

Geothermal Energy

6.1 INTRODUCTION AND WHY GEOTHERMAL IS IMPORTANT

6.1.1 History and Growth of Usage

Geothermal power has been used since ancient times—at least in those places on Earth where geysers and hot springs spontaneously bubble up from the Earth. In fact, the town of Bath in England is named for the thermal baths developed there by the ancient Romans. It was, however, not until the twentieth century that the vastness of the reserves of underground heat throughout the planet's interior was appreciated and the first usage of geothermal power for electricity generation was demonstrated in 1904 and put to use, generating significant amounts in 1911 when a power plant was built in Larderello, Italy. Subsequent progress of installing geothermal electrical generating capacity has continued since that pioneering development at an exponentially expanding rate. In fact, the capacity during the 80 years preceding 2000 has been growing approximately exponentially with an 8.5% annual growth rate. Since 2000, the global usage of geothermal has accelerated just as rapidly both because of the push for energy alternatives and recent technological advances.

6.1.2 Geographic Distribution

As far as the direct heating usage of geothermal is concerned, the leading nation is China in 2010, with the United States a close second. Interestingly, China is nowhere to be found among the top 15 nations in geothermal electricity production for which 10.7 GW was produced in 2010 worldwide—a 20% increase over the last 5 years. Despite the past rapid growth, geothermal now accounts for a meager 0.5% of the world's electricity, which is about the same as solar cells. The United States produces the most geothermal electricity (3.1 GW), with the Philippines in second place at 1.9 GW. Strictly in terms of percentages, Iceland is the world leader, where approximately 53.4% of the total national consumption of primary energy is from geothermal (Figure 6.1).

The global distribution of the most productive geothermal sources is largely dictated by geography, since this energy source is most abundant and accessible at places on Earth near tectonic plate boundaries, or in major volcanic regions. In most other places, geothermal has not yet proven economically competitive to exploit—at least for electricity generation. Given that many of the best sites for geothermal electricity generation have already been exploited, its future rapid growth is contingent on making technological advances that will allow lower grade resources to be exploited at costs competitive with other sources.

Chapter



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Figure 6.1 Geothermal borehole outside the Reykjavik Power Station. (Photo taken by Yomangani, and released to the public domain; http://en.wikipedia.org/wiki/Geothermal_power_in_Iceland)

6.1.3 Sources of the Earth's Thermal Energy

Much of the Earth's stored heat never makes it to the surface spontaneously. In fact, on average only around $0.06 \pm 0.02 \text{ W/m}^2$ geothermal power reaches the Earth's surface on its own, which is a tiny fraction of the energy reaching the surface from the sun. However, the available stored thermal energy in the Earth's interior is enormous. According to an MIT study, the U.S. total geothermal energy that could feasibly be extracted with improved technology in the upper 10 km of the Earth's crust is over 2000 ZJ or $2 \times 10^{24} \text{ J}$, or 4000 times the energy humans use per year (Tester, 2006).

Heat is created underground by at least six different mechanisms, but around 80% of it is generated due to radioactive decay mostly from very long-lived isotopes of uranium and thorium—although that estimated 80% could be as little as 45% or as much as 90%. Given that decay of radioisotopes having multibillion-year half-life is the primary

source of the Earth's heat, even if it were extracted in sizable quantities there need be no concern of it running out, since it is being continually replenished.

6.1.4 Comparison with Other Energy Sources

Geothermal energy as a means for generating electricity has the great advantage of not being intermittent like most other renewable sources, such as wind and solar; in fact, its average “capacity factor” is around 73%. This means that a plant produces full power 73% of the time—far higher than wind turbines, for example. As a result, geothermal electrical plants are capable of providing base load electricity, which is not the case for intermittent renewable sources like wind. Moreover, in places where conditions are favorable, geothermal electric power can be produced at a very cost-competitive basis compared to other methods, either renewable or nonrenewable. As with other forms of renewable energy, the cost of electricity from an existing plant does not fluctuate like gas or oil, since the fuel is free. However, the cost of new geothermal plants is strongly dependent on the price of oil and gas because those costs influence the competition for drilling equipment—and drilling is the main contributor to capital costs (Figure 6.2).

The International Geothermal Association expects that in the coming 5 years geothermal electric generating capacity might be expected to expand by as much as 80%, with much of the expansion taking place in areas previously not considered favorable—a development made possible by recent technological improvements. However, even in areas where conditions are not favorable for generating electricity because drilling to

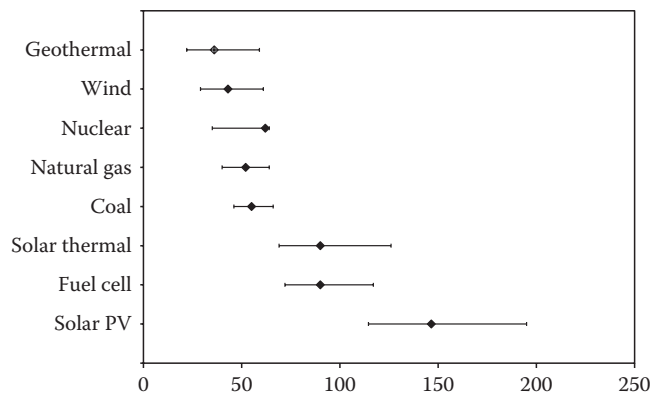


Figure 6.2 Comparison of electricity generation costs in \$/MW-h for eight fuel sources. The bars show high and low estimates for the ranges. The costs are for plants in the United States and they include a \$19/MW-h tax incentive for renewable sources. Moreover, these are “levelized” costs, which assume that the same interest rates can be obtained for highly capital-intensive sources compared to others. These data are from a 2009 publication, and the relative costs of electricity from different sources can change appreciably over time. (From Mims, C., *Sci. Am.*, March 2, 2009.)

reach high enough temperatures would be cost-prohibitive, geothermal always can be used for residential heating, which does not require very high temperatures. Geothermal heat has also proven useful for a wide range of other nonresidential uses, including district heating, hot water heating, horticulture, industrial processes, and even tourism, i.e., hot thermal baths.

6.2 GEOPHYSICS OF THE EARTH'S INTERIOR

The underground composition, temperature, and pressure throughout the Earth's interior is a challenging problem that geophysicists have solved only indirectly, since direct underground exploration is limited by the depths to which boreholes can be drilled. Few oil and gas wells go deeper than 6 km, although the current depth record is the Kola research borehole at 12.262 km in Russia. Despite having only explored directly partway through continental crust, geophysicists are confident that they understand the entire interior of the Earth, based on results from the science of seismology. The chief method is to create seismic waves at one point on the Earth by detonating an explosive charge, and then record the arrival time at many other locations around the globe. These waves will be reflected at discontinuous interior boundaries and refracted in media whose properties change continuously. Moreover, the two types of seismic waves known as "s-waves" and "p-waves" differ in their nature, since while the former is longitudinal, the latter is transverse. This distinction is important since while both s-waves and p-waves can pass through solids, only the former can pass through liquids, allowing seismologists to deduce which interior layers of the Earth are liquid, and which solid. As a result of seismology studies, geophysicists now believe that the Earth's interior consists of the following three main regions:

Core: which extends out to half the Earth's radius (6400 km) and is made mostly of iron (80%) and nickel (20%), whose inner half (by radius) is solid and whose outer half is liquid. This iron and nickel core is the source of the Earth's magnetic field, which is believed to be created by electric currents in the core.

Mantle: which makes up most of the rest (83%) of the Earth's volume, and made mostly of rocky material, whose inner part is semirigid, and whose outer and cooler part is plastic and, therefore, can flow (think lava).

Crust: the outermost thin layer (1% of the Earth's volume), whose average thickness is 15 km. The crustal thickness ranges from a high of 90 km under continental mountains to as little as 5 km under some parts of the oceans. On a scale where the Earth is the size of a soccer ball, the crust would be a mere 0.25 mm thick.

