CHAPTER 10

Solutions Key
Spatial Reasoning

ARE YOU READY? PAGE 651

1. D **2.** C

3. A **4.** E

5. $b = AB = 5 - 0 = 5;\ h = 3 - (-1) = 4$
$A = \frac{1}{2}bh = \frac{1}{2}(5)(4) = 10 \text{ units}^2$

6. $b = LM = 6 - (-2) = 8,\ h = KL = 7 - 3 = 4$
$A = bh = (8)(4) = 32 \text{ units}^2$

7. $r = PQ = 2 - (-6) = 8$
$A = \pi r^2 = \pi(8)^2 = 64\pi \text{ units}^2$

8. $C = 2\pi(8) = 16\pi$ cm
$A = \pi(8)^2 = 64\pi \text{ cm}^2$

9. $C = \pi(21) = 21\pi$ ft
$A = \pi(10.5)^2 = 110.25\pi \text{ ft}^2$

10. $C = \pi\left(\frac{32}{\pi}\right) = 32$ in.
$A = \pi\left(\frac{16}{\pi}\right)^2 = \frac{256}{\pi} \text{ in}^2$

11. $AB = \sqrt{(5 - (-3))^2 + (6 - 2)^2}$
$= \sqrt{80} = 4\sqrt{5} \approx 8.9$ units
$M = \left(\frac{-3 + 5}{2}, \frac{2 + 6}{2}\right) = \left(\frac{2}{2}, \frac{8}{2}\right) = (1, 4)$

12. $CD = \sqrt{(2 - (-4))^2 + (-3 - (-4))^2}$
$= \sqrt{37} \approx 6.1$ units
$M = \left(\frac{-4 + 2}{2}, \frac{-4 + (-3)}{2}\right) = \left(\frac{-2}{2}, \frac{-7}{2}\right) = (-1, -3.5)$

13. $EF = \sqrt{(-3 - 0)^2 + (4 - 1)^2}$
$= \sqrt{18} = 3\sqrt{2} \approx 4.2$ units
$M = \left(\frac{0 + (-3)}{2}, \frac{1 + 4}{2}\right) = \left(\frac{-3}{2}, \frac{5}{2}\right) = (-1.5, 2.5)$

14. $GH = \sqrt{(-2 - 2)^2 + (-2 - (-5))^2}$
$= \sqrt{25} = 5$ units
$M = \left(\frac{2 + (-2)}{2}, \frac{-5 + (-2)}{2}\right) = \left(\frac{0}{2}, \frac{-7}{2}\right) = (0, -3.5)$

15. $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{121\pi}{\pi}} = \sqrt{121} = 11$ cm

16. $a = \frac{2A}{P} = \frac{2(128)}{32} = 8$ ft

17. $b = \sqrt{c^2 - a^2} = \sqrt{(17)^2 - (8)^2} = \sqrt{225} = 15$ m

18. $b_2 = \frac{2A}{h} - b_1 = \frac{2(60)}{6} - 8 = 20 - 8 = 12$ in.

10-1 SOLID GEOMETRY, PAGES 654–660

CHECK IT OUT! PAGES 655–656

1a. cone
vertex: N
edges: none
base: circle M

b. triangular prism
vertices: T, U, V, W, X, Y
edges: $\overline{TU}, \overline{TV}, \overline{UV}, \overline{TW}, \overline{UX}, \overline{VY}, \overline{WX}, \overline{WY}, \overline{XY}$
bases: $\triangle TUV$, $\triangle WXY$

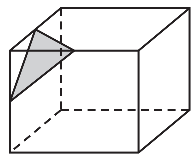
2a. The net has a triangular central face and 3 other triangular faces. So, it forms a triangular pyramid.

b. The net has a rectangular face between 2 circular faces. So, it forms a cylinder.

3a. The cross section is a hexagon.

b. The cross section is a triangle.

4.

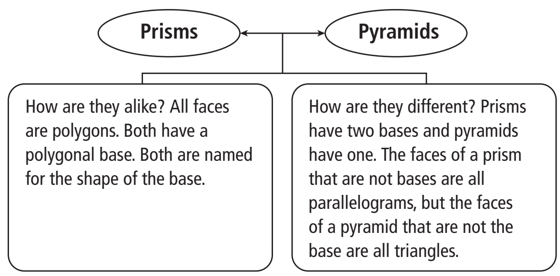


Cut through mdpts. of three edges that meet at one vertex.

THINK AND DISCUSS, PAGE 656

1. Both prisms and cylinders have 2 congruent parallel bases. The bases of a prism are polygons, and the bases of a cylinder are circles. The bases of a prism are connected by , and the bases of a cylinder are connected by a curved surface.

2.



EXERCISES, PAGES 657–660

GUIDED PRACTICE, PAGE 657

1. cylinder

2. cone
vertex: A
edges: none
base: circle B

3. rect. prism
vertices: C, D, E, F, G, H, J, K
edges: $\overline{GH}, \overline{GK}, \overline{HJ}, \overline{JK}, \overline{GF}, \overline{HE}, \overline{JD}, \overline{KC}, \overline{FC}, \overline{CD}, \overline{DE}, \overline{EF}$
bases: rect. $CDEF$, rect. $GHJK$

4. triangular pyramid
vertices: L, M, N, P
edges: $\overline{LM}$, $\overline{LN}$, $\overline{LP}$, $\overline{MN}$, $\overline{MP}$, $\overline{NP}$
base: triangle LMP

5. The net has 2 $\cong$, nonadjacent rect. faces, and remaining faces are ▱. So, the net forms a rectangular prism.

6. The net has 1 circular face and 1 other curved face that is a sector of a circle. So, the net forms a cone.

7. The net has 6 $\cong$ square faces and folds up without overlapping. So, the net forms a cube.

8. cross section is a circle

9. cross section is a pentagon

10. cross section is a rectangle

11. cut $\parallel$ to bases

12. cut $\perp$ to bases

PRACTICE AND PROBLEM SOLVING, PAGES 657–659

13. cube
vertices: S, T, U, V, W, X, Y, Z
edges: $\overline{ST}$, $\overline{TU}$, $\overline{UV}$, $\overline{VS}$, $\overline{SW}$, $\overline{TX}$, $\overline{UY}$, $\overline{VZ}$, $\overline{WX}$, $\overline{XY}$, $\overline{YZ}$, $\overline{ZW}$
bases: $STUV$, $WXYZ$

14. rect, pyramid
vertices: A, B, C, D, E
edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{AD}$, $\overline{AE}$, $\overline{BE}$, $\overline{CE}$, $\overline{DE}$
base: $ABCD$

15. cylinder
vertices: none
edges: none
bases: circle Q, circle R

16. The net has 2 $\cong$ pentagonal faces and 5 ▭ faces. So, it forms a pentagonal prism.

17. The net has a central triangular face, surrounded by 3 other triangular faces. So, net forms a triangular pyramid.

18. The net has a rectangular face between 2 circular faces. So, it forms a cylinder.

19. square

20. rectangle

21. rectangle

22. cut $\parallel$ to ground

23. cut $\perp$ to ground

24–27. Possible answers:

24. cube

25. rectangular prism

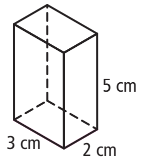
26. cylinder

27. hexagonal prism

28. Possible answer: The figure is a hexagonal prism whose bases are regular hexagons with 7-in. sides. Height of the prism is 13 in.

29. Possible answer: The figure is a cylinder whose bases each have radius 12 ft. Height of the cylinder is 9 ft.

30. Possible answer: The figure is a square prism with 36 cm by 36 cm bases and a height of 108 cm.

31.


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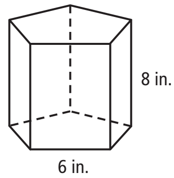

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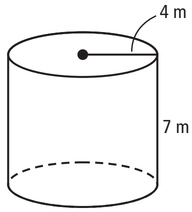

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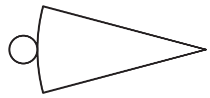

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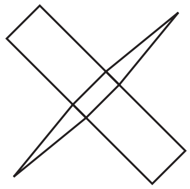

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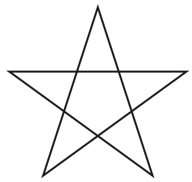
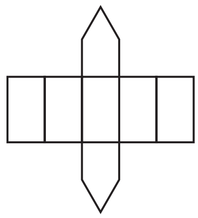
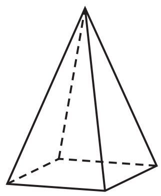


37a. pentagonal prism

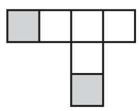
c.


one

38. B is incorrect; bases are $\cong$ reg. hexagons. So, opposite sides of cross section must be congruent.

39.


40. Figure **b**; when figure is folded, shaded faces will overlap.



TEST PREP, PAGE 659

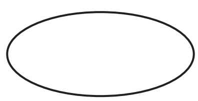
41. D

42. F

43. B

44. G

CHALLENGE AND EXTEND, PAGE 660

45.


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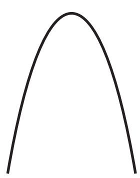

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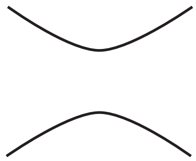

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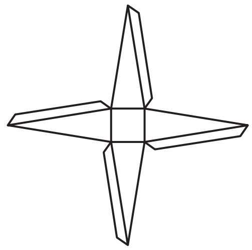

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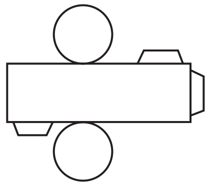

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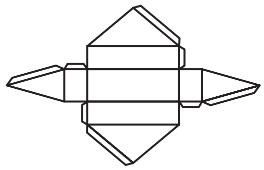
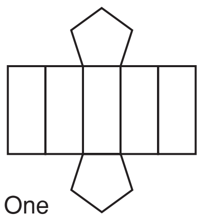


51a. *A* and *B*, *C* and *F*, *D* and *G*, *E* and *H*

b.



One

SPIRAL REVIEW, PAGE 660

52. $y = -x^2$ **53.** $y = x^2 + 6$

54. Possible answer: $y = \frac{1}{2}x^2$

55. largest ∠ is opp. longest side: ∠*B*
smallest ∠ is opp. shortest side: ∠*C*

56. largest: ∠*E*
smallest: ∠*D*

57. largest: ∠*I*
smallest: ∠*H*

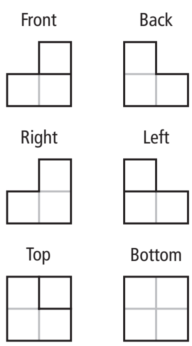
58. no; 2 : 4 ≠ 8 : 12

59. yes; 7 : 11.9 = 24 : 40.8 = 25 : 42.5 = 10 : 17

10-2 REPRESENTATIONS OF THREE-DIMENSIONAL FIGURES, PAGES 661–668

CHEC K IT OUT! PAGES 661–664

1.



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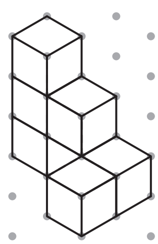
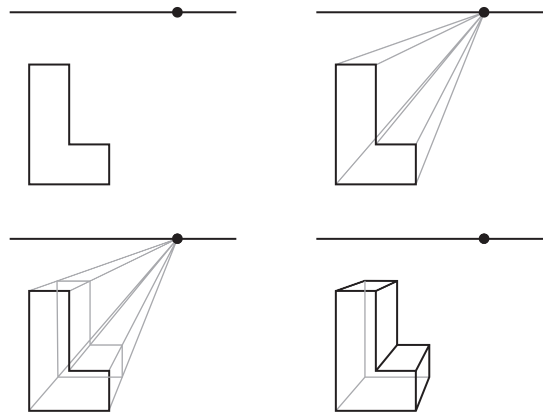
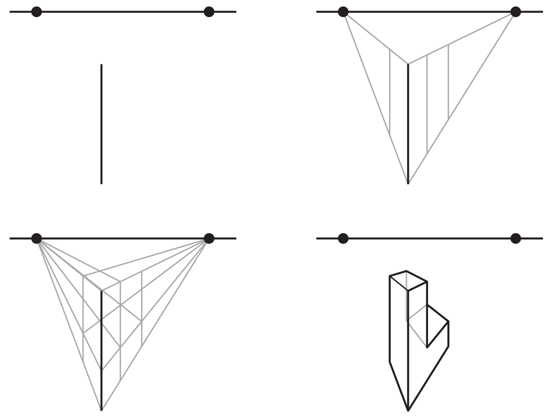


3a.



b.



4. no

THINK AND DISCUSS, PAGE 664

1. All 6 views are squares.

2. No; vertical lines do not meet at a vanishing point.

3.

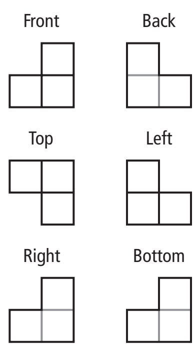
Type of Drawing	Description
isometric	Corner view
orthographic	Top, bottom, front, back, left, and right views
perspective	Parallel lines are drawn so that they meet at vanishing point(s).

EXERCISES, PAGES 665–668

GUIDED PRACTICE, PAGE 665

1. perspective

2.



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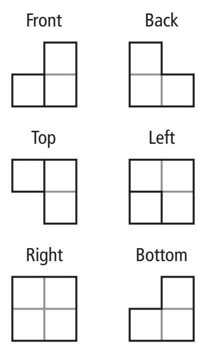

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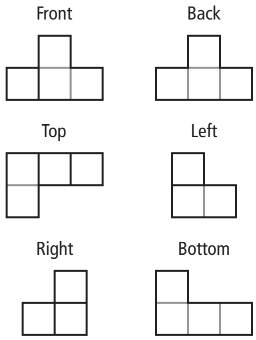

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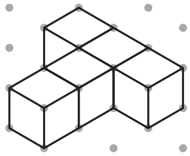

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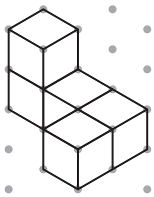

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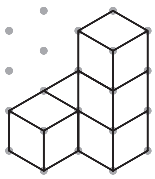
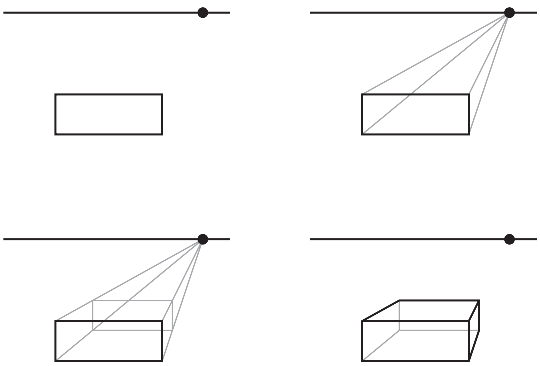
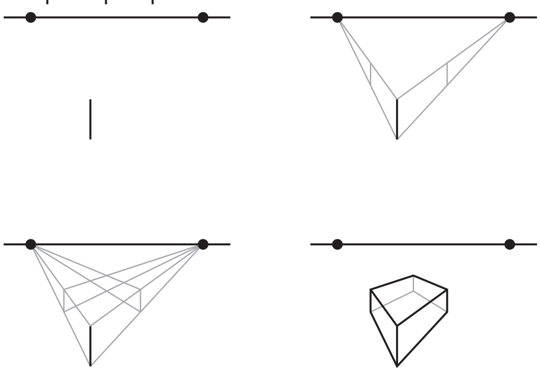


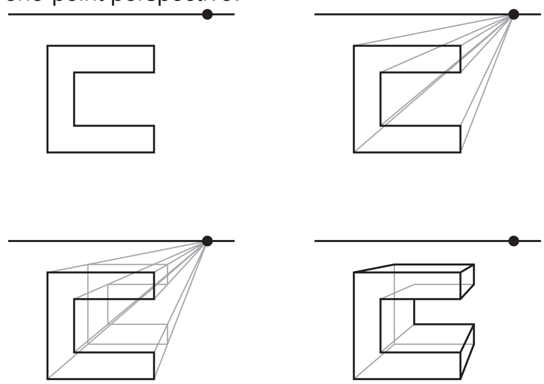
8. one-point perspective:



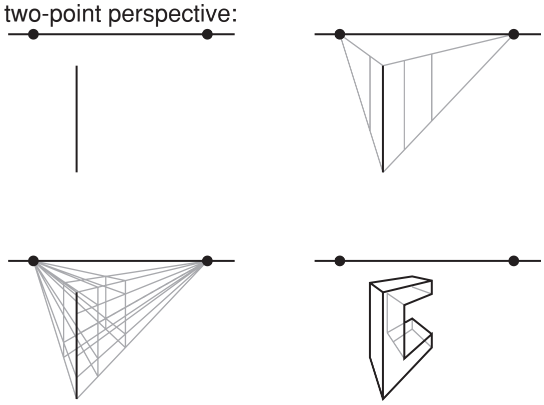
two-point perspective:



9. one-point perspective:



two-point perspective:



10. yes

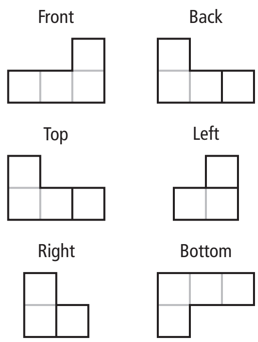
11. no

12. no

13. no

PRACTICE AND PROBLEM SOLVING, PAGES 665–667

14.



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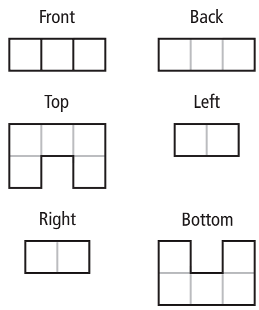

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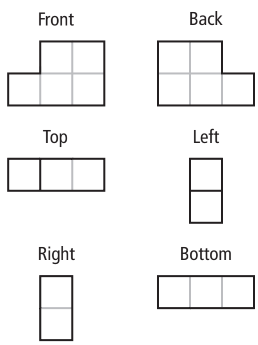

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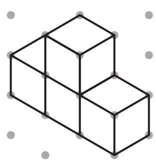

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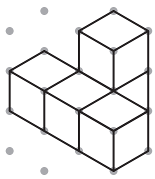

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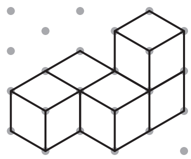
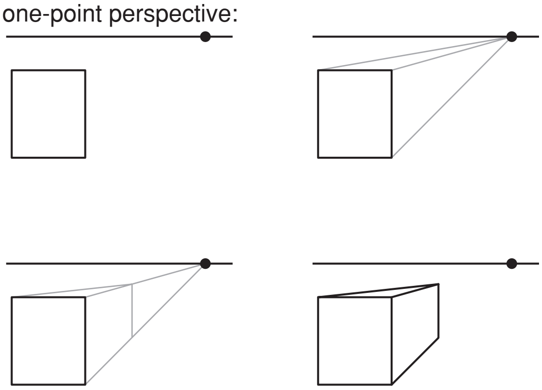
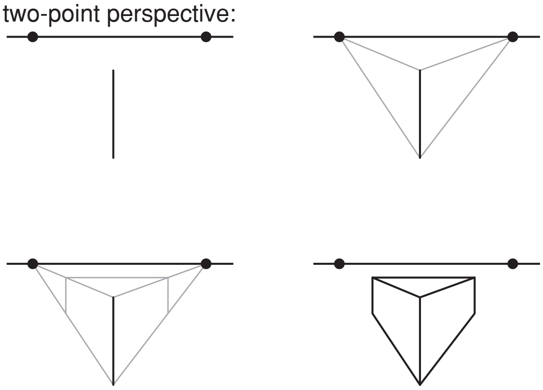


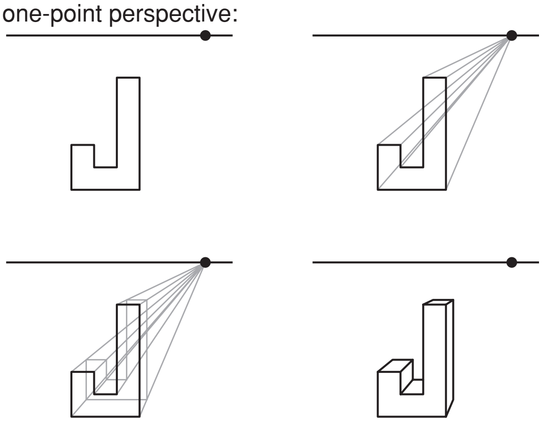
20. one-point perspective:



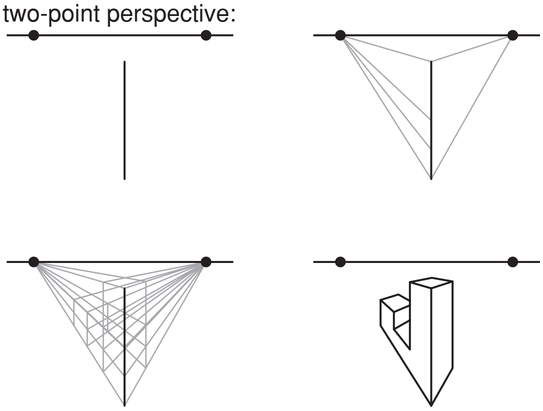
two-point perspective:



21. one-point perspective:



two-point perspective:



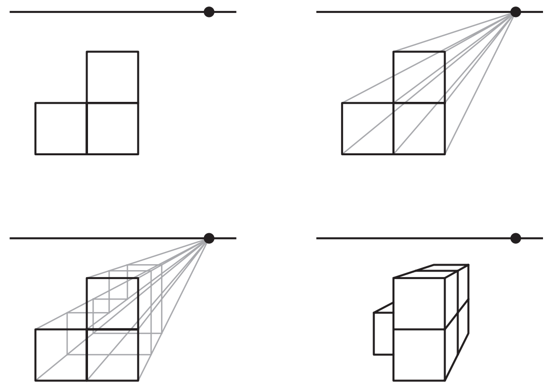
22. yes

23. yes

24. no

25. no

26.



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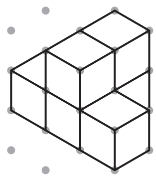

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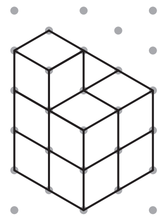
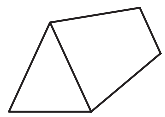


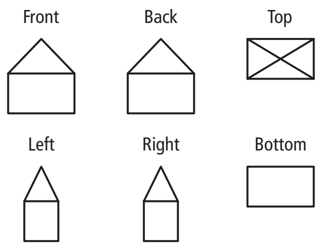
29a. Bottom

b.



c. 3 at front + 3 along sides + 3 at rear = 9

30.



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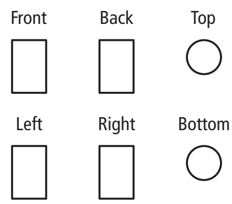
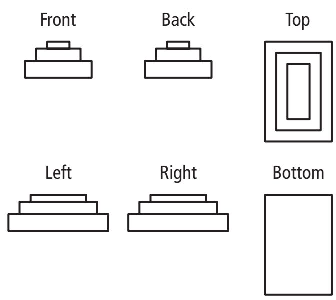


32.



33. Possible answer: a right cylinder and a right square prism in which the diameter of the cylinder is equal to the side length of the square base of the prism and the heights are the same.

