros of the function are 0 and 2.



The zeros of the function are $-\frac{1}{2}$ and 5.

80. a. $2x^2 + x + 3k = 0$

$$b^2 - 4ac > 0$$
$$1^2 - 4(2)(3k) > 0$$
$$1 - 24k > 0$$
$$\underline{-1} \qquad \underline{-1}$$
$$-24k > -1$$
$$\frac{-24k}{-24} < \frac{-1}{-24}$$
$$k < \frac{1}{24}$$

The equation $2x^2 + x + 3k = 0$ has two solutions when $k < \frac{1}{24}$.

b.
$$b^2 - 4ac = 0$$
$$1^2 - 4(2)(3k) = 0$$
$$1 - 24k = 0$$
$$\underline{-1} \qquad \underline{-1}$$
$$-24k = -1$$
$$\frac{-24k}{-24} = \frac{-1}{-24}$$
$$k = \frac{1}{24}$$

The equation $2x^2 + x + 3k = 0$ has one solution when $k = \frac{1}{24}$.

c.
$$b^2 - 4ac < 0$$
$$1^2 - 4(2)(3k) < 0$$
$$1 - 24k < 0$$
$$\underline{-1} \qquad \underline{-1}$$
$$-24k < -1$$
$$\frac{-24k}{-24} > \frac{-1}{-24}$$
$$k > \frac{1}{24}$$

The equation $2x^2 + x + 3k = 0$ has no solutions when $k > \frac{1}{24}$.

81. a. $x^2 - 4kx + 36 = 0$

$$b^2 - 4ac > 0$$
$$(-4k)^2 - 4(1)(36) > 0$$
$$16k^2 - 144 > 0$$
$$\underline{+ 144} \qquad \underline{+ 144}$$
$$16k^2 > 144$$
$$\frac{16k^2}{16} > \frac{144}{16}$$
$$k^2 > 9$$
$$\sqrt{k^2} > \sqrt{9}$$
$$|k| > 3$$

So, when $k > 3$ or $k < -3$, the equation $x^2 - 4kx + 36 = 0$ has two solutions.

b.
$$b^2 - 4ac = 0$$
$$(-4k)^2 - 4(1)(36) = 0$$
$$16k^2 - 144 = 0$$
$$\underline{+ 144} \qquad \underline{+ 144}$$
$$16k^2 = 144$$
$$\frac{16k^2}{16} = \frac{144}{16}$$
$$k^2 = 9$$
$$\sqrt{k^2} = \sqrt{9}$$
$$k = \pm 3$$

So, when $k = 3$ or $k = -3$, the equation $x^2 - 4kx + 36 = 0$ has one solution.

c.
$$b^2 - 4ac < 0$$
$$(-4k)^2 - 4(1)(36) < 0$$
$$16k^2 - 144 < 0$$
$$\underline{+ 144} \qquad \underline{+ 144}$$
$$16k^2 < 144$$
$$\frac{16k^2}{16} < \frac{144}{16}$$
$$k^2 < 9$$
$$\sqrt{k^2} < \sqrt{9}$$
$$|k| < 3$$

So, when $-3 < k < 3$, the equation $x^2 - 4kx + 36 = 0$ has no solutions.

82. $kx^2 + 5x - 16 = 0$

a.
$$b^2 - 4ac > 0$$
$$5^2 - 4(k)(-16) > 0$$
$$25 + 64k > 0$$
$$\underline{-25} \qquad \underline{-25}$$
$$64k > -25$$
$$\frac{64k}{64} > \frac{-25}{64}$$
$$k > -\frac{25}{64}$$

So, the equation $kx^2 + 5x - 16 = 0$ has two solutions when $k > -\frac{25}{64}$.

b.
$$b^2 - 4ac = 0$$
$$5^2 - 4(k)(-16) = 0$$
$$25 + 64k = 0$$
$$\underline{-25} \qquad \underline{-25}$$
$$64k = -25$$
$$\frac{64k}{64} = \frac{-25}{64}$$
$$k = -\frac{25}{64}$$

So, the equation $kx^2 + 5x - 16 = 0$ has one solution when $k = -\frac{25}{64}$.

c.
$$b^2 - 4ac < 0$$
$$5^2 - 4(k)(-16) < 0$$
$$25 + 64k < 0$$
$$\underline{-25} \qquad \underline{-25}$$
$$64k < -25$$
$$\frac{64k}{64} < \frac{-25}{64}$$
$$k < -\frac{25}{64}$$

So, the equation $kx^2 + 5x - 16 = 0$ has no solution when $k < -\frac{25}{64}$.

Maintaining Mathematical Proficiency

83. *Sample answer:* Use substitution because both equations are solved for y.

$y = -x + 4$

$y = 2x - 8$

Step 2 $y = -x + 4$
$$2x - 8 = -x + 4$$
$$\underline{+x} \qquad \underline{+x}$$
$$3x - 8 = 4$$
$$\underline{+8} \qquad \underline{+8}$$
$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$

Step 3 $y = -x + 4$
$$y = -4 + 4$$
$$y = 0$$

The solution is $(4, 0)$.

84. *Sample answer:* Use substitution because one equation is solved for x.

$x = 16 - 4y$

$3x + 4y = 8$

Step 2 $3x + 4y = 8$
$$3(16 - 4y) + 4y = 8$$
$$3(16) - 3(4y) + 4y = 8$$
$$48 - 12y + 4y = 8$$
$$48 - 8y = 8$$
$$\underline{-48} \qquad \underline{-48}$$
$$-8y = -40$$
$$\frac{-8y}{-8} = \frac{-40}{-8}$$
$$y = 5$$

Step 3 $x = 16 - 4y$
$$x = 16 - 4(5)$$
$$x = 16 - 20$$
$$x = -4$$

The solution is $(-4, 5)$.

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85. *Sample answer:* Use elimination because one pair of like terms has the same coefficient.

Step 2
$$\begin{array}{r} 2x - y = 7 \\ \underline{-(2x + 7y = 31)} \\ -8y = -24 \end{array}$$

Step 3
$$-8y = -24$$
$$\frac{-8y}{-8} = \frac{-24}{-8}$$
$$y = 3$$

Step 4
$$2x - y = 7$$
$$2x - 3 = 7$$
$$\underline{+3} \quad \underline{+3}$$
$$2x = 10$$
$$\frac{2x}{2} = \frac{10}{2}$$
$$x = 5$$

The solution is $(5, 3)$.

86. *Sample answer:* Use substitution because one of the variables has a coefficient of 1.

$3x - 2y = -20$

$x + 1.2y = 6.4$

Step 1
$$x + 1.2y = 6.4$$
$$x + 1.2y - 1.2y = 6.4 - 1.2y$$
$$x = 6.4 - 1.2y$$

Step 2
$$3x - 2y = -20$$
$$3(6.4 - 1.2y) - 2y = -20$$
$$3(6.4) - 3(1.2y) - 2y = -20$$
$$19.2 - 3.6y - 2y = -20.$$
$$19.2 - 5.6y = -20$$
$$\underline{-19.2} \qquad \underline{-19.2}$$
$$-5.6y = -39.2$$
$$\frac{-5.6y}{-5.6} = \frac{-39.2}{-5.6}$$
$$y = 7$$

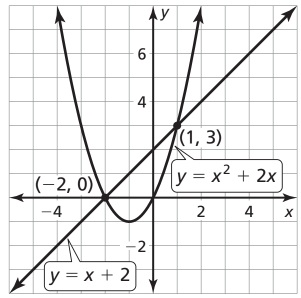
Step 3
$$x + 1.2y = 6.4$$
$$x + 1.2(7) = 6.4$$
$$x + 8.4 = 6.4$$
$$\underline{-8.4} \quad \underline{-8.4}$$
$$x = -2$$

The solution is $(-2, 7)$.

9.6 Explorations (p. 525)

1. $y = x + 2$

$y = x^2 + 2x$



The graphs appear to intersect at $(-2, 0)$ and $(1, 3)$.

Check

$y = x + 2$	$y = x^2 + 2x$
$0 \stackrel{?}{=} -2 + 2$	$0 \stackrel{?}{=} (-2)^2 + 2(-2)$
$0 = 0$ ✓	$0 \stackrel{?}{=} 4 - 4$
	$0 = 0$ ✓

$y = x + 2$	$y = x^2 + 2x$
$3 \stackrel{?}{=} 1 + 2$	$3 \stackrel{?}{=} 1^2 + 2(1)$
$3 = 3$ ✓	$3 \stackrel{?}{=} 1 + 2$
	$3 = 3$ ✓

The solutions are $(-2, 0)$ and $(1, 3)$.

2. a. A; The graph of $y = x^2 - 4$ is a vertical shift 4 units down of the graph of $y = x^2$. The graph of $y = -x - 2$ is a straight line with a negative slope and a y-intercept of -2. So, this system matches graph A.

The graph appears to intersect at $(-2, 0)$ and $(1, -3)$.

Check

$y = x^2 - 4$	$y = -x - 2$
$0 \stackrel{?}{=} (-2)^2 - 4$	$0 \stackrel{?}{=} -(-2) - 2$
$0 \stackrel{?}{=} 4 - 4$	$0 \stackrel{?}{=} 2 - 2$
$0 = 0$ ✓	$0 = 0$ ✓

$y = x^2 - 4$	$y = -x - 2$
$-3 \stackrel{?}{=} 1^2 - 4$	$-3 \stackrel{?}{=} -1 - 2$
$-3 \stackrel{?}{=} 1 - 4$	$-3 = -3$ ✓
$-3 = -3$ ✓	

The solutions are $(-2, 0)$ and $(1, -3)$.

b. C; The graph of $y = x^2 - 2x + 2$ is a parabola with a y-intercept of $c = 2$. The graph of $y = 2x - 2$ is a straight line with a slope of 2 and a y-intercept of -2. So, this system matches graph C.

The graph appears to intersect at $(2, 2)$

Check

$y = x^2 - 2x + 2$	$y = 2x - 2$
$2 \stackrel{?}{=} 2^2 - 2(2) + 2$	$2 \stackrel{?}{=} 2(2) - 2$
$2 \stackrel{?}{=} 4 - 4 + 2$	$2 \stackrel{?}{=} 4 - 2$
$2 \stackrel{?}{=} 0 + 2$	$2 = 2$ ✓
$2 = 2$ ✓	

The solution is $(2, 2)$.

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c. B; The graph of $y = x^2 + 1$ is a vertical shift 1 unit up of the graph of $y = x^2$. The graph of $y = x - 1$ is a straight line with a slope of 1 and a y-intercept of -1. So, this system matches graph B.

The graphs do not intersect. So, the system has no solution.

d. D; The graph of $y = x^2 - x - 6$ is a parabola with a y-intercept of $c = -6$. The graph of $y = 2x - 2$ is a straight line with a slope of 2 and a y-intercept of -2. So, this system matches graph D.

The graphs appear to intersect at $(-1, -4)$ and $(4, 6)$.

Check $y = x^2 - x - 6$ $\qquad$ $y = 2x - 2$

$-4 \stackrel{?}{=} (-1)^2 - (-1) - 6$ $\qquad$ $-4 \stackrel{?}{=} 2(-1) - 2$

$-4 \stackrel{?}{=} 1 + 1 - 6$ $\qquad$ $-4 \stackrel{?}{=} -2 - 2$

$-4 \stackrel{?}{=} 2 - 6$ $\qquad$ $-4 = -4$ ✓

$-4 = -4$ ✓

$y = x^2 - x - 6$ $\qquad$ $y = 2x - 2$

$6 \stackrel{?}{=} 4^2 - 4 - 6$ $\qquad$ $6 \stackrel{?}{=} 2(4) - 2$

$6 \stackrel{?}{=} 16 - 4 - 6$ $\qquad$ $6 \stackrel{?}{=} 8 - 2$

$6 \stackrel{?}{=} 12 - 6$ $\qquad$ $6 = 6$ ✓

$6 = 6$ ✓

The solutions are $(-1, -4)$ and $(4, 6)$.

3. In order to solve a system of two equations, graph the equations in the same coordinate plane and find the point(s) of intersection.

4. a. *Sample answer:* A system of equations that has no solutions is $y = x + 1$, $y = x^2 + 5$.

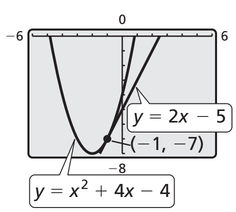
b. *Sample answer:* A system of equations that has one solution is $y = 5$, $y = x^2 + 5$.

c. *Sample answer:* A system of equations that has two solutions is $y = x + 8$, $y = x^2 + 5$.

9.6 Monitoring Progress (pp. 526–529)

1. $y = x^2 + 4x - 4$

$y = 2x - 5$



The graphs appear to intersect at $(-1, -7)$.

Check $y = x^2 + 4x - 4$ $\qquad$ $y = 2x - 5$

$-7 \stackrel{?}{=} (-1)^2 + 4(-1) - 4$ $\qquad$ $-7 \stackrel{?}{=} 2(-1) - 5$

$-7 \stackrel{?}{=} 1 - 4 - 4$ $\qquad$ $-7 \stackrel{?}{=} 2 - 5$

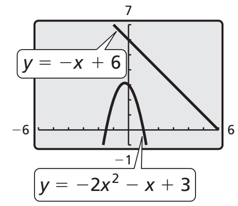
$-7 \stackrel{?}{=} -3 - 4$ $\qquad$ $-7 = -7$ ✓

$-7 = -7$ ✓

The solution is $(-1, -7)$.

2. $y = -x + 6$

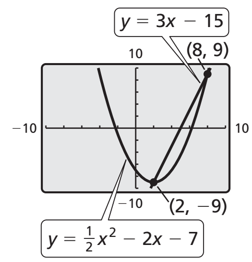
$y = -2x^2 - x + 3$



The graphs do not intersect. So, the system has no real solutions.

3. $y = 3x - 15$

$y = \frac{1}{2}x^2 - 2x - 7$



The graphs appear to intersect at $(2, -9)$ and $(8, 9)$.

Check $y = 3x - 15$ $\qquad$ $y = \frac{1}{2}x^2 - 2x - 7$

$-9 \stackrel{?}{=} 3(2) - 15$ $\qquad$ $-9 \stackrel{?}{=} \frac{1}{2}(2)^2 - 2(2) - 7$

$-9 \stackrel{?}{=} 6 - 15$ $\qquad$ $-9 \stackrel{?}{=} \frac{1}{2}(4) - 4 - 7$

$-9 = -9$ ✓ $\qquad$ $-9 \stackrel{?}{=} 2 - 4 - 7$

$-9 \stackrel{?}{=} -2 - 7$

$-9 = -9$ ✓

$y = 3x - 15$ $\qquad$ $y = \frac{1}{2}x^2 - 2x - 7$

$9 \stackrel{?}{=} 3(8) - 15$ $\qquad$ $9 \stackrel{?}{=} \frac{1}{2}(8)^2 - 2(8) - 7$

$9 \stackrel{?}{=} 24 - 15$ $\qquad$ $9 \stackrel{?}{=} \frac{1}{2}(64) - 16 - 7$

$9 = 9$ ✓ $\qquad$ $9 \stackrel{?}{=} 32 - 16 - 7$

$9 \stackrel{?}{=} 16 - 7$

$9 = 9$ ✓

The solutions are $(2, -9)$ and $(8, 9)$.

4. $y = x^2 + 9$

$y = 9$

Step 2 $y = x^2 + 9$

$$9 = x^2 + 9$$
$$\underline{-9} \qquad \underline{-9}$$
$$0 = x^2$$
$$\sqrt{0} = \sqrt{x^2}$$
$$0 = x$$

Step 3 $y = 9$

The solution is $(0, 9)$.

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5. $y = -5x$

$y = x^2 - 3x - 3$

Step 2
$$\begin{aligned} y &= x^2 - 3x - 3 \\ -5x &= x^2 - 3x - 3 \\ -5x + 5x &= x^2 - 3x - 3 + 5x \\ 0 &= x^2 + 2x - 3 \\ 0 &= (x + 3)(x - 1) \end{aligned}$$
$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$
$$\underline{-3 \quad -3} \qquad \underline{+1 \quad +1}$$
$$x = -3 \qquad x = 1$$

Step 3
$$\begin{aligned} y &= -5x \\ y &= -5(-3) \\ y &= 15 \\ y &= -5x \\ y &= -5(1) \\ y &= -5 \end{aligned}$$

The solutions are $(-3, 15)$ and $(1, -5)$.

