CHAPTER 7

Hypothesis Testing with One Sample

The Entertainment Software Rating Board (ESRB) assigns ratings to video games to indicate the appropriate ages for players. These ratings include EC (early childhood), E (everyone), E10+ (everyone 10+), T (teen), M (mature), and AO (adults only).

7.1 Introduction to Hypothesis Testing

7.2 Hypothesis Testing for the Mean (σ Known)

7.3 Hypothesis Testing for the Mean (σ Unknown)
Activity
Case Study

7.4 Hypothesis Testing for Proportions
Activity

7.5 Hypothesis Testing for Variance and Standard Deviation
Uses and Abuses
Real Statistics—Real Decisions
Technology

Where You've Been

In Chapter 6, you began your study of inferential statistics. There, you learned how to form a confidence interval to estimate a population parameter, such as the proportion of people in the United States who agree with a certain statement. For instance, in a nationwide poll conducted by Pew Research Center, 2001 U.S. adults were asked whether they agreed or disagreed with the statement, "People who play violent video games are more likely to be violent themselves." Out of those surveyed, 800 adults agreed with the statement.

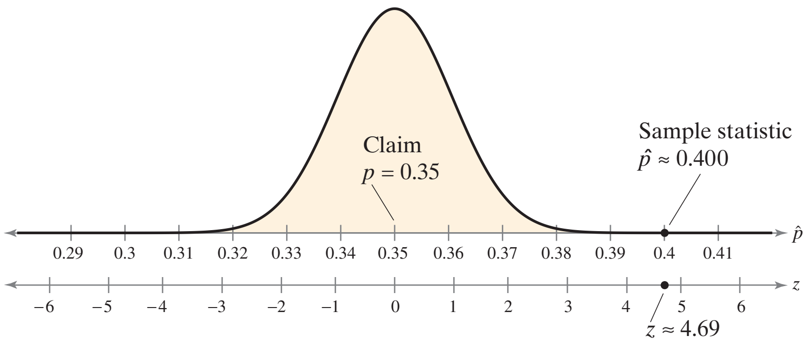
You have learned how to use these results to state with 95% confidence that the population proportion of U.S. adults who agree that people who play violent video games are more likely to be violent themselves is between 37.9% and 42.1%.

Where You're Going

In this chapter, you will continue your study of inferential statistics. But now, instead of making an estimate about a population parameter, you will learn how to test a claim about a parameter.

For instance, suppose that you work for Pew Research Center and are asked to test a claim that the proportion of U.S. adults who agree that people who play violent video games are more likely to be violent themselves is $p = 0.35$. To test the claim, you take a random sample of $n = 2001$ U.S. adults and find that 800 of them think that people who play violent video games are more likely to be violent themselves. Your sample statistic is $\hat{p} \approx 0.400$.

Is your sample statistic different enough from the claim $(p = 0.35)$ to decide that the claim is false? The answer lies in the sampling distribution of sample proportions taken from a population in which $p = 0.35$. The figure below shows that your sample statistic is more than 4 standard errors from the claimed value. If the claim is true, then the probability of the sample statistic being 4 standard errors or more from the claimed value is extremely small. Something is wrong! If your sample was truly random, then you can conclude that the actual proportion of the adult population is not 0.35. In other words, you tested the original claim (hypothesis), and you decided to reject it.



Sampling Distribution

7.1 Introduction to Hypothesis Testing

What You Should Learn

- A practical introduction to hypothesis tests
- How to state a null hypothesis and an alternative hypothesis
- How to identify type I and type II errors and interpret the level of significance
- How to know whether to use a one-tailed or two-tailed statistical test and find a *P*-value
- How to make and interpret a decision based on the results of a statistical test
- How to write a claim for a hypothesis test

Hypothesis Tests ■ Stating a Hypothesis ■ Types of Errors and Level of Significance ■ Statistical Tests and *P*-Values ■ Making a Decision and Interpreting the Decision ■ Strategies for Hypothesis Testing

Hypothesis Tests

Throughout the remainder of this text, you will study an important technique in inferential statistics called hypothesis testing. A **hypothesis test** is a process that uses sample statistics to test a claim about the value of a population parameter. Researchers in fields such as medicine, psychology, and business rely on hypothesis testing to make informed decisions about new medicines, treatments, and marketing strategies.

For instance, consider a manufacturer that advertises its new hybrid car has a mean gas mileage of 50 miles per gallon. If you suspect that the mean mileage is not 50 miles per gallon, how could you show that the advertisement is false?

Obviously, you cannot test *all* the vehicles, but you can still make a reasonable decision about the mean gas mileage by taking a random sample from the population of vehicles and measuring the mileage of each. If the sample mean differs enough from the advertisement's mean, you can decide that the advertisement is wrong.

For instance, to test that the mean gas mileage of all hybrid vehicles of this type is $\mu = 50$ miles per gallon, you take a random sample of $n = 30$ vehicles and measure the mileage of each. You obtain a sample mean of $\bar{x} = 47$ miles per gallon with a sample standard deviation of $s = 5.5$ miles per gallon. Does this indicate that the manufacturer's advertisement is false?

To decide, you do something unusual—*you assume the advertisement is correct!* That is, you assume that $\mu = 50$. Then, you examine the sampling distribution of sample means (with $n = 30$) taken from a population in which $\mu = 50$ and $\sigma = 5.5$. From the Central Limit Theorem, you know this sampling distribution is normal with a mean of 50 and standard error of

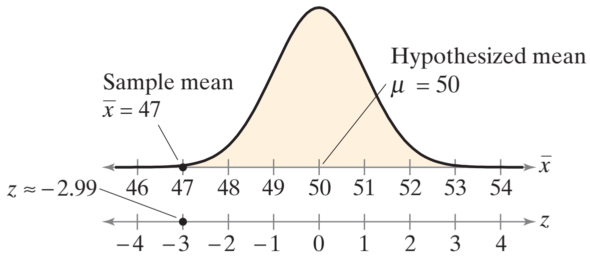
$$\frac{5.5}{\sqrt{30}} \approx 1.$$

In the figure below, notice that the sample mean of $\bar{x} = 47$ miles per gallon is highly unlikely—it is about 3 standard errors ($z \approx -2.99$) from the claimed mean! Using the techniques you studied in Chapter 5, you can determine that if the advertisement is true, then the probability of obtaining a sample mean of 47 or less is about 0.001. This is an unusual event! Your assumption that the company's advertisement is correct has led you to an improbable result. So, either you had a very unusual sample, or the advertisement is probably false. The logical conclusion is that the advertisement is probably false.

Study Tip

As you study this chapter, do not get confused regarding concepts of certainty and importance. For instance, even if you were very certain that the mean gas mileage of a type of hybrid vehicle is not 50 miles per gallon, the actual mean mileage might be very close to this value and the difference might not be important.

Sampling Distribution of $\bar{x}$



Stating a Hypothesis

A statement about a population parameter is called a **statistical hypothesis.** To test a population parameter, you should carefully state a pair of hypotheses—one that represents the claim and the other, its complement. When one of these hypotheses is false, the other must be true. Either hypothesis—the **null hypothesis** or the **alternative hypothesis**—may represent the original claim.

DEFINITION

1. A **null hypothesis** $\boldsymbol{H_0}$ is a statistical hypothesis that contains a statement of equality, such as $\leq$, $=$, or $\geq$.
2. The **alternative hypothesis** $\boldsymbol{H_a}$ is the complement of the null hypothesis. It is a statement that must be true if H_0 is false and it contains a statement of strict inequality, such as $>$, $\neq$, or $<$.

The symbol H_0 is read as "H sub-zero" or "H naught" and H_a is read as "H sub-a."

To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement. Then, write its complement. For instance, if the claim value is k and the population parameter is μ, then some possible pairs of null and alternative hypotheses are

$$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}, \quad \begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}, \quad \text{and} \quad \begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}.$$

Regardless of which of the three pairs of hypotheses you use, you always assume $\mu = k$ and examine the sampling distribution on the basis of this assumption. Within this sampling distribution, you will determine whether or not a sample statistic is unusual.

The table shows the relationship between possible verbal statements about the parameter μ and the corresponding null and alternative hypotheses. Similar statements can be made to test other population parameters, such as p, σ, or σ^2.

Verbal Statement H_0 *The mean is . . .*	Mathematical Statements	Verbal Statement H_a *The mean is . . .*
. . . greater than or equal to k. *. . . at least k.* *. . . not less than k.* *. . . not shorter than k.*	$\begin{cases} H_0: \mu \geq k \\ H_a: \mu < k \end{cases}$	*. . . less than k.* *. . . below k.* *. . . fewer than k.* *. . . shorter than k.*
. . . less than or equal to k. *. . . at most k.* *. . . not more than k.* *. . . not longer than k.*	$\begin{cases} H_0: \mu \leq k \\ H_a: \mu > k \end{cases}$	*. . . greater than k.* *. . . above k.* *. . . more than k.* *. . . longer than k.*
. . . equal to k. *. . . k.* *. . . exactly k.* *. . . the same as k.* *. . . not changed from k.*	$\begin{cases} H_0: \mu = k \\ H_a: \mu \neq k \end{cases}$	*. . . not equal to k.* *. . . different from k.* *. . . not k.* *. . . different from k.* *. . . changed from k.*

Note to Instructor

Some texts state the null hypothesis using the strict equality symbol. We use the symbol that is complementary to the alternative hypothesis.

Study Tip

The term *null hypothesis* was introduced by Ronald Fisher (see page 35). If the statement in the null hypothesis is not true, then the alternative hypothesis must be true.

Picturing the World

A study was done on the effect of a wearable fitness device combined with a low-calorie diet on weight loss. The study used a random sample of 237 adults. At the end of the study, the adults had a mean weight loss of 3.5 kilograms. So, it is claimed that the mean weight loss is 3.5 kilograms for all adults who use a wearable fitness device combined with a low-calorie diet. (Adapted from The Journal of the American Medical Association)

Determine a null hypothesis and alternative hypothesis for this claim.

$H_0: \mu = 3.5$, $H_a: \mu \neq 3.5$

Note to Instructor

Begin with a hypothesis statement and ask students to state its logical complement. Some students will have difficulty with the fact that the complement of $\mu \neq k$ is $\mu = k$. Discuss the role of a double negative in English. The important point is that if you conclude that H_0 is false, then you are also concluding that H_a is true.

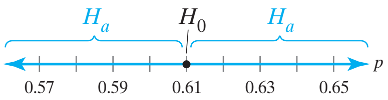
EXAMPLE 1

Stating the Null and Alternative Hypotheses

Write each claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.

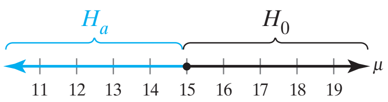
SOLUTION



1. The claim "the proportion . . . is 61%" can be written as $p = 0.61$. Its complement is $p \neq 0.61$, as shown in the figure at the left. Because $p = 0.61$ contains the statement of equality, it becomes the null hypothesis. In this case, the null hypothesis represents the claim. You can write the null and alternative hypotheses as shown.

   H_0: $p = 0.61$ (Claim)

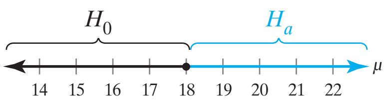
   H_a: $p \neq 0.61$



2. The claim "the mean . . . is less than 15 minutes" can be written as $\mu < 15$. Its complement is $\mu \geq 15$, as shown in the figure at the left. Because $\mu \geq 15$ contains the statement of equality, it becomes the null hypothesis. In this case, the alternative hypothesis represents the claim. You can write the null and alternative hypotheses as shown.

   H_0: $\mu \geq 15$ minutes

   H_a: $\mu < 15$ minutes (Claim)



3. The claim "the mean . . . is more than 18 years" can be written as $\mu > 18$. Its complement is $\mu \leq 18$, as shown in the figure at the left. Because $\mu \leq 18$ contains the statement of equality, it becomes the null hypothesis. In this case, the alternative hypothesis represents the claim. You can write the null and alternative hypotheses as shown.

   H_0: $\mu \leq 18$ years

   H_a: $\mu > 18$ years (Claim)

In the three figures at the left, notice that each point on the number line is in either H_0 or H_a, but no point is in both.

TRY IT YOURSELF 1

Write each claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.

1. A consumer analyst reports that the mean life of a certain type of automobile battery is not 74 months.
2. An electronics manufacturer publishes that the variance of the life of its home theater systems is less than or equal to 2.7.
3. A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

Answer: Page A36

In Example 1, notice that the claim is represented by either the null hypothesis *or* the alternative hypothesis.

Types of Errors and Level of Significance

No matter which hypothesis represents the claim, you always begin a hypothesis test by assuming that the equality condition in the null hypothesis is true. So, when you perform a hypothesis test, you make one of two decisions:

1. reject the null hypothesis

or

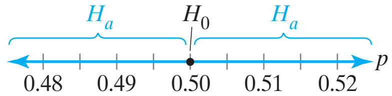
2. fail to reject the null hypothesis.

Because your decision is based on a sample rather than the entire population, there is always the possibility you will make the wrong decision.

For instance, you claim that a coin is not fair. To test your claim, you toss the coin 100 times and get 49 heads and 51 tails. You would probably agree that you do not have enough evidence to support your claim. Even so, it is possible that the coin is actually not fair and you had an unusual sample.

But then you toss the coin 100 times and get 21 heads and 79 tails. It would be a rare occurrence to get only 21 heads out of 100 tosses with a fair coin. So, you probably have enough evidence to support your claim that the coin is not fair. However, you cannot be 100% sure. It is possible that the coin is fair and you had an unusual sample.

Letting p represent the proportion of heads, the claim that "the coin is not fair" can be written as the mathematical statement $p \neq 0.5$. Its complement, "the coin is fair," is written as $p = 0.5$, as shown in the figure.



So, the null hypothesis is

$$H_0: p = 0.5$$

and the alternative hypothesis is

$$H_a: p \neq 0.5. \quad \text{(Claim)}$$

Remember, the only way to be absolutely certain of whether H_0 is true or false is to test the entire population. Because your decision—to reject H_0 or to fail to reject H_0—is based on a sample, you must accept the fact that your decision might be incorrect. You might reject a null hypothesis when it is actually true. Or, you might fail to reject a null hypothesis when it is actually false. These types of errors are summarized in the next definition.

DEFINITION

A **type I error** occurs if the null hypothesis is rejected when it is true.

A **type II error** occurs if the null hypothesis is not rejected when it is false.

The table shows the four possible outcomes of a hypothesis test.

	Truth of H_0	
Decision	**H_0 is true.**	**H_0 is false.**
Do not reject H_0.	Correct decision	Type II error
Reject H_0.	Type I error	Correct decision

Hypothesis testing is sometimes compared to the legal system used in the United States. Under this system, these steps are used.

1. A carefully worded accusation is written.
2. The defendant is assumed innocent (H_0) until proven guilty. The burden of proof lies with the prosecution. If the evidence is not strong enough, then there is no conviction. A "not guilty" verdict does not prove that a defendant is innocent.
3. The evidence needs to be conclusive beyond a reasonable doubt. The system assumes that more harm is done by convicting the innocent (type I error) than by not convicting the guilty (type II error).

	Truth about defendant	
Verdict	**Innocent**	**Guilty**
Not guilty	Justice	Type II error
Guilty	Type I error	Justice

The table at the left shows the four possible outcomes.

EXAMPLE 2

Identifying Type I and Type II Errors

The USDA limit for salmonella contamination for ground beef is 7.5%. A meat inspector reports that the ground beef produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which error is more serious? *(Source: U.S. Department of Agriculture)*

SOLUTION

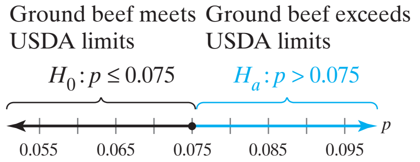
Let p represent the proportion of the ground beef that is contaminated. The meat inspector's claim is "more than 7.5% is contaminated." You can write the null hypothesis as

H_0: $p \leq 0.075$ The proportion is less than or equal to 0.075.

and the alternative hypothesis is

H_a: $p > 0.075$. (Claim) The proportion is greater than 0.075.

You can visualize the null and alternative hypotheses using a number line, as shown below.



A type I error will occur when the actual proportion of contaminated ground beef is less than or equal to 0.075, but you reject H_0. A type II error will occur when the actual proportion of contaminated ground beef is greater than 0.075, but you do not reject H_0. With a type I error, you might create a health scare and hurt the sales of ground beef producers who were actually meeting the USDA limits. With a type II error, you could be allowing ground beef that exceeded the USDA contamination limit to be sold to consumers. A type II error is more serious because it could result in sickness or even death.

TRY IT YOURSELF 2

A company specializing in parachute assembly states that its main parachute failure rate is not more than 1%. You perform a hypothesis test to determine whether the company's claim is false. When will a type I or type II error occur? Which error is more serious?

Answer: Page A36

You will reject the null hypothesis when the sample statistic from the sampling distribution is unusual. You have already identified unusual events to be those that occur with a probability of 0.05 or less. When statistical tests are used, an unusual event is sometimes required to have a probability of 0.10 or less, 0.05 or less, or 0.01 or less. Because there is variation from sample to sample, there is always a possibility that you will reject a null hypothesis when it is actually true. In other words, although the null hypothesis is true, your sample statistic is determined to be an unusual event in the sampling distribution. You can decrease the probability of this happening by lowering the **level of significance.**

Study Tip

When you decrease α (the maximum allowable probability of making a type I error), you are likely to be increasing β. The value $1 - \beta$ is called the *power of the test.* It represents the probability of rejecting the null hypothesis when it is false. The value of the power is difficult (and sometimes impossible) to find in most cases.

Note to Instructor

You can use an example of "false positive" and "false negative" results for a medical test (for example, cancer) to discuss type I and type II errors. You might also want to point out that the computation of β is beyond the scope of this text.

DEFINITION

In a hypothesis test, the **level of significance** is your maximum allowable probability of making a type I error. It is denoted by α, the lowercase Greek letter alpha.

The probability of a type II error is denoted by β, the lowercase Greek letter beta.

By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small. Three commonly used levels of significance are

$$\alpha = 0.10, \quad \alpha = 0.05, \quad \text{and} \quad \alpha = 0.01.$$

Statistical Tests and *P*-Values

After stating the null and alternative hypotheses and specifying the level of significance, the next step in a hypothesis test is to obtain a random sample from the population and calculate the sample statistic (such as $\bar{x}$, $\hat{p}$, or s^2) corresponding to the parameter in the null hypothesis (such as μ, p, or σ^2). This sample statistic is called the **test statistic.** With the assumption that the null hypothesis is true, the test statistic is then converted to a **standardized test statistic,** such as z, t, or χ^2. The standardized test statistic is used in making the decision about the null hypothesis.

In this chapter, you will learn about several one-sample statistical tests. The table shows the relationships between population parameters and their corresponding test statistics and standardized test statistics.

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 7.2, σ known), t (Section 7.3, σ unknown)
p	$\hat{p}$	z (Section 7.4)
σ^2	s^2	χ^2 (Section 7.5)

One way to decide whether to reject the null hypothesis is to determine whether the probability of obtaining the standardized test statistic (or one that is more extreme) is less than the level of significance.

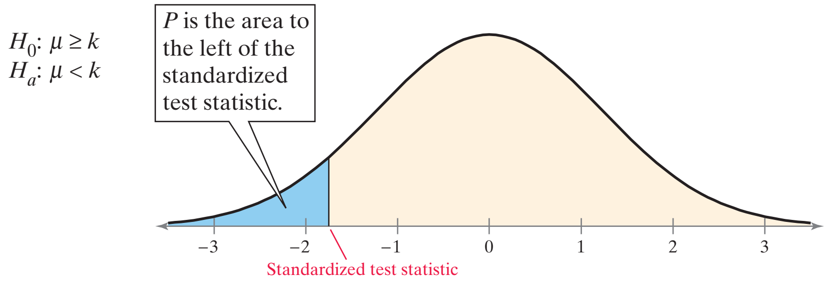
DEFINITION

If the null hypothesis is true, then a ***P*-value** (or **probability value**) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

The P-value of a hypothesis test depends on the nature of the test. There are three types of hypothesis tests—**left-tailed, right-tailed,** and **two-tailed.** The type of test depends on the location of the region of the sampling distribution that favors a rejection of H_0. This region is indicated by the alternative hypothesis.

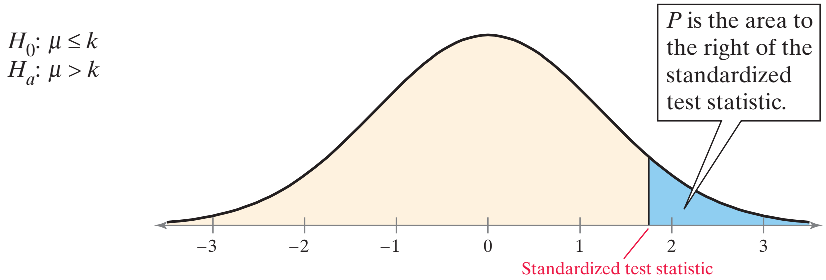
DEFINITION

1. If the alternative hypothesis H_a contains the less-than inequality symbol $(<)$, then the hypothesis test is a **left-tailed test.**



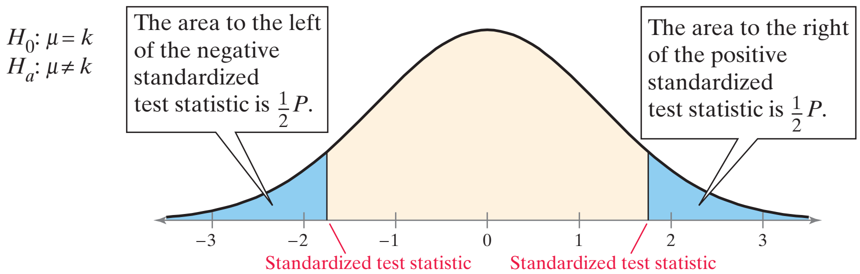
Left-Tailed Test

2. If the alternative hypothesis H_a contains the greater-than inequality symbol $(>)$, then the hypothesis test is a **right-tailed test.**



Right-Tailed Test

3. If the alternative hypothesis H_a contains the not-equal-to symbol $(\neq)$, then the hypothesis test is a **two-tailed test.** In a two-tailed test, each tail has an area of $\frac{1}{2}P$.



Two-Tailed Test

The smaller the P-value of the test, the more evidence there is to reject the null hypothesis. A very small P-value indicates an unusual event. Remember, however, that even a very low P-value does not constitute proof that the null hypothesis is false, only that it is probably false.

Study Tip

The third type of test is called a two-tailed test because evidence that would support the alternative hypothesis could lie in either tail of the sampling distribution.

EXAMPLE 3

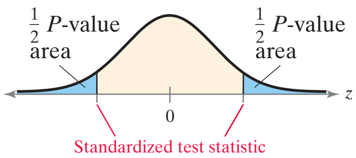
Identifying the Nature of a Hypothesis Test

For each claim, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the *P*-value.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. A car dealership announces that the mean time for an oil change is less than 15 minutes.
3. A company advertises that the mean life of its furnaces is more than 18 years.

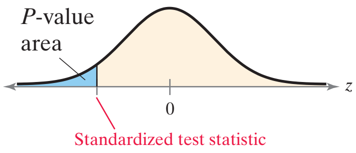
SOLUTION

	In Symbols	*In Words*
1.	H_0: $p = 0.61$	The proportion of students who are involved in at least one extracurricular activity is 61%.
	H_a: $p \neq 0.61$	The proportion of students who are involved in at least one extracurricular activity is not 61%.



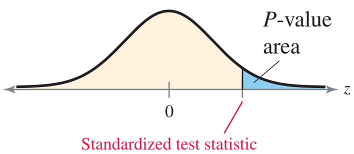
Because H_a contains the $\neq$ symbol, the test is a two-tailed hypothesis test. The figure at the left shows the normal sampling distribution with a shaded area for the *P*-value.

	In Symbols	*In Words*
2.	H_0: $\mu \geq 15$ min	The mean time for an oil change is greater than or equal to 15 minutes.
	H_a: $\mu < 15$ min	The mean time for an oil change is less than 15 minutes.



Because H_a contains the $<$ symbol, the test is a left-tailed hypothesis test. The figure at the left shows the normal sampling distribution with a shaded area for the *P*-value.

	In Symbols	*In Words*
3.	H_0: $\mu \leq 18$ yr	The mean life of the furnaces is less than or equal to 18 years.
	H_a: $\mu > 18$ yr	The mean life of the furnaces is more than 18 years.



Because H_a contains the $>$ symbol, the test is a right-tailed hypothesis test. The figure at the left shows the normal sampling distribution with a shaded area for the *P*-value.

TRY IT YOURSELF 3

For each claim, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is a left-tailed test, right-tailed test, or two-tailed test. Sketch a normal sampling distribution and shade the area for the *P*-value.

1. A consumer analyst reports that the mean life of a certain type of automobile battery is not 74 months.
2. An electronics manufacturer publishes that the variance of the life of its home theater systems is less than or equal to 2.7.
3. A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

Answer: Page A36

Making a Decision and Interpreting the Decision

To conclude a hypothesis test, you make a decision and interpret that decision. For any hypothesis test, there are two possible outcomes: (1) reject the null hypothesis or (2) fail to reject the null hypothesis. To decide to reject H_0 or fail to reject H_0, you can use the following **decision rule.**

Study Tip

In this chapter, you will learn that there are two types of decision rules for deciding whether to reject H_0 or fail to reject H_0. The decision rule described on this page is based on P-values. The second type of decision rule is based on rejection regions. When the standardized test statistic falls in the rejection region, the observed probability (P-value) of a type I error is less than α. You will learn more about rejection regions in the next section.

Decision Rule Based on P-Value

To use a P-value to make a decision in a hypothesis test, compare the P-value with α.

1. If $P \leq \alpha$, then reject H_0.
2. If $P > \alpha$, then fail to reject H_0.

Failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis. To support a claim, state it so that it becomes the alternative hypothesis. To reject a claim, state it so that it becomes the null hypothesis. The table will help you interpret your decision.

	Claim	
Decision	**Claim is H_0.**	**Claim is H_a.**
Reject H_0.	There is enough evidence to reject the claim.	There is enough evidence to support the claim.
Fail to reject H_0.	There is not enough evidence to reject the claim.	There is not enough evidence to support the claim.

EXAMPLE 4

Interpreting a Decision

You perform a hypothesis test for each claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

1. H_0 (Claim): A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.
2. H_a (Claim): A car dealership announces that the mean time for an oil change is less than 15 minutes.

SOLUTION

1. The claim is represented by H_0. If you reject H_0, then you should conclude "there is enough evidence to reject the school's claim that the proportion of students who are involved in at least one extracurricular activity is 61%." If you fail to reject H_0, then you should conclude "there is not enough evidence to reject the school's claim that the proportion of students who are involved in at least one extracurricular activity is 61%."
2. The claim is represented by H_a, so the null hypothesis is "the mean time for an oil change is greater than or equal to 15 minutes." If you reject H_0, then you should conclude "there is enough evidence to support the dealership's claim that the mean time for an oil change is less than 15 minutes." If you fail to reject H_0, then you should conclude "there is not enough evidence to support the dealership's claim that the mean time for an oil change is less than 15 minutes."

