CHAPTER 2

Trigonometry

What You'll Learn

- Determine the measure of an acute angle in a right triangle using the lengths of two sides.
- Determine the length of a side in a right triangle using the length of another side and the measure of an acute angle.
- Solve problems that involve more than one right triangle.

Why It's Important

Trigonometric ratios are used by:

- surveyors, to determine the distance across a river or a very busy street
- pilots, to determine flight paths and measure crosswinds
- forestry technicians, to calculate the heights of trees

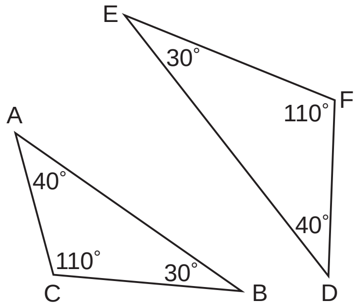
Key Words

tangent ratio
angle of inclination
indirect measurement
sine ratio
cosine ratio
angle of elevation
angle of depression

2.1 Skill Builder

Similar Triangles

Similar triangles have:
- the measures of matching angles equal OR
- the ratios of matching sides equal

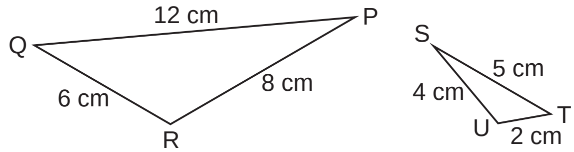


These triangles are similar because matching angles are equal.

$\angle A = \angle D = 40°$

$\angle B = \angle E = 30°$

$\angle C = \angle F = 110°$



These triangles are not similar because the ratios of matching sides are different.

$\frac{PQ}{ST} = \frac{12}{5} = 2.4$

$\frac{QR}{TU} = \frac{6}{2} = 3$

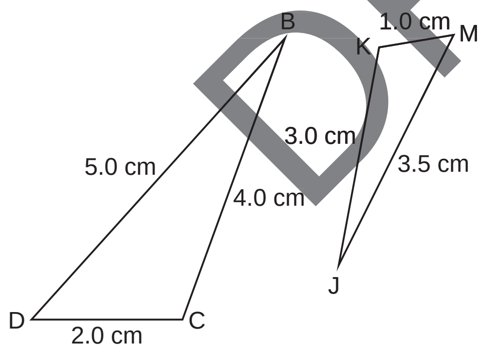
$\frac{RP}{US} = \frac{8}{4} = 2$

Compare the longest sides, compare the shortest sides, then compare the third pair of sides.

Check

1. Which triangles in each pair are similar?

a)



Compare the ratios of matching sides.

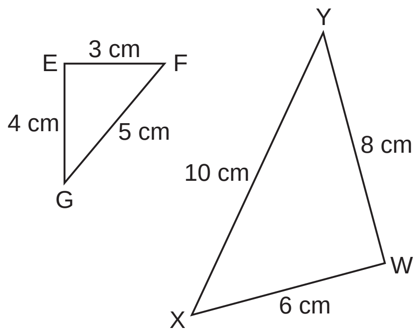
$\frac{DB}{\textbf{MJ}} = \frac{\textbf{5.0}}{\textbf{3.5}} \doteq \textbf{1.4}$

$\frac{CD}{\textbf{KM}} = \frac{\textbf{2.0}}{\textbf{1.0}} = \textbf{2}$

$\frac{BC}{\textbf{JK}} = \frac{\textbf{4.0}}{\textbf{3.0}} = \textbf{1.}\overline{\textbf{3}}$

The triangles **are not** similar.

b)



Compare the ratios of matching sides.

$\frac{\textbf{FG}}{\textbf{XY}} = \frac{\textbf{5}}{\textbf{10}} = \textbf{0.5}$

$\frac{\textbf{EF}}{\textbf{WX}} = \frac{\textbf{3}}{\textbf{6}} = \textbf{0.5}$

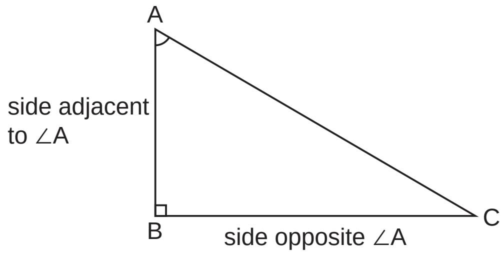
$\frac{\textbf{GE}}{\textbf{YW}} = \frac{\textbf{4}}{\textbf{8}} = \textbf{0.5}$

The triangles **are** similar.

2.1 The Tangent Ratio

FOCUS **Use the tangent ratio to find an angle measure.**

The Tangent Ratio

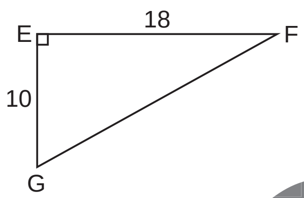


*An **acute angle** is less than 90°.*

If ∠A is an acute angle in a right triangle, then $\tan A = \dfrac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$

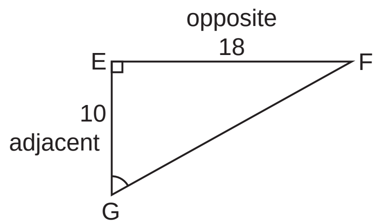
Example 1 Finding the Tangent Ratio

Find the tangent ratio for ∠G.



Solution

Draw an arc at ∠G.
The side opposite ∠G is EF.
The side adjacent to ∠G is GE.



$\tan G = \dfrac{\text{length of side opposite } \angle G}{\text{length of side adjacent to } \angle G}$

$\tan G = \dfrac{EF}{GE}$ Substitute: EF = 18 and GE = 10

$\tan G = \dfrac{18}{10}$

$\tan G = 1.8$

The side opposite the right angle is always the hypotenuse.

Check

1. a) Find tan P.

The side opposite ∠P is **QR**.

The side adjacent to ∠P is **RP**.

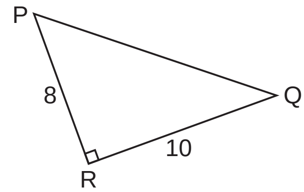
$$\tan P = \frac{\text{length of side } \textbf{opposite } \angle P}{\text{length of side } \textbf{adjacent to } \angle P}$$

$$\tan P = \frac{\textbf{QR}}{\textbf{RP}}$$

$$\tan P = \frac{\textbf{10}}{\textbf{8}}$$

tan P = **1.25**

b) Find tan Q.



The side opposite ∠Q is **RP**.

The side adjacent to ∠Q is **QR**.

$$\tan Q = \frac{\textbf{length of side opposite } \angle Q}{\textbf{length of side adjacent to } \angle Q}$$

$$\tan Q = \frac{\textbf{RP}}{\textbf{QR}}$$

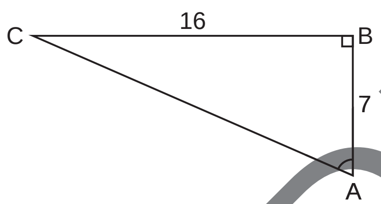
$$\tan Q = \frac{\textbf{8}}{\textbf{10}}$$

tan Q = **0.8**

To find the measure of an angle, use the $\tan^{-1}$ key on a scientific calculator.

Example 2 Using the Tangent Ratio to Find the Measure of an Angle

Find the measure of ∠A to the nearest degree.



Solution

The side opposite ∠A is BC.

The side adjacent to ∠A is AB.

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

$\tan A = \frac{BC}{AB}$ Substitute: BC = 16 and AB = 7

$\tan A = \frac{16}{7}$

If you are using a different calculator, consult the user's manual.

To find ∠A using a TI-30XIIS calculator, enter:

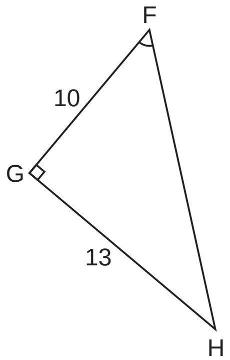
[2nd] [TAN] [1] [6] [÷] [7] [)] [ENTER]

∠A ≐ 66°

Check

1. Find the measure of each indicated angle to the nearest degree.

a) ∠F



The side opposite ∠F is **GH**.

The side adjacent to ∠F is **FG**.

$$\tan F = \frac{\text{length of side } \textbf{opposite } \angle F}{\text{length of side } \textbf{adjacent to } \angle F}$$

$$\tan F = \frac{\mathbf{GH}}{\mathbf{FG}}$$

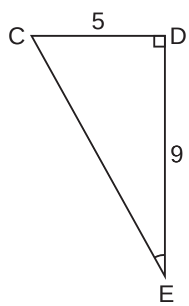
$$\tan F = \frac{\mathbf{13}}{\mathbf{10}}$$

$\tan F =$ **1.3** Use a calculator.

$\angle F = \tan^{-1}$ **(1.3)**

$\angle F \doteq$ **52°**

b) ∠E



The side opposite ∠E is **CD**.

The side adjacent to ∠E is **DE**.

$$\tan E = \frac{\textbf{length of side opposite } \angle E}{\textbf{length of side adjacent to } \angle E}$$

$$\tan E = \frac{\mathbf{CD}}{\mathbf{DE}}$$

$$\tan E = \frac{\mathbf{5}}{\mathbf{9}}$$

$$\angle E = \tan^{-1}\left(\frac{5}{9}\right)$$

$\angle E \doteq$ **29°**

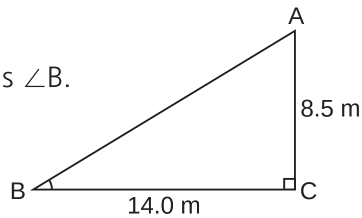
Example 3 Using the Tangent Ratio to Find an Angle of Inclination

A guy wire is fastened to a cell-phone tower 8.5 m above the ground. The wire is anchored to the ground 14.0 m from the base of the tower. What angle, to the nearest degree, does the wire make with the ground?

*The angle the wire makes with the ground is called the **angle of inclination.***

Solution

Draw a diagram.
The angle the wire makes with the ground is ∠B.
To find ∠B, use the tangent ratio.



Assume the tower is perpendicular to the ground.

$\tan B = \dfrac{\text{length of side opposite } \angle B}{\text{length of side adjacent to } \angle B}$ The side opposite ∠B is CA. The side adjacent to ∠B is BC.

$\tan B = \dfrac{CA}{BC}$ Substitute: CA = 8.5 and BC = 14.0

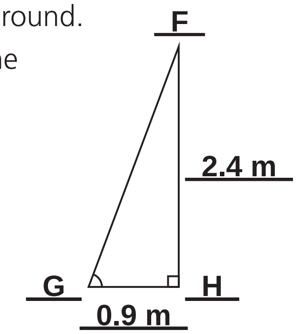
$\tan B = \dfrac{8.5}{14.0}$ Use a calculator.

$\angle B \doteq 31°$

The angle between the ground and the wire is about 31°.

Check

1. A ladder leans against a house. The top of the ladder is 2.4 m above the ground. Its base is 0.9 m from the wall. What angle, to the nearest degree, does the ladder make with the ground?



Label the given triangle FGH.
Label G where the ladder meets the ground.
Label F where it meets the wall.
We want to find the measure of ∠G.

The side opposite ∠G is **HF**.

The side adjacent to ∠G is **GH**.

$\tan G = \dfrac{\mathbf{HF}}{\mathbf{GH}}$

$\tan G = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan G = \dfrac{\mathbf{2.4}}{\mathbf{0.9}}$

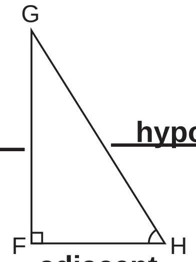
$\angle G \doteq$ **69°**

The angle between the ground and the ladder is about **69°**.

Practice

1. Label the hypotenuse, opposite, and adjacent sides of each right triangle in relation to the given angle.

a) ∠H

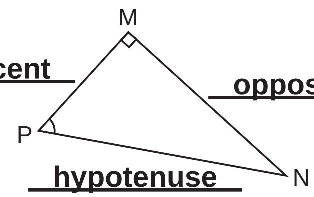


opposite (GF)

hypotenuse (GH)

adjacent (FH)

b) ∠P



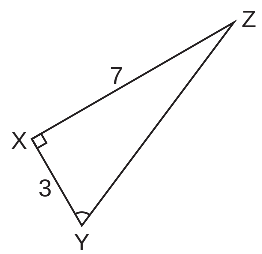
adjacent (PM)

opposite (MN)

hypotenuse (PN)

2. Find the tangent ratio for each indicated angle. Leave the ratio in fraction form.

a)



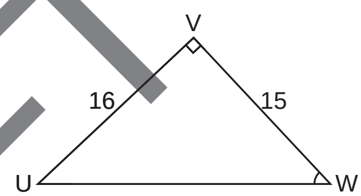
The side opposite ∠Y is **ZX**.
The side adjacent to ∠Y is **XY**.

tan Y = $\dfrac{\text{length of side } \textbf{opposite } \angle Y}{\text{length of side } \textbf{adjacent to } \angle Y}$

tan Y = $\dfrac{\mathbf{ZX}}{\mathbf{XY}}$

tan Y = $\dfrac{\mathbf{7}}{\mathbf{3}}$

b)



The side opposite ∠W is **UV**.
The side adjacent to ∠W is **VW**.

tan W = $\dfrac{\textbf{length of side opposite } \angle W}{\textbf{length of side adjacent to } \angle W}$

tan W = $\dfrac{\mathbf{UV}}{\mathbf{VW}}$

tan W = $\dfrac{\mathbf{16}}{\mathbf{15}}$

3. Find the measure of ∠A for each value of tan A. Give your answer to the nearest degree.

a) tan A = 0.5

∠A = $\tan^{-1}$(**0.5**) Use a calculator.

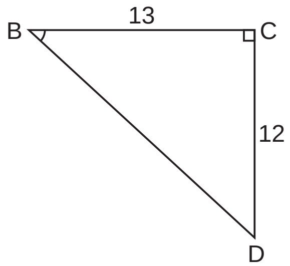
∠A ≐ **27°**

b) tan A = $\dfrac{5}{6}$

∠A = $\mathbf{\tan^{-1}\left(\dfrac{5}{6}\right)}$

∠A ≐ **40°**

4. Find the measure of ∠B to the nearest degree.



The side opposite ∠B is **CD**.
The side adjacent to ∠B is **BC**.

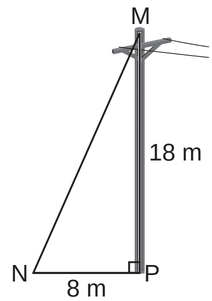
$$\tan B = \frac{\text{length of side } \textbf{opposite } \angle\textbf{B}}{\text{length of side } \textbf{adjacent to } \angle\textbf{B}}$$

$$\tan B = \frac{\textbf{CD}}{\textbf{BC}}$$

$$\tan B = \frac{\textbf{12}}{\textbf{13}}$$

$$\angle B \doteq \textbf{43°}$$

5. A telephone pole is supported by a wire, as shown.
What angle, to the nearest degree, does the wire make with the ground?



We want to find the measure of ∠N.
Use the tangent ratio.

$$\tan \textbf{N} = \frac{\text{length of side } \textbf{opposite } \angle\textbf{N}}{\text{length of side } \textbf{adjacent to } \angle\textbf{N}}$$

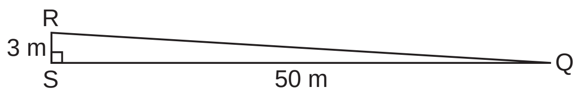
$$\tan \textbf{N} = \frac{\textbf{PM}}{\textbf{NP}}$$

$$\tan \textbf{N} = \frac{\textbf{18}}{\textbf{8}}$$

$$\angle \textbf{N} \doteq \textbf{66°}$$

The angle between the ground and the wire is about **66°**.

6. Victor is building a wheelchair ramp to an entranceway that is 3 m above the sidewalk. The ramp will cover a horizontal distance of 50 m. What angle, to the nearest degree, will the ramp make with the ground?



We want to find the measure of ∠Q.
Use the tangent ratio.

$$\tan Q = \frac{\textbf{length of side opposite } \angle\textbf{Q}}{\textbf{length of side adjacent to } \angle\textbf{Q}}$$

$$\tan Q = \frac{\textbf{RS}}{\textbf{SQ}}$$

$$\tan Q = \frac{\textbf{3}}{\textbf{50}}$$

$$\angle Q \doteq \textbf{3°}$$

The angle between the ground and the ramp is about **3°**.

TEACHER NOTE

Next Steps: Have students complete questions 6, 8, 10, 14, and 15 on pages 75 and 76 of the Student Text.
For students experiencing success, introduce Example 4, on page 74 of the Student Text, and assign Practice questions 13, 19, and 20.

2.2 Skill Builder

Solving Equations

Inverse operations "undo" each other's results.
Multiplication and division are inverse operations.
We can use inverse operations to solve some equations.

To solve $\frac{a}{5} = 4$:

$$\frac{a}{5} = 4$$

Undo the division.
Multiply each side by 5.

$$5 \times \frac{a}{5} = 5 \times 4$$
$$a = 20$$

To solve $\frac{36}{b} = 9$:

$$b \times \frac{36}{b} = b \times 9$$

Undo the division.
Multiply each side by b.