6. $y = -3x^2 + 2x + 1$

$y = 5 - 3x$

Step 2
$$\begin{aligned} y &= -3x^2 + 2x + 1 \\ 5 - 3x &= -3x^2 + 2x + 1 \\ 5 - 5 - 3x + 3x &= -3x^2 + 2x + 1 - 5 + 3x \\ 0 &= -3x^2 + 5x - 4 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{5^2 - 4(-3)(-4)}}{2(-3)} \\ &= \frac{-5 \pm \sqrt{25 - 48}}{-6} \\ &= \frac{-5 \pm \sqrt{-23}}{-6} \end{aligned}$$

The discriminant is negative. So, the equation has no real solutions, which means the system has no real solutions.

7. Step 2
$$\begin{aligned} y &= x^2 + x \\ \underline{-(y} &\underline{= \qquad x + 5)} \\ 0 &= x^2 \qquad - 5 \end{aligned}$$

Step 3
$$\begin{aligned} 0 &= x^2 - 5 \\ \underline{+5} &\quad \underline{+5} \\ 5 &= x^2 \\ \sqrt{5} &= \sqrt{x^2} \\ \pm\sqrt{5} &= x \end{aligned}$$

Step 4 $y = x + 5$ $\qquad$ $y = x + 5$

$y = \sqrt{5} + 5$ $\qquad$ $y = -\sqrt{5} + 5$

The solutions are $\left(\sqrt{5}, 5 + \sqrt{5}\right) \approx (2.24, 7.24)$ and $\left(-\sqrt{5}, 5 - \sqrt{5}\right) \approx (-2.24, 2.76)$.

8. Step 2
$$\begin{aligned} y &= 9x^2 + 8x - 6 \\ \underline{-(y} &\underline{= \qquad 5x - 4)} \\ 0 &= 9x^2 + 3x - 2 \end{aligned}$$

Step 3 $0 = (3x + 2)(3x - 1)$
$$3x + 2 = 0 \quad \text{or} \quad 3x - 1 = 0$$
$$\underline{-2 \quad -2} \qquad \underline{+1 \quad +1}$$
$$3x = -2 \qquad 3x = 1$$
$$\frac{3x}{3} = -\frac{2}{3} \qquad \frac{3x}{3} = \frac{1}{3}$$
$$x = -\frac{2}{3} \qquad x = \frac{1}{3}$$

Step 4
$$\begin{aligned} y &= 5x - 4 \\ y &= 5\left(-\tfrac{2}{3}\right) - 4 \\ &= -\tfrac{10}{3} - 4 \\ &= -\tfrac{22}{3} \end{aligned} \qquad \begin{aligned} y &= 5x - 4 \\ &= 5\left(\tfrac{1}{3}\right) - 4 \\ &= \tfrac{5}{3} - 4 \\ &= -\tfrac{7}{3} \end{aligned}$$

The solutions are $\left(-\frac{2}{3}, -\frac{22}{3}\right)$ and $\left(\frac{1}{3}, -\frac{7}{3}\right)$.

9. Step 2
$$\begin{aligned} y &= -3x^2 + x - 4 \\ \underline{-(y} &\underline{= \qquad 2x + 5)} \\ 0 &= -3x^2 - x - 9 \end{aligned}$$

Step 3
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-3)(-9)}}{2(-1)} \\ &= \frac{1 \pm \sqrt{1 - 108}}{-2} \\ &= \frac{1 \pm \sqrt{-107}}{-2} \end{aligned}$$

The discriminant is negative. So, the equation has no real solutions, which means the system has no real solutions.

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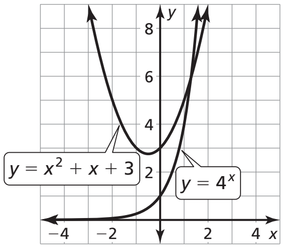
10. $y = 4^x$

$y = x^2 + x + 3$



The graphs intersect between $x = 1$ and $x = 2$.

$$y = x^2 + x + 3$$
$$4^x = x^2 + x + 3$$
$$4^x - x^2 - x - 3 = x^2 - x^2 + x - x + 3 - 3$$
$$4^x - x^2 - x - 3 = 0$$

Let $f(x) = 4^x - x^2 - x - 3$.

$f(1.2) = -0.362$

$f(1.29) \approx 0.025$

$f(1.288) \approx 0.016$

$f(1.286) \approx 0.007$

$f(1.284) \approx -0.003$

$f(1.3) \approx 0.073$

$f(1.289) \approx 0.021$

$f(1.287) \approx 0.011$

$f(1.285) \approx 0.002$

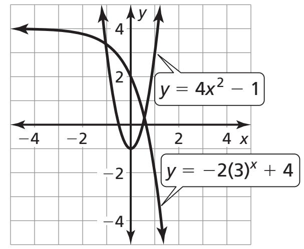
Because $f(1.285)$ is closest to 0, $x \approx 1.285$.

$y = 4^x \approx 4^{1.285} \approx 5.938$

So, the solution of the system is about (1.285, 5.938).

11. $y = 4x^2 - 1$

$y = -2(3)^x + 4$



The graphs intersect between $x = -2$ and $x = -1$ and again between $x = 0$ and $x = 1$.

$$y = -2(3)^x + 4$$
$$4x^2 - 1 = -2(3)^x + 4$$
$$4x^2 - 1 + 2(3)^x - 4 = -2(3)^x + 2(3)^x + 4 - 4$$
$$2(3)^x + 4x^2 - 5 = 0$$

Let $f(x) = 2(3)^x + 4x^2 - 5$.

$f(-1.1) \approx 0.437$

$f(-1.0) \approx -0.333$

$f(-1.01) \approx -0.260$

$f(-1.02) \approx -0.186$

$f(-1.03) \approx -0.111$

$f(-1.04) \approx -0.036$

$f(-1.041) \approx -0.028$

$f(-1.042) \approx -0.020$

$f(-1.043) \approx -0.013$

$f(-1.044) \approx -0.005$

$f(-1.045) \approx 0.003$

So, $x \approx -1.045$.

$y = 4x^2 - 1$

$\approx 4(-1.045)^2 - 1$

≈ 3.368

$f(0.5) \approx -0.536$

$f(0.6) \approx 0.306$

$f(0.59) \approx 0.217$

$f(0.58) \approx 0.128$

$f(0.57) \approx 0.041$

$f(0.569) \approx 0.032$

$f(0.568) \approx 0.023$

$f(0.567) \approx 0.015$

$f(0.566) \approx 0.006$

$f(0.565) \approx -0.003$

So, $x \approx 0.565$.

$y = 4x^2 - 1$

$= 4(0.565)^2 - 1$

≈ 0.277

So, the solutions of the system are about $(-1.045, 3.368)$ and about (0.565, 0.277).

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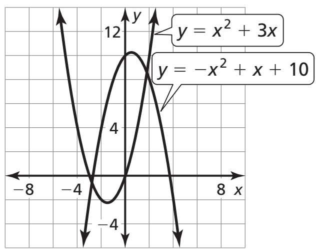
12. $y = x^2 + 3x$

$y = -x^2 + x + 10$



The graphs intersect between $x = -3$ and $x = -2$ and again between $x = 1$ and $x = 2$.

$$y = -x^2 + x + 10$$
$$x^2 + 3x = -x^2 + x + 10$$
$$x^2 + 3x + x^2 - x - 10 = -x^2 + x^2 + x - x + 10 - 10$$
$$2x^2 + 2x - 10 = 0$$

Let $f(x) = 2x^2 + 2x - 10$.

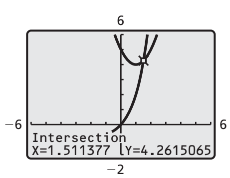
$f(1.7) \approx -0.82$	$f(-2.8) \approx 0.08$
$f(1.8) \approx 0.08$	$f(-2.7) \approx -0.82$
$f(1.79) \approx -0.012$	$f(-2.79) \approx -0.012$
$f(1.791) \approx -0.003$	$f(-2.791) \approx -0.003$
$f(1.792) \approx 0.007$	$f(-2.792) \approx 0.007$
$f(1.791)$ is closest to 0.	$f(-2.791)$ is closest to 0.
So, $x \approx 1.791$.	So, $x \approx -2.791$.
$y = x^2 + 3x$	$y = x^2 + 3x$
$\approx (1.791)^2 + 3(1.791)$	$\approx (-2.791)^2 + 3(-2.791)$
≈ 8.581	≈ -0.583

The solutions are about $(1.791, 8.581)$ and about $(-2.791, -0.583)$.

13. $3^x - 1 = x^2 - 2x + 5$

$y = 3^x - 1$

$y = x^2 - 2x + 5$

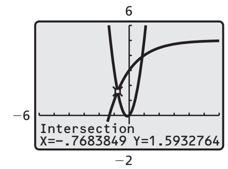


The point of intersection is about $(1.51, 4.26)$. So, the solution of the equation is $x \approx 1.51$.

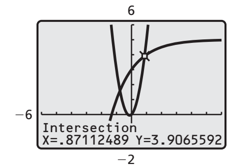
14. $4x^2 + x = -2\left(\frac{1}{2}\right)^x + 5$

$y = 4x^2 + x$

$y = -2\left(\frac{1}{2}\right)^x + 5$



The point of intersection is about $(-0.77, 1.59)$.



The point of intersection is about $(0.87, 3.91)$.

So, the solutions of the equation are $x \approx -0.77$ and $x \approx 0.87$.

9.6 Exercises (pp. 530–532)

Vocabulary and Core Concept Check

1. In order to use substitution to solve a system of nonlinear equations, solve one of the equations for one of the variables, substitute into the other equation, and solve.

2. *Sample answer:* Both types of systems can be solved by graphing. Some nonlinear systems cannot be solved algebraically.

Monitoring Progress and Modeling with Mathematics

3. B; The graph of $y = x^2 - 2x + 1$ is a parabola that opens up (because $a > 0$) and has a y-intercept of 1 (because $c = 1$). The graph of $y = x + 1$ is a straight line with a slope of 1 and a y-intercept of 1. So, the system matches graph B.

The graphs appear to intersect at $(0, 1)$ and $(3, 4)$.

Check

$y = x^2 - 2x + 1$	$y = x + 1$
$1 \stackrel{?}{=} 0^2 - 2(0) + 1$	$1 \stackrel{?}{=} 0 + 1$
$1 \stackrel{?}{=} 0 - 0 + 1$	$1 = 1$ ✓
$1 \stackrel{?}{=} 0 + 1$	
$1 = 1$ ✓	
$y = x^2 - 2x + 1$	$y = x + 1$
$4 \stackrel{?}{=} 3^2 - 2(3) + 1$	$4 \stackrel{?}{=} 3 + 1$
$4 \stackrel{?}{=} 9 - 6 + 1$	$4 = 4$ ✓
$4 \stackrel{?}{=} 3 + 1$	
$4 = 4$ ✓	

The solutions are $(0, 1)$ and $(3, 4)$.

4. D; The graph of $y = x^2 + 3x + 2$ is a parabola that opens up (because $a > 0$) and has a y-intercept of 2 (because $c = 2$). The graph of $y = -x - 3$ is a straight line with a slope of -1 and a y-intercept of -3. So, the system matches graph D.

The graphs do not intersect. So, the system has no real solutions.

Chapter 9

5. A; The graph of $y = x - 1$ is a straight line with a slope of 1 and a y-intercept of -1. The graph of $y = -x^2 + x - 1$ is a parabola that opens down (because $a < 0$) and has a y-intercept of -1 (because $c = -1$). So, the system matches graph A.

The graphs have the same y-intercept and intersect at this point $(0, -1)$.

Check $y = x - 1$ | $y = -x^2 + x - 1$

$-1 \stackrel{?}{=} 0 - 1$ | $-1 \stackrel{?}{=} -0^2 + 0 - 1$

$-1 = -1$ ✓ | $-1 \stackrel{?}{=} 0 + 0 - 1$

$-1 = -1$ ✓

The solution of the system is $(0, -1)$.

6. C; The graph of $y = -x + 3$ is a straight line with a slope of -1 and a y-intercept of 3. The graph of $y = -x^2 - 2x + 5$ is a parabola that opens down (because $a < 0$) and has a y-intercept of 5 (because $c = 5$). So, the system matches graph C.

The graphs appear to intersect at $(-2, 5)$ and $(1, 2)$.

Check $y = -x + 3$ | $y = -x^2 - 2x + 5$

$5 \stackrel{?}{=} -(-2) + 3$ | $5 \stackrel{?}{=} -(-2)^2 - 2(-2) + 5$

$5 \stackrel{?}{=} 2 + 3$ | $5 \stackrel{?}{=} -4 + 4 + 5$

$5 = 5$ ✓ | $5 \stackrel{?}{=} 0 + 5$

$5 = 5$ ✓

$y = -x + 3$ | $y = -x^2 - 2x + 5$

$2 \stackrel{?}{=} -1 + 3$ | $2 \stackrel{?}{=} -1^2 - 2(1) + 5$

$2 = 2$ ✓ | $2 \stackrel{?}{=} -1 - 2 + 5$

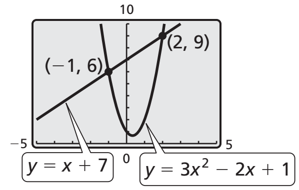
$2 \stackrel{?}{=} -3 + 5$

$2 = 2$ ✓

The solutions of the system are $(-2, 5)$ and $(1, 2)$.

7. $y = 3x^2 - 2x + 1$

$y = x + 7$



The graphs appear to intersect at $(-1, 6)$ and $(2, 9)$.

Check $y = 3x^2 - 2x + 1$ | $y = x + 7$

$6 \stackrel{?}{=} 3(-1)^2 - 2(-1) + 1$ | $6 \stackrel{?}{=} -1 + 7$

$6 \stackrel{?}{=} 3(1) + 2 + 1$ | $6 = 6$ ✓

$6 \stackrel{?}{=} 3 + 2 + 1$

$6 \stackrel{?}{=} 5 + 1$

$6 = 6$ ✓

$y = 3x^2 - 2x + 1$ | $y = x + 7$

$9 \stackrel{?}{=} 3(2)^2 - 2(2) + 1$ | $6 \stackrel{?}{=} -1 + 7$

$9 \stackrel{?}{=} 3(4) - 4 + 1$ | $6 = 6$ ✓

$9 \stackrel{?}{=} 12 - 4 + 1$

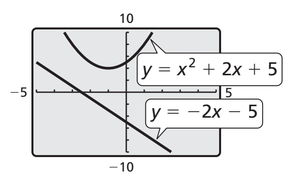
$9 \stackrel{?}{=} 8 + 1$

$9 = 9$ ✓

The solutions are $(2, 9)$ and $(-1, 6)$.

8. $y = x^2 + 2x + 5$

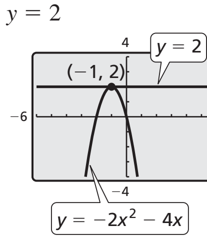
$y = -2x - 5$



The graphs do not intersect. So, the system has no solutions.

9. $y = -2x^2 - 4x$

$y = 2$



The graphs appear to intersect at $(-1, 2)$.

Check $y = -2x^2 - 4x$ | $y = 2$

$2 \stackrel{?}{=} -2(-1)^2 - 4(-1)$ | $2 = 2$ ✓

$2 \stackrel{?}{=} -2(1) + 4$

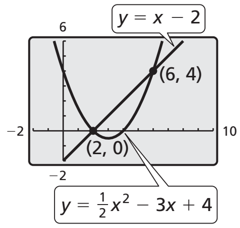
$2 \stackrel{?}{=} -2 + 4$

$2 = 2$ ✓

The solution is $(-1, 2)$.

10. $y = \frac{1}{2}x^2 - 3x + 4$

$y = x - 2$



The graphs appear to intersect at (2, 0) and (6, 4).