TRY IT YOURSELF 4

You perform a hypothesis test for each claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

1. A consumer analyst reports that the mean life of a certain type of automobile battery is not 74 months.
2. H_a (Claim): A realtor publicizes that the proportion of homeowners who feel their house is too small for their family is more than 24%.

Answer: Page A36

The general steps for a hypothesis test using *P*-values are summarized below. Note that when performing a hypothesis test, you should always state the null and alternative hypotheses before collecting data. You should not collect the data first and then create a hypothesis based on something unusual in the data.

Steps for Hypothesis Testing

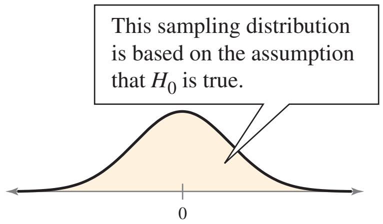
1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.

   H_0: **?** H_a: **?**

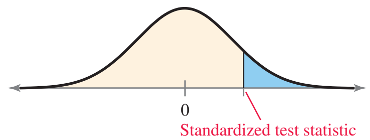
2. Specify the level of significance.

   $\alpha =$ **?**

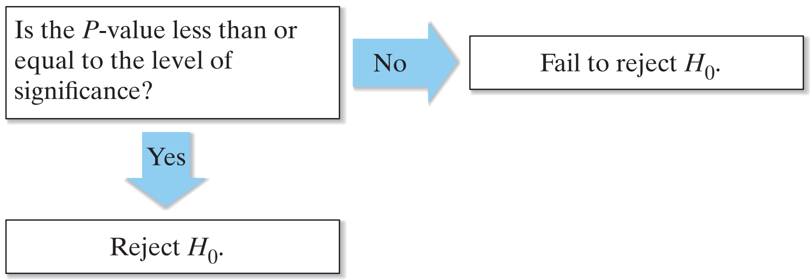
3. Determine the standardized sampling distribution and sketch its graph.



4. Calculate the test statistic and its corresponding standardized test statistic. Add it to your sketch.



5. Find the *P*-value.
6. Use this decision rule.



7. Write a statement to interpret the decision in the context of the original claim.

In Step 4 above, the figure shows a right-tailed test. However, the same basic steps also apply to left-tailed and two-tailed tests.

Strategies for Hypothesis Testing

In a courtroom, the strategy used by an attorney depends on whether the attorney is representing the defense or the prosecution. In a similar way, the strategy that you will use in hypothesis testing should depend on whether you are trying to support or reject a claim. Remember that you cannot use a hypothesis test to support your claim when your claim is the null hypothesis. So, as a researcher, to perform a hypothesis test where the possible outcome will support a claim, word the claim so it is the alternative hypothesis. To perform a hypothesis test where the possible outcome will reject a claim, word it so the claim is the null hypothesis.

EXAMPLE 5

Writing the Hypotheses

A medical research team is investigating the benefits of a new surgical treatment. One of the claims is that the mean recovery time for patients after the new treatment is less than 96 hours.

1. How would you write the null and alternative hypotheses when you are on the research team and want to support the claim? How should you interpret a decision that rejects the null hypothesis?
2. How would you write the null and alternative hypotheses when you are on an opposing team and want to reject the claim? How should you interpret a decision that rejects the null hypothesis?

SOLUTION

1. To answer the question, first think about the context of the claim. Because you want to support this claim, make the alternative hypothesis state that the mean recovery time for patients is less than 96 hours. So, H_a: $\mu < 96$ hours. Its complement, H_0: $\mu \geq 96$ hours, would be the null hypothesis. If you reject H_0, then you will support the claim that the mean recovery time is less than 96 hours.

   H_0: $\mu \geq 96$ and H_a: $\mu < 96$ (Claim)

2. First think about the context of the claim. As an opposing researcher, you do not want the recovery time to be less than 96 hours. Because you want to reject this claim, make it the null hypothesis. So, H_0: $\mu \leq 96$ hours. Its complement, H_a: $\mu > 96$ hours, would be the alternative hypothesis. If you reject H_0, then you will reject the claim that the mean recovery time is less than or equal to 96 hours.

   H_0: $\mu \leq 96$ (Claim) and H_a: $\mu > 96$

TRY IT YOURSELF 5

1. You represent a chemical company that is being sued for paint damage to automobiles. You want to support the claim that the mean repair cost per automobile is less than \$650. How would you write the null and alternative hypotheses? How should you interpret a decision that rejects the null hypothesis?

2. You are on a research team that is investigating the mean temperature of adult humans. The commonly accepted claim is that the mean temperature is about 98.6°F. You want to show that this claim is false. How would you write the null and alternative hypotheses? How should you interpret a decision that rejects the null hypothesis?

Answer: Page A36

7.1 EXERCISES

For Extra Help: MyLab Statistics

Building Basic Skills and Vocabulary

1. What are the two types of hypotheses used in a hypothesis test? How are they related?

2. Describe the two types of errors possible in a hypothesis test decision.

3. What are the two decisions that you can make from performing a hypothesis test?

4. Does failing to reject the null hypothesis mean that the null hypothesis is true? Explain.

True or False? *In Exercises 5–10, determine whether the statement is true or false. If it is false, rewrite it as a true statement.*

5. In a hypothesis test, you assume the alternative hypothesis is true.

6. A statistical hypothesis is a statement about a sample.

7. If you decide to reject the null hypothesis, then you can support the alternative hypothesis.

8. The level of significance is the maximum probability you allow for rejecting a null hypothesis when it is actually true.

9. A large *P*-value in a test will favor rejection of the null hypothesis.

10. To support a claim, state it so that it becomes the null hypothesis.

Stating Hypotheses *In Exercises 11–16, the statement represents a claim. Write its complement and state which is H_0 and which is H_a.*

11. $\mu \leq 645$ **12.** $\mu < 128$

13. $\sigma \neq 5$ **14.** $\sigma^2 \geq 1.2$

15. $p < 0.45$ **16.** $p = 0.21$

Graphical Analysis *In Exercises 17–20, match the alternative hypothesis with its graph. Then state the null hypothesis and sketch its graph.*

17. H_a: $\mu > 3$

18. H_a: $\mu < 3$

19. H_a: $\mu \neq 3$

20. H_a: $\mu > 2$

(a)


(b)


(c)


(d)


Identifying a Test *In Exercises 21–24, determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed.*

21. H_0: $\mu \leq 8.0$
H_a: $\mu > 8.0$

22. H_0: $\sigma \geq 5.2$
H_a: $\sigma < 5.2$

23. H_0: $\sigma^2 = 142$
H_a: $\sigma^2 \neq 142$

24. H_0: $p = 0.25$
H_a: $p \neq 0.25$

---

1. The two types of hypotheses used in a hypothesis test are the null hypothesis and the alternative hypothesis.

The alternative hypothesis is the complement of the null hypothesis.

2. A type I error occurs if the null hypothesis is rejected when it is true.

A type II error occurs if the null hypothesis is not rejected when it is false.

3. You can reject the null hypothesis, or you can fail to reject the null hypothesis.

4. No; Failing to reject the null hypothesis means that there is not enough evidence to reject it.

5. False. In a hypothesis test, you assume the null hypothesis is true.

6. False. A statistical hypothesis is a statement about a population.

7. True **8.** True

9. False. A small *P*-value in a test will favor rejection of the null hypothesis.

10. False. To support a claim, state it so that it becomes the alternative hypothesis.

11. H_0: $\mu \leq 645$ (claim); H_a: $\mu > 645$

12. H_0: $\mu \geq 128$; H_a: $\mu < 128$ (claim)

13. H_0: $\sigma = 5$; H_a: $\sigma \neq 5$ (claim)

14. H_0: $\sigma^2 \geq 1.2$ (claim); H_a: $\sigma^2 < 1.2$

15. H_0: $p \geq 0.45$; H_a: $p < 0.45$ (claim)

16. H_0: $p = 0.21$ (claim); H_a: $p \neq 0.21$

17. c; H_0: $\mu \leq 3$

18. d; H_0: $\mu \geq 3$

19. b; H_0: $\mu = 3$

20. a; H_0: $\mu \leq 2$

21. Right-tailed **22.** Left-tailed

23. Two-tailed **24.** Two-tailed

Using and Interpreting Concepts

Stating the Null and Alternative Hypotheses *In Exercises 25–30, write the claim as a mathematical statement. State the null and alternative hypotheses, and identify which represents the claim.*

25. Tablets A tablet manufacturer claims that the mean life of the battery for a certain model of tablet is more than 8 hours.

26. Shipping Errors As stated by a company's shipping department, the number of shipping errors per million shipments has a standard deviation that is less than 3.

27. Base Price of an ATV The standard deviation of the base price of an all-terrain vehicle is no more than $320.

28. Attendance An amusement park claims that the mean daily attendance at the park is at least 20,000 people.

29. Paying for College According to a recent survey, 73% of college students did not use student loans to pay for college. *(Source: Sallie Mae)*

30. Paying for College According to a recent survey, 52% of college students used their own income or savings to pay for college. *(Source: Sallie Mae)*

Identifying Type I and Type II Errors *In Exercises 31–36, describe type I and type II errors for a hypothesis test of the indicated claim.*

31. Repeat Customers A used textbook selling website claims that at least 60% of its new customers will return to buy their next textbook.

32. Flow Rate An urban planner claims that the noontime mean traffic flow rate on a busy downtown college campus street is 35 cars per minute.

33. Chess A local chess club claims that the length of time to play a game has a standard deviation of more than 12 minutes.

34. Video Game Systems A researcher claims that the percentage of adults in the United States who own a video game system is not 26%.

35. Security A campus security department publicizes that at most 25% of applicants become campus security officers.

36. Phone Repairs A cellphone repair shop advertises that the mean cost of repairing a phone screen is less than $75.

Identifying the Nature of a Hypothesis Test *In Exercises 37–42, state H_0 and H_a in words and in symbols. Then determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed. Explain your reasoning. Sketch a normal sampling distribution and shade the area for the P-value.*

37. Security Alarms A security expert claims that at least 14% of all homeowners have a home security alarm.

38. Clocks A manufacturer of grandfather clocks claims that the mean time its clocks lose is no more than 0.02 second per day.

39. Golf A golf analyst claims that the standard deviation of the 18-hole scores for a golfer is less than 2.1 strokes.

40. Lung Cancer A report claims that lung cancer accounts for 25% of all cancer diagnoses. *(Source: American Cancer Society)*

25. $\mu > 8$
H_0: $\mu \le 8$; H_a: $\mu > 8$ (claim)

26. $\sigma < 3$
H_0: $\sigma \ge 3$; H_a: $\sigma < 3$ (claim)

27. $\sigma \le 320$
H_0: $\sigma \le 320$ (claim); H_a: $\sigma > 320$

28. $\mu \ge 20{,}000$
H_0: $\mu \ge 20{,}000$ (claim); H_a: $\mu < 20{,}000$

29. $p = 0.73$
H_0: $p = 0.73$ (claim); H_a: $p \ne 0.73$

30. $p = 0.52$
H_0: $p = 0.52$ (claim); H_a: $p \ne 0.52$

31. A type I error will occur when the actual proportion of new customers who return to buy their next textbook is at least 0.60, but you reject H_0: $p \ge 0.60$.

A type II error will occur when the actual proportion of new customers who return to buy their next textbook is less than 0.60, but you fail to reject H_0: $p \ge 0.60$.

32. A type I error will occur when the actual noontime mean traffic flow rate is 35 cars per minute, but you reject H_0: $\mu = 35$.

A type II error will occur when the actual noontime mean traffic flow rate is not 35 cars per minute, but you fail to reject H_0: $\mu = 35$.

33. A type I error will occur when the actual standard deviation of the length of time to play a game is less than or equal to 12 minutes, but you reject H_0: $\sigma \le 12$.

A type II error will occur when the actual standard deviation of the length of time to play a game is greater than 12 minutes, but you fail to reject H_0: $\sigma \le 12$.

34. See Selected Answers, page A99.

35. See Odd Answers, page A68.

36. See Selected Answers, page A99.

37. See Odd Answers, page A68.

38. See Selected Answers, page A100.

39. See Odd Answers, page A68.

40. See Selected Answers, page A100.

41. High School Graduation Rate A high school claims that its mean graduation rate is more than 97%.

42. Survey A polling organization reports that the number of responses to a survey mailed to 100,000 U.S. residents is not 100,000.

Interpreting a Decision *In Exercises 43–48, determine whether the claim represents the null hypothesis or the alternative hypothesis. If a hypothesis test is performed, how should you interpret a decision that*

(a) rejects the null hypothesis?

(b) fails to reject the null hypothesis?

43. Swans A scientist claims that the mean incubation period for swan eggs is less than 40 days.

44. Affording Basic Necessities A report claims that more than 40% of households in a New York county struggle to afford basic necessities. *(Source: Niagara Frontier Publications)*

45. Lawn Mowers A researcher claims that the standard deviation of the life of a brand of lawn mower is at most 2.8 years.

46. Gas Mileage An automotive manufacturer claims that the standard deviation for the gas mileage of one of the vehicles it manufactures is 3.9 miles per gallon.

47. Terrorism Convictions A report claims that at least 65% of individuals convicted of terrorism or terrorism-related offenses in the United States are foreign born. *(Source: Hannity.com)*

48. Minimum Wage A marketing organization claims that none of its employees are paid minimum wage.

49. Writing Hypotheses: Medicine A medical research team is investigating the mean cost of a 30-day supply of a heart medication. A pharmaceutical company thinks that the mean cost is less than \$60. You want to support this claim. How would you write the null and alternative hypotheses?

50. Writing Hypotheses: Transportation Network Company A transportation network company claims that the mean travel time between two destinations is about 16 minutes. You work for one of the company's competitors and want to reject this claim. How would you write the null and alternative hypotheses?

51. Writing Hypotheses: Backpack Manufacturer A backpack manufacturer claims that the mean life of its competitor's backpacks is less than 5 years. You are asked to perform a hypothesis test to test this claim. How would you write the null and alternative hypotheses when

(a) you represent the manufacturer and want to support the claim?

(b) you represent the competitor and want to reject the claim?

52. Writing Hypotheses: Internet Provider An Internet provider is trying to gain advertising deals and claims that the mean time a customer spends online per day is greater than 28 minutes. You are asked to test this claim. How would you write the null and alternative hypotheses when

(a) you represent the Internet provider and want to support the claim?

(b) you represent a competing advertiser and want to reject the claim?

41. See Odd Answers, page A68.

42. See Selected Answers, page A100.

43. See Odd Answers, page A68.

44. See Selected Answers, page A100.

45. See Odd Answers, page A68.

46. Null hypothesis

(a) There is enough evidence to reject the automotive manufacturer's claim that the standard deviation for the gas mileage of its vehicles is 3.9 miles per gallon.

(b) There is not enough evidence to reject the automotive manufacturer's claim that the standard deviation for the gas mileage of its vehicles is 3.9 miles per gallon.

47. Null hypothesis

(a) There is enough evidence to reject the report's claim that at least 65% of individuals convicted of terrorism or terrorism-related offenses in the United States are foreign born.

(b) There is not enough evidence to reject the report's claim that at least 65% of individuals convicted of terrorism or terrorism-related offenses in the United States are foreign born.

48. Null hypothesis

(a) There is enough evidence to reject the organization's claim that none of its employees are paid minimum wage.

(b) There is not enough evidence to reject the organization's claim that none of its employees are paid minimum wage.

49. H_0: $\mu \geq 60$; H_a: $\mu < 60$

50. H_0: $\mu = 16$; H_a: $\mu \neq 16$

51. (a) H_0: $\mu \geq 5$; H_a: $\mu < 5$

(b) H_0: $\mu \leq 5$; H_a: $\mu > 5$

52. (a) H_0: $\mu \leq 28$; H_a: $\mu > 28$

(b) H_0: $\mu \geq 28$; H_a: $\mu < 28$

53. If you decrease α, then you are decreasing the probability that you will reject H_0. Therefore, you are increasing the probability of failing to reject H_0. This could increase β, the probability of failing to reject H_0 when H_0 is false.

54. If $\alpha = 0$, then the null hypothesis cannot be rejected and the hypothesis test is useless.

55. Yes; If the P-value is less than $\alpha = 0.05$, then it is also less than $\alpha = 0.10$.

56. Not necessarily; A P-value less than $\alpha = 0.10$ may or may not also be less than $\alpha = 0.05$.

57. (a) Fail to reject H_0 because the confidence interval includes values greater than 70.

(b) Reject H_0 because the confidence interval is located entirely to the left of 70.

(c) Fail to reject H_0 because the confidence interval includes values greater than 70.

58. (a) Fail to reject H_0 because the confidence interval includes values less than 54.

(b) Fail to reject H_0 because the confidence interval includes values less than 54.

(c) Reject H_0 because the confidence interval is located entirely to the right of 54.

59. (a) Reject H_0 because the confidence interval is located entirely to the right of 0.20.

(b) Fail to reject H_0 because the confidence interval includes values less than 0.20.

(c) Fail to reject H_0 because the confidence interval includes values less than 0.20.

60. (a) Fail to reject H_0 because the confidence interval includes values greater than 0.73.

(b) Reject H_0 because the confidence interval is located entirely to the left of 0.73.

(c) Fail to reject H_0 because the confidence interval includes values greater than 0.73.

Extending Concepts

53. Getting at the Concept Why can decreasing the probability of a type I error cause an increase in the probability of a type II error?

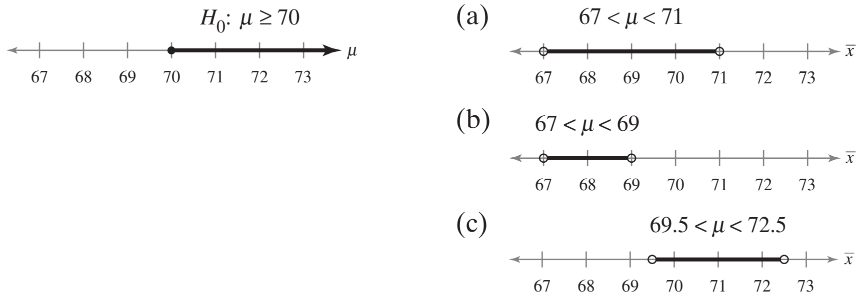
54. Getting at the Concept Explain why a level of significance of $\alpha = 0$ is not used.

55. Writing A null hypothesis is rejected with a level of significance of 0.05. Is it also rejected at a level of significance of 0.10? Explain.

56. Writing A null hypothesis is rejected with a level of significance of 0.10. Is it also rejected at a level of significance of 0.05? Explain.

Graphical Analysis *In Exercises 57–60, you are given a null hypothesis and three confidence intervals that represent three samplings. Determine whether each confidence interval indicates that you should reject H_0. Explain your reasoning.*

57.



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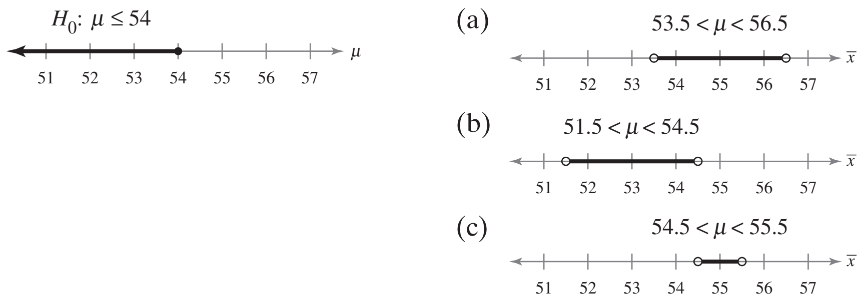

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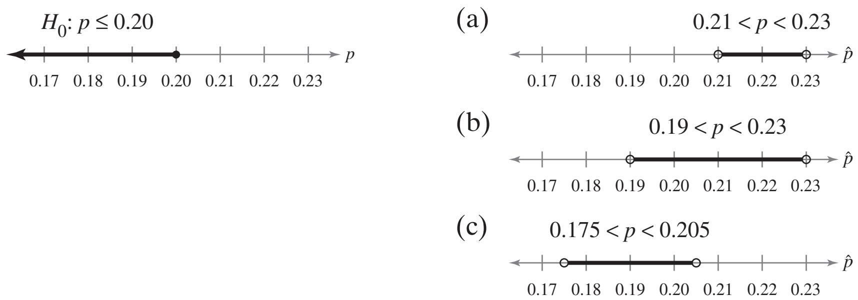

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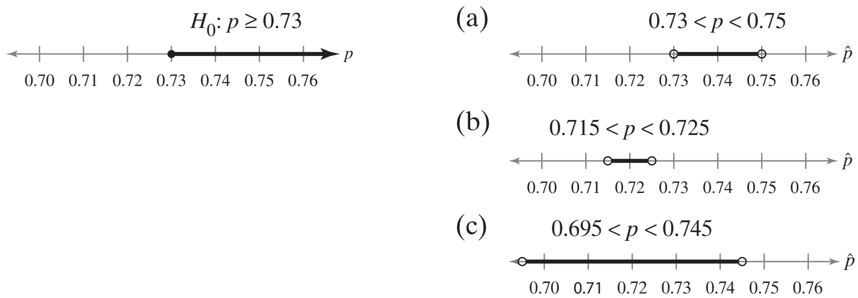


7.2 Hypothesis Testing for the Mean (σ Known)

What You Should Learn

- How to find and interpret *P*-values
- How to use *P*-values for a *z*-test for a mean μ when σ is known
- How to find critical values and rejection regions in the standard normal distribution
- How to use rejection regions for a *z*-test for a mean μ when σ is known

Using *P*-Values to Make Decisions ■ Using *P*-Values for a *z*-Test ■ Rejection Regions and Critical Values ■ Using Rejection Regions for a *z*-Test

Using *P*-Values to Make Decisions

In Chapter 5, you learned that when the sample size is at least 30, the sampling distribution for $\bar{x}$ (the sample mean) is normal. In Section 7.1, you learned that a way to reach a conclusion in a hypothesis test is to use a *P*-value for the sample statistic, such as $\bar{x}$. Recall that when you assume the null hypothesis is true, a *P*-value (or probability value) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data. The decision rule for a hypothesis test based on a *P*-value is shown below.

Decision Rule Based on *P*-Value

To use a *P*-value to make a decision in a hypothesis test, compare the *P*-value with α.

1. If $P \leq \alpha$, then reject H_0.

2. If $P > \alpha$, then fail to reject H_0.

Note to Instructor

If a *P*-value is less than 0.01, then the null hypothesis will be rejected at the common levels of $\alpha = 0.01$, $\alpha = 0.05$, and $\alpha = 0.10$. If the *P*-value is greater than 0.10, then you would fail to reject H_0 for these common levels. Make sure students know that the same conclusion will be reached regardless of whether they use the critical value method or the *P*-value method.

EXAMPLE 1

Interpreting a *P*-Value

The *P*-value for a hypothesis test is $P = 0.0237$. What is your decision when the level of significance is (1) $\alpha = 0.05$ and (2) $\alpha = 0.01$?

SOLUTION

1. Because $0.0237 < 0.05$, you reject the null hypothesis.

2. Because $0.0237 > 0.01$, you fail to reject the null hypothesis.

TRY IT YOURSELF 1

The *P*-value for a hypothesis test is $P = 0.0745$. What is your decision when the level of significance is (1) $\alpha = 0.05$ and (2) $\alpha = 0.10$? *Answer: Page A37*

The lower the *P*-value, the more evidence there is in favor of rejecting H_0. The *P*-value gives you the lowest level of significance for which the sample statistic allows you to reject the null hypothesis. In Example 1, you would reject H_0 at any level of significance greater than or equal to 0.0237.

Finding the *P*-Value for a Hypothesis Test

After determining the hypothesis test's standardized test statistic and the standardized test statistic's corresponding area, do one of the following to find the *P*-value.

a. For a left-tailed test, $P =$ (Area in left tail).

b. For a right-tailed test, $P =$ (Area in right tail).

c. For a two-tailed test, $P = 2$(Area in tail of standardized test statistic).

EXAMPLE 2

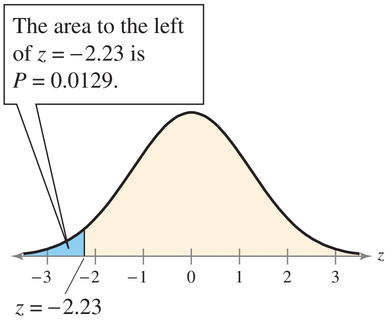
Finding a *P*-Value for a Left-Tailed Test

Find the *P*-value for a left-tailed hypothesis test with a standardized test statistic of $z = -2.23$. Decide whether to reject H_0 when the level of significance is $\alpha = 0.01$.

SOLUTION

The figure at the left shows the standard normal curve with a shaded area to the left of $z = -2.23$. For a left-tailed test,

$$P = (\text{Area in left tail}).$$



Left-Tailed Test

Using Table 4 in Appendix B, the area corresponding to $z = -2.23$ is 0.0129, which is the area in the left tail. So, the *P*-value for a left-tailed hypothesis test with a standardized test statistic of $z = -2.23$ is $P = 0.0129$. You can check your answer using technology, as shown below.

EXCEL

	A	B
1	NORM.DIST(-2.23,0,1,TRUE)	
2		0.012873721

Interpretation Because the *P*-value of 0.0129 is greater than 0.01, you fail to reject H_0.

TRY IT YOURSELF 2

Find the *P*-value for a left-tailed hypothesis test with a standardized test statistic of $z = -1.71$. Decide whether to reject H_0 when the level of significance is $\alpha = 0.05$.

Answer: Page A37

EXAMPLE 3

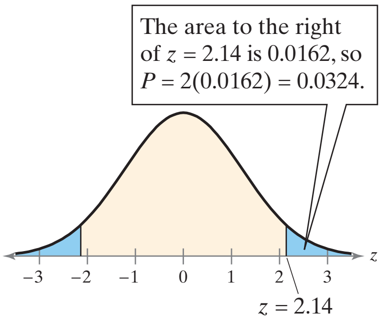
Finding a *P*-Value for a Two-Tailed Test

Find the *P*-value for a two-tailed hypothesis test with a standardized test statistic of $z = 2.14$. Decide whether to reject H_0 when the level of significance is $\alpha = 0.05$.

SOLUTION

The figure at the left shows the standard normal curve with shaded areas to the left of $z = -2.14$ and to the right of $z = 2.14$. For a two-tailed test,

$$P = 2(\text{Area in tail of standardized test statistic}).$$



Two-Tailed Test

Using Table 4, the area corresponding to $z = 2.14$ is 0.9838. The area in the right tail is $1 - 0.9838 = 0.0162$. So, the *P*-value for a two-tailed hypothesis test with a standardized test statistic of $z = 2.14$ is

$$P = 2(0.0162) = 0.0324.$$

Interpretation Because the *P*-value of 0.0324 is less than 0.05, you reject H_0.

TRY IT YOURSELF 3

Find the *P*-value for a two-tailed hypothesis test with a standardized test statistic of $z = 1.64$. Decide whether to reject H_0 when the level of significance is $\alpha = 0.10$.

Answer: Page A37

Using *P*-Values for a *z*-Test

You will now learn how to perform a hypothesis test for a mean μ assuming the standard deviation σ is known. When σ is known, you can use a z-test for the mean. To use the z-test, you need to find the standardized value for the test statistic $\bar{x}$. The standardized test statistic takes the form of

$$z = \frac{(\text{Sample mean}) - (\text{Hypothesized mean})}{\text{Standard error}}.$$

z-Test for a Mean μ

The **z-test for a mean μ** is a statistical test for a population mean. The **test statistic** is the sample mean $\bar{x}$. The **standardized test statistic** is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Standardized test statistic for μ (σ known)

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or $n \geq 30$.