Recall: $b \times 9 = 9b$

$$36 = 9b$$

$$\frac{36}{9} = \frac{9b}{9}$$
$$4 = b$$

Undo the multiplication.
Divide each side by 9.

Check

1. Solve each equation.

a) $\frac{a}{7} = 5$

Multiply each side by **7**.

$\mathbf{7} \times \frac{a}{7} = \mathbf{7} \times 5$

$a = \mathbf{35}$

b) $9 = \frac{c}{8}$

Multiply each side by **8**.

$\mathbf{8 \times 9 = 8 \times \frac{c}{8}}$

$\mathbf{72 = c}$

c) $13 = \frac{156}{f}$

Multiply each side by $\boldsymbol{f}$.

$\boldsymbol{f} \times 13 = \boldsymbol{f} \times \frac{156}{f}$

$\mathbf{13}\boldsymbol{f} = \mathbf{156}$

Divide each side by **13**.

$\mathbf{\frac{13\boldsymbol{f}}{13}} = \mathbf{\frac{156}{13}}$

$\boldsymbol{f} = \mathbf{12}$

d) $\frac{15}{b} = 6$

$\boldsymbol{b \times \frac{15}{b} = b \times 6}$

$\mathbf{15 = 6}\boldsymbol{b}$

$\mathbf{\frac{15}{6} = \frac{6\boldsymbol{b}}{6}}$

$\mathbf{2.5 =} \boldsymbol{b}$

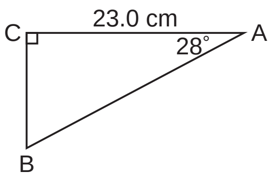
2.2 Using the Tangent Ratio to Calculate Lengths

FOCUS Use the tangent ratio to calculate lengths.

When we know the measure of an acute angle and the length of a leg of a right triangle, we can use the tangent ratio to find the length of the other leg.

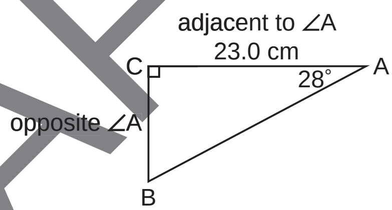
Example 1 Finding the Length of an Opposite Side

Find the length of BC to the nearest tenth of a centimetre.



Solution

We know the measure of $\angle$A.
BC is the side opposite $\angle$A.
CA is the side adjacent to $\angle$A.
Use the tangent ratio to write an equation.



$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}$$

$$\tan A = \frac{BC}{CA}$$ Substitute: $\angle A = 28°$ and $CA = 23$

$$\tan 28° = \frac{BC}{23}$$

Solve the equation for BC.
Multiply each side by 23.

$$23 \times \tan 28° = 23 \times \frac{BC}{23}$$

$$23 \tan 28° = BC$$ Use a calculator. Enter: $\boxed{2}\ \boxed{3}\ \boxed{\times}\ \boxed{\text{TAN}}\ \boxed{2}\ \boxed{8}\ \boxed{)}\ \boxed{\text{ENTER}}$

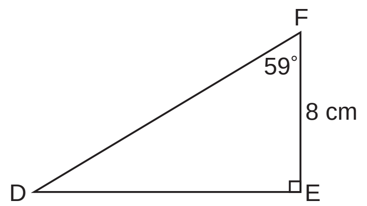
$$BC = 12.2293...$$

BC is about 12.2 cm long.

Check

1. Find the length of each indicated side to the nearest tenth of a centimetre.

a) Side ED



The given angle is ∠F.

DE is the side opposite ∠F.

EF is the side adjacent to ∠F.

$\tan F = \dfrac{\text{side } \textbf{opposite } \angle \textbf{F}}{\text{side } \textbf{adjacent to } \angle \textbf{F}}$

$\tan F = \dfrac{\textbf{DE}}{\textbf{EF}}$

$\tan \textbf{59°} = \dfrac{\textbf{DE}}{\textbf{8}}$

Solve the equation for DE.

Multiply each side by **8**.

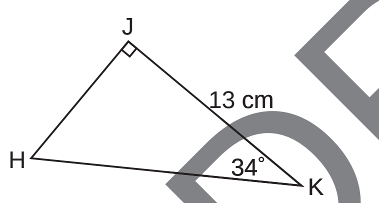
$\textbf{8} \times \tan \textbf{59°} = \textbf{8} \times \dfrac{\textbf{DE}}{\textbf{8}}$

$\textbf{8} \times \tan \textbf{59°} = \textbf{DE}$

$\text{DE} = \textbf{13.3142...}$

DE is about **13.3 cm** long.

b) Side HJ



The given angle is **∠K**.

HJ is the side opposite **∠K**.

JK is the side adjacent to **∠K**.

$\tan \textbf{K} = \dfrac{\textbf{side opposite } \angle \textbf{K}}{\textbf{side adjacent to } \angle \textbf{K}}$

$\tan \textbf{K} = \dfrac{\textbf{HJ}}{\textbf{JK}}$

$\tan \textbf{34°} = \dfrac{\textbf{HJ}}{\textbf{13}}$

$\mathbf{13 \times \tan 34° = 13 \times \dfrac{HJ}{13}}$

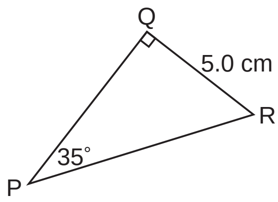
$\mathbf{13 \tan 34° = HJ}$

$\mathbf{HJ = 8.7686...}$

HJ is about **8.8 cm** long.

Example 2 Finding the Length of an Adjacent Side

Find the length of PQ to the nearest tenth of a centimetre.



Solution

Use the tangent ratio to write an equation.

We know $\angle P = 35°$.
QR is opposite $\angle P$.
PQ is adjacent $\angle P$.

$$\tan P = \frac{\text{side opposite } \angle P}{\text{side adjacent to } \angle P}$$

$$\tan P = \frac{QR}{PQ}$$ Substitute: QR = 5 and $\angle P = 35°$

$$\tan 35° = \frac{5}{PQ}$$

Solve the equation for PQ.
Multiply each side by PQ.

$$PQ \times \tan 35° = PQ \times \frac{5}{PQ}$$

$$PQ \tan 35° = 5$$ Divide each side by tan 35°.

$$\frac{PQ \tan 35°}{\tan 35°} = \frac{5}{\tan 35°}$$

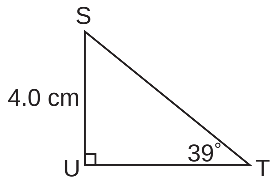
$$PQ = \frac{5}{\tan 35°}$$ Use a calculator.

$$PQ = 7.1407\ldots$$

So, PQ is about 7.1 cm long.

Check

1. Find the length of TU to the nearest tenth of a centimetre.



The given angle is ∠T.

US is the side opposite ∠T.

TU is the side adjacent to ∠T.

$$\tan T = \frac{\text{side } \underline{\textbf{opposite } \angle T}}{\text{side } \underline{\textbf{adjacent to } \angle T}}$$

$$\tan T = \underline{\frac{\textbf{US}}{\textbf{TU}}}$$ Substitute: $\underline{\angle \textbf{T} = \textbf{39°}}$ and $\underline{\textbf{US} = \textbf{4}}$

$$\tan \underline{\textbf{39°}} = \underline{\frac{\textbf{4}}{\textbf{TU}}}$$ Multiply each side by $\underline{\textbf{TU}}$.

$$\underline{\textbf{TU}} \times \tan \underline{\textbf{39°}} = \underline{\textbf{TU}} \times \underline{\frac{\textbf{4}}{\textbf{TU}}}$$

$$\underline{\textbf{TU}} \times \tan \underline{\textbf{39°}} = \underline{\textbf{4}}$$ Divide each side by $\underline{\textbf{tan 39°}}$.

$$\frac{\underline{\textbf{TU}} \tan \underline{\textbf{39°}}}{\underline{\textbf{tan 39°}}} = \underline{\frac{\textbf{4}}{\textbf{tan 39°}}}$$

$$\text{TU} = \underline{\frac{\textbf{4}}{\textbf{tan 39°}}}$$ Use a calculator.

$$\text{TU} = \underline{\textbf{4.9395...}}$$

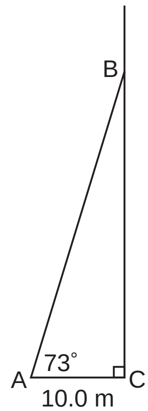
TU is about **4.9 cm** long.

Example 3 Using the Tangent Ratio to Solve a Problem

A wire supports a flagpole. The angle between the wire and the level ground is 73°. The wire is anchored to the ground 10 m from the base of the pole. How high up the pole does the wire reach? Give the answer to the nearest tenth of a metre.

Solution

Sketch and label a diagram.
Assume the flagpole meets the ground at a right angle.



The given angle is ∠A. We want to find the length of BC.

$$\tan A = \frac{\text{side opposite } \angle A}{\text{side adjacent to } \angle A}$$

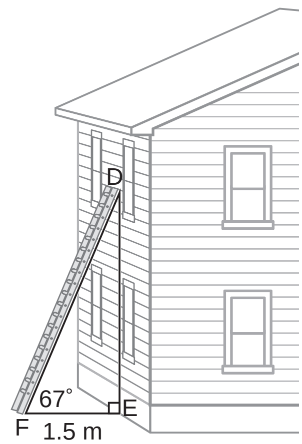
BC is opposite ∠A.
CA is adjacent to ∠A.

$\tan A = \frac{BC}{CA}$	Substitute: ∠A = 73° and CA = 10
$\tan 73° = \frac{BC}{10}$	Solve the equation for BC. Multiply each side by 10.
$10 \tan 73° = BC$	Use a calculator.
$BC = 32.7085\ldots$	

The wire reaches the flagpole at a height of about 32.7 m.

Check

1. A ladder leans on a wall, as shown. How far up the wall does the ladder reach? Give your answer to the nearest tenth of a metre.



The given angle is ∠F.
We want to find the length of **DE**.

$\tan F = \dfrac{\text{side } \underline{\textbf{opposite } \angle \textbf{F}}}{\text{side } \underline{\textbf{adjacent to } \angle \textbf{F}}}$

$\tan F = \dfrac{\textbf{DE}}{\textbf{EF}}$ Substitute: **∠F = 67°** and **EF = 1.5**

$\tan \underline{\textbf{67°}} = \dfrac{\textbf{DE}}{\textbf{1.5}}$ Multiply each side by **1.5**.

$\underline{\textbf{1.5}} \tan \underline{\textbf{67°}} = \underline{\textbf{DE}}$

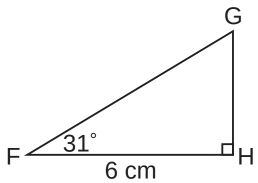
$DE = \underline{\textbf{3.5337...}}$

The ladder reaches the wall at a height of about **3.5 m**.

Practice

1. Find the length of the side opposite the given angle to the nearest tenth of a centimetre.

a)



The given angle is ∠F.
The side opposite ∠F is **GH**.
The side adjacent to ∠F is **HF**.

$\tan F = \dfrac{\text{side } \underline{\textbf{opposite } \angle \textbf{F}}}{\text{side } \underline{\textbf{adjacent to } \angle \textbf{F}}}$

$\tan F = \dfrac{\textbf{GH}}{\textbf{HF}}$

$\tan \underline{\textbf{31°}} = \dfrac{\textbf{GH}}{\textbf{6}}$

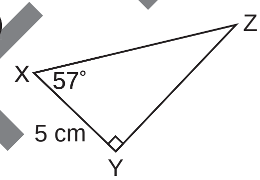
$\underline{\textbf{6}} \times \tan \underline{\textbf{31°}} = \textbf{6} \times \dfrac{\textbf{GH}}{\textbf{6}}$

$\underline{\textbf{6}} \tan \underline{\textbf{31°}} = \underline{\textbf{GH}}$

$GH = \underline{\textbf{3.6051...}}$

GH is about **3.6 cm** long.

b)



The given angle is ∠**X**.
The side opposite ∠**X** is **YZ**.
The side adjacent to ∠**X** is **XY**.

$\tan \underline{\textbf{X}} = \dfrac{\text{side } \underline{\textbf{opposite } \angle \textbf{X}}}{\text{side } \underline{\textbf{adjacent to } \angle \textbf{X}}}$

$\mathbf{\tan X = \dfrac{YZ}{XY}}$

$\mathbf{\tan 57° = \dfrac{YZ}{5}}$

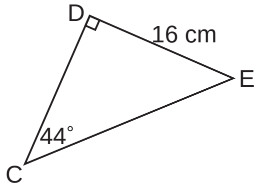
$\mathbf{5 \times \tan 57° = 5 \times \dfrac{YZ}{5}}$

$\mathbf{5 \tan 57° = YZ}$

$\mathbf{YZ = 7.6993...}$

YZ is about **7.7 cm** long.

2. Find the length of CD to the nearest tenth of a centimetre.



The given angle is ∠C.

The side opposite ∠C is **DE**.

The side adjacent to ∠C is **CD**.

$$\tan C = \frac{\text{side } \textbf{opposite } \angle \textbf{C}}{\text{side } \textbf{adjacent to } \angle \textbf{C}}$$

$$\tan C = \frac{\textbf{DE}}{\textbf{CD}}$$

$$\tan \textbf{44°} = \frac{\textbf{16}}{\textbf{CD}}$$

Multiply each side by **CD**.

$$\textbf{CD} \times \textbf{tan 44°} = \textbf{CD} \times \frac{\textbf{16}}{\textbf{CD}}$$

$$\textbf{CD tan 44°} = \textbf{16}$$

Divide each side by **tan 44°**.

$$\frac{\textbf{CD tan 44°}}{\textbf{tan 44°}} = \frac{\textbf{16}}{\textbf{tan 44°}}$$

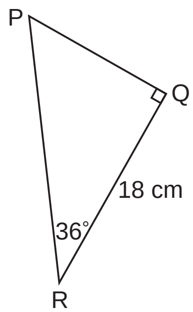
$$\textbf{CD} = \frac{\textbf{16}}{\textbf{tan 44°}}$$

$$\textbf{CD} = \textbf{16.5684...}$$

CD is about **16.6 cm** long.

3. Find the length of the indicated side to the nearest tenth of a centimetre.

a) Side PQ



$$\tan R = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \textbf{36°} = \frac{\textbf{PQ}}{\textbf{18}}$$

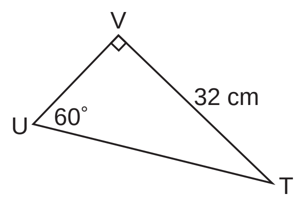
$$\textbf{18} \times \textbf{tan 36°} = \frac{\textbf{PQ}}{\textbf{18}} \times \textbf{18}$$

$$\textbf{18 tan 36°} = \textbf{PQ}$$

$$\textbf{PQ} = \textbf{13.0777...}$$

PQ is about **13.1 cm** long.

b) Side UV



$$\tan \textbf{U} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\tan \textbf{60°} = \frac{\textbf{32}}{\textbf{UV}}$$

$$\textbf{UV} \times \textbf{tan 60°} = \textbf{UV} \times \frac{\textbf{32}}{\textbf{UV}}$$

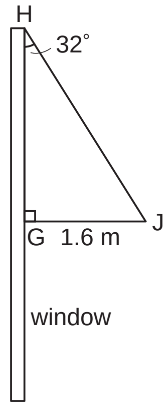
$$\textbf{UV tan 60°} = \textbf{32}$$

$$\frac{\textbf{UV tan 60°}}{\textbf{tan 60°}} = \frac{\textbf{32}}{\textbf{tan 60°}}$$

$$\textbf{UV} = \textbf{18.4752...}$$

UV is about **18.5 cm** long.

4. This diagram shows an awning over the window of a house. Find the height of the awning, GH, to the nearest tenth of a metre.



$$\tan H = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\tan 32° = \frac{1.6}{GH}$$

$$GH \times \tan 32° = GH \times \frac{1.6}{GH}$$

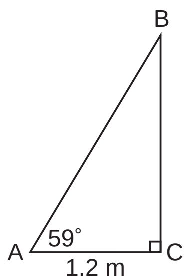
$$GH \tan 32° = 1.6$$

$$\frac{GH \tan 32°}{\tan 32°} = \frac{1.6}{\tan 32°}$$

$$GH = 2.5605\ldots$$

The height of the awning is about **2.6 m**.

5. A rope supports a tent. The angle between the rope and the level ground is 59°. The rope is attached to the ground 1.2 m from the base of the tent. At what height above the ground is the rope attached to the tent? Give your answer to the nearest tenth of a metre.



We want to find the length of BC.

$$\tan A = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\tan A = \frac{BC}{AC}$$

$$\tan 59° = \frac{BC}{1.2}$$

$$1.2 \times \tan 59° = 1.2 \times \frac{BC}{1.2}$$

$$1.2 \tan 59° = BC$$

$$BC = 1.9971\ldots$$

The rope is attached to the tent at a height of about **2.0 m**.

TEACHER NOTE

Next Steps: Have students complete questions 6, 8, 10, 11, and 13 on pages 82 and 83 of the Student Text.