34a. two-point perspective

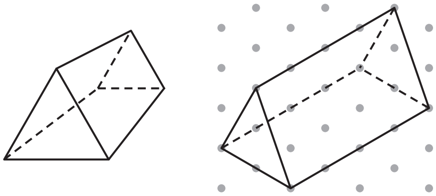
b. Extend a pair of || lines to meet at each vanishing point.

TEST PREP, PAGE 667

35. B

36. H

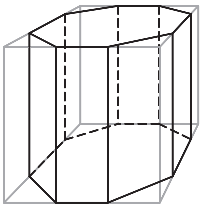
37. Possible answer:



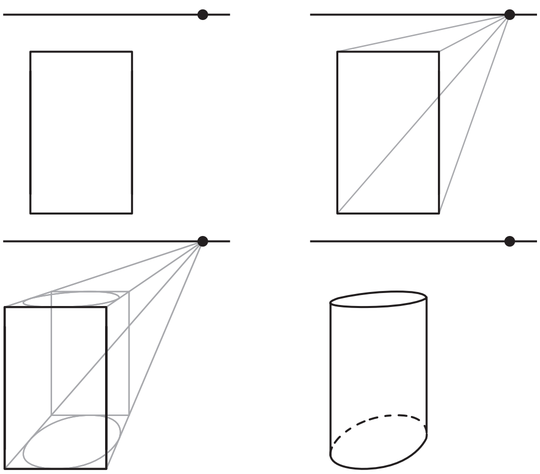
Some edges that are || on the 3-dimensional object are not || in perspective drawing. If they were extended, they would meet at the vanishing point of drawing. All the || edges of prism are also || in the isometric drawing.

CHALLENGE AND EXTEND, PAGE 668

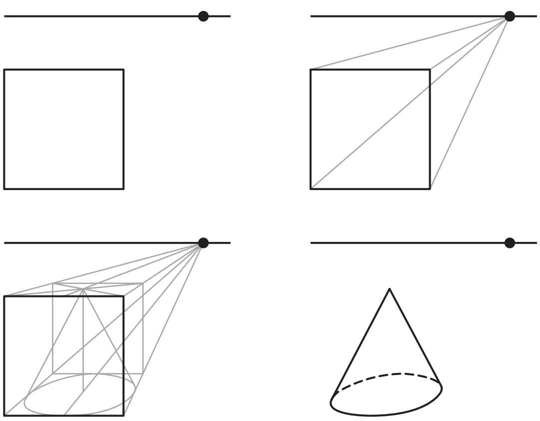
38.



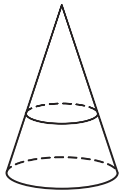
39.



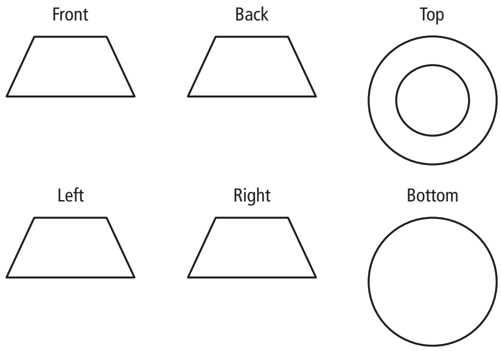
40.



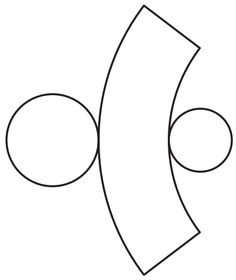
41a.



b.



c.



42. Check students' drawings.

SPIRAL REVIEW, PAGE 668

43.
$$\begin{aligned} x + y &= 30 && (1)\\ 2x - 2y &= 20 && (2)\\ x - y &= 10 && (2) \div 2\\ x + y &= 30 && + (1)\\ 2x &= 40 &&\\ x &= 20 &&\\ y = 30 - x &= 30 - 20 = 10 && \end{aligned}$$

44.
$$\begin{aligned} x - y &= 7\\ 4x + y &= 38\\ 5x &= 45\\ x &= 9\\ (9) - y &= 7\\ y &= 2 \end{aligned}$$

45.
$$\begin{aligned} y &= x + 5\\ x + y &= 5\\ x + 2y &= x + 10\\ 2y &= 10\\ y &= 5\\ x = y - 5 &= 5 - 5 = 0 \end{aligned}$$

46. slope of $\overline{AB} = \dfrac{1 - 2}{6 - 4} = -\dfrac{1}{2}$

47. slope of $\overline{AC} = \dfrac{0 - 2}{3 - 4} = \dfrac{-2}{-1} = 2$

48. slope of $\overline{AD} = \dfrac{0 - 2}{2 - 4} = \dfrac{-2}{-2} = 1$

49. 2 pentagons and 5 parallelograms

50. 6 squares

51. 4 triangles

USING TECHNOLOGY, PAGE 668

1. Check students' drawings.

2. Possible answer: First, draw a horiz. line to represent horizon and locate 2 vanishing points A and B on the line. Then, draw a vert. seg. $\overline{CD}$ and draw segs. $\overline{CA}$, $\overline{CB}$, $\overline{DA}$, and $\overline{DB}$. Draw two more vert. segs., one with endpoints on $\overline{CA}$ and $\overline{DA}$, and other with endpoints on $\overline{CB}$ and $\overline{DB}$. Connect each endpoint of these segs. to both vanishing points to form a rectangular prism.

GEOMETRY LAB: USE NETS TO CREATE POLYHEDRONS, PAGE 669

TRY THIS, PAGE 669

1.

Polyhedron	V	E	F	$V - E + F$
Tetrahedron	4	6	4	2
Octahedron	6	12	8	2
Icosahedron	12	30	20	2
Cube	8	12	6	2
Dodecahedron	20	30	12	2

2. $V - E + F$ is always equal to 2.

10-3 FORMULAS IN THREE DIMENSIONS, PAGES 670–677

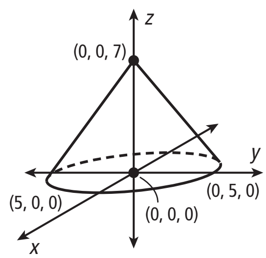
CHECK IT OUT! PAGES 670–673

1a. $V = 6, E = 12, F = 8$
$6 - 12 + 8 \stackrel{?}{=} 2$
$2 = 2$

b. $V = 7, E = 12, F = 7$
$7 - 12 + 7 \stackrel{?}{=} 2$
$2 = 2$

2. $d = \sqrt{5^2 + 5^2 + 5^2}$
$= \sqrt{25 + 25 + 25}$
$= \sqrt{75}$
$= 5\sqrt{3} \approx 8.7$ cm

3. Graph center of base at (0, 0, 0). Since height is 7, graph vertex at (0, 0, 7). Radius is 5, so, base will cross x-axis at (0, 5, 0) and y-axis at (5, 0, 0). Connect vertex to base.



4a. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(6 - 0)^2 + (0 - 9)^2 + (12 - 5)^2}$
$= \sqrt{36 + 81 + 49}$
$= \sqrt{166} \approx 12.9$ units
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
$M\left(\frac{0 + 6}{2}, \frac{9 + 0}{2}, \frac{5 + 12}{2}\right)$
$M(3, 4.5, 8.5)$

b. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(12 - 5)^2 + (16 - 8)^2 + (20 - 16)^2}$
$= \sqrt{49 + 64 + 16}$
$= \sqrt{129} \approx 11.4$ units
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
$M\left(\frac{5 + 12}{2}, \frac{8 + 16}{2}, \frac{16 + 20}{2}\right)$
$M(8.5, 12, 18)$

5. Locations of divers on surface can be represented by ordered triples (18, 9, 0) and (−15, −6, 0).
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(-15 - 18)^2 + (-6 - 9)^2 + (0 - 0)^2}$
$= \sqrt{1314} \approx 36.2$ ft

THINK AND DISCUSS, PAGE 673

1. Find the difference of x-coordinates, the difference of y-coordinates, and the difference of z-coordinates. Square each result, and add. The distance is the square root of the sum.

2.

	Rectangular Prism	Rectangular Pyramid
Vertices V	8	5
Edges E	12	8
Faces F	6	5
$V - E + F$	2	2

EXERCISES, PAGES 674–677

GUIDED PRACTICE, PAGE 674

1. because the bases are circles, which are not polygons

2. $V = 6, E = 9, F = 5$
$6 - 9 + 5 \stackrel{?}{=} 2$
$2 = 2$

3. $V = 6, E = 10, F = 6$
$6 - 10 + 6 \stackrel{?}{=} 2$
$2 = 2$

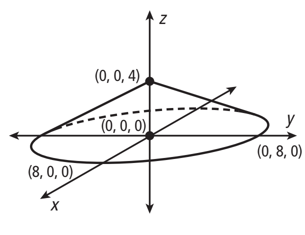
4. $V = 10, E = 20, F = 12$
$10 - 20 + 12 \stackrel{?}{=} 2$
$2 = 2$

5. $d = \sqrt{4^2 + 8^2 + 12^2}$
$= \sqrt{16 + 64 + 144}$
$= \sqrt{224}$
$= 4\sqrt{16} \approx 15.0$ ft

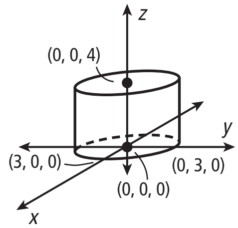
6. $13^2 = 6^2 + 10^2 + h^2$
$h^2 = 13^2 - 6^2 - 10^2$
$h = \sqrt{13^2 - 6^2 - 10^2}$
$= \sqrt{169 - 36 - 100}$
$= \sqrt{33} \approx 5.7$ in.

7. $d = \sqrt{12^2 + 12^2 + 1^2}$
$= \sqrt{144 + 144 + 1}$
$= \sqrt{289} = 17$ in.

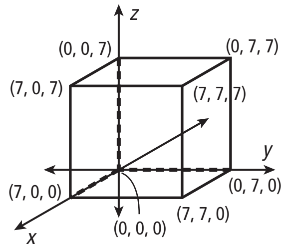
8. Graph the center of base at (0, 0, 0). Since the height is 4, graph the vertex at (0, 0, 4). The radius is 8, so, the base will cross the x-axis at (0, 8, 0) and the y-axis at (8, 0, 0). Connect the vertex to base.



9. Graph the center of the bottom base at (0, 0, 0). Since the height is 4, graph the center of the top base at (0, 0, 4). The radius is 3, so the bottom base will cross the x-axis at (0, 3, 0) and the y-axis at (3, 0, 0). Draw the top base parallel to the bottom base and connect the bases.



10. cube has 8 vertices: (0, 0, 0), (7, 0, 0), (7, 7, 0), (0, 7, 0), (0, 0, 7), (7, 0, 7), (7, 7, 7), (0, 7, 7)



11. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(5 - 0)^2 + (9 - 0)^2 + (10 - 0)^2}$
$= \sqrt{25 + 81 + 100}$
$= \sqrt{206} \approx 14.4$ units
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
$M\left(\frac{0 + 5}{2}, \frac{0 + 9}{2}, \frac{0 + 10}{2}\right)$
$M(2.5, 4.5, 5)$

12. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(7 - 0)^2 + (0 - 3)^2 + (14 - 8)^2}$
$= \sqrt{49 + 9 + 36}$
$= \sqrt{94} \approx 9.7$ units
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
$M\left(\frac{0 + 7}{2}, \frac{3 + 0}{2}, \frac{8 + 14}{2}\right)$
$M(3.5, 1.5, 11)$

13. $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(9 - 4)^2 + (12 - 6)^2 + (15 - 10)^2}$
$= \sqrt{25 + 36 + 25}$
$= \sqrt{86} \approx 9.3$ units
$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$
$M\left(\frac{4 + 9}{2}, \frac{6 + 12}{2}, \frac{10 + 15}{2}\right)$
$M(6.5, 9, 12.5)$

14. Represent the locations of the starting point and the camp by ordered triples (0, 0, 0) and (3, 7, 0.6).
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
$= \sqrt{(3 - 0)^2 + (7 - 0)^2 + (0.6 - 0)^2}$
$= \sqrt{58.36} \approx 7.6$ km

PRACTICE AND PROBLEM SOLVING, PAGES 674–676

15. $V = 8$, $E = 12$, $F = 6$
$8 - 12 + 6 \stackrel{?}{=} 2$
$2 = 2$

16. $V = 8$, $E = 18$, $F = 12$
$8 - 18 + 12 \stackrel{?}{=} 2$
$2 = 2$

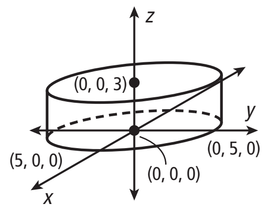
17. $V = 11$, $E = 20$, $F = 11$
$11 - 20 - 11 \stackrel{?}{=} 2$
$2 = 2$

18. $d = \sqrt{7^2 + 8^2 + 16^2}$
$= \sqrt{49 + 64 + 256}$
$= \sqrt{369}$
$= 3\sqrt{41} \approx 19.2$ yd

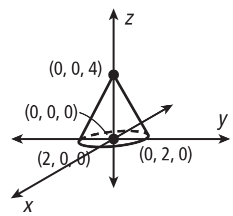
19. $17^2 = 15^2 + 6^2 + h^2$
$h^2 = 17^2 - 15^2 - 6^2$
$h = \sqrt{17^2 - 15^2 - 6^2}$
$= \sqrt{289 - 225 - 36}$
$= \sqrt{28}$
$= 2\sqrt{7} \approx 5.3$ m

20. $8^2 = s^2 + s^2 + s^2$
$64 = 3s^2$
$8 = s\sqrt{3}$
$s = \frac{8}{\sqrt{3}}$
$= \frac{8\sqrt{3}}{3} \approx 4.6$ cm

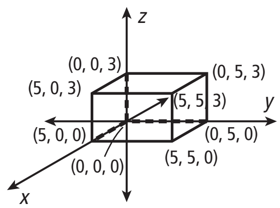
21. Graph the center of the bottom base at (0, 0, 0). Since the height is 3, graph the center of the top base at (0, 0, 3). The radius is 5, so, the bottom base will cross the x-axis at (0, 5, 0) and the y-axis at (5, 0, 0). Draw the top base ‖ to the bottom base and connect bases.



22. Graph the center of the base at (0, 0, 0). Since the height is 4, graph the vertex at (0, 0, 4). The radius is 2, so, the base will cross the x-axis at (0, 2, 0) and the y-axis at (2, 0, 0). Connect the vertex to the base.



23. prism has 8 vertices: (0, 0, 0), (5, 0, 0), (5, 5, 0), (0, 5, 0), (0, 0, 3), (5, 0, 3), (5, 5, 3), (0, 5, 3),



24. $d = \sqrt{(4 - 0)^2 + (4 - 0)^2 + (4 - 0)^2}$
$= \sqrt{16 + 16 + 16}$
$= \sqrt{48}$
$= 4\sqrt{3} \approx 6.9$ units
$M\left(\frac{0 + 4}{2}, \frac{0 + 4}{2}, \frac{0 + 4}{2}\right) = M(2, 2, 2)$

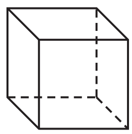
25. $d = \sqrt{(9 - 2)^2 + (10 - 3)^2 + (10 - 7)^2}$
$= \sqrt{49 + 49 + 9}$
$= \sqrt{107} \approx 10.3$ units
$M\left(\frac{2 + 9}{2}, \frac{3 + 10}{2}, \frac{10 + 7}{2}\right) = M(5.5, 6.5, 8.5)$

26. $d = \sqrt{(8-2)^2 + (8-5)^2 + (10-3)^2}$
$= \sqrt{36 + 9 + 49}$
$= \sqrt{94} \approx 9.7$ units
$M\left(\frac{2+8}{2}, \frac{5+8}{2}, \frac{3+10}{2}\right) = M(5, 6.5, 6.5)$

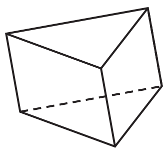
27. Let the cloud and the raindrop hitting ground have the coordinates (0, 0, 6500) and (−700, 500, 0). The distance traveled by the raindrop is
$d = \sqrt{(-700-0)^2 + (500-0)^2 + (0-6500)^2}$
$= \sqrt{42{,}990{,}000} \approx 6557$ ft.

28. original:
$d = \sqrt{12^2 + 3^2 + 4^2}$
$= \sqrt{144 + 9 + 16}$
$= \sqrt{169} = 13$ ft
doubled:
$d = \sqrt{24^2 + 6^2 + 8^2}$
$= \sqrt{576 + 36 + 64}$
$= \sqrt{676} = 36$ ft
The diagonal is doubled.

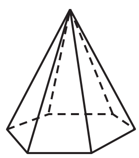
29. $V - E + F = 2$
$8 - 12 + F = 2$
$-4 + F = 2$
$F = 6$



30. $V - E + F = 2$
$V - 9 + 5 = 2$
$V - 4 = 2$
$V = 6$



31. $V - E + F = 2$
$7 - E + 7 = 2$
$14 - E = 2$
$E = 12$



32. $V = n + n = 2n$
$E = n + n + n = 3n$
$F = 1 + n + 1 = n + 2$
$V - E + F = 2n - 3n + (n + 2) = 2$

33. $V = n + 1$
$E = n + n = 2n$
$F = n + 1$
$V - E + F = (n + 1) - 2n + (n + 1) = 2$

34a. $K(0, 2.5, 6)$, $L(0, 5, 0)$, $N(7, 2.5, 6)$, $P(7, 5, 0)$

b. $KP = \sqrt{(7-0)^2 + (5-2.5)^2 + (0-6)^2}$
$= \sqrt{91.25} \approx 9.6$ ft

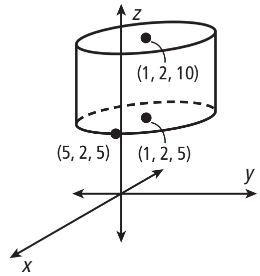
35. $d = \sqrt{6^2 + 6^2 + 6^2}$
$= \sqrt{108} = 6\sqrt{3}$

36. $w = \sqrt{65^2 - 24^2 - 60^2}$
$= \sqrt{49} = 7$

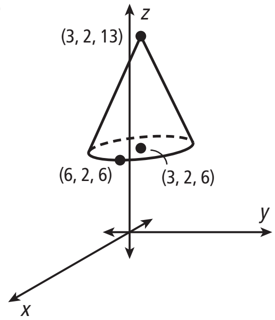
37. $h = \sqrt{24^2 - 12^2 - 18^2}$
$= \sqrt{108} = 6\sqrt{3}$

38. $\ell = \sqrt{4^2 - 2^2 + 3^2}$
$= \sqrt{3}$

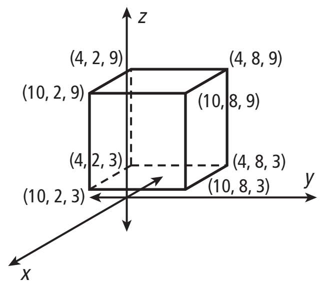
39. bottom base extends forward to (5, 2, 5)
center of top base: (1, 2, 10)
Draw the top base ∥ to bottom base.



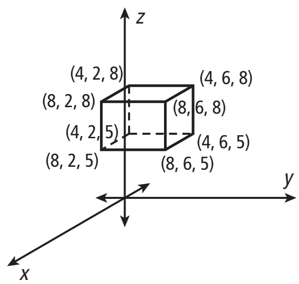
40. base extends forward to (6, 2, 6)
vertex: (3, 2, 13)



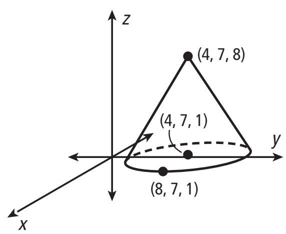
41. vertices of cube:
(4, 2, 3), (10, 2, 3), (10, 8, 3), (4, 8, 3), (4, 2, 9), (10, 2, 9), (10, 8, 9), (4, 8, 9)



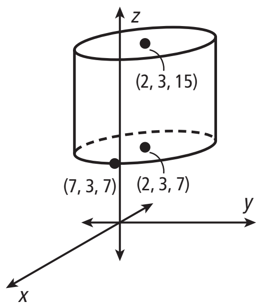
42.



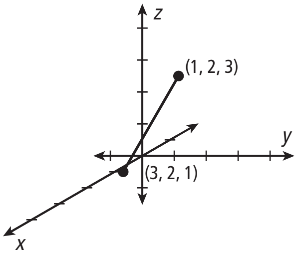
43. base extends forward to (8, 7, 1)



44. bottom base extends forward to (7, 3, 7)
Draw the top base parallel to bottom base.

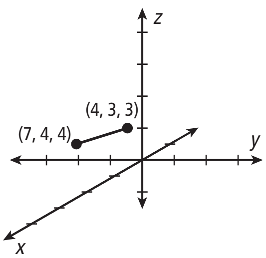


45.



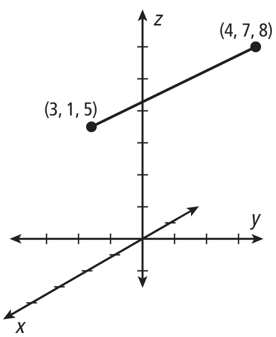
$$d = \sqrt{(3-1)^2 + (2-2)^2 + (1-3)^2}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2} \approx 2.8 \text{ units}$$
$$M\left(\frac{1+3}{2}, \frac{2+2}{2}, \frac{3+1}{2}\right) = M(2, 2, 2)$$

46.



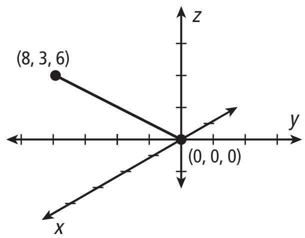
$$d = \sqrt{(7-4)^2 + (4-3)^2 + (4-3)^2}$$
$$= \sqrt{11} \approx 3.3 \text{ units}$$
$$M\left(\frac{4+7}{2}, \frac{3+4}{2}, \frac{3+4}{2}\right) = M(5.5, 3.5, 3.5)$$

47.



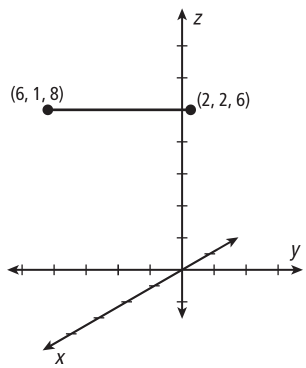
$$d = \sqrt{(3-4)^2 + (1-7)^2 + (5-8)^2}$$
$$= \sqrt{46} \approx 6.8 \text{ units}$$
$$M\left(\frac{4+3}{2}, \frac{7+1}{2}, \frac{8+5}{2}\right) = M(3.5, 4, 6.5)$$

48.