Recall that $\sigma/\sqrt{n}$ is the standard error of the mean, $\sigma_{\bar{x}}$.

Note to Instructor

We use the same format for all hypothesis testing throughout the text. Using the same format makes it easier for students to understand the logic of the test. Emphasize that the sampling distribution and, consequently, the logic of the test are based on the assumption that the equality condition of the null hypothesis is true.

GUIDELINES

Using *P*-Values for a *z*-Test for a Mean μ (σ Known)

In Words	In Symbols
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Find the standardized test statistic.	$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$
5. Find the area that corresponds to z.	Use Table 4 in Appendix B.
6. Find the P-value. **a.** For a left-tailed test, P = (Area in left tail). **b.** For a right-tailed test, P = (Area in right tail). **c.** For a two-tailed test, P = 2(Area in tail of standardized test statistic).	
7. Make a decision to reject or fail to reject the null hypothesis.	If $P \leq \alpha$, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

With all hypothesis tests, it is helpful to sketch the sampling distribution. Your sketch should include the standardized test statistic.

EXAMPLE 4

Hypothesis Testing Using a *P*-Value

In auto racing, a pit stop is where a racing vehicle stops for new tires, fuel, repairs, and other mechanical adjustments. The efficiency of a pit crew that makes these adjustments can affect the outcome of a race. A pit crew claims that its mean pit stop time (for 4 new tires and fuel) is less than 13 seconds. A random sample of 32 pit stop times has a sample mean of 12.9 seconds. Assume the population standard deviation is 0.19 second. Is there enough evidence to support the claim at $\alpha = 0.01$? Use a *P*-value.

SOLUTION

Because σ is known ($\sigma = 0.19$), the sample is random, and $n = 32 \geq 30$, you can use the z-test. The claim is "the mean pit stop time is less than 13 seconds." So, the null and alternative hypotheses are

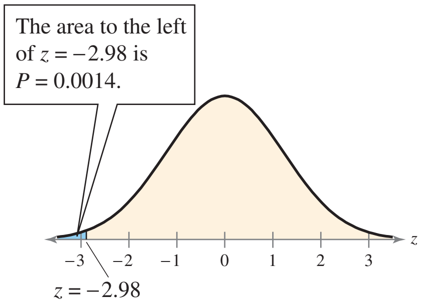
H_0: $\mu \geq 13$ seconds and H_a: $\mu < 13$ seconds. (Claim)

The level of significance is $\alpha = 0.01$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$ Because σ is known and $n \geq 30$, use the z-test.

$$= \frac{12.9 - 13}{0.19 / \sqrt{32}}$$ Assume $\mu = 13$.

$$\approx -2.98.$$ Round to two decimal places.



Left-Tailed Test

Using Table 4 in Appendix B, the area corresponding to $z = -2.98$ is 0.0014. Because this test is a left-tailed test, the *P*-value is equal to the area to the left of $z = -2.98$, as shown in the figure at the left. So, $P = 0.0014$. Because the *P*-value is less than $\alpha = 0.01$, you reject the null hypothesis. You can check your answer using technology, as shown below. Note that the *P*-value differs slightly from the one you found due to rounding.

STATCRUNCH

One sample Z hypothesis test:
μ : Mean of population
H_0 : μ = 13
H_A : μ < 13
Standard deviation = 0.19

Hypothesis test results:

Mean	n	Sample Mean	Std. Err.	Z-Stat	P-value
μ	32	12.9	0.033587572	−2.9772917	0.0015

Interpretation There is enough evidence at the 1% level of significance to support the claim that the mean pit stop time is less than 13 seconds.

TRY IT YOURSELF 4

Homeowners claim that the mean speed of automobiles traveling on their street is greater than the speed limit of 35 miles per hour. A random sample of 100 automobiles has a mean speed of 36 miles per hour. Assume the population standard deviation is 4 miles per hour. Is there enough evidence to support the claim at $\alpha = 0.05$? Use a *P*-value.

Answer: Page A37

EXAMPLE 5

See Minitab steps on page 414.

Hypothesis Testing Using a *P*-Value

According to a study of U.S. homes that use heating equipment, the mean indoor temperature at night during winter is 68.3°F. You think this information is incorrect. You randomly select 25 U.S. homes that use heating equipment in the winter and find that the mean indoor temperature at night is 67.2°F. From past studies, the population standard deviation is known to be 3.5°F and the population is normally distributed. Is there enough evidence to support your claim at $\alpha = 0.05$? Use a *P*-value. *(Adapted from U.S. Energy Information Administration)*

SOLUTION

Because σ is known ($\sigma = 3.5$°F), the sample is random, and the population is normally distributed, you can use the *z*-test. The claim is "the mean is different from 68.3°F." So, the null and alternative hypotheses are

H_0: $\mu = 68.3$°F and H_a: $\mu \neq 68.3$°F. (Claim)

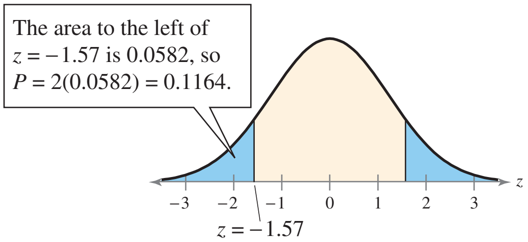
The level of significance is $\alpha = 0.05$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$ Because σ is known and the population is normally distributed, use the *z*-test.

$$= \frac{67.2 - 68.3}{3.5 / \sqrt{25}}$$ Assume $\mu = 68.3$°F.

$$\approx -1.57.$$ Round to two decimal places.

In Table 4, the area corresponding to $z = -1.57$ is 0.0582. Because the test is a two-tailed test, the *P*-value is equal to twice the area to the left of $z = -1.57$, as shown in the figure.



Two-Tailed Test

So, the *P*-value is $P = 2(0.0582) = 0.1164$. Because the *P*-value is greater than $\alpha = 0.05$, you fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 5% level of significance to support the claim that the mean indoor temperature at night during winter is different from 68.3°F for U.S. homes that use heating equipment.

TRY IT YOURSELF 5

According to a study of employed U.S. adults ages 18 and over, the mean number of workdays missed due to illness or injury in the past 12 months is 3.5 days. You randomly select 25 employed U.S. adults ages 18 and over and find that the mean number of workdays missed is 4 days. Assume the population standard deviation is 1.5 days and the population is normally distributed. Is there enough evidence to doubt the study's claim at $\alpha = 0.01$? Use a *P*-value. *(Adapted from U.S. National Center for Health Statistics)*

Answer: Page A37

Tech Tip

Using a TI-84 Plus, you can either enter the original data into a list to find a *P*-value or enter the descriptive statistics.

STAT

Choose the TESTS menu.

1: Z-Test...

Select the *Data* input option when you use the original data. Select the *Stats* input option when you use the descriptive statistics. In each case, enter the appropriate values including the corresponding type of hypothesis test indicated by the alternative hypothesis. Then select *Calculate*.

EXAMPLE 6

Using Technology to Find a *P*-Value

Use the TI-84 Plus displays to make a decision to reject or fail to reject the null hypothesis at a level of significance of $\alpha = 0.05$.

TI-84 PLUS

Z-Test
Inpt:Data Stats
μ_0:6.2
σ:.47
$\bar{x}$:6.07
n:53
μ:$\neq\mu_0$ $<\mu_0$ $>\mu_0$
Calculate Draw

TI-84 PLUS

Z-Test
$\mu\neq6.2$
z=−2.013647416
p=.0440464253
$\bar{x}$=6.07
n=53

SOLUTION

The *P*-value for this test is 0.0440464253. Because the *P*-value is less than $\alpha = 0.05$, you reject the null hypothesis.

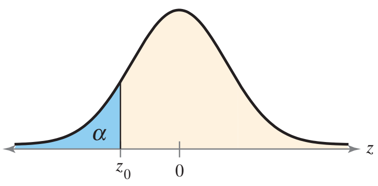
TRY IT YOURSELF 6

Repeat Example 6 using a level of significance of $\alpha = 0.01$.

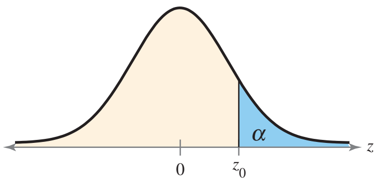
Answer: Page A37

Rejection Regions and Critical Values

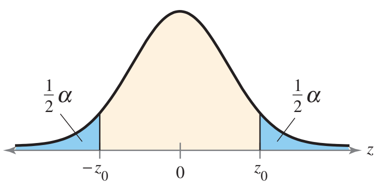
Another method to decide whether to reject the null hypothesis is to determine whether the standardized test statistic falls within a range of values called the **rejection region** of the sampling distribution.



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

DEFINITION

A **rejection region** (or **critical region**) of the sampling distribution is the range of values for which the null hypothesis is not probable. If a standardized test statistic falls in this region, then the null hypothesis is rejected. A **critical value** z_0 separates the rejection region from the nonrejection region.

GUIDELINES

Finding Critical Values in the Standard Normal Distribution

1. Specify the level of significance α.
2. Determine whether the test is left-tailed, right-tailed, or two-tailed.
3. Find the critical value(s) z_0. When the hypothesis test is
   a. *left-tailed,* find the z-score that corresponds to an area of α.
   b. *right-tailed,* find the z-score that corresponds to an area of $1 - \alpha$.
   c. *two-tailed,* find the z-scores that correspond to $\frac{1}{2}\alpha$ and $1 - \frac{1}{2}\alpha$.
4. Sketch the standard normal distribution. Draw a vertical line at each critical value and shade the rejection region(s). (See the figures at the left.)

Note that a standardized test statistic that falls in a rejection region is considered an unusual event.

When you cannot find the exact area in Table 4, use the area that is closest. For an area that is exactly midway between two areas in the table, use the z-score midway between the corresponding z-scores.

EXAMPLE 7

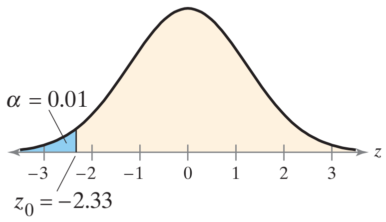
Finding a Critical Value for a Left-Tailed Test

Find the critical value and rejection region for a left-tailed test with $\alpha = 0.01$.

SOLUTION

The figure shows the standard normal curve with a shaded area of 0.01 in the left tail. In Table 4, the z-score that is closest to an area of 0.01 is -2.33. So, the critical value is

$$z_0 = -2.33.$$



1% Level of Significance

The rejection region is to the left of this critical value. You can check your answer using technology, as shown below.

EXCEL

	A	B
1	NORM.S.INV(0.01)	
2		−2.32634787

TRY IT YOURSELF 7

Find the critical value and rejection region for a left-tailed test with $\alpha = 0.10$.

Answer: Page A37

Because normal distributions are symmetric, in a two-tailed test the critical values are opposites, as shown in the next example.

Study Tip

The table lists the critical values for commonly used levels of significance.

Alpha	Tail	z
0.10	Left	−1.28
	Right	1.28
	Two	±1.645
0.05	Left	−1.645
	Right	1.645
	Two	±1.96
0.01	Left	−2.33
	Right	2.33
	Two	±2.575

EXAMPLE 8

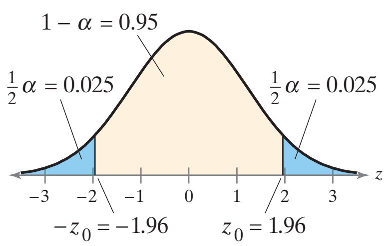
Finding Critical Values for a Two-Tailed Test

Find the critical values and rejection regions for a two-tailed test with $\alpha = 0.05$.

SOLUTION

The figure shows the standard normal curve with shaded areas of $\frac{1}{2}\alpha = 0.025$ in each tail. The area to the left of $-z_0$ is $\frac{1}{2}\alpha = 0.025$, and the area to the left of z_0 is $1 - \frac{1}{2}\alpha = 0.975$. In Table 4, the z-scores that correspond to the areas 0.025 and 0.975 are -1.96 and 1.96, respectively. So, the critical values are

$$-z_0 = -1.96 \quad \text{and} \quad z_0 = 1.96.$$



5% Level of Significance

The rejection regions are to the left of -1.96 and to the right of 1.96.

TRY IT YOURSELF 8

Find the critical values and rejection regions for a two-tailed test with $\alpha = 0.08$.

Answer: Page A37

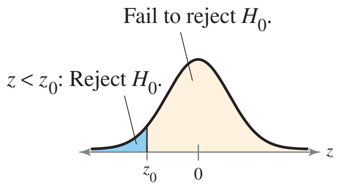
Using Rejection Regions for a *z*-Test

To conclude a hypothesis test using rejection region(s), you make a decision and interpret the decision according to the next rule.

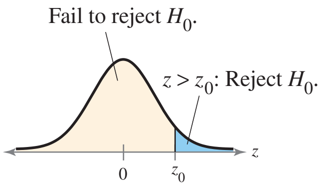
Decision Rule Based on Rejection Region

To use a rejection region to conduct a hypothesis test, calculate the standardized test statistic z. If the standardized test statistic

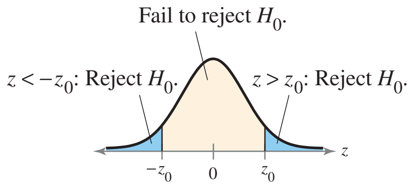
1. is in the rejection region, then reject H_0.
2. is *not* in the rejection region, then fail to reject H_0.



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

Remember, failing to reject the null hypothesis does not mean that you have accepted the null hypothesis as true. It simply means that there is not enough evidence to reject the null hypothesis.

GUIDELINES

Using Rejection Regions for a *z*-Test for a Mean μ (σ Known)

In Words	In Symbols
1. Verify that σ is known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.
5. Determine the rejection region(s).	
6. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

Picturing the World

Each year, the Environmental Protection Agency (EPA) publishes reports of gas mileage for all makes and models of passenger vehicles. In a recent year, the small station wagon with an automatic transmission that posted the best mileage had a mean mileage of 52 miles per gallon (city) and 49 miles per gallon (highway). An auto manufacturer claims its station wagons exceed 49 miles per gallon on the highway. To support its claim, it tests 36 vehicles on highway driving and obtains a sample mean of 51.2 miles per gallon. Assume the population standard deviation is 4.8 miles per gallon. (Source: U.S. Department of Energy)

Is the evidence strong enough to support the claim that the station wagon's highway miles per gallon exceeds the EPA estimate? Use a *z*-test with $\alpha = 0.01$.

There is enough evidence at the 1% level of significance to conclude that the mean mileage of the station wagon is greater than 49 miles per gallon on the highway.

EXAMPLE 9

See TI-84 Plus steps on page 415.

Hypothesis Testing Using a Rejection Region

Employees at a construction and mining company claim that the mean salary of the company's mechanical engineers is less than that of one of its competitors, which is \$88,200. A random sample of 20 of the company's mechanical engineers has a mean salary of \$85,900. Assume the population standard deviation is \$9500 and the population is normally distributed. At $\alpha = 0.05$, test the employees' claim.

SOLUTION

Because σ is known ($\sigma = \$9500$), the sample is random, and the population is normally distributed, you can use the z-test. The claim is "the mean salary is less than \$88,200." So, the null and alternative hypotheses can be written as

$$H_0: \mu \geq \$88{,}200 \quad \text{and} \quad H_a: \mu < \$88{,}200. \text{ (Claim)}$$

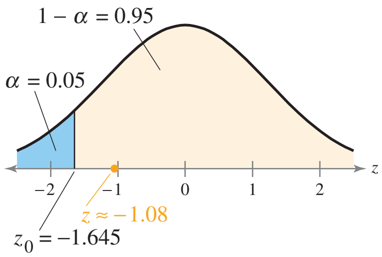
Because the test is a left-tailed test and the level of significance is $\alpha = 0.05$, the critical value is $z_0 = -1.645$ and the rejection region is $z < -1.645$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$ Because σ is known and the population is normally distributed, use the z-test.

$$= \frac{85{,}900 - 88{,}200}{9500/\sqrt{20}}$$ Assume $\mu = \$88{,}200$.

$$\approx -1.08.$$ Round to two decimal places.

The figure shows the location of the rejection region and the standardized test statistic z. Because z is not in the rejection region, you fail to reject the null hypothesis.



5% Level of Significance

Interpretation There is not enough evidence at the 5% level of significance to support the employees' claim that the mean salary is less than \$88,200.

Be sure you understand the decision made in this example. Even though your sample has a mean of \$85,900, you cannot (at a 5% level of significance) support the claim that the mean of all the mechanical engineers' salaries is less than \$88,200. The difference between your test statistic ($\bar{x} = \$85{,}900$) and the hypothesized mean ($\mu = \$88{,}200$) is probably due to sampling error.

TRY IT YOURSELF 9

The CEO of the company in Example 9 claims that the mean workday of the company's mechanical engineers is less than 8.5 hours. A random sample of 25 of the company's mechanical engineers has a mean workday of 8.2 hours. Assume the population standard deviation is 0.5 hour and the population is normally distributed. At $\alpha = 0.01$, test the CEO's claim.

Answer: Page A37

EXAMPLE 10

Hypothesis Testing Using Rejection Regions

A researcher claims that the mean annual cost of raising a child (age 2 and under) by married-couple families in the U.S. is $14,050. In a random sample of married-couple families in the U.S., the mean annual cost of raising a child (age 2 and under) is $13,795. The sample consists of 500 children. Assume the population standard deviation is $2875. At $\alpha = 0.10$, is there enough evidence to reject the claim? *(Adapted from U.S. Department of Agriculture Center for Nutrition Policy and Promotion)*

SOLUTION

Because σ is known $(\sigma = \$2875)$, the sample is random, and $n = 500 \geq 30$, you can use the z-test. The claim is "the mean annual cost is $14,050." So, the null and alternative hypotheses are

H_0: $\mu = \$14{,}050$ (Claim)   and H_a: $\mu \neq \$14{,}050$.

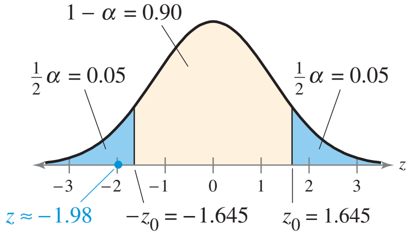
Because the test is a two-tailed test and the level of significance is $\alpha = 0.10$, the critical values are $-z_0 = -1.645$ and $z_0 = 1.645$. The rejection regions are $z < -1.645$ and $z > 1.645$. The standardized test statistic is

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$ Because σ is known and $n \geq 30$, use the z-test.

$$= \frac{13{,}795 - 14{,}050}{2875 / \sqrt{500}}$$ Assume $\mu = \$14{,}050$.

$$\approx -1.98.$$ Round to two decimal places.

The figure shows the location of the rejection regions and the standardized test statistic z. Because z is in the rejection region, you reject the null hypothesis.



10% Level of Significance

You can check your answer using technology, as shown below.

MINITAB

One-Sample Z

Test of μ = 14050 vs ≠ 14050
The assumed standard deviation = 2875

N	Mean	SE Mean	90% CI	Z	P
500	13795	129	(13584, 14006)	−1.98	0.047

Interpretation There is enough evidence at the 10% level of significance to reject the claim that the mean annual cost of raising a child (age 2 and under) by married-couple families in the U.S. is $14,050.

TRY IT YOURSELF 10

In Example 10, at $\alpha = 0.01$, is there enough evidence to reject the claim?

Answer: Page A37

7.2 EXERCISES

For Extra Help: MyLab Statistics

Building Basic Skills and Vocabulary

1. Explain the difference between the z-test for μ using a P-value and the z-test for μ using rejection region(s).

2. In hypothesis testing, does using the critical value method or the P-value method affect your conclusion? Explain.

Interpreting a *P*-Value *In Exercises 3–8, the P-value for a hypothesis test is shown. Use the P-value to decide whether to reject H_0 when the level of significance is (a) $\alpha = 0.01$, (b) $\alpha = 0.05$, and (c) $\alpha = 0.10$.*

3. $P = 0.0461$

4. $P = 0.0691$

5. $P = 0.1271$

6. $P = 0.0107$

7. $P = 0.0838$

8. $P = 0.0062$

Finding a *P*-Value *In Exercises 9–14, find the P-value for the hypothesis test with the standardized test statistic z. Decide whether to reject H_0 for the level of significance α.*

9. Left-tailed test
$z = -1.32$
$\alpha = 0.10$

10. Left-tailed test
$z = -1.55$
$\alpha = 0.05$

11. Right-tailed test
$z = 2.46$
$\alpha = 0.01$

12. Right-tailed test
$z = 1.23$
$\alpha = 0.10$

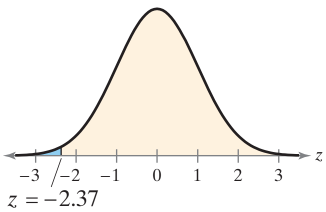
13. Two-tailed test
$z = -1.68$
$\alpha = 0.05$

14. Two-tailed test
$z = 1.95$
$\alpha = 0.08$

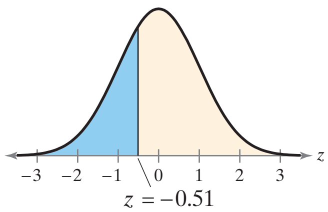
Graphical Analysis *In Exercises 15 and 16, match each P-value with the graph that displays its area without performing any calculations. Explain your reasoning.*

15. $P = 0.0089$ and $P = 0.3050$

(a)

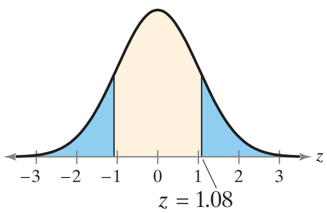


(b)

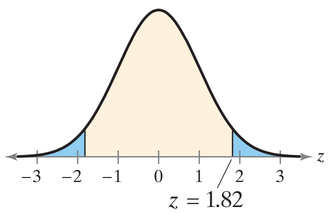


16. $P = 0.0688$ and $P = 0.2802$

(a)



(b)



1. The z-test using a P-value compares the P-value with the level of significance α. In the z-test using rejection region(s), the test statistic is compared with critical values.

2. No; Both involve comparing the test statistic's probability with the level of significance. The P-value method converts the standardized test statistic to a probability (P-value) and compares this with the level of significance, whereas the critical value method converts the level of significance to a z-score and compares this with the standardized test statistic.

3. (a) Fail to reject H_0. (b) Reject H_0. (c) Reject H_0.

4. (a) Fail to reject H_0. (b) Fail to reject H_0. (c) Reject H_0.

5. (a) Fail to reject H_0. (b) Fail to reject H_0. (c) Fail to reject H_0.

6. (a) Fail to reject H_0. (b) Reject H_0. (c) Reject H_0.

7. (a) Fail to reject H_0. (b) Fail to reject H_0. (c) Reject H_0.

8. (a) Reject H_0. (b) Reject H_0. (c) Reject H_0.

9. $P = 0.0934$; Reject H_0.

10. $P = 0.0606$; Fail to reject H_0.

11. $P = 0.0069$; Reject H_0.

12. $P = 0.1093$; Fail to reject H_0.

13. $P = 0.0930$; Fail to reject H_0.

14. $P = 0.0512$; Reject H_0.

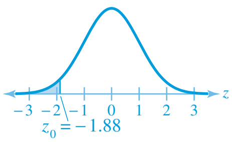
15. (a) $P = 0.0089$
(b) $P = 0.3050$
The larger P-value corresponds to the larger area.

16. (a) $P = 0.2802$
(b) $P = 0.0688$
The larger P-value corresponds to the larger area.

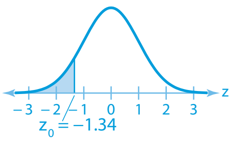
17. Fail to reject H_0.

18. Reject H_0.

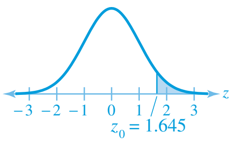
19. Critical value: $z_0 = -1.88$
Rejection region: $z < -1.88$



20. Critical value: $z_0 = -1.34$
Rejection region: $z < -1.34$



21. Critical value: $z_0 = 1.645$
Rejection region: $z > 1.645$



22. See Selected Answers, page A100.

23. See Odd Answers, page A69.

24. See Selected Answers, page A100.

25. (a) Fail to reject H_0 because $z < 1.285$.

(b) Fail to reject H_0 because $z < 1.285$.

(c) Fail to reject H_0 because $z < 1.285$.

(d) Reject H_0 because $z > 1.285$.

26. (a) Reject H_0 because $z > 1.96$.

(b) Fail to reject H_0 because $-1.96 < z < 1.96$.

(c) Fail to reject H_0 because $-1.96 < z < 1.96$.

(d) Reject H_0 because $z < -1.96$.

27. Reject H_0. There is enough evidence at the 5% level of significance to reject the claim.

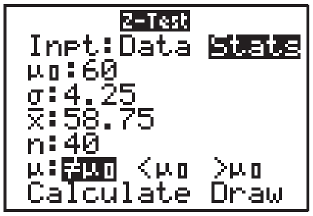
28. Fail to reject H_0. There is not enough evidence at the 7% level of significance to support the claim.

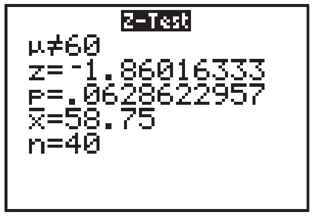
29. Fail to reject H_0. There is not enough evidence at the 3% level of significance to support the claim.

30. Reject H_0. There is enough evidence at the 1% level of significance to reject the claim.

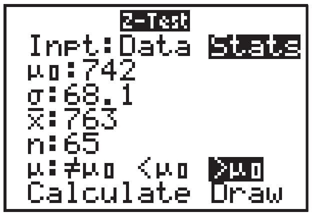
In Exercises 17 and 18, use the TI-84 Plus displays to make a decision to reject or fail to reject the null hypothesis at the level of significance.

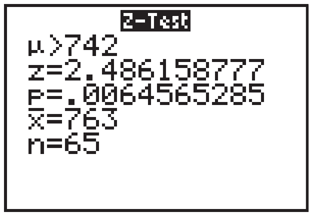
17. $\alpha = 0.05$





18. $\alpha = 0.01$





Finding Critical Values and Rejection Regions *In Exercises 19–24, find the critical value(s) and rejection region(s) for the type of z-test with level of significance α. Include a graph with your answer.*

19. Left-tailed test, $\alpha = 0.03$

20. Left-tailed test, $\alpha = 0.09$

21. Right-tailed test, $\alpha = 0.05$

22. Right-tailed test, $\alpha = 0.08$

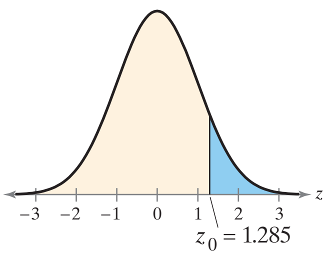
23. Two-tailed test, $\alpha = 0.02$

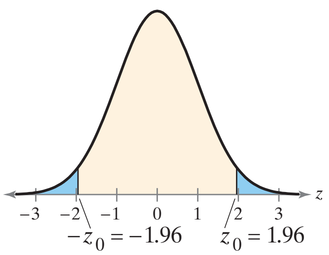
24. Two-tailed test, $\alpha = 0.12$

Graphical Analysis *In Exercises 25 and 26, state whether each standardized test statistic z allows you to reject the null hypothesis. Explain your reasoning.*

25. (a) $z = -1.301$
(b) $z = 1.203$
(c) $z = 1.280$
(d) $z = 1.286$

26. (a) $z = 1.98$
(b) $z = -1.89$
(c) $z = 1.65$
(d) $z = -1.99$





In Exercises 27–30, test the claim about the population mean μ at the level of significance α. Assume the population is normally distributed.