2.3 Math Lab: Measuring an Inaccessible Height

FOCUS **Determine a height that cannot be measured directly.**

When we find a length or an angle without using a measuring instrument, we are using **indirect measurement.**

Try This

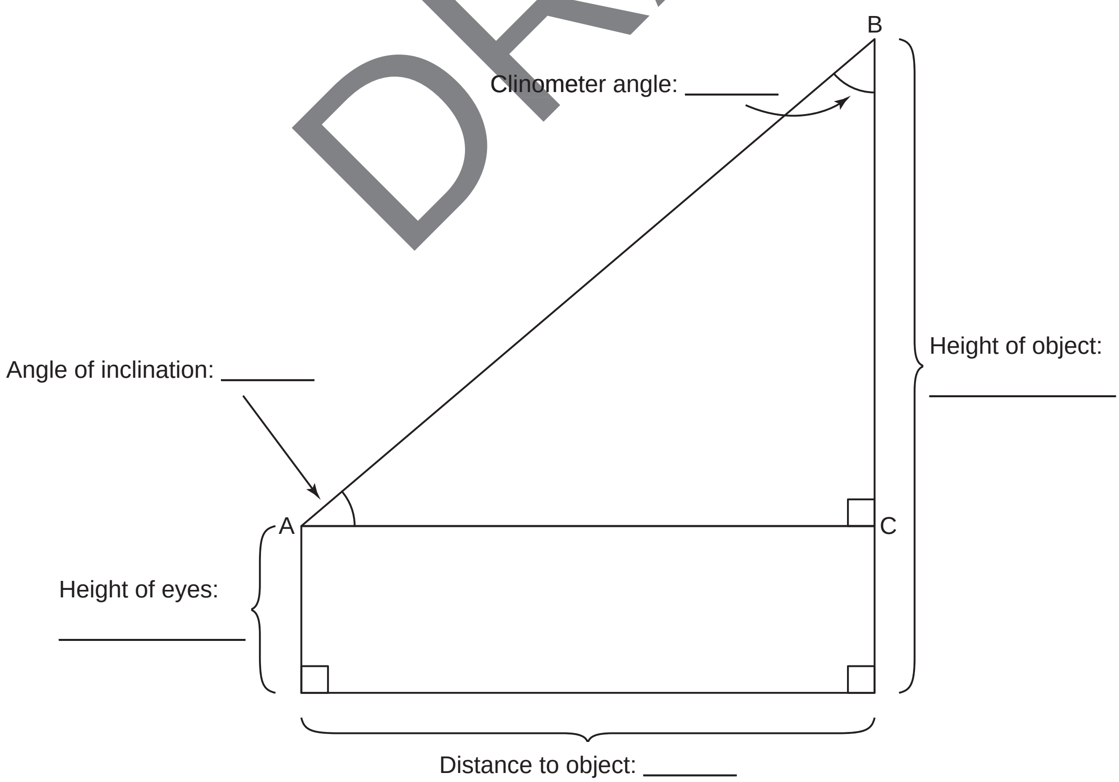
Work with a partner.
Follow the instructions in Part A on Student Text page 85 to make a clinometer.
The materials you need are listed on Student Text page 84.
Record all your measurements on the diagram below.

Choose a tall object; for example, a tree or a flagpole.
Object: ____________________

Mark a point on the ground.
Measure the distance to the base of the object.

One person stands at the point. He holds the clinometer, then looks at the top of the object through the straw. The other person records the angle shown by the thread on the protractor. Then that person measures the height of the eyes above the ground of the person holding the clinometer.

Subtract the clinometer angle from 90°.
This is the angle of inclination of the straw.



Use the tangent ratio to calculate the length of BC:

$\tan A = \frac{BC}{AC}$

$\tan \_\_\_\_\_\_ = \frac{BC}{\_\_\_\_\_\_\_\_}$

BC = ______ × tan ______

BC ≐ ______

Height of object = length of BC + height of eyes above the ground

Height of object = __________ + __________

Height of object = __________

Change places with your partner.

Repeat the activity.

Does the height of your eyes affect the measurements? Explain. **Sample response:**

Yes, the height of your eyes affects the angle of inclination of the object.

The taller you are, the less you have to look up to the object.

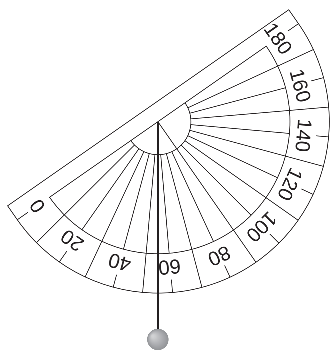
Does the height of your eyes affect the final result? Explain. **Sample response:**

No, a taller person will calculate a greater height above the horizontal but will have a lesser eye height. But, when the two heights are added, both results should be the same.

Practice

1. Which angle of inclination does each clinometer measure?

a)



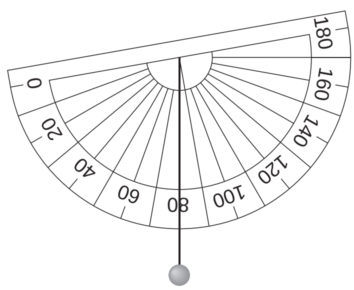
Angle of inclination

= 90° − angle on clinometer

= 90° − **55°**

= **35°**

b)

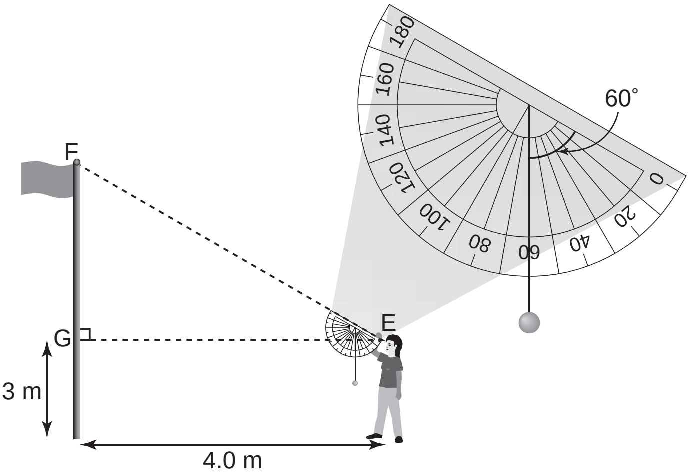


Angle of inclination

= 90° − **80°**

= **10°**

2. Use the information in the diagram to find the height of the flagpole to the nearest tenth of a metre.



Angle on clinometer: **60°**

Angle of inclination: 90° − **60°** = **30°**

So, ∠E = **30°**

$$\tan E = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan \mathbf{30°} = \frac{\mathbf{FG}}{\mathbf{EG}}$$

$$\tan \mathbf{30°} = \frac{\mathbf{FG}}{\mathbf{4}}$$

$$\mathbf{4 \tan 30° = 4\left(\frac{FG}{4}\right)}$$

$$\mathbf{4 \tan 30° = FG}$$

FG = **2.3094…**

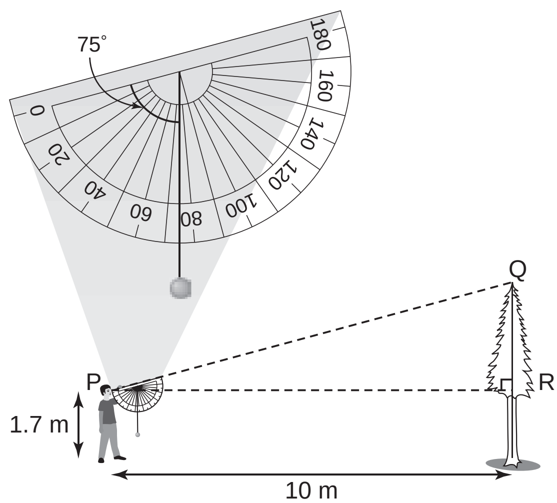
So, height of flagpole = **FG** + height of eyes above ground

= **2.3094…** + **1.3**

= **3.6094…**

The height of the flagpole is about **3.6 m**.

3. Use the information in the diagram to find the height of the tree to the nearest tenth of a metre.



This diagram is not drawn to scale.

∠P is: 90° − **75°** = **15°**

$$\tan P = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\tan \mathbf{15°} = \frac{\mathbf{QR}}{\mathbf{PR}}$$

$$\mathbf{\tan 15° = \frac{QR}{10}}$$

$$\mathbf{10 \tan 15° = QR}$$

QR = **2.6794…**

So, height of tree = **2.6794… + 1.7**

= **4.3794…**

The height of the tree is about **4.4 m**.

TEACHER NOTE

Next Steps: Have students complete question 1 on page 86 of the Student Text.

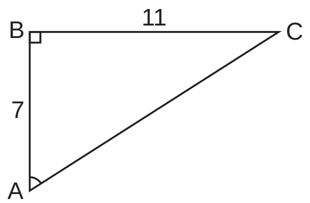
CHECKPOINT 1

Can you ...

- use the tangent ratio to find an angle measure?
- use the tangent ratio to calculate a length?
- use the tangent ratio to solve a problem?

2.1 **1.** Find the tangent ratio for each indicated angle. Leave the ratio in fraction form.

a)



The side opposite ∠A is **BC**.

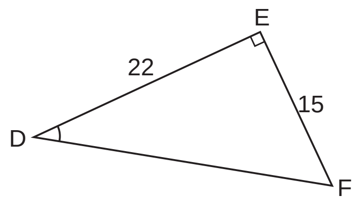
The side adjacent to ∠A is **AB**.

tan A = $\dfrac{\text{side } \textbf{opposite } \angle A}{\text{side } \textbf{adjacent to } \angle A}$

tan A = $\dfrac{\textbf{BC}}{\textbf{AB}}$

tan A = $\dfrac{\textbf{11}}{\textbf{7}}$

b)



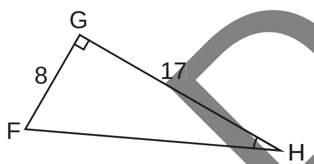
tan D = $\dfrac{\textbf{side opposite } \angle D}{\textbf{side adjacent to } \angle D}$

tan D = $\dfrac{\textbf{EF}}{\textbf{DE}}$

tan D = $\dfrac{\textbf{15}}{\textbf{22}}$

2. Find the measure of each indicated angle to the nearest degree.

a)



The side opposite ∠H is **FG**.

The side adjacent to ∠H is **GH**.

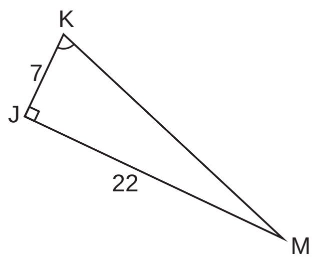
tan H = $\dfrac{\text{side } \textbf{opposite } \angle H}{\text{side } \textbf{adjacent to } \angle H}$

tan H = $\dfrac{\textbf{FG}}{\textbf{GH}}$

tan H = $\dfrac{\textbf{8}}{\textbf{17}}$

∠H ≐ **25°**

b)



tan **K** = $\dfrac{\textbf{side opposite } \angle K}{\textbf{side adjacent to } \angle K}$

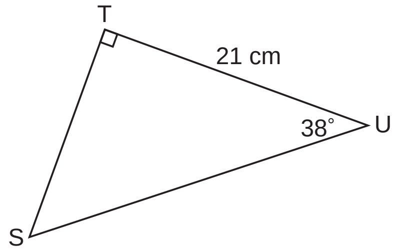
tan **K** = $\dfrac{\textbf{MJ}}{\textbf{JK}}$

tan **K** = $\dfrac{\textbf{22}}{\textbf{7}}$

∠K ≐ **72°**

2.2 **3.** Find the length of each indicated side to the nearest tenth of a centimetre.

a) Side ST



$\tan U = \dfrac{\text{side } \mathbf{opposite\ \angle U}}{\text{side } \mathbf{adjacent\ to\ \angle U}}$

$\tan U = \dfrac{\mathbf{ST}}{\mathbf{TU}}$

$\tan \mathbf{38°} = \dfrac{\mathbf{ST}}{\mathbf{21}}$

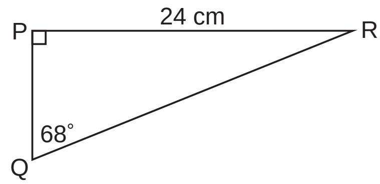
$\mathbf{21 \times \tan 38° = 21 \times \dfrac{ST}{21}}$

$\mathbf{21 \tan 38° = ST}$

$ST = \mathbf{16.4069...}$

ST is about **16.4 cm** long.

b) Side PQ



$\tan \mathbf{Q} = \dfrac{\mathbf{side\ opposite\ \angle Q}}{\mathbf{side\ adjacent\ to\ \angle Q}}$

$\mathbf{\tan Q = \dfrac{RP}{PQ}}$

$\mathbf{\tan 68° = \dfrac{24}{PQ}}$

$\mathbf{PQ \times \tan 68° = PQ \times \dfrac{24}{PQ}}$

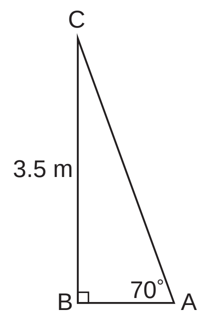
$\mathbf{PQ \tan 68° = 24}$

$\mathbf{\dfrac{PQ \tan 68°}{\tan 68°} = \dfrac{24}{\tan 68°}}$

$\mathbf{PQ = 9.6966...}$

PQ is about **9.7 cm** long.

4. Margy is building a support brace to reach the top of a wall, as shown. How far from the wall should the brace be anchored to the ground? Give your answer to the nearest tenth of a metre.



We want to find the length of AB.

The side opposite ∠A is BC.

The side adjacent to ∠A is AB.

$\mathbf{\tan A = \dfrac{side\ opposite\ \angle A}{side\ adjacent\ to\ \angle A}}$

$\mathbf{\tan A = \dfrac{BC}{AB}}$

$\mathbf{\tan 70° = \dfrac{3.5}{AB}}$

$\mathbf{AB \times \tan 70° = AB \times \dfrac{3.5}{AB}}$

$\mathbf{AB \tan 70° = 3.5}$

$\mathbf{\dfrac{AB \tan 70°}{\tan 70°} = \dfrac{3.5}{\tan 70°}}$

$\mathbf{AB = \dfrac{3.5}{\tan 70°}}$

$\mathbf{AB = 1.2738...}$

The brace should be anchored to the ground **about 1.3 m** from the wall.

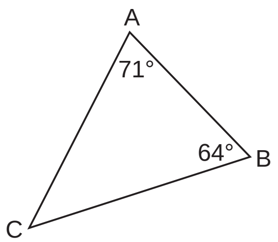
TEACHER NOTE

Next Steps: Have students complete questions 3 and 5 on page 88 of the Student Text.

2.4 Skill Builder

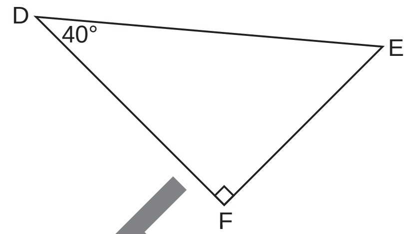
Sum of the Angles in a Triangle

In any triangle, the sum of the angle measures is 180°.
So, to find an unknown angle measure:
- start with 180°
- subtract the known measures



$\angle C = 180° - 71° - 64°$
$\angle C = 45°$

In any right triangle, the sum of the measures of the acute angles is 90°.
So, to find the measure of an acute angle:
- start with 90°
- subtract the known acute angle

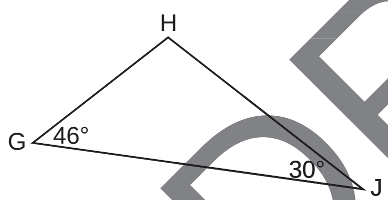


$\angle E = 90° - 40°$
$\angle E = 50°$

Check

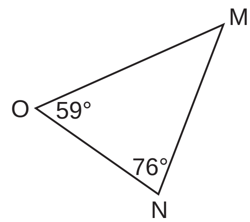
1. Find the measure of the third angle.

a)



$\angle H = 180° -$ **46°** $-$ **30°**
$\angle H =$ **104°**

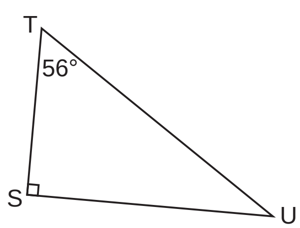
b)



$\angle M =$ **180° − 76° − 59°**
$\angle M =$ **45°**

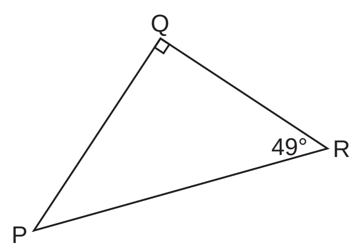
2. Find the measure of the third angle.

a)



$\angle U = 90° -$ **56°**
$\angle U =$ **34°**

b)



$\angle P =$ **90° − 49°**
$\angle P =$ **41°**

2.4 The Sine and Cosine Ratios

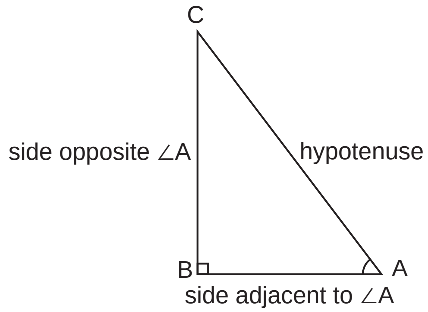
FOCUS **Use the sine and cosine ratios to determine angle measures.**

The Sine and Cosine Ratios

In a right triangle, if $\angle$A is an acute angle, then

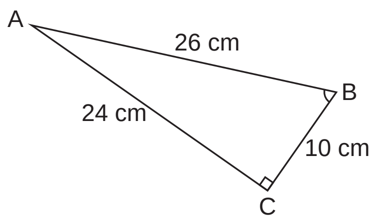
$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}$$

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}$$



Example 1 Finding the Sine and Cosine of an Angle

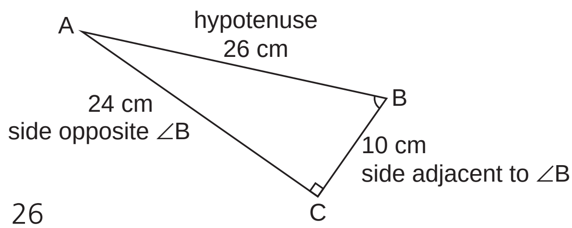
Find sin B and cos B to the nearest hundredth.