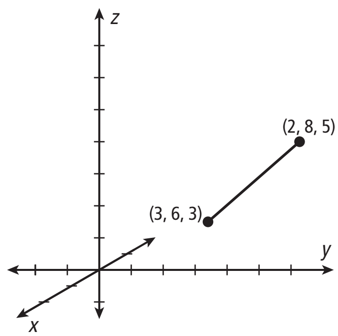
$$d = \sqrt{(8-0)^2 + (3-0)^2 + (6-0)^2}$$
$$= \sqrt{109} \approx 10.4 \text{ units}$$
$$M\left(\frac{0+8}{2}, \frac{0+3}{2}, \frac{0+6}{2}\right) = M(4, 1.5, 3)$$

49.



$$d = \sqrt{(2-6)^2 + (2-1)^2 + (6-8)^2}$$
$$= \sqrt{21} \approx 4.6 \text{ units}$$
$$M\left(\frac{6+2}{2}, \frac{1+2}{2}, \frac{8+6}{2}\right) = M(4, 1.5, 7)$$

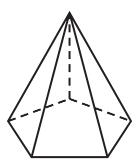
50.



$$d = \sqrt{(3-2)^2 + (6-8)^2 + (3-5)^2}$$
$$= \sqrt{9} = 3 \text{ units}$$
$$M\left(\frac{2+3}{2}, \frac{8+6}{2}, \frac{5+3}{2}\right) = M(2.5, 7, 4)$$

51. $13 = \sqrt{(3-6)^2 + (3-(-1))^2 + (z-(-3))^2}$
$13^2 = (3-6)^2 + (3-(-1))^2 + (z-(-3))^2$
$169 = 9 + 16 + z^2 + 6z + 9$
$0 = z^2 + 6z - 135$
$0 = (z+15)(z-9)$
$z = 9$ or -15

52. Possible answer:



53. Possible answer:
$d \approx \sqrt{0.4^2 + 0.9^2 + 1.5^2} \approx \sqrt{32.2} \approx 1.8$ in.

54. Possible answer: a segment that connects a vertex of one base to the opposite vertex of the other base
$AB = \sqrt{(1-0)^2 + (0-0)^2 + (0-0)^2} = 1$ unit
$AC = \sqrt{(1-0)^2 + (2-0)^2 + (0-0)^2} = \sqrt{5}$
≈ 2.2 units
$AG = \sqrt{(1-0)^2 + (2-0)^2 + (2-0)^2} = 3$ units

55. $AB = \sqrt{(5-3)^2 + (8-2)^2 + (6-(-3))^2}$
$= \sqrt{121} = 11$
$AC = \sqrt{(-3-3)^2 + (-5-2)^2 + (3-(-3))^2}$
$= \sqrt{121} = 11$
$BC = \sqrt{(-3-5)^2 + (-5-8)^2 + (3-6)^2}$
$= \sqrt{242} = 11\sqrt{2}$
$AB = AC$ and $AB^2 + AC^2 = BC^2$
$\triangle ABC$ is a right isoceles $\triangle$.

56. 10 cm; the segment is the hypotenuse of a right $\triangle$ in which one leg is a diameter of one base, and the opposite vertex is on the other base. Legs measure 6 cm and 2(4) = 8 cm, so, segment length is 10 cm. Segment is longest because a diameter is longest possible segment in a circle.

TEST PREP, PAGE 677

57. C

58. H
$d = \sqrt{12^2 + 8^2 + 6^2}$
$= \sqrt{244} \approx 15.6$ m

59. B
$d = \sqrt{(9-7)^2 + (3-14)^2 + (12-8)^2}$
$= \sqrt{189} = 11.9$ units

CHALLENGE AND EXTEND, PAGE 677

60. legs of rt. $\triangle$ measure h and $\frac{a}{2} + a + \frac{a}{2} = 2a$
$d = \sqrt{(2a)^2 + h^2} = \sqrt{4a^2 + h^2}$

61. $AB = \sqrt{(1-(-1))^2 + (-2-2)^2 + (6-4)^2}$
$= \sqrt{24} = 2\sqrt{6}$
$AC = \sqrt{(3-(-1))^2 + (-6-2)^2 + (8-4)^2}$
$= \sqrt{96} = 4\sqrt{6}$
$BC = \sqrt{(3-1)^2 + (-6-(-2))^2 + (8-6)^2}$
$= \sqrt{24} = 2\sqrt{6}$
$AB + BC = AC$. So, points are collinear.

62. AB
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
AM
$= \sqrt{\left(\frac{x_1 + x_2}{2} - x_1\right)^2 + \left(\frac{y_1 + y_2}{2} - y_1\right)^2 + \left(\frac{z_1 + z_2}{2} - z_1\right)^2}$
$= \sqrt{\left(\frac{x_2}{2} - \frac{x_1}{2}\right)^2 + \left(\frac{y_2}{2} - \frac{y_1}{2}\right)^2 + \left(\frac{z_2}{2} - \frac{z_1}{2}\right)^2}$, and
MB
$= \sqrt{\left(x_2 - \frac{x_1 + x_2}{2}\right)^2 + \left(y_2 - \frac{y_1 + y_2}{2}\right)^2 + \left(z_2 - \frac{z_1 + z_2}{2}\right)^2}$
$= \sqrt{\left(\frac{x_2}{2} - \frac{x_1}{2}\right)^2 + \left(\frac{y_2}{2} - \frac{y_1}{2}\right)^2 + \left(\frac{z_2}{2} - \frac{z_1}{2}\right)^2}$
So $AM = MB$. Also, $AM + MB$
$= 2\sqrt{\left(\frac{x_2}{2} - \frac{x_1}{2}\right)^2 + \left(\frac{y_2}{2} - \frac{y_1}{2}\right)^2 + \left(\frac{z_2}{2} - \frac{z_1}{2}\right)^2}$
$= 2\sqrt{\frac{1}{4}(x_2 - x_1)^2 + \frac{1}{4}(y_2 - y_1)^2 + \frac{1}{4}(z_2 - z_1)^2}$
$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} = AB$
So A, M, and B are collinear. Since M is on $\overline{AB}$ and $AM = MB$, M is the midpoint of $\overline{AB}$ by def. midpt.

63. $AG = \sqrt{(a-0)^2 + (b-0)^2 + (c-0)^2}$
$= \sqrt{a^2 + b^2 + c^2}$
$BH = \sqrt{(0-a)^2 + (b-0)^2 + (c-0)^2}$
$= \sqrt{a^2 + b^2 + c^2}$
$AG = BH$, so $\overline{AG} \cong \overline{BH}$. By the definition of $\cong$ segs.
Let M and N be mdpts. of $\overline{AG}$ and $\overline{BH}$.
$M = \left(\frac{0+a}{2}, \frac{0+b}{2}, \frac{0+c}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$
$N = \left(\frac{0+a}{2}, \frac{0+b}{2}, \frac{0+c}{2}\right) = \left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$
$M = N$. These segs. have the same midpoint, so they bisect each other..

SPIRAL REVIEW, PAGE 677

64. $30 + 25 = 55$

65. 0–9 yr old

66. $A = b(2h) = 2bh$

67. $A = \frac{1}{2}h\left(\frac{1}{2}b_1 + b_2\right)$

68. $A = \pi(3r)^2 = 9\pi r^2$

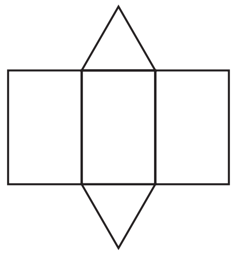
69. cone

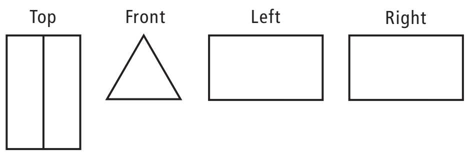
70. none

71. $\odot C$

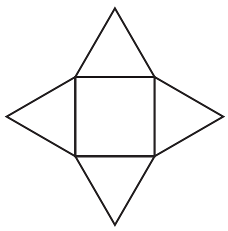
10A MULTI-STEP TEST PREP, PAGE 678

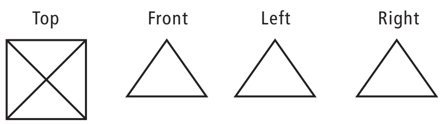
1. A-frame tent:





pyramid tent:





2. A-frame: floor is a rectangle *ABED*;
sleeping area = 8(7) = 56 ft
pyramid: floor is a square *PQRS*;
sleeping area = 8^2 = 56 ft
The pyramid tent has greater sleeping area.

3. A-frame: vertical distance from *A* to *C* is 7 − 0 = 7 ft
pyramid: vertical distance from *P* to *T* is 8 − 0 = 8 ft
The pyramid tent has more headroom.

4. $EF = \sqrt{(8-8)^2 + (3.5-7)^2 + (7-0)^2}$
$= \sqrt{61.25} \approx 7.8$ ft
$TR = \sqrt{(8-4)^2 + (8-4)^2 + (8-0)^2}$
$= \sqrt{96} \approx 9.8$ ft
The camper should purchase the A-frame tent.

10A READY TO GO ON? PAGE 679

1. hexagonal pyramid
vertices: *A*, *B*, *C*, *D*, *E*, *F*, *G*
edges: $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, $\overline{AG}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FG}$, $\overline{BG}$
base: hexagon *BCDEFG*

2. cone
vertices: *H*
edges: none
base: circle *J*

3. rectangular prism
vertices: *K*, *L*, *M*, *N*, *P*, *Q*, *R*, *S*
edges: $\overline{KL}$, $\overline{LM}$, $\overline{MN}$, $\overline{NK}$, $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, $\overline{PS}$, $\overline{KP}$, $\overline{LQ}$, $\overline{MR}$, $\overline{NS}$;
bases: *KLMN*, *PQRS*

4. cube

5. cone

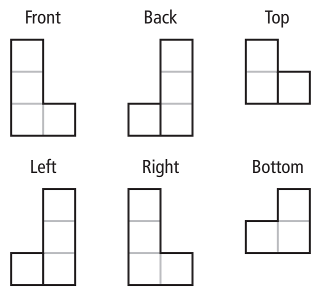
6. pentagonal pyramid

7. square

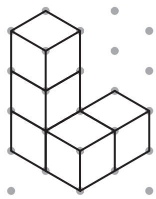
8. rectangle

9. circle

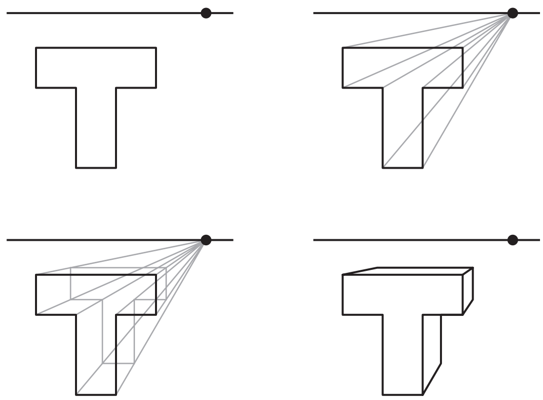
10.



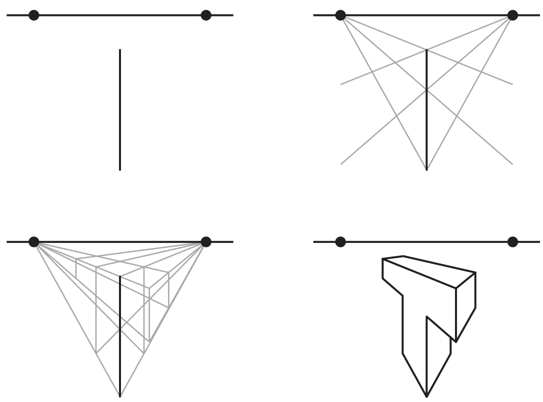
11.



12.



13.



14. $V = 8$, $E = 12$, $F = 6$
$V - E + F = 8 - 12 + 6 = 2$

15. $V = 7$, $E = 12$, $F = 7$
$V - E + F = 7 - 12 + 7 = 2$

16. $V = 4$, $E = 6$, $F = 4$
$V - E + F = 4 - 6 + 4 = 2$

17. Let the coordinates of the nest and the bird be $(0, 0, 0)$ and $(6, -7, 6)$.

$$d = \sqrt{(6-0)^2 + (-7-0)^2 + (6-0)^2}$$
$$= \sqrt{121} = 11 \text{ ft}$$

18. $d = \sqrt{(4-0)^2 + (6-0)^2 + (12-0)^2}$
$$= \sqrt{196} = 14 \text{ units}$$
$$M\left(\frac{0+4}{2}, \frac{0+6}{2}, \frac{0+12}{2}\right) = M(2, 3, 6)$$

19. $d = \sqrt{(5-3)^2 + (-5-1)^2 + (7-(-2))^2}$
$$= \sqrt{121} = 11 \text{ units}$$
$$M\left(\frac{3+5}{2}, \frac{1+(-5)}{2}, \frac{-2+7}{2}\right) = M(4, -2, 2.5)$$

20. $d = \sqrt{(7-3)^2 + (2-5)^2 + (0-9)^2}$
$$= \sqrt{106} \approx 10.3 \text{ units}$$
$$M\left(\frac{3+7}{2}, \frac{5+2}{2}, \frac{9+0}{2}\right) = M(5, 3.5, 4.5)$$

10-4 SURFACE AREA OF PRISMS AND CYLINDERS, PAGES 680–687

CHECK IT OUT! PAGES 681–683

1. $L = Ph$
$= (4s)(s)$
$= 4(8)(8) = 256 \text{ cm}^2$

$S = L + 2B$
$= L + 2s^2$
$= 256 + 2(8)^2$
$= 384 \text{ cm}^2$

2. **Step 1** Use the base area to find the radius.
$A = \pi r^2$
$49\pi = \pi r^2$
$49 = r^2$
$r = 7$ in.

Step 2 Use the radius to find the lateral area and the base area.
height is 2 times radius, or 14 in.
$L = 2\pi rh = 2\pi(7)(14) = 196\pi \text{ in}^2$
$S = 2\pi rh + 2\pi r^2 = 196\pi + 2(49\pi) = 294\pi \text{ in}^2$

3. Surface area of right rectangular prism is
$S = Ph + 2B$
$= 26(5) + 2(9)(4) = 202 \text{ cm}^2$.
A right cylinder is added to a rectangular prism.
Lateral area of cylinder is
$L = 2\pi rh$
$= 2\pi(2)(3) = 12\pi \text{ cm}^2$.
The base area of the cylinder is not added, just raised through 3 cm.
The surface area of the composite figure is the sum of the areas of all surfaces on the exterior of figure.
$S =$ (prism surface area) + (cylinder lateral area)
$= 202 + 12\pi \approx 239.7 \text{ cm}^2$

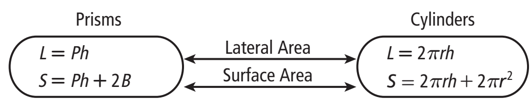
4. original dimensions:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(11)(14) + 2\pi(11)^2$
$= 550\pi \text{ cm}^2$
height, diameter multiplied by $\frac{1}{2}$:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(5.5)(7) + 2\pi(5.5)^2$
$= 137.5\pi \text{ cm}^2$
Notice $137.5\pi = \frac{1}{4}(550\pi)$. If the height and diameter are multiplied by $\frac{1}{2}$, the surface area is multiplied by $\frac{1}{4}$.

5. The 5 cm by 5 cm by 1 cm prism has a surface area of $S = Ph + 2B = 20(1) + 2(5)(5) = 70 \text{ cm}^2$, which is greater than the 2 cm by 3 cm by 4 cm prism and about the same as the half cylinder. It will melt faster than 2 cm by 3 cm by 4 cm prism and at about same rate as the half cylinder.

THINK AND DISCUSS, PAGE 683

1. Use the radius of the base to find the base area, then add twice the base area to the lateral area.

2. An oblique prism has at least one lateral face that is not a rectangle. All the lateral faces of a right prism are rectangles.

3.


EXERCISES, PAGES 684–687

GUIDED PRACTICE, PAGE 684

1. 5 lateral faces

2. $L = Ph$
$= 24(3) = 72 \text{ ft}^2$

$S = L + 2B$
$= 72 + 2(5)(7) = 142 \text{ ft}^2$

3. The base is a rt. △, because 3, 4, 5 is a Pythag. triple.
$L = Ph$
$= 12(2) = 24 \text{ cm}^2$

$S = L + 2B$
$= 24 + 2\left(\frac{1}{2}(3)(4)\right)$
$= 36 \text{ cm}^2$

4. $L = Ph$
$= (4s)(s)$
$= 4(9)(9) = 324 \text{ in}^2$

$S = L + 2B$
$= L + 2s^2$
$= 324 + 2(9)^2 = 486 \text{ in}^2$

5. $L = 2\pi rh = 2\pi(3)(4) = 24\pi \text{ ft}^2$
$S = 2\pi rh + 2\pi r^2 = 24\pi + 2\pi(3)^2 = 42\pi \text{ ft}^2$

6. $L = 2\pi rh = 2\pi(7.5)(12) = 180\pi \text{ yd}^2$
$S = 2\pi rh + 2\pi r^2 = 180\pi + 2\pi(7.5)^2 = 292.5\pi \text{ yd}^2$

7. **Step 1** Use the base area to find the radius.
$A = \pi r^2$
$64\pi = \pi r^2$
$64 = r^2$
$r = 8$ m

Step 2 Use the radius to find the lateral area and base area. The height is 3 less than the radius, or 5 m.
$L = 2\pi rh = 2\pi(8)(5) = 80\pi \text{ m}^2$
$S = 2\pi rh + 2\pi r^2 = 80\pi + 2\pi(8)^2 = 208\pi \text{ m}^2$

8. surface area of the right rectangular prism is
$S = Ph + 2B$
$= 44(12) + 2(14)(8) = 752 \text{ ft}^2$.
A right cylinder is added to right rectangular prism.
The lateral area of the cylinder is
$L = 2\pi rh$
$= 2\pi(4)(8) = 64\pi \text{ ft}^2$.
The base area of the cylinder is not added, just raised through 8 ft.
The surface area of the composite figure is the sum of the areas of all the surfaces on the exterior of the figure.
$S =$ (prism surface area) + (cylinder lateral area)
$= 752 + 64\pi \approx 953.1 \text{ ft}^2$

9. surface area of cylinder is
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(14)(14) + 2\pi(14)^2 = 784\pi \text{ ft}^2$
A right rectangular prism is removed from the cylinder.
The lateral area is $L = Ph = 40(14) = 560 \text{ ft}^2$.
The base area is $B = 14(6) = 84 \text{ ft}^2$.
The surface area of the composite figure is the sum of the areas of all the surfaces on the exterior of the figure.
$S =$ (cylinder surface area) + (prism rectangle area) − (prism base area)
$= 784\pi + 560 - 2(84)$
$= 784\pi + 392 \approx 2855.0 \text{ ft}^2$

10. original dimensions:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(8)(4) + 2\pi(8)^2$
$= 192\pi \text{ yd}^2$
dimensions multiplied by $\frac{1}{2}$:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(4)(2) + 2\pi(4)^2$
$= 48\pi \text{ yd}^2$
Notice $48\pi = \frac{1}{4}(192\pi)$. If the dimensions are multiplied by $\frac{1}{2}$, the surface area is multiplied by $\frac{1}{4}$.

11. original dimensions:
$S = Ph + 2B$
$= 32(6) + 2(8)(8) = 320 \text{ yd}^2$
dimensions multiplied by $\frac{2}{3}$:
$S = Ph + 2B$
$= \frac{64}{3}(4) + 2\left(\frac{16}{3}\right)\left(\frac{16}{3}\right) = \frac{1280}{9} \text{ yd}^2$
Notice $\frac{1280}{9} = \frac{4}{9}(320)$. If the dimensions are multiplied by $\frac{2}{3}$, the surface area is multiplied by $\frac{4}{9}$.

12. 16-in. bulb: $L = 2\pi rh = 2\pi(1)(16) = 32\pi \text{ in}^2$
23-in. bulb: $L = 2\pi rh = 2\pi 3/4(23) = 34.5\pi \text{ in}^2$
23-in. bulb will produce more light

PRACTICE AND PROBLEM SOLVING, PAGES 685–686

13. $L = Ph$
$= 20(10) = 200 \text{ cm}^2$
$S = L + 2B$
$= 200 + 2(5)(5)$
$= 250 \text{ cm}^2$

14. $L = Ph$
$= 6(12)(15)$
$= 1080 \text{ m}^2$
apothem of base is $6\sqrt{3}$ m
$S = L + 2B$
$= 1080 + 2\left(\frac{1}{2}\left(6\sqrt{3}\right)(72)\right)$
$= 1080 + 432\sqrt{3}$
$\approx 1828.2 \text{ m}^2$

15. $L = Ph$
$= 3(8)(14)$
$= 336 \text{ ft}^2$
altitude of base is $4\sqrt{3}$ ft
$S = L + 2B$
$= 336 + 2\left(\frac{1}{2}(8)\left(4\sqrt{3}\right)\right)$
$= 336 + 32\sqrt{3}$
$\approx 391.4 \text{ ft}^2$

16. $L = 2\pi rh = 2\pi(5.5)(7) = 77\pi \text{ in}^2$
$S = 2\pi rh + 2\pi r^2 = 77\pi + 2\pi(5.5)^2 = 137.5\pi \text{ in}^2$

17. $L = 2\pi rh = 2\pi(4)(23) = 184\pi \text{ cm}^2$
$S = 2\pi rh + 2\pi r^2 = 184\pi + 2\pi(4)^2 = 216\pi \text{ cm}^2$

18. **Step 1** Use the base circumference to find the radius.
$C = 2\pi r$
$16\pi = 2\pi r$
$r = 8$ yd
Step 2 Use the radius to find the lateral area and the base area. The height is 3 times the radius, or 24 yd.
$L = 2\pi rh = 2\pi(8)(24) = 384\pi \text{ yd}^2$
$S = 2\pi rh + 2\pi r^2 = 384\pi + 2\pi(8)^2 = 512\pi \text{ yd}^2$

19. The base of the right triangular prism is a rt. △, because 6, 8, 10 is a Pythagorean triple.
The surface area of the right triangular prism is
$S = Ph + 2B$
$= 24(9) + 2\left(\frac{1}{2}(6)(8)\right) = 264 \text{ cm}^2$.
A right cylinder is removed from the triangular prism.
The lateral area of cylinder is
$L = 2\pi rh$
$= 2\pi(2)(9) = 36\pi \text{ cm}^2$.
The base area of cylinder is
$B = \pi r^2 = \pi(2)^2 = 4\pi \text{ cm}^2$.
The surface area of the composite figure is the sum of areas of all surfaces on the exterior of the figure.
$S =$ (prism surface area) + (cylinder lateral area) − (cylinder base area)
$= 264 + 36\pi - 2(4\pi)$
$= 264 + 28\pi \approx 352.0 \text{ cm}^2$

20. The surface area of right rectangular prism is
$S = Ph + 2B$
$= 8(0.5) + 2(2)(2) = 12 \text{ ft}^2$.
A right cylinder is added to the rectangular prism.
The lateral area of cylinder is
$L = 2\pi rh$
$= 2\pi(0.5)(2) = 2\pi \text{ ft}^2$.
The base area of cylinder is not added, just lowered through 2 ft.
The surface area of the composite figure is the sum of the areas of all surfaces on the exterior of figure.
$S =$ (prism surface area) + (cylinder lateral area)
$= 12 + 2\pi \approx 18.3 \text{ ft}^2$

21. original:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(4.5)(11) + 2\pi(4.5)^2$
$= 139.5\pi \text{ ft}^2$
dimensions tripled:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(13.5)(33) + 2\pi(13.5)^2$
$= 1255.5\pi \text{ ft}^2$
$1255.5\pi = 9(139.5\pi)$. So, surface area is multiplied by 9.

22. original:
$S = Ph + 2B$
$= 42(3) + 2(12)(9) = 342 \text{ ft}^2$
dimensions doubled:
$S = Ph + 2B$
$= 84(6) + 2(24)(18) = 1368 \text{ ft}^2$
$1368 = 4(342)$. So, surface area is multiplied by 4.