27. Claim: $\mu = 40$; $\alpha = 0.05$; $\sigma = 1.97$
Sample statistics: $\bar{x} = 39.2$, $n = 25$

28. Claim: $\mu \geq 1475$; $\alpha = 0.07$; $\sigma = 29$
Sample statistics: $\bar{x} = 1468$, $n = 26$

29. Claim: $\mu \neq 5880$; $\alpha = 0.03$; $\sigma = 413$
Sample statistics: $\bar{x} = 5771$, $n = 67$

30. Claim: $\mu \leq 22{,}500$; $\alpha = 0.01$; $\sigma = 1200$
Sample statistics: $\bar{x} = 23{,}500$, $n = 45$

31. (a) The claim is "the mean total score for the school's applicants is more than 499."

H_0: $\mu \leq 499$
H_a: $\mu > 499$ (claim)

(b) 2.83 (c) 0.0023

(d) Reject H_0.

(e) There is enough evidence at the 1% level of significance to support the report's claim that the mean total score for the school's applicants is more than 499.

32. (a) The claim is "the average activating temperature is at least 135°F."

H_0: $\mu \geq 135$ (claim)
H_a: $\mu < 135$

(b) −3.43 (c) 0.0003

(d) Reject H_0.

(e) There is enough evidence at the 10% level of significance to reject the manufacturer's claim that the average activating temperature is at least 135°F.

33. (a) The claim is "the mean winning times for Boston Marathon women's open division champions is at least 2.68 hours."

H_0: $\mu \geq 2.68$ (claim)
H_a: $\mu < 2.68$

(b) −1.37 (c) 0.0853

(d) Fail to reject H_0.

(e) There is not enough evidence at the 5% level of significance to reject the statistician's claim that the mean winning times for Boston Marathon women's open division champions is at least 2.68 hours.

34. See Selected Answers, page A100.

35. See Odd Answers, page A69.

36. See Selected Answers, page A100.

Using and Interpreting Concepts

Hypothesis Testing Using a *P*-Value *In Exercises 31–36,*

(a) *identify the claim and state H_0 and H_a.*

(b) *find the standardized test statistic z.*

(c) *find the corresponding P-value.*

(d) *decide whether to reject or fail to reject the null hypothesis.*

(e) *interpret the decision in the context of the original claim.*

31. MCAT Scores A random sample of 100 medical school applicants at a university has a mean total score of 502 on the MCAT. According to a report, the mean total score for the school's applicants is more than 499. Assume the population standard deviation is 10.6. At $\alpha = 0.01$, is there enough evidence to support the report's claim? *(Source: Association of American Medical Colleges)*

32. Sprinkler Systems A manufacturer of sprinkler systems designed for fire protection claims that the average activating temperature is at least 135°F. To test this claim, you randomly select a sample of 32 systems and find the mean activation temperature to be 133°F. Assume the population standard deviation is 3.3°F. At $\alpha = 0.10$, do you have enough evidence to reject the manufacturer's claim?

33. Boston Marathon A sports statistician claims that the mean winning times for Boston Marathon women's open division champions is at least 2.68 hours. The mean winning time of a sample of 30 randomly selected Boston Marathon women's open division champions is 2.60 hours. Assume the population standard deviation is 0.32 hour. At $\alpha = 0.05$, can you reject the claim? *(Source: Boston Athletic Association)*

34. Acceleration Times A consumer group claims that the mean acceleration time from 0 to 60 miles per hour for a sedan is 6.3 seconds. A random sample of 33 sedans has a mean acceleration time from 0 to 60 miles per hour of 7.2 seconds. Assume the population standard deviation is 2.5 seconds. At $\alpha = 0.05$, can you reject the claim? *(Source: Zero to 60 Times)*

35. Roller Coasters The heights (in feet) of 36 randomly selected top-rated roller coasters are listed. Assume the population standard deviation is 71.6 feet. At $\alpha = 0.05$, is there enough evidence to reject the claim that the mean height of top-rated roller coasters is 160 feet? *(Source: POP World Media, LLC)*

325	188	306	107	208	167	105	78	140
232	230	170	170	205	305	135	200	200
100	223	135	195	80	90	120	210	82
161	245	88	70	116	121	146	149	124

36. Salaries An analyst claims that the mean annual salary for intermediate level architects in Wichita, Kansas, is more than the national mean, $52,000. The annual salaries (in dollars) for a random sample of 21 intermediate level architects in Wichita are listed. Assume the population is normally distributed and the population standard deviation is $8000. At $\alpha = 0.09$, is there enough evidence to support the analyst's claim? *(Adapted from Salary.com)*

47,066	58,955	59,774	56,016	52,487	41,258	43,806
44,291	44,063	44,365	40,120	49,853	50,233	43,827
56,085	48,967	57,983	60,295	57,776	46,500	47,658

37. (a) The claim is "the mean caffeine content per 12-ounce bottle of a population of caffeinated soft drinks is 37.7 milligrams."

H_0: $\mu = 37.7$ (claim)
H_a: $\mu \neq 37.7$

(b) $-z_0 = -2.575$, $z_0 = 2.575$
Rejection regions: $z < -2.575$, $z > 2.575$

(c) -0.72 (d) Fail to reject H_0.

(e) There is not enough evidence at the 1% level of significance to reject the consumer research organization's claim that the mean caffeine content per 12-ounce bottle of a population of caffeinated soft drinks is 37.7 milligrams.

38. (a) The claim is "the mean high school graduation rate per state in the United States is 80%."

H_0: $\mu = 80$ (claim)
H_a: $\mu \neq 80$

(b) $-z_0 = -1.96$, $-z_0 = 1.96$
Rejection regions: $z < -1.96$, $z > 1.96$

(c) 2.15 (d) Reject H_0.

(e) There is enough evidence at the 5% level of significance to reject the education researcher's claim that the mean high school graduation rate per state in the United States is 80%.

39. See Odd Answers, page A69.

40. See Selected Answers, page A100.

41. See Odd Answers, page A69.

Carbon dioxide emissions (in megatons)					
340	76	46	44	75	1617
34	43	23	0.5	0.3	6
0.3	0.7	11	0.1	0.2	7.6
0.6	0.6	26	9.9	2.3	8.2
3.4	0.1	472	4.2	4.2	0
113	21	7.2	5	0.1	16
0.2	45	5.1	175	0	4.1

TABLE FOR EXERCISE 42

42. See Selected Answers, page A100.

43. See Odd Answers, page A69.

44. See Selected Answers, page A100.

Hypothesis Testing Using Rejection Region(s) *In Exercises 37–42, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic z, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.*

37. Caffeine Content A consumer research organization states that the mean caffeine content per 12-ounce bottle of a population of caffeinated soft drinks is 37.7 milligrams. You want to test this claim. During your tests, you find that a random sample of thirty-six 12-ounce bottles of caffeinated soft drinks has a mean caffeine content of 36.4 milligrams. Assume the population standard deviation is 10.8 milligrams. At $\alpha = 0.01$, can you reject the research organization's claim? *(Source: National Soft Drink Association)*

38. High School Graduation Rate An education researcher claims that the mean high school graduation rate per state in the United States is 80%. You want to test this claim. You find that a random sample of 30 states has a mean high school graduation rate of 82%. Assume the population standard deviation is 5.1%. At $\alpha = 0.05$, do you have enough evidence to support the researcher's claim? *(Source: U.S. Department of Education)*

39. Fast Food A fast food restaurant estimates that the mean sodium content in one of its breakfast sandwiches is no more than 920 milligrams. A random sample of 44 breakfast sandwiches has a mean sodium content of 925 milligrams. Assume the population standard deviation is 18 milligrams. At $\alpha = 0.10$, do you have enough evidence to reject the restaurant's claim?

40. Light Bulbs A light bulb manufacturer guarantees that the mean life of a certain type of light bulb is at least 750 hours. A random sample of 25 light bulbs has a mean life of 745 hours. Assume the population is normally distributed and the population standard deviation is 60 hours. At $\alpha = 0.02$, do you have enough evidence to reject the manufacturer's claim?

41. Fluorescent Lamps A fluorescent lamp manufacturer guarantees that the mean life of a fluorescent lamp is at least 10,000 hours. You want to test this guarantee. To do so, you record the lives of a random sample of 32 fluorescent lamps. The results (in hours) are listed. Assume the population standard deviation is 1850 hours. At $\alpha = 0.11$, do you have enough evidence to reject the manufacturer's claim?

8,800	9,155	13,001	10,250	10,002	11,413	8,234	10,402
10,016	8,015	6,110	11,005	11,555	9,254	6,991	12,006
10,420	8,302	8,151	10,980	10,186	10,003	8,814	11,445
6,277	8,632	7,265	10,584	9,397	11,987	7,556	10,380

42. Carbon Dioxide Emissions A scientist estimates that the mean carbon dioxide emissions per country in a recent year are greater than 150 megatons. You want to test this estimate. To do so, you determine the carbon dioxide emissions for 42 randomly selected countries for that year. The results (in megatons) are shown in the table at the left. Assume the population standard deviation is 816 megatons. At $\alpha = 0.06$, can you support the scientist's estimate? *(Source: Global Carbon Project)*

Extending Concepts

43. Writing When $P > \alpha$, does the standardized test statistic lie inside or outside of the rejection region(s)? Explain your reasoning.

44. Writing In a right-tailed test where $P < \alpha$, does the standardized test statistic lie to the left or the right of the critical value? Explain your reasoning.

7.3 Hypothesis Testing for the Mean (σ Unknown)

What You Should Learn

- How to find critical values in a *t*-distribution
- How to use the *t*-test to test a mean μ when σ is not known
- How to use technology to find *P*-values and use them with a *t*-test to test a mean μ when σ is not known

Critical Values in a *t*-Distribution ■ The *t*-Test for a Mean μ ■ Using *P*-Values with *t*-Tests

Critical Values in a *t*-Distribution

In Section 7.2, you learned how to perform a hypothesis test for a population mean when the population standard deviation is known. In many real-life situations, the population standard deviation in *not* known. When either the population has a normal distribution or the sample size is at least 30, you can still test the population mean μ. To do so, you can use the *t*-distribution with $n - 1$ degrees of freedom.

Note to Instructor

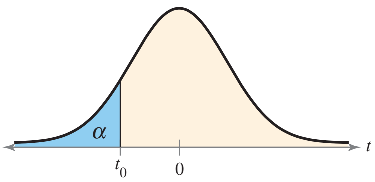
A thoughtful student might ask what should be done if the sample size is small, the standard deviation is not known, and you cannot assume that the population is normally distributed. Chapter 11 will cover this case (see nonparametric tests). You can cover these tests immediately after this section if desirable.

GUIDELINES

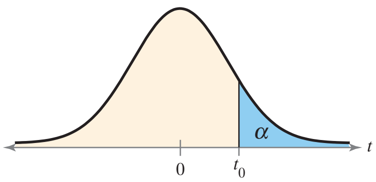
Finding Critical Values in a *t*-Distribution

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. $= n - 1$.
3. Find the critical value(s) using Table 5 in Appendix B in the row with $n - 1$ degrees of freedom. When the hypothesis test is
   a. *left-tailed,* use the "One Tail, α" column with a negative sign.
   b. *right-tailed,* use the "One Tail, α" column with a positive sign.
   c. *two-tailed,* use the "Two Tails, α" column with a negative and a positive sign.

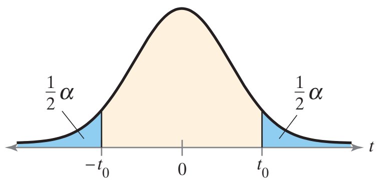
See the figures below.



Left-Tailed Test



Right-Tailed Test



Two-Tailed Test

EXAMPLE 1

Finding a Critical Value for a Left-Tailed Test

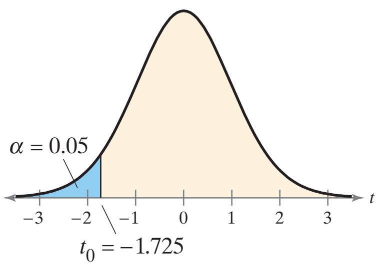
Find the critical value t_0 for a left-tailed test with $\alpha = 0.05$ and $n = 21$.

SOLUTION

The degrees of freedom are

$$\text{d.f.} = n - 1 = 21 - 1 = 20.$$

To find the critical value, use Table 5 in Appendix B with d.f. = 20 and $\alpha = 0.05$ in the "One Tail, α" column. Because the test is left-tailed, the critical value is negative. So, $t_0 = -1.725$, as shown in the figure at the left.



5% Level of Significance

TRY IT YOURSELF 1

Find the critical value t_0 for a left-tailed test with $\alpha = 0.01$ and $n = 14$.

Answer: Page A37

EXAMPLE 2

Finding a Critical Value for a Right-Tailed Test

Find the critical value t_0 for a right-tailed test with $\alpha = 0.01$ and $n = 17$.

SOLUTION

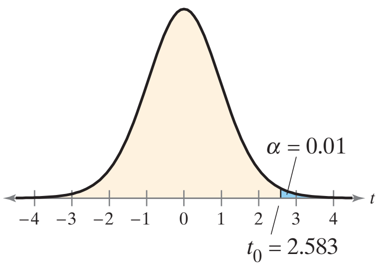
The degrees of freedom are

$$\text{d.f.} = n - 1 = 17 - 1 = 16.$$

To find the critical value, use Table 5 with d.f. $= 16$ and $\alpha = 0.01$ in the "One Tail, α" column. Because the test is right-tailed, the critical value is positive. So,

$$t_0 = 2.583$$

as shown in the figure.



1% Level of Significance

TRY IT YOURSELF 2

Find the critical value t_0 for a right-tailed test with $\alpha = 0.10$ and $n = 9$.

Answer: Page A37

Because t-distributions are symmetric, in a two-tailed test the critical values are opposites, as shown in the next example.

EXAMPLE 3

Finding Critical Values for a Two-Tailed Test

Find the critical values $-t_0$ and t_0 for a two-tailed test with $\alpha = 0.10$ and $n = 26$.

SOLUTION

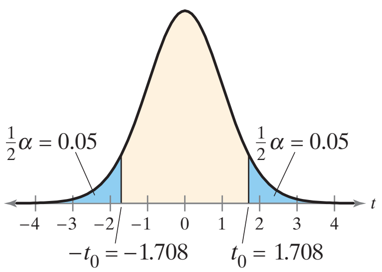
The degrees of freedom are

$$\text{d.f.} = n - 1 = 26 - 1 = 25.$$

To find the critical values, use Table 5 with d.f. $= 25$ and $\alpha = 0.10$ in the "Two Tails, α" column. Because the test is two-tailed, one critical value is negative and one is positive. So,

$$-t_0 = -1.708 \quad \text{and} \quad t_0 = 1.708$$

as shown in the figure at the left. You can check your answer using technology, as shown below.



10% Level of Significance

EXCEL

	A	B
1	T.INV.2T(0.1,25)	
2		1.708140761

TRY IT YOURSELF 3

Find the critical values $-t_0$ and t_0 for a two-tailed test with $\alpha = 0.05$ and $n = 16$.

Answer: Page A37

Picturing the World

Exposure to lead may cause health problems ranging from stomach distress to brain damage. The Environmental Protection Agency established rules that require water systems to monitor drinking water at customer taps. If lead concentrations exceed 0.015 milligram per liter in more than 10% of customer taps sampled, the system must undertake a number of actions, such as source water treatment, public education, and lead service line replacement. On the basis of a *t*-test, a water system makes a decision on whether the mean level of lead in the water exceeds the allowable amount of 0.015 milligram per liter. Assume the null hypothesis is $\mu \leq 0.015$. (Source: Environmental Protection Agency)

	H_0 True	H_0 False
Fail to reject H_0.		
Reject H_0.		

Describe the possible type I and type II errors of this situation.

A type I error will occur when the actual amount of lead is less than or equal to 0.015 milligram per liter, but you reject H_0: $\mu \leq 0.015$. So, even though the water is safe, the water system will undertake actions that are not needed and possibly cause a public panic. A type II error will occur when the actual amount of lead is greater than 0.015 milligram per liter, but you fail to reject H_0: $\mu \leq 0.015$. So, the water system will not undertake actions to protect the public from water that has too much lead, which could cause health problems.

The *t*-Test for a Mean μ

To test a claim about a mean μ when σ is *not* known, you can use a *t*-sampling distribution. The standardized test statistic takes the form of

$$t = \frac{(\text{Sample mean}) - (\text{Hypothesized mean})}{\text{Standard error}}.$$

Because σ is not known, the standardized test statistic is calculated using the sample standard deviation s, as shown in the next definition.

t-Test for a Mean μ

The ***t*-test for a mean μ** is a statistical test for a population mean. The **test statistic** is the sample mean $\bar{x}$. The **standardized test statistic** is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \qquad \text{Standardized test statistic for } \mu \ (\sigma \text{ unknown})$$

when these conditions are met.

1. The sample is random.
2. At least one of the following is true: The population is normally distributed or $n \geq 30$.

The degrees of freedom are d.f. $= n - 1$.

GUIDELINES

Using the *t*-Test for a Mean μ (σ Unknown)

In Words	In Symbols
1. Verify that σ is not known, the sample is random, and either the population is normally distributed or $n \geq 30$.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= n - 1$
5. Determine the critical value(s).	Use Table 5 in Appendix B.
6. Determine the rejection region(s).	
7. Find the standardized test statistic and sketch the sampling distribution.	$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
8. Make a decision to reject or fail to reject the null hypothesis.	If t is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

In Step 8 of the guidelines, the decision rule uses rejection regions. You can also test a claim using *P*-values, as shown on page 382. Also, when the number of degrees of freedom you need is not in Table 5, use the closest number in the table that is less than the value you need (or use technology). For instance, for d.f. $= 57$, use 50 degrees of freedom.

EXAMPLE 4

See Minitab steps on page 414.

Hypothesis Testing Using a Rejection Region

A used car dealer says that the mean price of used cars sold in the last 12 months is at least \$21,000. You suspect this claim is incorrect and find that a random sample of 14 used cars sold in the last 12 months has a mean price of \$19,189 and a standard deviation of \$2950. Is there enough evidence to reject the dealer's claim at $\alpha = 0.05$? Assume the population is normally distributed. *(Adapted from Edmunds.com)*

SOLUTION

Because σ is unknown, the sample is random, and the population is normally distributed, you can use the t-test. The claim is "the mean price is at least \$21,000." So, the null and alternative hypotheses are

H_0: $\mu \geq \$21{,}000$ (Claim)

and

H_a: $\mu < \$21{,}000$.

The test is a left-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are

$$\text{d.f.} = 14 - 1 = 13.$$

So, using Table 5, the critical value is $t_0 = -1.771$. The rejection region is $t < -1.771$. The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$ Because σ is unknown and the population is normally distributed, use the t-test.

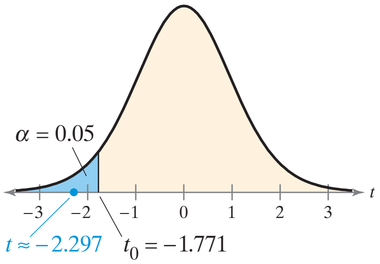
$$= \frac{19{,}189 - 21{,}000}{2950/\sqrt{14}}$$ Assume $\mu = 21{,}000$.

$$\approx -2.297.$$ Round to three decimal places.

7.3 To explore this topic further, see **Activity 7.3** on page 386.

The figure shows the location of the rejection region and the standardized test statistic t. Because t is in the rejection region, you reject the null hypothesis.

Interpretation There is enough evidence at the 5% level of significance to reject the claim that the mean price of used cars sold in the last 12 months is at least \$21,000.



5% Level of Significance

TRY IT YOURSELF 4

An industry analyst says that the mean age of a used car sold in the last 12 months is less than 4.1 years. A random sample of 25 used cars sold in the last 12 months has a mean age of 3.7 years and a standard deviation of 1.3 years. Is there enough evidence to support the analyst's claim at $\alpha = 0.10$? Assume the population is normally distributed. *(Adapted from Edmunds.com)*

Answer: Page A37

Remember that when you make a decision, the possibility of a type I or a type II error exists. For instance, in Example 4, a type I error is possible when you reject H_0, because $\mu \geq \$21{,}000$ may be true.

EXAMPLE 5

See TI-84 Plus steps on page 415.

Hypothesis Testing Using Rejection Regions

An industrial company claims that the mean pH level of the water in a nearby river is 6.8. You randomly select 39 water samples and measure the pH of each. The sample mean and standard deviation are 6.7 and 0.35, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.05$?

SOLUTION

Because σ is unknown, the sample is random, and $n = 39 \geq 30$, you can use the t-test. The claim is "the mean pH level is 6.8." So, the null and alternative hypotheses are

H_0: $\mu = 6.8$ (Claim) and H_a: $\mu \neq 6.8$.

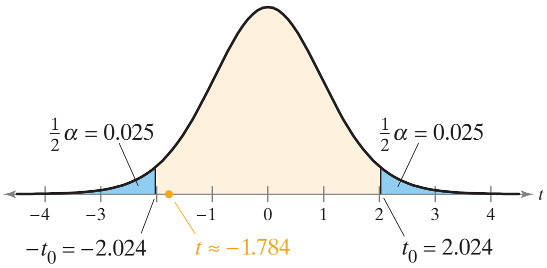
The test is a two-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are d.f. $= 39 - 1 = 38$. So, using Table 5, the critical values are $-t_0 = -2.024$ and $t_0 = 2.024$. The rejection regions are $t < -2.024$ and $t > 2.024$. The standardized test statistic is

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$ Because σ is unknown and $n \geq 30$, use the t-test.

$$= \frac{6.7 - 6.8}{0.35/\sqrt{39}}$$ Assume $\mu = 6.8$.

$$\approx -1.784.$$ Round to three decimal places.

The figure shows the location of the rejection regions and the standardized test statistic t. Because t is not in the rejection region, you fail to reject the null hypothesis. You can confirm this decision using technology, as shown below. Note that the standardized statistic t differs from the one found using Table 5 due to rounding.



5% Level of Significance

MINITAB

One-Sample T

Test of μ = 6.8 vs ≠ 6.8

N	Mean	StDev	SE Mean	95% CI	T	P
39	6.7000	0.3500	0.0560	(6.5865, 6.8135)	−1.78	0.082

Interpretation There is not enough evidence at the 5% level of significance to reject the claim that the mean pH level is 6.8.

TRY IT YOURSELF 5

The company in Example 5 claims that the mean conductivity of the river is 1890 milligrams per liter. The conductivity of a water sample is a measure of the total dissolved solids in the sample. You randomly select 39 water samples and measure the conductivity of each. The sample mean and standard deviation are 2350 milligrams per liter and 900 milligrams per liter, respectively. Is there enough evidence to reject the company's claim at $\alpha = 0.01$?

Answer: Page A37

Tech Tip

Using a TI-84 Plus, you can either enter the original data into a list to find a *P*-value or enter the descriptive statistics.

STAT

Choose the TESTS menu.

2: T-Test...

Select the *Data* input option when you use the original data. Select the *Stats* input option when you use the descriptive statistics. In each case, enter the appropriate values, including the corresponding type of hypothesis test indicated by the alternative hypothesis. Then select *Calculate*.

Using *P*-Values With *t*-Tests

You can also use *P*-values for a *t*-test for a mean μ. For instance, consider finding a *P*-value given $t = 1.98$, 15 degrees of freedom, and a right-tailed test. Using Table 5 in Appendix B, you can determine that *P* falls between

$$\alpha = 0.025 \quad \text{and} \quad \alpha = 0.05$$

but you cannot determine an exact value for *P*. In such cases, you can use technology to perform a hypothesis test and find exact *P*-values.

EXAMPLE 6

Using *P*-Values with a *t*-Test

A department of motor vehicles office claims that the mean wait time is less than 14 minutes. A random sample of 10 people has a mean wait time of 13 minutes with a standard deviation of 3.5 minutes. At $\alpha = 0.10$, test the office's claim. Assume the population is normally distributed.

SOLUTION

Because σ is unknown, the sample is random, and the population is normally distributed, you can use the *t*-test. The claim is "the mean wait time is less than 14 minutes." So, the null and alternative hypotheses are

$$H_0: \mu \geq 14 \text{ minutes}$$

and

$$H_a: \mu < 14 \text{ minutes.} \qquad \text{(Claim)}$$

The TI-84 Plus display at the far left shows how to set up the hypothesis test. The two displays on the right show the possible results, depending on whether you select *Calculate* or *Draw*.

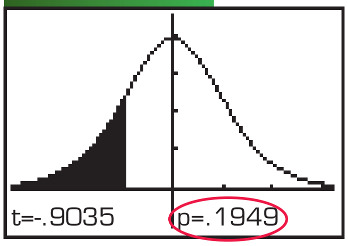
TI-84 PLUS

```
T-Test
Inpt:Data Stats
μ0:14
x̄:13
Sx:3.5
n:10
μ:≠μ0 <μ0 >μ0
Calculate Draw
```

TI-84 PLUS

```
T-Test
μ<14
t=−.9035079029
p=.1948994027
x̄=13
Sx=3.5
n=10
```

TI-84 PLUS



From the displays, you can see that

$$P \approx 0.1949.$$

Because the *P*-value is greater than $\alpha = 0.10$, you fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 10% level of significance to support the office's claim that the mean wait time is less than 14 minutes.

TRY IT YOURSELF 6

Another department of motor vehicles office claims that the mean wait time is at most 18 minutes. A random sample of 12 people has a mean wait time of 15 minutes with a standard deviation of 2.2 minutes. At $\alpha = 0.05$, test the office's claim. Assume the population is normally distributed.

Answer: Page A37

7.3 EXERCISES

For Extra Help: MyLab Statistics

1. See Odd Answers, page A69.
2. See Selected Answers, page A100.
3. Critical value: $t_0 = -1.328$
   Rejection region: $t < -1.328$
4. Critical value: $t_0 = -2.441$
   Rejection region: $t < -2.441$
5. Critical value: $t_0 = 1.717$
   Rejection region: $t > 1.717$
6. Critical value: $t_0 = 2.457$
   Rejection region: $t > 2.457$
7. Critical values: $-t_0 = -2.056$, $t_0 = 2.056$
   Rejection regions: $t < -2.056$, $t > 2.056$
8. Critical values: $-t_0 = -1.687$, $t_0 = 1.687$
   Rejection regions: $t < -1.687$, $t > 1.687$
9. (a) Fail to reject H_0 because $t > -2.086$.
   (b) Fail to reject H_0 because $t > -2.086$.
   (c) Reject H_0 because $t < -2.086$.
10. (a) Fail to reject H_0 because $t < 1.402$.
    (b) Reject H_0 because $t > 1.402$.
    (c) Fail to reject H_0 because $t < 1.402$.
11. (a) Reject H_0 because $t < -1.725$.
    (b) Fail to reject H_0 because $-1.725 < t < 1.725$.
    (c) Reject H_0 because $t > 1.725$.
12. (a) Reject H_0 because $t < -1.071$.
    (b) Fail to reject H_0 because $-1.071 < t < 1.071$.
    (c) Reject H_0 because $t > 1.071$.
13. Fail to reject H_0. There is not enough evidence at the 1% level of significance to reject the claim.
14. See Selected Answers, page A100.
15. See Odd Answers, page A69.
16. See Selected Answers, page A100.
17. See Odd Answers, page A69.
18. See Selected Answers, page A100.