Check $y = \frac{1}{2}x^2 - 3x + 4$ $\qquad y = x - 2$

$0 \stackrel{?}{=} \frac{1}{2}(2)^2 - 3(2) + 4 \qquad 0 \stackrel{?}{=} 2 - 2$

$0 \stackrel{?}{=} \frac{1}{2}(4) - 6 + 4 \qquad 0 = 0$ ✓

$0 \stackrel{?}{=} 2 - 6 + 4$

$0 \stackrel{?}{=} -4 + 4$

$0 = 0$ ✓

$4 \stackrel{?}{=} \frac{1}{2}(6)^2 - 3(6) + 4 \qquad y = x - 2$

$4 \stackrel{?}{=} \frac{1}{2}(36) - 18 + 4 \qquad 4 \stackrel{?}{=} 6 - 2$

$4 \stackrel{?}{=} 18 - 18 + 4 \qquad 4 = 4$ ✓

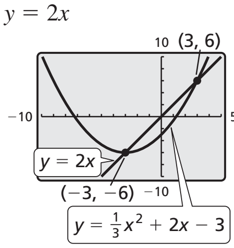
$4 \stackrel{?}{=} 0 + 4$

$4 = 4$ ✓

The solutions are (2, 0) and (6, 4).

11. $y = \frac{1}{3}x^2 + 2x - 3$

$y = 2x$



The graphs appear to intersect at (−3, −6) and (3, 6).

Check $y = \frac{1}{3}x^2 + 2x - 3 \qquad y = 2x$

$-6 \stackrel{?}{=} \frac{1}{3}(-3)^2 + 2(-3) - 3 \qquad -6 \stackrel{?}{=} 2(-3)$

$-6 \stackrel{?}{=} \frac{1}{3}(9) - 6 - 3 \qquad -6 = -6$ ✓

$-6 \stackrel{?}{=} 3 - 6 - 3$

$-6 \stackrel{?}{=} -3 - 3$

$-6 = -6$ ✓

$y = \frac{1}{3}x^2 + 2x - 3 \qquad y = 2x$

$6 \stackrel{?}{=} \frac{1}{3}(3)^2 + 2(3) - 3 \qquad 6 \stackrel{?}{=} 2(3)$

$6 \stackrel{?}{=} \frac{1}{3}(9) + 6 - 3 \qquad 6 = 6$ ✓

$6 \stackrel{?}{=} 3 + 6 - 3$

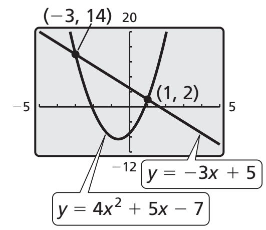
$6 \stackrel{?}{=} 9 - 3$

$6 = 6$ ✓

The solutions are (−3, −6) and (3, 6).

12. $y = 4x^2 + 5x - 7$

$y = -3x + 5$



The graphs appear to intersect at (−3, 14) and (1, 2).

Check $y = 4x^2 + 5x - 7 \qquad y = -3x + 5$

$14 \stackrel{?}{=} 4(-3)^2 + 5(-3) - 7 \qquad 14 \stackrel{?}{=} -3(-3) + 5$

$14 \stackrel{?}{=} 4(9) - 15 - 7 \qquad 14 \stackrel{?}{=} 9 + 5$

$14 \stackrel{?}{=} 36 - 15 - 7 \qquad 14 = 14$ ✓

$14 \stackrel{?}{=} 21 - 7$

$14 = 14$ ✓

$y = 4x^2 + 5x - 7 \qquad y = -3x + 5$

$2 \stackrel{?}{=} 4(1)^2 + 5(1) - 7 \qquad 2 \stackrel{?}{=} -3(1) + 5$

$2 \stackrel{?}{=} 4(1) + 5 - 7 \qquad 2 \stackrel{?}{=} -3 + 5$

$2 \stackrel{?}{=} 4 + 5 - 7 \qquad 2 = 2$ ✓

$2 \stackrel{?}{=} 9 - 7$

$2 = 2$ ✓

The solutions are (−3, 14) and (1, 2).

13. $y = x - 5$

$y = x^2 + 4x - 5$

Step 2

$$\begin{aligned} y &= x^2 + 4x - 5 \\ x - 5 &= x^2 + 4x - 5 \\ x - x - 5 + 5 &= x^2 + 4x - 5 - x + 5 \\ 0 &= x^2 + 3x \\ 0 &= x(x + 3) \end{aligned}$$

$x = 0 \quad$ or $\quad x + 3 = 0$

$\qquad\qquad\qquad\; \underline{-3 \quad -3}$

$\qquad\qquad\qquad\quad x = -3$

Step 3 $y = x - 5 \qquad y = x - 5$

$y = -3 - 5 \qquad y = 0 - 5$

$y = -8 \qquad y = -5$

The solutions are (−3, −8) and (0, −5).

Chapter 9

14. $y = -3x^2$

$y = 6x + 3$

Step 2
$$\begin{aligned} y &= 6x + 3 \\ -3x^2 &= 6x + 3 \\ -3x^2 + 3x^2 &= 6x + 3 + 3x^2 \\ 0 &= 3x^2 + 6x + 3 \\ \frac{0}{3} &= \frac{3x^2 + 6x + 3}{3} \\ 0 &= x^2 + 2x + 1 \\ 0 &= x^2 + 2(x)(1) + 1^2 \\ 0 &= (x + 1)^2 \\ x + 1 &= 0 \\ -1 \quad & \quad -1 \\ x &= -1 \end{aligned}$$

Step 3 $y = -3x^2$
$$\begin{aligned} y &= -3(-1)^2 \\ y &= -3(1) \\ y &= -3 \end{aligned}$$

The solution is $(-1, -3)$.

15. $y = -x + 7$

$y = -x^2 - 2x - 1$

Step 2
$$\begin{aligned} y &= -x^2 - 2x - 1 \\ -x + 7 &= -x^2 - 2x - 1 \\ -x + 7 + x^2 + 2x + 1 &= -x^2 + x^2 - 2x + 2x - 1 + 1 \\ x^2 + x + 8 &= 0 \end{aligned}$$
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1 \pm \sqrt{1^2 - 4(1)(8)}}{2(1)} \\ &= \frac{-1 \pm \sqrt{1 - 32}}{2} \\ &= \frac{-1 \pm \sqrt{-31}}{2} \end{aligned}$$

The discriminant is negative. So, the equation has no real solutions, and the system has no solutions.

16. $y = -x^2 + 7$

$y = 2x + 4$

Step 2
$$\begin{aligned} y &= -x^2 + 7 \\ 2x + 4 &= -x^2 + 7 \\ 2x + 4 + x^2 - 7 &= -x^2 + x^2 + 7 - 7 \\ x^2 + 2x - 3 &= 0 \\ (x + 3)(x - 1) &= 0 \end{aligned}$$
$$\begin{aligned} x + 3 &= 0 & \text{or} \quad x - 1 &= 0 \\ -3 \quad & \quad -3 & +1 \quad & \quad +1 \\ x &= -3 & x &= 1 \end{aligned}$$

Step 3 $y = 2x + 4$
$$\begin{aligned} y &= 2(-3) + 4 \\ y &= -6 + 4 \\ y &= -2 \end{aligned}$$

$$\begin{aligned} y &= 2x + 4 \\ y &= 2(1) + 4 \\ y &= 2 + 4 \\ y &= 6 \end{aligned}$$

The solutions are $(-3, -2)$ and $(1, 6)$.

17. $y - 5 = -x^2$

$y = 5$

Step 2 $y - 5 = -x^2$
$$\begin{aligned} 5 - 5 &= -x^2 \\ 0 &= -x^2 \\ \frac{0}{-1} &= \frac{-x^2}{-1} \\ 0 &= x^2 \\ \sqrt{0} &= \sqrt{x^2} \\ 0 &= x \end{aligned}$$

Step 3 $y = 5$

The solution is $(0, 5)$.

Chapter 9

18. $y = 2x^2 + 3x - 4$

$y - 4x = 2$

Step 2

$$\begin{aligned} y - 4x &= 2 \\ 2x^2 + 3x - 4 - 4x &= 2 \\ 2x^2 - x - 4 &= 2 \\ 2x^2 - x - 4 - 2 &= 2 - 2 \\ 2x^2 - x - 6 &= 0 \\ (2x + 3)(x - 2) &= 0 \end{aligned}$$

$$2x + 3 = 0 \quad \text{or} \quad x - 2 = 0$$

$$\underline{-3 \quad -3} \qquad \underline{+2 \quad +2}$$

$$2x = -3 \qquad x = 2$$

$$\frac{2x}{2} = \frac{-3}{2}$$

$$x = -\frac{3}{2}$$

Step 3

$$\begin{aligned} y - 4x &= 2 \\ y - 4\left(-\tfrac{3}{2}\right) &= 2 \\ y + 6 &= 2 \\ \underline{-6} \quad & \underline{-6} \\ y &= -4 \end{aligned} \qquad \begin{aligned} y - 4x &= 2 \\ y - 4(2) &= 2 \\ y - 8 &= 2 \\ \underline{+8} \quad & \underline{+8} \\ y &= 10 \end{aligned}$$

The solutions are $\left(-\frac{3}{2}, -4\right)$ and $(2, 10)$.

19. Step 2

$$\begin{aligned} y &= x^2 - 5x - 7 \\ -(y &= \quad\;\; -5x + 9) \\ \hline 0 &= x^2 \qquad\quad - 16 \end{aligned}$$

Step 3

$$0 = x^2 - 16$$

$$0 = (x + 4)(x - 4)$$

$$x + 4 = 0 \quad \text{or} \quad x - 4 = 0$$

$$\underline{-4 \quad -4} \qquad \underline{+4 \quad +4}$$

$$x = -4 \qquad x = 4$$

Step 4

$$\begin{aligned} y &= -5x + 9 \\ y &= -5(-4) + 9 \\ y &= 20 + 9 \\ y &= 29 \end{aligned} \qquad \begin{aligned} y &= -5x + 9 \\ y &= -5(4) + 9 \\ y &= -20 + 9 \\ y &= -11 \end{aligned}$$

The solutions are $(-4, 29)$ and $(4, -11)$.

20. Step 2

$$\begin{aligned} y &= -3x^2 + x + 2 \\ -(y &= \qquad\quad\;\; x + 4) \\ \hline 0 &= -3x^2 \qquad - 2 \end{aligned}$$

Step 3

$$0 = -3x^2 - 2$$

$$\underline{+2} \qquad \underline{+2}$$

$$2 = -3x^2$$

$$\frac{2}{-3} = \frac{-3x^2}{-3}$$

$$-\frac{2}{3} = x^2$$

No real number multiplied by itself produces a negative value. So, the equation has no real solutions, and the system has no real solutions.

21. Step 2

$$\begin{aligned} y &= -x^2 - 2x + 2 \\ -(y &= \qquad\quad 4x + 2) \\ \hline 0 &= -x^2 - 6x \end{aligned}$$

Step 3

$$0 = -x^2 - 6x$$

$$0 = -x(x + 6)$$

$$-x = 0 \quad \text{or} \quad x + 6 = 0$$

$$\frac{-x}{-1} = \frac{0}{-1} \qquad \underline{-6 \quad -6}$$

$$x = 0 \qquad x = -6$$

Step 4

$$\begin{aligned} y &= 4x + 2 \\ y &= 4(0) + 2 \\ y &= 0 + 2 \\ y &= 2 \end{aligned} \qquad \begin{aligned} y &= 4x + 2 \\ y &= 4(-6) + 2 \\ y &= -24 + 2 \\ y &= -22 \end{aligned}$$

The solutions are $(0, 2)$ and $(-6, -22)$.

22. Step 2

$$\begin{aligned} y &= -2x^2 + x - 3 \\ -(y &= \qquad\quad 2x - 2) \\ \hline 0 &= -2x^2 - x - 1 \end{aligned}$$

Step 3

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(-2)(-1)}}{2(-2)} \\ &= \frac{1 \pm \sqrt{1 - 8}}{-4} \\ &= \frac{1 \pm \sqrt{-7}}{-4} \end{aligned}$$

The discriminant is zero. So, the equation has no real solutions, and the system has no real solutions.

23. Step 2

$$\begin{aligned} y &= x^2 \\ -(y &= \qquad 2x - 1) \\ \hline 0 &= x^2 - 2x + 1 \end{aligned}$$

Step 3

$$0 = x^2 - 2(x)(1) + 1^2$$

$$0 = (x - 1)^2$$

$$x - 1 = 0$$

$$\underline{+1 \quad +1}$$

$$x = 1$$

Step 4

$$\begin{aligned} y &= x^2 \\ y &= 1^2 \\ y &= 1 \end{aligned}$$

The solution is $(1, 1)$.

Chapter 9

24. Step 2

$$\begin{aligned} y &= x^2 + x + 1 \\ -(y &= \quad\; - x - 2) \\ \hline 0 &= x^2 + 2x + 3 \end{aligned}$$

Step 3

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(1)(3)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{4 - 12}}{2} \\ &= \frac{-2 \pm \sqrt{-8}}{2} \end{aligned}$$

The discriminant is negative. So, the equation has no real solutions, and the system has no real solutions.

25. Step 1

$$\begin{aligned} y + 2x &= 0 \\ y + 2x - 2x &= 0 - 2x \\ y &= -2x \end{aligned}$$

Step 2

$$\begin{aligned} y &= x^2 + 4x - 6 \\ -(y &= \quad\; - 2x) \\ \hline 0 &= x^2 + 6x - 6 \end{aligned}$$

Step 3

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-6 \pm \sqrt{6^2 - 4(1)(-6)}}{2(1)} \\ &= \frac{-6 \pm \sqrt{36 + 24}}{2} \\ &= -\frac{6 \pm \sqrt{60}}{2} \\ &= \frac{-6 \pm \sqrt{4} \bullet \sqrt{15}}{2} \\ &= \frac{-6 \pm 2\sqrt{15}}{2} \\ &= -3 \pm \sqrt{15} \end{aligned}$$

Step 4

$$\begin{aligned} y &= -2x & y &= -2x \\ y &= -2\left(-3 + \sqrt{15}\right) & y &= -2\left(-3 - \sqrt{15}\right) \\ &= -2(-3) - 2\sqrt{15} & &= -2(-3) - 2 - \sqrt{15} \\ &= 6 - 2\sqrt{15} & &= 6 + 2\sqrt{15} \end{aligned}$$

The solutions are $\left(-3 + \sqrt{15}, 6 - 2\sqrt{15}\right) \approx (0.87, -1.75)$ and $\left(-3 - \sqrt{15}, 6 + 2\sqrt{15}\right) \approx (-6.87, 13.75)$.

26. Step 1

$$\begin{aligned} y + 5x &= x^2 - 2 \\ y + 5x - 5x &= x^2 - 2 - 5x \\ y &= x^2 - 5x - 2 \end{aligned}$$

Step 2

$$\begin{aligned} y &= x^2 - 5x - 2 \\ -(y &= \qquad\; 2x - 7) \\ \hline 0 &= x^2 - 7x + 5 \end{aligned}$$

Step 3

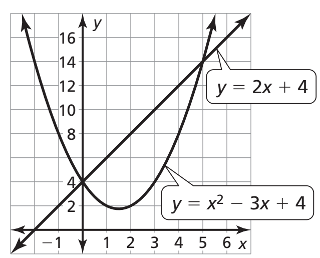
$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(5)}}{2(1)} \\ &= \frac{7 \pm \sqrt{49 - 20}}{2} \\ &= \frac{7 \pm \sqrt{29}}{2} \end{aligned}$$

Step 4

$$\begin{aligned} y &= 2x - 7 & y &= 2x - 7 \\ y &= 2\left(\frac{7 + \sqrt{29}}{2}\right) - 7 & y &= 2\left(\frac{7 - \sqrt{29}}{2}\right) - 7 \\ &= 7 + \sqrt{29} - 7 & &= 7 - \sqrt{29} - 7 \\ &= \sqrt{29} + 7 - 7 & &= -\sqrt{29} + 7 - 7 \\ &= \sqrt{29} & &= -\sqrt{29} \end{aligned}$$

The solutions are $\left(\frac{7 + \sqrt{29}}{2}, \sqrt{29}\right) \approx (6.19, 5.39)$ and $\left(\frac{7 - \sqrt{29}}{2}, -\sqrt{29}\right) \approx (0.81, -5.39)$.