23. left cell:
$S = Ph + 2B$
$= 90(7) + 2(35)(10) = 1330 \ \mu\text{m}^2$
right cell:
$S = Ph + 2B$
$= 52(15) + 2(15)(11) = 1110 \ \mu\text{m}^2$
The cell that measures 35 μm by 7 μm by 10 μm should absorb at a greater rate.

24. $S = 2\pi rh + 2\pi r^2$
$160\pi = 2\pi(5)h + 2\pi(5)^2$
$160\pi = 10\pi h + 50\pi$
$110\pi = 10\pi h$
$h = 11$ ft

25. $S = Ph + 2B$
$286 = 36h + 2(10)(8)$
$286 = 36h + 160$
$126 = 36h$
$h = 3.5$ m

26. $L = Ph$
$1368 = 6(12)h$
$1368 = 72h$
$h = 19$ m

27. The bases are rt. $\triangle$s with leg lengths 2 and 5 units, and hypotenuse length $\sqrt{2^2 + 5^2} = \sqrt{29}$ units; height is 9 units.
$S = Ph + 2B$
$= \left(2 + 5 + \sqrt{29}\right)(9) + 2\left(\frac{1}{2}(2)(5)\right)$
$= 63 + 9\sqrt{29} + 10$
$= 73 + 9\sqrt{29} \approx 121.5 \text{ units}^2$

28. $S = 2\pi rh + 2\pi r^2$
$= 2\pi(9.525)(1.55) + 2\pi(9.525)^2$
$\approx 662.81 \text{ mm}^2$

29. $S = 2\pi rh + 2\pi r^2$
$= 2\pi(10.605)(1.95) + 2\pi(10.605)^2$
$\approx 836.58 \text{ mm}^2$

30. $S = 2\pi rh + 2\pi r^2$
$= 2\pi(8.955)(1.35) + 2\pi(8.955)^2$
$\approx 579.82 \text{ mm}^2$

31. $S = 2\pi rh + 2\pi r^2$
$= 2\pi(12.13)(1.75) + 2\pi(12.13)^2$
$\approx 1057.86 \text{ mm}^2$

32. Possible answer: triple edge lengths
Let ℓ, w, and h represent the original dimensions.
original:
$S = Ph + 2B$
$= (2\ell + 2w)h + 2\ell w$
$= 2\ell h + 2wh + 2\ell w$
tripled:
$S = Ph + 2B$
$= (2(3\ell) + 2(3w))(3h) + 2(3\ell)(3w)$
$= 18\ell h + 18wh + 18\ell w$
$= 9(2\ell h + 2wh + 2\ell w)$

33. Possible answer: Multiply the radius and the height by $\frac{1}{2}$.
original: $S = 2\pi rh + 2\pi r^2$
halved: $S = 2\pi\left(\frac{1}{2}r\right)\left(\frac{1}{2}h\right) + 2\pi\left(\frac{1}{2}r\right)^2$
$= \frac{1}{2}\pi rh + \frac{1}{2}\pi r^2$
$= \frac{1}{4}\left(2\pi rh + 2\pi r^2\right)$

34. the triangular prism shaped frame
$S =$ (area of 2 sides) + (area of 2 ends)
$= 2(10)(10) + 2\left(\frac{1}{2}(10)5\sqrt{3}\right)$
$= 200 + 50\sqrt{3} \approx 286.6 \text{ ft}^2$
half cylinder:
$S =$ (area of curved panel) + (area of 2 ends)
$= (10)\left(\frac{1}{2}\pi(10)\right) + 2\left(\frac{1}{2}\pi(5)^2\right)$
$= 50\pi + 25\pi$
$= 75\pi \approx 235.6 \text{ ft}^2$
The triangular-prism-shaped frame requires more plastic.

35. area with given measurements:
$S = Ph + 2B$
$= 12(6) + 2(3)(3) = 90 \text{ cm}^2$
least possible area:
$S = 4\ell w + 2w$
$= 4(5.95)(2.95) + 2(2.95)(2.95) = 87.615 \text{ cm}^2$
greatest possible area:
$S = 4\ell w + 2w$
$= 4(6.05)(3.05) + 2(3.05)(3.05) = 92.415 \text{ cm}^2$
max. error below 90 cm^2 is $90 - 87.615 = 2.385 \text{ cm}^2$
max. error above 90 cm^2 is $92.415 - 90 = 2.415 \text{ cm}^2$
max. error $< 2.415 \text{ cm}^2$

36. Find the area of each part of the net and add the areas.

37a. $AB = 8 - 1 = 7$ in.
$BC = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$ in. ≈ 5.7 in.

b. $AC^2 + BC^2 = AB^2$
$AC^2 + \left(4\sqrt{2}\right)^2 = 7^2$
$AC^2 + 32 = 49$
$AC^2 = 17$
$AC = \sqrt{17}$ in. ≈ 4.1 in.

c. $S = Ph + 2B$
$= 16\,(4.1) + 2(16) \approx 97.6 \text{ in}^2$

TEST PREP, PAGE 687

38. A
$S =$ (area of rectangle) + (area of circles)
$= (3.0)(8.1) + 2\pi(1.35)^2 \approx 35.8 \text{ cm}^2$

39. F
$L = Ph = 24(5) = 120 \text{ in}^2$

40. 414.5
$S = 2(3.14)rh + 2(3.14)r^2$
$= 2(3.14)(6)(5) + 2(3.14)(6)^2 \approx 414.5 \text{ in}^2$

CHALLENGE AND EXTEND, PAGE 687

41. 1st cylinder:
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(8)(3) + 2\pi(8)^2 = 176\pi \text{ cm}^2$
2nd cylinder:
$S = 2\pi rh + 2\pi r^2$
$176\pi = 2\pi(4)h + 2\pi(4)^2$
$176\pi = 2\pi(4)h + 32\pi$
$144\pi = 8\pi h$
$h = 18$ cm

42. $S =$ (area of 2 sides) + (area of 2 ends)
$= 2(12)(12) + 2\left(2\left(\frac{1}{2}(12 + 18)(9)\right)\right)$
$= 288 + 540 = 828 \text{ ft}^2$
amount of paint $= \frac{828}{250} \approx 3.3$
4 gal of paint are needed, costing 4(\$25) = \$100.

43. $L = Ph$
$144 = (2\ell + 2w)h$
$144 = (2(3w) + 2w)(2w)$
$144 = 16w^2$
$9 = w^2$
$w = 3$ cm
$\ell = 3w = 3(3) = 9$ cm, $h = 2w = 2(3) = 6$ cm
$S = L + 2B$
$= 144 + 2(9)(3) = 198 \text{ cm}^2$

SPIRAL REVIEW, PAGE 687

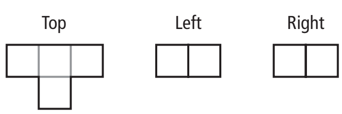
44. $154 + m \leq 250$

45. $70 \leq s \leq 110$

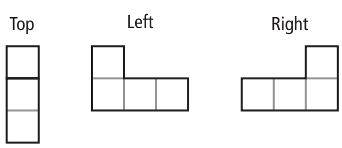
46. $BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$
$a^2 = 7^2 + 8^2 - 2(7)(8)\cos 45°$
$a^2 \approx 33.804$
$a \approx 5.8$

47. $\frac{\sin B}{AC} = \frac{\sin A}{BC}$
$\frac{\sin B}{8} \approx \frac{\sin 45°}{5.81}$
$m\angle B \approx \sin^{-1}\left(\frac{8 \sin 45°}{5.81}\right) \approx 77°$

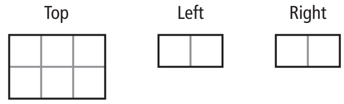
48.



49.



50.



GEOMETRY LAB: MODEL RIGHT AND OBLIQUE CYLINDERS, PAGE 688

ACTIVITY 1, TRY THIS, PAGE 688

1. Each cross section is a circle.

2. The cross section is an ellipse.

ACTIVITY 2, TRY THIS, PAGE 688

3. The base of rectangle is the distance around the cylinder, the height of the rectangle is the slant height of the cylinder, and the area of the rectangle is the lateral area of the cylinder. Check students' estimates.

10-5 SURFACE AREA OF PYRAMIDS AND CONES, PAGES 689–696

CHECK IT OUT! PAGES 690–692

1. **Step 1** Find the base perimeter and the altitude.
The base perimeter is $3(6) = 18$ ft.
The altitude is $3\sqrt{3}$ ft, so, the base area is $\frac{1}{2}bh = \frac{1}{2}(6)\left(3\sqrt{3}\right) = 9\sqrt{3} \text{ ft}^2.$
Step 2 Find the lateral area.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(18)(10) = 90 \text{ ft}^2$
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 90 + 9\sqrt{3} \approx 105.6 \text{ ft}^2$

2. **Step 1** Use the Pythag. Thm. to find ℓ.
$\ell = \sqrt{8^2 + 6^2} = 10$ cm
Step 2 Find the lateral area and the surface area.
$L = \pi r\ell$
$= \pi(8)(10) = 80\pi \text{ cm}^2$
$S = \pi r\ell + \pi r^2$
$= 80\pi + \pi(8)^2 = 144\pi \text{ cm}^2$

3. original:
$S = \frac{1}{2}P\ell + B$
$= \frac{1}{2}(60)(12) + (15)^2 = 585 \text{ ft}^2$
base edge length and slant height multiplied by 2/3:
$S = \frac{1}{2}P\ell + B$
$= \frac{1}{2}(40)(8) + (10)^2 = 260 \text{ ft}^2$
Notice that $260 = \frac{4}{9}(585)$. If the base edge length and the slant height are multiplied by $\frac{2}{3}$, the surface area is multiplied by $\frac{4}{9}$.

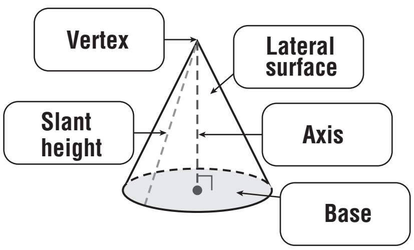
4. The base area of the cube is $B = (2)^2 = 4$ yd^2.
The lateral area of the cube is
$L = Ph = (8)(2) = 16$ yd^2.
By the Pythag. Thm., $\ell = \sqrt{1^2 + 2^2} = \sqrt{5}$ yd.
The lateral area of the pyramid is $L = \frac{1}{2}P\ell = \frac{1}{2}(8)\sqrt{5} = 4\sqrt{5}$ yd^2.
A = (base area of cube) + (lateral area of cube) + (lateral area of pyramid)
$= 4 + 16 + 4\sqrt{5}$
$= 20 + 4\sqrt{5} \approx 28.9$ yd^2

5. The radius of the large circle used to create the pattern is the slant height of the cone.
The area of the pattern is the lateral area of the cone. The area of the pattern is also $\frac{3}{4}$ of the area of the large circle. So, $\pi r\ell = \frac{3}{4}\pi\ell^2$.
$\pi r(12) = \frac{3}{4}\pi(12)^2$
$r = 9$ in.

THINK AND DISCUSS, PAGE 692

1. The lateral faces are all $\triangle$s with an area of $\frac{1}{2}$ the base edge length times the slant height, the perimeter P is the sum of the base edge lengths. So, $L = \frac{1}{2}P\ell$.

2. A radius and the axis of a right cone form the legs of a right $\triangle$. Length of the hypotenuse is the slant height of the cone. The hypotenuse of a right $\triangle$ is always the longest side, so slant height is greater than the height.

3.


EXERCISES, PAGES 693–696

GUIDED PRACTICE, PAGE 693

1. vertex and center of base

2. **Step 1** Find the base perimeter and the apothem.
The base perimeter is $6(8) = 48$ cm.
The apothem is $4\sqrt{3}$ cm. So, base area is $\frac{1}{2}aP = \frac{1}{2}(4\sqrt{3})(48) = 96\sqrt{3}$ cm^2.
Step 2 Find the lateral area.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(48)(12) = 288$ cm^2
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 288 + 96\sqrt{3} \approx 454.3$ cm^2

3. **Step 1** Find the base perimeter and the area.
The base perimeter is $4(16) = 64$ ft.
The base area is $(16)^2 = 256$ ft^2.
Step 2 Find the lateral area.
The slant height is $\sqrt{8^2 + 15^2} = 17$ ft.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(64)(17) = 544$ ft^2
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 544 + 256 = 800$ ft^2

4. **Step 1** Find the base perimeter and the altitude.
The base perimeter is $3(15) = 45$ in.
The altitude of base is $7.5\sqrt{3}$ in. So, base area is $\frac{1}{2}bh = \frac{1}{2}(15)(7.5\sqrt{3}) = 56.25\sqrt{3}$ in^2.
Step 2 Find the lateral area.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(45)(20) = 450$ in^2
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 450 + 56.25\sqrt{3} \approx 547.4$ in^2

5. **Step 1** Use the Pythogorean Theorem to find the base radius r.
$r = \sqrt{25^2 - 24^2} = 7$ in.
Step 2 Find the lateral area and the surface area.
$L = \pi r\ell$
$= \pi(7)(25) = 175\pi$ in^2
$S = \pi r\ell + \pi r^2$
$= 175\pi + \pi(7)^2 = 224\pi$ in^2

6. $L = \pi r\ell$
$= \pi(14)(22) = 308\pi$ m^2
$S = \pi r\ell + \pi r^2$
$= 308\pi + \pi(14)^2 = 504\pi$ m^2

7. **Step 1** Use the base area to find the base radius r.
$A = 36\pi = \pi r^2$
$36 = r^2$
$r = 6$ ft
Step 2 Find the lateral area and the surface area.
$L = \pi r\ell$
$= \pi(6)(8) = 48\pi$ ft^2
$S = \pi r\ell + \pi r^2$
$= 48\pi + \pi(6)^2 = 84\pi$ ft^2

8. original:
$S = \frac{1}{2}P\ell + B$
$= \frac{1}{2}(24)(10) + (6)^2 = 156$ in^2
dimensions halved:
$S = \frac{1}{2}P\ell + B$
$= \frac{1}{2}(12)(5) + (3)^2 = 39$ in^2
Notice that $39 = \frac{1}{4}(156)$. If the dimensions are cut in half, the surface area is multiplied by $\frac{1}{4}$.

9. original:
$S = \pi r\ell + \pi r^2$
$= \pi(9)(15) + \pi(9)^2 = 216 \text{ cm}^2$
dimensions tripled:
$S = \pi r\ell + \pi r^2$
$= \pi(27)(45) + \pi(27)^2 = 1944 \text{ cm}^2$
Notice that $1944 = 9(216)$. If the dimensions are tripled, the surface area is multiplied by 9.

10. The lateral area of the upper pyramid is
$L = \frac{1}{2}Ph = \frac{1}{2}(32)(15) = 240 \text{ ft}^2$.
The lateral area of the upper pyramid is
$L = \frac{1}{2}Ph = \frac{1}{2}(32)(18) = 288 \text{ ft}^2$.
$S =$ (lateral area of upper pyramid)
+ (lateral area of lower pyramid)
$= 240 + 288 = 528 \text{ ft}^2$

11. The lateral area of upper cone is
$L = \pi r\ell = \pi(12)(26) = 312\pi \text{ m}^2$.
The lateral area of cylinder is
$L = 2\pi rh = 2\pi(12)(15) = 360\pi \text{ m}^2$.
The lateral area of upper cone is
$L = \pi r\ell = \pi(12)(32) = 384\pi \text{ m}^2$.
$S =$ (lateral area of upper cone)
+ (lateral area of cylinder)
+ (lateral area of lower cone)
$= 312\pi + 360\pi + 384\pi = 1056\pi \text{ m}^2$

12. The radius of the large circle used to create the hat is the slant height of the cone.
The area of the hat is the lateral area of the cone.
The area of the hat is also $\frac{3}{4}$ of the area of the large circle. So, $\pi r\ell = \frac{3}{4}\pi\ell^2$.
$\pi r(6) = \frac{3}{4}\pi(6)^2$
$r = 4.5$
$d = 2(4.5) = 9$ in. So, the hat will be too large.

PRACTICE AND PROBLEM SOLVING, PAGES 694–695

13. **Step 1** Find the base perimeter and the area.
The base perimeter is $4(6) = 24$ ft.
The base area is $(6)^2 = 36 \text{ ft}^2$.
Step 2 Find the lateral area.
The slant height is $\sqrt{3^2 + 4^2} = 5$ ft.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(24)(5) = 60 \text{ ft}^2$
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 60 + 36 = 96 \text{ ft}^2$

14. **Step 1** Find the base perimeter and the altitude.
The base perimeter is $3(40) = 120$ cm.
The altitude of base is $20\sqrt{3}$ ft. So, the base area is $\frac{1}{2}bh = \frac{1}{2}(40)\left(20\sqrt{3}\right) = 400\sqrt{3} \text{ cm}^2$.
Step 2 Find the lateral area.
The slant height is $\sqrt{25^2 - 20^2} = 15$ cm.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(120)(15) = 900 \text{ cm}^2$
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 900 + 400\sqrt{3} \approx 1592.8 \text{ cm}^2$

15. **Step 1** Find the base perimeter and the apothem.
The base perimeter is $6(7) = 42$ ft.
The apothem is $3.5\sqrt{3}$ ft. So, the base area is
$\frac{1}{2}aP = \frac{1}{2}\left(3.5\sqrt{3}\right)(42) = 73.5\sqrt{3} \text{ ft}^2$.
Step 2 Find the lateral area.
$L = \frac{1}{2}P\ell$
$= \frac{1}{2}(42)(15) = 315 \text{ ft}^2$
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= 315 + 73.5\sqrt{3} \approx 442.3 \text{ ft}^2$

16. $L = \pi r\ell = \pi(11.5)(23) = 264.5\pi \text{ cm}^2$
$S = \pi r\ell + \pi r^2 = 264.5\pi + \pi(11.5)^2 = 396.75\pi \text{ cm}^2$

17. $\ell = \sqrt{12^2 + 35^2} = 37$ in.
$L = \pi r\ell = \pi(12)(37) = 444\pi \text{ in}^2$
$S = \pi r\ell + \pi r^2 = 444\pi + \pi(12)^2 = 588\pi \text{ in}^2$

18. $h = 2(8) - 1 = 15$ m; $\ell = \sqrt{8^2 + 15^2} = 17$ m
$L = \pi r\ell = \pi(8)(17) = 136\pi \text{ m}^2$
$S = \pi r\ell + \pi r^2 = 136\pi + \pi(8)^2 = 200\pi \text{ m}^2$

19. original:
$S = \frac{1}{2}P\ell + \frac{1}{2}aP$
$= \frac{1}{2}(24)(12) + \frac{1}{2}\left(2\sqrt{3}\right)(24)$
$= \left(144 + 24\sqrt{3}\right) \text{ ft}^2$
dimensions divided by 3:
$S = \frac{1}{2}P\ell + \frac{1}{2}aP$
$= \frac{1}{2}(8)(4) + \frac{1}{2}\left(\frac{2\sqrt{3}}{3}\right)(8)$
$= \left(16 + \frac{8\sqrt{3}}{3}\right) \text{ ft}^2$
$\left(16 + \frac{8\sqrt{3}}{3}\right) = \left(144 + 24\sqrt{3}\right) \div 9$. So, the surface area is divided by 9.

20. original:
$S = \pi r\ell + \pi r^2 = \pi(2)(5) + \pi(2)^2 = 14\pi \text{ m}^2$
dimensions doubled:
$S = \pi r\ell + \pi r^2 = \pi(4)(10) + \pi(4)^2 = 56\pi \text{ m}^2$
$56\pi = 4(14\pi)$. So, the surface area is multiplied by 4.

21. lateral area of left cone $= \pi(7)(24) = 168\pi \text{ in}^2$
lateral area of right cone $= \pi(7)(17) = 119\pi \text{ in}^2$
$S =$ (lateral area of left cone)
+ (lateral area of right cone)
$= 168\pi + 119\pi = 287\pi \text{ in}^2$

22. lateral area of left pyramid
$= \frac{1}{2}P\ell = \frac{1}{2}(36)(15) = 270 \text{ cm}^2$
lateral area of cube $= Ph = (36)(9) = 324 \text{ cm}^2$
lateral area of right pyramid
$= \frac{1}{2}P\ell = \frac{1}{2}(36)(19) = 342 \text{ cm}^2$
$S =$ (lateral area of left pyramid)
+ (lateral area of cube)
+ (lateral area of right pyramid)
$= 270 + 324 + 342 = 936 \text{ cm}^2$

23. $S = \pi r\ell = \frac{1}{2}\pi\ell^2$
$\pi r(6) = \frac{1}{2}\pi(6)^2$
$r = 3$
$d = 2(3) = 6$ in.

24. $B = s^2 = 36$. So, $s = 6$ cm and $P = 24$ cm
$S = \frac{1}{2}P\ell + B$
$= \frac{1}{2}(24)(5) + 36 = 96 \text{ cm}^2$

25. $B = \frac{1}{2}as$
$\sqrt{3} = \frac{1}{2}\left(\frac{s\sqrt{3}}{2}\right)s = \frac{1}{4}s^2\sqrt{3}$
$4 = s^2$
$s = 2$ m
$P = 3(2) = 6$ m
$S = \frac{1}{2}P\ell + B$
$= \frac{1}{2}(6)\left(\sqrt{3}\right) + \sqrt{3} = 4\sqrt{3} \text{ m}^2$

26. $B = \pi r^2 = 16\pi$. So, $r = 4$ in.
$S = \pi(4)(7) + \pi(4)^2 = 44\pi \text{ in}^2$

27. $B = \pi r^2 = \pi$, *so* $r = 1$ ft
$S = \pi(1)(2) + \pi(1)^2 = 3\pi \text{ ft}^2$

28a. $\ell = \sqrt{4^2 + 10^2} = \sqrt{116} = 2\sqrt{29}$ cm
$S = \frac{1}{2}P\ell + s^2$
$= \frac{1}{2}(32)\left(2\sqrt{29}\right) + (8)^2$
$= 32\sqrt{29} + 64 \approx 236.3 \text{ cm}^2$

b. $S = \pi r\ell + \pi r^2$
$32\sqrt{29} + 64 = \pi(4.5)\ell + \pi(4.5)^2$
$32\sqrt{29} + 64 - 20.25\pi = 4.5\pi\ell$
$\ell = \dfrac{32\sqrt{29} + 64 - 20.25\pi}{4.5\pi} \approx 12.2$ cm

29. $S = \pi r\ell + \pi r^2$
$232\pi = \pi r(21) + \pi r^2$
$232 = 21r + r^2$
$0 = r^2 + 21r - 232$
$0 = (r - 8)(r + 29)$
$r = 8$ m (since r is positive)

30. $P = 4s = 32$. So, $s = 8$ ft
$S = \frac{1}{2}P\ell + s^2$
$256 = \frac{1}{2}(32)\ell + (8)^2$
$256 = 16\ell + 64$
$192 = 16\ell$
$\ell = 12$ ft

31. $L = \frac{1}{2}P\ell$
$120 = \frac{1}{2}P(10) = 5P$
$P = 24$ cm

32. $r^2 + h^2 = \ell^2$
$r^2 + 7^2 = 25^2$
$r = \sqrt{25^2 - 7^2} = 24$ units
$S = \pi r\ell + \pi r^2$
$= \pi(24)(25) + \pi(24)^2 = 1176\pi \text{ units}^2$

33. left pyramid:
$L_{left} = \frac{1}{2}P\ell = \frac{1}{2}(30)(14) = 210 \text{ cm}^2$
right pyramid:
$L_{right} = \frac{1}{2}P\ell = \frac{1}{2}(30)(8) = 120 \text{ cm}^2$
$S = L_{left} + L_{right} = 210 + 210 = 330 \text{ cm}^2$

34. left cone: $\ell = \sqrt{6^2 + 8^2} = 10$ m
$S = \pi r\ell + \pi r^2 = \pi(6)(10) + \pi(6)^2 = 96\pi \text{ m}^2$
right cone: $\ell = \sqrt{10^2 + 24^2} = 26$ m
$S = \pi r\ell + \pi r^2 = \pi(10)(26) + \pi(10)^2 = 360\pi \text{ m}^2$
$S = 96\pi + 360\pi = 456\pi \text{ m}^2$

35. $s = 200(3 \text{ ft}) = 600$ ft and $h = 32(10) = 320$ ft. So,
$\ell = \sqrt{300^2 + 320^2} = \sqrt{192{,}400} \approx 438.6$ ft
$L = \frac{1}{2}P\ell \approx \frac{1}{2}(2400)(438.6) \approx 526{,}000 \text{ ft}^2$

36. A triangle is formed with 2 vertices at the midpoints of opposite sides of the square base and the third vertex at the vertex of the pyramid. The side lengths of the triangle are ℓ, ℓ, and s, the edge length of the base. By the Triangle Inequality Theorem, $\ell + \ell > s$, so $2\ell > s$. Therefore, $\ell > \frac{1}{2}s$.