Building Basic Skills and Vocabulary

1. Explain how to find critical values for a *t*-distribution.

2. Explain how to use a *t*-test to test a hypothesized mean μ when σ is unknown. What assumptions are necessary?

In Exercises 3–8, find the critical value(s) and rejection region(s) for the type of t-test with level of significance α and sample size n.

3. Left-tailed test, $\alpha = 0.10$, $n = 20$

4. Left-tailed test, $\alpha = 0.01$, $n = 35$

5. Right-tailed test, $\alpha = 0.05$, $n = 23$

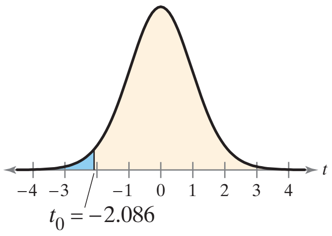
6. Right-tailed test, $\alpha = 0.01$, $n = 31$

7. Two-tailed test, $\alpha = 0.05$, $n = 27$

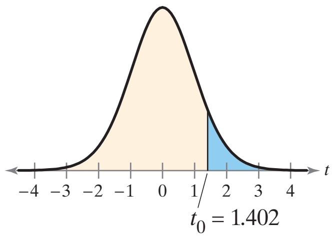
8. Two-tailed test, $\alpha = 0.10$, $n = 38$

Graphical Analysis *In Exercises 9–12, state whether each standardized test statistic t allows you to reject the null hypothesis. Explain.*

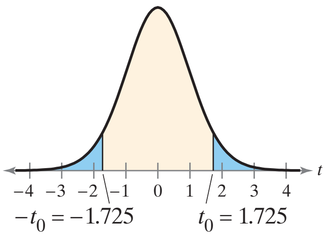
9. (a) $t = 2.091$
(b) $t = 0$
(c) $t = -2.096$



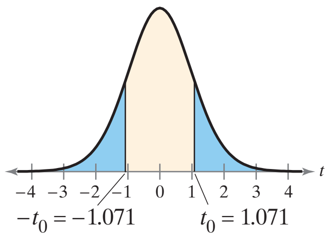
10. (a) $t = 1.4$
(b) $t = 1.42$
(c) $t = -1.402$



11. (a) $t = -1.755$
(b) $t = -1.585$
(c) $t = 1.745$



12. (a) $t = -1.1$
(b) $t = 1.01$
(c) $t = 1.7$



In Exercises 13–18, test the claim about the population mean μ at the level of significance α. Assume the population is normally distributed.

13. Claim: $\mu = 15$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 13.9$, $s = 3.23$, $n = 36$

14. Claim: $\mu > 25$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 26.2$, $s = 2.32$, $n = 17$

15. Claim: $\mu \geq 8000$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 7700$, $s = 450$, $n = 25$

16. Claim: $\mu \leq 1600$; $\alpha = 0.02$. Sample statistics: $\bar{x} = 1550$, $s = 165$, $n = 46$

17. Claim: $\mu < 4915$; $\alpha = 0.02$. Sample statistics: $\bar{x} = 5017$, $s = 5613$, $n = 51$

18. Claim: $\mu \neq 52{,}200$; $\alpha = 0.05$. Sample statistics: $\bar{x} = 53{,}220$, $s = 2700$, $n = 34$

19. (a) The claim is "the mean price of a three-year-old sport utility vehicle (in good condition) is \$20,000."

H_0: $\mu = 20{,}000$ (claim)
H_a: $\mu \neq 20{,}000$

(b) $-t_0 = -2.080$, $t_0 = 2.080$
Rejection regions: $t < -2.080$, $t > 2.080$

(c) 1.51 (d) Fail to reject H_0.

(e) There is not enough evidence at the 5% level of significance to reject the claim that the mean price of a three-year-old sport utility vehicle (in good condition) is \$20,000.

20. (a) The claim is "the mean wait time for various services at different locations is at most 6 minutes."

H_0: $\mu \leq 6$ (claim); H_a: $\mu > 6$

(b) $t_0 = 2.445$
Rejection region: $t > 2.445$

(c) 3.13 (d) Reject H_0.

(e) There is enough evidence at the 1% level of significance to reject the state Department of Transportation's claim that the mean wait time for various services at different locations is at most 6 minutes.

21. See Odd Answers, page A70.

22. See Selected Answers, page A101.

23. See Odd Answers, page A70.

24. See Selected Answers, page A101.

25. See Odd Answers, page A70.

26. See Selected Answers, page A101..

Using and Interpreting Concepts

Hypothesis Testing Using Rejection Regions *In Exercises 19–26, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic t, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.*

19. Used Car Cost A used car dealer says that the mean price of a three-year-old sport utility vehicle (in good condition) is \$20,000. You suspect this claim is incorrect and find that a random sample of 22 similar vehicles has a mean price of \$20,640 and a standard deviation of \$1990. Is there enough evidence to reject the claim at $\alpha = 0.05$?

20. DMV Wait Times A state Department of Transportation claims that the mean wait time for various services at its different locations is at most 6 minutes. A random sample of 34 services at different locations has a mean wait time of 10.3 minutes and a standard deviation of 8.0 minutes. Is there enough evidence to reject the claim at $\alpha = 0.01$?

21. Credit Card Debt A credit reporting agency claims that the mean credit card debt by state is greater than \$5500 per person. You want to test this claim. You find that a random sample of 30 states has a mean credit card debt of \$5594 per person and a standard deviation of \$597 per person. At $\alpha = 0.05$, can you support the claim? *(Adapted from TransUnion)*

22. Battery Life A company claims that the mean battery life of their MP3 player is at least 30 hours. You suspect this claim is incorrect and find that a random sample of 18 MP3 players has a mean battery life of 28.5 hours and a standard deviation of 1.7 hours. Is there enough evidence to reject the claim at $\alpha = 0.01$?

23. Carbon Monoxide Levels As part of your work for an environmental awareness group, you want to test a claim that the mean amount of carbon monoxide in the air in U.S. cities is less than 2.34 parts per million. You find that the mean amount of carbon monoxide in the air for a random sample of 64 U.S. cities is 2.37 parts per million and the standard deviation is 2.11 parts per million. At $\alpha = 0.10$, can you support the claim? *(Adapted from U.S. Environmental Protection Agency)*

24. Lead Levels As part of your work for an environmental awareness group, you want to test a claim that the mean amount of lead in the air in U.S. cities is less than 0.036 microgram per cubic meter. You find that the mean amount of lead in the air for a random sample of 56 U.S. cities is 0.039 microgram per cubic meter and the standard deviation is 0.069 microgram per cubic meter. At $\alpha = 0.01$, can you support the claim? *(Adapted from U.S. Environmental Protection Agency)*

25. Annual Salary An employment information service claims the mean annual salary for senior level product engineers is \$98,000. The annual salaries (in dollars) for a random sample of 16 senior level product engineers are shown in the table at the left. At $\alpha = 0.05$, test the claim that the mean salary is \$98,000. *(Adapted from Salary.com)*

Annual salaries			
100,651	82,505	102,450	91,091
96,309	74,193	76,184	82,088
93,551	77,012	104,020	85,063
112,717	80,970	103,982	110,316

TABLE FOR EXERCISE 25

26. Annual Salary An employment information service claims the mean annual salary for home care physical therapists is more than \$80,000. The annual salaries (in dollars) for a random sample of 12 home care physical therapists are shown in the table at the left. At $\alpha = 0.10$, is there enough evidence to support the claim that the mean salary is more than \$80,000? *(Adapted from Salary.com)*

Annual salaries			
89,245	86,013	83,151	69,771
87,834	67,964	76,523	90,268
90,440	93,538	76,999	68,257

TABLE FOR EXERCISE 26

Using a *P*-Value with a *t*-Test *In Exercises 27–30, (a) identify the claim and state H_0 and H_a, (b) use technology to find the P-value, (c) decide whether to reject or fail to reject the null hypothesis, and (d) interpret the decision in the context of the original claim. Assume the population is normally distributed.*

27. Quarter Mile Times A consumer group claims that the mean minimum time it takes for a sedan to travel a quarter mile is greater than 14.7 seconds. A random sample of 22 sedans has a mean minimum time to travel a quarter mile of 15.4 seconds and a standard deviation of 2.10 seconds. At $\alpha = 0.10$, do you have enough evidence to support the consumer group's claim? *(Adapted from Zero to 60 Times)*

28. Dive Duration An oceanographer claims that the mean dive duration of a North Atlantic right whale is 11.5 minutes. A random sample of 34 dive durations has a mean of 12.2 minutes and a standard deviation of 2.2 minutes. Is there enough evidence to reject the claim at $\alpha = 0.10$? *(Source: Marine Ecology Progress Series)*

29. Class Size You receive a brochure from a large university. The brochure indicates that the mean class size for full-time faculty is fewer than 32 students. You want to test this claim. You randomly select 18 classes taught by full-time faculty and determine the class size of each. The results are shown in the table at the left. At $\alpha = 0.05$, can you support the university's claim?

Class sizes					
35	28	29	33	32	40
26	25	29	28	30	36
33	29	27	30	28	25

TABLE FOR EXERCISE 29

30. Faculty Classroom Hours The dean of a university estimates that the mean number of classroom hours per week for full-time faculty is 11.0. As a member of the student council, you want to test this claim. A random sample of the number of classroom hours for eight full-time faculty for one week is shown in the table at the left. At $\alpha = 0.01$, can you reject the dean's claim?

Classroom hours			
11.8	8.6	12.6	7.9
6.4	10.4	13.6	9.1

TABLE FOR EXERCISE 30

Extending Concepts

Deciding on a Distribution *In Exercises 31 and 32, decide whether you should use the standard normal sampling distribution or a t-sampling distribution to perform the hypothesis test. Justify your decision. Then use the distribution to test the claim. Write a short paragraph about the results of the test and what you can conclude about the claim.*

31. Gas Mileage A car company claims that the mean gas mileage for its luxury sedan is at least 23 miles per gallon. You believe the claim is incorrect and find that a random sample of 5 cars has a mean gas mileage of 22 miles per gallon and a standard deviation of 4 miles per gallon. At $\alpha = 0.05$, test the company's claim. Assume the population is normally distributed.

32. Tuition and Fees An education publication claims that the mean in-state tuition and fees at public four-year institutions by state is more than \$9000 per year. A random sample of 30 states has a mean in-state tuition and fees at public four-year institutions of \$9231 per year. Assume the population standard deviation is \$2380. At $\alpha = 0.01$, test the publication's claim. *(Adapted from The College Board)*

33. Writing You are testing a claim and incorrectly use the standard normal sampling distribution instead of the *t*-sampling distribution. Does this make it more or less likely to reject the null hypothesis? Is this result the same no matter whether the test is left-tailed, right-tailed, or two-tailed? Explain your reasoning.

27. (a) The claim is "the mean minimum time it takes for a sedan to travel a quarter mile is greater than 14.7 seconds."
H_0: $\mu \leq 14.7$
H_a: $\mu > 14.7$ (claim)
(b) 0.0664 (c) Reject H_0.
(d) There is enough evidence at the 10% level of significance to support the consumer group's claim that the mean minimum time it takes for a sedan to travel a quarter mile is greater than 14.7 seconds.

28. (a) The claim is "the mean dive duration of a North Atlantic right whale is 11.5 minutes."
H_0: $\mu = 11.5$ (claim)
H_a: $\mu \neq 11.5$
(b) 0.0725 (c) Reject H_0.
(d) There is enough evidence at the 10% level of significance to reject the oceanographer's claim that the mean dive duration of a North Atlantic right whale is 11.5 minutes.

29. See Odd Answers, page A70.

30. See Selected Answers, page A101.

31. See Odd Answers, page A70.

32. See Selected Answers, page A101.

33. See Odd Answers, page A70.

7.3 ACTIVITY Hypothesis Tests for a Mean

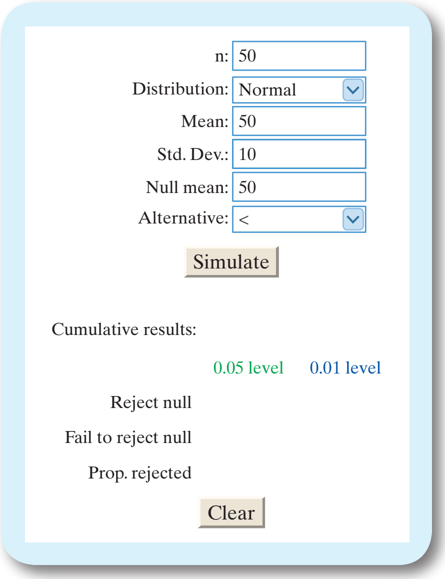
APPLET

You can find the interactive applet for this activity within MyLab Statistics or at *www.pearsonhighered.com/mathstatsresources.*

The *hypothesis tests for a mean* applet allows you to visually investigate hypothesis tests for a mean. You can specify the sample size n, the shape of the distribution (Normal or Right skewed), the true population mean (Mean), the true population standard deviation (Std. Dev.), the null value for the mean (Null mean), and the alternative for the test (Alternative). When you click SIMULATE, 100 separate samples of size n will be selected from a population with these population parameters. For each of the 100 samples, a hypothesis test based on the T statistic is performed, and the results from each test are displayed in the plots at the right. The test statistic for each test is shown in the top plot and the *P*-value is shown in the bottom plot. The green and blue lines represent the cutoffs for rejecting the null hypothesis with the 0.05 and 0.01 level tests, respectively. Additional simulations can be carried out by clicking SIMULATE multiple times. The cumulative number of times that each test rejects the null hypothesis is also shown. Press CLEAR to clear existing results and start a new simulation.

EXPLORE

Step 1 Specify a value for n.
Step 2 Specify a distribution.
Step 3 Specify a value for the mean.
Step 4 Specify a value for the standard deviation.
Step 5 Specify a value for the null mean.
Step 6 Specify an alternative hypothesis.
Step 7 Click SIMULATE to generate the hypothesis tests.



DRAW CONCLUSIONS

APPLET

1. Set $n = 15$, Mean $= 40$, Std. Dev. $= 5$, and the distribution to "Normal." Test the claim that the mean is equal to 40. What are the null and alternative hypotheses? Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Is this what you would expect? Explain.

2. Suppose a null hypothesis is rejected at the 0.01 level. Will it be rejected at the 0.05 level? Explain. Suppose a null hypothesis is rejected at the 0.05 level. Will it be rejected at the 0.01 level? Explain.

3. Set $n = 25$, Mean $= 25$, Std. Dev. $= 3$, and the distribution to "Normal." Test the claim that the mean is at least 27. What are the null and alternative hypotheses? Run the simulation so that at least 1000 hypothesis tests are run. Compare the proportion of null hypothesis rejections for the 0.05 level and the 0.01 level. Is this what you would expect? Explain.

CASE STUDY

Human Body Temperature: What's Normal?

In an article in the *Journal of Statistics Education* (vol. 4, no. 2), Allen Shoemaker describes a study that was reported in the Journal of the American Medical Association (JAMA).* It is generally accepted that the mean body temperature of an adult human is 98.6°F. In his article, Shoemaker uses the data from the JAMA article to test this hypothesis. Here is a summary of his test.

Claim: The body temperature of adults is 98.6°F.

H_0: $\mu = 98.6$°F (Claim) H_a: $\mu \neq 98.6$°F

Sample Size: $n = 130$

Population: Adult human temperatures (Fahrenheit)

Distribution: Approximately normal

Test Statistics: $\bar{x} \approx 98.25$, $s \approx 0.73$

* Data for the JAMA article were collected from healthy men and women, ages 18 to 40, at the University of Maryland Center for Vaccine Development, Baltimore.

Men's Temperatures (in degrees Fahrenheit)

96	3
96	7 9
97	0 1 1 1 2 3 4 4 4 4
97	5 5 6 6 6 7 8 8 8 8 9 9
98	0 0 0 0 0 0 1 1 2 2 2 2 3 3 4 4 4 4
98	5 5 6 6 6 6 6 6 7 7 8 8 8 9
99	0 0 0 1 2 3 4
99	5
100	
100	

Key: 96|3 = 96.3

Women's Temperatures (in degrees Fahrenheit)

96	4
96	7 8
97	2 2 4
97	6 7 7 8 8 8 9 9 9
98	0 0 0 0 0 1 2 2 2 2 2 2 3 3 3 4 4 4 4 4
98	5 6 6 6 6 7 7 7 7 7 7 8 8 8 8 8 8 8 9
99	0 0 1 1 2 2 3 4
99	9
100	0
100	8

Key: 96|4 = 96.4

EXERCISES

1. Complete the hypothesis test for all adults (men and women) by performing the following steps. Use a level of significance of $\alpha = 0.05$.

(a) Sketch the sampling distribution.

(b) Determine the critical values and add them to your sketch.

(c) Determine the rejection regions and shade them in your sketch.

(d) Find the standardized test statistic. Plot and label it in your sketch.

(e) Make a decision to reject or fail to reject the null hypothesis.

(f) Interpret the decision in the context of the original claim.

2. If you lower the level of significance to $\alpha = 0.01$, does your decision change? Explain your reasoning.

3. Test the hypothesis that the mean temperature of men is 98.6°F. What can you conclude at a level of significance of $\alpha = 0.01$?

4. Test the hypothesis that the mean temperature of women is 98.6°F. What can you conclude at a level of significance of $\alpha = 0.01$?

5. Use the sample of 130 temperatures to form a 99% confidence interval for the mean body temperature of adult humans.

6. The conventional "normal" body temperature was established by Carl Wunderlich over 100 years ago. What were possible sources of error in Wunderlich's sampling procedure?

7.4 Hypothesis Testing for Proportions

What You Should Learn

- How to use the z-test to test a population proportion p

Hypothesis Test for Proportions

Hypothesis Test for Proportions

In Sections 7.2 and 7.3, you learned how to perform a hypothesis test for a population mean μ. In this section, you will learn how to test a population proportion p.

Hypothesis tests for proportions can be used when politicians want to know the proportion of their constituents who favor a certain bill or when quality assurance engineers test the proportion of parts that are defective.

If $np \geq 5$ and $nq \geq 5$ for a binomial distribution, then the sampling distribution for $\hat{p}$ is approximately normal with a mean of $\mu_{\hat{p}} = p$ and a standard error of

$$\sigma_{\hat{p}} = \sqrt{pq/n}.$$

z-Test for a Proportion p

The **z-test for a proportion p** is a statistical test for a population proportion. The z-test can be used when a binomial distribution is given such that $np \geq 5$ and $nq \geq 5$. The **test statistic** is the sample proportion $\hat{p}$ and the **standardized test statistic** is

$$z = \frac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$ Standardized test statistic for p

GUIDELINES

Using a z-Test for a Proportion p

In Words	In Symbols
1. Verify that the sampling distribution of $\hat{p}$ can be approximated by a normal distribution.	$np \geq 5,\ nq \geq 5$
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Determine the critical value(s).	Use Table 4 in Appendix B.
5. Determine the rejection region(s).	
6. Find the standardized test statistic and sketch the sampling distribution.	$z = \dfrac{\hat{p} - p}{\sqrt{pq/n}}$.
7. Make a decision to reject or fail to reject the null hypothesis.	If z is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
8. Interpret the decision in the context of the original claim.	

In Step 7 of the guidelines, the decision rule uses rejection regions. You can also test a claim using P-values, as shown in the Study Tip at the left.

Study Tip

A hypothesis test for a proportion p can also be performed using P-values. Use the guidelines on page 365 for using P-values for a z-test for a mean μ, but in Step 4 find the standardized test statistic by using the formula

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}.$$

The other steps in the test are the same.

7.4 To explore this topic further, see **Activity 7.4** on page 393.

EXAMPLE 1

See TI-84 Plus steps on page 415.

Hypothesis Test for a Proportion

A researcher claims that less than 45% of U.S. adults use passwords that are less secure because complicated ones are too hard to remember. In a random sample of 100 adults, 41% say they use passwords that are less secure because complicated ones are too hard to remember. At $\alpha = 0.01$, is there enough evidence to support the researcher's claim? *(Adapted from Pew Research Center)*

SOLUTION

The products $np = 100(0.45) = 45$ and $nq = 100(0.55) = 55$ are both greater than 5. So, you can use a z-test. The claim is "less than 45% of U.S. adults use passwords that are less secure because complicated ones are too hard to remember." So, the null and alternative hypotheses are

H_0: $p \geq 0.45$ and H_a: $p < 0.45$. (Claim)

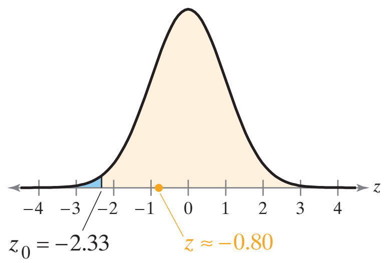
Because the test is a left-tailed test and the level of significance is $\alpha = 0.01$, the critical value is $z_0 = -2.33$ and the rejection region is $z < -2.33$. The standardized test statistic is

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$ Because $np \geq 5$ and $n \geq 5$, you can use the z-test.

$$= \frac{0.41 - 0.45}{\sqrt{(0.45)(0.55)/100}}$$ Assume $p = 0.45$.

$$\approx -0.80.$$ Round to two decimal places.

The figure shows the location of the rejection region and the standardized test statistic z. Because z is not in the rejection region, you fail to reject the null hypothesis.



1% Level of Significance

Interpretation There is not enough evidence at the 1% level of significance to support the claim that less than 45% of U.S. adults use passwords that are less secure because complicated ones are too hard to remember.

Study Tip

Remember that when you fail to reject H_0, a type II error is possible. For instance, in Example 1 the null hypothesis, $p \geq 0.45$, may be false.

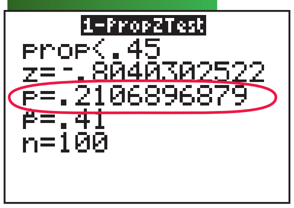
TRY IT YOURSELF 1

A researcher claims that more than 90% of U.S. adults have access to a smartphone. In a random sample of 150 adults, 87% say they have access to a smartphone. At $\alpha = 0.01$, is there enough evidence to support the researcher's claim? *(Adapted from Nielsen Mobile Insights)*

Answer: Page A37

To use a P-value to perform the hypothesis test in Example 1, you can use technology, as shown at the right, or you can use Table 4. Using Table 4, the area corresponding to $z = -0.80$ is 0.2119. Because this is a left-tailed test, the P-value is equal to the area to the left of $z = -0.80$. So, $P = 0.2119$. (This value differs from the one found using technology due to rounding.) Because the P-value is greater than $\alpha = 0.01$, you fail to reject the null hypothesis. Note that this is the same result obtained in Example 1.

TI-84 PLUS



Picturing the World

According to a survey, at least 35% of smartphone owners say the first thing they access on their phones each day is texts or instant messages. To test this claim, you randomly select 300 smartphone owners. In the sample, you find that 93 of them say the first thing they access on their phones each day is texts or instant messages. (Adapted from Deloitte's 2016 Global Mobile Consumer Survey: U.S. edition)

At $\alpha = 0.05$, is there enough evidence to reject the claim?

No, there is not enough evidence at the 5% level of significance to reject the claim that at least 35% of smartphone owners say the first thing they access on their phones each day is texts or instant messages.

Recall from Section 6.3 that when the sample proportion is not given, you can find it using the formula

$$\hat{p} = \frac{x}{n} \qquad \text{Sample proportion}$$

where x is the number of successes in the sample and n is the sample size.

EXAMPLE 2

See Minitab steps on page 414.

Hypothesis Test for a Proportion

A researcher claims that 51% of U.S. adults believe, incorrectly, that antibiotics are effective against viruses. In a random sample of 2202 adults, 1161 say antibiotics are effective against viruses. At $\alpha = 0.10$, is there enough evidence to support the researcher's claim? *(Source: HealthDay/Harris Poll)*

SOLUTION

The products $np = 2202(0.51) \approx 1123$ and $nq = 2202(0.49) \approx 1079$ are both greater than 5. So, you can use a z-test. The claim is "51% of U.S. adults believe, incorrectly, that antibiotics are effective against viruses." So, the null and alternative hypotheses are

$$H_0\colon p = 0.51 \text{ (Claim)} \qquad \text{and} \qquad H_a\colon p \neq 0.51.$$

Because the test is a two-tailed test and the level of significance is $\alpha = 0.10$, the critical values are $-z_0 = -1.645$ and $z_0 = 1.645$. The rejection regions are $z < -1.645$ and $z > 1.645$. Because the number of successes is $x = 1161$ and $n = 2202$, the sample proportion is

$$\hat{p} = \frac{x}{n} = \frac{1161}{2202} \approx 0.527.$$

The standardized test statistic is

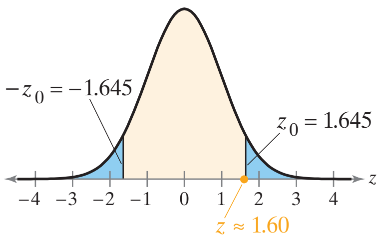
$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} \qquad \text{Because } np \geq 5 \text{ and } nq \geq 5, \text{ you can use the } z\text{-test.}$$

$$= \frac{0.527 - 0.51}{\sqrt{(0.51)(0.49)/2202}} \qquad \text{Assume } p = 0.51.$$

$$\approx 1.60. \qquad \text{Round to two decimal places.}$$

The figure shows the location of the rejection regions and the standardized test statistic z. Because z is not in the rejection region, you fail to reject the null hypothesis.

Interpretation There is not enough evidence at the 10% level of significance to reject the claim that 51% of U.S. adults believe, incorrectly, that antibiotics are effective against viruses.



10% Level of Significance

TRY IT YOURSELF 2

A researcher claims that 67% of U.S. adults believe that doctors prescribing antibiotics for viral infections for which antibiotics are not effective is a significant cause of drug-resistant superbugs. (Superbugs are bacterial infections that are resistant to many or all antibiotics.) In a random sample of 1768 adults, 1150 say they believe that doctors prescribing antibiotics for viral infections for which antibiotics are not effective is a significant cause of drug-resistant superbugs. At $\alpha = 0.10$, is there enough evidence to support the researcher's claim? *(Source: HealthDay/Harris Poll)* *Answer: Page A37*

7.4 EXERCISES

For Extra Help: MyLab Statistics

Building Basic Skills and Vocabulary

1. Explain how to determine whether a normal distribution can be used to approximate a binomial distribution.

2. Explain how to test a population proportion p.

In Exercises 3–6, determine whether a normal sampling distribution can be used. If it can be used, test the claim.