6.3 THERMAL GRADIENT

The thermal gradient is the rate of change of temperature with depth. The Earth has a radius of 6400 km and at its center the temperature is believed to be 7000 K, giving the convenient value of about 1 K/km or 1°C/km for the average gradient. The gradient, however, does vary enormously both as a function of depth and as a function of the particular location on Earth. **Figure 6.3** illustrates the former variation, which is strongly correlated with the composition of each interior region. The largest gradient (the topmost section of the graph) is on the Earth's crust, where the gradient averages 25–30 K/km. Since the crust is solid and heat cannot be transferred by convection, we may apply the heat conduction equation for the flow across a layer (slab) of thickness, Δz , to find for the thermal gradient. The heat flow per unit area across the slab is given by $\dot{q} = k\Delta T/\Delta z$, where k is the thermal conductivity and ΔT is the temperature difference across the slab. Hence,

$$\frac{\Delta T}{\Delta z} = \frac{\dot{q}}{k} \quad (6.1)$$

For the inner core of the Earth, we see in **Figure 6.3** that there is a rise in temperature of about 1200 K in the first 1000 km out from the center for a gradient of 1.2 K/km—a value that is 20–25 times smaller than that for the crust. This difference can be explained using **Equation 6.1**, since the thermal conductivities of iron and rocks are 55 W/m-K and around 4 W/m-K, respectively—making k for rock around 13 times smaller than for iron. If we were to assume that the heat flow out of the core is the same as that which eventually passes through the crust, then based on the preceding ratio of k values, we would predict the thermal gradient in the crust to be around 13 times larger than that in the inner core, or around 15 K/km, which is within a factor of 2 with what is actually found. Expecting any better agreement than this is unrealistic given the large variation in conductivities for different types of rock.

How can we explain the sudden changes in thermal gradient (slope) that occur at the two boundaries of the outer core? Recall that the outer core is liquid not solid and that convective and conductive heat flow occur in parallel there. As a result, the thermal gradient will be smaller for the outer core than the inner core. So far we have been considering how the thermal gradients vary with depth on a very large scale—from the surface to the center of the Earth—most of which is inaccessible for energy extraction. For geothermal energy to be accessible, we are primarily concerned about the Earth's crust, and how gradients there vary from place to place. As we can see from **Figure 6.4**, it can vary quite a bit—both from place to place and also as a function of depth. Not surprisingly, the known location that has the highest thermal gradient, Lardorello, Italy, was the site for the first geothermal electricity source because the high thermal gradient found there means that one can reach very high temperatures (close to 200°C) in a mere 0.25 km.

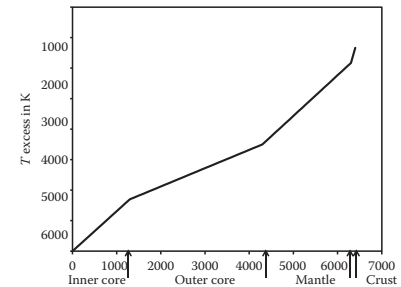


Figure 6.3 Excess temperature in K or °C above that on the surface as a function of distance from the center in kilometers showing the boundaries between interior regions of the Earth.

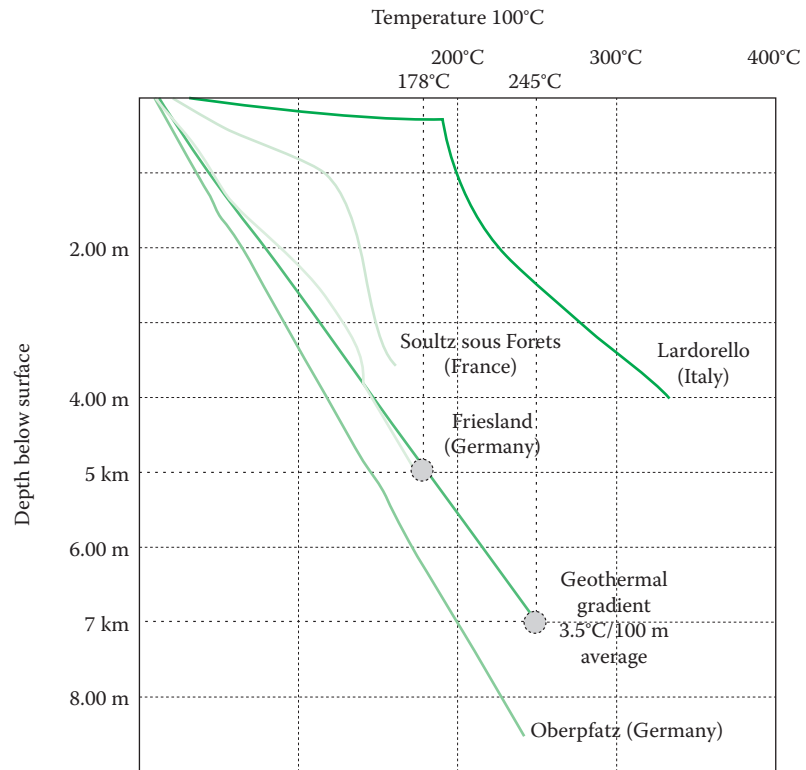


Figure 6.4 Earth's crust temperature in degrees Celsius versus depth in km in selected places. (Courtesy of Geohil AG (modified by Electropaedia); http://www.mpoweruk.com/geothermal_energy.htm)

The abrupt changes seen in thermal gradients for Lardorello and Oberpfalz in Figure 6.4 are a consequence of radical changes in rock composition at certain depths. For example, the sharp discontinuity seen for the Lardorello curve is due to a magma intrusion into a region where the rocks (such as granite) have high values of specific heat and density (so they hold a lot of thermal energy per unit volume). In addition, there is a layer of sedimentary rock above the granite having low thermal conductivity that tends to trap the stored heat below. It is natural to ask whether the abrupt changes in gradient (slope discontinuity) such as those occurring at Lardorello (initial gradient of an astounding $680^{\circ}\text{C}/\text{km}$) and Soutz-sous are rare or common. It must be the case anywhere on Earth where the thermal gradient is initially very high that its value will change radically at some deeper depth. Were this not the case, then given the initial gradient of $680^{\circ}\text{C}/\text{km}$ found at Lardorello, the temperature would reach nearly that at the center of the Earth in a mere 10 km, which is clearly impossible.

Obviously, the most promising places to build geothermal plants are where the gradient is highest, and the depth of wells to access high temperatures is the least. Figure 6.5 shows how the gradient varies across the continental United States, based on data from drilling numerous boreholes—sometimes in connection with gas and oil exploration.

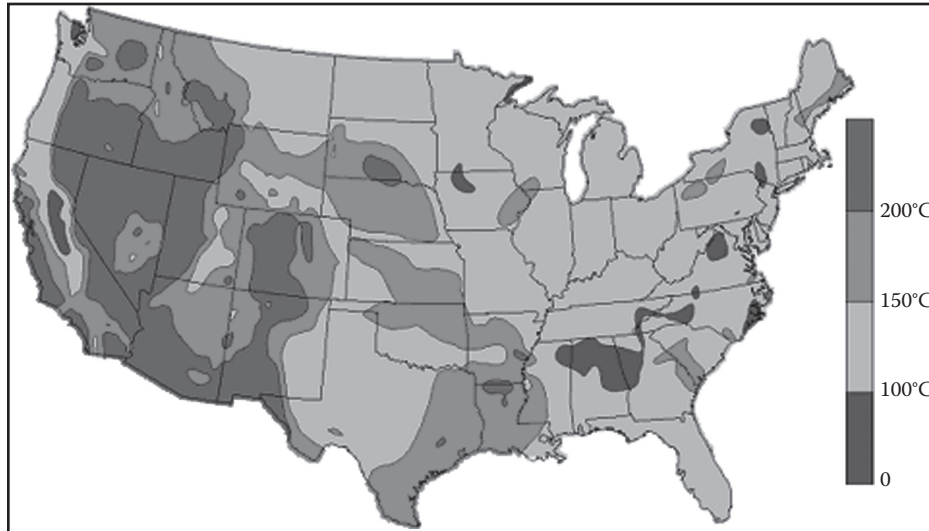


Figure 6.5 Temperatures at 6 km depth beneath the surface of the earth. The thermal gradient in $^{\circ}\text{C}/\text{km}$ over that distance therefore equals the indicated gray-scale-coded temperatures divided by 6. (Courtesy of U.S. Department of Energy, Washington, DC, public domain image.)