37. In an oblique cone, the distance from a point on edge of the base to the vertex is not the same for each point on the edge of the base.

TEST PREP, PAGE 695

38. D
$A = \left(\frac{1}{4}t\right)^2 + \frac{1}{2}t\ell$
$= \dfrac{t^2}{16} + \dfrac{t\ell}{2}$ (II)
$= \dfrac{t}{2}\left(\dfrac{t}{8} + \ell\right)$ (III)

39. F
$L = \frac{1}{2}P\ell$
$216 = \frac{1}{2}(4s)(18)$
$432 = 72s$
$s = 6$ cm
$S = L + B = 216 + (6)^2 = 252 \text{ cm}^2$

40. B
$\ell = \sqrt{9^2 + 40^2} = 41$ cm
$L = \pi r\ell = \pi(9)(41) = 369\pi \text{ cm}^2$

CHALLENGE AND EXTEND, PAGE 696

41a. $\dfrac{\ell}{10} = \dfrac{\ell - 20}{5}$
$5\ell = 10(\ell - 20)$
$5\ell = 10\ell - 200$
$200 = 5\ell$
$\ell = 40$ cm
$S = \pi r\ell + \pi r^2 = \pi(10)(40) + \pi(10)^2 = 500\pi \text{ cm}^2$

b. $\ell = 40 - 20 = 20$ cm
$L = \pi r\ell = \pi(5)(20) = 100\pi \text{ cm}^2$

c. $B = \pi(5)^2 = 25\pi \text{ cm}^2$

d. S = (surface area of cone) + (area of top base) − (lateral area of top of cone)
$= 500\pi + 25\pi - 100\pi = 425\pi \text{ cm}^2$

42. $L = 4\left(\frac{1}{2}(b_1 + b_2)\ell\right) = 2(b_1 + b_2)\ell$

43a. $c = 2\pi r$

b. $C = 2\pi\ell$

c. $\dfrac{c}{C} = \dfrac{2\pi r}{2\pi\ell} = \dfrac{r}{\ell}$

d. $A = \pi\ell^2$
$L = \dfrac{c}{C}(A) = \pi\ell^2\left(\dfrac{r}{\ell}\right) = \pi r\ell$

SPIRAL REVIEW, PAGE 696

44. Since the area of a circle depends on the square of its radius, the surface area of a cone cannot be described by a linear function.

45. Since perimeter is 1-dimensional, the perimeter of a rectangle can be described by a linear function.

46. Since the area of a circle depends on the square of its radius, it cannot be described by a linear function.

47. area of $ACEF = 4^2 = 16 \text{ cm}^2$
area of $\triangle BDG = \frac{1}{2}(4)(2) = 4 \text{ cm}^2$
$P = \frac{4}{16} = 0.25$

48. area of circle $H = \pi(2)^2 = 4\pi \text{ cm}^2$
$P = \frac{4\pi}{16} = \frac{\pi}{4} \approx 0.79$

49. area of shaded region $= (16 - 4\pi) \text{ cm}^2$
$P = \frac{16 - 4\pi}{16} = \frac{4 - \pi}{4} \approx 0.21$

50. $S = L + B$
$= \frac{1}{2}(8 + 15 + 17)(10) + 2\left(\frac{1}{2}(8)(15)\right)$
$= 400 + 120 = 520 \text{ in}^2$

51. $S = L + B$
$= (2(8) + 2(10))(15) + 2((8)(10))$
$= 540 + 160 = 700 \text{ cm}^2$

52. $S = 2\pi rh + 2\pi r^2$
$= 2\pi(2)(3) + 2\pi(2)^2$
$= 20\pi \approx 62.8 \text{ cm}^2$

10-6 VOLUME OF PRISMS AND CYLINDERS, PAGES 697–704

CHECK IT OUT! PAGES 698–700

1. $V = Bh$
$= \left(\frac{1}{2}(5)(7)\right)(9) = 157.5 \text{ yd}^3$

2. **Step 1** Find the volume of the aquarium in cubic feet. $V = \ell wh = (120)(60)(16) = 115{,}200 \text{ ft}^3$

Step 2 Use the conversion factor $\frac{1 \text{ gal}}{0.134 \text{ ft}^3}$ to estimate the volume in gallons.

$115{,}200 \text{ ft}^3 \bullet \frac{1 \text{ gal}}{0.134 \text{ ft}^3} \approx 859{,}702 \text{ gal}$

Step 3 Use the conversion factor $\frac{8.33 \text{ lb}}{1 \text{ gal}}$ to estimate the weight of water.

$859{,}702 \text{ gal} \bullet \frac{8.33 \text{ lb}}{1 \text{ gal}} \approx 7{,}161{,}318 \text{ lb}$

3. $V = \pi r^2 h$
$= \pi(8)^2(17) = 1088\pi \text{ in}^3$
$\approx 3418.1 \text{ in}^3$

4. original dimensions:
$V = \ell wh = (4)(3)(1.5) = 18 \text{ ft}^3$
length, width, and height doubled:
$V = \ell wh = (8)(6)(3) = 144 \text{ ft}^3$
Notice that $144 = 8(18)$. If the length, width, and height are doubled, the volume is multiplied by 8.

5. The volume of cylinder is
$V = \pi r^2 h = \pi(3)^2(5) = 45\pi \text{ cm}^3$.
The volume of prism is $V = Bh = \left(3\sqrt{2}\right)^2(5) = 90 \text{ cm}^3$.
The net volume of figure is difference of volumes.
$V = 45\pi - 90 \approx 51.4 \text{ cm}^3$

THINK AND DISCUSS, PAGE 700

1. In both formulas, the volume equals the base area times the height. The base area of a cylinder $= \pi r^2$, and the base area of a prism is given by the area formula for that polygon type.

2. An oblique prism has the same cross-sectional area at every level as a right prism with the same base area and height. By Cavalieri's Principle, the oblique prism and the right prism have the same volume.

3.

Shape	Volume
Prism	$V = Bh$
Cube	$V = s^3$
Cylinder	$V = \pi r^2 h$

EXERCISES, PAGES 701–704

GUIDED PRACTICE, PAGE 701

1. the same length as

2. $V = \ell wh$
$= (9)(4)(6) = 216 \text{ cm}^3$

3. $V = Bh$
$= \frac{1}{2}aP \bullet h$
$= \frac{1}{2}\left(3\sqrt{3}\right)(36)(8)$
$= 432\sqrt{3} \approx 748.2 \text{ m}^3$

4. $V = s^3$
$= (8)^3 = 512 \text{ ft}^3$

5. **Step 1** Find the volume of the ice cream cake in cubic feet.
$V = \ell wh = (19)(9)(2) = 342 \text{ ft}^3$

Step 2 Use the conversion factor $\frac{1 \text{ gal}}{0.134 \text{ ft}^3}$ to estimate the volume in gallons.

$342 \text{ ft}^3 \bullet \frac{1 \text{ gal}}{0.134 \text{ ft}^3} \approx 2552 \text{ gal}$

Step 3 Use the conversion factor $\frac{8.33 \text{ lb}}{1 \text{ gal}}$ to estimate the weight of water.

$2552 \text{ gal} \bullet \frac{4.73 \text{ lb}}{1 \text{ gal}} \approx 12{,}071 \text{ lb}$

6. $V = \pi r^2 h$
$= \pi(6)^2(10) = 360\pi \text{ ft}^3$
$\approx 1131.0 \text{ ft}^3$

7. $V = \pi r^2 h$
$= \pi(3)^2(5) = 45\pi \text{ m}^3$
$\approx 141.4 \text{ m}^3$

8. $B = \pi r^2$
$25\pi = \pi r^2$
$25 = r^2$
$r = 5$ cm
$h = 5 + 3 = 8$ cm
$V = \pi r^2 h$
$= \pi(5)^2(8) = 200\pi$ cm^3
≈ 628.3 cm^3

9. original dimensions:
$V = \ell wh = (12)(4)(8) = 384$ ft^3
dimensions multiplied by $\frac{1}{4}$:
$V = \ell wh = (3)(1)(2) = 6$ ft^3
Notice $6 = \frac{1}{64}(384)$. If the dimensions are multiplied by $\frac{1}{4}$, the volume is multiplied by $\frac{1}{64}$.

10. original dimensions:
$V = \pi r^2 h = \pi(2)^2(7) = 28\pi$ in^3
dimensions tripled:
$V = \pi r^2 h = \pi(6)^2(21) = 756\pi$ in^3
Notice $756\pi = 27(28\pi)$. If the dimensions are tripled, the volume is multiplied by 27.

11. The volume of rect. prism is
$V = \ell wh = (12)(14)(6) = 1008$ ft^3.
The volume of cylinder is
$V = \pi r^2 h = \pi(4)^2(4) = 624\pi$ ft^3.
The total volume of the figure is the sum of its volumes.
$V = 1008 + 64\pi \approx 1209.1$ ft^3

12. The volume of outside cylinder is
$V = \pi r^2 h = \pi(10)^2(15) = 1500\pi$ in^3.
The volume of inside cylinder is
$V = \pi r^2 h = \pi(5)^2(15) = 375\pi$ in^3.
The net volume of the figure is the difference of the volumes.
$V = 1500\pi - 375\pi = 1125\pi \approx 3534.3$ in^3

PRACTICE AND PROBLEM SOLVING, PAGES 702–703

13. $V = Bh = \left(\frac{1}{2}(9)(15)\right)(12) = 810$ yd^3

14. $V = Bh$
$= \frac{1}{2}aP \cdot h$
$= \frac{1}{2}\left(\frac{5}{\tan 36°}\right)(50)(15) \approx 2580.7$ m^3

15. $B = s^2$
$49 = s^2$
$s = 7$ ft
$h = 7 - 2 = 5$ ft
$V = Bh = (49)(5) = 245$ ft^3

16. **Step 1** Find the volume in cubic feet.
$V = \ell wh = (9)(16)\left(\frac{1}{3}\right) = 48$ ft^3
Step 2 Use the conversion factor $\frac{1 \text{ yd}^3}{27 \text{ ft}^3}$ to find the volume in cubic yards. Then round up to nearest cubic yard.
$V = 48 \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 1.78$ yd^3
Colin must buy 2 yd^3 of dirt.
Step 3 Use the cost per yard to find the cost of dirt.
Cost = 2($25) = $50

17. $V = \pi r^2 h$
$= \pi(14)^2(9) = 1764\pi$ cm^3
≈ 5541.8 cm^3

18. $V = \pi r^2 h$
$= \pi(6)^2(3) = 108\pi$ in^3
≈ 339.3 in^3

19. $V = Bh$
$= 24\pi(16) = 384\pi$ cm^3
≈ 1206.4 cm^3

20. original dimensions:
$V = \pi r^2 h = \pi(2)^2(3) = 12\pi$ yd^3
dimensions multiplied by 5:
$V = \pi r^2 h = \pi(10)^2(15) = 1500\pi$ yd^3
$1500\pi = 125(12\pi)$. So, the volume is multiplied by 125.

21. original dimensions:
$V = Bh = (5)^2(10) = 250\pi$ m^3
dimensions multiplied by $\frac{3}{5}$:
$V = \pi r^2 h = \pi(3)^2(6) = 54\pi$ m^3
$54\pi = \frac{27}{125}(250\pi)$. So, the volume is multiplied by $\frac{27}{125}$.

22. volume of lower cube: $V = s^3 = (8)^3 = 512$ cm^3
volume of middle cube: $V = s^3 = (6)^3 = 216$ cm^3
volume of upper cube: $V = s^3 = (4)^3 = 64$ cm^3
total volume: $V = 512 + 216 + 64 = 792$ cm^3

23. volume of square-based prism:
$V = Bh = (4)^2(12) = 192$ ft^3
volume of each half-cylinder:
$V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(2)^2(4) = 8\pi$ ft^3
total volume:
$V = 8\pi + 192 + 8\pi = 192 + 16\pi \approx 242.3$ ft^3

24. radius $r = 2$ in.:
$V = \pi r^2 h$
$14.4375 = \pi(2)^2 h = 4\pi h$
$h = \frac{14.4375}{4\pi} \approx 1.1489$ in.
radius $r = 1.5$ in.:
$V = \pi r^2 h$
$14.4375 = \pi(1.5)^2 h = 2.25\pi h$
$h = \frac{14.4375}{2.25\pi} \approx 2.0425$ in.

25a. $V = \pi r^2 h = \pi(5)^2(3) = 75\pi \approx 235.6$ in^2

b. $V = \ell wh = (3)(1)(3) = 9 \text{ in}^3$
$P = \frac{9}{75\pi} \approx 0.04$

26a. $(3 - 1)^2 + h^2 = 5^2$
$4 + h^2 = 25$
$h^2 = 21$
$h = \sqrt{21} \approx 4.6$ in.

b. $V = \pi r^2 h = \pi(1.5)^2\left(\sqrt{21}\right) \approx 32.5 \text{ in}^3$

c. $32.5 \text{ in}^3 \cdot \frac{0.55 \text{ oz}}{1 \text{ in}^3} \approx 17.9$ oz

27. $V = \ell wh$
$495 = (5)(9)h = 45h$
$h = 11$ ft.

28. $V = Bh$
$360 = B(9)$
$B = 40 \text{ in}^2$

29. $S = 2\pi rh + 2\pi r^2$
$210\pi = 2\pi r(8) + 2\pi r^2$
$210 = 16r + 2r^2$
$105 = 8r + r^2$
$0 = r^2 + 8r - 105$
$0 = (r - 7)(r + 15)$
$r = 7$ m
$V = \pi r^2 h = \pi(7)^2(8) = 392\pi \text{ m}^3$

30. $\ell = 7$ units, $w = 3$ units, $h = 6$ units
$V = \ell wh = (7)(3)(6) = 126 \text{ units}^3$

31. $V = \ell wh = (1)(2)\left(\frac{1}{6}\right) = \frac{1}{3} \text{ ft}^3$
or
$V = (12)(24)(2) = 576 \text{ in}^3$

32. $V = Bh = (4)^2\left(\frac{1}{4}\right) = 4 \text{ in}^3 = 4\left(1 \text{ in}^3\right)$
4 servings

33. **Step 1** Find the volume in cubic feet.
$V = \pi r^2 h = \pi(45)^2(52) = 105{,}300\pi \text{ ft}^3$
Step 2 Convert the volume to gallons.
$105{,}300\pi \text{ ft}^3 \cdot \frac{1 \text{ gal}}{0.134 \text{ ft}^3} \approx 2{,}468{,}729$ gal

34. **Step 1** Find the volume in cubic feet.
$V = \ell wh = (50)(100)\left(\frac{1}{4}\right) = 1250 \text{ ft}^3$
Step 2 Convert the volume to gallons.
$1250 \text{ ft}^3 \cdot \frac{1 \text{ gal}}{0.134 \text{ ft}^3} \approx 9328$ gal
Step 3 Convert to weight in pounds.
$9328 \text{ gal} \cdot \frac{8.33 \text{ lb}}{1 \text{ gal}} \approx 77{,}705$ lb

35. For a scale factor of k, the ratio of surface area to volume of the new prism is $\frac{1}{k}$ times the ratio of the surface area to the volume of the old prism. A cube with edge length 1 has a surface area-to-volume ratio of 6:1. If each edge length is multiplied by 2, the surface area-to-volume ratio is 24:8 = 3:1.

36. $V_{new} = 2V_{old}$
$s_{new}^{\ 3} = 2\left(s_{old}^{\ 3}\right)$
$s_{new} = \sqrt[3]{2}\left(s_{old}\right)$
Multiply edge length by $\sqrt[3]{2}$.

TEST PREP, PAGES 703–704

37. A
$V_{candle} = \pi r^2 h = \pi(3.4)^2(6.0) = 69.36\pi \text{ cm}^3$
$V_{wax} = \ell wh = (15)(12)(18) = 3240 \text{ cm}^3$
candles $\leq \frac{3240}{69.36\pi} \approx 14.9$
candles = 14

38. F
wire length = (# edges)(edge length)
$96 = 12s$
$s = 8$ in.
$V = s^3 = (8)^3 = 512 \text{ in}^3$

39. B
$V_{prism} = Bh = (3)^2(9) = 81 \text{ in}^3$
$V_{cylinder} = \pi r^2 h = \pi(1.75)^2(9) \approx 86.6 \text{ in}^3$

40. H
upper rect. prism: $V = (10 - 4)(10)(4) = 240 \text{ cm}^3$
lower rect. prism: $V = (10)(10)(10 - 4) = 600 \text{ cm}^3$
total: $V = 240 + 600 = 800 \text{ cm}^3$

CHALLENGE AND EXTEND, PAGE 704

41. $V = \ell wh$
$= (x + 2)(x - 1)(x)$
$= x^3 + x^2 - 2x$

42. $V = \pi r^2 h$
$= \pi(x + 1)^2(x)$
$= \pi x^3 + 2\pi x^2 + \pi x$

43. $B = \frac{1}{2}(x)\left(\frac{x\sqrt{3}}{2}\right) = \frac{x^2\sqrt{3}}{4}$
$V = Bh = \left(\frac{x^2\sqrt{3}}{4}\right)(x + 1) = \frac{x^3\sqrt{3} + x^2\sqrt{3}}{4}$

44. The volume is equal to the surface area, so $\pi r^2 h = 2\pi r^2 + 2\pi rh$. Solve for r to get $r = \frac{2h}{h - 2}$.
If $h < 2$, then $r < 0$, so h must be greater than 2.
Similarly, if you solve for h, you get $h = \frac{2r}{r - 2}$.
If $r < 2$, then $h < 0$, so r must be greater than 2.

SPIRAL REVIEW, PAGE 704

45. $\begin{cases} m = 2r - 100 & (1) \\ t = r + 40 & (2) \\ r > m & (3) \end{cases}$
substitute (1) in (3): $r > 2r - 100$
$100 > r \quad (4)$
substitute (4) in (2): $t = r + 40 < 140$
$t \leq 139$
So $t = 139$

46. typing time is
$\frac{5000 \text{ words}}{40 \text{ wpm}} = 125 \text{ min} = (45 + 45 + 35)$ min
So, he takes 2 15-min breaks. Total time is
$125 + 2(15) = 155$ min or 2 h 35 min.

47. $\angle ABC \cong \angle CDA$
$m\angle ABC = m\angle CDA$
$30x - 10 = 23x + 4$
$7x = 14$
$x = 2$
$m\angle ABC = 30(2) - 10 = 50°$

48. $\overline{BC} \cong \overline{AD}$
$BC = AD$
$\frac{1}{4}z + 11 = \frac{3}{4}z + 3$
$8 = \frac{1}{2}z$
$z = 16$
$BC = \frac{1}{4}(16) + 11 = 15$

49. $\overline{AB} \cong \overline{DC}$
$AB = DC$
$y + 6 = 4y$
$6 = 3y$
$y = 2$
$AB = (2) + 6 = 8$

50. $S = L + B$
$= \frac{1}{2}P\ell + s^2$
$= \frac{1}{2}(32)(10) + (8)^2 = 224 \text{ in}^2$

51. $S = L + B$
$= \frac{1}{2}P\ell + \frac{1}{2}aP$
$= \frac{1}{2}(30)(8) + \frac{1}{2}\left(\frac{3}{\tan 36°}\right)(30)$
$= 120 + \frac{45}{\tan 36°} \approx 181.9 \text{ cm}^2$

52. $C = \pi = 2\pi r$
$r = 0.5 \text{ ft}$
$S = \pi rh + \pi r^2$
$= \pi(0.5)(2) + \pi(0.5)^2 = 1.25\pi \text{ ft}^2$
$\approx 3.9 \text{ ft}^2$

10-7 VOLUME OF PYRAMIDS AND CONES, PAGES 705–712

CHECK IT OUT! PAGES 706–708

1. **Step 1** Find the area of the base.
$B = \frac{1}{2}aP$
$= \frac{1}{2}\left(\sqrt{3}\right)(12) = 6\sqrt{3} \text{ cm}^2$
Step 2 Use the base and the height to find the volume. The height is equal to the base.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}\left(6\sqrt{3}\right)\left(6\sqrt{3}\right) = 36 \text{ cm}^3$

2. First find the volume in cubic yards.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(70^2)(66) = 107{,}800 \text{ yd}^3$
Then, convert the answer to find the volume in cubic feet.
Use the conversion factor $\frac{27 \text{ ft}^3}{1 \text{ yd}^3}$ to find the volume in cubic feet.
$107{,}800 \text{ yd}^3 \cdot \frac{27 \text{ ft}^3}{1 \text{ yd}^3} = 2{,}910{,}600 \text{ ft}^3$

3. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(9)^2(8) = 216\pi \approx 678.6 \text{ m}^3$

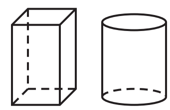
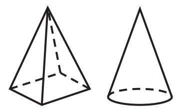
4. original dimensions:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(9)^2(18) = 486\pi \text{ cm}^3$
radius and height doubled:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(18)^2(36) = 3888\pi \text{ cm}^3$
Notice that $3888\pi = 8(486\pi)$. If the radius and height are doubled, the volume is multiplied by 8.

5. The volume of the rectangular prism is
$V = \ell wh = (25)(12)(15) = 4500 \text{ ft}^3$.
The volume of the rectangular pyramid is
$V = \frac{1}{3}Bh = \frac{1}{3}((25)(12))(15) = 1500 \text{ ft}^3$.
The volume of the composite figure is the difference of the volumes.
$V = 4500 - 1500 = 3000 \text{ ft}^3$

THINK AND DISCUSS, PAGE 708

1. The volume of a pyramid is $\frac{1}{3}$ the volume of a prism with the same base and height.

2.