3. Claim: $p < 0.12$; $\alpha = 0.01$. Sample statistics: $\hat{p} = 0.10$, $n = 40$

4. Claim: $p \geq 0.48$; $\alpha = 0.08$. Sample statistics: $\hat{p} = 0.40$, $n = 90$

5. Claim: $p \neq 0.15$; $\alpha = 0.05$. Sample statistics: $\hat{p} = 0.12$, $n = 500$

6. Claim: $p > 0.70$; $\alpha = 0.04$. Sample statistics: $\hat{p} = 0.64$, $n = 225$

Using and Interpreting Concepts

Hypothesis Testing Using Rejection Regions *In Exercises 7–12, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic z, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.*

7. **Vaccination Requirement** A medical researcher says that less than 80% of U.S. adults think that healthy children should be required to be vaccinated. In a random sample of 200 U.S. adults, 82% think that healthy children should be required to be vaccinated. At $\alpha = 0.05$, is there enough evidence to support the researcher's claim? *(Adapted from Pew Research Center)*

8. **Internal Revenue Service Audits** A research center claims that at least 27% of U.S. adults think that the IRS will audit their taxes. In a random sample of 1000 U.S. adults in a recent year, 23% say they are concerned that the IRS will audit their taxes. At $\alpha = 0.01$, is there enough evidence to reject the center's claim? *(Source: Rasmussen Reports)*

9. **Student Employment** An eduction researcher claims that at most 3% of working college students are employed as teachers or teaching assistants. In a random sample of 200 working college students, 4% are employed as teachers or teaching assistants. At $\alpha = 0.01$, is there enough evidence to reject the researcher's claim? *(Adapted from Sallie Mae)*

10. **Working Students** An education researcher claims that 57% of college students work year-round. In a random sample of 300 college students, 171 say they work year-round. At $\alpha = 0.10$, is there enough evidence to support the researcher's claim? *(Adapted from Sallie Mae)*

11. **Zika Virus** A researcher claims that 85% percent of Americans think they are unlikely to contract the Zika virus. In a random sample of 250 Americans, 225 think they are unlikely to contract the Zika virus. At $\alpha = 0.05$, is there enough evidence to reject the researcher's claim? *(Adapted from Gallup)*

12. **Changing Jobs** A research center claims that more than 29% of U.S. employees have changed jobs in the past three years. In a random sample of 180 U.S. employees, 63 have changed jobs in the past three years. At $\alpha = 0.10$, is there enough evidence to support the center's claim? *(Adapted from Gallup)*

---

1. If $np \geq 5$ and $nq \geq 5$, then the normal distribution can be used.
2. Verify that $np \geq 5$ and $nq \geq 5$. State H_0 and H_a. Specify the level of significance α. Determine the critical value(s) and rejection region(s). Find the standardized test statistic. Make a decision and interpret it in the context of the original claim.
3. Cannot use normal distribution.
4. Can use normal distribution.
   Reject H_0. There is enough evidence at the 8% level of significance to reject the claim.
5. Can use normal distribution.
   Fail to reject H_0. There is not enough evidence at the 5% level of significance to support the claim.
6. Can use normal distribution.
   Fail to reject H_0. There is not enough evidence at the 4% level of significance to support the claim.
7. (a) The claim is "less than 80% of U.S. adults think that healthy children should be required to be vaccinated."
   H_0: $p \geq 0.80$
   H_a: $p < 0.80$ (claim)
   (b) $z_0 = -1.645$
   Rejection region: $z < -1.645$
   (c) 0.707 (d) Fail to reject H_0.
   (e) There is not enough evidence at the 5% level of significance to support the medical researcher's claim that less than 80% of U.S. adults think that healthy children should be required to be vaccinated.
8. See Selected Answers, page A101.
9. See Odd Answers, page A70.
10. See Selected Answers, page A101.
11. See Odd Answers, page A70.
12. See Selected Answers, page A101.

Hypothesis Testing Using a *P*-Value *In Exercises 13–16, (a) identify the claim and state H_0 and H_a, (b) use technology to find the P-value, (c) decide whether to reject or fail to reject the null hypothesis, and (d) interpret the decision in the context of the original claim.*

13. Space Travel A research center claims that 27% of U.S. adults would travel into space on a commercial flight if they could afford it. In a random sample of 1000 U.S. adults, 30% say that they would travel into space on a commercial flight if they could afford it. At $\alpha = 0.05$, is there enough evidence to reject the research center's claim? *(Source: Rasmussen Reports)*

14. Purchasing Food Online A research center claims that at most 18% of U.S. adults' online food purchases are for snacks. In a random sample of 1995 U.S. adults, 20% say their online food purchases are for snacks. At $\alpha = 0.10$, is there enough evidence to support the center's claim? *(Source: The Harris Poll)*

15. Pet Ownership A humane society claims that less than 67% of U.S. households own a pet. In a random sample of 600 U.S. households, 390 say they own a pet. At $\alpha = 0.10$, is there enough evidence to support the society's claim? *(Adapted from The Humane Society of the United States)*

16. Stray dogs A humane society claims that 5% of U.S. households have taken in a stray dog. In a random sample of 200 U.S. households, 12 say they have taken in a stray dog. At $\alpha = 0.05$, is there enough evidence to reject the society's claim? *(Adapted from The Humane Society of the United States)*

13. (a) The claim is "27% of U.S. adults would travel into space on a commercial flight if they could afford it."
H_0: $p = 0.27$ (claim)
H_a: $p \neq 0.27$
(b) 0.03 (c) Reject H_0.
(d) There is enough evidence at the 5% level of significance to reject the research center's claim that 27% of U.S. adults would travel into space on a commercial flight if they could afford it.

14. See Selected Answers, page A101.

15. See Odd Answers, page A71.

16. See Selected Answers, page A101.

17. Fail to reject H_0. There is not enough evidence at the 5% level of significance to reject the claim that at least 63% of adults make an effort to live in ways that help protect the environment some of the time.

Protecting the Environment *In Exercises 17 and 18, use the figure at the left, which suggests what adults think about protecting the environment. (Source: Pew Research Center)*

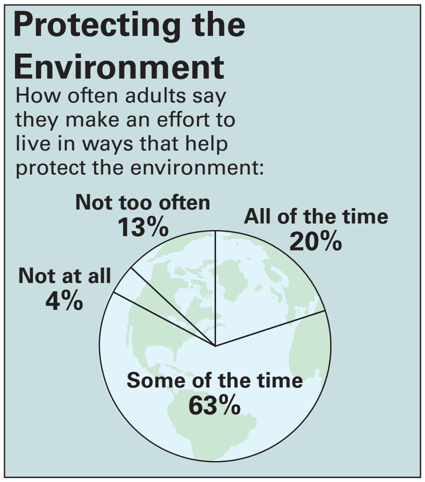


FIGURE FOR EXERCISES 17 AND 18

17. Are People Concerned About Protecting the Environment? You interview a random sample of 100 adults. The results of the survey show that 59% of the adults said they live in ways that help protect the environment some of the time. At $\alpha = 0.05$, can you reject the claim that at least 63% of adults make an effort to live in ways that help protect the environment some of the time?

18. What Are People's Attitudes About Protecting the Environment? Use your conclusion from Exercise 17 to write a paragraph on people's attitudes about protecting the environment.

18. Answers will vary.

19. See Odd Answers, page A71.

20. See Selected Answers, page A101.

Extending Concepts

Alternative Formula *In Exercises 19 and 20, use the information below. When you know the number of successes x, the sample size n, and the population proportion p, it can be easier to use the formula*

$$z = \frac{x - np}{\sqrt{npq}}$$

to find the standardized test statistic when using a z-test for a population proportion p.

19. Rework Exercise 7 using the alternative formula and verify that the results are the same.

20. The alternative formula is derived from the formula

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} = \frac{(x/n) - p}{\sqrt{pq/n}}.$$

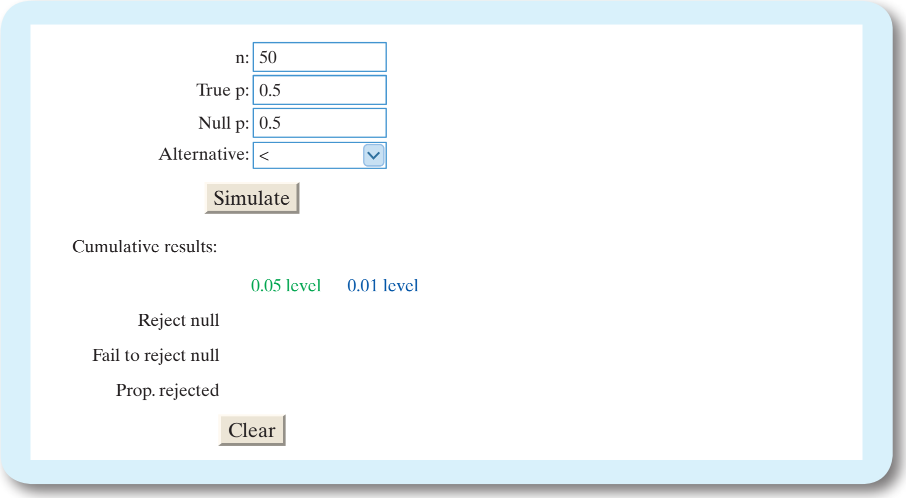
Use this formula to derive the alternative formula. Justify each step.

7.4 ACTIVITY Hypothesis Tests for a Proportion

APPLET

You can find the interactive applet for this activity within MyLab Statistics or at *www.pearsonhighered.com/mathstatsresources.*

The *hypothesis tests for a proportion* applet allows you to visually investigate hypothesis tests for a population proportion. You can specify the sample size n, the population proportion (True p), the null value for the proportion (Null p), and the alternative for the test (Alternative). When you click SIMULATE, 100 separate samples of size n will be selected from a population with a proportion of successes equal to True p. For each of the 100 samples, a hypothesis test based on the Z statistic is performed, and the results from each test are displayed in plots at the right. The standardized test statistic for each test is shown in the top plot and the P-value is shown in the bottom plot. The green and blue lines represent the cutoffs for rejecting the null hypothesis with the 0.05 and 0.01 level tests, respectively. Additional simulations can be carried out by clicking SIMULATE multiple times. The cumulative number of times that each test rejects the null hypothesis is also shown. Press CLEAR to clear existing results and start a new simulation.



EXPLORE

Step 1 Specify a value for n.
Step 2 Specify a value for True p.
Step 3 Specify a value for Null p.
Step 4 Specify an alternative hypothesis.
Step 5 Click SIMULATE to generate the hypothesis tests.

DRAW CONCLUSIONS

APPLET

1. Set $n = 25$ and True $p = 0.35$. Test the claim that the proportion is equal to 35%. What are the null and alternative hypotheses? Run the simulation so that at least 1000 tests are run. Compare the proportion of null hypothesis rejections for the 0.05 and 0.01 levels. Is this what you would expect? Explain.

2. Set $n = 50$ and True $p = 0.6$. Test the claim that the proportion is at least 40%. What are the null and alternative hypotheses? Run the simulation so that at least 1000 tests are run. Compare the proportion of null hypothesis rejections for the 0.05 and 0.01 levels. Perform a hypothesis test for each level. Use the results of the hypothesis tests to explain the results of the simulation.

7.5 Hypothesis Testing for Variance and Standard Deviation

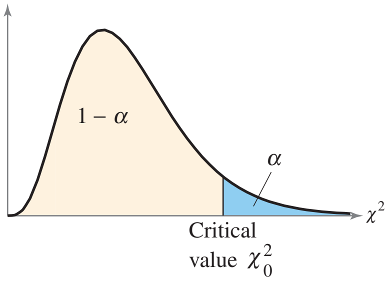
What You Should Learn

- How to find critical values for a chi-square test
- How to use the chi-square test to test a variance σ^2 or a standard deviation σ

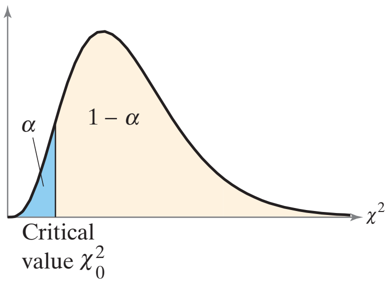
Critical Values for a Chi-Square Test ■ The Chi-Square Test

Critical Values for a Chi-Square Test

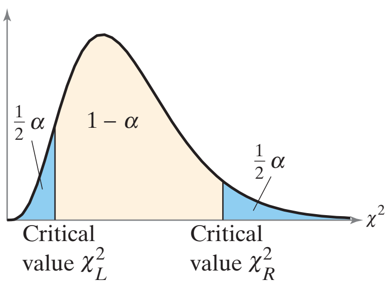
In real life, it is important to produce consistent, predictable results. For instance, consider a company that manufactures golf balls. The manufacturer must produce millions of golf balls, each having the same size and the same weight. There is a very low tolerance for variation. For a normally distributed population, you can test the variance and standard deviation of the process using the chi-square distribution with $n - 1$ degrees of freedom. Before learning how to do the test, you must know how to find the critical values, as shown in the guidelines.



Right-Tailed Test



Left-Tailed Test



Two-Tailed Test

GUIDELINES

Finding Critical Values for a Chi-Square Test

1. Specify the level of significance α.
2. Identify the degrees of freedom, d.f. $= n - 1$.
3. The critical values for the chi-square distribution are found in Table 6 in Appendix B. To find the critical value(s) for a
   a. *right-tailed test,* use the value that corresponds to d.f. and α.
   b. *left-tailed test,* use the value that corresponds to d.f. and $1 - \alpha$.
   c. *two-tailed test,* use the values that correspond to d.f. and $\frac{1}{2}\alpha$, and d.f. and $1 - \frac{1}{2}\alpha$.

See the figures at the left.

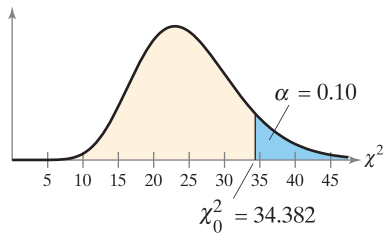
EXAMPLE 1

Finding a Critical Value for a Right-Tailed Test

Find the critical value χ^2_0 for a right-tailed test when $n = 26$ and $\alpha = 0.10$.

SOLUTION

The degrees of freedom are d.f. $= n - 1 = 26 - 1 = 25$. The figure below shows a chi-square distribution with 25 degrees of freedom and a shaded area of $\alpha = 0.10$ in the right tail. Using Table 6 in Appendix B with d.f. $= 25$ and $\alpha = 0.10$, the critical value is $\chi^2_0 = 34.382$.



TRY IT YOURSELF 1

Find the critical value χ^2_0 for a right-tailed test when $n = 18$ and $\alpha = 0.01$.

Answer: Page A37

Note to Instructor

This section can be omitted or covered later (with Chapter 10) without loss of continuity.

EXAMPLE 2

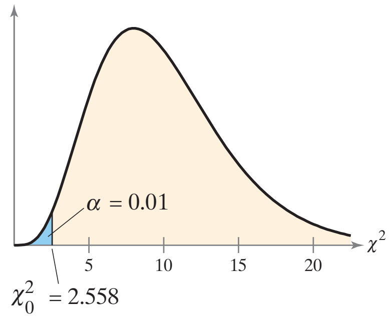
Finding a Critical Value for a Left-Tailed Test

Find the critical value χ_0^2 for a left-tailed test when $n = 11$ and $\alpha = 0.01$.

SOLUTION

The degrees of freedom are

$$\text{d.f.} = n - 1 = 11 - 1 = 10.$$



The figure at the left shows a chi-square distribution with 10 degrees of freedom and a shaded area of $\alpha = 0.01$ in the left tail. The area to the right of the critical value is

$$1 - \alpha = 1 - 0.01 = 0.99.$$

Using Table 6 with d.f. $= 10$ and the area 0.99, the critical value is $\chi_0^2 = 2.558$. You can check your answer using technology, as shown below.

MINITAB

Inverse Cumulative Distribution Function

Chi-Square with 10 DF

P (X ≤ x)	x
0.01	2.55821

TRY IT YOURSELF 2

Find the critical value χ_0^2 for a left-tailed test when $n = 30$ and $\alpha = 0.05$.

Answer: Page A37

Note that because chi-square distributions are not symmetric (like normal or *t*-distributions), in a two-tailed test the two critical values are not opposites. Each critical value must be calculated separately, as shown in the next example.

EXAMPLE 3

Finding Critical Values for a Two-Tailed Test

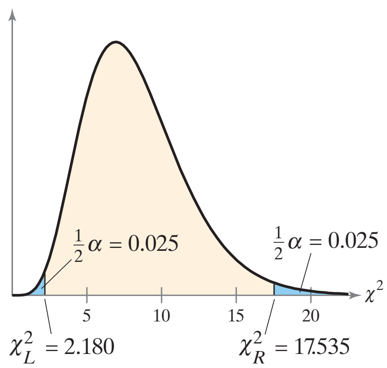
Find the critical values χ_L^2 and χ_R^2 for a two-tailed test when $n = 9$ and $\alpha = 0.05$.

SOLUTION

The degrees of freedom are

$$\text{d.f.} = n - 1 = 9 - 1 = 8.$$

The figure shows a chi-square distribution with 8 degrees of freedom and a shaded area of $\frac{1}{2}\alpha = 0.025$ in each tail. The area to the right of χ_R^2 is $\frac{1}{2}\alpha = 0.025$, and the area to the right of χ_L^2 is $1 - \frac{1}{2}\alpha = 0.975$. Using Table 6 with d.f. $= 8$ and the areas 0.025 and 0.975, the critical values are $\chi_R^2 = 17.535$ and $\chi_L^2 = 2.180$. You can check you answer using technology, as shown at the left.



EXCEL

	A	B
1	CHISQ.INV(0.025,8)	
2		2.179730747
3	CHISQ.INV.RT(0.025,8)	
4		17.53454614

TRY IT YOURSELF 3

Find the critical values χ_L^2 and χ_R^2 for a two-tailed test when $n = 51$ and $\alpha = 0.01$.

Answer: Page A37

The Chi-Square Test

To test a variance σ^2 or a standard deviation σ of a population that is normally distributed, you can use the chi-square test. The chi-square test for a variance or standard deviation is not as robust as the tests for the population mean μ or the population proportion p. So, it is essential in performing a chi-square test for a variance or standard deviation that the population be normally distributed. The results can be misleading when the population is not normal.

Chi-Square Test for a Variance σ^2 or Standard Deviation σ

The **chi-square test for a variance σ^2 or standard deviation σ** is a statistical test for a population variance or standard deviation. The chi-square test can only be used when the population is normal. The **test statistic** is s^2 and the **standardized test statistic**

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ Standardized test statistic for σ^2 or σ

follows a chi-square distribution with degrees of freedom

d.f. $= n - 1$.

Note to Instructor

Review the properties of chi-square distributions. Tell students that this family of distributions will be used in later chapters, but the degrees of freedom for those tests are not necessarily $n - 1$.

In Step 8 of the guidelines below, the decision rule uses rejection regions. You can also test a claim using P-values (see Exercises 31-34).

GUIDELINES

Using the Chi-Square Test for a Variance σ^2 or a Standard Deviation σ

In Words	In Symbols
1. Verify that the sample is random and the population is normally distributed.	
2. State the claim mathematically and verbally. Identify the null and alternative hypotheses.	State H_0 and H_a.
3. Specify the level of significance.	Identify α.
4. Identify the degrees of freedom.	d.f. $= n - 1$
5. Determine the critical value(s).	Use Table 6 in Appendix B.
6. Determine the rejection region(s).	
7. Find the standardized test statistic and sketch the sampling distribution.	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$
8. Make a decision to reject or fail to reject the null hypothesis.	If χ^2 is in the rejection region, then reject H_0. Otherwise, fail to reject H_0.
9. Interpret the decision in the context of the original claim.	

For Step 5 of the guidelines, in addition to using Table 6 in Appendix B, you can use technology to find the critical value(s). Also, some technology tools allow you to perform a hypothesis test for a variance (or a standard deviation) using only the descriptive statistics.

EXAMPLE 4

Using a Hypothesis Test for the Population Variance

A dairy processing company claims that the variance of the amount of fat in the whole milk processed by the company is no more than 0.25. You suspect this is wrong and find that a random sample of 41 milk containers has a variance of 0.27. At $\alpha = 0.05$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

SOLUTION

Because the sample is random and the population is normally distributed, you can use the chi-square test. The claim is "the variance is no more than 0.25." So, the null and alternative hypotheses are

H_0: $\sigma^2 \leq 0.25$ (Claim) and H_a: $\sigma^2 > 0.25$.

The test is a right-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are d.f. $= 41 - 1 = 40$. So, using Table 6, the critical value is

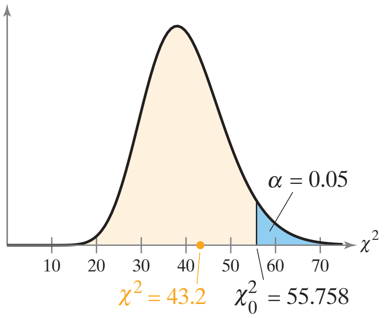
$$\chi_0^2 = 55.758.$$

The rejection region is $\chi^2 > 55.758$. The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ Use the chi-square test.

$$= \frac{(41-1)(0.27)}{0.25}$$ Assume $\sigma^2 = 0.25$.

$$= 43.2.$$



The figure at the left shows the location of the rejection region and the standardized test statistic χ^2. Because χ^2 is not in the rejection region, you fail to reject the null hypothesis. You can check your answer using technology, as shown below. Note that the test statistic, 43.2, is the same as what you found above.

STATCRUNCH

One sample variance hypothesis test:
σ^2 : Variance of population
H_0 : $\sigma^2 = 0.25$
H_A : $\sigma^2 > 0.25$

Hypothesis test results:

Variance	Sample Var.	DF	Chi-square Stat	P-value
σ^2	0.27	40	43.2	0.3362

Interpretation There is not enough evidence at the 5% level of significance to reject the company's claim that the variance of the amount of fat in the whole milk is no more than 0.25.

TRY IT YOURSELF 4

A bottling company claims that the variance of the amount of sports drink in a 12-ounce bottle is no more than 0.40. A random sample of 31 bottles has a variance of 0.75. At $\alpha = 0.01$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

Answer: Page A37

EXAMPLE 5

Using a Hypothesis Test for the Standard Deviation

A company claims that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes. A random sample of 25 incoming telephone calls has a standard deviation of 1.1 minutes. At $\alpha = 0.10$, is there enough evidence to support the company's claim? Assume the population is normally distributed.

SOLUTION

Because the sample is random and the population is normally distributed, you can use the chi-square test. The claim is "the standard deviation is less than 1.4 minutes." So, the null and alternative hypotheses are

H_0: $\sigma \geq 1.4$ minutes and H_a: $\sigma < 1.4$ minutes. (Claim)

The test is a left-tailed test, the level of significance is $\alpha = 0.10$, and the degrees of freedom are

$$\text{d.f.} = 25 - 1 = 24.$$

So, using Table 6, the critical value is

$$\chi_0^2 = 15.659.$$

Study Tip

Although you are testing a standard deviation in Example 5, the standardized test statistic χ^2 requires variance. Remember to square the standard deviation to calculate the variance.

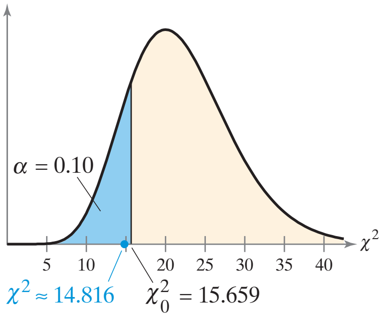
The rejection region is $\chi^2 < 15.659$. The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \quad \text{Use the chi-square test.}$$

$$= \frac{(25-1)(1.1)^2}{(1.4)^2} \quad \text{Assume } \sigma = 1.4.$$

$$\approx 14.816. \quad \text{Round to three decimal places.}$$

The figure below shows the location of the rejection region and the standardized test statistic χ^2. Because χ^2 is in the rejection region, you reject the null hypothesis.



Interpretation There is enough evidence at the 10% level of significance to support the claim that the standard deviation of the lengths of time it takes an incoming telephone call to be transferred to the correct office is less than 1.4 minutes.

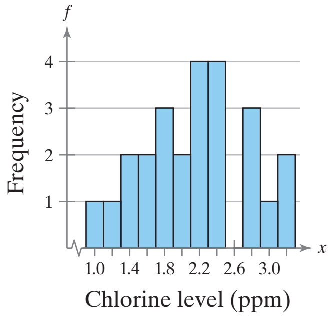
TRY IT YOURSELF 5

A police chief claims that the standard deviation of the lengths of response times is less than 3.7 minutes. A random sample of 9 response times has a standard deviation of 3.0 minutes. At $\alpha = 0.05$, is there enough evidence to support the police chief's claim? Assume the population is normally distributed.

Answer: Page A37

Picturing the World

A community center claims that the chlorine level in its pool has a standard deviation of 0.46 parts per million (ppm). A sampling of the pool's chlorine levels at 25 random times during a month yields a standard deviation of 0.61 ppm. (Adapted from American Pool Supply)



At 0.05, is there enough evidence to reject the claim?

Yes, there is enough evidence at the 5% level of significance to reject the claim that the chlorine level in the pool has a standard deviation of 0.46 parts per million.

EXAMPLE 6

Using a Hypothesis Test for the Population Variance

A sporting goods manufacturer claims that the variance of the strengths of a certain fishing line is 15.9. A random sample of 15 fishing line spools has a variance of 21.8. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim? Assume the population is normally distributed.

SOLUTION

Because the sample is random and the population is normally distributed, you can use the chi-square test. The claim is "the variance is 15.9." So, the null and alternative hypotheses are

H_0: $\sigma^2 = 15.9$ (Claim) and H_a: $\sigma^2 \neq 15.9$.

The test is a two-tailed test, the level of significance is $\alpha = 0.05$, and the degrees of freedom are

$$\text{d.f.} = 15 - 1 = 14.$$

Using Table 6, the critical values are $\chi^2_L = 5.629$ and $\chi^2_R = 26.119$. The rejection regions are

$\chi^2 < 5.629$ and $\chi^2 > 26.119$.

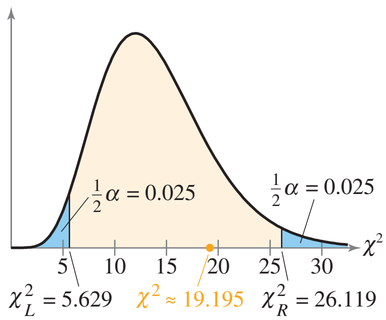
The standardized test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$ Use the chi-square test.

$$= \frac{(15-1)(21.8)}{(15.9)}$$ Assume $\sigma^2 = 15.9$.

$$\approx 19.195.$$ Round to three decimal places.

The figure below shows the location of the rejection regions and the standardized test statistic χ^2. Because χ^2 is not in the rejection regions, you fail to reject the null hypothesis.



Interpretation There is not enough evidence at the 5% level of significance to reject the claim that the variance of the strengths of the fishing line is 15.9.

TRY IT YOURSELF 6

A company that offers dieting products and weight loss services claims that the variance of the weight losses of their users is 25.5. A random sample of 13 users has a variance of 10.8. At $\alpha = 0.10$, is there enough evidence to reject the company's claim? Assume the population is normally distributed.

Answer: Page A37

7.5 EXERCISES

For Extra Help: MyLab Statistics

Building Basic Skills and Vocabulary

1. Explain how to find critical values in a chi-square distribution.

2. Can a critical value for the chi-square test be negative? Explain.

3. How do the requirements for a chi-square test for a variance or standard deviation differ from a z-test or a t-test for a mean?