6.4 CHARACTERIZATION AND RELATIVE ABUNDANCE OF THE RESOURCE

6.4.1 Impact of the Thermal Gradient

As we have noted, to generate electricity the most important characteristic is the thermal gradient because this quantity will determine the well depth that needs to be reached to access temperatures above some minimum needed for a power plant—typically 150°C , even though some types of power plants can operate at lower temperatures. For this reason geothermal resources are often put into three grades: high, medium, and low, based on the gradient. High grade resources have gradients in excess of $250^{\circ}\text{C}/\text{km}$, medium grade have gradients $150\text{--}250^{\circ}\text{C}/\text{km}$, and low grade have gradients below $150^{\circ}\text{C}/\text{km}$. These grades are quite arbitrary and depend on the intended usage of the resource—in this case electricity generation. Thus, it is not surprising that some experts prefer other categories, such as “hyperthermal” (above $80^{\circ}\text{C}/\text{km}$), “semithermal” ($40\text{--}80^{\circ}\text{C}/\text{km}$), and “normal” (below $40^{\circ}\text{C}/\text{km}$). It is clear from [Figure 6.5](#) that in the case of the United States, it is mainly in some of the Western states that geothermal energy can most easily be exploited for producing electricity.

Let us now evaluate the amount of the thermal resource available under the assumption that the thermal gradient G at some location is constant with depth z , although as already noted this assumption is questionable—especially for the case of a very high initial gradient. Suppose that z_1 is the minimum depth needed to reach temperatures of $T_1 = 150^{\circ}\text{C}$, and z_2 is the maximum depth to which current technology allows wells to be drilled. Recall that by the definition of the specific heat of a substance, c , the stored thermal energy in a mass m whose temperature excess ΔT above some reference temperature can be expressed as $E = mc\Delta T = mc(T - T_1)$. Thus, the amount of stored thermal energy below a surface area A between a depth z and $z + dz$ can be expressed as

$dE = \rho Ac \Delta T dz = \rho Ac(T - T_1) dz$, where ρ is the rock density. Finally, given the definition of the thermal gradient $G = dT/dz = (T - T_1)/(z - z_1)$, we can integrate dE to find the total energy stored between depths z_1 , and z_2 :

$$\begin{aligned} E &= \int_{z_1}^{z_2} \rho Ac(T - T_1) dz = \int_{z_1}^{z_2} \rho Ac G(z - z_1) dz = \frac{1}{2} \rho Ac G(z_2 - z_1)^2 \\ &= \frac{1}{2} \rho Ac G \left(z_2 - \frac{T_1}{G} \right)^2 \end{aligned} \quad (6.2)$$

6.4.2 Example 1: Relative Energy Content for Two Gradients

Suppose we have two locations A and B for which the gradients are $G_A = 100^\circ\text{C}/\text{km}$ and $G_B = 50^\circ\text{C}/\text{km}$. What are the ratios of the energy content per unit area of surface down to a depth of 6 km at the two places, assuming that a minimum temperature of 150°C is needed? How can this be illustrated graphically?

Solution

$$E_A = \frac{1}{2} \rho Ac G \left(z_2 - \frac{T_{\min}}{G_A} \right)^2 = \frac{1}{2} \rho Ac (100) \left(6 - \frac{150}{100} \right)^2 = 703 \rho Ac$$

$$E_B = \frac{1}{2} \rho Ac G \left(z_2 - \frac{T_{\min}}{G_B} \right)^2 = \frac{1}{2} \rho Ac (50) \left(6 - \frac{150}{50} \right)^2 = 225 \rho Ac$$

Thus, the energy per unit area at A is 3.12 times that at B . The respective energies at A and B correspond to the areas of the white and shaded triangles (Figure 6.6).

6.4.3 Questioning Our Assumptions

Recall that the preceding analysis made the assumption that it is the maximum drillable depth that is the main limitation in exploiting a geothermal resource. This assumption may be incorrect. For example, the Kola borehole (the world's deepest) reached a far greater depth (12.3 km) than would be possible in most places on Earth. This was possible in Kola only due to the exceptionally low gradient there ($13^\circ\text{C}/\text{km}$), so that even at 12.3 km the temperature did not yet exceed 200°C . It therefore seems reasonable to believe that the state of drilling technology in reality does not limit the maximum drillable depth at all, but rather the maximum temperature, which currently seems to be around 300°C . This fact has a surprising impact on our earlier assessment of how the amount of available energy depends on thermal gradient.

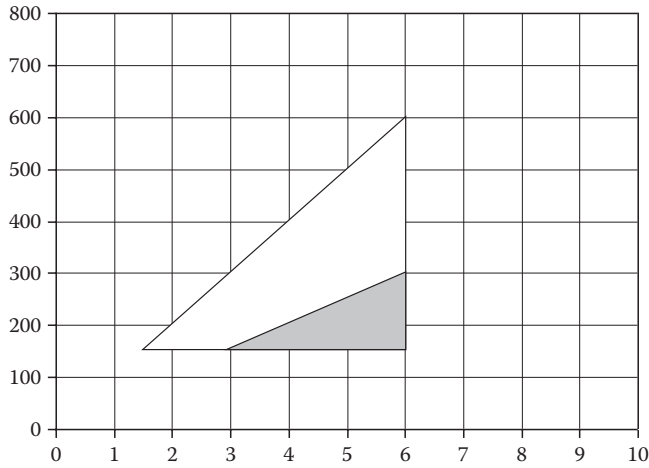


Figure 6.6 Temperature in °C versus depth for two different gradients: (a) 100°C/km (slope of hypotenuse of white triangle) and (b) 50°C/km (slope of hypotenuse of shaded triangle). It is assumed that for both cases wells have the same maximum depth (6 km), but different minimum depths since $z_1 = 150^\circ\text{C}/G$. We have also assumed for simplicity that the surface temperature is 0°C.

It can easily be shown by integration that when T is the limiting factor, instead of Equation 6.2, we have

$$E = \frac{1}{2G} \rho A c (T_2 - T_1)^2 \tag{6.3}$$

We can again use a graphical representation to understand this result. As illustrated in Figure 6.7, the respective areas for the high and low gradient cases now favor the low gradient case by a 2 to 1 margin! Moreover, this very surprising result holds irrespective of the specific choice of maximum temperature (an unrealistic 600°C in the figure). Of course, the economic

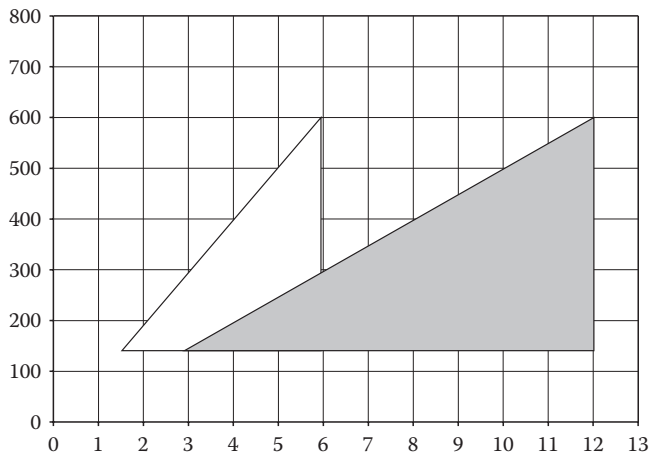


Figure 6.7 Temperature in °C versus depth for two different gradients: (a) 100°C/km (slope of hypotenuse of white triangle), and (b) 50°C/km (slope of hypotenuse of shaded triangle). It is assumed that for both cases wells have the same maximum temperature (600°C), and the surface temperature (for $z = 0$) is 0°C for simplicity.

feasibility of extraction may make the exploitation of the low gradient location (with the need for deeper wells) out of the question—but much more on this topic later. To summarize the main point of the preceding discussion: the conventional wisdom that high thermal gradient resources are more worthwhile to exploit in terms of extracting energy at reasonable cost depends crucially on whether the limitation on drilling technology is a matter of (a) maximum depth or (b) maximum temperature, and the precise the manner in which drilling costs depend on depth.

6.4.4 Other Geologic Factors Affecting the Amount of the Resource

In addition to the thermal gradient at a given location, a geothermal prospector will also want to know many other geological characteristics, including these five properties of the underlying rock formations: hardness, thermal conductivity, specific heat, density, and porosity, the latter being the fraction of the rock volume that is empty space, which is often filled with fluids, usually water with dissolved salts. A particularly desirable choice would be rocks with high values of the density, specific heat, and thermal conductivity (such as granite). The best locations are also important: that the rock be overlain by a layer of sedimentary rocks having low thermal conductivity, which acts to trap the heat. Rocks that are permeable, meaning that fluids can flow through them, are also more desirable. If there is in fact fluid present in the rocks we have by definition an aquifer or hydrothermal system. A confined aquifer is one that is overlain by a nonporous rock layer.