Volumes of Three-Dimensional Figures		
Formula	$V = Bh$	$V = \frac{1}{3}Bh$
Shapes	Prism, cylinder	Pyramid, cone
Examples		

EXERCISES, PAGES 709–712

GUIDED PRACTICE, PAGE 709

1. perpendicular

2. **Step 1** Find the area of base.
$B = \ell w$
$= (6)(4) = 24 \text{ in.}^2$
Step 2 Use the base and the height to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(24)(17) = 136 \text{ in}^3$

3. **Step 1** Find the area of base.
$B = \frac{1}{2}aP$
$= \frac{1}{2}\left(2\sqrt{3}\right)(24) = 24\sqrt{3} \text{ cm}^2$
Step 2 Use the base and the height to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}\left(24\sqrt{3}\right)\left(4\sqrt{3}\right) = 96 \text{ cm}^3$

4. $V = \frac{1}{3}Bh$
$= \frac{1}{3}(25)(9) = 75 \text{ ft}^3$

5. $V = 2\left(\frac{1}{3}Bh\right)$
$= 2\left(\frac{1}{3}(5.7^2)(3)\right) \approx 65 \text{ mm}^3$

6. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(9)^2(14) = 378\pi \text{ cm}^3$
$\approx 1187.5 \text{ cm}^3$

7. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(12)^2(30) = 1440\pi \text{ in}^3$
$\approx 4523.9 \text{ in}^3$

8. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(12)^2(20) = 960\pi \text{ m}^3$
$\approx 3015.9 \text{ m}^3$

9. original dimensions:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(5)^2(3) = 25\pi \text{ cm}^3$
dimensions tripled:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(15)^2(9) = 675\pi \text{ cm}^3$
Notice that $675\pi = 27(25\pi)$. If the dimensions are tripled, the volume is multiplied by 27.

10. original dimensions:
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(9^2)(15) = 405 \text{ cm}^3$
dimensions multiplied by $\frac{1}{2}$:
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(4.5^2)(7.5) = 50.625 \text{ cm}^3$
Notice that $405 = \frac{1}{8}(50.625)$. If the dimensions are multiplied by $\frac{1}{2}$, the volume is multiplied by $\frac{1}{8}$.

11. The volume of the cube is
$V = s^3 = (12)^3 = 1728 \text{ cm}^3$.
The volume of the rectangular pyramid is
$V = \frac{1}{3}Bh = \frac{1}{3}(12^2)(18) = 864 \text{ cm}^3$.
The volume of the composite figure is the sum of the volumes.
$V = 1728 + 864 = 2592 \text{ cm}^3$

12. The volume of the outer cone is
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(12) = 256\pi \text{ in}^3$.
The volume of the inner cone is
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2(6) = 32\pi \text{ in}^3$.
The volume of the composite figure is the difference of the volumes.
$V = 256\pi - 32\pi = 224\pi \text{ in}^3$
$\approx 703.7 \text{ in}^3$

PRACTICE AND PROBLEM SOLVING, PAGES 710–711

13. **Step 1** Find the area of the base.
$B = \ell w$
$= (8)(6) = 48 \text{ ft}^2$
Step 2 Use the base and height to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(48)(10) = 160 \text{ ft}^3$

14. **Step 1** Find the area of the base. $5^2 + 12^2 = 13^2$. So, the base is a right triangle with $b = 12$ m and h = 5 m.
$B = \frac{1}{2}bh$
$= \frac{1}{2}(12)(5) = 30 \text{ m}^2$
Step 2 Use the base and height to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(30)(9) = 90 \text{ m}^2$

15. **Step 1** Find the height.
$6^2 + h^2 = 10^2$
$h = 8$ ft
Step 2 Use the height and base edge length to find the volume.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(12^2)(8) = 384 \text{ ft}^3$

16. **Step 1** Find the volume in cubic feet. The height is 5(3) = 15 ft.
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(45^2)(15) = 10{,}125 \text{ ft}^3$
Step 2 Convert the volume to cubic yards.
Use the conversion factor $\frac{1 \text{ yd}^3}{27 \text{ ft}^3}$.
$10{,}125 \text{ ft}^3 \cdot \frac{1 \text{ yd}^3}{27 \text{ ft}^3} \approx 375 \text{ yd}^3$

17. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(9)^2(41) = 1107\pi \text{ m}^3$
$\approx 3477.7 \text{ m}^3$

18. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(2)^2(4) = \frac{16}{3}\pi \text{ in}^3$
$\approx 16.8 \text{ in}^3$

19. $B = \pi r^2$
$36\pi = \pi r^2$
$r = 6$ ft
$h = 2r = 2(6) = 12$ ft
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(36\pi)(12) = 144\pi \text{ ft}^3$
$\approx 452.4 \text{ ft}^3$

20. original dimensions:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(15)^2(21) = 1575\pi \text{ in}^3$
dimensions multiplied by $\frac{1}{3}$:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(5)^2(7) = \frac{175}{3}\pi \text{ in}^3$
Notice that $\frac{175}{3}\pi = \frac{1}{27}(1575\pi)$. If the dimensions are multiplied by $\frac{1}{3}$, the volume is multiplied by $\frac{1}{27}$.

21. original dimensions:
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(7^2)(4) = \frac{196}{3} \text{ ft}^3$
dimensions multiplied by 6:
$V = \frac{1}{3}Bh$
$= \frac{1}{3}(42^2)(24) = 14{,}112 \text{ cm}^3$
Notice that $14{,}112 = 216\left(\frac{196}{3}\right)$. If the dimensions are multiplied by 6, the volume is multiplied by 216.

22. The volume of the cylinder is
$V = \pi r^2 h = \pi(6)^2(10) = 360\pi \text{ ft}^3$.
The volume of the cone is
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(10) = 120\pi \text{ ft}^3$.
The volume of the composite figure is the difference of the volumes.
$V = 360\pi - 120\pi = 240\pi \text{ ft}^3$
$\approx 754.0 \text{ ft}^3$

23. The volume of the rectangular prism is
$V = \ell wh = (10)(5)(2) = 100 \text{ ft}^3$.
The volume of each square-based pyramid is
$V = \frac{1}{3}Bh = \frac{1}{3}(5^2)(3) = 25 \text{ ft}^3$.
The volume of the composite figure is the sum of the volumes.
$V = 100 + 25 + 25 = 150 \text{ ft}^3$

24. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(7) = 21\pi \text{ in}^3$

25. $r = \frac{1}{2}d = \frac{5}{2} \text{ m}$
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2(2) = \frac{25}{6}\pi \text{ m}^3$

26. $r^2 + h^2 = \ell^2$
$28^2 + h^2 = 53^2$
$h = 45 \text{ ft}$
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(28)^2(45) = 11{,}760\pi \text{ ft}^3$

27. $r = \frac{1}{2}d = \frac{1}{2}(6) = 12 \text{ cm}$
$r^2 + h^2 = \ell^2$
$12^2 + h^2 = 13^2$
$h = 5 \text{ cm}$
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(12)^2(5) = 240\pi \text{ ft}^3$

28. $B = \frac{1}{2}bh_{base} = \frac{1}{2}(10)\left(5\sqrt{3}\right) = 25\sqrt{3} \text{ ft}^2$
$V = \frac{1}{3}Bh = \frac{1}{3}\left(25\sqrt{3}\right)(6) \approx 86.6 \text{ ft}^3$

29. $V = \frac{1}{3}Bh = \frac{1}{3}(15^2)(18) = 1350 \text{ m}^3$

30. **Step 1** Find the apothem of the base.
$\tan 36° = \frac{4.5}{a}$
$a = \frac{4.5}{\tan 36°} \text{ in.}$
Step 2 Find the area of the base.
$B = \frac{1}{2}aP = \frac{1}{2}\left(\frac{4.5}{\tan 36°}\right)(45) = \frac{101.25}{\tan 36°} \text{ in}^2$
Step 3 Find the volume of the pyramid.
$V = \frac{1}{3}Bh = \frac{1}{3}\left(\frac{101.25}{\tan 36°}\right)(12) \approx 557.4 \text{ in}^3$

31. $B = \frac{1}{2}aP = \frac{1}{2}\left(4\sqrt{3}\right)(48) = 96\sqrt{3} \text{ cm}^2$
$V = \frac{1}{3}Bh = \frac{1}{3}\left(96\sqrt{3}\right)(3) \approx 166.3 \text{ cm}^3$

32. $V = \frac{1}{3}Bh$
$V = \frac{1}{3}lwh$
$112 = \frac{1}{3}(3)(8)h$
$112 = 8h$
$h = 14 \text{ m}$

33. $V = \frac{1}{3}\pi r^2 h$
$125\pi = \frac{1}{3}\pi r^2(5)$
$125\pi = \frac{5}{3}\pi r^2$
$75 = r^2$
$r = 5\sqrt{3} \text{ cm}$
$C = 2\pi r = 2\pi\left(5\sqrt{3}\right) = 10\pi\sqrt{3} \text{ cm}$

34. $r^2 + h^2 = \ell^2$
$r^2 + 8^2 = 10^2$
$r = 6 \text{ ft}$
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(8) = 96\pi \text{ ft}^3$

35. $S = \frac{1}{2}P\ell + B$
$800 = \frac{1}{2}(4s)(17) + s^2$
$800 = 34s + s^2$
$0 = s^2 + 34s - 800$
$0 = (s - 16)(s + 50)$
$s = 16 \text{ in.}$
$\left(\frac{1}{2}s\right)^2 + h^2 = \ell^2$
$8^2 + h^2 = 17^2$
$h = 15 \text{ in.}$
$V = \frac{1}{3}Bh = \frac{1}{3}(16^2)(15) = 1280 \text{ in}^3$

36. $V = \frac{1}{3}\pi r^2 h$
$1500\pi = \frac{1}{3}\pi r^2(20)$
$4500\pi = 20\pi r^2$
$225 = r^2$
$r = 15 \text{ yd}$
$r^2 + h^2 = \ell^2$
$15^2 + 20^2 = \ell^2$
$\ell = 25 \text{ yd}$
$S = \pi r\ell + \pi r^2$
$= \pi(15)(25) + \pi(15)^2 = 600\pi \text{ yd}^2$

37. The base is a right triangle with a base length of 5 units and a height of 3 units. The height of the pyramid is 7 units.
$B = \frac{1}{2}(5)(3) = 7.5 \text{ units}^2$
$V = \frac{1}{3}Bh = \frac{1}{3}(7.5)(7) = 17.5 \text{ units}^3$

38. A is incorrect because it uses the slant height of the cone instead of the height.

39. 3 : 2; The base areas are the same for both figures The volume of the prism is By, and the volume of the figure formed by 2 pyramids is $\frac{1}{3}B(2y)$.
The ratio of the volumes is $By : \frac{1}{3}B(2y)$, which is equivalent to 3:2.

40. Possible answer: Substitute the given values for r and S into the surface area formula and solve for ℓ. Then, use the Pythagorean Theorem and the values for r and ℓ to solve for h. Substitute the values for r and h into the volume formula.

41a. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2)^2(8) = \frac{32}{3}\pi \approx 33.5 \text{ in}^3$

b. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2(8) = \frac{128}{3}\pi \approx 134.0 \text{ in}^3$

c. The large size holds 4 times as much. So, the price should be 4($1.25) = $5.

TEST PREP, PAGE 712

42. A

$12^2 + h^2 = 15^2$

$h = 9$ cm

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(12)^2(9) = 432\pi \text{ cm}^3$

43. H

$L = \frac{1}{2}P\ell$

$350 = \frac{1}{2}(4s)(25)$

$350 = 50s$

$s = 7$ m

$3.5^2 + h^2 = 25^2$

$h = \sqrt{612.75} \approx 24.75$

$V = \frac{1}{3}Bh \approx \frac{1}{3}(7^2)(24.75) \approx 404 \text{ m}^3$

44. B

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(6) = 18\pi \text{ in}^3$

45. 9

$V = \frac{1}{3}Bh$

$243 = \frac{1}{3}(h^2)h$

$729 = h^3$

$h = 9$ cm

CHALLENGE AND EXTEND, PAGE 712

46. radius = apothem of regular triangle

$\frac{1}{2}s = 1$. So, $r = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ft

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{\sqrt{3}}{3}\right)^2(2) = \frac{2}{9}\pi \text{ ft}^3$

47. $r = \frac{1}{2}s = 1$ ft

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(1)^2(2) = \frac{2}{3}\pi \text{ ft}^3$

48. radius = apothem of regular hexagon

$s = 2$. So, $r = \sqrt{3}$ ft

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\sqrt{3}\right)^2(2) = 2\pi \text{ ft}^3$

49. slant height is altitude of a face; $s = 10$.
So, $\ell = 5\sqrt{3}$ cm.

$\left(\frac{1}{2}s\right)^2 + h^2 = \ell^2$

$5^2 + h^2 = \left(5\sqrt{3}\right)^2$

$h = \sqrt{50} = 5\sqrt{2}$ cm

$V = 2\left(\frac{1}{3}Bh\right) = \frac{2}{3}(10^2)\left(5\sqrt{2}\right) = \frac{1000\sqrt{2}}{3} \text{ cm}^3$

50. $h = 9$ in.; the volume of a cone with the same base and height as the cylinder is $\frac{1}{3}$ the volume of the cylinder. For a cone to have the same volume as the cylinder, the height of cone must be 3 times the height of the cylinder.

SPIRAL REVIEW, PAGE 712

51. $\begin{cases} x - y = 24 & (1) \\ x = 3y - 4 & (2) \end{cases}$

Substitute (2) into (1):

$3y - 4 - y = 24$

$2y = 28$

$y = 14$

$x = 3(14) - 4 = 38$

The numbers are 38 and 14.

52. $\begin{cases} 3x + y = 88 & (1) \\ 10x = 4y & (2) \end{cases}$

Solve (2) for y and substitute into (1):

$y = \frac{10x}{4} = \frac{5x}{2}$

$3x + \frac{5x}{2} = 88$

$6x + 5x = 176$

$11x = 176$

$x = 16$

$y = \frac{10(16)}{4} = 40$

The numbers are 16 and 40.

53. $\begin{cases} x + y = 197 & (1) \\ x = \frac{1}{2}y + 20 & (2) \end{cases}$

Substitute (2) into (1):

$\frac{1}{2}y + 20 + y = 197$

$y + 40 + 2y = 394$

$3y = 354$

$y = 118$

$x = \frac{1}{2}(118) + 20 = 79$

The numbers are 79 and 118.

54. $\frac{DF}{AC} = \frac{25.5}{17} = \frac{3}{2}$; $\frac{EF}{BC} = \frac{31.5}{21} = \frac{3}{2}$; $\angle C \cong \angle F$

triangles are $\sim$ by $\sim$ SAS

$\frac{AB}{10} = \frac{3}{2}$

$2AB = 3(10) = 30$

$AB = 15$

55. $\overline{LM}$ and $\overline{PQ}$ are both perpendicular to $\overline{LN}$. So, $\overline{LM}$ parallel $\overline{PQ}$. By Corr. ∡ Post., $\angle M \cong \angle NQP$; by Reflex. Prop. of $\cong$, $\angle N \cong \angle N$. So ⚠ are $\sim$ by AA $\sim$.

$\frac{PQ}{LM} = \frac{PN}{LN}$

$\frac{PQ}{9} = \frac{16}{24}$

$24PQ = 9(16) = 144$

$PQ = 6$

56. $AB = \sqrt{(8-1)^2 + (9-1)^2 + (10-2)^2}$

$= \sqrt{49 + 64 + 64} = \sqrt{177} \approx 13.3$

$M\left(\frac{1+8}{2}, \frac{1+9}{2}, \frac{2+10}{2}\right) = M(4.5, 5, 6)$

57. $AB = \sqrt{(5-(-4))^2 + (1-(-1))^2 + (-4-0)^2}$

$= \sqrt{81 + 4 + 16} = \sqrt{102} \approx 10.0$

$M\left(\frac{-4+5}{2}, \frac{-1+1}{2}, \frac{0+-4}{2}\right) = M(0.5, 0, -2)$

58. $AB = \sqrt{(-2-2)^2 + (2-(-2))^2 + (-4-4)^2}$

$= \sqrt{16 + 16 + 64} = \sqrt{96} \approx 9.8$

$M\left(\frac{2+(-2)}{2}, \frac{-2+2}{2}, \frac{4+(-4)}{2}\right) = M(0, 0, 0)$

59. $AB = \sqrt{(-1-(-3))^2 + (5-(-1))^2 + (5-2)^2}$

$= \sqrt{4 + 36 + 9} = \sqrt{49} = 7$

$M\left(\frac{-3+(-1)}{2}, \frac{-1+5}{2}, \frac{2+5}{2}\right) = M(-2, 2, 3.5)$

CONNECTING GEOMETRY TO ALGEBRA: FUNCTIONAL RELATIONSHIPS IN FORMULAS, PAGE 713

TRY THIS, PAGE 713

1. First use the surface area formula to write an equation.

$S = \pi r\ell + \pi r^2$

$S = \pi(10)\ell + \pi(10)^2$

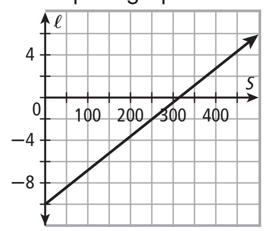
$S = 10\pi\ell + 100\pi$

Then, solve for ℓ to get an equation for ℓ in terms of S.

$S - 100\pi = 10\pi\ell$

$\ell = \dfrac{S}{10\pi} - 10$

Since the equation is linear, use slope and y-intercept to graph it.



The slant height increases as the surface area increases.

2. First use the volume formula to write an equation.

$V = \pi r^2 h$

$V = \pi r^2(5)$

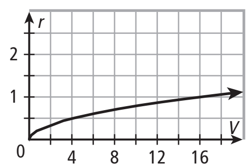
Then, solve for ℓ to get an equation for ℓ in terms of S.

$\dfrac{V}{5\pi} = r^2$

$r = \sqrt{\dfrac{V}{5\pi}}$

Make a table of V- and r-values. Then, plot points and draw a smooth curve through them.

V	r
0	0
5	0.46
10	0.80
15	0.98
20	1.13



The radius increases as the volume increases.

10-8 SPHERES, PAGES 714–721

CHECK IT OUT! PAGES 715–717

1. $V = \frac{4}{3}\pi r^3$

$2304\pi = \frac{4}{3}\pi r^3$

$1728 = r^3$

$r = 12$ ft

2. hummingbird eyeball:

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(1.25)^3 \approx 2.604\pi \text{ cm}^3$

human eyeball:

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(0.3)^3 = 0.036\pi \text{ cm}^3$

A human eyeball is about $\dfrac{2.604\pi}{0.036\pi} \approx 72.3$ times as great in volume as a hummingbird eyeball.

3. $S = 4\pi r^2$

$= 4\pi(25)^2 = 2500\pi \text{ cm}^2$

4. original dimensions:

$S = 4\pi r^2$

$= 4\pi(3)^2$

$= 36\pi \text{ m}^3$

radius multiplied by $\frac{1}{3}$:

$S = 4\pi r^2$

$= 4\pi(1)^2$

$= 4\pi \text{ m}^3$

Notice that $4\pi = \frac{1}{9}(36\pi)$. If the radius is multiplied by $\frac{1}{3}$, the surface area is divided by 9.

5. **Step 1** Find the surface area of the composite figure. The surface area of the composite figure is the sum of the surface area of the hemisphere, the lateral area of cylinder, and the area of the lower base of cylinder.

$S_{\text{hemisphere}} = \frac{1}{2}(4\pi r^2) = 2\pi(3)^2 = 18\pi \text{ ft}^2$

$L_{\text{cylinder}} = 2\pi rh = 2\pi(3)(5) = 30\pi \text{ ft}^2$

$B_{\text{cylinder}} = \pi r^2 = \pi(3)^2 = 9\pi \text{ ft}^2$

The surface area of composite figure is $18\pi + 30\pi + 9\pi = 57\pi \text{ ft}^2$.

Step 2 Find the volume of the composite figure. The volume of composite figure is difference of volume of cylinder and volume of hemisphere.

$V_{\text{cylinder}} = \pi r^2 h = \pi(3)^2(5) = 45\pi \text{ ft}^3$

$V_{\text{hemisphere}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(3)^3 = 18\pi \text{ ft}^3$

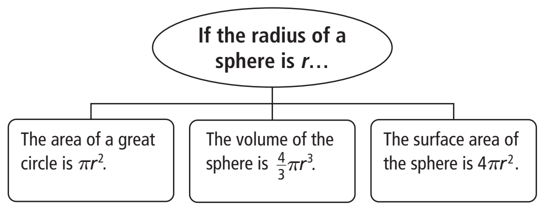
The volume of composite figure is $45\pi - 18\pi = 27\pi \text{ ft}^3$.

THINK AND DISCUSS, PAGE 717

1. The surface area is 4 times the area of the great circle.

2. Both the area of sphere and the area of the composite figure have a volume of $\frac{4}{3}\pi r^3$.

3.



EXERCISES, PAGES 718–721

GUIDED PRACTICE, PAGE 718

1. One endpoint is the center of the sphere, and the other is a point on the sphere.

2. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(11)^3 = \frac{2662}{3}\pi \text{ in}^3$

3. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \text{ m}^3$

4. $V = \frac{4}{3}\pi r^3$
$288\pi = \frac{4}{3}\pi r^3$
$216 = r^3$
$r = 6 \text{ cm}$

5. grapefruit:
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(5)^3 = \frac{50}{3}\pi \text{ cm}^3$
lime:
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi\left(\frac{5}{2}\right)^3 = \frac{25}{12}\pi \text{ cm}^3$
The grapefruit is 8 times as great in volume as the lime.

6. $S = 4\pi r^2 = 4\pi(8)^2 = 256\pi \text{ yd}^3$

7. $A = \pi r^2$
$S = 4\pi r^2 = 4A = 4(49\pi) = 196\pi \text{ cm}^2$

8. $S = 4\pi r^2$
$6724\pi = 4\pi r^2$
$1681 = r^2$
$r = 41 \text{ ft}$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(41)^3 = \frac{275{,}684}{3}\pi \text{ ft}^3$

9. original dimensions:
$S = 4\pi r^2$
$= 4\pi(15)^2$
$= 900\pi \text{ in}^2$

dimensions doubled:
$S = 4\pi r^2$
$= 4\pi(30)^2$
$= 3600\pi \text{ in}^2$

Notice that $3600\pi = 4(900\pi)$. If dimensions are doubled, the surface area is multiplied by 4.

10. original dimensions:
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(8)^3$
$= \frac{2048}{3}\pi \text{ cm}^3$

dimensions multiplied by $\frac{1}{4}$:
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(8)^3$
$= \frac{32}{3}\pi \text{ cm}^3$

Notice that $\frac{32}{3}\pi = \frac{1}{64}\left(\frac{2048}{3}\pi\right)$. If the radius is multiplied by $\frac{1}{4}$, the volume is multiplied by $\frac{1}{64}$.

11. **Step 1** Find the surface area of the composite figure. The surface area of the composite figure is the sum of the surface area of two hemispheres and the lateral area of cylinder.
$S_{\text{hemisphere}} = \frac{1}{2}(4\pi r^2) = 2\pi(2)^2 = 8\pi \text{ ft}^2$
$L_{\text{cylinder}} = 2\pi rh = 2\pi(2)(5) = 20\pi \text{ ft}^2$
The surface area of the composite figure is $2(8\pi) + 20\pi = 36\pi \text{ ft}^2$.
Step 2 Find the volume of the composite figure. The volume of the composite figure is the sum of the volume of the cylinder and the volume of the two hemispheres.
$V_{\text{cylinder}} = \pi r^2 h = \pi(2)^2(5) = 20\pi \text{ ft}^3$
$V_{\text{hemisphere}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(2)^3 = \frac{16}{3}\pi \text{ ft}^3$
The volume of the composite figure is
$20\pi + 2\left(\frac{16}{3}\pi\right) = \frac{92}{3}\pi \text{ ft}^3$.