4. Explain how to test a population variance or a population standard deviation.

In Exercises 5–12, find the critical value(s) and rejection region(s) for the type of chi-square test with sample size n and level of significance α.

5. Right-tailed test, $n = 27, \alpha = 0.05$

6. Right-tailed test, $n = 10, \alpha = 0.10$

7. Left-tailed test, $n = 7, \alpha = 0.01$

8. Left-tailed test, $n = 24, \alpha = 0.05$

9. Two-tailed test, $n = 81, \alpha = 0.10$

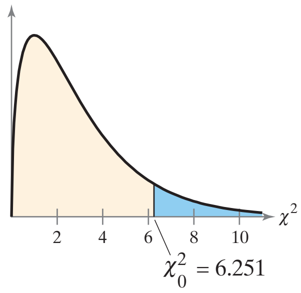
10. Two-tailed test, $n = 61, \alpha = 0.01$

11. Right-tailed test, $n = 30, \alpha = 0.01$

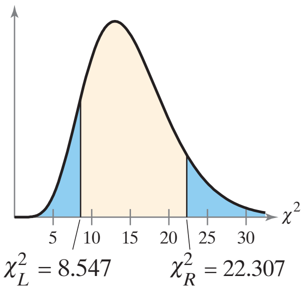
12. Two-tailed test, $n = 31, \alpha = 0.05$

Graphical Analysis *In Exercises 13 and 14, state whether each standardized test statistic χ^2 allows you to reject the null hypothesis. Explain.*

13. (a) $\chi^2 = 2.091$
(b) $\chi^2 = 0$
(c) $\chi^2 = 1.086$
(d) $\chi^2 = 6.3471$



14. (a) $\chi^2 = 22.302$
(b) $\chi^2 = 23.309$
(c) $\chi^2 = 8.457$
(d) $\chi^2 = 8.577$



In Exercises 15–22, test the claim about the population variance σ^2 or standard deviation σ at the level of significance α. Assume the population is normally distributed.

15. Claim: $\sigma^2 = 0.52$; $\alpha = 0.05$. Sample statistics: $s^2 = 0.508, n = 18$

16. Claim: $\sigma^2 \geq 8.5$; $\alpha = 0.05$. Sample statistics: $s^2 = 7.45, n = 23$

17. Claim: $\sigma^2 \leq 17.6$; $\alpha = 0.01$. Sample statistics: $s^2 = 28.33, n = 41$

18. Claim: $\sigma^2 > 19$; $\alpha = 0.1$. Sample statistics: $s^2 = 28, n = 17$

19. Claim: $\sigma^2 \neq 32.8$; $\alpha = 0.1$. Sample statistics: $s^2 = 40.9, n = 101$

20. Claim: $\sigma^2 = 63$; $\alpha = 0.01$. Sample statistics: $s^2 = 58, n = 29$

21. Claim: $\sigma < 40$; $\alpha = 0.01$. Sample statistics: $s = 40.8, n = 12$

22. Claim: $\sigma = 24.9$; $\alpha = 0.10$. Sample statistics: $s = 29.1, n = 51$

1. See Odd Answers, page A71.
2. See Selected Answers, page A101.
3. See Odd Answers, page A71.
4. See Selected Answers, page A101.
5. Critical value: $\chi^2_0 = 38.885$
Rejection region: $\chi^2 > 38.885$
6. Critical value: $\chi^2_0 = 14.684$
Rejection region: $\chi^2 > 14.684$
7. Critical value: $\chi^2_0 = 0.872$
Rejection region: $\chi^2 < 0.872$
8. Critical value: $\chi^2_0 = 13.091$
Rejection region: $\chi^2 < 13.091$
9. Critical values: $\chi^2_L = 60.391$, $\chi^2_R = 101.879$
Rejection regions: $\chi^2 < 60.391$, $\chi^2 > 101.879$
10. Critical values: $\chi^2_L = 35.534$, $\chi^2_R = 91.952$
Rejection regions: $\chi^2 < 35.534$, $\chi^2 > 91.952$
11. Critical value: $\chi^2_0 = 49.588$
Rejection region: $\chi^2 > 49.588$
12. Critical values: $\chi^2_L = 16.791$, $\chi^2_R = 46.979$
Rejection regions: $\chi^2 < 16.791$, $\chi^2 > 46.979$
13. (a) Fail to reject H_0 because $\chi^2 < 6.251$.
(b) Fail to reject H_0 because $\chi^2 < 6.251$.
(c) Fail to reject H_0 because $\chi^2 < 6.251$.
(d) Reject H_0 because $\chi^2 > 6.251$.
14. (a) Fail to reject H_0 because $8.547 < \chi^2 < 22.307$.
(b) Reject H_0 because $\chi^2 > 22.307$.
(c) Reject H_0 because $\chi^2 < 8.547$.
(d) Fail to reject H_0 because $8.547 < \chi^2 < 22.307$.
15. See Odd Answers, page A71.
16. See Selected Answers, page A101.
17. See Odd Answers, page A71.
18. See Selected Answers, page A101.
19. See Odd Answers, page A71.
20. See Selected Answers, page A101.
21. See Odd Answers, page A71.
22. Reject H_0. There is enough evidence at the 10% level of significance to reject the claim.

23. See Odd Answers, page A71.
24. See Selected Answers, page A102.
25. See Odd Answers, page A71.
26. See Selected Answers, page A102.
27. See Odd Answers, page A71.
28. See Selected Answers, page A102.
29. See Odd Answers, page A71.
30. See Selected Answers, page A102.
31. *P*-value = 0.4524
Fail to reject H_0.
32. *P*-value = 0.4014
Fail to reject H_0.
33. *P*-value = 0.0033
Reject H_0.
34. *P*-value = 0.0060
Reject H_0.

Using and Interpreting Concepts

Hypothesis Testing Using Rejection Regions *In Exercises 23–30, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic χ^2, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.*

23. Tires A tire manufacturer claims that the variance of the diameters in a tire model is 8.6. A random sample of 10 tires has a variance of 4.3. At $\alpha = 0.01$, is there enough evidence to reject the claim?

24. Gas Mileage An auto manufacturer claims that the variance of the gas mileages in a model of hybrid vehicle is 0.16. A random sample of 30 vehicles has a variance of 0.26. At $\alpha = 0.05$, is there enough evidence to reject the claim? *(Adapted from Green Hybrid)*

25. Mathematics Assessment Tests A school administrator claims that the standard deviation for grade 12 students on a mathematics assessment test is less than 35 points. A random sample of 28 grade 12 test scores has a standard deviation of 34 points. At $\alpha = 0.10$, is there enough evidence to support the claim? *(Adapted from National Center for Educational Statistics)*

26. Vocabulary Assessment Tests A school administrator claims that the standard deviation for grade 12 students on a vocabulary assessment test is greater than 45 points. A random sample of 25 grade 12 test scores has a standard deviation of 46 points. At $\alpha = 0.01$, is there enough evidence to support the claim? *(Adapted from National Center for Educational Statistics)*

27. Waiting Times A hospital claims that the standard deviation of the waiting times for patients in its emergency department is no more than 0.5 minute. A random sample of 25 waiting times has a standard deviation of 0.7 minute. At $\alpha = 0.10$, is there enough evidence to reject the claim?

28. Hotel Room Rates A travel analyst claims that the standard deviation of the room rates for two adults at three-star hotels in Denver is at least \$68. A random sample of 18 three-star hotels has a standard deviation of \$40. At $\alpha = 0.01$, is there enough evidence to reject the claim? *(Adapted from Expedia)*

Annual salaries		
47,262	67,363	81,246
65,876	59,649	78,268
88,549	52,130	73,955
91,288	54,476	86,787
66,923	48,337	70,172

TABLE FOR EXERCISE 29

29. Salaries The annual salaries (in dollars) of 15 randomly chosen senior level graphic design specialists are shown in the table at the left. At $\alpha = 0.05$, is there enough evidence to support the claim that the standard deviation of the annual salaries is different from \$10,300? *(Adapted from Salary.com)*

Annual salaries		
59,922	99,493	98,221
90,143	65,106	78,975
74,644	107,817	85,492
87,179	90,505	71,090

TABLE FOR EXERCISE 30

30. Salaries The annual salaries (in dollars) of 12 randomly chosen nursing supervisors are shown in the table at the left. At $\alpha = 0.10$, is there enough evidence to reject the claim that the standard deviation of the annual salaries is \$16,500? *(Adapted from Salary.com)*

Extending Concepts

TI-84 PLUS

```
χ²cdf(0,43.2,40)
           .6637768667
```

***P*-Values** *You can calculate the P-value for a chi-square test using technology. After calculating the standardized test statistic, use the cumulative distribution function (CDF) to calculate the area under the curve. From Example 4 on page 397, $\chi^2 = 43.2$. Using a TI-84 Plus (choose 8 from the DISTR menu), enter 0 for the lower bound, 43.2 for the upper bound, and 40 for the degrees of freedom, as shown at the left. Because it is a right-tailed test, the P-value is approximately $1 - 0.6638 = 0.3362$. Because $P > \alpha = 0.05$, fail to reject H_0. In Exercises 31–34, use the P-value method to perform the hypothesis test for the indicated exercise.*

31. Exercise 25 **32.** Exercise 26 **33.** Exercise 27 **34.** Exercise 28

7 A Summary of Hypothesis Testing

With hypothesis testing, perhaps more than any other area of statistics, it can be difficult to see the forest for all the trees. To help you see the forest—the overall picture—a summary of what you studied in this chapter is provided.

Writing the Hypotheses

- You are given a claim about a population parameter μ, p, σ^2, or σ.
- Rewrite the claim and its complement using $\underbrace{\leq, \geq, =}_{H_0}$ and $\underbrace{>, <, \neq}_{H_a}$.
- Identify the claim. Is it H_0 or H_a?

Specifying a Level of Significance

- Specify α, the maximum acceptable probability of rejecting a valid H_0 (a type I error).

Specifying the Sample Size

- Specify your sample size n.

Study Tip

Large sample sizes will usually increase the cost and effort of testing a hypothesis, but they also tend to make your decision more reliable.

Choosing the Test ▲ Normally distributed population ● Any population

- **Mean:** H_0 describes a hypothesized population mean μ.
  - ▲ Use a ***z*-test** when σ is known and the population is normal.
  - ● Use a ***z*-test** for any population when σ is known and $n \geq 30$.
  - ▲ Use a ***t*-test** when σ is not known and the population is normal.
  - ● Use a ***t*-test** for any population when σ is not known and $n \geq 30$.
- **Proportion:** H_0 describes a hypothesized population proportion p.
  - ● Use a ***z*-test** for any population when $np \geq 5$ and $nq \geq 5$.
- **Variance or Standard Deviation:** H_0 describes a hypothesized population variance σ^2 or standard deviation σ.
  - ▲ Use a **chi-square test** when the population is normal.

Sketching the Sampling Distribution

- Use H_a to decide whether the test is left-tailed, right-tailed, or two-tailed.

Finding the Standardized Test Statistic

- Take a random sample of size n from the population.
- Compute the test statistic $\bar{x}$, $\hat{p}$, or s^2.
- Find the standardized test statistic z, t, or χ^2.

Making a Decision

Option 1. Decision based on rejection region

- Use α to find the critical value(s) z_0, t_0, or χ^2_0 and rejection region(s).
- **Decision Rule:**

  Reject H_0 when the standardized test statistic is in the rejection region.
  Fail to reject H_0 when the standardized test statistic is not in the rejection region.

Option 2. Decision based on P-value

- Use the standardized test statistic or technology to find the P-value.
- **Decision Rule:**

  Reject H_0 when $P \leq \alpha$.
  Fail to reject H_0 when $P > \alpha$.

z-Test for a Hypothesized Mean μ (σ Known) *(Section 7.2)*

Test statistic: $\bar{x}$

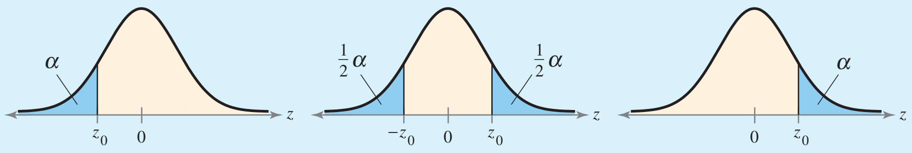
Critical value: z_0 (Use Table 4.)

Sampling distribution of sample means is a normal distribution.

Standardized test statistic: z

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Sample mean: $\bar{x}$; Hypothesized mean: μ; Population standard deviation: σ; Sample size: n



Left-Tailed | Two-Tailed | Right-Tailed

z-Test for a Hypothesized Proportion p *(Section 7.4)*

Test statistic: $\hat{p}$

Critical value: z_0 (Use Table 4.)

Sampling distribution of sample proportions is a normal distribution.

Standardized test statistic: z

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

Sample proportion: $\hat{p}$; Hypothesized proportion: p; $q = 1 - p$; Sample size: n

Study Tip

When your standardized test statistic is z or t, remember that these values measure standard deviations from the mean. Values that are outside of ± 3 indicate that H_0 is very unlikely. Values that are outside of ± 5 indicate that H_0 is almost impossible.

t-Test for a Hypothesized Mean μ (σ Unknown) *(Section 7.3)*

Test statistic: $\bar{x}$

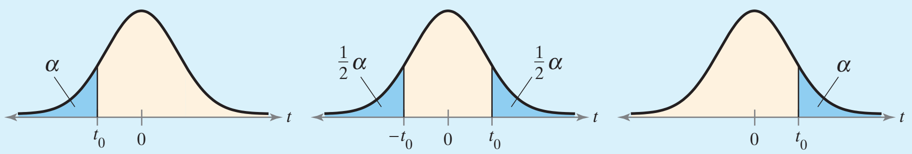
Critical value: t_0 (Use Table 5.)

Sampling distribution of sample means is approximated by a t-distribution with d.f. $= n - 1$.

Standardized test statistic: t

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Sample mean: $\bar{x}$; Hypothesized mean: μ; Sample standard deviation: s; Sample size: n



Left-Tailed | Two-Tailed | Right-Tailed

Chi-Square Test for a Hypothesized Variance σ^2 or Standard Deviation σ *(Section 7.5)*

Test statistic: s^2

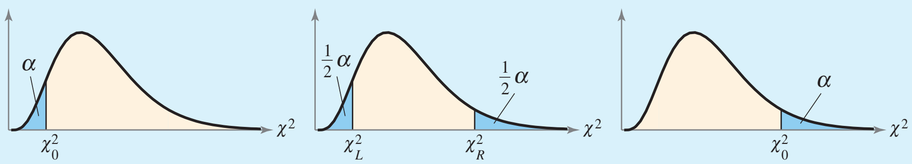
Critical value: χ^2_0 (Use Table 6.)

Sampling distribution is approximated by a chi-square distribution with d.f. $= n - 1$.

Standardized test statistic: χ^2

$$\chi^2 = \frac{(n - 1)s^2}{\sigma^2}$$

Sample size: n; Sample variance: s^2; Hypothesized variance: σ^2



Left-Tailed | Two-Tailed | Right-Tailed

USES AND ABUSES

Statistics in the Real World

Uses

Hypothesis testing is important in many different fields because it gives a scientific procedure for assessing the validity of a claim about a population. Some of the concepts in hypothesis testing are intuitive, but some are not. For instance, the *American Journal of Clinical Nutrition* suggests that eating dark chocolate can help prevent heart disease. A random sample of healthy volunteers were assigned to eat 3.5 ounces of dark chocolate each day for 15 days. After 15 days, the mean systolic blood pressure of the volunteers was 6.4 millimeters of mercury lower. A hypothesis test could show whether this drop in systolic blood pressure is significant or simply due to sampling error.

Careful inferences must be made concerning the results. The study only examined the effects of dark chocolate, so the inference of health benefits cannot be extended to all types of chocolate. You also would not infer that you should eat large quantities of chocolate because the benefits must be weighed against known risks, such as weight gain and acid reflux.

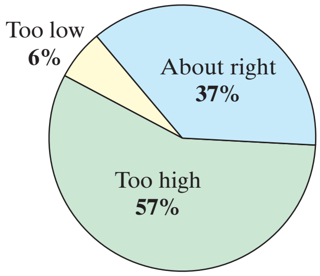
Abuses

Not Using a Random Sample The entire theory of hypothesis testing is based on the fact that the sample is randomly selected. If the sample is not random, then you cannot use it to infer anything about a population parameter.

Attempting to Prove the Null Hypothesis When the P-value for a hypothesis test is greater than the level of significance, you have not proven the null hypothesis is true—only that there is not enough evidence to reject it. For instance, with a P-value higher than the level of significance, a researcher could not prove that there is no benefit to eating dark chocolate—only that there is not enough evidence to support the claim that there is a benefit.

Making Type I or Type II Errors Remember that a type I error is rejecting a null hypothesis that is true and a type II error is failing to reject a null hypothesis that is false. You can decrease the probability of a type I error by lowering the level of significance α. Generally, when you decrease the probability of making a type I error, you increase the probability β of making a type II error. Which error is more serious? It depends on the situation. In a criminal trial, a type I error is considered worse, as explained on page 352. If you are testing a person for a disease and they are assumed to be disease-free (H_0), then a type II error is more serious because you would fail to detect the disease even though the person has it. You can decrease the chance of making both types of errors by increasing the sample size.

Do You Consider the Amount of Federal Income Tax You Pay as Too High, About Right, or Too Low?



EXERCISES

In Exercises 1–3, assume that you work for the Internal Revenue Service. You are asked to write a report about the claim that 57% of U.S. adults think the amount of federal income tax they pay is too high. *(Source: Gallup)*

1. What is the null hypothesis in this situation? Describe how your report could be incorrect by trying to prove the null hypothesis.

2. Describe how your report could make a type I error.

3. Describe how your report could make a type II error.

7 Chapter Summary

What Did You Learn?	Example(s)	Review Exercises
Section 7.1		
▶ How to state a null hypothesis and an alternative hypothesis	1	1–6
▶ How to identify type I and type II errors	2	7–10
▶ How to know whether to use a one-tailed or a two-tailed statistical test and find a *P*-value	3	7–10
▶ How to interpret a decision based on the results of a statistical test	4	7–10
▶ How to write a claim for a hypothesis test	5	7–10
Section 7.2		
▶ How to find and interpret *P*-values	1–3	11, 12
▶ How to use *P*-values for a *z*-test for a mean μ when σ is known	4–6	25, 26
▶ How to find critical values and rejection regions in the standard normal distribution	7, 8	13–16
▶ How to use rejection regions for a *z*-test for a mean μ when σ is known	9, 10	17–24, 27, 28
Section 7.3		
▶ How to find critical values in a *t*-distribution	1–3	29–34
▶ How to use the *t*-test to test a mean μ when σ is not known	4, 5	35–42
▶ How to use technology to find *P*-values and use them with a *t*-test to test a mean μ when σ is not known	6	43, 44
Section 7.4		
▶ How to use the *z*-test to test a population proportion *p*	1, 2	45–50
Section 7.5		
▶ How to find critical values for a chi-square test	1–3	51–54
▶ How to use the chi-square test to test a variance σ^2 or a standard deviation σ	4–6	55–62

7 Review Exercises

Section 7.1

In Exercises 1–6, the statement represents a claim. Write its complement and state which is H_0 and which is H_a.

1. $\mu \le 375$ **2.** $\mu = 82$ **3.** $p < 0.205$

4. $\mu \ne 150{,}020$ **5.** $\sigma > 1.9$ **6.** $p \ge 0.64$

In Exercises 7–10, (a) state the null and alternative hypotheses and identify which represents the claim, (b) describe type I and type II errors for a hypothesis test of the claim, (c) explain whether the hypothesis test is left-tailed, right-tailed, or two-tailed, (d) explain how you should interpret a decision that rejects the null hypothesis, and (e) explain how you should interpret a decision that fails to reject the null hypothesis.

7. A polling organization reports that the proportion of U.S. adults who have volunteered their time or donated money to help clean up the environment is 65%. *(Source: Rasmussen Reports)*

8. An agricultural cooperative guarantees that the mean shelf life of a type of dried fruit is at least 400 days.

9. A nonprofit consumer organization says that the standard deviation of the fuel economies of its top-rated vehicles for a recent year is no more than 9.5 miles per gallon. *(Adapted from Consumer Reports)*

10. An energy bar maker claims that the mean number of grams of carbohydrates in one bar is less than 25.

Section 7.2

In Exercises 11 and 12, find the P-value for the hypothesis test with the standardized test statistic z. Decide whether to reject H_0 for the level of significance α.

11. Left-tailed test, $z = -0.94$, $\alpha = 0.05$

12. Two-tailed test, $z = 2.57$, $\alpha = 0.10$

In Exercises 13–16, find the critical value(s) and rejection region(s) for the type of z-test with level of significance α. Include a graph with your answer.

13. Left-tailed test, $\alpha = 0.02$ **14.** Two-tailed test, $\alpha = 0.005$

15. Right-tailed test, $\alpha = 0.025$ **16.** Two-tailed test, $\alpha = 0.03$

In Exercises 17–20, state whether the standardized test statistic z allows you to reject the null hypothesis. Explain your reasoning.

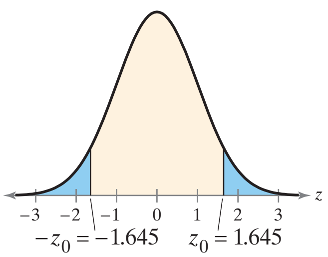


FIGURE FOR EXERCISES 17–20

17. $z = 1.631$ **18.** $z = 1.723$ **19.** $z = -1.464$ **20.** $z = -1.655$

In Exercises 21–24, test the claim about the population mean μ at the level of significance α. Assume the population is normally distributed.

21. Claim: $\mu \le 45$; $\alpha = 0.05$; $\sigma = 6.7$. Sample statistics: $\bar{x} = 47.2$, $n = 22$

22. Claim: $\mu \ne 8.45$; $\alpha = 0.03$; $\sigma = 1.75$. Sample statistics: $\bar{x} = 7.88$, $n = 60$

23. Claim: $\mu < 5.500$; $\alpha = 0.01$; $\sigma = 0.011$. Sample statistics: $\bar{x} = 5.497$, $n = 36$

24. Claim: $\mu = 7450$; $\alpha = 0.10$; $\sigma = 243$. Sample statistics: $\bar{x} = 7495$, $n = 27$

1. H_0: $\mu \le 375$ (claim); H_a: $\mu > 375$
2. H_0: $\mu = 82$ (claim); H_a: $\mu \ne 82$
3. H_0: $p \ge 0.205$
   H_a: $p < 0.205$ (claim)
4. H_0: $\mu = 150{,}020$
   H_a: $\mu \ne 150{,}020$ (claim)
5. H_0: $\sigma \le 1.9$; H_a: $\sigma > 1.9$ (claim)
6. H_0: $p \ge 0.64$ (claim); H_a: $p < 0.64$
7. See Odd Answers, page A72.
8. See Selected Answers, page A102.
9. See Odd Answers, page A72.
10. See Selected Answers, page A102.
11. 0.1736; Fail to reject H_0.
12. 0.0102; Reject H_0.
13. See Odd Answers, page A72.
14. See Selected Answers, page A102.
15. See Odd Answers, page A72.
16. See Selected Answers, page A102.
17. Fail to reject H_0 because $-1.645 < z < 1.645$.
18. Reject H_0 because $z > 1.645$.
19. Fail to reject H_0 because $-1.645 < z < 1.645$.
20. Reject H_0 because $z < -1.645$.
21. Fail to reject H_0. There is not enough evidence at the 5% level of significance to reject the claim.
22. Reject H_0. There is enough evidence at the 3% level of significance to support the claim.
23. Fail to reject H_0. There is not enough evidence at the 1% level of significance to support the claim.
24. See Selected Answers, page A102.

25. (a) The claim is "the mean annual production of cotton is 3.5 million bales per country."

H_0: $\mu = 3.5$ (claim)
H_a: $\mu \neq 3.5$

(b) -2.06 (c) 0.0394

(d) Reject H_0.

(e) There is enough evidence at the 5% level of significance to reject the researcher's claim that the mean annual production of cotton is 3.5 million bales per country.

26. See Selected Answers, page A102.

27. See Odd Answers, page A72.

28. See Selected Answers, page A102.

29. Critical values: $-t_0 = -2.093$, $t_0 = 2.093$
Rejection regions: $t < -2.093$, $t > 2.093$

30. Critical value: $t_0 = 2.449$
Rejection region: $t > 2.449$

31. Critical value: $t_0 = 2.098$
Rejection region: $t > 2.098$

32. Critical value: $t_0 = -1.678$
Rejection region: $t < -1.678$

33. Critical value: $t_0 = -2.977$
Rejection region: $t < -2.977$

34. Critical values: $-t_0 = -2.718$, $t_0 = 2.718$
Rejection regions: $t < -2.718$, $t > 2.718$

35. Reject H_0. There is enough evidence at the 0.5% level of significance to support the claim.

36. Fail to reject H_0. There is not enough evidence at the 10% level of significance to reject the claim.

37. Reject H_0. There is enough evidence at the 1% level of significance to reject the claim.

38. Fail to reject H_0. There is not enough evidence at the 2.5% level of significance to support the claim.

39. Fail to reject H_0. There is not enough evidence at the 10% level of significance to reject the claim.

40. Reject H_0. There is enough evidence at the 5% level of significance to support the claim.

In Exercises 25 and 26, (a) identify the claim and state H_0 and H_a, (b) find the standardized test statistic z, (c) find the corresponding P-value, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.

25. Cotton Production A researcher claims that the mean annual production of cotton is 3.5 million bales per country. A random sample of 44 countries has a mean annual production of 2.1 million bales. Assume the population standard deviation is 4.5 million bales. At $\alpha = 0.05$, can you reject the claim? *(Source: U.S. Department of Agriculture)*

26. Cotton Consumption A researcher claims that the mean annual consumption of cotton is greater than 1.1 million bales per country. A random sample of 67 countries has a mean annual consumption of 1.0 million bales. Assume the population standard deviation is 4.3 million bales. At $\alpha = 0.01$, can you support the claim? *(Source: U.S. Department of Agriculture)*

In Exercises 27 and 28, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic z, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.

27. An environmental researcher claims that the mean amount of sulfur dioxide in the air in U.S. cities is 1.15 parts per billion. In a random sample of 134 U.S. cities, the mean amount of sulfur dioxide in the air is 0.93 parts per billion. Assume the population standard deviation is 2.62 parts per billion. At $\alpha = 0.01$, is there enough evidence to reject the claim? *(Source: U.S. Environmental Protection Agency)*

28. A travel analyst claims that the mean price of a round trip flight from New York City to Los Angeles is less than \$507. In a random sample of 55 round trip flights from New York City to Los Angeles, the mean price is \$502. Assume the population standard deviation is \$111. At $\alpha = 0.05$, is there enough evidence to support the travel analyst's claim? *(Adapted from Expedia)*

Section 7.3

In Exercises 29–34, find the critical value(s) and rejection region(s) for the type of t-test with level of significance α and sample size n.

29. Two-tailed test, $\alpha = 0.05$, $n = 20$

30. Right-tailed test, $\alpha = 0.01$, $n = 33$

31. Right-tailed test, $\alpha = 0.02$, $n = 63$

32. Left-tailed test, $\alpha = 0.05$, $n = 48$

33. Left-tailed test, $\alpha = 0.005$, $n = 15$

34. Two-tailed test, $\alpha = 0.02$, $n = 12$

In Exercises 35–40, test the claim about the population mean μ at the level of significance α. Assume the population is normally distributed.