27. The graph does not show both solutions.



The solutions are (0, 4) and (5, 14).

28. The number 4 should be substituted for y, not for x.

$y = 3x^2 - 6x + 4$

$y = 4$

Step 2

$$\begin{aligned} y &= 3x^2 - 6x + 4 \\ 4 &= 3x^2 - 6x + 4 \\ 4 - 4 &= 3x^2 - 6x + 4 - 4 \\ 0 &= 3x^2 - 6x \\ 0 &= 3x(x - 2) \end{aligned}$$

$$\begin{aligned} 3x &= 0 & \text{or} \qquad x - 2 &= 0 \\ \frac{3x}{3} &= \frac{0}{3} & \underline{+2} \quad &\;\; \underline{+2} \\ x &= 0 & x &= 2 \end{aligned}$$

The solutions are $x = 0$ and $x = 2$.

Chapter 9

29. Because there is a change of signs between where $x = -4$ and $x = -3$, and $f(-4)$ and $f(-3)$ are the same distance from 0, the first solution is halfway between -4 and -3. Similarly, because there is a change of signs between where $x = 0$ and $x = 1$, and $f(0)$ and $f(1)$ are the same distance from 0, the other solution is halfway between 0 and 1.

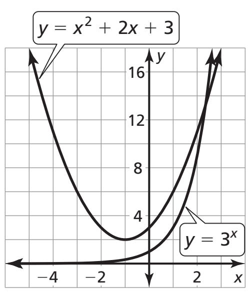
30. Because there is a change of signs between where $x = 1$ and $x = 2$, and $f(1)$ and $f(2)$ are the same distance from 0, the first solution is halfway between 1 and 2. Similarly, because there is a change of signs between where $x = 3$ and $x = 4$, and $f(3)$ and $f(4)$ are the same distance from 0, the other solution is halfway between 3 and 4.

31. Because there is a change of signs between where $x = -4$ and $x = -3$ and again between where $x = -2$ and $x = -1$, one solution is between -4 and -3 and the other solution is between -2 and -1. Because $f(-3)$ is closer to 0 than $f(-4)$ and $f(-2)$ is closer to 0 than $f(-1)$, one solution is closer to -3 than -4 and the other solution is closer to -2 than -1.

32. Because there is a change of signs between where $x = 2$ and $x = 3$ and again between where $x = 5$ and $x = 6$, one solution is between 2 and 3 and the other solution is between 5 and 6. Because $f(3)$ is closer to 0 than $f(2)$ and $f(5)$ is closer to 0 than $f(6)$, one solution is closer to 3 than 2 and the other solution is closer to 5 than 6.

33. $y = x^2 + 2x + 3$

$y = 3^x$



The graphs intersect between $x = 2$ and $x = 3$.

$$y = x^2 + 2x + 3$$
$$3^x = x^2 + 2x + 3$$
$$3^x - x^2 - 2x - 3 = x^2 - x^2 + 2x - 2x + 3 - 3$$
$$3^x - x^2 - 2x - 3 = 0$$

Let $f(x) = 3^x - x^2 - 2x - 3$.

$f(2.3) \approx -0.377$

$f(2.4) \approx 0.407$

$f(2.35) \approx -0.002$

$f(2.351) \approx 0.005$

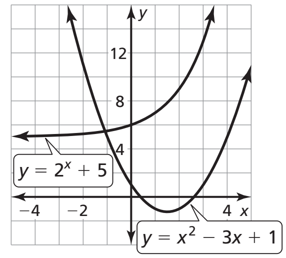
Because $f(2.350)$ is closest to 0, $x \approx 2.350$.

$y = 3^x \approx 3^{2.350} \approx 13.220$

So, the solution of the system is about (2.350, 13.220).

34. $y = 2^x + 5$

$y = x^2 - 3x + 1$



The graphs intersect between $x = -2$ and $x = -1$.

$$y = x^2 - 3x + 1$$
$$2^x + 5 = x^2 - 3x + 1$$
$$2^x + 5 - x^2 + 3x - 1 = x^2 - 3x + 1 - x^2 + 3x - 1$$
$$2^x - x^2 + 3x + 4 = 0$$

Let $f(x) = 2^x - x^2 + 3x + 4$.

$f(-1) \approx 0.5$

$f(-1.1) \approx -0.0435$

$f(-1.09) \approx 0.012$

$f(-1.091) \approx 0.006$

$f(-1.092) \approx 0.001$

$f(-1.093) \approx -0.005$

Because $f(-1.092)$ is closest to 0, $x \approx -1.092$.

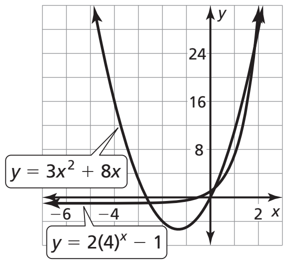
$y = 2^x + 5 \approx 2^{-1.092} + 5 \approx 5.469$

So, the solution of the system is about $(-1.092, 5.469)$.

Chapter 9

35. $y = 2(4)^x - 1$

$y = 3x^2 + 8x$



The graphs intersect between $x = -3$ and $x = -2$, between $x = 0$ and $x = 1$, and between $x = 1$ and $x = 2$.

$$y = 3x^2 + 8x$$
$$2(4)^x - 1 = 3x^2 + 8x$$
$$2(4)^x - 1 - 3x^2 - 8x = 3x^2 - 3x^2 + 8x - 8x$$
$$2(4)^x - 3x^2 - 8x - 1 = 0$$

Let $f(x) = 2(4)^x - 3x^2 - 8x - 1$.

$f(-2.6) \approx -0.426$

$f(-2.5) \approx 0.313$

$f(-2.54) \approx 0.024$

$f(-2.541) \approx 0.017$

$f(-2.542) \approx 0.010$

$f(-2.543) \approx 0.002$

$f(-2.544) \approx -0.005$

$f(-2.543)$ is closest to 0.

So, $x \approx -2.543$.

$y = 3x^2 + 8x$

$\approx 3(-2.543)^2 + 8(-2.543)$

≈ -0.943

$f(0.1) \approx 0.467$

$f(0.2) \approx -0.081$

$f(0.19) \approx -0.026$

$f(0.189) \approx -0.020$

$f(0.188) \approx -0.015$

$f(0.187) \approx -0.009$

$f(0.186) \approx -0.004$

$f(0.185) \approx 0.002$

$f(0.185)$ is closest to 0.

So, $x \approx 0.185$.

$y = 3x^2 + 8x$

$\approx 3(0.185)^2 + 8(0.185)$

≈ 1.583

$f(1.8) \approx -0.869$

$f(1.9) \approx 0.828$

$f(1.85) \approx -0.0754$

$f(1.851) \approx -0.059$

$f(1.852) \approx -0.042$

$f(1.853) \approx -0.024$

$f(1.854) \approx -0.007$

$f(1.855) \approx 0.010$

$f(1.854)$ is closest to 0. So, $x \approx 1.854$.

$y = 3x^2 + 8x$

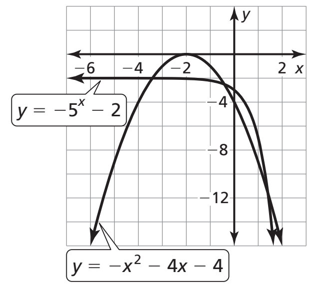
$\approx 3(1.854)^2 + 8(1.854)$

≈ 25.144

So, the solutions of the system are about $(-2.543, -0.943)$, about $(0.185, 1.583)$, and about $(1.854, 25.144)$.

36. $y = -x^2 - 4x - 4$

$y = -5^x - 2$



The graphs intersect between $x = -4$ and $x = -3$, between $x = -1$ and $x = 0$, and between $x = 1$ and $x = 2$.

$$y = -5^x - 2$$
$$-x^2 - 4x - 4 = -5^x - 2$$
$$-x^2 - 4x - 4 + 5^x + 2 = -5^x - 2 + 5^x + 2$$
$$5^x - x^2 - 4x - 2 = 0$$

Let $f(x) = 5^x - x^2 - 4x - 2$.

$f(-3.5) \approx -0.246$

$f(-3.4) \approx 0.044$

$f(-3.41) \approx 0.016$

$f(-3.42) \approx -0.012$

$f(-3.411) \approx 0.013$

$f(-3.412) \approx 0.010$

$f(-3.413) \approx 0.008$

$f(-3.414) \approx 0.005$

$f(-3.415) \approx 0.002$

$f(-3.416) \approx -0.001$

$f(-3.416)$ is closest to 0.

So, $x \approx -3.416$.

$y \approx -5^{(-3.416)} - 2$

≈ -2.004

$f(-0.5) \approx 0.197$

$f(-0.4) \approx -0.035$

$f(-0.41) \approx -0.011$

$f(-0.411) \approx -0.009$

$f(-0.412) \approx -0.007$

$f(-0.413) \approx -0.004$

$f(-0.414) \approx -0.002$

$f(-0.415) \approx 0.001$

$f(-0.415)$ is closest to 0.

So, $x \approx -0.415$.

$y \approx -5^{(-0.415)} - 2$

≈ -2.513

$f(1.4) \approx -0.042$

$f(1.5) \approx 0.930$

$f(1.41) \approx 0.045$

$f(1.401) \approx -0.033$

$f(1.402) \approx -0.025$

$f(1.403) \approx -0.016$

$f(1.404) \approx -0.008$

$f(1.405) \approx 0.001$

$f(1.405)$ is closest to 0. So, $x \approx 1.405$.

$y \approx -5^{(1.405)} - 2$

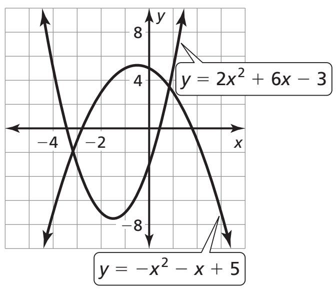
≈ -11.595

So, the solutions of the system are about $(-3.416, -2.004)$, about $(-0.415, -2.513)$, and about $(1.405, -11.595)$.

Chapter 9

37. $y = -x^2 - x + 5$

$y = 2x^2 + 6x - 3$



The graphs intersect between $x = -4$ and $x = -3$ and between $x = 0$ and $x = 1$.

$$\begin{aligned} y &= -x^2 - x + 5 \\ 2x^2 + 6x - 3 &= -x^2 - x + 5 \\ 2x^2 + 6x - 3 + x^2 + x - 5 &= -x^2 - x + 5 + x^2 + x - 5 \\ 3x^2 + 7x - 8 &= 0 \end{aligned}$$

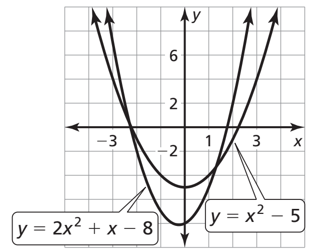
Let $f(x) = 3x^2 + 7x - 8$.

$f(0.8) = -0.48$	$f(-3.2) = 0.32$
$f(0.9) = 0.73$	$f(-3.1) = -0.87$
$f(0.81) \approx -0.362$	$f(-3.19) \approx 0.198$
$f(0.82) \approx -0.243$	$f(-3.18) \approx 0.077$
$f(0.83) \approx -0.123$	$f(-3.17) \approx -0.043$
$f(0.84) \approx -0.003$	$f(-3.179) \approx 0.065$
$f(0.841) \approx 0.009$	$f(-3.178) \approx 0.053$
$f(0.85) \approx 0.118$	$f(-3.177) \approx 0.041$
$f(0.840)$ is closest to 0.	$f(-3.176) \approx 0.029$
So, $x \approx 0.840$.	$f(-3.175) \approx 0.017$
$y = -x^2 - x + 5$	$f(-3.174) \approx 0.005$
$y \approx -(0.840)^2 - 0.840 + 5$	$f(-3.173) \approx -0.007$
≈ 3.454	$f(-3.174)$ is closest to 0.
	So, $x \approx -3.174$.
	$y = -x^2 - x + 5$
	$y \approx -(-3.174)^2 - (-3.174) + 5$
	≈ -1.900

So, the solutions of the system are about $(-3.174, -1.900)$ and about $(0.840, 3.454)$.

38. $y = 2x^2 + x - 8$

$y = x^2 - 5$



The graphs intersect between $x = -3$ and $x = -2$ and between $x = 1$ and $x = 2$.

$$\begin{aligned} y &= x^2 - 5 \\ 2x^2 + x - 8 &= x^2 - 5 \\ 2x^2 + x - 8 - x^2 + 5 &= x^2 - x^2 - 5 + 5 \\ x^2 + x - 3 &= 0 \end{aligned}$$

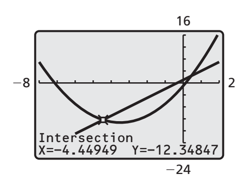
Let $f(x) = x^2 + x - 3$.

$f(-2.3) = -0.01$	$f(1.3) = -0.01$
$f(-2.4) = 0.36$	$f(1.4) = 0.36$
$f(-2.301) \approx -0.006$	$f(1.301) \approx -0.006$
$f(-2.302) \approx -0.003$	$f(1.302) \approx -0.003$
$f(-2.303) \approx 0.001$	$f(1.303) \approx 0.001$
$f(-2.303)$ is closest to 0.	$f(1.303)$ is closest to 0.
So, $x \approx -2.303$.	So, $x \approx 1.303$.
$y = x^2 - 5$	$y = x^2 - 5$
$y \approx (-2.303)^2 - 5$	$y \approx 1.303^2 - 5$
≈ 0.304	≈ -3.302

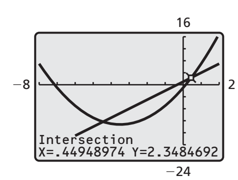
So, the solutions of the system are about $(-2.303, 0.304)$ and about $(1.303, -3.302)$.

39. $3x + 1 = x^2 + 7x - 1$

Graph the system $y = 3x + 1$ and $y = x^2 + 7x - 1$.



One point of intersection is about $(-4.45, -12.35)$.

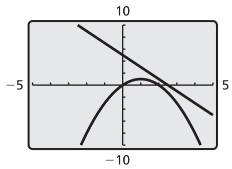


Another point of intersection is about $(0.45, 2.35)$. So, the solutions of the equation are $x \approx -4.45$ and $x \approx 0.45$.

Chapter 9

40. $-x^2 + 2x = -2x + 5$

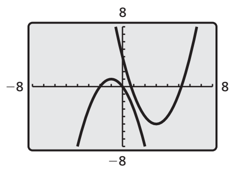
Graph the system $y = -x^2 + 2x$ and $y = -2x + 5$.



The graphs do not intersect. So, the equation has no solutions.

41. $x^2 - 6x + 4 = -x^2 - 2x$

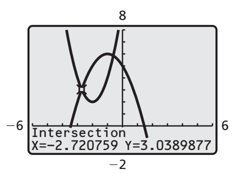
Graph the system $y = x^2 - 6x + 4$ and $y = -x^2 - 2x$.



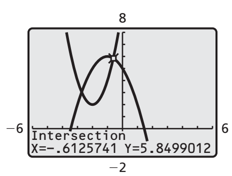
The graphs do not intersect. So, the equation has no solutions.

42. $2x^2 + 8x + 10 = -x^2 - 2x + 5$

Graph the system $y = 2x^2 + 8x + 10$ and $y = -x^2 - 2x + 5$.



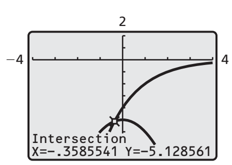
One point of intersection is about $(-2.72, 3.04)$.



The other point of intersection is about $(-0.61, 5.85)$. So, the solutions of the equation are $x \approx -2.72$ and $x \approx 0.61$.

43. $-4\left(\frac{1}{2}\right)^x = -x^2 - 5$

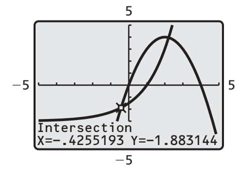
Graph the system $y = -4\left(\frac{1}{2}\right)^x$ and $y = -x^2 - 5$.