The nearest hundredth means two decimal places.

Solution

AC is the side opposite $\angle$B.
AB is the hypotenuse.



$$\sin B = \frac{\text{length of side opposite } \angle B}{\text{length of hypotenuse}}$$

$\sin B = \frac{AC}{AB}$ Substitute: AC = 24 and AB = 26

$\sin B = \frac{24}{26}$

$\sin B = 0.9230\ldots$

$\sin B \doteq 0.92$

BC is the side adjacent to $\angle$B.
AB is the hypotenuse.

$$\cos B = \frac{\text{length of side adjacent to } \angle B}{\text{length of hypotenuse}}$$

$\cos B = \frac{BC}{AB}$ Substitute: BC = 10 and AB = 26

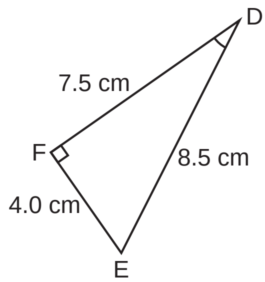
$\cos B = \frac{10}{26}$

$\cos B = 0.3846\ldots$

$\cos B \doteq 0.38$

Check

1. Find sin D and cos D to the nearest hundredth.



$\sin D = \dfrac{\text{side opposite } \angle D}{\text{hypotenuse}}$ $\qquad$ $\cos D = \dfrac{\text{side adjacent to } \angle D}{\text{hypotenuse}}$

$\sin D = \dfrac{\mathbf{EF}}{\mathbf{DE}}$ $\qquad$ $\cos D = \dfrac{\mathbf{FD}}{\mathbf{DE}}$

$\sin D = \dfrac{\mathbf{4.0}}{\mathbf{8.5}}$ $\qquad$ $\cos D = \dfrac{\mathbf{7.5}}{\mathbf{8.5}}$

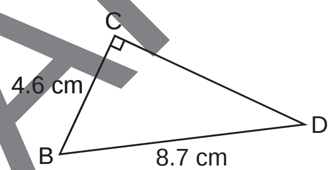
$\sin D =$ **0.4705…** $\qquad$ $\cos D =$ **0.8823…**

$\sin D \doteq$ **0.47** $\qquad$ $\cos D \doteq$ **0.88**

To find the measure of an angle, use the $\sin^{-1}$ or $\cos^{-1}$ key on a scientific calculator.

Example 2 Using the Sine or Cosine Ratio to Find the Measure of an Angle

Find the measures of ∠B and ∠D to the nearest degree.



Solution

Find the measure of ∠B first.
BC is adjacent to ∠B. BD is the hypotenuse.
So, use the cosine ratio to write an equation.

We could use the sine ratio to find the measure of ∠D first.

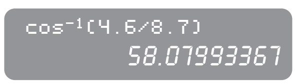
$\cos B = \dfrac{\text{side adjacent to } \angle B}{\text{hypotenuse}}$

$\cos B = \dfrac{BC}{BD}$ $\qquad$ Substitute: BC = 4.6 and BD = 8.7

$\cos B = \dfrac{4.6}{8.7}$

To find ∠B using a TI-30XIIS calculator, enter:

[2nd] [COS] [4] [·] [6] [÷] [8] [·] [7] [)] [ENTER]



∠B ≐ 58°

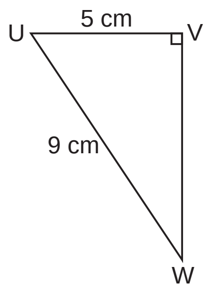
Since the sum of the acute angles in a right triangle is 90°,
∠D = 90° − ∠B
So, ∠D ≐ 90° − 58°
∠D ≐ 32°

Since the measure of ∠B is an estimate, so is the measure of ∠D.

Check

1. Find the measure of each acute angle to the nearest degree.

a)



Find the measure of ∠U first.

UV is the **side adjacent to** ∠U.

UW is the **hypotenuse**.

So, use the cosine ratio to write an equation.

cos U = $\dfrac{\textbf{side adjacent to } \angle \textbf{U}}{\textbf{hypotenuse}}$

cos U = $\dfrac{\mathbf{UV}}{\mathbf{UW}}$

cos U = $\dfrac{\mathbf{5}}{\mathbf{9}}$

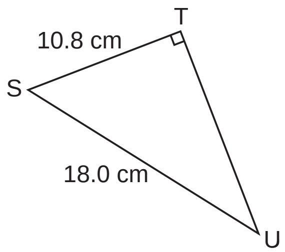
∠U ≐ **56°**

So, ∠W = 90° − ∠U

∠W ≐ 90° − **56°**

∠W ≐ **34°**

b)



Find the measure of ∠U first.

ST is the **side opposite ∠U**.

SU is the **hypotenuse**.

So, use the **sine** ratio to write an equation.

sin U = $\dfrac{\textbf{side opposite } \angle \textbf{U}}{\textbf{hypotenuse}}$

sin U = $\dfrac{\mathbf{ST}}{\mathbf{US}}$

sin U = $\dfrac{\mathbf{10.8}}{\mathbf{18.0}}$

∠U ≐ **37°**

So, ∠S = 90° − **∠U**

∠S ≐ 90° − **37°**

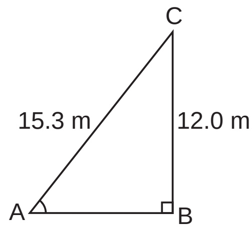
∠S ≐ **53°**

Example 3 Using Sine or Cosine to Solve a Problem

A storm caused a 15.3-m hydro pole to lean over. The top of the pole is now 12.0 m above the ground. What angle does the pole make with the ground? Give the answer to the nearest degree.

Solution

Draw a diagram. AC represents the pole.
The pole meets the ground at A.
BC is the side opposite ∠A. AC is the hypotenuse.
So, use the sine ratio to find ∠A.



$\sin A = \dfrac{\text{side opposite } \angle A}{\text{hypotenuse}}$

$\sin A = \dfrac{BC}{AC}$ Substitute: BC = 12.0 and AC = 15.3

Assume the ground is horizontal.

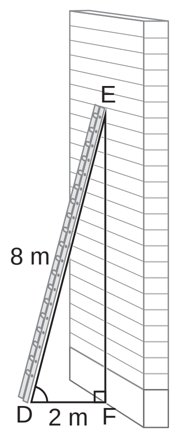
$\sin A = \dfrac{12.0}{15.3}$ Use a calculator.

$\angle A \doteq 52°$

So, the hydro pole makes an angle of about 52° with the ground.

Check

1. A ladder leans on a wall as shown.
What angle does the ladder make with the ground?
Give your answer to the nearest degree.
We want to find the measure of ∠D.
DF is **adjacent to ∠D**.
DE is **the hypotenuse**.
So, use the **cosine** ratio to find ∠D.



Assume the ground is horizontal.

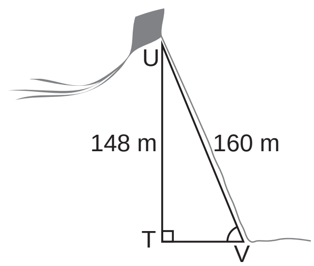
$\underline{\textbf{cos}}\ D = \dfrac{\textbf{side adjacent to } \angle \textbf{D}}{\textbf{hypotenuse}}$

$\underline{\textbf{cos}}\ D = \dfrac{\textbf{DF}}{\textbf{DE}}$ Substitute: **DF = 2** and **DE = 8**

$\underline{\textbf{cos}}\ D = \dfrac{\textbf{2}}{\textbf{8}}$

$\angle D \doteq$ **76°** So, the ladder makes an angle of about **76°** with the ground.

2. The string of a kite is 160 m long. The string is anchored to the ground. The kite is 148 m high. What angle does the string make with the ground? Give your answer to the nearest degree.



We want to find the measure of ∠V.

TU is the **side opposite ∠V**.

UV is the **hypotenuse**.

So, use the **sine** ratio to write an equation.

$\textbf{sin V} = \dfrac{\textbf{side opposite } \angle \textbf{V}}{\textbf{hypotenuse}}$

$\textbf{sin V} = \dfrac{\textbf{TU}}{\textbf{UV}}$

$\textbf{sin V} = \dfrac{\textbf{148}}{\textbf{160}}$

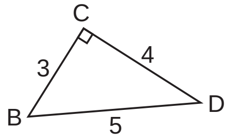
$\angle \textbf{V} \doteq \textbf{68°}$

The angle the string makes with the ground is about **68°**.

Practice

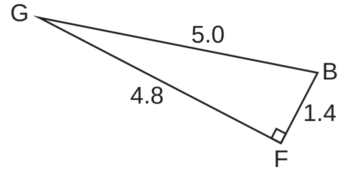
1. Fill in the blanks.

a)



The side opposite ∠B is **CD**.
The side adjacent to ∠B is **BC**.
The hypotenuse is **BD**.

b)



The side opposite ∠B is **FG**.
The side adjacent to ∠B is **BF**.
The hypotenuse is **GB**.

2. For each triangle in question 1, find sin B and cos B as decimals.

a) sin B = side **opposite** ∠B / hypotenuse

sin B = **CD / BD**

sin B = **4 / 5**

sin B = **0.8**

cos B = side **adjacent to** ∠B / hypotenuse

cos B = **BC / BD**

cos B = **3 / 5**

cos B = **0.6**

b) sin B = **side opposite ∠B / hypotenuse**

sin B = FG / GB

sin B = 4.8 / 5.0

sin B = **0.96**

cos B = **side adjacent to ∠B / hypotenuse**

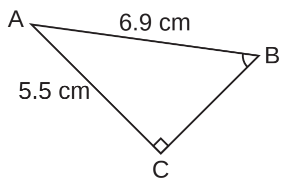
cos B = BF / GB

cos B = 1.4 / 5.0

cos B = **0.28**

3. Find the measure of each indicated angle to the nearest degree.

a)



AC is the **side opposite ∠B**.

AB is the **hypotenuse**.

So, use the **sine** ratio.

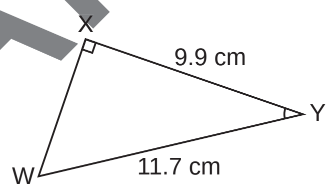
sin B = side opposite ∠B / hypotenuse

sin B = AC / AB

sin B = 5.5 / 6.9

∠B ≐ **53°**

b)



XY is the **side adjacent to ∠Y**.

WY is the **hypotenuse**.

So, use the **cosine** ratio.

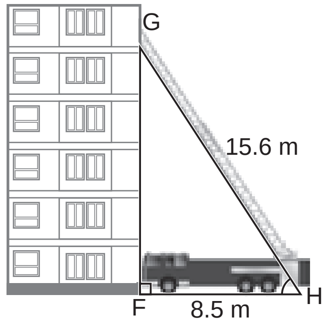
cos Y = side adjacent to ∠Y / hypotenuse

cos Y = XY / WY

cos Y = 9.9 / 11.7

∠Y ≐ **32°**

4. A firefighter rests a 15.6-m ladder against a building, as shown. What angle does the ladder make with the ground? Give your answer to the nearest degree.



We want to find the measure of ∠H.
FH is the **side adjacent to ∠H**.
GH is the **hypotenuse**.
So, use the **cosine** ratio.

cos H = $\dfrac{\textbf{side adjacent to } \angle \textbf{H}}{\textbf{hypotenuse}}$

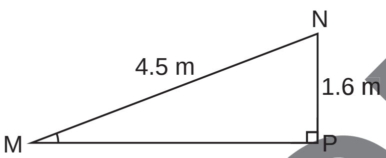
cos H = $\dfrac{\textbf{FH}}{\textbf{GH}}$

cos H = $\dfrac{\textbf{8.5}}{\textbf{15.6}}$

∠H ≐ **57°**

The angle the ladder makes with the ground is about **57°**.

5. A loading ramp is 4.5 m long. The top of the ramp has height 1.6 m. What angle does the ramp make with the ground? Give your answer to the nearest degree.



We want to find the measure of ∠M.
NP is the side opposite ∠M.
MN is the hypotenuse.
So, use the sine ratio.

sin M = $\dfrac{\textbf{side opposite } \angle \textbf{M}}{\textbf{hypotenuse}}$

sin M = $\dfrac{\textbf{NP}}{\textbf{MN}}$

sin M = $\dfrac{\textbf{1.6}}{\textbf{4.5}}$

∠M ≐ 21°

The angle the ramp makes with the ground is about **21°**.

TEACHER NOTE

Next Steps: Have students complete questions 7, 8, 10, 11, 13, and 14 on pages 95 and 96 of the Student Text.

2.5 Using the Sine and Cosine Ratios to Calculate Lengths

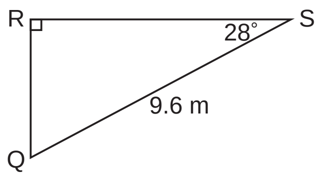
FOCUS **Use the sine and cosine ratios to determine lengths.**

To use the sine or cosine ratio to find the length of a leg, we need to know:

- the measure of an acute angle, and
- the length of the hypotenuse

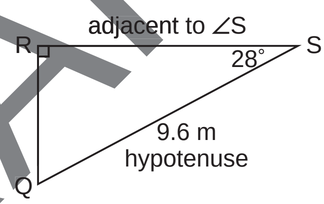
Example 1 Using the Sine or Cosine Ratio to Find the Length of a Leg

Find the length of RS to the nearest tenth of a metre.



Solution

The measure of $\angle S$ is known.
RS is the side adjacent to $\angle S$.
QS is the hypotenuse.
So, use the cosine ratio.



$$\cos S = \frac{\text{side adjacent to } \angle S}{\text{hypotenuse}}$$

$$\cos S = \frac{RS}{QS}$$ Substitute: $\angle S = 28°$ and $QS = 9.6$

$$\cos 28° = \frac{RS}{9.6}$$ Multiply both sides by 9.6.

The cosine ratio compares the adjacent side to the hypotenuse.

$$9.6 \times \cos 28° = 9.6 \times \frac{RS}{9.6}$$

$$9.6 \cos 28° = RS$$ Use a calculator.

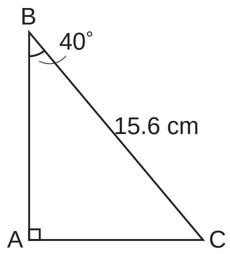
$$RS = 8.4762\ldots$$

RS is about 8.5 m long.

Check

1. Find the length of each indicated side to the nearest tenth of a centimetre.

a) AC



The measure of ∠B is known.

AC is the **side opposite ∠B**.

BC is the **hypotenuse**.

So, use the **sine** ratio.

$$\textbf{sin}\ B = \frac{\text{side } \textbf{opposite } \angle \textbf{B}}{\text{hypotenuse}}$$

$$\textbf{sin}\ B = \frac{\textbf{AC}}{\text{BC}}$$

$$\textbf{sin } \mathbf{40°} = \frac{\textbf{AC}}{\mathbf{15.6}}$$

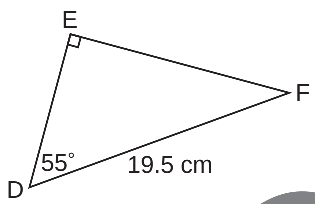
$$\mathbf{15.6 \times \textbf{sin } 40° = 15.6 \times \frac{\textbf{AC}}{15.6}}$$

$$\mathbf{15.6\ \textbf{sin } 40° = \textbf{AC}}$$

AC = **10.0274…**

AC is about **10.0 cm** long.

b) DE



The measure of ∠**D** is known.

DE is the **side adjacent to ∠D**.

DF is the **hypotenuse**.

So, use the **cosine** ratio.

$$\textbf{cos D} = \frac{\textbf{side adjacent to } \angle \textbf{D}}{\textbf{hypotenuse}}$$

$$\textbf{cos D} = \frac{\textbf{DE}}{\textbf{DF}}$$

$$\textbf{cos } \mathbf{55°} = \frac{\textbf{DE}}{\mathbf{19.5}}$$

19.5 cos 55° = DE

DE = 11.1847…

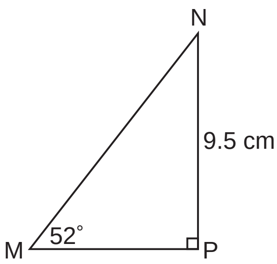
DE is about **11.2 cm** long.