In order to evaluate the thermal energy content of an aquifer using Equation 6.2, it would only be necessary to use the average value of c_p for the rock and fluid, i.e., to make the substitution:

$$c_p = p' \rho_w c_w + (1 - p') \rho_r c_r \quad (6.4)$$

where

the subscripts refer to water and rock

p' is the porosity of the rock, or the fraction of the aquifer volume taken up by fluid assumed to be water

6.4.5 Hot Dry Rock Formations

Aquifers are the easiest geothermal resource to exploit for extracting energy, because they already contain fluid, but most of the stored thermal energy in the Earth's crust is in dry rock formations which may lack porosity. The porosity of rocks can occur in one of two ways: either because of the spaces between grains of the rocks or because of large-scale fractures, which are far more favorable in terms of yielding greater permeability, making them less prone to clogging up over time when fluid flows through the rock. Starting in the 1970s, hydrofracture (or “fracking”) of rock

was pioneered (using injected water under pressure to create rock fractures). This technique allowed engineers to extract energy from hot dry rock (HDR) formations, using what has also more recently been termed enhanced geothermal systems (EGS). Although the fracking technique has been controversial in connection with oil and gas exploration, its use with geothermal power is not nearly as problematic since the chemical additives used to free up oil and gas in rock pores are not needed.

As explained earlier, in an EGS system, water needs to be pumped down a well under pressure to induce thermal stresses in HDRs, causing them to fracture and create porosity. In addition to this “injection well,” additional “production” wells must be dug some distance away—but close enough for the injected water to reach them by flowing through the rock fissures—see [Figure 6.8](#).

EGS geothermal tends to be much more expensive in terms of its initial investment (about five times more than hydrothermal systems) because wells usually must go deeper to reach higher temperatures. Extra production and injection wells may be required if the induced pores in the rocks should clog. Such extra wells may also be useful in order to increase the extracted power. In the case of multiple injection and production wells, their spacing is quite important—if they are too close they draw on the same thermal energy store, while if they are too far apart, they fail to draw adequately on a thermal energy store between them.

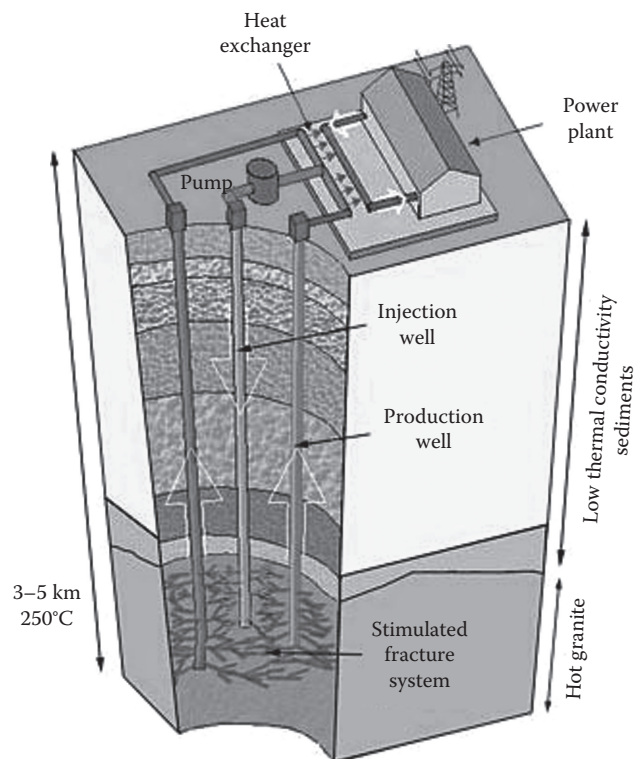


Figure 6.8 An EGS system having one injection well and two production wells. (Courtesy of Australian National University, Canberra, Australia (modified by Geothermal Resources Ltd).)

There are, of course, many other nongeological factors that a geothermal “prospector” need to consider that will determine whether it makes sense to exploit a geothermal resource. These include the current state of drilling technology, the state of the economy (and whether funds will be available for a large initial investment), the cost of natural gas (which affects the cost and availability of drilling equipment), the adequacy of existing transmission lines, the price of land or drilling rights, and the closeness to population centers.

6.5 GEOTHERMAL ELECTRICITY POWER PLANTS

There are three main types of geothermal power plants: dry steam, flash, and binary cycle, with the flash type being most common. The basics of the last two of these three types are depicted in Figure 6.9a and b. In the flash type of power plant, high pressure water comes up the production well and vaporizes (“flashes”) when its pressure is reduced to produce a flow of steam that drives a turbine, which then generates electricity. The dry steam type (not depicted) is similar to the flash type, but without the first step, since the dry steam coming up directly from the production well drives the turbine directly. This type of plant is rare because it generally is used in very high gradient locations where steam spontaneously rises out of the production well. Binary cycle power plants involve one additional step in the process. For these plants high temperature fluid coming up from a production well passes through a heat exchanger in which the secondary loop contains a low boiling point liquid such as butane or pentane, which can vaporize at a lower temperature than water. This added step allows such plants to generate electricity at much lower temperatures than the other types. The current low

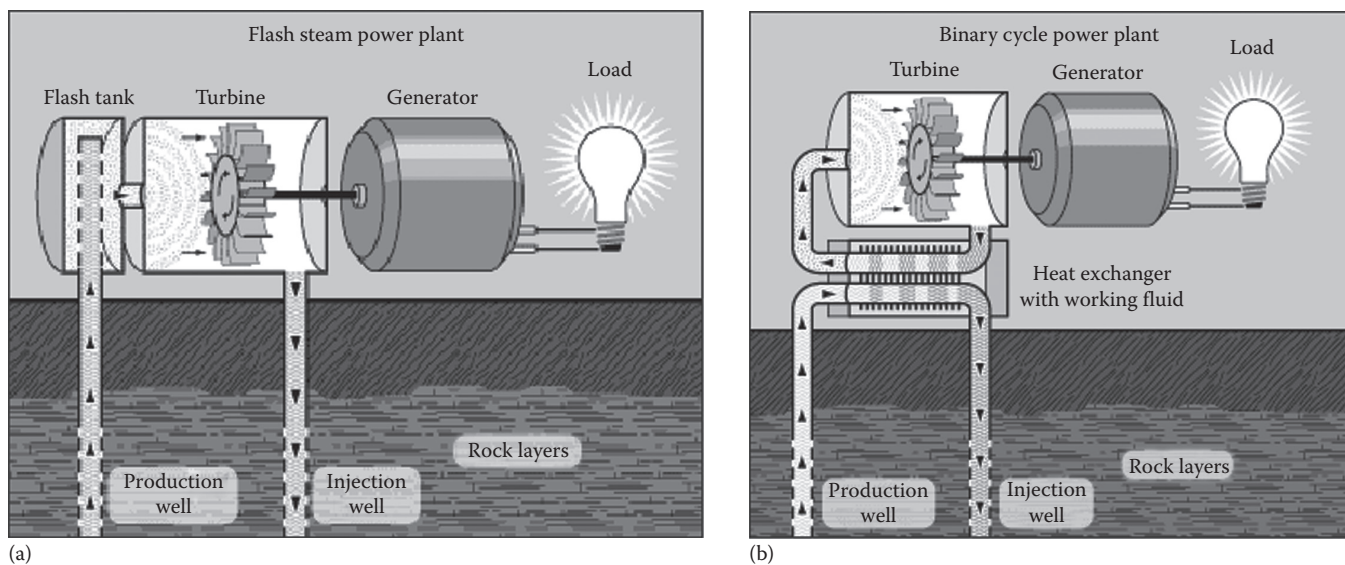


Figure 6.9 (a) Flash steam power plant; (b) Binary cycle power plant. (Courtesy of U.S. Department of Energy, Washington, DC, public domain images.)

temperature record for a binary plant is 57°C!—but, of course, the thermodynamic efficiency is very low due to Carnot's theorem. For example, a plant planned for Alaska using a Rankine cycle uses a 75°C hot spring that ejects heat to a 3°C river water, and it has an expected efficiency of only 8%. Some binary plants have several flash loops in series, each using a lower boiling point liquid, which allows thermal energy to be extracted several times, and results in higher efficiency. Low efficiency is a significant detriment in geothermal plants because even if the fuel is free, this raises the cost per MW/h of electricity to the point where it may be uneconomical. Thus, binary cycle plants tend to be significantly more expensive on a per MWh basis than the other types, even though they are useful in expanding the region over which geothermal electricity can be generated.