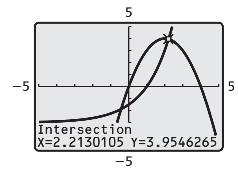
The only point of intersection is about $(-0.36, -5.13)$. So, the solution of the equation is $x \approx -0.36$.

44. $1.5(2)^x - 3 = -x^2 + 4x$

Graph the system $y = 1.5(2)^x - 3$ and $y = -x^2 + 4x$.



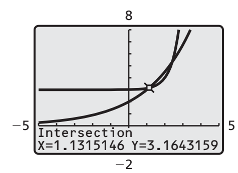
One point of intersection is about $(-0.43, -1.88)$.



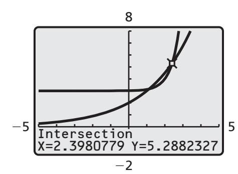
The other point of intersection is about $(2.21, 3.95)$. So, the solutions of the equation are $x \approx -0.43$ and $x \approx 2.21$.

45. $8^{x-2} + 3 = 2\left(\frac{3}{2}\right)^x$

Graph the system $y = 8^{x-2} + 3$ and $y = 2\left(\frac{3}{2}\right)^x$.



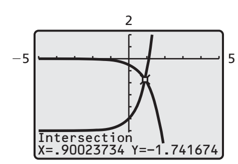
One point of intersection is about $(1.13, 3.16)$.



The other point of intersection is about $(2.40, 5.29)$. So, the solutions of the equation are $x \approx 1.13$ and $x \approx 2.40$.

46. $-0.5(4)^x = 5^x - 6$

Graph the system $y = -0.5(4)^x$ and $y = 5^x - 6$.



The only point of intersection is about $(0.90, -1.74)$. So, the solution of the equation is $x \approx 0.90$.

Chapter 9

47. $3x^2 + 7x - 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-7 \pm \sqrt{7^2 - 4(3)(-8)}}{2(3)}$$
$$= \frac{-7 \pm \sqrt{49 + 96}}{6}$$
$$= \frac{-7 \pm \sqrt{145}}{6}$$

$y = 2x^2 + 6x - 3$

$$y = 2\left(\frac{-7 + \sqrt{145}}{6}\right)^2 + 6\left(\frac{-7 + \sqrt{145}}{6}\right) - 3$$
$$= \frac{2\left[(-7)^2 + 2(-7)\sqrt{145} + \left(\sqrt{145}\right)^2\right]}{36 - 7 + \sqrt{145} - 3}$$
$$= \frac{49 - 14\sqrt{145} + 145}{18} + \sqrt{145} - 10$$
$$= \frac{194 - 14\sqrt{145}}{18} + \sqrt{145} - 10$$
$$= \frac{97 - 7\sqrt{145}}{9} + \frac{9\sqrt{145} - 90}{9}$$
$$= \frac{97 - 90 - 7\sqrt{145} + 9\sqrt{145}}{9}$$
$$= \frac{7 + 2\sqrt{145}}{9}$$

$y = 2x^2 + 6x - 3$

$$y = 2\left(\frac{-7 - \sqrt{145}}{6}\right)^2 + 6\left(\frac{-7 - \sqrt{145}}{6}\right) - 3$$
$$= \frac{2\left[(-7)^2 - 2(-7)\left(\sqrt{145}\right) + \left(\sqrt{145}\right)^2\right]}{36} - 7 - \sqrt{145} - 3$$
$$= \frac{49 + 14\sqrt{145} + 145}{18} - 10 - \sqrt{145}$$
$$= \frac{194 + 14\sqrt{145}}{18} - 10 - \sqrt{145}$$
$$= \frac{97 + 7\sqrt{145}}{9} - \frac{90 - 9\sqrt{145}}{9}$$
$$= \frac{97 - 90 + 7\sqrt{145} - 9\sqrt{145}}{9}$$
$$= \frac{7 - 2\sqrt{145}}{9}$$

So, the exact solutions are $\left(\frac{-7 + \sqrt{145}}{6}, \frac{7 + 2\sqrt{145}}{9}\right)$ and $\left(\frac{-7 - \sqrt{145}}{6}, \frac{7 - 2\sqrt{145}}{9}\right)$. These values, $\left(\frac{-7 + \sqrt{145}}{6}, \frac{7 + 2\sqrt{145}}{9}\right) \approx (0.840, 3.454)$ and $\left(\frac{-7 - \sqrt{145}}{6}, \frac{7 - 2\sqrt{145}}{9}\right) \approx (-3.174, -1.898)$ are about the same as the approximated solutions found in Exercise 37. One of the y-values is slightly different due to rounding.

48. Step 2

$$\begin{array}{r} y = 2x^2 + x - 8 \\ -(y = x^2 - 5) \\ \hline 0 = x^2 + x - 3 \end{array}$$

Step 3 $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{-1 \pm \sqrt{1^2 - 4(1)(-3)}}{2(1)}$$
$$= \frac{-1 \pm \sqrt{1 + 12}}{2}$$
$$= \frac{-1 \pm \sqrt{13}}{2}$$

$y = x^2 - 5$

$$y = \left(\frac{-1 + \sqrt{13}}{2}\right)^2 - 5$$
$$= \frac{(-1)^2 + 2(-1)\left(\sqrt{13}\right) + \left(\sqrt{13}\right)^2}{4} - 5$$
$$= \frac{1 - 2\sqrt{13} + 13}{4} - \frac{20}{4}$$
$$= \frac{1 + 13 - 20 - 2\sqrt{13}}{4}$$
$$= \frac{-6 - 2\sqrt{13}}{4}$$
$$= \frac{-3 - \sqrt{13}}{2}$$

$y = x^2 - 5$

$$y = \left(\frac{-1 - \sqrt{13}}{2}\right)^2 - 5$$
$$= \frac{(-1)^2 - 2(-1)\left(\sqrt{13}\right) + \left(-\sqrt{13}\right)^2}{4} - 5$$
$$= \frac{1 + 2\sqrt{13} + 13}{4} - \frac{20}{4}$$
$$= \frac{1 + 13 - 20 + 2\sqrt{13}}{4}$$
$$= \frac{-6 + 2\sqrt{13}}{4}$$
$$= \frac{-3 + \sqrt{13}}{2}$$

The exact solutions are $\left(\frac{-1 + \sqrt{13}}{2}, \frac{-3 - \sqrt{13}}{2}\right)$ and $\left(\frac{-1 - \sqrt{13}}{2}, \frac{-3 + \sqrt{13}}{2}\right)$. These values, $\left(\frac{-1 + \sqrt{13}}{2}, \frac{-3 - \sqrt{13}}{2}\right) \approx (1.303, -3.03)$ and $\left(\frac{-1 - \sqrt{13}}{2}, \frac{-3 + \sqrt{13}}{2}\right) \approx (-2.303, 0.303)$, are about the same as the approximated solutions found in Exercise 38. The y-values are slightly different due to rounding.

Chapter 9

49. Step 2

$$\begin{aligned} y &= \quad\quad -5x + 275 \\ -(y &= -x^2 + 35x + 100) \\ \hline 0 &= x^2 - 40x + 175 \end{aligned}$$

Step 3 $0 = (x - 5)(x + 35)$

$$\begin{array}{lll} x - 5 = 0 & \text{or} & x - 35 = 0 \\ \underline{+5 \quad +5} & & \underline{+35 \quad +35} \\ x = 5 & & x = 35 \end{array}$$

The solutions of the system are $x = 5$ and $x = 35$. So, the attendance for each movie is the same 5 days after the movies opened and again 35 days after the movies opened.

50. Step 2

$$\begin{aligned} y &= \quad\quad 2x + 8 \\ -(y &= -x^2 - 4x - 1) \\ \hline 0 &= x^2 + 6x + 9 \end{aligned}$$

Step 3

$$\begin{aligned} 0 &= x^2 + 2(x)(3) + 3^2 \\ 0 &= (x + 3)^2 \end{aligned}$$

$$\begin{aligned} x + 3 &= 0 \\ \underline{-3} \quad & \underline{-3} \\ x &= -3 \end{aligned}$$

Step 4

$$\begin{aligned} y &= 2x + 8 \\ y &= 2(-3) + 8 \\ y &= -6 + 8 \\ y &= 2 \end{aligned}$$

The coordinates of the point where the paths meet are $(-3, 2)$.

51. $y = -0.002x^2 + 1.06x$

$y = 52$

Step 2

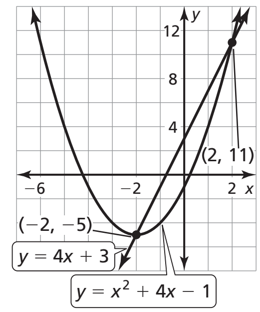
$$\begin{aligned} y &= -0.002x^2 + 1.06x \\ 52 &= -0.002x^2 + 1.06x \\ 52 - 52 &= -0.002x^2 + 1.06x - 52 \\ 0 &= -0.002x^2 + 1.06x - 52 \\ x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-1.06 \pm \sqrt{1.06^2 - 4(-0.002)(-52)}}{2(-0.002)} \\ &= \frac{-1.06 \pm \sqrt{1.1236 - 0.416}}{-0.004} \\ &= \frac{-1.06 \pm \sqrt{0.7076}}{-0.004} \end{aligned}$$

The solutions are $x = \dfrac{-1.06 + \sqrt{0.7076}}{-0.004} \approx 54.7$ and $x = \dfrac{-1.06 - \sqrt{0.7076}}{-0.004} \approx 475.3$. So, the two points are about 55 meters and about 475 meters from the left pylons.

52. A linear equation is represented by a line and a quadratic equation is represented by a parabola. Because a line and a parabola cannot be the same, a linear equation and a quadratic equation cannot have infinitely many solutions in common. So, your friend is incorrect.

53. a. $y = 4x + 3$

$y = x^2 + 4x - 1$



The points of intersection are $(-2, -5)$ and $(2, 11)$.

b. Step 2

$$\begin{aligned} y &= x^2 + 4x - 1 \\ 4x + 3 &= x^2 + 4x - 1 \\ 4x - 4x + 3 - 3 &= x^2 + 4x - 1 - 4x - 3 \\ 0 &= x^2 - 4 \\ 0 &= x^2 - 2^2 \\ 0 &= (x + 2)(x - 2) \end{aligned}$$

$$\begin{array}{lll} x + 2 = 0 & \text{or} & x - 2 = 0 \\ \underline{-2 \quad -2} & & \underline{+2 \quad +2} \\ x = -2 & & x = 2 \end{array}$$

Step 3

$$\begin{array}{ll} y = 4x + 3 & y = 4x + 3 \\ y = 4(-2) + 3 & y = 4(2) + 3 \\ y = (-8) + 3 & y = 8 + 3 \\ y = -5 & y = 11 \end{array}$$

The solutions are $(-2, -5)$ and $(2, 11)$.

c. Step 2

$$\begin{aligned} y &= x^2 + 4x - 1 \\ -(y &= \quad\quad 4x + 3) \\ \hline 0 &= x^2 \quad\quad - 4 \end{aligned}$$

Step 3

$$\begin{aligned} 0 &= x^2 - 4 \\ \underline{+4} \quad & \underline{+4} \\ 4 &= x^2 \\ \sqrt{4} &= \sqrt{x^2} \\ \pm 2 &= x \end{aligned}$$

Step 4

$$\begin{array}{ll} y = 4x + 3 & y = 4x + 3 \\ y = 4(2) + 3 & y = 4(-2) + 3 \\ y = 8 + 3 & y = -8 + 3 \\ y = 11 & y = -5 \end{array}$$

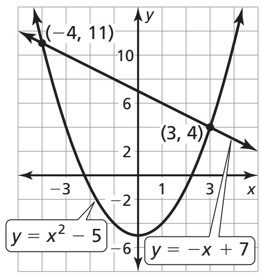
The solutions are $(-2, -5)$ and $(2, 11)$.

Sample answer: For this system, elimination is preferred because the resulting equation can be written in the form $x^2 = d$.

Chapter 9

54. a. $y = x^2 - 5$

$y = -x + 7$



The points of intersection are $(-4, 11)$ and $(3, 4)$.

b. Step 2

$$
\begin{aligned}
y &= -x + 7 \\
x^2 - 5 &= -x + 7 \\
x^2 - 5 + x - 7 &= -x + 7 + x - 7 \\
x^2 + x - 12 &= 0 \\
(x + 4)(x - 3) &= 0
\end{aligned}
$$

$$
\begin{array}{lcl}
x + 4 = 0 & \text{or} & x - 3 = 0 \\
\underline{-4 \quad -4} & & \underline{+3 \quad +3} \\
x = -4 & & x = 3
\end{array}
$$

Step 3

$$
\begin{array}{ll}
y = -x + 7 & y = -x + 7 \\
y = -(-4) + 7 & y = -3 + 7 \\
y = 4 + 7 & y = 4 \\
y = 11 &
\end{array}
$$

The solutions are $(-4, 11)$ and $(3, 4)$.

c. Step 2

$$
\begin{array}{rl}
& y = x^2 \qquad - 5 \\
-(& \underline{y = \qquad - x + 7)} \\
& 0 = x^2 + x - 12
\end{array}
$$

Step 3 $0 = (x + 4)(x - 3)$

$$
\begin{array}{lcl}
x + 4 = 0 & \text{or} & x - 3 = 0 \\
\underline{-4 \quad -4} & & \underline{+3 \quad +3} \\
x = -4 & & x = 3
\end{array}
$$

Step 4

$$
\begin{array}{ll}
y = -x + 7 & y = -x + 7 \\
y = -(-4) + 7 & y = -3 + 7 \\
y = 4 + 7 & y = 4 \\
y = 11 &
\end{array}
$$

The solutions are $(-4, 11)$ and $(3, 4)$.

Sample answer: For this system, substitution is preferred because the resulting equation is easy to factor.

.

55. a.

$$
\begin{aligned}
y &= -x^2 + 65x + 256 \\
y &= -1^2 + 65(1) + 256 \\
y &= -1 + 65 + 256 \\
y &= 320 \\
y &= -x^2 + 65x + 256 \\
y &= -34^2 + 65(34) + 256 \\
y &= -1156 + 2210 + 256 \\
y &= 1310
\end{aligned}
$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1310 - 320}{34 - 1} = \frac{990}{33} = 30$$

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - 320 &= 30(x - 1) \\
y - 320 &= 30(x) - 30(1) \\
y - 320 &= 30x - 30 \\
\underline{+320} \quad & \quad \underline{+320} \\
y &= 30x + 290
\end{aligned}
$$

A linear function that models the number of subscribers of the competitor's website is $y = 30x + 290$.

b. Step 2

$$
\begin{array}{rl}
& y = \qquad 30x + 290 \\
-(& \underline{y = -x^2 + 65x + 256)} \\
& 0 = x^2 \quad - 35x + 34 \\
& 0 = (x - 1)(x - 34)
\end{array}
$$

$$
\begin{array}{lcl}
x - 1 = 0 & \text{or} & x - 34 = 0 \\
\underline{+1 \quad +1} & & \underline{+34 \quad +34} \\
x = 1 & & x = 34
\end{array}
$$

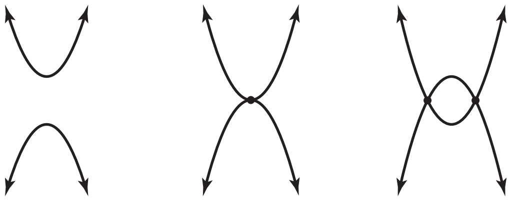
The solutions are $x = 1$ and $x = 34$, which confirms that the websites have the same number of subscribers on Days 1 and 34.

56. a. Changing the equation to $y = c + 2$ will cause a vertical translation 2 units up of the line $y = c$. So, the line will intersect the parabola in two points, and the system will have two solutions.

b. Changing the equation to $y = c - 2$ will cause a vertical translation 2 units down of the line $y = c$. So, the line will not intersect the parabola, and the system will have no solutions.

Chapter 9

57. The first graph shows that the parabolas may not intersect at all, in which case the system has no solutions. The second graph shows that the parabolas may intersect in one point, in which case the system has one solution. The third graph shows that the parabolas may intersect in two points, in which case the system has two solutions.