12. **Step 1** Find the surface area of the composite figure. The surface area of the composite figure is the sum of the lateral area of the hemisphere and the surface area of the cylinder, less the area of the base of the hemisphere.
$S_{\text{hemisphere}} = \frac{1}{2}(4\pi r^2) = 2\pi(2)^2 = 8\pi \text{ in}^2$
$S_{\text{cylinder}} = 2\pi rh + 2\pi r^2$
$= 2\pi(8)(3) + 2\pi(8)^2 = 176\pi \text{ in}^2$
$B_{\text{hemisphere}} = \pi r^2 = \pi(2)^2 = 4\pi \text{ in}^2$
The surface area of the composite figure is
$8\pi + 176\pi - 4\pi = 180\pi \text{ in}^2$.
Step 2 Find the volume of the composite figure. The volume of the composite figure is the difference of the volume of the cylinder and the volume of the hemisphere.
$V_{\text{cylinder}} = \pi r^2 h = \pi(8)^2(3) = 192\pi \text{ ft}^3$
$V_{\text{hemisphere}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(2)^3 = \frac{16}{3}\pi \text{ in}^3$
The volume of the composite figure is
$192\pi - \frac{16}{3}\pi = \frac{560}{3}\pi \text{ in}^3$.

PRACTICE AND PROBLEM SOLVING, PAGES 719–720

13. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi \text{ cm}^3$

14. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(7)^3 = \frac{686}{3}\pi \text{ ft}^3$

15. $V = \frac{4}{3}\pi r^3$
$7776\pi = \frac{4}{3}\pi r^3$
$5832 = r^3$
$r = 18$
$d = 36 \text{ in.}$

16. 9-mm pearl:
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4.5)^3 = 121.5\pi \text{ mm}^3$
6-mm pearl:
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 108\pi \text{ mm}^3$
The 9-mm pearl is 3.375 times as great in volume.

17. $S = 4\pi r^2 = 4\pi(21)^2 = 1764\pi \text{ in}^2$

18. $A = \pi r^2$
$S = 4\pi r^2 = 4A = 4(81\pi) = 324\pi \text{ in}^2$

19. $S = 4\pi r^2$
$625\pi = 4\pi r^2$
$156.25 = r^2$
$r = 12.5 \text{ m}$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.5)^3 = \frac{15{,}625}{6}\pi \text{ m}^3$

20. original dimensions:
$S = 4\pi r^2$
$= 4\pi(0.6)^2$
$= 1.44\pi \text{ ft}^2$

dimensions multiplied by $\frac{1}{5}$:
$S = 4\pi r^2$
$= 4\pi(0.12)^2$
$= 0.0576\pi \text{ in.}^2$

Notice that $0.0576\pi = \frac{1}{25}(1.44\pi)$. If the dimensions are multiplied by $\frac{1}{5}$, the surface area is multiplied by $\frac{1}{25}$.

21. original dimensions:
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(14)^3$
$= \frac{10{,}976}{3}\pi \text{ mm}^3$

dimensions multiplied by 6:
$V = \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(84)^3$
$= 790{,}272\pi \text{ mm}^3$

Notice that $790{,}272\pi = 216\left(\frac{10{,}976}{3}\pi\right)$. If the radius is multiplied by 6, the volume is multiplied by 216.

22. **Step 1** Find the surface area.
$S_{\text{prism}} = Ph + 2\ell w$
$= 2(10)+ 2(5))(4) + 2(10)(5) = 220 \text{ cm}^2$
$L_{\text{hemisphere}} = \frac{1}{2}(4\pi r^2) = 2\pi(3)^2 = 18\pi \text{ cm}^2$
$B_{\text{hemisphere}} = \pi r^2 = \pi(3)^2 = 9\pi$
$S = S_{\text{prism}} + L_{\text{hemisphere}} - B_{\text{hemisphere}}$
$= 220 + 18\pi - 9\pi$
$= 220 + 9\pi \approx 248.3 \text{ cm}^2$
Step 2 Find the volume.
$V_{\text{prism}} = \ell wh = (10)(5)(4) = 200 \text{ cm}^3$
$V_{\text{hemisphere}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(3)^3 = 18\pi \text{ cm}^3$
$V = V_{\text{prism}} + V_{\text{hemisphere}} = 200 + 18\pi \approx 256.5 \text{ cm}^3$

23. **Step 1** Find the surface area. The slant height of the cone is $\sqrt{20^2 + 24^2} = 26$ mm.
$S_{\text{cone}} = \pi r\ell + \pi r^2$
$= \pi(10)(26) + \pi(10)^2 = 360\pi \text{ mm}^2$
$L_{\text{hemisphere}} = \frac{1}{2}(4\pi r^2) = 2\pi(8)^2 = 128\pi \text{ mm}^2$
$B_{\text{hemisphere}} = \pi r^2 = \pi(8)^2 = 64\pi$
$S = S_{\text{prism}} + L_{\text{hemisphere}} - B_{\text{hemisphere}}$
$= 360\pi + 128\pi - 64\pi$
$= 424\pi \approx 1332.0 \text{ mm}^2$
Step 2 Find the volume.
$V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3}(10)^2(24) = 800\pi \text{ mm}^3$
$V_{\text{hemisphere}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(8)^3 = \frac{1024}{3}\pi \text{ mm}^3$
$V = V_{\text{cone}} - V_{\text{hemisphere}}$
$= 800\pi - \frac{1024}{3}\pi = \frac{1376}{3}\pi \approx 1440.9 \text{ mm}^3$

24. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$144\pi = \frac{2}{3}\pi r^3$
$216 = r^3$
$r = 6$ cm

25. $S = 4\pi r^2$
$60\pi = 4\pi r^2$
$15 = r^2$
$r = \sqrt{15}$ in.
$C = 2\pi r = 2\pi\sqrt{15}$ in.

26. $C = 2\pi r$
$36\pi = 2\pi r$
$r = 18$ ft
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(18)^3 = 7776\pi \text{ ft}^3$

27. $r = \sqrt{(2-0)^2 + (3-0)^2 + (6-0)^2} = \sqrt{49} = 7$
$S = 4\pi r^2 = 4\pi(7)^2 = 196\pi \text{ units}^2$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(7)^3 = \frac{1372}{3}\pi \text{ units}^3$

28. Possible answer: Use 8 mm as the estimated height of the cylinder.
$S_{\text{sphere}} = 4\pi r^2 = 4\pi(4)^2 = 64\pi \text{ mm}^2$
$L_{\text{cylinder}} = 2\pi rh = 2\pi(1)(8) = 16\pi \text{ mm}^2$
$B_{\text{cylinder}} = 2\pi r^2 = 2\pi(1)^2 = 2\pi \text{ mm}^2$
$S \approx S_{\text{sphere}} + L_{\text{cylinder}} - B_{\text{cylinder}}$
$= 64\pi + 16\pi - 2\pi = 78\pi \approx 245 \text{ mm}^2$
$V_{\text{sphere}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(4)^3 = \frac{256}{3}\pi \text{ mm}^3$
$V_{\text{cylinder}} = \pi r^2 h = \pi(1)^2(8) = 8\pi \text{ mm}^3$
$V \approx V_{\text{sphere}} - V_{\text{cylinder}}$
$= \frac{256}{3}\pi - 8\pi$
$= \frac{232}{3}\pi \approx 243 \text{ mm}^3$

29. $C = \pi d = \pi(1.68) \approx 5.28$ in.
$S = 4\pi r^2 = 4\pi(0.84)^2 \approx 8.87 \text{ in}^2$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.84)^3 \approx 2.48 \text{ in}^3$

30. $C = \pi d$
$9 = \pi d$
$d = \frac{9}{\pi} \approx 2.86$ in.
$S = 4\pi r^2 = 4\pi\left(\frac{9}{2\pi}\right)^2 \approx 25.78 \text{ in}^2$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{9}{2\pi}\right)^3 \approx 12.31 \text{ in}^3$

31. $C = \pi d = \pi(2.5) \approx 7.85$ in.
$S = 4\pi r^2 = 4\pi(1.25)^2 \approx 19.63 \text{ in}^2$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(1.25)^3 \approx 8.18 \text{ in}^3$

32. $C = \pi d = \pi(74) \approx 232.48$ mm
$S = 4\pi r^2 = 4\pi(37)^2 \approx 17{,}203.36 \text{ mm}^2$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(37)^3 \approx 212{,}174.79 \text{ mm}^3$

33. $V_{\text{outer}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(28.5)^3 = 30{,}865.5\pi \text{ in}^3$
$V_{\text{inner}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(27)^3 = 26{,}244\pi \text{ in}^3$
$V_{\text{window}} = \pi r^2 h = \pi(4)^2(1.5) = 24\pi \text{ in}^3$
$V \approx V_{\text{outer}} - V_{\text{inner}} - 3V_{\text{window}}$
$= 30{,}865.5\pi - 26{,}244\pi - 3(24\pi)$
$= 4549.5\pi \approx 14{,}293 \text{ in}^3$

34. $S_{\text{land}} = \frac{1}{3}S_{\text{Earth}}$
$= 4\pi r^2 \approx 4\pi(4000)^2 \approx 67{,}000{,}000 \text{ mi}^2$

35. $\frac{V_{\text{Jupiter}}}{V_{\text{Earth}}} = \frac{\frac{4}{3}\pi(44{,}423)^3}{\frac{4}{3}\pi(3963)^3} \approx 1408$
Volume of Jupiter is about 1408 times as great as the volume of Earth.

36. $V_{\text{Venus}} + V_{\text{Mars}} = \frac{4}{3}\pi(3760.5)^3 + \frac{4}{3}\pi(2111)^3$
$\approx 2.62 \times 10^{11} \text{ mi}^3$
$V_{\text{Earth}} = \frac{4}{3}\pi(3963)^3 \approx 2.61 \times 10^{11} \text{ mi}^3$
The sum of the volumes of Venus and Mars is about equal to the volume of the Earth.

37. $S_{\text{Uranus}} + S_{\text{Neptune}} = 4\pi(15{,}881.5)^2 + 4\pi(15{,}387.5)^2 \approx 6.14 \times 10^9 \text{ mi}^2$

$S_{\text{Saturn}} = 4\pi(37{,}449)^2 \approx 1.76 \times 10^{10} \text{ mi}^2$

The surface area of Saturn is greater.

38. $\dfrac{S_{\text{Earth}}}{S_{\text{Mars}}} = \dfrac{4\pi(3963)^2}{4\pi(2111)^2} \approx 4$

The surface area of Earth is about 4 times as great as the surface area of Mars.

39. The cross section of the hemisphere is a circle with radius $\sqrt{r^2 - x^2}$, so its area is $A = \pi(r^2 - x^2)$. The cross section of the cylinder with the cone removed has an outer radius of r and an inner radius of x, so the area is $A = \pi r^2 - \pi x^2 = \pi(r^2 - x^2)$.

40. $4\pi r^2 = 6s^2$

$\dfrac{4\pi r^2}{6} = s^2$

$s = 2r \cdot \sqrt{\dfrac{\pi}{6}}$ or $s \approx 1.4r$

41a. $S = 4\pi r^2$

$50.3 \approx 4\pi r^2$

$\dfrac{50.3}{4\pi} \approx r^2$

$r \approx \sqrt{\dfrac{50.3}{4\pi}}$ in.

$V \approx \dfrac{4}{3}\pi\left(\sqrt{\dfrac{50.3}{4\pi}}\right)^3 \approx 33.5 \text{ in}^3$

b. $V \approx \dfrac{4}{3}\pi\left(1.1\sqrt{\dfrac{50.3}{4\pi}}\right)^3 \approx 44.6 \text{ in}^3$

TEST PREP, PAGE 721

42. A

$(16)^3 : \frac{4}{3}\pi(8)^3$

$4096 : \frac{2048}{3}\pi$

$2 : \frac{1}{3}\pi$

43. H

$V = \frac{4}{3}\pi r^3$

$\frac{32}{3}\pi = \frac{4}{3}\pi r^3$

$8 = r^3$

$r = 2$ in.

$S = 4\pi(2)^2 = 16\pi \text{ in}^2$

44. A

$V = (2r)^3 + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= 8r^3 + \frac{2}{3}\pi r^3$

$= r^3\left(8 + \frac{2}{3}\pi\right)$

CHALLENGE AND EXTEND, PAGE 721

45. 3300 gumballs take up about 57% of the volume of sphere.

$3300\left(\frac{4}{3}\pi r^3\right) \approx 0.57\left(\frac{4}{3}\pi(9)^3\right)$

$3300r^3 \approx 0.57(9)^3 = 415.53$

$r^3 \approx \dfrac{415.53}{3300}$

$r \approx \sqrt[3]{\dfrac{415.53}{3300}} \approx 0.50$

$d = 2(0.50) \approx 1.0$ in.

46a. $S = 4\pi r^2$

$\dfrac{S}{4\pi} = r^2$

$r = \sqrt{\dfrac{S}{4\pi}} = \dfrac{\sqrt{S\pi}}{2\pi}$

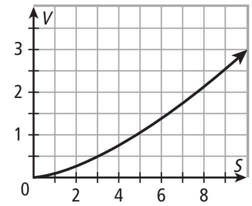
b. $V = \frac{4}{3}\pi r^3$

$= \dfrac{4}{3}\pi\left(\dfrac{\sqrt{S\pi}}{2\pi}\right)^3$

$= \dfrac{4}{3}\pi\left(\dfrac{S\pi\sqrt{S\pi}}{8\pi^3}\right)$

$= \dfrac{S\sqrt{S\pi}}{6\pi}$

c.



Possible answer: The shape of the graph is similar to half of a parabola.

47. $\dfrac{V_{\text{cylinder}}}{V_{\text{sphere}}} = \dfrac{\pi r^2(2r)}{\frac{4}{3}\pi r^3} = \dfrac{2}{\frac{4}{3}} = 1.5$

The volume of the cylinder is 1.5 times the volume of the sphere.

48. $\dfrac{L_{\text{cylinder}}}{S_{\text{sphere}}} = \dfrac{2\pi r(2r)}{4\pi r^2} = \dfrac{4\pi r^2}{4\pi r^2} = 1$

The surface area of the sphere is equal to the lateral area of the cylinder.

SPIRAL REVIEW, PAGE 721

49. The graph resembles a parabola. The data fits the equation $y = x^2 + 1$.

50. The graph is a straight line with slope 1 and y-intercept 10. The equation is $y = x + 10$.

51. quarter-circle: $A = \frac{1}{4}\pi(4)^2 = 4\pi \text{ in}^2$

$\triangle$: $A = \frac{1}{2}(4)(4) = 8 \text{ in}^2$

shaded area: $A = 4\pi - 8 \approx 4.6 \text{ in}^2$

52. rectangle: $A = (10)(6) = 60 \text{ cm}^2$

trapezoid: $A = \frac{1}{2}(1 + 5)(4) = 12 \text{ cm}^2$

shaded area: $A = 60 - 12 = 48 \text{ cm}^2$

53. $V = \left(\frac{3}{4}s\right)^3 = \frac{27}{64}s^3$

The volume is multiplied by $\frac{27}{64}$.

54. $V = (5B)(5h) = 25Bh$

The volume is multiplied by 25.

10-8 TECHNOLOGY LAB: COMPARE SURFACE AREAS AND VOLUMES, PAGES 722–723

ACTIVITY 1, TRY THIS, PAGE 723

1. The cylinder with the minimum surface area will be the one in which the height and diameter are the closest to each other.
2. Possible answer: No; a package such as a cereal box might be designed to have a large surface area so that it stands out on the shelf.

ACTIVITY 2, TRY THIS, PAGE 723

3. The cylinder with the maximum volume will have a height equal to its diameter.
4. SA = 2LW + 2LH + 2WH becomes
$$H = \frac{\left(\frac{SA}{2} - LW\right)}{L + W}$$ when solved for H.
5. Possible answer: A sphere will have the least surface area and a pyramid will have the greatest surface area for a given volume.
6. Possible answer: If dimensions of a rectangular prism are doubled, the surface area is multiplied by 4 and the volume is multiplied by 8. So the ratio of the surface area to the volume is multiplied by $\frac{4}{8} = \frac{1}{2}$. Set up a spreadsheet as in Activity 1. Enter the values for L, W, and H in one row, and enter the values 2L, 2W, and 2H in the next row. Repeat for several values of L, W, and H.

10B MULTI-STEP TEST PREP, PAGE 724

1. $4^2 + h^2 = 5^2$
$h = 3$ in.
$S = 2\pi rh + 2\pi r^2$
$= 2\pi(2)(3) + 2\pi(2)^2$
$= 20\pi \approx 62.8$ in^2
2. volume in cubic inches:
$V = \pi r^2 h = \pi(2)^2(3) = 12\pi$ in^3
volume in ounces:
$12\pi \text{ in}^3 \cdot \frac{0.55 \text{ oz}}{1 \text{ in}^3} \approx 20.7$ oz
3. $3^2 + 3^2 + h^2 = 5^2$
$h = \sqrt{7}$ in.
$S = Ph + B$
$= (4s)h + 2(s^2)$
$= 4(3)(\sqrt{7}) + 2(3)^2$
$= 18 + 12\sqrt{7} \approx 49.7$ in^2
4. volume in cubic inches:
$V = s^2 h = (3)^2(\sqrt{7}) = 9\sqrt{7}$ in^3
volume in ounces:
$9\sqrt{7} \text{ in}^3 \cdot \frac{0.55 \text{ oz}}{1 \text{ in}^3} \approx 13.1$ oz
5. Possible answer: I would recommend the cylinder, because the ratio of the surface area to the volume is about 3 in^2 per oz, and the ratio of the surface area to the volume of prism is about 3.8 in^2 per oz. Thus, the cylinder costs less to produce per oz of volume.

READY TO GO ON? PAGE 725

1. $S = L + B$
$= (2\ell + 2w)h + 2(\ell w)$
$= (2(12) + 2(8))(8) + 2(12)(8) = 512$ cm^2
2. $S = 2\pi rh + 2\pi r^2$
$= 2\pi(6)(10) + 2\pi(6)^2$
$= 192\pi \approx 603.2$ ft^2
3. $S_{prism} = Ph + 2s^2$
$= (140)(10) + 2(35)^2 = 3850$ in^2
$L_{cylinder} = 2\pi rh = 2\pi(5)(15) = 150\pi$ in^2
$S = S_{prism} - B_{cylinder} + L_{cylinder} + B_{cylinder}$
$= S_{prism} + L_{cylinder}$
$= 3850 + 150\pi \approx 4321.2$ in^2
4. original dimensions:
$S = (2\ell + 2w)h + 2(\ell w)$
$= (2(12) + 2(8))(24) + 2(12)(8) = 1152$ cm^2
dimensions multiplied by $\frac{3}{4}$:
$S = (2\ell + 2w)h + 2(\ell w)$
$= (2(9) + 2(6))(18) + 2(9)(6) = 648$ cm^2
$648 = \frac{9}{16}(1152)$. So, surface area is multiplied by $\frac{9}{16}$.
5. **Step 1** Find the apothem and the perimeter of the base.
$\tan 36° = \frac{9}{a}$
$a = \frac{9}{\tan 36°}$ yd
$P = 5(18) = 90$ yd
Step 2 Find the base area.
$B = \frac{1}{2}aP = \frac{1}{2}\left(\frac{9}{\tan 36°}\right)(90) = \frac{405}{\tan 36°}$ yd^2
Step 3 Find the surface area.
$S = \frac{1}{2}P\ell + B$
$= \frac{1}{2}(90)(20) + \frac{405}{\tan 36°} \approx 1457.4$ yd^2
6. $15^2 + 8^2 = \ell^2$
$\ell = 17$ in.
$S = \pi r\ell + \pi r^2$
$= \pi(15)(17) + \pi(15)^2$
$= 480\pi \approx 1508.0$ in^2
7. $L_{left} = \pi r\ell = \pi(10)(34) = 340\pi$ ft^2
$L_{right} = \pi r\ell = \pi(10)(26) = 260\pi$ ft^2
$S = L_{left} + L_{right}$
$= 340\pi + 260\pi$
$= 600\pi \approx 1885.0$ ft^2
8. $V = Bh = (23)(9) = 207$ in^3
9. $V = \pi r^2 h = \pi(8)^2(14) = 896\pi \approx 2814.9$ yd^3
10. **Step 1** Find the volume of the bricks.
$V = \ell wh = (10)(12)\left(\frac{1}{3}\right) = 40$ ft^3
Step 2 Use the conversion factor $\frac{130 \text{ lb}}{1 \text{ ft}^3}$ to find the weight of the bricks.
$40 \text{ ft}^3 \cdot \frac{130 \text{ lb}}{1 \text{ ft}^3} = 5200$ lb
11. original dimensions:
$V = \pi r^2 h = \pi(2)^2(1) = 4\pi$ ft^3
dimensions doubled:
$V = \pi r^2 h = \pi(4)^2(2) = 32\pi$ ft^3
$32\pi = 8(4\pi)$. So, volume is multiplied by 8.
12. $V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(12)^2(16) = 768\pi$ ft^3
≈ 2412.7 ft^3
13. $V = \frac{1}{3}Bh = \frac{1}{3}((39)(15))(16) = 3120$ m^3

14. $V_{\text{prism}} = Bh = \left(\frac{1}{2}(13)(20)\right)(9) = 1170 \text{ yd}^3$
$V_{\text{pyramid}} = \frac{1}{3}Bh = \frac{1}{3}\left(\frac{1}{2}(13)(20)\right)(27 - 9) = 780 \text{ yd}^3$
$V = V_{\text{prism}} + V_{\text{pyramid}} = 1170 + 780 = 1950 \text{ yd}^3$

15. $S = 4\pi r^2 = 4\pi(100)^2 = 400\pi \text{ in}^2$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(10)^3 = \frac{4000}{3}\pi \text{ in}^3$

16. $S = \frac{1}{2}\left(4\pi r^2\right) + \pi r^2 = 3\pi(12)^2 = 432\pi \text{ in}^2$
$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(12)^3 = 1152\pi \text{ in}^3$

17. $\dfrac{V_{\text{softball}}}{V_{\text{baseball}}} = \dfrac{\frac{4}{3}\pi(2.5)^3}{\frac{4}{3}\pi(1.5)^3} = \dfrac{(2.5)^3}{(1.5)^3} \approx 4.6$ times as great

STUDY GUIDE: REVIEW, PAGES 730–733

VOCABULARY, PAGE 730

1. oblique prism **2.** cross section

LESSON 10-1, PAGE 730

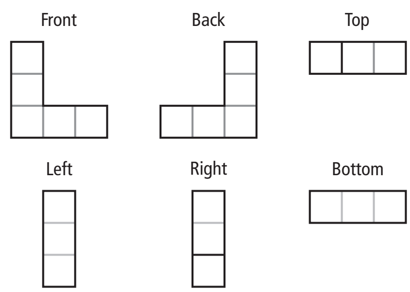
3. cone; vertex: M; edges: none; base: circle L

4. rectangular pyramid: vertices: N, P, Q, R, S; edges: $\overline{NP}$, $\overline{NQ}$, $\overline{NR}$, $\overline{NS}$, $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$, $\overline{SP}$; base: $PQRS$.