6.5.1 Example 2: Efficiency of a Geothermal Power Plant

Show that the efficiency of the proposed Alaskan power plant is a bit worse than half the maximum possible value and explain why that fraction would be even worse were the hot spring slightly colder or a nearby cold river unavailable.

Solution

The Carnot efficiency is given by

$$e_C = 1 - \frac{T_L}{T_H} = 1 - \frac{3 + 273}{75 + 273} = 0.207 \text{ (20.7\%)} \quad (6.5)$$

which is a bit more than twice the actual efficiency. Note that if either T_H were lower or T_C higher, the efficiency would be less.

6.6 RESIDENTIAL AND COMMERCIAL GEOTHERMAL HEATING

The direct use of geothermal energy especially for home heating is probably the fastest growing application, primarily because it can be implemented virtually anywhere, and requires neither high thermal gradients nor deeply drilled holes. In fact, it is not necessary to dig deeply enough to access temperatures even as warm as the desired temperature of your home, merely below the point where the ground temperature year-round is approximately the same. Given average soil conditions, this requirement means a depth of about 3 m where the annual variation is perhaps $\pm 3^\circ\text{C}$ from summer to winter.

A conventional heat pump works by extracting heat from the outside air (which in winter is probably colder than your house) and expelling the extracted heat to your home. In order to make heat “flow the wrong way,” i.e., from cold to hot, it is of course necessary to have an input of energy in the form of electricity that drives a compressor. How exactly does this work? In a conventional heat pump, a volatile fluid (basically a

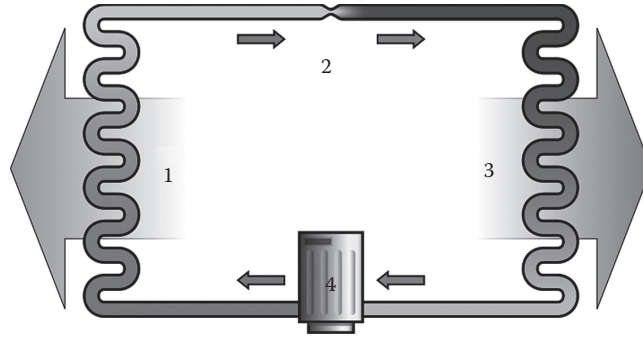


Figure 6.10 A simple stylized diagram of a heat pump's vapor-compression refrigeration cycle: 1, condenser; 2, expansion valve; 3, evaporator; 4, compressor expelling heat on the left (hot) side and absorbing heat or expelling cold on the right (cold) side.

refrigerant) in its vapor state is compressed by a compressor (#4 in [Figure 6.10](#)) so that it releases heat to its surroundings in the process of liquefying—the red coils in the figure. The high pressure liquid then passes through a valve where the pressure drop allows it to vaporize and cool below the temperature of the ground which acts to heat it—the blue section of the coil on the right. The cyclic process continues as long as electrical energy is supplied to the compressor.

The performance of a heat pump is measured in terms of its “coefficient of performance,” which is the ratio of the heat released to the home divided by the electrical energy supplied, a figure that is usually greater than 4, although much depends on how warm you want your home to be. In general, the coefficient of performance becomes greater the smaller the difference between the ground and home temperatures, with the maximum possible (Carnot) value being given by

$$COP_{\max} = \frac{T_H}{T_H - T_G} \quad (6.6)$$

In [Equation 6.6](#), both the high (H) and the ground (G) temperatures must be in Kelvins. Heat pumps are not specific to geothermal power, and in fact many homes use electric heat pumps that use heat extracted from the cold outside air to heat the home.

Geothermal heat pumps are more efficient than conventional ones since they extract thermal energy from the ground rather than the outside air and the ground below a few meters depth is warmer than the outside air in winter, and also because the circulating fluid is a liquid, not air which has a lower specific heat. In addition, unlike conventional heat pumps, the circulating fluid can be simply water or water plus antifreeze rather than a refrigerant. An unfortunate aspect of standard heat pumps is that the colder it is outside, i.e., the more you need them, the lower their ideal coefficient of performance by [Equation 6.6](#). This flaw, however, does not apply to geothermal heat pumps—do you see why? Heat pumps can also operate as air conditioners as well as suppliers of heat. In fact, the exact

same schematic diagram would describe how this works. The only difference is that now the hot (red) coil is understood to be inside your home and the cold (blue) one is in the ground. Essentially, the volatile fluid needs to circulate in the opposite direction in [Figure 6.10](#).

6.6.1 Economics of Residential Geothermal

There are several ways to characterize geothermal heat pumps, one being the layout of the pipes—either horizontal or vertical. Horizontal systems generally cost less than half the cost of vertical systems, given the much greater depth needed in the latter case. However, horizontal systems, such as the one shown in [Figure 6.11](#) before it was buried, are not as suitable on many lots because of the greater land area they require. Here are some approximate cost figures in 2010 U.S. dollars for these two types of systems.

Horizontal system: The up front costs of a residential system is about \$2500 per ton ($1\text{ T} = 3.517\text{ kW}$), or roughly \$7500 for a 3 ton unit on an average residential size system. This figure is about double the cost of a gas furnace, but since the fuel is free, typical annual energy savings are perhaps \$450/year, although, obviously it depends on many factors: the price of gas, the size of your house, and the temperature you choose to keep it at. Given the preceding cost figures, the breakeven time would be 7 years—longer if gas is very cheap, shorter if a sizable tax credit is allowed for the geothermal system.

Vertical system: In this case, drilling can run anywhere from \$10,000 to \$30,000, so the cost of a vertical system might be \$25,000, and so to make up the difference between this initial cost and that of a gas furnace would take $20,000/450$, or around 45 years.



Figure 6.11 A horizontal closed loop field is composed of pipes that run horizontally in the ground below the frost line—photo shows a “slinky” (overlapping coils) arrangement before it has been covered by dirt. The slinky style layout is a more space-efficient version of a horizontal ground loop than straight piping. (Image by Marktj released to the public domain; http://en.wikipedia.org/wiki/Geothermal_heat_pump)

The other way to characterize geothermal heat pumps is whether they are open or closed loop systems, with the former only used when the system is of the vertical type. In this case, water is pumped down a vertical injection well and passes through HDR to reach the production well in much the same way as in EGS systems used for electric power generation.

6.7 SUSTAINABILITY OF GEOTHERMAL

Given the way heat pumps work, and the relatively small amount of energy extracted from the ground, there is no question that a residential system can operate indefinitely without exhausting the heat stored there. What about the sustainability of large-scale geothermal extraction of heat from the Earth for producing electricity? Would the Earth begin to cool down in that case? Currently, geothermal accounts for only around 10GW electricity around the world. Since radioactive decay is continually replenishing most of the Earth's heat at about 30TW—or 3000 times as much—there is no need to worry about running out on a timescale of billions of years even if we extracted a far greater percentage than at present.

Nevertheless, depletion of an individual geothermal field is another story, which we shall now consider with the aid of a simplified model.

6.7.1 Depletion of a Geothermal Field

Here we shall assume that some number N production wells are sunk over a surface area A from which geothermal energy is extracted down to a depth z . The spacing between the wells is assumed to be such that the thermal energy in a given region is extracted by only one well. Let us further artificially assume that the entire resource from which energy is extracted is at a common temperature T , whose initial value at time $t = 0$ is taken to be T_0 . The geothermal field is stimulated when water is injected, and the thermal power extracted from the HDRs of density ρ_r and specific heat c_r can be expressed as

$$\dot{q} = \frac{dq}{dt} = -m_r c_r \frac{dT}{dt} = -Az\rho_r c_r \frac{dT}{dt} \quad (6.7)$$

This thermal power equals that in the water rising out of the production well, ignoring any losses or replenishment of the thermal energy from surrounding rock regions. Thus,

$$\dot{q} = \dot{m}c_w(T - T_S) = N\rho_w a v c_w(T - T_S) \quad (6.8)$$

where

v is the speed of the water rising out of the N pipes, each of cross-sectional area a