To use the sine or cosine ratio to find the length of the hypotenuse, we need to know:

- the measure of an acute angle, and
- the length of one leg

Example 2 Using the Sine or Cosine Ratio to Find the Length of the Hypotenuse

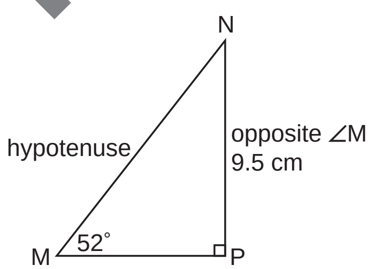
Find the length of the hypotenuse to the nearest tenth of a centimetre.



The hypotenuse is the side opposite the right angle.

Solution

The measure of $\angle$M is known.
NP is the side opposite $\angle$M.
MN is the hypotenuse.
So, use the sine ratio to write an equation.



$$\sin M = \frac{\text{side opposite } \angle M}{\text{hypotenuse}}$$

$$\sin M = \frac{NP}{MN}$$ Substitute: $\angle M = 52°$ and $NP = 9.5$

$$\sin 52° = \frac{9.5}{MN}$$ Multiply both sides by MN.

$$MN \sin 52° = 9.5$$ Divide both sides by sin 52°.

The sine ratio compares the opposite side to the hypotenuse.

$$\frac{MN \sin 52°}{\sin 52°} = \frac{9.5}{\sin 52°}$$

$$MN = \frac{9.5}{\sin 52°}$$ Use a calculator.

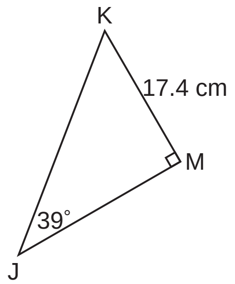
$$MN = 12.0556\ldots$$

MN is about 12.1 cm long.

Check

1. Find the length of each hypotenuse to the nearest tenth of a centimetre.

a)



The measure of ∠J is known.
The side opposite ∠J is: **KM**
The hypotenuse is: **JK**
Use the sine ratio.

$\sin J = \dfrac{\text{side } \textbf{opposite } \angle J}{\textbf{hypotenuse}}$

$\sin J = \dfrac{\textbf{KM}}{\textbf{JK}}$

$\sin \mathbf{39°} = \dfrac{\mathbf{17.4}}{\textbf{JK}}$

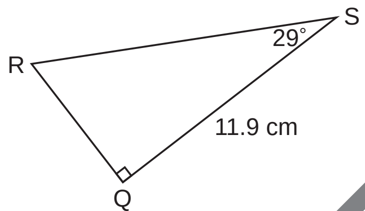
$\textbf{JK} \sin \mathbf{39°} = \mathbf{17.4}$

$JK = \dfrac{\mathbf{17.4}}{\mathbf{\sin 39°}}$

$JK = \mathbf{27.6488...}$

JK is about **27.6 cm** long.

b)



The measure of ∠ **S** is known.
QS is the side **adjacent to ∠S**.
RS is the **hypotenuse**.
So, use the **cosine** ratio.

$\mathbf{\cos S = \dfrac{\textbf{side adjacent to } \angle S}{\textbf{hypotenuse}}}$

$\mathbf{\cos S = \dfrac{QS}{RS}}$

$\mathbf{\cos 29° = \dfrac{11.9}{RS}}$

$\mathbf{RS \cos 29° = 11.9}$

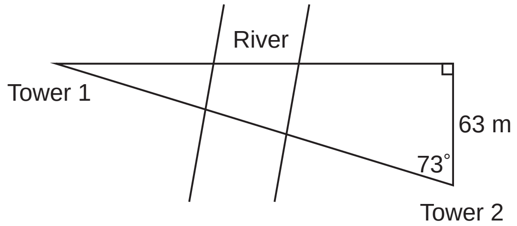
$\mathbf{RS = \dfrac{11.9}{\cos 29°}}$

$RS = \mathbf{13.6059...}$

RS is about **13.6 cm** long.

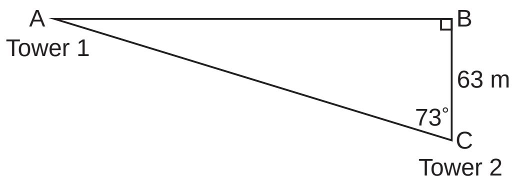
Example 3 Using Sine or Cosine to Solve a Problem

A surveyor makes the measurements shown in the diagram to find the distance between two observation towers on opposite sides of a river. How far apart are the towers? Give the answer to the nearest metre.



Solution

The distance between the towers is the hypotenuse, AC.



The measure of ∠C is known.
BC is the side adjacent to ∠C.
AC is the hypotenuse.
So, use the cosine ratio.

$$\cos C = \frac{\text{side adjacent to } \angle C}{\text{hypotenuse}}$$

$$\cos C = \frac{BC}{AC}$$ Substitute: ∠C = 73° and BC = 63

$$\cos 73° = \frac{63}{AC}$$ Multiply both sides by AC.

$$AC \times \cos 73° = 63$$ Divide both sides by cos 73°.

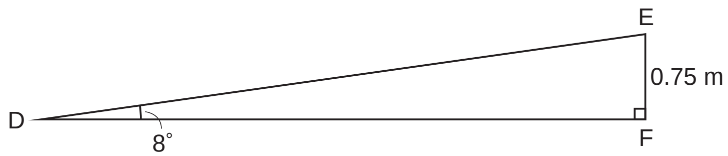
$$AC = \frac{63}{\cos 73°}$$ Use a calculator.

$$AC = 215.4791\ldots$$

The distance between the towers is about 215 m.

Check

1. Sam and Sofia are building a wooden ramp for skateboarding. The height of the ramp is 0.75 m. The ramp makes an angle of 8° with the ground. What length of plywood do Sam and Sofia need for the top of the ramp? Give your answer to the nearest tenth of a metre.



We want to find the length of DE.
The measure of ∠D is known.
The side opposite ∠D is: **EF**
The hypotenuse is: **DE**
So, use the sine ratio.

$\sin D = \dfrac{\mathbf{EF}}{\mathbf{DE}}$ Substitute: **∠D = 8° and EF = 0.75**

$\sin \mathbf{8°} = \dfrac{\mathbf{0.75}}{\mathbf{DE}}$ Multiply both sides by **DE**.

$\mathbf{DE} \sin \mathbf{8°} = \mathbf{0.75}$ Divide both sides by **sin 8°**.

$\mathbf{DE} = \dfrac{\mathbf{0.75}}{\mathbf{\sin 8°}}$

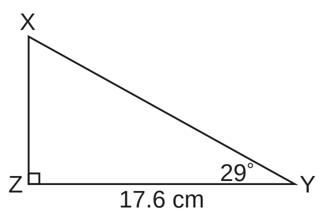
DE = **5.3889…**

Sam and Sofia need about **5.4 m** of plywood.

Practice

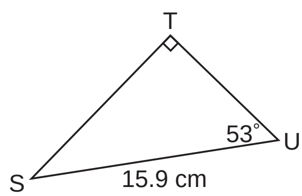
1. Which ratio would you use to find each length?

a) XY



The measure of ∠ **Y** is known.
YZ is the side **adjacent to ∠Y**.
XY is the **hypotenuse**.
So, use the **cosine** ratio.

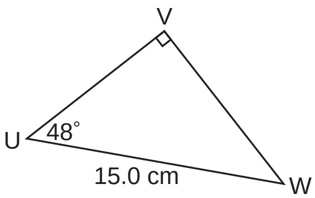
b) ST



The measure of ∠ **U** is known.
ST is the side **opposite ∠U**.
SU is the **hypotenuse**.
So, use the **sine** ratio.

2. Find the length of each indicated side to the nearest tenth of a centimetre.

a) VW



The measure of ∠ **U** is known.

The side opposite ∠ **U** is **VW**.

The hypotenuse is **UW**.

So, use the **sine** ratio.

$$\sin \textbf{U} = \frac{\text{side } \textbf{opposite } \angle \textbf{U}}{\text{hypotenuse}}$$

$$\sin \textbf{U} = \frac{\textbf{VW}}{\textbf{UW}}$$

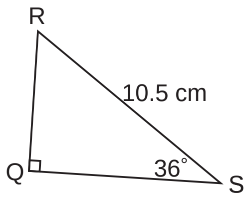
$$\sin \textbf{48°} = \frac{\textbf{VW}}{\textbf{15.0}}$$

$$\textbf{15.0} \sin \textbf{48°} = \textbf{VW}$$

$$\text{VW} = \textbf{11.1471...}$$

VW is about **11.1 cm** long.

b) QR



The measure of ∠ **S** is known.

QR is the **side opposite ∠S**.

RS is the **hypotenuse**.

So, use the **sine** ratio.

$$\textbf{sin S} = \frac{\textbf{side opposite } \angle \textbf{S}}{\textbf{hypotenuse}}$$

$$\textbf{sin S} = \frac{\textbf{QR}}{\textbf{RS}}$$

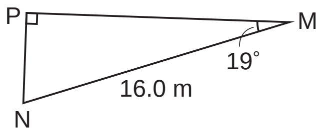
$$\textbf{sin 36°} = \frac{\textbf{QR}}{\textbf{10.5}}$$

$$\textbf{10.5 sin 36°} = \textbf{QR}$$

$$\textbf{QR} = \textbf{6.1717...}$$

QR is about **6.2 cm** long.

3. Find the length of side PM to the nearest tenth of a metre.



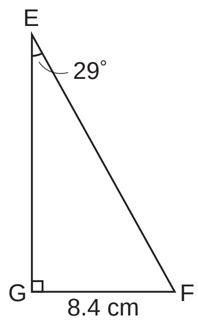
The measure of ∠ **M** is known.
PM is the side **adjacent to ∠M**.
MN is the **hypotenuse**.
So, use the **cosine** ratio.

$$\cos M = \frac{PM}{MN}$$
$$\cos 19° = \frac{PM}{16.0}$$
$$16.0 \cos 19° = PM$$
$$PM = 15.1282\ldots$$

PM is about **15.1 m** long.

4. Find the length of each hypotenuse to the nearest tenth of a centimetre.

a)

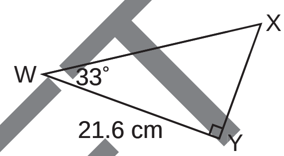


The measure of ∠ **E** is known.
The side opposite ∠ **E** is: **FG**
The hypotenuse is: **EF**
So, use the sine ratio.

$$\sin E = \frac{FG}{EF}$$
$$\sin 29° = \frac{8.4}{EF}$$
$$EF \sin 29° = 8.4$$
$$EF = \frac{8.4}{\sin 29°}$$
$$EF = 17.3263\ldots$$

EF is about **17.3 cm** long.

b)

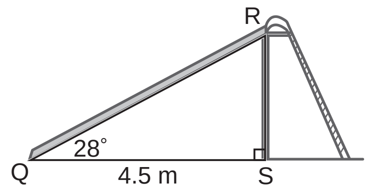


The measure of ∠ **W** is known.
WY is the side **adjacent to ∠W**.
WX is the **hypotenuse**.
So, use the **cosine** ratio.

$$\cos W = \frac{WY}{WX}$$
$$\cos 33° = \frac{21.6}{WX}$$
$$WX \cos 33° = 21.6$$
$$WX = \frac{21.6}{\cos 33°}$$
$$WX = 25.7550\ldots$$

WX is about **25.8 cm** long.

5. A straight slide in a playground makes an angle of 28° with the ground. The slide covers a horizontal distance of 4.5 m. How long is the slide? Give your answer to the nearest tenth of a metre.



The measure of ∠Q is known.

The side adjacent to ∠Q is: **QS**

The hypotenuse is: **QR**

So, use the **cosine** ratio.

$$\cos Q = \frac{QS}{QR}$$

$$\cos 28° = \frac{4.5}{QR}$$

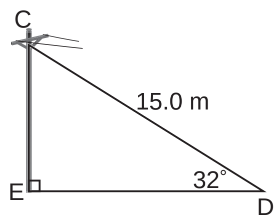
$$QR \cos 28° = 4.5$$

$$QR = \frac{4.5}{\cos 28°}$$

QR = **5.0965…**

The slide is about **5.1 m** long.

6. A 15-m support cable joins the top of a telephone pole to a point on the ground. The cable makes an angle of 32° with the ground. Find the height of the pole to the nearest tenth of a metre.



Use the sine ratio.

$$\sin D = \frac{CE}{CD}$$

$$\sin 32° = \frac{CE}{15.0}$$

$$15.0 \sin 32° = CE$$

$$CE = 7.9487\ldots$$

The height of the pole is about **7.9 m**.

TEACHER NOTE

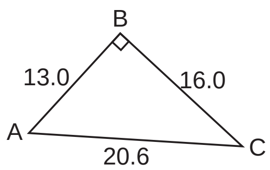
Next Steps: Have students complete questions 6–11 on pages 101 and 102 of the Student Text.

CHECKPOINT 2

Can you ...

- use the sine or cosine ratio to find an angle measure?
- use the sine or cosine ratio to calculate a length?
- use the sine or cosine ratio to solve a problem?

2.4 **1.** Find sin A and cos A to the nearest hundredth.



$\sin A = \dfrac{\textbf{side opposite } \angle \textbf{A}}{\textbf{hypotenuse}}$ $\cos A = \dfrac{\textbf{side adjacent to } \angle \textbf{A}}{\textbf{hypotenuse}}$

$\sin A = \dfrac{\textbf{BC}}{\textbf{AC}}$ $\cos A = \dfrac{\textbf{AB}}{\textbf{AC}}$

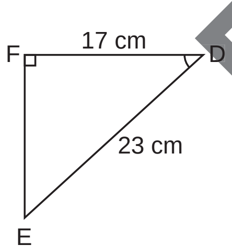
$\sin A = \dfrac{\textbf{16.0}}{\textbf{20.6}}$ $\cos A = \dfrac{\textbf{13.0}}{\textbf{20.6}}$

$\sin A =$ **0.7766...** $\cos A =$ **0.6310...**

$\sin A \doteq$ **0.78** $\cos A \doteq$ **0.63**

2. Find the measure of each indicated angle to the nearest degree.

a)



FD is the **side adjacent to ∠D**.
DE is the **hypotenuse**.
So, use the **cosine** ratio.

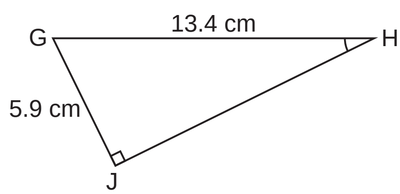
cos $D = \dfrac{\textbf{side adjacent to } \angle \textbf{D}}{\textbf{hypotenuse}}$

cos $D = \dfrac{\textbf{FD}}{\textbf{DE}}$

cos $D = \dfrac{\textbf{17}}{\textbf{23}}$

$\angle D \doteq$ **42°**

b)



GJ is the **side opposite ∠H**.
GH is the **hypotenuse**.
So, use the **sine** ratio.

sin $H = \dfrac{\textbf{side opposite } \angle \textbf{H}}{\textbf{hypotenuse}}$

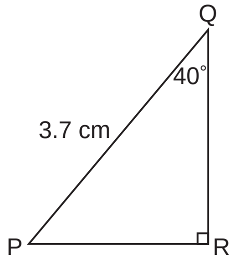
sin $H = \dfrac{\textbf{GJ}}{\textbf{GH}}$

sin $H = \dfrac{\textbf{5.9}}{\textbf{13.4}}$

$\angle H \doteq$ **26°**

2.5 **3.** Find the length of each indicated side to the nearest tenth of a centimetre.

a) PR



The measure of ∠ **Q** is known.

The side opposite ∠ **Q** is **PR**.

The hypotenuse is **PQ**.

So, use the **sine** ratio.

$$\sin Q = \frac{\text{side opposite } \angle Q}{\text{hypotenuse}}$$

$$\sin Q = \frac{PR}{PQ}$$

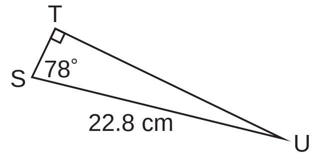
$$\sin 40° = \frac{PR}{3.7}$$

$$3.7 \sin 40° = PR$$

$$PR = 2.3783\ldots$$

PR is about **2.4 cm** long.

b) ST



The measure of ∠ **S** is known.

ST is the side **adjacent to ∠S**.

SU is the **hypotenuse**.

So, use the **cosine** ratio.

$$\cos S = \frac{\text{side adjacent to } \angle S}{\text{hypotenuse}}$$

$$\cos S = \frac{ST}{SU}$$

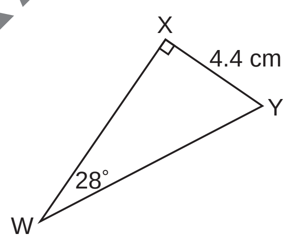
$$\cos 78° = \frac{ST}{22.8}$$

$$22.8 \cos 78° = ST$$

$$ST = 4.7403\ldots$$

ST is about **4.7 cm** long.