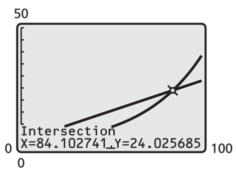


58. a. Let x be the number of years.

Let y be the number of people (in millions).

$y = 2(1.03)^x$

$y = 3 + 0.25x$

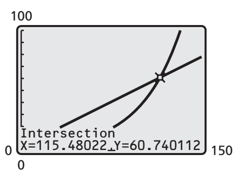


The point of intersection is about (84.1, 24). So, the country will experience a food shortage in year 84.

b. $2(0.25) = 0.5$

$y = 2(1.03)^x$

$y = 3 + 0.5x$



Because the population growth is defined by an exponential function, eventually it will surpass the food supply, which is defined by a linear function. In this case, the graphs intersect at about (115.5, 60.7). So, in year 115, a food shortage will occur.

59. a. To find A, let $x = 0$ because A is on the y-axis.

$y = -x^2 + 6x + 4$

$y = -0^2 + 6(0) + 4$

$= 0 + 4$

$= 4$

So, the coordinates of point A are (0, 4).

b. Write the equation of the red line.

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{6-4}{2-0} = \dfrac{2}{2} = 1$

$y - y_1 = m(x - x_1)$

$y - 6 = 1(x - 2)$

$y - 6 = x - 2$

$\underline{+6} \qquad \underline{+6}$

$y = x + 4$

Solve by elimination.

Step 2

$$\begin{array}{r} y = x + 4 \\ \underline{-(y = -x^2 + 6x + 4)} \\ 0 = x^2 - 5x \end{array}$$

$0 = x(x - 5)$

$x = 0 \quad$ or $\quad x - 5 = 0$

$\underline{+5} \quad \underline{+5}$

$x = 5$

Step 3 $y = x + 4$

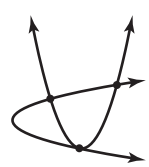
$y = 5 + 4$

$y = 9$

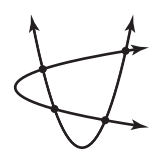
So, the coordinates of point B are (5, 9).

60. If one equation is quadratic in x and the other is quadratic in y, then the one parabola opens either up or down and the other opens to the right or left. So, there can be 3 or 4 points of intersection as shown.

Sample answer:



3 points of intersection



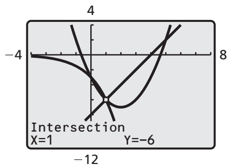
4 points of intersection

Chapter 9

61. $y = 2x - 8$

$y = x^2 - 4x - 3$

$y = -3(2)^x$



Using the *intersect* feature of a graphing calculator three times for each pair of equations, you find that they all intersect at $(1, -6)$. So, the solution of the system is $(1, -6)$.

62. Solve by substitution.

Step 2

$$\begin{aligned} x^2 + y^2 &= 41 \\ x^2 + (-x - 1)^2 &= 41 \\ x^2 + (-x)^2 - 2(-x)(1) + 1^2 &= 41 \\ x^2 + x^2 + 2x + 1 &= 41 \\ 2x^2 + 2x + 1 &= 41 \\ 2x^2 + 2x + 1 - 41 &= 41 - 41 \\ 2x^2 + 2x - 40 &= 0 \\ \frac{2x^2 + 2x - 40}{2} &= \frac{0}{2} \\ x^2 + x - 20 &= 0 \\ (x + 5)(x - 4) &= 0 \end{aligned}$$

$$\begin{aligned} x + 5 &= 0 \\ \underline{-5} \quad &\ \ \underline{-5} \\ x &= -5 \end{aligned}$$

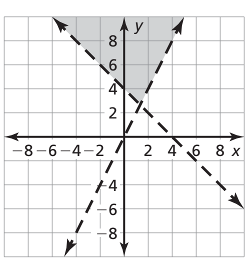
or

$$\begin{aligned} x - 4 &= 0 \\ \underline{+4} \quad &\ \ \underline{+4} \\ x &= 4 \end{aligned}$$

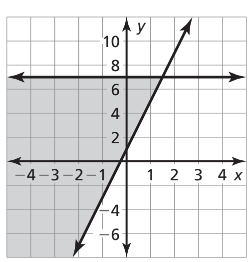
The points of intersection are $(-5, 4)$ and $(4, -5)$.

Maintaining Mathematical Proficiency

63.

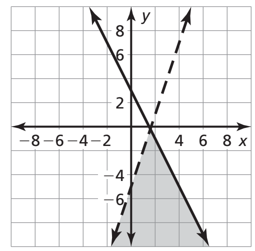


64.



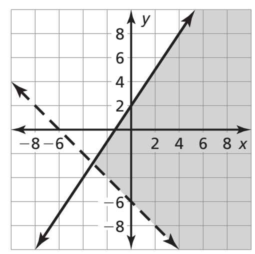
65.

$$\begin{aligned} y - 3 &\le -2x \\ y - 3 + 3 &\le -2x + 3 \\ y &\le -2x + 3 \end{aligned} \qquad \begin{aligned} y + 5 &< 3x \\ y + 5 - 5 &< 3x - 5 \\ y &< 3x - 5 \end{aligned}$$



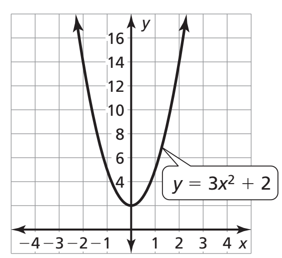
66.

$$\begin{aligned} x + y &> -6 \\ x - x + y &> -6 - x \\ y &> -x - 6 \end{aligned} \qquad \begin{aligned} 2y &\le 3x + 4 \\ \frac{2y}{2} &\le \frac{3x + 4}{2} \\ y &\le \tfrac{3}{2}x + 2 \end{aligned}$$



67. $y = 3x^2 + 2$

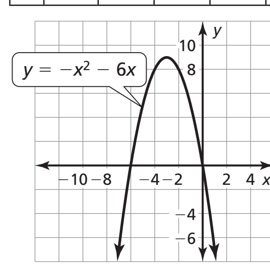
x	-2	-1	0	1	2
y	14	5	2	5	14



The domain is all real numbers. The range is $y \ge 2$.

68. $y = -x^2 - 6x$

x	-7	-6	-5	-4	-3	-2	-1	0	1
y	-7	0	5	8	9	8	5	0	-7

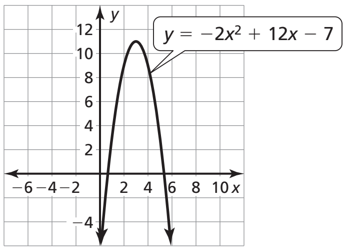


The domain is all real numbers. The range is $y \le 9$.

Chapter 9

69. $y = -2x^2 + 12x - 7$

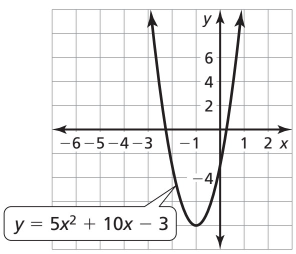
x	0	1	2	3	4	5	6
y	-7	3	9	11	9	3	-7



The domain is all real numbers. The range is $y \le -11$.

70. $y = 5x^2 + 10x - 3$

x	-3	-2	-1	0	1
y	12	-3	-8	-3	12



The domain is all real numbers. The range is $y \ge -8$.

9.4–9.6 What Did You Learn? (p. 533)

1. It provides a quick way to check for equations with no solutions.

2. When a and c have different signs, ac is negative, so subtracting $-4ac$ is subtracting a negative, which is the same as adding a positive. So, $b^2 - 4ac$ is positive.

3. *Sample answer:* All the methods had the same results. Substitution and elimination both resulted in equations that could easily be put in the form $x^2 = d$. Graphing did not require algebraic manipulation.

Chapter 9 Review (pp. 534–536)

1. $$\begin{aligned}\sqrt{72p^7} &= \sqrt{36 \cdot 2 \cdot p^6 \cdot p}\\ &= \sqrt{36} \cdot \sqrt{2} \cdot \sqrt{p^6} \cdot \sqrt{p}\\ &= 6 \cdot \sqrt{2} \cdot p^3 \cdot \sqrt{p}\\ &= 6 \cdot p^3 \cdot \sqrt{2} \cdot \sqrt{p}\\ &= 6p^3\sqrt{2p}\end{aligned}$$

2. $$\begin{aligned}\sqrt{\frac{45}{7y}} &= \frac{\sqrt{45}}{\sqrt{7y}}\\ &= \frac{\sqrt{9 \cdot 5}}{\sqrt{7y}} \cdot \frac{\sqrt{7y}}{\sqrt{7y}}\\ &= \frac{\sqrt{9} \cdot \sqrt{5} \cdot \sqrt{7y}}{\sqrt{49y^2}}\\ &= \frac{3\sqrt{35y}}{7y}\end{aligned}$$

3. $$\begin{aligned}\sqrt[3]{\frac{125x^{11}}{4}} &= \frac{\sqrt[3]{125x^{11}}}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}\\ &= \frac{\sqrt[3]{125 \cdot x^9 \cdot x^2} \cdot \sqrt[3]{2}}{\sqrt[3]{8}}\\ &= \frac{\sqrt[3]{125} \cdot \sqrt[3]{x^9} \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{2}}{2}\\ &= \frac{5 \cdot x^3 \cdot \sqrt[3]{x^2} \cdot \sqrt[3]{2}}{2}\\ &= \frac{5x^3\sqrt[3]{2x^2}}{2}\end{aligned}$$

4. $$\begin{aligned}\frac{8}{\sqrt{6} + 2} &= \frac{8}{\sqrt{6} + 2} \cdot \frac{\sqrt{6} - 2}{\sqrt{6} - 2}\\ &= \frac{8(\sqrt{6} - 2)}{(\sqrt{6})^2 - 2^2}\\ &= \frac{8\sqrt{6} - 16}{6 - 4}\\ &= \frac{8\sqrt{6} - 16}{2}\\ &= 4\sqrt{6} - 8\end{aligned}$$

5. $$\begin{aligned}4\sqrt{3} + 5\sqrt{12} &= 4\sqrt{3} + 5 \cdot \sqrt{4 \cdot 3}\\ &= 4\sqrt{3} + 5 \cdot \sqrt{4} \cdot \sqrt{3}\\ &= 4\sqrt{3} + 5 \cdot 2 \cdot \sqrt{3}\\ &= 4\sqrt{3} + 10\sqrt{3}\\ &= (4 + 10)\sqrt{3}\\ &= 14\sqrt{3}\end{aligned}$$

6. $$\begin{aligned}15\sqrt[3]{2} - 2\sqrt[3]{54} &= 15\sqrt[3]{2} - 2\sqrt[3]{27 \cdot 2}\\ &= 15\sqrt[3]{2} - 2 \cdot \sqrt[3]{27} \cdot \sqrt[3]{2}\\ &= 15\sqrt[3]{2} - 2 \cdot 3 \cdot \sqrt[3]{2}\\ &= 15\sqrt[3]{2} - 6\sqrt[3]{2}\\ &= (15 - 6)\sqrt[3]{2}\\ &= 9\sqrt[3]{2}\end{aligned}$$

Chapter 9

7. $(3\sqrt{7}+5)^2 = (3\sqrt{7})^2 + 2(3\sqrt{7})(5) + 5^2$

$= 3^2 \bullet (\sqrt{7})^2 + 2 \bullet 3 \bullet 5 \bullet \sqrt{7} + 25$

$= 9 \bullet 7 + 30\sqrt{7} + 25$

$= 63 + 30\sqrt{7} + 25$

$= 63 + 25 + 30\sqrt{7}$

$= 88 + 30\sqrt{7}$

8. $\sqrt{6}(\sqrt{18}+\sqrt{8}) = \sqrt{6} \bullet \sqrt{18} + \sqrt{6} \bullet \sqrt{8}$

$= \sqrt{108} + \sqrt{48}$

$= \sqrt{36 \bullet 3} + \sqrt{16 \bullet 3}$

$= \sqrt{36} \bullet \sqrt{3} + \sqrt{16} \bullet \sqrt{3}$

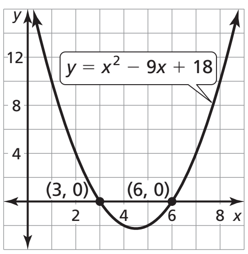
$= 6\sqrt{3} + 4\sqrt{3}$

$= (6+4)\sqrt{3}$

$= 10\sqrt{3}$

9. $x^2 - 9x + 18 = 0$

Graph $y = x^2 - 9x + 18$.



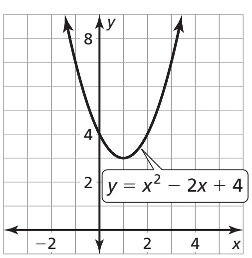
The x-intercepts are 3 and 6. So, the solutions of the equation are $x = 3$ and $x = 6$.

10. $x^2 - 2x = -4$

$x^2 - 2x + 4 = -4 + 4$

$x^2 - 2x + 4 = 0$

Graph $y = x^2 - 2x + 4$.



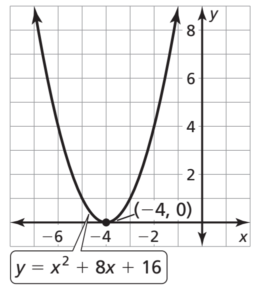
The graph does not cross the x-axis. So, the equation has no solutions.

11. $-8x - 16 = x^2$

$-8x + 8x - 16 + 16 = x^2 + 8x + 16$

$0 = x^2 + 8x + 16$

Graph $y = x^2 + 8x + 16$.



The only x-intercept is $(-4, 0)$. So, The solution is $x = -4$.

12. The graph appears to cross the x-axis at $(-3, 0)$, $(-1, 0)$, and $(1, 0)$.

Check $f(x) = (x+1)(x^2+2x-3)$

$0 \stackrel{?}{=} (-3+1)[(-3)^2 + 2(-3) - 3]$

$0 \stackrel{?}{=} (-2)(9 - 6 - 3)$

$0 \stackrel{?}{=} (-2)(3 - 3)$

$0 \stackrel{?}{=} (-2)(0)$

$0 = 0$ ✓

$f(x) = (x+1)(x^2+2x-3)$

$0 \stackrel{?}{=} (-1+1)[(-1)^2 + 2(-1) - 3]$

$0 \stackrel{?}{=} (0)(1 - 2 - 3)$

$0 \stackrel{?}{=} (0)(-1 - 3)$

$0 \stackrel{?}{=} (0)(-4)$

$0 = 0$ ✓

$f(x) = (x+1)(x^2+2x-3)$

$0 \stackrel{?}{=} (1+1)[1^2 + 2(1) - 3]$

$0 \stackrel{?}{=} (2)(1 + 2 - 3)$

$0 \stackrel{?}{=} (2)(3 - 3)$

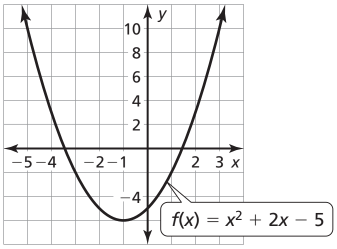
$0 \stackrel{?}{=} (2)(0)$

$0 = 0$ ✓

The solutions are $x = -3$, $x = -1$, and $x = 1$.

Chapter 9

13. Graph $f(x) = x^2 + 2x - 5$.



The graph crosses the x-axis between $x = -4$ and $x = -3$ and again between $x = 1$ and $x = 2$.

x	-3.9	-3.8	-3.7	-3.6	-3.5	-3.4
y	2.41	1.84	1.29	0.76	0.25	-0.24

x	-3.3	-3.2	-3.1
y	-0.71	-1.16	-1.59

change in signs

x	1.1	1.2	1.3	1.4	1.5
y	-1.59	-1.16	-0.71	-0.24	0.25

x	1.6	1.7	1.8	1.9
y	0.76	1.29	1.84	2.41

change in signs

In each table, the function value that is closest to 0 is -0.24. So, the solutions of the equation are $x \approx -3.4$ and $x \approx 1.4$.