T_S is the surface temperature

Combining Equations 6.6 and 6.7 and rearranging terms yield

$$\frac{d(T - T_S)}{(T - T_S)} = -\frac{dt}{\tau} \quad (6.9)$$

where the lifetime τ of the geothermal field is given by

$$\tau = \frac{\rho_r A z c_r}{N \rho_w a v c_w} \quad (6.10)$$

Upon integrating both sides of Equation 6.9, we find that

$$T - T_S = (T_0 - T_S)e^{-t/\tau} \quad (6.11)$$

Note that the lifetime of the resource can also be characterized in terms of a half-life given by $T_{1/2} = \tau \ln 2$. Furthermore, note that the thermal power extracted declines with the same exponential time dependence, assuming v is constant. Thus, if E_0 is the initial energy content at $t = 0$, we have for the power extraction at any given time t

$$\frac{dE}{dt} = -\frac{E_0 e^{-t/\tau}}{\tau} \quad (6.12)$$

In practice, however, the actual lifetime will likely be greater than τ since we have ignored the replenishment from surrounding regions as energy is extracted. Typical values of the replenishment time (once energy extraction ceases) range from 1 to 10 times the lifetime. One can, of course, choose to extract energy at a lower rate to prolong the drawdown time, but then the power output becomes proportionately smaller. Note that geothermal power plants do tend to have much smaller power output in any case than either fossil fuel or nuclear plants—50–100 MW being typical. Another approach to prolonging the drawdown time is to extract the energy over a larger surface area, but that would mean many more extraction wells, which tend to increase costs, although not necessarily on a per MWh basis.

6.7.2 Example 3: Lengthening the Lifetime

Suppose a given geothermal resource has a lifetime of 20 years, and a replenishment time of 60 years. What are two ways to extend the lifetime of the resource to over 1000 years?

Solution

Given that the replenishment time is three times longer than the lifetime, thermal energy is being restored to the geothermal field at a third the rate it is being extracted, so if the power drawn were cut by two-thirds, the two would be in balance, and the field would essentially supply energy for many millions of years. An even better choice might be

to supply electricity only at peak times each day, when the demand is highest and when it is in short supply. Still another option would be to sink more extraction wells, but then run the water through them at a lower rate so the same amount of power extracted is now extracted over an area three times as large, resulting in a drawdown of the thermal energy at one-third the original rate and a near infinite lifetime would be the result.

6.7.3 Example 4: A 100 MW Power Plant

(a) Find the useful heat content per square km to a depth of 7 km. Assume a thermal gradient of $40^{\circ}\text{C}/\text{km}$, a minimum useful $T = 140^{\circ}\text{C}$ above that on the surface, a rock density of $2700\text{kg}/\text{m}^3$, and rock specific heat of $820\text{J}/\text{kg}\cdot\text{K}$. (b) What volume flow rate of injected water is needed for this power plant if it extracts heat over a surface area of 0.5km^2 ? (c) After how many years will the power produced be half its initial value, assuming a constant water flow rate?

Solution

(a) Using Equation 6.2 yields $5.4 \times 10^{17}\text{ J}/\text{km}^2$. (b) Using Equation 6.12, we first find the lifetime in terms of the initial power extraction ($dE/dt = 100\text{MW}$) and the initial total energy stored $E = 0.5\text{km}^2 \times 5.4 \times 10^{17}\text{ J}/\text{km}^2$ to be $\tau = 5.4 \times 10^9\text{ s}$. Finally, using Equation 6.10, we may solve for the mass flow rate of the water (the product \dot{m}) to be $3500\text{kg}/\text{s}$. (c) Using $T_{1/2} = \tau \ln 2$, we find a half-life of 118 years.

6.8 ENVIRONMENTAL IMPACTS

6.8.1 Released Gases

A number of noxious gases are emitted during operation of a geothermal plant, but in relatively low concentrations. Plants where this is a problem may be required to install emission controls. The emitted gases may include a small amount of radon, which is a by-product of the decay of uranium—one of the main isotopes accounting for much of the Earth's stored heat. Radon (the second leading cause of lung cancer) is a well-recognized problem in some homes where it can seep up through cracks and become concentrated in the home, especially in the basement, and even more especially in "tight," well-insulated homes. However, it is relatively harmless when it comes up from the Earth and is released to the outdoors. Moreover, areas having good geothermal potential are not associated with higher than average uranium concentrations. Radon gas can become dissolved in the drilling fluid used in geothermal wells, but in EGS systems this fluid is continually recycled back down the well, and hence it is not released to the environment. Greenhouse gases (especially carbon dioxide) are also an issue of possible concern. However, production of geothermal electricity usually results in far less CO_2 emitted than fossil fuel sources, but there is some variation depending on the type of power plant and the characteristics of the geothermal field. Typically,

geothermal power plants have less than a tenth that of coal-fired plants (Bloomfield, 2003). The situation with respect to CO₂ emissions when geothermal is used for residential heating is more complex and is discussed in Section 6.8.3.

6.8.2 Impact on Land and Freshwater

Small amounts of some harmful substances are found in the fluid after water is injected into a well during the hydrofracture process. However, these can be injected and recycled back from the production hole to reduce risk. Normally with HDR stimulation, the wells are deep enough so that groundwater should not be affected, unlike the case of using hydrofracture to extract natural gas. Geothermal power plants tend to occupy a relatively small land area (a small “footprint”), especially in comparison to other energy sources. For example, the comparable figures in units of km²/GW are 3.5 (geothermal), 12 (wind farms), 32 (coal), 20 (nuclear), and (20–50) solar.

Geothermal plants do require a source of freshwater; however, unlike nuclear, coal, gas, or oil, the water is continually recycled, so that the amount used per MW-h generated is negligible. Subsistence of land has occurred in some places due to operation of a geothermal plant in New Zealand, and several locations in Germany. Even worse, one geothermal plant built in Basel, Switzerland, was shut down after many small earthquakes were observed—10,000 of them up to magnitude 3.4 during its first week of operation. On the other hand, it must also be remembered that seismologists have been inducing artificial earthquakes since the 1960s, with the largest having a magnitude 4.5, but no large earthquakes have occurred as a result. During the hydrofracture process, earthquakes can be induced when the size of the fractures created are large, but can be controlled by adjusting water flow rates and pressures in fracturing the rocks. Most importantly, one must avoid intersecting a large natural fault that could trigger a large earthquake.

BOX 6.1 THE BASEL EXPERIENCE

After a series of small earthquakes occurred, the Swiss government did a study that concluded that if the project had been allowed to continue, there was a 15% chance of it triggering a quake powerful enough to cause damage of up to \$500 million. As a result, the government brought criminal charges against the head of the company Geopower that did the drilling. However, the trial found that the company had not deliberately damaged property or acted carelessly. Nevertheless, the Swiss government’s fears may not have been unfounded because there is an earthquake fault in Basel, and in the year 1356 the town experienced what may have been the most significant seismological event in Central Europe in recorded history. That earthquake led to the destruction of the town and all major churches and castles within a 30 km radius.

6.8.3 Do Heat Pumps Cut Down on CO₂ Emissions?

We have seen that geothermal plants emit negligible amounts of CO₂, but for geothermal residential heating the issue is less clear. Geothermal heating systems usually rely on heat pumps that extract heat from the ground and deliver it to a higher temperature, i.e., the interior of your house. Thus, they make heat flow the “wrong way” through the input of work. The COP of ground source heat pumps is usually above 4. The actual average COP tends to be somewhat lower than 4 when we include the energy needed to power the water pumps. Let us assume a COP of 3—meaning that the heat supplied is three times the electrical energy used to power the compressor.

Let us further assume that electricity consumed was generated by a gas-fueled power station. Given that the electrical energy generated at the power plant and transmitted to your home is typically 31% of the thermal energy at the plant from burning the gas or $0.31Q$ and given that this electricity powers a geothermal heat pump with $COP = 3$, the thermal power it delivers to your home is three times as much or $0.93Q$. But, now suppose instead of a geothermal heating system you chose to install a high efficiency gas furnace with an efficiency of 95%. For the same amount of gas burned, Q , the furnace delivers more heat: $0.95Q$, so it will require less gas to deliver the same heat and would emit less CO₂. This comparison is, of course, invalid if the power plant uses coal (where the situation is much worse in terms of emissions) or if it uses nuclear or renewable energy, where it is much better. The comparison also is invalid if you chose not a high efficiency gas furnace, but instead an electric heat pump or an oil burning furnace.

6.9 ECONOMICS OF GEOTHERMAL ELECTRICITY

A major factor determining the cost of electricity from geothermal resources is the high initial costs associated with drilling wells which can be very substantial—particularly with deeper wells where they may amount to over 60% of the initial investment, with the rest being mostly construction of the power plant.