4. Find the length of the hypotenuse to the nearest tenth of a centimetre.



The measure of ∠W is known.

The side opposite ∠W is: **XY**

The hypotenuse is: **WY**

Use the sine ratio.

$$\sin W = \frac{XY}{WY}$$

$$\sin 28° = \frac{4.4}{WY}$$

Multiply both sides by **WY**.

$$WY \sin 28° = 4.4$$

Divide both sides by **sin 28°**.

$$WY = \frac{4.4}{\sin 28°}$$

$$WY = 9.3722\ldots$$

WY is about **9.4 cm** long.

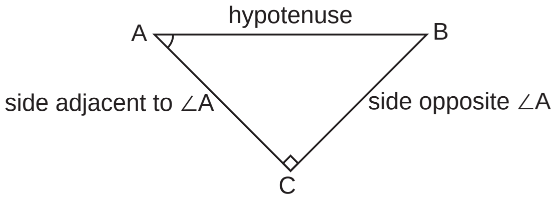
TEACHER NOTE

Next Steps: Have students complete questions 2 and 5 on page 104 of the Student Text.

2.6 Applying the Trigonometric Ratios

FOCUS **Use trigonometric ratios to solve a right triangle.**

When we **solve a triangle**, we find the measures of all the angles and the lengths of all the sides.
To do this we use any of the sine, cosine, and tangent ratios.

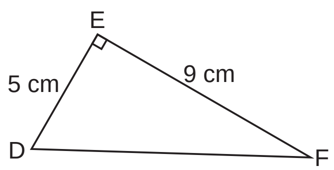


We can use the acronym SOH-CAH-TOA to help us remember these ratios.

$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \dfrac{\text{opposite}}{\text{adjacent}}$

Example 1 Finding the Measures of All Angles

Find all unknown angle measures to the nearest degree.



Solution

Find the measure of ∠D.
EF is the side opposite ∠D.
DE is the side adjacent to ∠D.
So, use the tangent ratio.

We could have used the tangent ratio to find ∠F.

$\tan D = \dfrac{\text{opposite}}{\text{adjacent}}$

$\tan D = \dfrac{EF}{DE}$ Substitute: EF = 9 and DE = 5

$\tan D = \dfrac{9}{5}$

$\angle D \doteq 61°$

The acute angles in a right triangle have a sum of 90°.
So, $\angle F = 90° - \angle D$

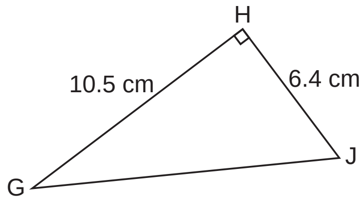
$\angle F \doteq 90° - 61°$

$\angle F \doteq 29°$

Check

1. Find all unknown angle measures to the nearest degree.

a)



Find the measure of ∠G.

HJ is the side **opposite ∠G**.

GH is the side **adjacent to ∠G**.

So, use the **tangent** ratio.

We could use the tangent ratio to check the measure of ∠J.

tan G = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan G = $\dfrac{\textbf{HJ}}{\textbf{GH}}$

tan G = $\dfrac{\textbf{6.4}}{\textbf{10.5}}$

∠G ≐ **31°**

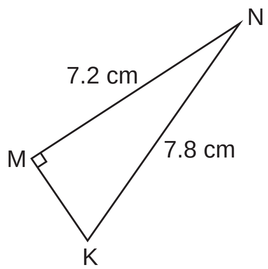
The acute angles have a sum of 90°.

So, ∠J = 90° − **∠G**

∠J ≐ 90° − **31°**

∠J ≐ **59°**

b)



Find the measure of ∠K.

MN is the side **opposite** ∠K.

NK is the **hypotenuse**.

So, use the **sine** ratio.

We could use the cosine ratio to check the measure of ∠N.

sin K = $\dfrac{\textbf{opposite}}{\textbf{hypotenuse}}$

sin K = $\dfrac{\textbf{MN}}{\textbf{NK}}$

sin K = $\dfrac{\textbf{7.2}}{\textbf{7.8}}$

∠K ≐ **67°**

The acute angles have a sum of 90°.

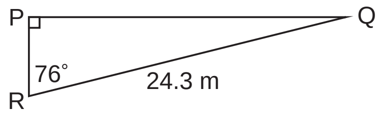
So, ∠N = 90° − **∠K**

∠N ≐ 90° − **67°**

∠N ≐ **23°**

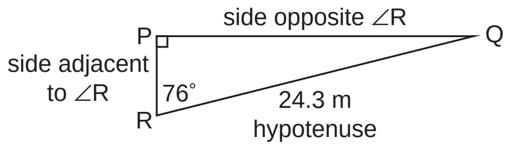
Example 2 Finding the Lengths of All Sides

Find all unknown side lengths to the nearest tenth of a metre.



Solution

Find the length of PR.
PR is the side adjacent to ∠R.
QR is the hypotenuse.
So, use the cosine ratio.



$$\cos R = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos R = \frac{PR}{QR}$$ Substitute: ∠R = 76° and QR = 24.3

$$\cos 76° = \frac{PR}{24.3}$$ Multiply both sides by 24.3.

$$24.3 \cos 76° = PR$$

$$PR = 5.8787\ldots$$

PR is about 5.9 m long.

Find the length of PQ.
PQ is the side opposite ∠R.
QR is the hypotenuse.
So, use the sine ratio.

$$\sin R = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin R = \frac{PQ}{QR}$$ Substitute: ∠R = 76° and QR = 24.3

$$\sin 76° = \frac{PQ}{24.3}$$ Multiply both sides by 24.3.

$$24.3 \sin 76° = PQ$$

$$PQ = 23.5781\ldots$$

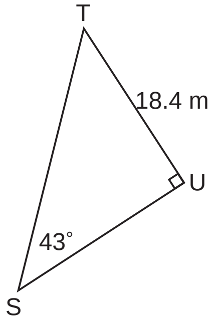
PQ is about 23.6 m long.

We could use the Pythagorean Theorem to check the side lengths.

TEACHER NOTE
If students need to review the Pythagorean Theorem, refer them to Chapter 1, Lesson 1.4 of this text.

Check

1. Find all unknown side lengths to the nearest tenth of a metre.



Find the length of ST.

TU is the side **opposite** ∠S.

ST is the **hypotenuse**.

So, use the **sine** ratio.

sin S = $\dfrac{\textbf{opposite}}{\textbf{hypotenuse}}$

sin S = $\dfrac{\textbf{TU}}{\textbf{ST}}$

$\textbf{sin } \mathbf{43^\circ} = \dfrac{\mathbf{18.4}}{\textbf{ST}}$

$\textbf{ST sin } \mathbf{43^\circ = 18.4}$

$\textbf{ST} = \dfrac{\mathbf{18.4}}{\textbf{sin } \mathbf{43^\circ}}$

ST = **26.9795…**

ST is about **27.0 m** long.

Find the length of SU.

TU is the side **opposite** ∠S.

SU is the side **adjacent to** ∠S.

So, use the **tangent** ratio.

tan S = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan S = $\dfrac{\textbf{TU}}{\textbf{SU}}$

$\textbf{tan } \mathbf{43^\circ} = \dfrac{\mathbf{18.4}}{\textbf{SU}}$

$\textbf{SU tan } \mathbf{43^\circ = 18.4}$

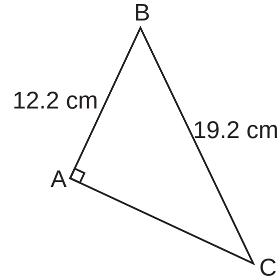
$\textbf{SU} = \dfrac{\mathbf{18.4}}{\textbf{tan } \mathbf{43^\circ}}$

SU = **19.7315…**

SU is about **19.7 m** long.

Example 3 Solving a Triangle

Solve this triangle.
Give angle measures to the nearest degree.
Give side lengths to the nearest tenth of a centimetre.



Solution

Find the measure of $\angle$B.
AB is the side adjacent to $\angle$B.
BC is the hypotenuse.
So, use the cosine ratio.

We know the lengths of two sides.

$$\cos B = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos B = \frac{AB}{BC}$$ Substitute: AB = 12.2 and BC = 19.2

$$\cos B = \frac{12.2}{19.2}$$

$\angle B = 50.5491\ldots°$
$\angle B \doteq 51°$

The acute angles in a right triangle have a sum of 90°.
So, $\angle C = 90° - \angle B$
$\angle C \doteq 90° - 51°$
$\angle C \doteq 39°$

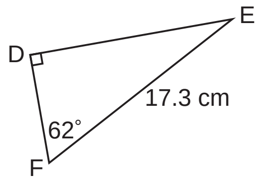
Find the length of AC.
Use the Pythagorean Theorem to find AC.

We could have used the sine ratio and the exact measure of $\angle$B to find AC.

$AC^2 = BC^2 - AB^2$
$AC^2 = 19.2^2 - 12.2^2$
$AC^2 = 219.8$
$AC = \sqrt{219.8}$
$AC = 14.8256\ldots$
AC is about 14.8 cm long.

Check

1. Solve this triangle. Give side lengths to the nearest tenth of a centimetre.



We know the length of one side and the measure of one acute angle.

The acute angles have a sum of **90°**.

So, ∠E = **90°** − **∠F**

∠E = **90°** − **62°**

∠E = **28°**

Find the length of DF.

DF is the side **adjacent to** ∠F.

EF is the **hypotenuse**.

So, use the **cosine** ratio.

$$\textbf{cos}\ F = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$$

$$\textbf{cos}\ F = \frac{\textbf{DF}}{\textbf{EF}}$$

$$\cos 62° = \frac{DF}{17.3}$$

$$17.3 \cos 62° = DF$$

$$DF = 8.1218\ldots$$

DF is about **8.1 cm** long.

Find the length of DE.

DE is the side **opposite** ∠F.

EF is the **hypotenuse**.

So, use the **sine** ratio.

$$\textbf{sin}\ F = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$$

$$\textbf{sin}\ F = \frac{\textbf{DE}}{\textbf{EF}}$$

$$\sin 62° = \frac{DE}{17.3}$$

$$17.3 \sin 62° = DE$$

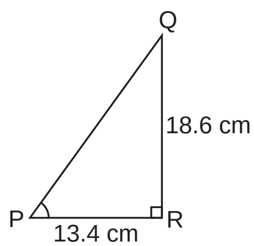
$$DE = 15.2749\ldots$$

DE is about **15.3 cm** long.

Practice

1. Which ratio would you use to find the measure of each angle?

a) ∠P

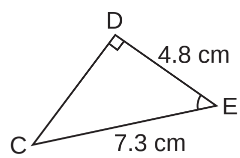


QR is the side **opposite ∠P**.

PR is the side **adjacent to ∠P**.

So, use the **tangent** ratio.

b) ∠E



Remember the acronym SOH–CAH–TOA.

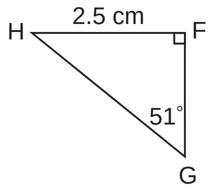
DE is the **side adjacent to ∠E**.

CE is the **hypotenuse**.

So, use the **cosine** ratio.

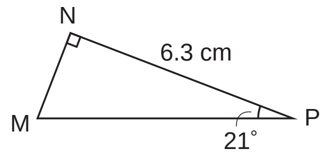
2. Which ratio would you use to find the length of each indicated side?

a) GH



HF is the **side opposite ∠G**.
GH is the **hypotenuse**.
So, use the **sine** ratio.

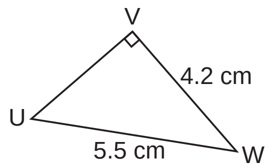
b) MN



MN is the **side opposite ∠P**.
NP is the **side adjacent to ∠P**.
So, use the **tangent** ratio.

3. Find all unknown angle measures to the nearest degree.

a)



Find the measure of ∠U.

sin U = $\dfrac{\textbf{opposite}}{\textbf{hypotenuse}}$

sin U = $\dfrac{\textbf{VW}}{\textbf{UW}}$

sin U = $\dfrac{\textbf{4.2}}{\textbf{5.5}}$

∠U ≐ **50°**

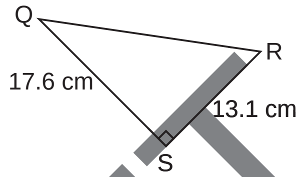
The acute angles have a sum of 90°.

So, ∠W = 90° − **∠U**

∠W ≐ 90° − **50°**

∠W ≐ **40°**

b)



Find the measure of ∠Q.

tan Q = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan Q = $\dfrac{\textbf{RS}}{\textbf{QS}}$

tan Q = $\dfrac{\textbf{13.1}}{\textbf{17.6}}$

∠Q ≐ **37°**

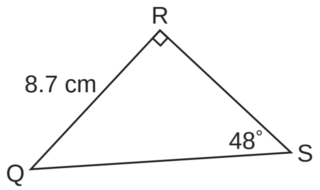
The acute angles have a sum of 90°.

So, ∠R = **90° − ∠Q**

∠R ≐ **90° − 37°**

∠R ≐ **53°**

4. Find all unknown side lengths to the nearest tenth of a centimetre.



Find the length of QS.

sin S = $\dfrac{\textbf{opposite}}{\textbf{hypotenuse}}$

sin S = $\dfrac{\textbf{QR}}{\textbf{QS}}$

$\mathbf{\sin 48° = \dfrac{8.7}{QS}}$

$\mathbf{QS \sin 48° = 8.7}$

$\mathbf{QS = \dfrac{8.7}{\sin 48°}}$

$\mathbf{QS = 11.7070...}$

QS is about **11.7 cm** long.

Find the length of RS.

tan S = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan S = $\dfrac{\textbf{QR}}{\textbf{RS}}$

$\mathbf{\tan 48° = \dfrac{8.7}{RS}}$

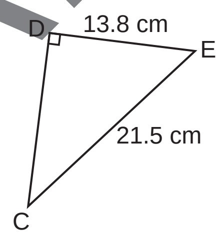
$\mathbf{RS \tan 48° = 8.7}$

$\mathbf{RS = \dfrac{8.7}{\tan 48°}}$

$\mathbf{RS = 7.8335...}$

RS is about **7.8 cm** long.

5. Solve this triangle. Give angle measures to the nearest degree. Give side lengths to the nearest tenth of a centimetre.



Find the measure of ∠E.

cos E = $\dfrac{\textbf{adjacent}}{\textbf{hypotenuse}}$

cos E = $\dfrac{\textbf{DE}}{\textbf{CE}}$

cos E = $\dfrac{\mathbf{13.8}}{\mathbf{21.5}}$

∠E ≐ **50°**

The acute angles have a sum of 90°.

So, ∠C = 90° − **∠E**

∠C ≐ 90° − **50°**

∠C ≐ **40°**

Find the length of CD.

Use the Pythagorean Theorem.

$\mathbf{CD^2 = CE^2 - DE^2}$

$\mathbf{CD^2 = 21.5^2 - 13.8^2}$

$\mathbf{CD^2 = 271.81}$

$\mathbf{CD = \sqrt{271.81}}$

$\mathbf{CD = 16.4866...}$

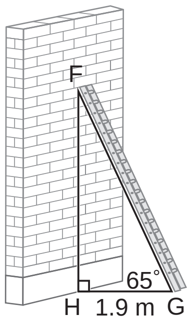
CD is about **16.5 cm** long.

6. The base of a ladder is on level ground 1.9 m from a wall. The ladder leans against the wall. The angle between the ladder and the ground is 65°.

a) How far up the wall does the ladder reach?

b) How long is the ladder?

Give your answers to the nearest tenth of a metre.



a) We want to find the length of FH.

The measure of ∠G is known.

FH is the side opposite ∠G.

GH is the side adjacent to ∠G.

So, use the tangent ratio.

$$\tan G = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan G = \frac{FH}{GH}$$

$$\tan 65° = \frac{FH}{1.9}$$

$$1.9 \tan 65° = FH$$

$$FH = 4.0745\ldots$$

The ladder reaches the wall at a height of about **4.1 m**.

b) We want to find the length of FG.

GH is the side adjacent to ∠G.

FG is the hypotenuse.

So, use the cosine ratio.

$$\cos G = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos G = \frac{GH}{FG}$$

$$\cos 65° = \frac{1.9}{FG}$$

$$FG \cos 65° = 1.9$$

$$FG = \frac{1.9}{\cos 65°}$$

$$FG = 4.4957\ldots$$

The ladder is about **4.5 m** long.

TEACHER NOTE

Next Steps: Have students complete questions 6, 7, 8, 12, and 13 on pages 111 and 112 of the Student Text.

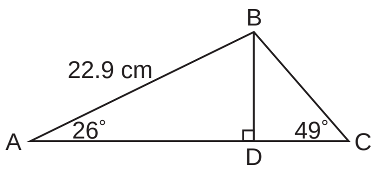
2.7 Solving Problems Involving More than One Right Triangle

FOCUS **Use trigonometric ratios to solve problems that involve more than one right triangle.**

When a problem involves more than one right triangle, we can use information from one triangle to solve the other triangle.

Example 1 Solving a Problem with Two Triangles

Find the length of BC to the nearest tenth of a centimetre.