35. Claim: $\mu > 12{,}700$; $\alpha = 0.005$.
Sample statistics: $\bar{x} = 12{,}855$, $s = 248$, $n = 21$

36. Claim: $\mu \geq 0$; $\alpha = 0.10$. Sample statistics: $\bar{x} = -0.45$, $s = 2.38$, $n = 31$

37. Claim: $\mu \leq 51$; $\alpha = 0.01$. Sample statistics: $\bar{x} = 52$, $s = 2.5$, $n = 40$

38. Claim: $\mu < 850$; $\alpha = 0.025$. Sample statistics: $\bar{x} = 875$, $s = 25$, $n = 14$

39. Claim: $\mu = 195$; $\alpha = 0.10$. Sample statistics: $\bar{x} = 190$, $s = 36$, $n = 101$

40. Claim: $\mu \neq 3{,}330{,}000$; $\alpha = 0.05$.
Sample statistics: $\bar{x} = 3{,}293{,}995$, $s = 12{,}801$, $n = 35$

In Exercises 41 and 42, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic t, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.

41. A fitness magazine advertises that the mean monthly cost of joining a health club is \$25. You want to test this claim. You find that a random sample of 18 clubs has a mean monthly cost of \$26.25 and a standard deviation of \$3.23. At $\alpha = 0.10$, do you have enough evidence to reject the advertisement's claim?

42. A fitness magazine claims that the mean cost of a yoga session is no more than \$14. You want to test this claim. You find that a random sample of 32 yoga sessions has a mean cost of \$15.59 and a standard deviation of \$2.60. At $\alpha = 0.025$, do you have enough evidence to reject the magazine's claim?

In Exercises 43 and 44, (a) identify the claim and state H_0 and H_a, (b) use technology to find the P-value, (c) decide whether to reject or fail to reject the null hypothesis, and (d) interpret the decision in the context of the original claim. Assume the population is normally distributed.

43. An education publication claims that the mean score for grade 12 students on a science achievement test is more than 145. You want to test this claim. You randomly select 36 grade 12 test scores. The results are listed below. At $\alpha = 0.1$, can you support the publication's claim? *(Adapted from National Center for Education Statistics)*

188	80	175	195	201	143	119	81	118	119	165	222
109	134	200	110	199	181	79	135	124	205	90	120
216	167	198	183	173	187	143	166	147	219	206	97

44. An education researcher claims that the overall average score of 15-year-old students on an international mathematics literacy test is 494. You want to test this claim. You randomly select the average scores of 33 countries. The results are listed below. At $\alpha = 0.05$, do you have enough evidence to reject the researcher's claim? *(Source: National Center for Education Statistics)*

561	554	536	531	523	518	515	511	506	500	499
493	490	489	485	482	482	479	477	466	453	448
439	432	423	421	413	407	394	388	386	376	368

Section 7.4

In Exercises 45–48, determine whether a normal sampling distribution can be used to approximate the binomial distribution. If it can, test the claim.

45. Claim: $p = 0.15$; $\alpha = 0.05$
Sample statistics: $\hat{p} = 0.09$, $n = 40$

46. Claim: $p = 0.65$; $\alpha = 0.03$
Sample statistics: $\hat{p} = 0.76$, $n = 116$

47. Claim: $p < 0.70$; $\alpha = 0.01$
Sample statistics: $\hat{p} = 0.50$, $n = 68$

48. Claim: $p \geq 0.04$; $\alpha = 0.10$
Sample statistics: $\hat{p} = 0.03$, $n = 30$

41. (a) The claim is "the mean monthly cost of joining a health club is \$25."
H_0: $\mu = 25$ (claim)
H_a: $\mu \neq 25$
(b) $-t_0 = -1.740$, $t_0 = 1.740$
Rejection regions: $t < -1.740$, $t > 1.740$
(c) 1.64 (d) Fail to reject H_0.
(e) There is not enough evidence at the 10% level of significance to reject the advertisement's claim that the mean monthly cost of joining a health club is \$25.

42. (a) The claim is "the mean cost of a yoga session is no more than \$14."
H_0: $\mu \leq 14$ (claim)
H_a: $\mu > 14$
(b) $t_0 = 2.040$
Rejection region: $t > 2.040$
(c) 3.46 (d) Reject H_0.
(e) There is enough evidence at the 2.5% level of significance to reject the fitness magazine's claim that the mean cost of a yoga session is no more than \$14.

43. (a) The claim is "the mean score for grade 12 students on a science achievement test is more than 145."
H_0: $\mu \leq 145$
H_a: $\mu > 145$ (claim)
(b) 0.0824 (c) Reject H_0.
(d) There is enough evidence at the 10% level of significance to support the education publication's claim that the mean score for grade 12 students on a science achievement test is more than 145.

44. See Selected Answers, page A102.

45. See Odd Answers, page A72.

46. Can use normal distribution. Reject H_0. There is enough evidence at the 3% level of significance to reject the claim.

47. Can use normal distribution. Reject H_0. There is enough evidence at the 1% level of significance to support the claim.

48. Cannot use normal distribution.

In Exercises 49 and 50, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic z, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim.

49. A polling agency reports that over 40% of U.S. adults say they are less likely to travel to Europe in the next six months for fear of terrorist attacks. In a random sample of 1000 U.S. adults, 42% said they are less likely to travel to Europe in the next six months for fear of terrorist attacks. At $\alpha = 0.01$, is there enough evidence to support the agency's claim? *(Adapted from Rasmussen Reports)*

50. A labor researcher claims that 6% of U.S. employees say it is likely they will be laid off in the next year. In a random sample of 547 U.S. employees, 44 said it is likely they will be laid off in the next year. At $\alpha = 0.05$, is there enough evidence to reject the researcher's claim? *(Adapted from Gallup)*

Section 7.5

In Exercises 51–54, find the critical value(s) and rejection region(s) for the type of chi-square test with sample size n and level of significance α.

51. Right-tailed test, $n = 20$, $\alpha = 0.05$

52. Two-tailed test, $n = 14$, $\alpha = 0.01$

53. Two-tailed test, $n = 41$, $\alpha = 0.10$

54. Left-tailed test, $n = 6$, $\alpha = 0.05$

In Exercises 55–58, test the claim about the population variance σ^2 or standard deviation σ at the level of significance α. Assume the population is normally distributed.

55. Claim: $\sigma^2 > 2$; $\alpha = 0.10$. Sample statistics: $s^2 = 2.95$, $n = 18$

56. Claim: $\sigma^2 \le 60$; $\alpha = 0.025$. Sample statistics: $s^2 = 72.7$, $n = 15$

57. Claim: $\sigma = 1.25$; $\alpha = 0.05$. Sample statistics: $s = 1.03$, $n = 6$

58. Claim: $\sigma \ne 0.035$; $\alpha = 0.01$. Sample statistics: $s = 0.026$, $n = 16$

In Exercises 59 and 60, (a) identify the claim and state H_0 and H_a, (b) find the critical value(s) and identify the rejection region(s), (c) find the standardized test statistic χ^2, (d) decide whether to reject or fail to reject the null hypothesis, and (e) interpret the decision in the context of the original claim. Assume the population is normally distributed.

59. A bolt manufacturer makes a type of bolt to be used in airtight containers. The manufacturer claims that the variance of the bolt widths is at most 0.01. A random sample of 28 bolts has a variance of 0.064. At $\alpha = 0.005$, is there enough evidence to reject the claim?

60. A restaurant claims that the standard deviation of the lengths of serving times is 3 minutes. A random sample of 27 serving times has a standard deviation of 3.9 minutes. At $\alpha = 0.01$, is there enough evidence to reject the claim?

61. In Exercise 59, is there enough evidence to reject the claim at the $\alpha = 0.01$ level? Explain.

62. In Exercise 60, is there enough evidence to reject the claim at the $\alpha = 0.05$ level? Explain.

49. (a) The claim is "over 40% of U.S. adults say they are less likely to travel to Europe in the next six months for fear of terrorist attacks."

H_0: $p \le 0.40$
H_a: $p > 0.40$ (claim)

(b) $z_0 = 2.33$
Rejection region: $z > 2.33$

(c) 1.29 (d) Fail to reject H_0.

(e) There is not enough evidence at the 1% level of significance to support the polling agency's claim that over 40% of U.S. adults say they are less likely to travel to Europe in the next six months for fear of terrorist attacks.

50. See Selected Answers, page A103.

51. Critical value: $\chi^2_0 = 30.144$
Rejection region: $\chi^2 > 30.144$

52. Critical values: $\chi^2_L = 3.565$, $\chi^2_R = 29.819$
Rejection regions: $\chi^2 < 3.565$, $\chi^2 > 29.819$

53. Critical values: $\chi^2_L = 26.509$, $\chi^2_R = 55.758$
Rejection regions: $\chi^2 < 26.509$, $\chi^2 > 55.758$

54. Critical value: $\chi^2_0 = 1.145$
Rejection region: $\chi^2 < 1.145$

55. Reject H_0. There is enough evidence at the 10% level of significance to support the claim.

56. Fail to reject H_0. There is not enough evidence at the 2.5% level of significance to reject the claim.

57. Fail to reject H_0. There is not enough evidence at the 5% level of significance to reject the claim.

58. Fail to reject H_0. There is not enough evidence at the 1% level of significance to support the claim.

59. See Odd Answers, page A73.

60. See Selected Answers, page A103.

61. You can reject H_0 at the 1% level of significance because $\chi^2 = 172.8 > 46.963$.

62. You can reject H_0 at the 5% level of significance because $\chi^2 = 43.94 > 41.923$.

7 Chapter Quiz

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

For each exercise, perform the steps below.

(a) Identify the claim and state H_0 and H_a.

(b) Determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed, and whether to use a z-test, a t-test, or a chi-square test. Explain your reasoning.

(c) Choose one of the options.

Option 1: Find the critical value(s), identify the rejection region(s), and find the appropriate standardized test statistic.

Option 2: Find the appropriate standardized test statistic and the P-value.

(d) Decide whether to reject or fail to reject the null hypothesis.

(e) Interpret the decision in the context of the original claim.

1. A hat company claims that the mean hat size for a male is at least 7.25. A random sample of 12 hat sizes has a mean of 7.15. At $\alpha = 0.01$, can you reject the company's claim? Assume the population is normally distributed and the population standard deviation is 0.27.

2. A travel analyst claims the mean daily base price for renting a full-size or less expensive vehicle in Vancouver, Washington, is more than \$36. You want to test this claim. In a random sample of 40 full-size or less expensive vehicles available to rent in Vancouver, Washington, the mean daily base price is \$42. Assume the population standard deviation is \$19. At $\alpha = 0.10$, do you have enough evidence to support the analyst's claim? *(Adapted from Expedia)*

3. A government agency reports that the mean amount of earnings for full-time workers ages 18 to 24 with a bachelor's degree in a recent year is \$47,254. In a random sample of 15 full-time workers ages 18 to 24 with a bachelor's degree, the mean amount of earnings is \$50,781 and the standard deviation is \$5290. At $\alpha = 0.05$, is there enough evidence to support the claim? Assume the population is normally distributed. *(Adapted from U.S. Census Bureau)*

4. A weight loss program claims that program participants have a mean weight loss of at least 10.5 pounds after 1 month. The weight losses after 1 month (in pounds) of a random sample of 40 program participants are listed below. At $\alpha = 0.01$, is there enough evidence to reject the program's claim?

4.7	6.0	7.2	8.3	9.2	10.1	14.0	11.7	12.8	10.8
11.0	7.2	8.0	4.7	11.8	10.7	6.1	8.8	7.7	8.5
9.5	10.2	5.6	6.9	7.9	8.6	10.5	9.6	5.7	9.6
12.6	12.9	6.8	12.0	5.1	14.0	9.7	10.8	9.1	12.9

5. A nonprofit consumer organization says that less than 18% of the vehicles the organization rated in a recent year have an overall score of 78 or more. In a random sample of 90 vehicles the organization rated in a recent year, 20% have an overall score of 78 or more. At $\alpha = 0.05$, can you support the organization's claim? *(Adapted from Consumer Reports)*

6. In Exercise 5, the nonprofit consumer organization says that the standard deviation of the vehicle rating scores is 11.90. A random sample of 90 vehicle rating scores has a standard deviation of 11.96. At $\alpha = 0.10$, is there enough evidence to reject the organization's claim? Assume the population is normally distributed. *(Adapted from Consumer Reports)*

1. (a) The claim is "the mean hat size for a male is at least 7.25."

H_0: $\mu \geq 7.25$ (claim)
H_a: $\mu < 7.25$

(b) Left-tailed because the alternative hypothesis contains $<$; z-test because σ is known and the population is normally distributed.

(c) *Sample answer:* $z_0 = -2.33$; Rejection region: $z < -2.33$; -1.28

(d) Fail to reject H_0.

(e) There is not enough evidence at the 1% level of significance to reject the company's claim that the mean hat size for a male is at least 7.25.

2. (a) The claim is "the mean daily base price for renting a full-size or less expensive vehicle in Vancouver, Washington, is more than \$36."

H_0: $\mu \leq 36$
H_a: $\mu > 36$ (claim)

(b) Right-tailed because the alternative hypothesis contains $>$; z-test because σ is known and $n \geq 30$.

(c) *Sample answer:* $z_0 = 1.28$; Rejection region: $z > 1.28$; 1.997

(d) Reject H_0.

(e) There is enough evidence at the 10% level of significance to support the travel analyst's claim that the mean daily base price for renting a full-size or less expensive vehicle in Vancouver, Washington, is more than \$36.

3. See Odd Answers, page A73.

4. See Odd Answers, page A73.

5. See Odd Answers, page A73.

6. See Odd Answers, page A73.

7 Chapter Test

1. (a) The claim is "more than 30% of adults have purchased a meal kit in a recent year."

H_0: $p \leq 0.30$
H_a: $p > 0.30$ (claim)

(b) Right-tailed because the alternative hypothesis contains $>$; z-test because $np \geq 5$ and $nq \geq 5$.

(c) *Sample answer:* $z_0 = 1.28$; Rejection region: $z > 1.28$; -0.65

(d) Fail to reject H_0.

(e) There is not enough evidence at the 10% level of significance to support the retail grocery chain's claim that more than 30% of adults have purchased a meal kit in a recent year.

2. (a) The claim is "the mean of the room rates for two adults at three-star hotels in Salt Lake City is \$134."

H_0: $\mu = 134$ (claim)
H_a: $\mu \neq 134$

(b) Two-tailed because the alternative hypothesis contains $\neq$; z-test because σ is known and $n \geq 30$.

(c) *Sample answer:* $-z_0 = -1.645$, $z_0 = 1.645$; Rejection regions: $z < -1.645$, $z > 1.645$; 1.82

(d) Reject H_0.

(e) There is enough evidence at the 10% level of significance to reject the travel analyst's claim that the mean of the room rates for two adults at three-star hotels in Salt Lake City is \$134.

3. See Selected Answers, page A103.

4. See Selected Answers, page A103.

5. See Selected Answers, page A103.

6. See Selected Answers, page A103.

7. See Selected Answers, page A103.

Take this test as you would take a test in class.

For each exercise, perform the steps below.

(a) *Identify the claim and state H_0 and H_a.*

(b) *Determine whether the hypothesis test is left-tailed, right-tailed, or two-tailed, and whether to use a z-test, a t-test, or a chi-square test. Explain your reasoning.*

(c) *Choose one of the options.*

Option 1: Find the critical value(s), identify the rejection region(s), and find the appropriate standardized test statistic.

Option 2: Find the appropriate standardized test statistic and the P-value.

(d) *Decide whether to reject or fail to reject the null hypothesis.*

(e) *Interpret the decision in the context of the original claim.*

1. A retail grocery chain owner claims that more than 30% of adults have purchased a meal kit in a recent year. In a random sample of 36 adults, 25% have purchased a meal kit in a recent year. At $\alpha = 0.10$, is there enough evidence to support the owner's claim? *(Adapted from Harris Interactive)*

2. A travel analyst claims that the mean of the room rates for two adults at three-star hotels in Salt Lake City is \$134. In a random sample of 37 three-star hotels in Salt Lake City, the mean room rate for two adults is \$143. Assume the population standard deviation is \$30. At $\alpha = 0.10$, is there enough evidence to reject the analyst's claim? *(Adapted from Expedia)*

3. A travel analyst says that the mean price of a meal for a family of 4 in a resort restaurant is at most \$100. A random sample of 33 meal prices for families of 4 has a mean of \$110 and a standard deviation of \$19. At $\alpha = 0.01$, is there enough evidence to reject the analyst's claim?

4. A research center claims that more than 80% of U.S. adults think that mothers should have paid maternity leave. In a random sample of 50 U.S. adults, 82% think that mothers should have paid maternity leave. At $\alpha = 0.05$, is there enough evidence to support the center's claim? *(Adapted from Pew Research Center)*

5. A nutrition bar manufacturer claims that the standard deviation of the number of grams of carbohydrates in a bar is 1.11 grams. A random sample of 26 bars has a standard deviation of 1.19 grams. At $\alpha = 0.05$, is there enough evidence to reject the manufacturer's claim? Assume the population is normally distributed.

6. A nonprofit consumer organization says that the mean price of the vehicles the organization rated in a recent year is at least \$41,000. In a random sample of 150 vehicles the organization rated in a recent year, the mean price is \$40,600 and the standard deviation is \$17,300. At $\alpha = 0.01$, is there enough evidence to reject the organization's claim? *(Adapted from Consumer Reports)*

7. A researcher claims that the mean age of the residents of a small town is more than 38 years. The ages (in years) of a random sample of 30 residents are listed below. At $\alpha = 0.10$, is there enough evidence to support the researcher's claim? Assume the population standard deviation is 9 years.

41	44	40	30	29	46	42	53	21	29	43	46	39	35	33
42	35	43	35	24	21	29	24	25	85	56	82	87	72	31

REAL STATISTICS REAL DECISIONS

Putting it all together

The charts show results of studies on four-year colleges in the United States. You want to portray your college in a positive light for an advertising campaign designed to attract high school students. You decide to use hypothesis tests to show that your college is better than the average in certain aspects.

EXERCISES

1. *What Would You Test?*

What claims could you test if you wanted to convince a student to come to your college? Suppose the student you are trying to convince is mainly concerned with (a) affordability, (b) having a good experience, and (c) graduating and starting a career. List one claim for each case. State the null and alternative hypotheses for each claim.

2. *Choosing a Random Sample*

Classmates suggest conducting the following sampling techniques to test various claims. Determine whether the sample will be random. If not, suggest an alternative.

(a) Survey all the students you have class with and ask about the average time they spend daily on different activities.

(b) Randomly select former students from a list of recent graduates and ask whether they are employed.

(c) Randomly select students from a directory, ask how much debt money they borrowed to pay for college this year, and multiply by four.

3. *Supporting a Claim*

You want your test to support a positive claim about your college, not just fail to reject one. Should you state your claim so that the null hypothesis contains the claim or the alternate hypothesis contains the claim? Explain.

4. *Testing a Claim*

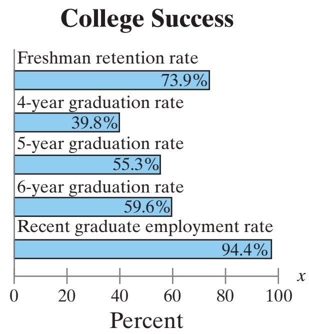
You want to claim that students at your college graduate with an average debt of less than \$25,000. A random sample of 40 recent graduates has a mean amount borrowed of \$23,475 and a standard deviation of \$8000. At $\alpha = 0.05$, is there enough evidence to support your claim?

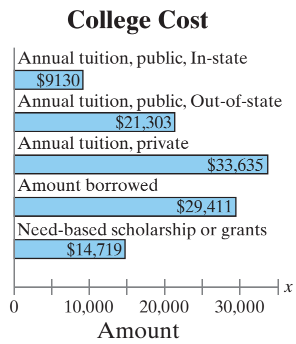
5. *Testing a Claim*

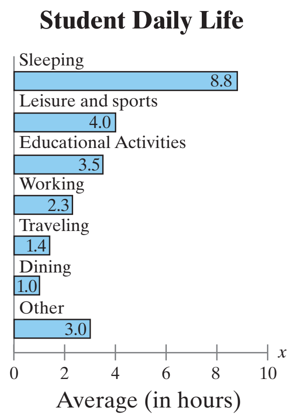
You want to claim that your college has a freshmen retention rate of at least 80%. You take a random sample of 60 of last year's freshmen and find that 54 of them still attend your college. At $\alpha = 0.05$, is there enough evidence to reject your claim?

6. *Conclusion*

Test one of the claims you listed in Exercise 1 and interpret the results. Discuss any limits of your sampling process.







TECHNOLOGY

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The Case of the Vanishing Women

53% → 29% → 9% → 0%

From 1966 to 1968, Dr. Benjamin Spock and others were tried for conspiracy to violate the Selective Service Act by encouraging resistance to the Vietnam War. By a series of three selections, no women ended up being on the jury. In 1969, Hans Zeisel wrote an article in *The University of Chicago Law Review* using statistics and hypothesis testing to argue that the jury selection was biased against Dr. Spock. Dr. Spock was a well-known pediatrician and author of books about raising children. Millions of mothers had read his books and followed his advice. Zeisel argued that, by keeping women off the jury, the court prejudiced the verdict.

The jury selection process for Dr. Spock's trial is shown at the right.

Stage 1. The clerk of the Federal District Court selected 350 people "at random" from the Boston City Directory. The directory contained several hundred names, 53% of whom were women. However, only 102 of the 350 people selected were women.

Stage 2. The trial judge, Judge Ford, selected 100 people "at random" from the 350 people. This group was called a venire and it contained only nine women.

Stage 3. The court clerk assigned numbers to the members of the venire and, one by one, they were interrogated by the attorneys for the prosecution and defense until 12 members of the jury were chosen. At this stage, only one potential female juror was questioned, and she was eliminated by the prosecutor under his quota of peremptory challenges (for which he did not have to give a reason).

EXERCISES

1. The Minitab display below shows a hypothesis test for a claim that the proportion of women in the city directory is $p = 0.53$. In the test, $n = 350$ and $\hat{p} \approx 0.291$. Should you reject the claim? What is the level of significance? Explain.

2. In Exercise 1, you rejected the claim that $p = 0.53$. But this claim was true. What type of error is this?

3. When you reject a true claim with a level of significance that is virtually zero, what can you infer about the randomness of your sampling process?

4. Describe a hypothesis test for Judge Ford's "random" selection of the venire. Use a claim of

$$p = \frac{102}{350} \approx 0.291.$$

(a) Write the null and alternative hypotheses.

(b) Use technology to perform the test.

(c) Make a decision.

(d) Interpret the decision in the context of the original claim. Could Judge Ford's selection of 100 venire members have been random?

MINITAB

Test and CI for One Proportion

Test of p = 0.53 vs p ≠ 0.53

Sample	X	N	Sample p	99 % CI	Z-Value	P-Value
1	102	350	0.291429	(0.228862, 0.353995)	−8.94	0.000

Using the normal approximation.

Extended solutions are given in the technology manuals that accompany this text. Technical instruction is provided for Minitab, Excel, and the TI-84 Plus.

7 Using Technology to Perform Hypothesis Tests

Here are some Minitab and TI-84 Plus printouts for some of the examples in this chapter.

See Example 5, page 367.

Display Descriptive Statistics...
Store Descriptive Statistics...
Graphical Summary...

1-Sample Z...
1-Sample t...
2-Sample t...
Paired t...

1 Proportion...
2 Proportions...

MINITAB

One-Sample Z

Test of μ = 68.3 vs ≠ 68.3
The assumed standard deviation = 3.5

N	Mean	SE Mean	95% CI	Z	P
25	67.200	0.700	(65.828, 68.572)	−1.57	0.116

See Example 4, page 380.

Display Descriptive Statistics...
Store Descriptive Statistics...
Graphical Summary...

1-Sample Z...
1-Sample t...
2-Sample t...
Paired t...

1 Proportion...
2 Proportions...

MINITAB

One-Sample T

Test of μ = 21000 vs < 21000

N	Mean	StDev	SE Mean	95% Upper Bound	T	P
14	19189	2950	788	20585	−2.30	0.019

See Example 2, page 390.

Display Descriptive Statistics...
Store Descriptive Statistics...
Graphical Summary...

1-Sample Z...
1-Sample t...
2-Sample t...
Paired t...

1 Proportion...
2 Proportions...

MINITAB

Test and CI for One Proportion

Test of p = 0.51 vs p ≠ 0.51

Sample	X	N	Sample p	90% CI	Z-Value	P-Value
1	1161	2202	0.527248	(0.509748, 0.544748)	1.62	0.105

Using the normal approximation.

See Example 9, page 371.

TI-84 PLUS

EDIT CALC TESTS
1: Z–Test…
2: T–Test…
3: 2–SampZTest…
4: 2–SampTTest…
5: 1–PropZTest…
6: 2–PropZTest…
7↓ ZInterval…

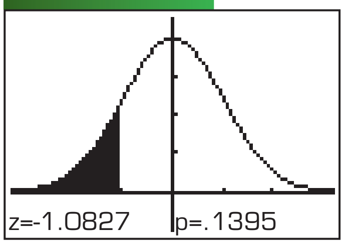
TI-84 PLUS

Z-Test
Inpt:Data Stats
μ_0:88200
σ:9500
$\bar{x}$:85900
n:20
μ:$\neq\mu_0$ $<\mu_0$ $>\mu_0$
Calculate Draw

TI-84 PLUS

Z-Test
$\mu<88200$
z=–1.082727652
p=.1394646984
$\bar{x}$=85900
n=20

TI-84 PLUS



See Example 5, page 381.

TI-84 PLUS

EDIT CALC TESTS
1: Z–Test…
2: T–Test…
3: 2–SampZTest…
4: 2–SampTTest…
5: 1–PropZTest…
6: 2–PropZTest…
7↓ ZInterval…

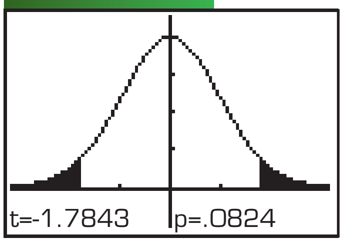
TI-84 PLUS

T-Test
Inpt:Data Stats
μ_0:6.8
$\bar{x}$:6.7
Sx:.35
n:39
μ:$\neq\mu_0$ $<\mu_0$ $>\mu_0$
Calculate Draw

TI-84 PLUS

T-Test
$\mu\neq6.8$
t=–1.784285142
p=.0823638462
$\bar{x}$=6.7
Sx=.35
n=39

TI-84 PLUS



See Example 1, page 389.

TI-84 PLUS

EDIT CALC TESTS
1: Z–Test…
2: T–Test…
3: 2–SampZTest…
4: 2–SampTTest…
5: 1–PropZTest…
6: 2–PropZTest…
7↓ ZInterval…

TI-84 PLUS

1-PropZTest
p_0:.45
x:41
n:100
prop$\neq p_0$ $<p_0$ $>p_0$
Calculate Draw

TI-84 PLUS

1-PropZTest
prop<.45
z=–.8040302522
p=.2106896879
$\hat{p}$=.41
n=100

TI-84 PLUS