6.9.1 Drilling Costs

The cost and technology for drilling geothermal wells have many similarities with drilling gas and oil wells, although they tend to be much higher—at least for shallow wells. Currently, there are not enough geothermal wells (especially at greater depths) to draw reliable conclusions on how their cost varies with depth over a wide range of depths. Therefore, it is common to examine the situation for gas and oil wells, and then make models that take into account the differences for the

geothermal case. The costs of wells having any given depth can vary enormously, depending on many factors. For example, for wells over 6 km depth, there is a difference of a factor of 10 between the least expensive and the most expensive wells. Another complication in estimating well drilling costs is that the cost of individual oil and gas is propriety information. Fortunately, however, the industry association (JAS) does provide average data for a given range of depths for a given year. As can be seen in Figure 6.12, the average well drilling costs C in dollars for 2004 are nicely fit by an exponential function of depth z of the form (Augustine, 2009)

$$C = Ae^{Bz} \quad (6.13)$$

where $A = \$200,000$ and $B = 0.75$ million/km. The values of the constants A and B can vary somewhat from year to year based on the availability of drilling equipment and labor, which correlate strongly with the price of oil and gas. Nevertheless, while the coefficients in the exponential function may change with time, the shape of the curve for gas and oil wells tends to remain exponential—at least over the range of depths drilled.

6.9.2 Beating the Exponential?

Beating the exponential increase in drilling cost with depth is of vital importance for tapping reserves that have a modest thermal gradient and are therefore found at deeper depths. For wells 5–6 km deep, drilling costs can be perhaps half the initial investment (the other half mostly being the plant itself). It may be true that reducing drilling costs will still leave all those other costs intact and therefore can have only a limited impact. However, if the exponential increase in cost with depth really can be avoided (made linear), vast reserves in the upper 10 km of the Earth might be accessed at only a modest increase in cost, and not just in places having a high gradient. Moreover, if it can be shown empirically that improved technology allows the drilling cost to be a linear function of depth, the gradient of the resource would have no effect on drilling costs per MW-h! This assertion is based on the discussion associated with Figure 6.7, where it was shown that if drilling depth is limited only by some maximum temperature (not depth), the size of the accessible geothermal resource is proportional to the depth, regardless of gradient. Under these assumptions it should be possible to exploit geothermal for electric power generation economically anywhere on land.

Unfortunately, there is not enough direct empirical evidence for the drilling costs of geothermal wells (especially for very deep ones) to directly check if they are also exponential functions of depth. Instead, what has been done is to develop a model (known as WellCost Lite) based on a detailed analysis of the time and costs of each step in the drilling process and then correct for the variations in costs from year to year. Note that while geothermal wells tend to be more expensive than oil and gas

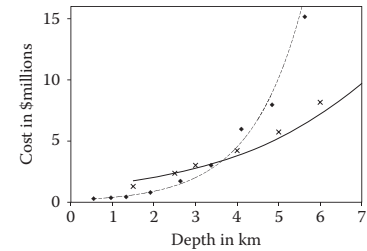


Figure 6.12 The black diamond points are the costs of drilling in millions of U.S. 2004 dollars versus depth of hole in km for oil and gas wells, and the dashed line is an exponential fit to them. The points shown by X's correspond to a much more limited number of geothermal wells and the solid curve is a fit to them based on the WellCost Lite model that takes into account differences between the two types of wells.

wells for lower depths, as depth increases they are less expensive with the crossover point being around 4 km. Is the WellCost Lite model predictions (checked against actual drilling costs only up to 5 km) consistent with an exponential function? At first sight they appear not to be—see Figure 6.12. On the other hand, as can be seen, the model results do fit an exponential—provided one subtracts off a constant dollar amount, meaning that the costs of drilling geothermal wells can be expressed as

$$C = D + Ae^{Bz} \quad (6.14)$$

6.9.3 Why the Exponential Dependence of Cost on Well Depth?

We have seen that drilling costs with depth empirically tend to be an exponential function of depth for oil and gas wells (upper dashed curve in Figure 6.12). Are there reasons to suspect such a dependence of cost with depth? As long as the cost to dig each additional increment of depth dz is p percent more than the previous increment, it is easy to show that the overall cost of drilling a well to any depth will be an exponential function of depth. This would seem to be true for any factors that become more time consuming the deeper you drill. Some examples might include

- Difficulty of the actual drilling as you go deeper as it gets hotter and there is more wear on the bit and also more likelihood of it getting stuck
- Flushing out debris—which has further to travel to surface
- Greater likelihood of losing circulation of drilling fluid at deeper depths
- Time needed to replace worn drilling bits that need to be hauled out

For such factors as these, the time (and cost) to drill an increment of depth dz is quite likely to be proportional to depth, so just like with compound interest, there will be an exponential growth in cost with depth, z .

6.9.4 Is Spallation Drilling the Answer?

Spallation is the process by which fragments of a piece of material are ejected due to either impact or stress. Spallation drilling involves no drill bit that can wear out due to contact with the rock. Instead, a flame jet makes contact with a small area of rock at the bottom of the borehole, and the induced thermal stresses in the rock cause small fragments of it (“spalls”) to be ejected. The fragments are small enough that the injection of high pressure water carries them up the water filled drilling pipe. Oxygen must be supplied to allow combustion underwater in a similar manner as done in underwater welding. The spallation technique has been demonstrated and found to work well, and it is likely to be a significant improvement over conventional drilling methods, in terms of both

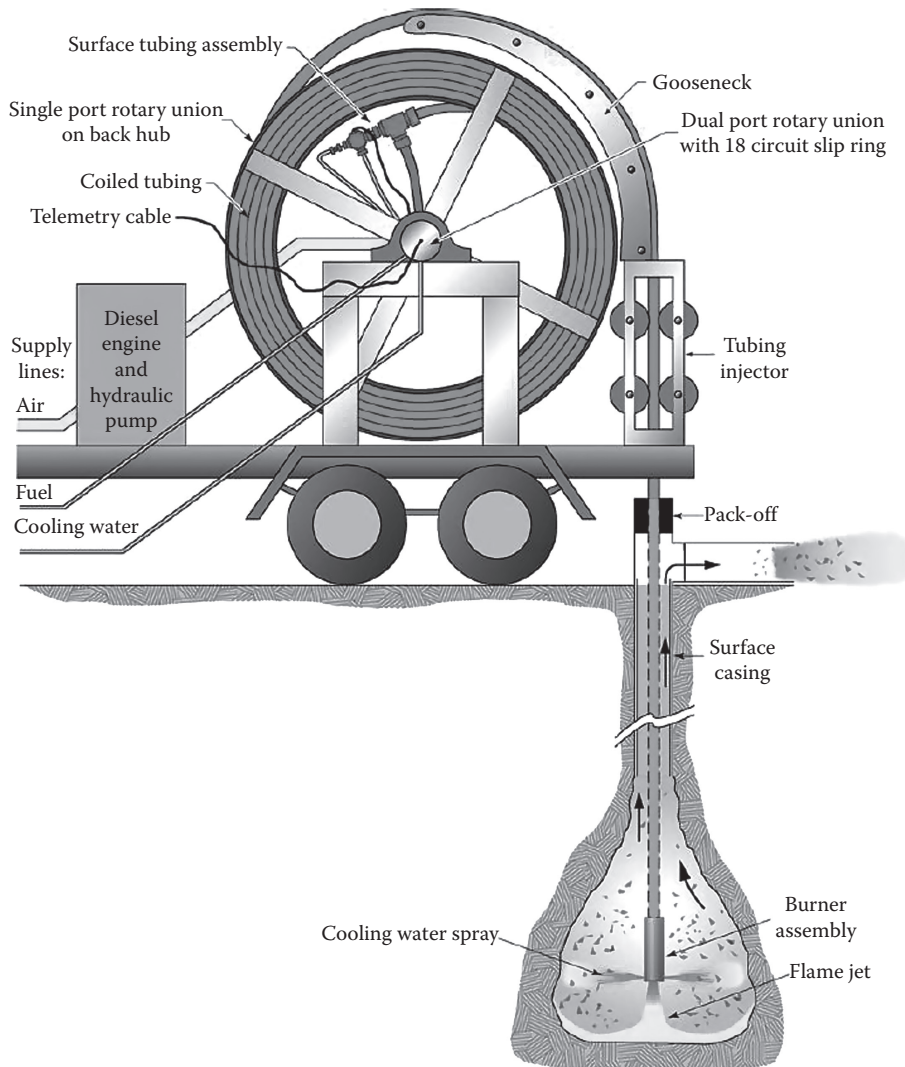


Figure 6.13 Diagram of spallation drilling of a borehole. (Courtesy of Los Alamos National Laboratory, Los Alamos, NM, a public domain image.)

speed and cost—especially if it can make the cost dependence on depth linear rather than exponential (Figure 6.13).