To solve a right triangle we must know:
- *the lengths of two sides, or*
- *the length of one side and the measure of one acute angle*

Solution

First use △ABD to find the length of BD.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

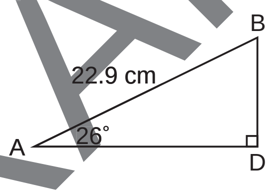
$$\sin A = \frac{BD}{AB}$$

$$\sin 26° = \frac{BD}{22.9}$$

$$22.9 \sin 26° = BD$$

$$BD = 10.0386\ldots$$

Do not clear the calculator screen.



Side BD is common to both triangles.

In △BCD, find the length of BC.

$$\sin C = \frac{\text{opposite}}{\text{hypotenuse}}$$

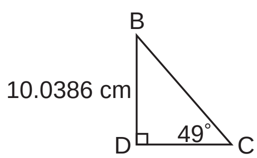
$$\sin C = \frac{BD}{BC}$$

$$\sin 49° = \frac{10.0386\ldots}{BC}$$

$$BC \sin 49° = 10.0386\ldots$$

$$BC = \frac{10.0386\ldots}{\sin 49°}$$

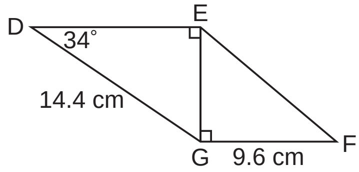
$$BC = 13.3014\ldots$$



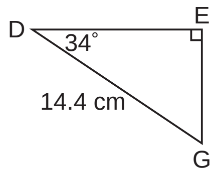
BC is about 13.3 cm long.

Check

1. Find the measure of ∠F to the nearest degree.



Use △DEG to find the length of EG.
Use the sine ratio.



Side EG is common to both triangles.

$$\sin D = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$$

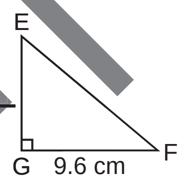
$$\sin D = \frac{\textbf{EG}}{\textbf{DG}}$$

$$\sin \ \textbf{34°} = \frac{\textbf{EG}}{\textbf{14.4}}$$

14.4 sin 34° = EG

EG = **8.0523...**

8.0523 cm



In △EFG, use the **tangent** ratio to find ∠F.

$$\textbf{tan F} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\textbf{tan F} = \frac{\textbf{EG}}{\textbf{FG}}$$

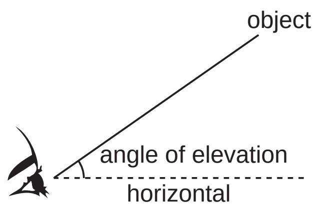
$$\textbf{tan F} = \frac{\textbf{8.0523...}}{\textbf{9.6}}$$

∠F = 39.9895...°

The measure of ∠F is about **40°**.

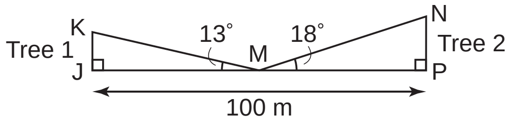
TEACHER NOTE
Show students how to use the ANS key to find ∠F directly. 8.0523... is already on the calculator screen. Input:
÷ 9 • 6 ENTER
2nd TAN 2nd (–) ENTER
to display 39.98956614.

The **angle of elevation** is the angle between the horizontal and a person's line of sight to an object above.



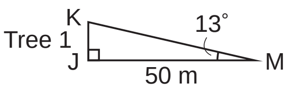
Example 2 Solving a Problem Involving Angle of Elevation

Jason is lying on the ground midway between two trees, 100 m apart.
The angles of elevation of the tops of the trees are 13° and 18°. How much taller is one tree than the other? Give the answer to the nearest tenth of a metre.



Solution

Jason is midway between the trees.
So, the distance from Jason to the base of each tree is: $\frac{100\text{ m}}{2} = 50\text{ m}$
Use $\triangle$JKM to find the length of JK.



We know $\angle$M = 13°.
JK is opposite $\angle$M.
JM is adjacent to $\angle$M.
Use the tangent ratio.

$$\tan M = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan M = \frac{JK}{JM}$$

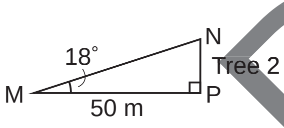
Substitute: $\angle$M = 13° and JM = 50

$$\tan 13° = \frac{JK}{50}$$

$$50 \tan 13° = JK$$

$$JK = 11.5434\ldots$$

Use $\triangle$MNP to find the length of NP.



We know $\angle$M = 18°.
NP is opposite $\angle$M.
MP is adjacent to $\angle$M.
Use the tangent ratio.

$$\tan M = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan M = \frac{NP}{MP}$$

Substitute: $\angle$M = 18° and MP = 50

$$\tan 18° = \frac{NP}{50}$$

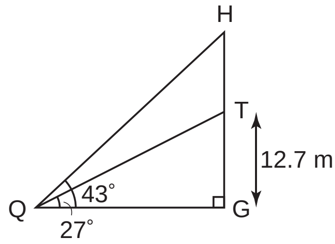
$$50 \tan 18° = NP$$

$$NP = 16.2459\ldots$$

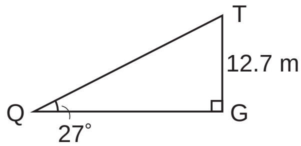
To find how much taller one tree is than the other, subtract:
16.2459 m − 11.5434 m = 4.7025 m
One tree is about 4.7 m taller than the other.

Check

1. The angle of elevation of the top of a tree, T, is 27°. From the same point on the ground, the angle of elevation of a hawk, H, flying directly above the tree is 43°. The tree is 12.7 m tall. How high is the hawk above the ground? Give your answer to the nearest tenth of a metre.



We want to find the length of HG.
Use △QTG to find the length of QG.
Use the tangent ratio.



$\tan Q = \dfrac{\textbf{opposite}}{\textbf{adjacent}}$

$\tan Q = \dfrac{\textbf{TG}}{\textbf{QG}}$ Substitute: **∠Q = 27°** and **TG = 12.7**

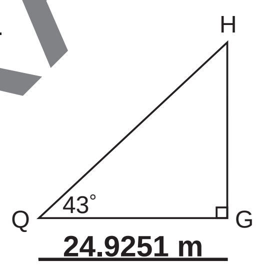
$\tan \underline{\textbf{27°}} = \dfrac{\textbf{12.7}}{\textbf{QG}}$

QG tan 27° = 12.7

$\textbf{QG} = \dfrac{\textbf{12.7}}{\textbf{tan 27°}}$

QG = **24.9251…**

In △QHG, use the tangent ratio to find HG.



$\textbf{tan Q} = \dfrac{\textbf{opposite}}{\textbf{adjacent}}$

$\textbf{tan Q} = \dfrac{\textbf{HG}}{\textbf{QG}}$

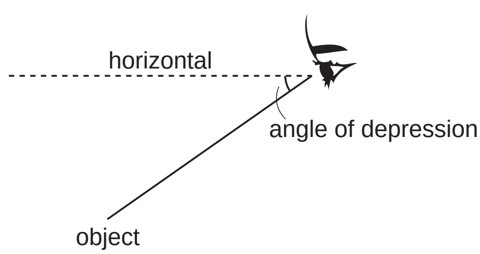
$\textbf{tan 43°} = \dfrac{\textbf{HG}}{\textbf{24.9251...}}$

24.9251… × tan 43° = HG

HG = **23.2430…**

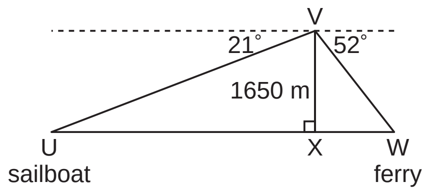
The hawk is about **23.2 m** above the ground.

The **angle of depression** is the angle between the horizontal and a person's line of sight to an object below.



Example 3 Solving a Problem Involving Angle of Depression

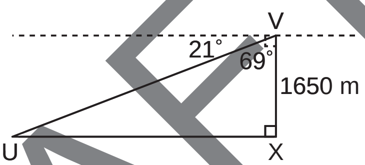
From a small plane, V, the angle of depression of a sailboat is 21°.
The angle of depression of a ferry on the other side of the plane is 52°.
The plane is flying at an altitude of 1650 m.
How far apart are the boats, to the nearest metre?



Solution

We want to find the length of UW.
The angle of depression of the sailboat is 21°.
So, in $\triangle$UVX, $\angle$V = 90° − 21°, or 69°.

Use $\triangle$UVX to find the length of UX.



We know $\angle V = 69°$.
UX is opposite $\angle V$.
VX is adjacent to $\angle V$.
So, use the tangent ratio.

$$\tan V = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan V = \frac{UX}{VX}$$ Substitute: $\angle V = 69°$ and VX = 1650

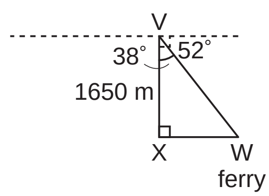
$$\tan 69° = \frac{UX}{1650}$$

$$1650 \tan 69° = UX$$

$$UX = 4298.3969\ldots$$

The angle of depression of the ferry is 52°.
So, $\angle$V in $\triangle$VWX is: 90° − 52°, or 38°.

Use $\triangle$VWX to find the length of WX.



We know $\angle V = 38°$.
WX is opposite $\angle V$.
VX is adjacent to $\angle V$.
So, use the tangent ratio.

$$\tan V = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan V = \frac{WX}{VX}$$ Substitute: $\angle V = 38°$ and VX = 1650

$$\tan 38° = \frac{WX}{1650}$$

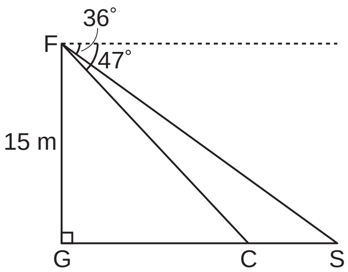
$$1650 \tan 38° = WX$$

$$WX = 1289.1212\ldots$$

To find the distance between the boats, add:
4298.3969 m + 1289.1212 m = 5587.5181 m
The boats are about 5588 m apart.

Check

1. This diagram shows a falcon, F, on a tree, with a squirrel, S, and a chipmunk, C, on the ground. From the falcon, the angles of depression of the animals are 36° and 47°. How far apart are the animals on the ground to the nearest tenth of a metre?

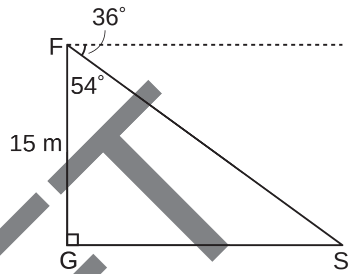


We want to find the length of CS.

CS = GS − GC

The angle of depression of the squirrel is **36°**.

So, ∠F in △FSG is: 90° − **36°**, or **54°**.



Use △FSG to find the length of GS.

$$\tan \mathbf{F} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\tan \mathbf{F} = \frac{\mathbf{GS}}{\mathbf{FG}}$$

$$\tan \mathbf{54°} = \frac{\mathbf{GS}}{\mathbf{15}}$$

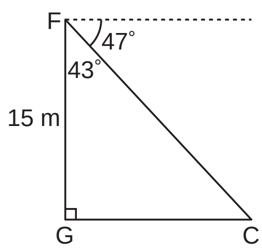
15 tan 54° = GS

GS = **20.6457…**

The angle of depression of the chipmunk is **47°**.

So, ∠F in △FCG is: 90° − **47°**, or **43°**.

Use △FCG to find the length of GC.



$$\textbf{tan F} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\textbf{tan F} = \frac{\mathbf{GC}}{\mathbf{FG}}$$

$$\textbf{tan 43°} = \frac{\mathbf{GC}}{\mathbf{15}}$$

15 tan 43° = GC

GC = **13.9877…**

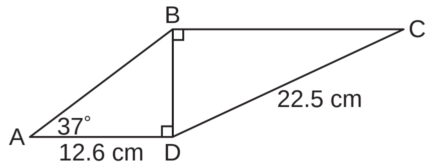
To find the distance between the animals, subtract:

20.6457 m − **13.9877 m** = **6.6580 m**

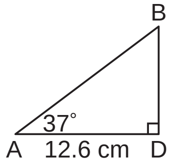
The animals on the ground are about **6.7 m** apart.

Practice

1. Find the measure of ∠C to the nearest degree.



Use △ABD to find the length of BD.



Use the tangent ratio.

tan A = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan A = $\dfrac{\textbf{BD}}{\textbf{AD}}$

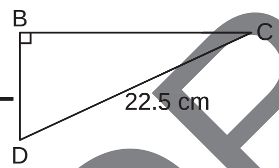
tan **37°** = $\dfrac{\textbf{BD}}{\textbf{12.6}}$

12.6 tan 37° = BD

BD = **9.4947…**

In △BCD, use the **sine** ratio to find ∠C.

9.4947 cm



sin C = $\dfrac{\textbf{opposite}}{\textbf{hypotenuse}}$

sin C = $\dfrac{\textbf{BD}}{\textbf{CD}}$

sin C = $\dfrac{\textbf{9.4947...}}{\textbf{22.5}}$

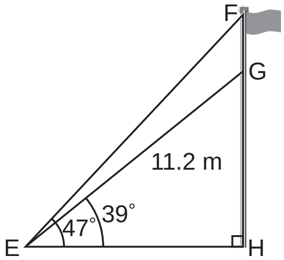
∠C = **24.9603…°**

The measure of ∠C is about **25°**.

TEACHER NOTE

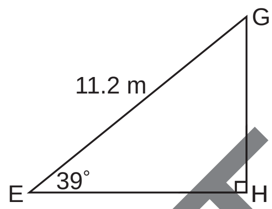
To find ∠C directly, with 9.4947... already on the calculator screen, input: ÷ 2 2 . 5 ENTER 2nd SIN 2nd (–) ENTER to display 24.96030534.

2. Two guy wires support a flagpole, FH. The first wire is 11.2 m long and has an angle of inclination of 39°. The second wire has an angle of inclination of 47°. How tall is the flagpole to the nearest tenth of a metre?



Recall that the angle the wire makes with the ground is called the ***angle of inclination****.*

We want to find the length of FH.
Use △EGH to find the length of EH.
Use the cosine ratio.



$$\cos E = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$$

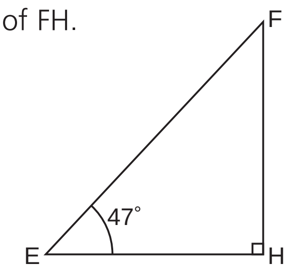
$$\cos E = \frac{\textbf{EH}}{\textbf{EG}}$$

$$\cos \underline{\ \textbf{39°}\ } = \frac{\textbf{EH}}{\textbf{11.2}}$$

11.2 cos 39° = EH

EH = **8.7040…**

In △EFH, use the **tangent** ratio to find the length of FH.



8.7040 m

$$\textbf{tan E} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\textbf{tan E} = \frac{\textbf{FH}}{\textbf{EH}}$$

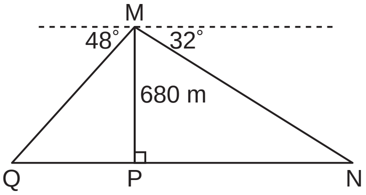
$$\textbf{tan 47°} = \frac{\textbf{FH}}{\textbf{8.7040...}}$$

8.7040… × tan 47° = FH

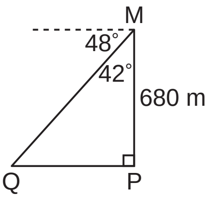
FH = **9.3339…**

The flagpole is about **9.3 m** tall.

3. A mountain climber is on top of a mountain that is 680 m high. The angles of depression of two points on opposite sides of the mountain are 48° and 32°. How long would a tunnel be that runs between the two points? Give your answer to the nearest metre.



We want to find the length of QN.
The angle of depression of point Q is **48°**.
So, ∠M in △PQM is: 90° − **48°**, or **42°**.



Use △PQM to find the length of PQ.
Use the **tangent** ratio.

$$\textbf{tan M} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

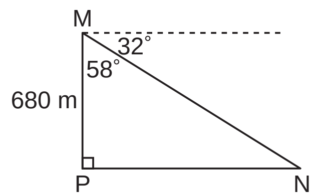
$$\textbf{tan M} = \frac{\textbf{PQ}}{\textbf{MP}}$$

$$\textbf{tan 42°} = \frac{\textbf{PQ}}{\textbf{680}}$$

680 tan 42° = PQ

PQ = **612.2747...**

The angle of depression of point N is **32°**.
So, ∠M in △PMN is: 90° − **32°**, or **58°**.



Use △PMN to find the length of PN.
Use the **tangent** ratio.

$$\textbf{tan M} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\textbf{tan M} = \frac{\textbf{NP}}{\textbf{MP}}$$

$$\textbf{tan 58°} = \frac{\textbf{NP}}{\textbf{680}}$$

680 tan 58° = NP

NP = **1088.2274...**

The length of the tunnel is: **QN** = **QP** + **PN**

QN = 612.2747 m + 1088.2274 m

QN = **1700.5021**

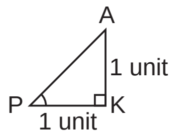
The tunnel would be about **1701 m** long.