14.
$$\begin{aligned} x^2 + 5 &= 17 \\ \underline{-5} \quad &\ \ \underline{-5} \\ x^2 &= 12 \\ \sqrt{x^2} &= \sqrt{12} \\ x &= \pm\sqrt{12} \\ x &= \pm\sqrt{4} \cdot \sqrt{3} \\ x &= \pm 2\sqrt{3} \end{aligned}$$

The solutions are $x = 2\sqrt{3} \approx 3.46$ and $x = -2\sqrt{3} \approx -3.46$.

15.
$$\begin{aligned} x^2 - 14 &= -14 \\ \underline{+14} \quad &\ \ \underline{+14} \\ x^2 &= 0 \\ \sqrt{x^2} &= \sqrt{0} \\ x &= 0 \end{aligned}$$

The solution is $x = 0$.

16.
$$\begin{aligned} (x + 2)^2 &= 64 \\ \sqrt{(x + 2)^2} &= \sqrt{64} \\ x + 2 &= \pm 8 \\ \underline{-2} \quad &\ \ \underline{-2} \\ x &= -2 \pm 8 \end{aligned}$$

The solutions are $x = -2 + 8 = 6$ and $x = -2 - 8 = -10$.

17.
$$\begin{aligned} 4x^2 + 25 &= -75 \\ \underline{-25} \quad &\ \ \underline{-25} \\ 4x^2 &= -100 \\ \frac{4x^2}{4} &= \frac{-100}{4} \\ x^2 &= -25 \end{aligned}$$

No real number multiplied by itself produces a negative number. So, the equation has no real solutions.

18.
$$\begin{aligned} (x - 1)^2 &= 0 \\ \sqrt{(x - 1)^2} &= \sqrt{0} \\ x - 1 &= 0 \\ \underline{+1} \quad &\ \ \underline{+1} \\ x &= 1 \end{aligned}$$

The solution is $x = 1$.

19.
$$\begin{aligned} 19 &= 30 - 5x^2 \\ \underline{-30} \quad &\ \ \underline{-30} \\ -11 &= -5x^2 \\ \frac{-11}{-5} &= \frac{-5x^2}{-5} \\ \frac{11}{5} &= x^2 \\ \sqrt{\frac{11}{5}} &= \sqrt{x^2} \\ \pm\sqrt{\frac{11}{5}} &= x \\ \pm\frac{\sqrt{11}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} &= x \\ \pm\frac{\sqrt{55}}{5} &= x \end{aligned}$$

The solutions are $x = \frac{\sqrt{55}}{5} \approx 1.48$ and $x = -\frac{\sqrt{55}}{5} \approx -1.48$.

20.
$$\begin{aligned} x^2 + 6x - 40 &= 0 \\ x^2 + 6x - 40 + 40 &= 0 + 40 \\ x^2 + 6x &= 40 \\ x^2 + 6x + 3^2 &= 40 + 3^2 \\ (x + 3)^2 &= 40 + 9 \\ (x + 3)^2 &= 49 \\ \sqrt{(x + 3)^2} &= \sqrt{49} \\ x + 3 &= \pm 7 \\ \underline{-3} \quad &\ \ \underline{-3} \\ x &= -3 \pm 7 \end{aligned}$$

So, the solutions are $x = -3 + 7 = 4$ and $x = -3 - 7 = -10$.

21.

$$\begin{aligned} x^2 + 2x + 5 &= 4 \\ x^2 + 2x + 5 - 5 &= 4 - 5 \\ x^2 + 2x &= -1 \\ x^2 + 2x + 1^2 &= -1 + 1^2 \\ (x + 1)^2 &= -1 + 1 \\ (x + 1)^2 &= 0 \\ \sqrt{(x + 1)^2} &= \sqrt{0} \\ x + 1 &= 0 \\ \underline{-1} \quad & \quad \underline{-1} \\ x &= -1 \end{aligned}$$

The solution is $x = -1$.

22.

$$\begin{aligned} 2x^2 - 4x &= 10 \\ \frac{2x^2 - 4x}{2} &= \frac{10}{2} \\ x^2 - 2x &= 5 \\ x^2 - 2x + (-1)^2 &= 5 + (-1)^2 \\ (x - 1)^2 &= 5 + 1 \\ (x - 1)^2 &= 6 \\ \sqrt{(x - 1)^2} &= \sqrt{6} \\ x - 1 &= \pm\sqrt{6} \\ \underline{+1} \quad & \quad \underline{+1} \\ x &= 1 \pm \sqrt{6} \end{aligned}$$

The solutions are $x = 1 + \sqrt{6} \approx 3.45$ and $x = 1 - \sqrt{6} \approx -1.45$.

23.

$$\begin{aligned} y &= -x^2 + 6x - 1 \\ y + 1 &= -x^2 + 6x - 1 + 1 \\ y + 1 &= -x^2 + 6x \\ y + 1 &= -(x^2 - 6x) \\ y + 1 - (-3)^2 &= -\left[x^2 - 6x + (-3)^2\right] \\ y + 1 - 9 &= -(x - 3)^2 \\ y - 8 &= -(x - 3)^2 \\ y - 8 + 8 &= -(x - 3)^2 + 8 \\ y &= -(x - 3)^2 + 8 \end{aligned}$$

The vertex is (3, 8). Because a is negative ($a = -1$), the parabola opens down, and the y-coordinate of the vertex is the maximum value. So, the function has a maximum value of 8.

24.

$$\begin{aligned} f(x) &= x^2 + 4x + 11 \\ f(x) - 11 &= x^2 + 4x + 11 - 11 \\ f(x) - 11 &= x^2 + 4x \\ f(x) - 11 + 2^2 &= x^2 + 4x + 2^2 \\ f(x) - 11 + 4 &= (x + 2)^2 \\ f(x) - 7 &= (x + 2)^2 \\ f(x) - 7 + 7 &= (x + 2)^2 + 7 \\ f(x) &= (x + 2)^2 + 7 \end{aligned}$$

The vertex is $(-2, 7)$. Because a is positive ($a = 1$), the parabola opens up, and the y-coordinate of the vertex is the minimum value. So, the function has a minimum value of 7.

25.

$$\begin{aligned} y &= 3x^2 - 24x + 15 \\ y - 15 &= 3x^2 - 24x + 15 - 15 \\ y - 15 &= 3x^2 - 24x \\ y - 15 &= 3(x^2 - 8x) \\ y - 15 + 3 \cdot (-4)^2 &= 3\left[x^2 - 8x + (-4)^2\right] \\ y - 15 + 3 \cdot 16 &= 3(x - 4)^2 \\ y - 15 + 48 &= 3(x - 4)^2 \\ y + 33 &= 3(x - 4)^2 \\ y + 33 - 33 &= 3(x - 4)^2 - 33 \\ y &= 3(x - 4)^2 - 33 \end{aligned}$$

The vertex is $(4, -33)$. Because a is positive ($a = 3$), the parabola opens up, and the y-coordinate of the vertex is the minimum value. So, the function has a minimum value of -33.

26. Let $\ell - 3$ be the width of the credit card.

$$\begin{aligned} \text{Area} &= \text{length} \cdot \text{width} \\ 46.75 &= \ell \cdot (\ell - 3) \\ 46.75 &= \ell(\ell) - \ell(3) \\ 46.75 &= \ell^2 - 3\ell \\ 46.75 + (-1.5)^2 &= \ell^2 - 3\ell + (-1.5)^2 \\ 46.75 + 2.25 &= (\ell - 1.5)^2 \\ 49 &= (\ell - 1.5)^2 \\ \sqrt{49} &= \sqrt{(\ell - 1.5)^2} \\ \pm 7 &= \ell - 1.5 \\ \underline{+1.5} \quad & \quad \underline{+1.5} \\ 1.5 \pm 7 &= \ell \end{aligned}$$

The solutions are $\ell = 1.5 + 7 = 8.5$ and $\ell = 1.5 - 7 = -5.5$. Disregard the negative solution, because a negative length does not make sense in this situation.

$w = \ell - 3 = 8.5 - 3 = 5.5$

$P = 2\ell + 2w = 2(8.5) + 2(5.5) = 17 + 11 = 28$

So, the perimeter of the credit card is 28 centimeters.

Chapter 9

27. $x^2 + 2x - 15 = 0$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{2^2 - 4(1)(-15)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{4 + 60}}{2} \\ &= \frac{-2 \pm \sqrt{64}}{2} \\ &= \frac{-2 \pm 8}{2} \end{aligned}$$

So, the solutions are $x = \frac{-2 + 8}{2} = \frac{6}{2} = 3$ and $x = \frac{-2 - 8}{2} = \frac{-10}{2} = -5$.

28.
$$\begin{aligned} 2x^2 - x + 8 &= 16 \\ 2x^2 - x + 8 - 16 &= 16 - 16 \\ 2x^2 - x - 8 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(-8)}}{2(2)} \\ &= \frac{1 \pm \sqrt{1 + 64}}{4} \\ &= \frac{1 \pm \sqrt{65}}{4} \end{aligned}$$

So, the solutions are $x = \frac{1 + \sqrt{65}}{4} \approx 2.3$ and $x = \frac{1 - \sqrt{65}}{4} \approx -1.8$.

29.
$$\begin{aligned} -5x^2 + 10x &= 5 \\ -5x^2 + 10x - 5 &= 5 - 5 \\ -5x^2 + 10x - 5 &= 0 \end{aligned}$$

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{10^2 - 4(-5)(-5)}}{2(-5)} \\ &= \frac{-10 \pm \sqrt{100 - 100}}{-10} \\ &= \frac{-10 \pm \sqrt{0}}{-10} \\ &= \frac{-10 \pm 0}{-10} \\ &= \frac{-10}{-10} \\ &= 1 \end{aligned}$$

The solution is $x = 1$.

30.
$$\begin{aligned} y &= -x^2 + 6x - 9 \\ b^2 - 4ac &= 6^2 - 4(-1)(-9) \\ &= 36 - 36 \\ &= 0 \end{aligned}$$

Because the discriminant is 0, the equation has one solution. So, the graph has one x-intercept.

31.
$$\begin{aligned} y &= 2x^2 + 4x + 8 \\ b^2 - 4ac &= 4^2 - 4(2)(8) \\ &= 16 - 64 \\ &= -48 \end{aligned}$$

Because the discriminant is negative, the equation has no solutions. So, the graph has no x-intercepts.

32.
$$\begin{aligned} y &= -\tfrac{1}{2}x^2 + 2x \\ b^2 - 4ac &= 2^2 - 4\left(-\tfrac{1}{2}\right)(0) \\ &= 4 - 0 \\ &= 4 \end{aligned}$$

Because the discriminant is positive, the equation has two solutions. So, the graph has two x-intercepts.

33. $y = x^2 - 2x - 4$

$y = -5$

Step 2
$$\begin{aligned} y &= x^2 - 2x - 4 \\ -5 &= x^2 - 2x - 4 \\ -5 + 5 &= x^2 - 2x - 4 + 5 \\ 0 &= x^2 - 2x + 1 \\ 0 &= x^2 - 2(x)(1) + (-1)^2 \\ 0 &= (x - 1)^2 \\ x - 1 &= 0 \\ +1 \quad & \quad +1 \\ x &= 1 \end{aligned}$$

Step 3 $y = -5$

So, the solution is $(1, -5)$.

34. Step 2
$$\begin{aligned} y &= x^2 \qquad\quad - 9 \\ -(y &= \qquad 2x + 5) \\ \hline 0 &= x^2 - 2x - 14 \end{aligned}$$

Step 3
$$\begin{aligned} 0 + 14 &= x^2 - 2x - 14 + 14 \\ 14 &= x^2 - 2x \\ 14 + (-1)^2 &= x^2 - 2x + (-1)^2 \\ 14 + 1 &= (x - 1)^2 \\ 15 &= (x - 1)^2 \\ \sqrt{15} &= \sqrt{(x - 1)^2} \\ \pm\sqrt{15} &= x - 1 \\ +1 \quad & \quad +1 \\ 1 \pm \sqrt{15} &= x \end{aligned}$$

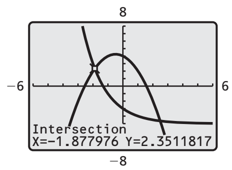
Chapter 9

Step 4 $y = 2x + 5$ $\qquad$ $y = 2x + 5$

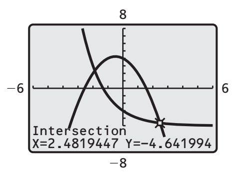
$$y = 2\left(1 + \sqrt{15}\right) + 5 \qquad y = 2\left(1 - \sqrt{15}\right) + 5$$
$$y = 2 + 2\sqrt{15} + 5 \qquad y = 2 - 2\sqrt{15} + 5$$
$$y = 2 + 5 + 2\sqrt{15} \qquad y = 2 + 5 - 2\sqrt{15}$$
$$y = 7 + 2\sqrt{15} \qquad y = 7 - 2\sqrt{15}$$

So, the solutions are $\left(1 + \sqrt{15}, 7 + 2\sqrt{15}\right) \approx (4.87, 14.75)$ and $\left(1 - \sqrt{15}, 7 - 2\sqrt{15}\right) \approx (-2.87, -0.75)$.

35. Graph the system $y = 2\left(\frac{1}{2}\right)^x - 5$ and $y = -x^2 - x + 4$.



One point of intersection is about $(-1.88, 2.35)$.



The other point of intersection is about $(2.48, -4.64)$.

So, the solutions are about $(-1.88, 2.35)$ and about $(2.48, -4.64)$.

Chapter 9 Test (p. 537)

1. $x^2 - 121 = 0$

$$\underline{+\,121} \quad \underline{+\,121}$$
$$x^2 = 121$$
$$\sqrt{x^2} = \sqrt{121}$$
$$x = \pm 11$$

The solutions are $x = 11$ and $x = -11$. *Sample answer:* The equation can be written in the form $x^2 = d$. So, solve using square roots.

2. $x^2 - 6x = 10$

$$x^2 - 6x + (-3)^2 = 10 + (-3)^2$$
$$(x - 3)^2 = 10 + 9$$
$$(x - 3)^2 = 19$$
$$\sqrt{(x - 3)^2} = \sqrt{19}$$
$$x - 3 = \pm\sqrt{19}$$
$$\underline{+\,3} \quad \underline{+\,3}$$
$$x = 3 \pm \sqrt{19}$$

The solutions are $x = 3 + \sqrt{19} \approx 7.36$ and $x = 3 - \sqrt{19} \approx -1.36$. *Sample answer:* Because $a = 1$ and b is even, solve by completing the square.

3. $-2x^2 + 3x + 7 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3 \pm \sqrt{3^2 - 4(-2)(7)}}{2(-2)}$$
$$= \frac{-3 \pm \sqrt{9 + 56}}{-4}$$
$$= \frac{-3 \pm \sqrt{65}}{-4}, \text{ or } \frac{3 \mp \sqrt{65}}{4}$$

So, the solutions are $x = \frac{3 - \sqrt{65}}{4} \approx -1.27$ and $x = \frac{3 + \sqrt{65}}{4} \approx 2.77$. *Sample answer:* The equation is not factorable, and $a \neq 1$. So, solve using the Quadratic Formula.

4. $x^2 - 7x + 12 = 0$

$$(x - 3)(x - 4) = 0$$
$$x - 3 = 0 \quad \text{or} \quad x - 4 = 0$$
$$\underline{+\,3} \quad \underline{+\,3} \qquad \underline{+\,4} \quad \underline{+\,4}$$
$$x = 3 \qquad x = 4$$

The solutions are $x = 3$ and $x = 4$. *Sample answer:* Because the equation is easily factorable, solve by factoring.

5. $5x^2 + x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4(5)(-4)}}{2(5)}$$
$$= \frac{-1 \pm \sqrt{1 + 80}}{10}$$
$$= \frac{-1 \pm \sqrt{81}}{10}$$
$$= \frac{-1 \pm 9}{10}$$

The solutions are $x = \frac{-1 + 9}{10} = \frac{8}{10} = \frac{4}{5}$ and $x = \frac{-1 - 9}{10} = \frac{-10}{10} = -1$. *Sample answer:* The equation is not easily factorable and $a \neq 1$. So, solve using the Quadratic Formula.