6.9.5 Why Spallation Drilling Cost Might Be a Linear Function of Depth

For a variety of reasons it might be reasonable to expect that the drilling cost associated with spallation drilling might prove to be a linear function of depth, rather than an exponential one. Recall that a linear cost function means that the time and cost to drill a segment of length dz does not depend on how deep that segment is. Unlike conventional drilling, which requires a rotary bit, the spallation process would be a continuous one with no need to haul out worn out drill bits. The continuous nature of the process and the small size of the spallated particles coming off

the rock mean that this debris is continuously removed with the water flushed down the hole. It would also permit a single diameter hole even at great depths. These factors give some hope that drilling deeper segments should take no more time than shallower ones of the same length.

Chad Augustine and his thesis advisor, MIT chemical engineering professor Jefferson Tester (who shares a patent on the spallation drilling process), produced the linear model shown as the dashed curve in Figure 6.12, using a similar approach as was used with the WellCost Lite model that fit actual drilling cost data fairly well—at least up to about 5 km depths. Unlike that earlier model, however, there is no actual drilling data to compare against the spallation linear model to check if the model conforms to reality. Thus, the spallation drilling process is still in the prototype stage and remains to be tested in real-world applications. However, should the linear dependence of drilling cost on depth prove to hold at great depths, the impact on the future of geothermal energy would be enormous, since it would mean that the drilling costs of geothermal electric power per MW-h would be the same for low thermal gradient regions as for high, and vast quantities of geothermal energy now prohibitively expensive to extract because of the exponential dependence on depth would become available. Geothermal produced electricity would then move from a source that is only economical at some special places having a very high geothermal gradient to one that is economically exploitable everywhere. Even more controversially, as Problem 11 illustrates, the cost of power could actually be less for areas having low thermal gradients than high, under the assumptions stated in the problem.

BOX 6.2 AS LONG AS THE HOLE IS ALREADY THERE...?

It has been estimated that the United States has 2.5 million abandoned oil and gas wells, some of which are miles deep. Given that drilling costs are a not insignificant fraction of total costs, Chinese scientists have come up with a way to retrofit these abandoned holes with new shafts containing pipes within pipes that would allow them to function as geothermal wells (Xianbiao et al., 2011). They estimate that the average abandoned well could generate 54 kW of electricity—not much compared to any central power station, but enough to make them collectively a source of a considerable amount of clean energy. They further estimate that the economic returns of electricity are about \$40,000 per year for each retrofitted well having a thermal gradient of 45°C/km.

6.10 SUMMARY

Geothermal power has many advantages as a renewable energy source: it is economical, environmentally fairly benign, and sustainable. It can also be used for either residential or commercial heating virtually anywhere,

and in places where conditions are right, it can generate electric power more cheaply than most other sources. Its usage over the last century has increased exponentially, even though it still accounts for a very small fraction of electric power production—about the same as solar. Geothermal electricity production has the potential for even more widespread usage into areas with low thermal gradient, but this development depends entirely on whether drilling costs can be made linear with depth for some novel drilling method such as spallation drilling.

PROBLEMS

1. Assume that the thermal gradient in a given location is not constant but rather a linear function of depth of the form $G = G_0(1 + \alpha z)$. (a) Find by integration the thermal energy per unit area between two depths z_1 and z_2 . (b) Find by integration the thermal energy per unit area between two temperatures T_1 and T_2 .
2. In Section 6.1 it was noted that according to an MIT study, the total geothermal energy that could feasibly be extracted in the United States with improved technology in the upper 10 km of the Earth's crust is over 2000 ZJ or 2×10^{24} J. Show that this is roughly correct by calculating the energy stored in the upper 10 km per unit area of surface above a temperature of 150°C using the average thermal gradient 30 K/km, values for the specific heat and density of average rocks, and the area of the United States found on the web.
3. Based on Equation 6.1, we explained the difference between the average thermal gradients in the Earth's inner core and its crust. However, that comparison assumed the same q for both cases (see discussion after Equation 6.1). How would this comparison change if we take into account that roughly 80% of the heat is generated from radioactive decay in the core and mantle?
4. Use the thermal conductivity of iron, and the radius of the Earth's inner core and its thermal gradient (seen in Figure 6.3) to find the total heat flow out of the core integrated over the area of the core. Compare that value to the total heat flow that reaches the Earth's surface about 0.06 W/m^2 integrated over the surface of the Earth. Explain the discrepancy. Hint: see previous problem.
5. Given the reserves of geothermal heat in Section 6.1 according to the MIT study, how long would they last at present annual world energy usage, taking into account the fact that 80% of the current heat supply is being continually replenished by radioactive decay of elements having multibillion-year half-lives?
6. (a) Justify Equation 6.3 by integration. (b) Consider the thermal energy in an aquifer for which the rocks have a porosity 0.2, a density 3000 kg/m^3 , and a specific heat 1000 J/kg-K , and for which the fluid is water. Find the average value of c_p for this aquifer and the total thermal energy to a depth 6 km that exceeds 150°C , assuming a gradient of 75°C/km .

7. A geothermal plant initially operates at 100 MW electric in its first year of operation. In the second year, the power is reduced by 2% due to drawdown from the geothermal reservoir. After 2 years to prolong the life of the plant, the circulating water flow is reduced to the point that the electrical power produced is only 50 MW, and it is then found that in the third year of operation the power reduction is only 0.7%. (a) Find the drawdown time for the plant power to be reduced to 25 MW. (b) Find the replenishment time if this geothermal field is left unused.
8. Using relevant data found on the web, determine the CO₂ emitted by a typical home heating system under these three choices: electric heat pump, geothermal heat pump, and high efficiency gas furnace, and these three assumptions as to the source of your home's electricity: gas-fired, coal-fired, and nuclear power plant—a total of $3 \times 3 = 9$ combinations.
9. Prove that as long as the cost dC to drill a very short section dz is proportional to both dz and the depth z of that section, then the overall cost of drilling a well will be an exponential function of depth, $C = Ae^{pz}$. What would p be if it cost \$10 million to drill a 5 km deep well and it cost \$100 million to drill a 10 km deep well? Why is it plausible that the actual cost of drilling a well to some depth z is fit better by including an additive constant—the D in Equation 6.14 rather than a pure exponential?
10. Explicitly show that if it can be shown empirically that improved technology allows the drilling cost to be a linear function of depth, the gradient of the resource doesn't matter in evaluating drilling costs per MW-h.
11. Consider the primary cost components of geothermal power, under the assumption that drilling costs can be made linear with depth, and use the parameters for the slope 0.433 \$/M/km and intercept 0.789 \$/M. Assume that owing to the economies of scale the costs of constructing a geothermal plant (on a per MW basis) decline somewhat as the power of the plant increases. Thus, assume the plant cost is a linear function of P , given by $A + BP$, with $A = \$15$ million, and $B = \$2$ million per MW. Let us further assume the plant operates for 25 years (over which half the energy in the reservoir has been extracted), with an annual operations and maintenance (O&M) cost of \$100,000 per MW. Now consider two places to site the plant, one having a gradient 50°C/km, and the other 100°C/km, and assume that the minimum and maximum well depths are both governed by the minimum and maximum temperatures of 150°C and 300°C. For each location calculate the total thermal energy available, and the total cost of the plant (drilling, plant construction, and O&M) on a per MW basis. Which location is the better choice—the one with the high or low gradient? This illustration presupposes a constant gradient for both locations, which may be less likely to be true in locations having a very high gradient.
12. Consider the comparison between a geothermal heat pump and a high efficiency gas furnace in Section 6.8.3. Find the COP of a geothermal

heat pump for which the amount of natural gas burned at the power plant to power the heat pump exactly equals that burned by the high efficiency gas furnace.

13. Show that the right hand side of Equation 6.10 has units of seconds, and explain why each term on the right hand side appears either in the numerator or the denominator.

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