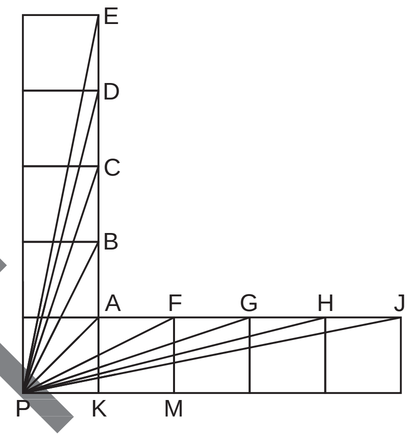
TEACHER NOTE
Next Steps: Have students complete questions 3, 4, 5, 6, 8, 9, and 11 on pages 118 and 119 of the Student Text.

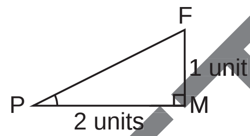
Chapter 2 Puzzle

Angle Mania!

A. Find the angles of inclination of the diagonals shown.
Assume the squares have side length 1 unit.







Sample Solution:

Diagonal PA: tan P = $\frac{1}{1}$; ∠P = 45°

Diagonal PB: tan P = $\frac{2}{1}$; ∠P ≐ 63°

Diagonal PC: tan P = 3; ∠P ≐ 72°

Diagonal PD: tan P = 4; ∠P ≐ 76°

Diagonal PE: tan P = 5; ∠P ≐ 79°

Diagonal PF: tan P = $\frac{1}{2}$; ∠P ≐ 27°

Diagonal PG: tan P = $\frac{1}{3}$; ∠P ≐ 18°

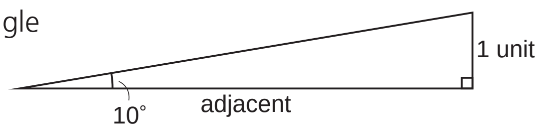
Diagonal PH: tan P = $\frac{1}{4}$; ∠P ≐ 14°

Diagonal PJ: tan P = $\frac{1}{5}$; ∠P ≐ 11°

B. How many squares would be needed on the vertical rectangle for a diagonal to have an angle of inclination greater than:

- 80°? **tan 80° = 5.6712…** **6 squares**
- 85°? **tan 85° = 11.4300…** **12 squares**
- 88°? **tan 88° = 28.6362…** **29 squares**
- 89°? **tan 89° = 57.2899…** **58 squares**

C. How many squares would be needed on the horizontal rectangle for a diagonal to have an angle of inclination less than:



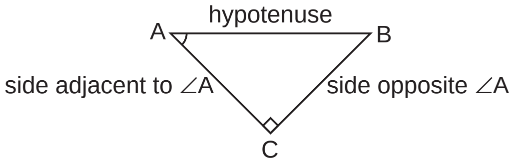
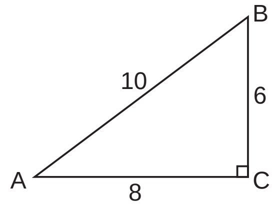
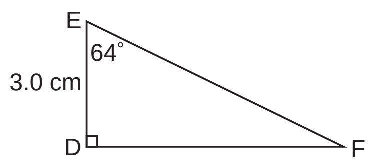
- 10°? **tan 10° = 0.1763…** $\frac{1}{\text{adjacent}} = 0.1763\ldots$

  adjacent $= \frac{1}{0.1763\ldots}$

  adjacent = 5.6712… **6 squares**
- 5°? **tan 5° = 0.0874…** $\frac{1}{0.0874\ldots} = 11.4300\ldots$ **12 squares**
- 2°? **tan 2° = 0.0349…** $\frac{1}{0.0349\ldots} = 28.6362\ldots$ **29 squares**
- 1°? **tan 1° = 0.0174…** $\frac{1}{0.0174\ldots} = 57.2899\ldots$ **58 squares**

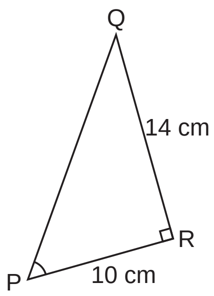
TEACHER NOTE
Strategies may vary. Some students may use guess and test.

Chapter 2 Study Guide

Skill	Description	Example
Find a trigonometric ratio.	In $\triangle ABC$,  $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \frac{\text{opposite}}{\text{adjacent}}$	 $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin A = \frac{BC}{AB}$ $\sin A = \frac{6}{10}$, or 0.6
Find the measure of an angle.	To find the measure of an acute angle in a right triangle: **1.** Use the given lengths to write a trigonometric ratio. **2.** Use the inverse function on a scientific calculator to find the measure of the angle.	To find the measure of $\angle B$ in $\triangle ABC$ above: $\tan B = \frac{\text{opposite}}{\text{adjacent}}$ $\tan B = \frac{AC}{BC}$ $\tan B = \frac{8}{6}$ $\angle B = \tan^{-1}\left(\frac{8}{6}\right)$ $\angle B \doteq 53°$
Find the length of a side.	To find the length of a side in a right triangle: **1.** Use the measure of an angle and the length of a related side to write an equation using a trigonometric ratio. **2.** Solve the equation.	To find the length of EF in $\triangle DEF$:  $\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos E = \frac{DE}{EF}$ $\cos 64° = \frac{3.0}{EF}$ $EF \cos 64° = 3.0$ $EF = \frac{3.0}{\cos 64°}$ $EF = 6.8435\ldots$ $EF \doteq 6.8$ cm

Chapter 2 Review

2.1 **1.** Find the measure of ∠P to the nearest degree.



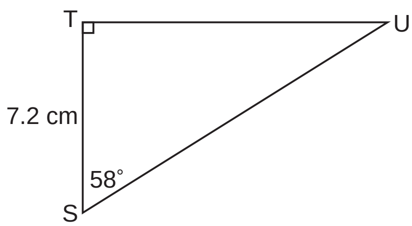
tan P = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan P = $\dfrac{\textbf{QR}}{\textbf{PR}}$

tan P = $\dfrac{\textbf{14}}{\textbf{10}}$

∠P ≐ **54°**

2.2 **2.** Find the length of TU to the nearest tenth of a centimetre.



tan S = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan S = $\dfrac{\textbf{TU}}{\textbf{ST}}$

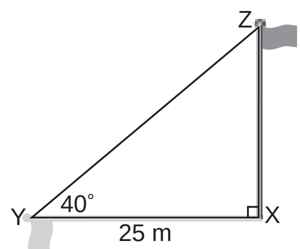
tan **58°** = $\dfrac{\textbf{TU}}{\textbf{7.2}}$

7.2 tan 58° = TU

TU = **11.5224…**

TU is about **11.5 cm** long.

3. A flagpole casts a shadow that is 25 m long when the angle between the sun's rays and the ground is 40°. What is the height of the flagpole to the nearest metre?



ZX is the side opposite ∠Y.

XY is the side adjacent to ∠Y.

tan Y = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan Y = $\dfrac{\textbf{ZX}}{\textbf{XY}}$

tan 40° = $\dfrac{\textbf{ZX}}{\textbf{25}}$

25 tan 40° = ZX

ZX = **20.9774…**

The flagpole is about **21 m** high.

2.3 **4.** Use the information in the diagram to find the height of the tower of a wind turbine observed with a drinking-straw clinometer. Give the answer to the nearest tenth of a metre.

∠A = 90° − **17°**, or **73°**

Side opposite ∠A: **BC**

Side adjacent to ∠A: **AC**

tan A = $\dfrac{\textbf{opposite}}{\textbf{adjacent}}$

tan **73°** = $\dfrac{\textbf{BC}}{\textbf{AC}}$

tan 73° = $\dfrac{\textbf{BC}}{\textbf{15}}$

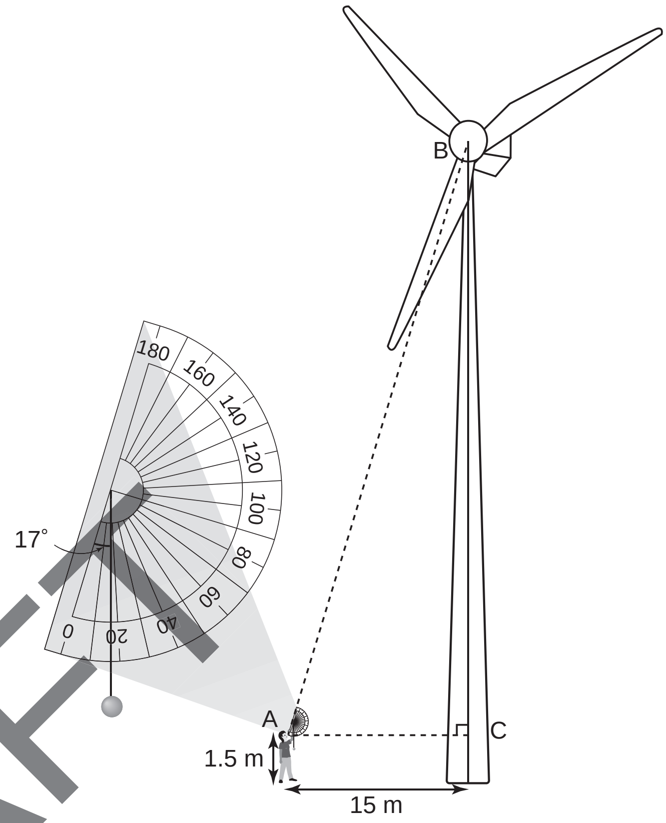
15 tan 73° = BC

BC = **49.0627…**

So, height of tower = **49.0627 m** + **1.5 m**

= **50.5627 m**

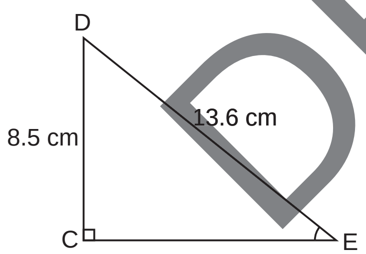
The height of the tower is about **50.6 m**.



This diagram is not drawn to scale.

2.4 **5.** Find the measure of each indicated angle to the nearest degree.

a)



CD is the **side opposite ∠E**.

DE is the **hypotenuse**.

So, use the **sine** ratio.

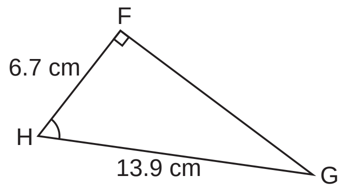
sin E = $\dfrac{\textbf{opposite}}{\textbf{hypotenuse}}$

sin E = $\dfrac{\textbf{CD}}{\textbf{DE}}$

sin E = $\dfrac{\textbf{8.5}}{\textbf{13.6}}$

∠E ≐ **39°**

b)



HF is the **side adjacent to ∠H**.

GH is the **hypotenuse**.

So, use the **cosine** ratio.

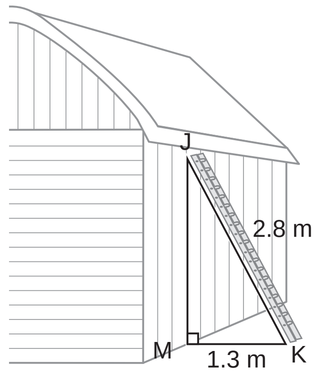
cos H = $\dfrac{\textbf{adjacent}}{\textbf{hypotenuse}}$

cos H = $\dfrac{\textbf{HF}}{\textbf{GH}}$

cos H = $\dfrac{\textbf{6.7}}{\textbf{13.9}}$

∠H ≐ **61°**

6. A 2.8-m ladder is leaning against a barn, as shown. What angle does the ladder make with the barn? Give your answer to the nearest degree.



Assume the ground is horizontal.

We want to find the measure of ∠ **J** .
KM is the **side opposite ∠J** .
JK is the **hypotenuse** .
So, use the **sine** ratio.

$$\sin J = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin J = \frac{KM}{JK}$$

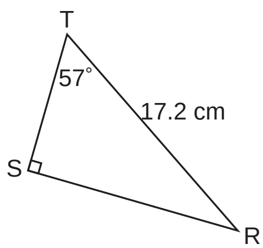
$$\sin J = \frac{1.3}{2.8}$$

$\angle J \doteq$ **28°**

The angle the ladder makes with the barn is about **28°** .

2.5 **7.** Find the length of each indicated side to the nearest tenth of a centimetre.

a) RS



The measure of ∠T is known.
Use the **sine** ratio.

$$\sin T = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin T = \frac{RS}{TR}$$

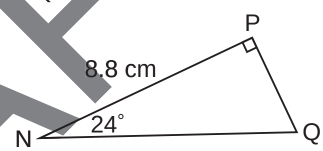
$$\sin 57° = \frac{RS}{17.2}$$

$$17.2 \sin 57° = RS$$

$$RS = 14.4251\ldots$$

RS is about **14.4 cm** long.

b) NQ



The measure of ∠N is known.
Use the **cosine** ratio.

$$\cos N = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos N = \frac{NP}{NQ}$$

$$\cos 24° = \frac{8.8}{NQ}$$

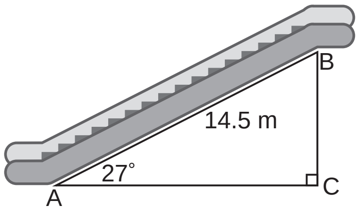
$$NQ \cos 24° = 8.8$$

$$NQ = \frac{8.8}{\cos 24°}$$

$$NQ = 9.6327\ldots$$

NQ is about **9.6 cm** long.

8. An escalator is 14.5 m long. The escalator makes an angle of 27° with the ground. What is the height of the escalator? Give your answer to the nearest tenth of a metre.



To find the length of BC, use the **sine** ratio.

$$\textbf{sin A} = \frac{\textbf{opposite}}{\textbf{hypotenuse}}$$

$$\textbf{sin A} = \frac{\textbf{BC}}{\textbf{AB}}$$

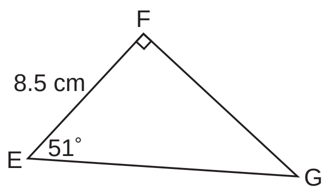
$$\textbf{sin 27°} = \frac{\textbf{BC}}{\textbf{14.5}}$$

14.5 sin 27° = BC

BC = **6.5828…**

The escalator is about **6.6 m** high.

2.6 **9.** Solve this triangle. Give side lengths to the nearest tenth of a centimetre.



Find the length of FG.

Use the **tangent** ratio.

$$\textbf{tan E} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\textbf{tan E} = \frac{\textbf{FG}}{\textbf{EF}}$$

$$\textbf{tan 51°} = \frac{\textbf{FG}}{\textbf{8.5}}$$

8.5 tan 51° = FG

FG = 10.4966…

FG is about **10.5 cm** long.

The acute angles have a sum of **90°**.

So, ∠G = 90° − **∠E**

∠G = 90° − **51°**

∠G = **39°**

Find the length of EG.

Use the **cosine** ratio.

$$\textbf{cos E} = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$$

$$\textbf{cos E} = \frac{\textbf{EF}}{\textbf{EG}}$$

$$\textbf{cos 51°} = \frac{\textbf{8.5}}{\textbf{EG}}$$

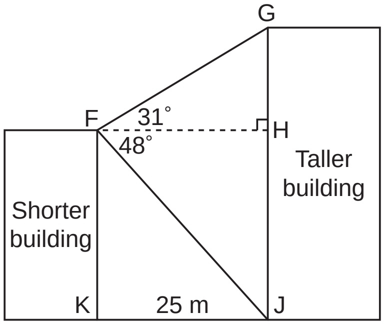
EG cos 51° = 8.5

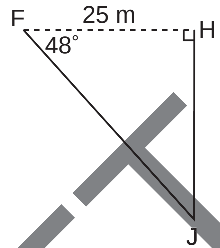
$$\textbf{EG} = \frac{\textbf{8.5}}{\textbf{cos 51°}}$$

EG = 13.5066…

EG is about **13.5 cm** long.

2.7 **10.** Two buildings are 25 m apart. From the top of the shorter building, the angles of elevation and depression of the top and bottom of the taller building are 31° and 48° respectively. What is the height of the taller building? Give your answer to the nearest metre.





We want to find the length of GJ.

GJ = **HJ** + **GH**

The angle of depression of point J is **48°**.

Use △FHJ to find the length of HJ.

Use the **tangent** ratio.

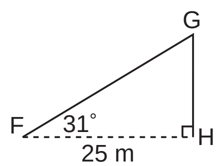
$$\textbf{tan F} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\textbf{tan F} = \frac{\textbf{HJ}}{\textbf{FH}}$$

$$\textbf{tan 48°} = \frac{\textbf{HJ}}{\textbf{25}}$$

25 tan 48° = HJ

HJ = **27.7653…**



The angle of elevation of point G is **31°**.

Use △FGH to find the length of GH.

Use the **tangent** ratio.

$$\textbf{tan F} = \frac{\textbf{opposite}}{\textbf{adjacent}}$$

$$\textbf{tan 31°} = \frac{\textbf{GH}}{\textbf{25}}$$

25 tan 31° = GH

GH = **15.0215…**

To find the height of the taller building, add:

27.7653 m + **15.0215 m** = **42.7868 m**

The taller building is about **43 m** tall.

TEACHER NOTE

Next Steps: Direct students to questions 1, 6, 8, 13, 15, 17, 19, 22, and 23 on pages 124–126 of the Student Text.