6. $(4x + 3)^2 = 16$

$$\sqrt{(4x + 3)^2} = \sqrt{16}$$
$$4x + 3 = \pm 4$$
$$\underline{-\,3} \quad \underline{-\,3}$$
$$4x = -3 \pm 4$$
$$\frac{4x}{4} = \frac{-3 \pm 4}{4}$$
$$x = \frac{-3 \pm 4}{4}$$

The solutions are $x = \frac{-3 + 4}{4} = \frac{1}{4}$ and $x = \frac{-3 - 4}{4} = -\frac{7}{4}$. *Sample answer:* The equation is in the form $x^2 = d$, where x is a binomial. So, solve using square roots.

Chapter 9

7. Use completing the square to rewrite the function in vertex form.

$$\begin{aligned}
f(x) &= 2x^2 + 4x - 6\\
f(x) + 6 &= 2x^2 + 4x - 6 + 6\\
f(x) + 6 &= 2x^2 + 4x\\
f(x) + 6 &= 2(x^2 + 2x)\\
f(x) + 6 + 2 \bullet 1^2 &= 2(x^2 + 2x + 1^2)\\
f(x) + 6 + 2 \bullet 1 &= 2(x + 1)^2\\
f(x) + 6 + 2 &= 2(x + 1)^2\\
f(x) + 8 &= 2(x + 1)^2\\
f(x) + 8 - 8 &= 2(x + 1)^2 - 8\\
f(x) &= 2(x + 1)^2 - 8
\end{aligned}$$

So, the vertex of the graph is $(-1, -8)$, which is in the third quadrant. The vertex of the graph shown is in the fourth quadrant. So, the function $f(x) = 2x^2 + 4x - 6$ cannot be represented by the graph shown.

8. *Sample answer:* $\dfrac{1}{2 + \sqrt{3}}$

9. Step 2

$$\begin{aligned}
y &= x^2 - 4x - 2\\
-(y &= - 4x + 2)\\
\hline
0 &= x^2 - 4
\end{aligned}$$

Step 3

$$\begin{aligned}
0 &= x^2 - 4\\
\underline{+ 4} & \quad \underline{+ 4}\\
4 &= x^2\\
\sqrt{4} &= \sqrt{x^2}\\
\pm 2 &= x
\end{aligned}$$

Step 4

$$\begin{aligned}
y &= -4x + 2 & y &= -4x + 2\\
y &= -4(2) + 2 & y &= -4(-2) + 2\\
y &= -8 + 2 & y &= 8 + 2\\
y &= -6 & y &= 10
\end{aligned}$$

The solutions are $(2, -6)$ and $(-2, 10)$.

10. $y = -5x^2 + x - 1$

$y = -7$

Step 2

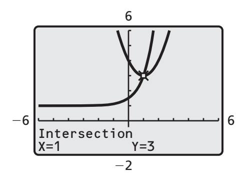
$$\begin{aligned}
y &= -5x^2 + x - 1\\
-7 &= -5x^2 + x - 1\\
-7 + 5x^2 - x + 1 &= -5x^2 + 5x^2 + x - x - 1 + 1\\
5x^2 - x - 6 &= 0\\
(5x - 6)(x + 1) &= 0
\end{aligned}$$

$$\begin{aligned}
5x - 6 &= 0 & \text{or} \qquad x + 1 &= 0\\
\underline{+ 6} \quad & \underline{+ 6} & \underline{- 1} \quad & \underline{- 1}\\
5x &= 6 & x &= -1\\
\frac{5x}{5} &= \frac{6}{5} & &\\
x &= \frac{6}{5} & &
\end{aligned}$$

The solutions are $(-1, -7)$ and $\left(\frac{6}{5}, -7\right)$.

11. $y = \frac{1}{2}(4)^x + 1$

$y = x^2 - 2x + 4$



The only point of intersection is $(1, 3)$. So, the solution is $(1, 3)$.

Chapter 9

12. $h = -16t^2 + 28t + 8$

$$0 = -16t^2 + 28t + 8$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-28 \pm \sqrt{28^2 - 4(-16)(8)}}{2(-16)}$$

$$= \frac{-28 \pm \sqrt{784 + 512}}{-32}$$

$$= \frac{-28 \pm \sqrt{1296}}{-32}$$

$$= \frac{-28 \pm 36}{-32}, \text{ or } \frac{7 \mp 9}{8}$$

The solutions are $t = \frac{7 - 9}{8} = \frac{-2}{8} = -\frac{1}{4}$ and $t = \frac{7 + 9}{8} = \frac{16}{8} = 2$. Disregard the negative solution because a negative time does not make sense in this situation. So, the skier is in the air for 2 seconds, which earns $2(5) = 10$ points.

$$h = -16t^2 + 28t + 8$$

$$h - 8 = -16t^2 + 28t + 8 - 8$$

$$h - 8 = -16t^2 + 28t$$

$$h - 8 = -16\left(t^2 - \tfrac{7}{4}t\right)$$

$$h - 8 - 16 \cdot \left(-\tfrac{7}{8}\right)^2 = -16\left[t^2 - \tfrac{7}{4}t + \left(-\tfrac{7}{8}\right)^2\right]$$

$$h - 8 - 16 \cdot \tfrac{49}{64} = -16\left(t - \tfrac{7}{4}\right)^2$$

$$h - 8 - \tfrac{49}{4} = -16\left(t - \tfrac{7}{4}\right)^2$$

$$h - \tfrac{81}{4} = -16\left(t - \tfrac{7}{4}\right)^2$$

$$h - \tfrac{81}{4} + \tfrac{81}{4} = -16\left(t - \tfrac{7}{4}\right)^2 + \tfrac{81}{4}$$

$$h = -16\left(t - \tfrac{7}{4}\right)^2 + \tfrac{81}{4}$$

The vertex is $\left(\frac{7}{4}, \frac{81}{4}\right)$. So, the skier reaches a maximum height of $\frac{81}{4}$, or 20.25 feet, which earns 20.25 points. So, the skier earns $25 + 10 + 20.25 = 55.25$ points.

13.

$$h = -16t^2 + 265$$

$$105 = -16t^2 + 265$$

$$\underline{-265} \qquad \underline{-265}$$

$$-160 = -16t^2$$

$$\frac{-160}{-16} = \frac{-16t^2}{-16}$$

$$10 = t^2$$

$$\sqrt{10} = \sqrt{t^2}$$

$$\pm\sqrt{10} = t$$

The solutions are $t = \sqrt{10} \approx 3.16$ and $t = -\sqrt{10} \approx -3.16$. Disregard the negative solution, because a negative time does not make sense in this situation. So, the riders experience free fall for about 3.16 seconds.

14. Area = length • width

$$= \sqrt{30x^7} \cdot \frac{36}{\sqrt{3}}$$

$$= \frac{\sqrt{30x^7} \cdot 36}{\sqrt{3}}$$

$$= 36 \cdot \sqrt{\frac{30x^7}{3}}$$

$$= 36 \cdot \sqrt{10x^7}$$

$$= 36 \cdot \sqrt{10 \cdot x^6 \cdot x}$$

$$= 36 \cdot \sqrt{10} \cdot \sqrt{x^6} \cdot \sqrt{x}$$

$$= 36 \cdot \sqrt{10} \cdot x^3 \cdot \sqrt{x}$$

$$= 36 \cdot x^3 \cdot \sqrt{10} \cdot \sqrt{x}$$

$$= 36x^3\sqrt{10x}$$

So, an expression that represents the area of the painting shown is $36x^3\sqrt{10x}$ square inches.

15. You can calculate the value of the discriminant $b^2 - 4ac$. If the discriminant is negative, then the equation has no solutions and the graph has no x-intercepts. If the discriminant is 0, then the equation has one solution and the graph has one x-intercept. If the discriminant is positive, then the equation has two solutions and the graph has two x-intercepts.

16. a. *Sample answer:* Let $a = 2$ and $b = 4$.

$$b^2 - 4ac > 0$$

$$4^2 - 4(2)c > 0$$

$$16 - 8c > 0$$

$$\underline{-16} \qquad \underline{-16}$$

$$-8c > -16$$

$$\frac{-8c}{-8} < \frac{-16}{-8}$$

$$c < 2$$

Let $c = 1$.

So, if $a = 2$, $b = 4$, and $c = 1$, then $ax^2 + bx + c = 0$ has two x-intercepts.

b. *Sample answer:* Let $a = 1$ and $b = 4$.

$$b^2 - 4ac = 0$$

$$4^2 - 4(1)c = 0$$

$$16 - 4c = 0$$

$$\underline{-16} \qquad \underline{-16}$$

$$-4c = -16$$

$$\frac{-4c}{-4} = \frac{-16}{-4}$$

$$c = 4$$

So, if $a = 1$, $b = 4$, and $c = 4$, then $ax^2 + bx + c = 0$ has one x-intercept.

Chapter 9

c. *Sample answer:* Let $a = 2$ and $b = 4$.

$$
\begin{aligned}
b^2 - 4ac &< 0 \\
4^2 - 4(2)c &< 0 \\
16 - 8c &< 0 \\
-16 \qquad & \quad -16 \\
-8c &< -16 \\
\frac{-8c}{-8} &> \frac{-16}{-8} \\
c &> 2
\end{aligned}
$$

Let $c = 3$.

So, if $a = 2$, $b = 4$, and $c = 3$, then $ax^2 + bx + c = 0$ has no x-intercepts.

17. a.

$$
\begin{aligned}
y &= 3x^2 + 8x + 20 \\
400 &= 3x^2 + 8x + 20 \\
400 - 400 &= 3x^2 + 8x + 20 - 400 \\
0 &= 3x^2 + 8x - 380 \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-8 \pm \sqrt{8^2 - 4(3)(-380)}}{2(3)} \\
&= \frac{-8 \pm \sqrt{64 + 4560}}{6} \\
&= \frac{-8 \pm \sqrt{4624}}{6} \\
&= \frac{-8 \pm 68}{6}, \text{ or } \frac{-4 \pm 34}{3}
\end{aligned}
$$

So, the solutions are $x = \frac{-4 + 34}{3} = \frac{30}{3} = 10$ and

$x = \frac{-4 - 34}{3} = -\frac{38}{3}$. Disregard the negative solution,

because a negative length of time does not make sense in this situation. So, there are 400 Type A bacteria after 10 hours.

b. Step 2

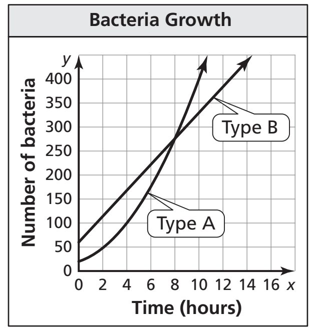
$$
\begin{aligned}
y &= 3x^2 + 8x + 20 \\
-(y &= \qquad\quad 27x + 60) \\
\hline
0 &= 3x^2 - 19x - 40
\end{aligned}
$$

Step 3 $0 = (3x + 5)(x - 8)$

$$
\begin{aligned}
3x + 5 &= 0 \quad \text{or} \quad x - 8 = 0 \\
-5 \quad & -5 \qquad\quad +8 \quad +8 \\
3x &= -5 \qquad\qquad x = 8 \\
\frac{3x}{3} &= \frac{-5}{3} \\
x &= \frac{-5}{3}
\end{aligned}
$$

Disregard the negative solution, because a negative length of time does not make sense in this situation. So, the number of Type A and Type B bacteria are the same after 8 hours.

c. Graph the system $y = 3x^2 + 8x + 20$ and $y = 27x + 60$.



You can see from the graph that there are more Type A bacteria than Type B after 8 hours, and there are more Type B bacteria than Type A before 8 hours.

Chapter 9 Standards Assessment (pp. 538–539)

1. Because f does not cross the x-axis, if the function is set equal to zero, its discriminant will be negative. Because g has two x-intercepts, if the function is set equal to zero, its discriminant will be positive. Because h and j each have one x-intercept, if each of their functions are set equal to zero, their discriminants will each be zero.

2. a. *Sample answer:* When $a = 400$ (or any number less than 600) and $b = 1.15$ (or any number greater than 1.08), Account B has a greater initial amount and increases at a faster rate than Account A.

b. *Sample answer:* When $a = 800$ (or any number greater than 600) and $b = 1.15$ (or any number greater than 1.08), Account B has a lesser initial amount than Account A but increases at a faster rate than Account A.

c. *Sample answer:* When $a = 600$ and $b = 1.05$ (or any number greater than 1 and less than 1.08), Account B and Account A have the same initial amount, and Account B increases at a slower rate than Account A.

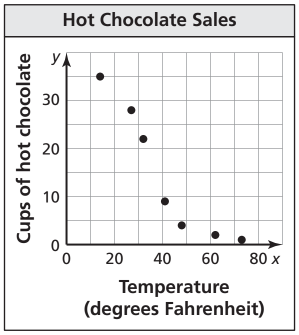
3. a. Your friend is incorrect. The height is unknown. So, you will not be able to calculate the radius when you are given only the surface area.

b. Your friend is correct that you can find the radius of a sphere when you are given the surface area.

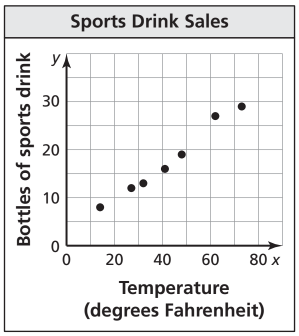
$$
\begin{aligned}
S &= 4\pi r^2 \\
\frac{S}{4\pi} &= \frac{4\pi r^2}{4\pi} \\
\frac{S}{4\pi} &= r^2 \\
\sqrt{\frac{S}{4\pi}} &= \sqrt{r} \\
\sqrt{\frac{S}{4\pi}} &= r
\end{aligned}
$$

So, you can use the formula $r = \sqrt{\frac{S}{4\pi}}$ to calculate the radius of a sphere when given the surface area.

Chapter 9

4. 

As x increases, y decreases. So, the data show a negative correlation.



As x increases, y increases. So, the data show a positive correlation.

5. A; Graph A is a curve that grows at an increasing rate as x increases. So, it shows exponential growth. Graph B shows exponential decay. Graph C shows linear growth. Graph D is a parabola, which represents a quadratic function.

6. B; Solve by elimination.

Step 2

$$\begin{aligned} y &= x^2 + 2x - 8 \\ -(y &= \phantom{x^2 + {}} 5x + 2) \\ \hline 0 &= x^2 - 3x - 10 \\ 0 &= (x - 5)(x + 2) \end{aligned}$$

$$\begin{array}{lcl} x - 5 = 0 & \text{or} & x + 2 = 0 \\ \underline{+5} \quad \underline{+5} & & \underline{-2} \quad \underline{-2} \\ x = 5 & & x = -2 \end{array}$$

Step 3

$$\begin{array}{ll} y = 5x + 2 & y = 5x + 2 \\ y = 5(5) + 2 & y = 5(-2) + 2 \\ y = 25 + 2 & y = -10 + 2 \\ y = 27 & y = -8 \end{array}$$

The solutions are $(5, 27)$ and $(-2, -8)$. So, statement B is correct.

7. The expressions that are in simplest form are $16\sqrt{5}$, $3x\sqrt{5x}$, $\frac{4\sqrt{7}}{3}$, and $2\sqrt[3]{x^2}$ because none of their radicands have perfect nth powers as factors other than 1, none of the radicands contain fractions, and no radicals appear in the denominator of a fraction.

8.

$$\begin{aligned} f(-2) &= 4(-2) - 5 \\ &= -8 - 5 \\ &= -13 \end{aligned} \qquad \begin{aligned} f(-1) &= 4(-1) - 5 \\ &= -4 - 5 \\ &= -9 \end{aligned}$$

$$\begin{aligned} f(0) &= 4(0) - 5 \\ &= 0 - 5 \\ &= -5 \end{aligned} \qquad \begin{aligned} f(1) &= 4(1) - 5 \\ &= 4 - 5 \\ &= -1 \end{aligned}$$

$$\begin{aligned} f(2) &= 4(2) - 5 \\ &= 8 - 5 \\ &= 3 \end{aligned} \qquad \begin{aligned} f(3) &= 4(3) - 5 \\ &= 12 - 5 \\ &= 7 \end{aligned}$$

So, the solutions are $(-2, -13)$, $(-1, -9)$, $(0, -5)$, $(1, -1)$, $(2, 3)$, and $(3, 7)$.