5. cylinder **6.** square pyramid

LESSON 10-2, PAGE 731

7.



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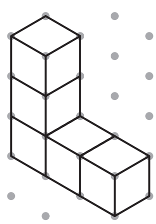

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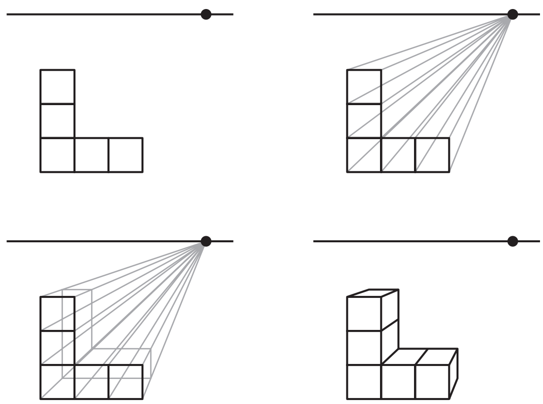

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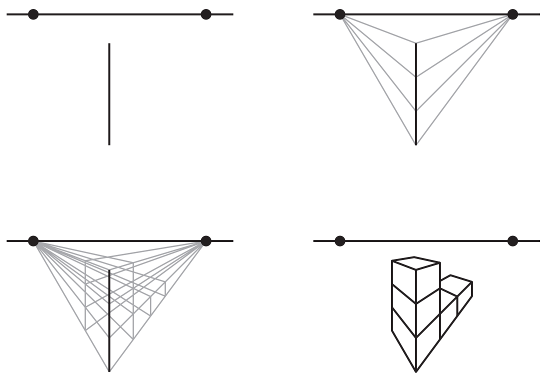


11. yes **12.** no

LESSON 10-3, PAGE 731

13. $V = 9$; $E = 16$; $F = 9$
$V - E + F = 9 - 16 + 9 = 2$

14. $V = 8$; $E = 12$; $F = 6$
$V - E + F = 8 - 12 + 6 = 2$

15. $d = \sqrt{(7-2)^2 + (1-6)^2 + (1-4)^2}$
$= \sqrt{59} \approx 7.7$ units
$M\left(\frac{2+7}{2}, \frac{6+1}{2}, \frac{4+1}{2}\right) = M(4.5, 3.5, 2.5)$

16. $d = \sqrt{(5-0)^2 + (7-3)^2 + (8-0)^2}$
$= \sqrt{105} \approx 10.2$ units
$M\left(\frac{0+5}{2}, \frac{3+7}{2}, \frac{0+8}{2}\right) = M(2.5, 5, 4)$

17. $d = \sqrt{(9-7)^2 + (1-2)^2 + (5-6)^2}$
$= \sqrt{6} \approx 2.4$ units
$M\left(\frac{7+9}{2}, \frac{2+1}{2}, \frac{6+5}{2}\right) = M(8, 1.5, 5.5)$

18. $d = \sqrt{(2-6)^2 + (7-2)^2 + (4-8)^2}$
$= \sqrt{57} \approx 7.5$ units
$M\left(\frac{6+2}{2}, \frac{2+7}{2}, \frac{8+4}{2}\right) = M(4, 4.5, 6)$

LESSON 10-4, PAGE 732

19. $L = 2\pi rh = 2\pi(5)(20) = 200\pi \approx 628.3 \text{ yd}^2$
$L = 2\pi rh + 2\pi r^2$
$= 200\pi + 2\pi(5)^2 = 250\pi \approx 785.4 \text{ yd}^2$

20. $L = Ph = (4s)s = 4(5)^2 = 100 \text{ ft}^2$
$S = Ph + B = 100 + 2(5^2) = 150 \text{ ft}^2$

21. $L = Ph = (18)(7) = 126 \text{ m}^2$
$S = L + B$
$= 126 + 2\left(\frac{1}{2}(6)\left(3\sqrt{3}\right)\right) = 126 + 18\sqrt{3} \approx 157.2 \text{ m}^2$

22. $L = Ph = (20)(8) = 160 \text{ cm}^2$
$S = Ph + 2\left(\frac{1}{2}aP\right)$
$= 160 + \left(\frac{2}{\tan 36°}\right)(20) \approx 215.1 \text{ cm}^2$

LESSON 10-5, PAGE 732

23. $L = \frac{1}{2}P\ell = \frac{1}{2}(60)(21) = 630 \text{ ft}^2$
$S = L + B = 630 + (15)^2 = 855 \text{ ft}^2$

24. $\ell^2 = 7^2 + 24^2$
$\ell = 25 \text{ m}$
$L = \pi r\ell = \pi(7)(25) = 175\pi \text{ m}^2$
$S = \pi r\ell + \pi r^2 = 175\pi + \pi(7)^2 = 224\pi \text{ m}^2$

25. $L = \pi r\ell = \pi(10)(15) = 150\pi \text{ in}^2$
$S = \pi r\ell + \pi r^2 = 150\pi + \pi(10)^2 = 250\pi \text{ in}^2$

26. $L_{\text{upper}} = \frac{1}{2}P\ell = \frac{1}{2}(32)(30) = 480 \text{ ft}^2$
$L_{\text{lower}} = \frac{1}{2}P\ell = \frac{1}{2}(32)(20) = 320 \text{ ft}^2$
$S = L_{\text{upper}} + L_{\text{lower}} = 480 + 320 = 800 \text{ ft}^2$

27. $L_{\text{cone}} = \pi r\ell = \pi(8)(12) = 96\pi \text{ m}^2$
$L_{\text{cylinder}} = 2\pi rh = 2\pi(8)(16) = 256\pi \text{ m}^2$
$S = L_{\text{cone}} + L_{\text{cylinder}} + L_{\text{cone}}$
$= 96\pi + 256\pi + 96\pi = 448\pi \text{ m}^2$

LESSON 10-6, PAGES 732–733

28. $V = Bh = (\ell w)h = (9)(12)(10) = 1080 \text{ ft}^3$

29. $V = Bh = \left(\frac{1}{2}aP\right)h = \frac{1}{2}\left(\frac{4}{\tan 36°}\right)(40)(15) \approx 1651.7 \text{ cm}^3$

30. $V = \pi r^2 h = \pi(7.5)^2(16) = 900\pi \text{ in}^3$

31. $V = \pi r^2 h = \pi(3)^2(5) = 45\pi \text{ m}^3$

LESSON 10-7, PAGE 733

32. $V = \frac{1}{3}Bh = \frac{1}{3}(42)(8) = 112 \text{ m}^3$

33. $V = \frac{1}{3}Bh = \frac{1}{3}\left(\frac{1}{2}(3)\left(1.5\sqrt{3}\right)\right)(8) = 6\sqrt{3} \approx 10.4 \text{ cm}^3$

34. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(6)^2(10) = 120\pi \text{ cm}^3$

35. $V = \frac{1}{3}Bh = \frac{1}{3}(16\pi)(9) = 48\pi \text{ ft}^3$

36. $V_{\text{cylinder}} = \pi r^2 h = \pi(8)^2(12) = 768\pi \text{ ft}^3$
$V_{\text{cone}} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(8)^2(12) = 256\pi \text{ ft}^3$
$V = V_{\text{cylinder}} - V_{\text{cone}} = 768\pi - 256\pi = 512\pi \text{ ft}^3$

37. $V_{\text{cube}} = s^3 = (10)^3 = 1000 \text{ cm}^3$
$V_{\text{pyramid}} = \frac{1}{3}Bh = \frac{1}{3}(10^2)(16) = \frac{1600}{3} \text{ cm}^3$
$V = V_{\text{cube}} + V_{\text{pyramid}}$
$= 1000 + \frac{1600}{3} = \frac{4600}{3} \approx 1533.3 \text{ cm}^3$

LESSON 10-8, PAGE 733

38. $S = 100\pi = 4\pi r^2$
$25 = r^2$
$r = 5 \text{ m}$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5)^3 = \frac{500}{3}\pi \text{ m}^3$

39. $V = 288\pi = \frac{4}{3}\pi r^3$
$216 = r^3$
$r = 6 \text{ in.}$
$S = 4\pi r^2 = 4\pi(6)^2 = 144\pi \text{ in}^2$

40. $S = 256\pi = 4\pi r^2$
$64 = r^2$
$r = 8$
$d = 2(8) = 16 \text{ ft}$

41. **Step 1** Find the surface area.
$S_{\text{prism}} = H + B = Ph + 2(\ell w)$
$= (2(10) + 2(7))(5) + 2(10)(7) = 310 \text{ cm}^2$
$L_{\text{hemisphere}} = \frac{1}{2}\left(4\pi r^2\right) = 2\pi(3)^2 = 18\pi \text{ cm}^2$
$B_{\text{hemisphere}} = \pi r^2 = \pi(3)^2 = 9\pi \text{ cm}^2$
$S = S_{\text{prism}} + L_{\text{hemisphere}} - B_{\text{hemisphere}}$
$= 310 + 18\pi - 9\pi = 310 + 9\pi \approx 338.3 \text{ cm}^2$
Step 2 Find the volume.
$V_{\text{prism}} = Bh = (\ell w)h = (10)(7)(5) = 350 \text{ cm}^3$
$V_{\text{hemisphere}} = \frac{1}{2}\left(4/3\pi r^3\right) = 2/3\pi(3)^3 = 18\pi \text{ cm}^3$
$V = V_{\text{prism}} - V_{\text{hemisphere}} = 350 - 18\pi \approx 293.5 \text{ cm}^3$

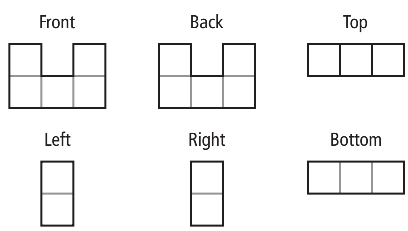
42. **Step 1** Find the surface area.
$L_{\text{cylinder}} = 2\pi rh = 2\pi(3)(7) = 42\pi \text{ ft}^2$
$L_{\text{hemisphere}} = \frac{1}{2}(4\pi r^2) = 2\pi(3)^2 = 18\pi \text{ ft}^2$
$S = L_{\text{cylinder}} + 2L_{\text{hemisphere}}$
$= 42\pi + 2(18\pi) = 78\pi \approx 245.0 \text{ ft}^2$
Step 2 Find the volume.
$V_{\text{cylinder}} = \pi r^2 h = \pi(3)^2(7) = 63\pi \text{ ft}^3$
$V_{\text{hemisphere}} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(3)^3 = 18\pi \text{ ft}^3$
$V = V_{\text{cylinder}} - 2V_{\text{hemisphere}}$
$= 63\pi - 2(18\pi) = 27\pi \approx 84.8 \text{ ft}^3$

CHAPTER TEST, PAGE 734

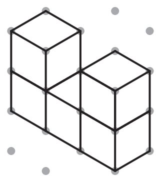
1. pentagonal pyramid; vertices: A, B, C, D, E, F; edges: $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$, $\overline{BC}$, $\overline{CD}$, $\overline{DE}$, $\overline{EF}$, $\overline{FB}$; faces: $\triangle ABC$, $\triangle ACD$, $\triangle ADE$, $\triangle AEF$, $\triangle AFB$, pentagon $BCDEF$

2. pentagon

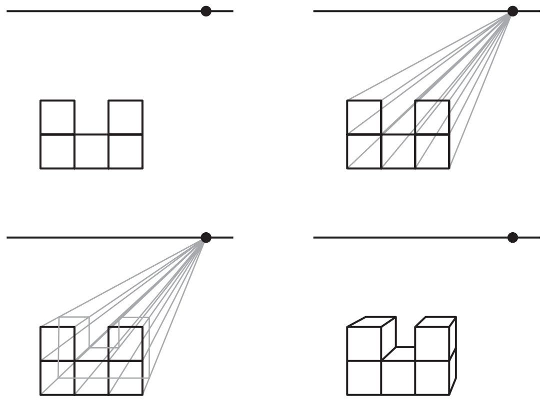
3. $V = 6$; $E = 10$; $F = 6$
$V - E + F = 6 - 10 + 6 = 2$

4.


5.



6.



7. $d = \sqrt{(5-0)^2 + (5-0)^2 + (5-0)^2}$
$= \sqrt{75} = 5\sqrt{3} \approx 8.7$ units
$M\left(\frac{0+5}{2}, \frac{0+5}{2}, \frac{0+5}{2}\right) = M(2.5, 2.5, 2.5)$

8. $d = \sqrt{(7-6)^2 + (1-0)^2 + (4-9)^2}$
$= \sqrt{27} = 3\sqrt{3} \approx 5.2$ units
$M\left(\frac{6+7}{2}, \frac{0+1}{2}, \frac{9+4}{2}\right) = M(6.5, 0.5, 6.5)$

9. $d = \sqrt{(2-(-1))^2 + (-5-4)^2 + (7-3)^2}$
$= \sqrt{106} \approx 10.3$ units
$M\left(\frac{-1+2}{2}, \frac{4+(-5)}{2}, \frac{3+7}{2}\right) = M(0.5, -0.5, 5)$

10. $S = L + B = Ph + 2\left(\frac{1}{2}aP\right)$
$= (48)(11) + \left(4\sqrt{3}\right)(48)$
$= 528 + 192\sqrt{3} \approx 860.6 \text{ ft}^2$

11. $S = 2\pi rh + 2\pi r^2$
$= 2\pi(10.5)(30) + 2\pi(10.5)^2 \approx 2671.9 \text{ in}^2$

12. $B = 36 = s^2$
$s = 6$ cm
$S = L + B = s^2 + \frac{1}{2}P\ell$
$= 36 + \frac{1}{2}(24)(14) = 204 \text{ cm}^2$

13. $C = 16\pi = 2\pi r$
$r = 8$ m
$\ell^2 = 8^2 + 15^2$
$\ell = 17$ m
$S = \pi r\ell + \pi r^2$
$= \pi(8)(17) + \pi(8)^2 = 200\pi \approx 628.3 \text{ m}^2$

14. $S = 4\pi r^2 = 4\pi(5)^2 = 100\pi \approx 314.2 \text{ yd}^2$

15. $L_{cube} = Ph = (4s)(s) = 4(6)^2 = 144 \text{ m}^2$
$B_{cube} = s^2 = (6)^2 = 36 \text{ m}^2$
$L_{pyramid} = \frac{1}{2}P\ell = \frac{1}{2}(24)(4) = 48 \text{ m}^{21}$
$S = L_{cube} + B_{cube} + L_{pyramid}$
$= 144 + 36 + 48 = 228 \text{ m}^2$

16. $V = \ \ell wh = (15)(9)(12) = 1620 \text{ ft}^3$

17. $7^2 + h^2 = 25^2$
$h = 24$ m
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(7)^2(24) = 392\pi \approx 1231.5 \text{ m}^3$

18. $V = \frac{1}{3}Bh = \frac{1}{3}(18^2)(20) = 2160 \text{ ft}^3$

19. $V = \pi r^2 h = \pi(3)^2(2) = 18\pi \approx 56.5 \text{ in}^3$

20. $4^2 + h^2 = 5^2$
$h = 3$ cm
$V_{cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2(3) = 16\pi \text{ cm}^3$
$V_{cylinder} = \pi r^2 h = \pi(4)^2(7) = 112\pi \text{ cm}^3$
$V_{hemisphere} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(4)^3 = \frac{128}{3}\pi \text{ cm}^3$
$V = V_{cone} + V_{cylinder} - V_{hemisphere}$
$= 16\pi + 112\pi - \frac{128}{3}\pi = \frac{256}{3}\pi \approx 268.1 \text{ cm}^3$

21. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = 288\pi \approx 904.8 \text{ cm}^3$

22. $\frac{V_{Earth}}{V_{Moon}} = \frac{\frac{4}{3}\pi(3965)^3}{\frac{4}{3}\pi(1080)^3} = \frac{(3965)^3}{(1080)^3} \approx 49.5$
The Earth's volume is about 49.5 times as great as the Moon's volume.

COLLEGE ENTRANCE EXAM PRACTICE, PAGE 735

1. D
A and B must be opposite vertices.
$AB = \sqrt{4^2 + 4^2 + 4^2} = \sqrt{48} = 4\sqrt{3}$ cm

2. C
$L = 3B$
$2\pi rh = 3\pi r^2$
$2h = 3r$
$h = \frac{3}{2}r$

3. C
$\ell^2 = 6^2 + 8^2$
$\ell = 10$ ft
$L = \pi r\ell = \pi(6)(10) = 60\pi \text{ ft}^2$

4. A
$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(5)^2(12) = 100\pi$ cubic units

5. D
The height of the cylinder is $h = 32 - 2(5) = 22$ in.
$V_{cylinder} = \pi r^2 h = \pi(5)^2(22) = 550\pi \text{ in}^3$
$V_{hemisphere} = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi(5)^3 = \frac{250}{3}\pi \text{ in}^3$
$V = \ + 2 = 550\pi + 2\left(\frac{250}{3}\pi\right) = \frac{2150}{3}\pi \text{ in}^3$

3. Let P and Q be pts. on $\overline{AD}$ directly below B and C. Then
$AP = PB = 310$ ft, $PQ = BC = 60$ ft, and QD satisfies

$$\begin{aligned} QD^2 + QC^2 &= CD^2 \\ QD^2 + 310^2 &= 314.8^2 \\ QD^2 &= 2999.04 \\ QD &= \sqrt{2999.04} \approx 54.8 \text{ ft} \\ AD &= AP + PQ + QD \\ &\approx 310 + 60 + 54.8 \approx 425 \text{ ft} \end{aligned}$$

4. $XY = \frac{1}{2}(AD + BC)$

$\approx \frac{1}{2}(424.8 + 60) \approx 242$ ft

CHAPTER 8, PAGES 582–583

THE JOHN HANCOCK CENTER, PAGE 582

1. By Alt. Int. ∡ Thm., height and horiz. dist. x are opp. and adj. sides for 10° ∠.

$\tan 10° = \frac{1000}{x}$

$x = \frac{1000}{\tan 10°} \approx 5671$ ft

2. $\tan 61° = \frac{h}{818.2}$

$h = 818.2 \tan 61° \approx 1476$ ft

3. $\tan 39° = \frac{(818.2 \tan 61°)}{x}$

$x = \frac{818.2 \tan 61°}{\tan 39°} \approx 1823$ ft

4. Shadow is longest when ∠ of elevation is smallest, on Dec 15.

$\tan 25° = \frac{(818.2 \tan 61°)}{x}$

$x = \frac{818.2 \tan 61°}{\tan 25°} \approx 3165$ ft

ERNEST HEMINGWAY'S BIRTHPLACE , PAGE 583

1. perim. of dining room on plan ≈ 3.5 in.

$\frac{3.5}{\text{actual length}} \approx \frac{1}{16}$

actual length ≈ 16(3.5) ≈ 56 ft

2. area of parlor and living room on plan ≈ $1\frac{1}{8}$ in.^{2}

$\frac{1.125}{\text{actual area}} \approx \left(\frac{1}{16}\right)^2 = \frac{1}{256}$

actual area ≈ 256(1.125) = 288 ft^2

3. $\ell = w + 4$ and $2\ell + 2w = 40$

$$\begin{aligned} 2(w + 4) + 2w &= 40 \\ 4w + 8 &= 40 \\ 4w &= 32 \\ w &= 8 \\ \ell &= 8 + 4 = 12 \end{aligned}$$

plan dimensions are $\frac{12}{16} = \frac{3}{4}$ in. by $\frac{8}{16} = \frac{1}{2}$ in.

CHAPTER 10, PAGES 740–741

THE MELLON ARENA, PAGE 581

1. The area is a circle with a diameter of 400 ft.
area in square feet:
$A = \pi r^2 = \pi(200)^2 \approx 126{,}000$ ft^2
Area in acres:
$A = 126{,}000 \text{ ft}^2 \cdot \frac{1 \text{ acre}}{43{,}560 \text{ ft}^2} \approx 3$ acres

2.
$$\begin{aligned} \frac{\ell}{w} &= \frac{40}{17} \\ \ell &= \frac{40}{17}w \\ P &= 2\ell + 2w \\ 570 &= 2\left(\frac{40}{17}w\right) + 2w \\ 570(17) &= 80w + 34w \\ 9690 &= 114w \\ w &= 85 \text{ ft} \\ \ell &= \frac{40}{17}(85) = 200 \text{ ft} \end{aligned}$$

The dimensions are 200 ft by 85 ft.

3.
$$\begin{aligned} \ell &= w + 130 \\ P &= 2\ell + 2w \\ 740 &= 2(w + 130) + 2w \\ 740 &= 2w + 260 + 2w \\ 480 &= 4w \\ w &= 120 \text{ ft} \\ \ell &= (120) + 130 = 250 \text{ ft} \end{aligned}$$

The dimensions are 250 ft by 120 ft.

4. **Step 1** Find the probability of sitting under the fixed sections.
area under fixed sections:
$A = \frac{2}{8}\pi(200)^2 = 10{,}000\pi$ ft^2
total area: $A = \pi(200)^2 = 40{,}000\pi$ ft^2
$P = \frac{10{,}000\pi}{40{,}000\pi} = \frac{1}{4}$
Step 1 Find the probability of sitting under the open sky.
area under open sky:
$A = 40{,}000\pi - 10{,}000\pi = 30{,}000\pi$ ft^2
$P = \frac{30{,}000\pi}{40{,}000\pi} = \frac{3}{4}$

THE U.S. MINT, PAGE 582

1. Assume the quarters are stamped out in a rectangular grid pattern, with each quarter occupying a 1-in. square. Then the number of quarters that can be stamped out of each strip equals the area of the strip in square inches.

$$\begin{aligned} \# \text{ quarters} = A &= \ell w \\ &= (13 \text{ in.})\left(1500 \text{ ft} \cdot \frac{12 \text{ in.}}{1 \text{ ft}}\right) \\ &\approx 234{,}000 \end{aligned}$$

2(234,000) < 700,000 < 3(234,000)
So, for 700,000 quarters, 3 strips are needed.

2. $V_{penny} = \pi r^2 h = \pi(9.525)^2(1.55) \approx 442$ mm^3

% (copper) $= \frac{V_{copper}}{V_{penny}} \cdot 100\% \approx \frac{11}{442} \cdot 100\% \approx 2.5\%$

3. Assume that all the metal is used up in making the nickels.
$V_{\text{nickel}} = \pi(10.605)^2(1.95) \approx 689 \text{ mm}^3$
$V_{\text{cube}} = (1000)^3 = 1 \times 10^9 \text{ mm}^3$
$\# \text{ nickels} \approx \dfrac{V_{\text{cube}}}{V_{\text{nickel}}} \approx \dfrac{1 \times 10^9}{689} \approx 1.45 \times 10^6$, or 1.45 million

4. The plastic forms the surface area of a cylinder with a height of 50 dimes.
$$\begin{aligned} S &= 2\pi rh + 2\pi r^2 \\ &= 2\pi(8.955)\big(50(1.35)\big) + 2\pi(8.955)^2 \\ &\approx 4302 \text{ mm}^2 \end{aligned}$$

CHAPTER 12, PAGES 894–895

SANDY HOOK LIGHTHOUSE, PAGE 894

1. $A = \pi r^2\left(\dfrac{m}{360}\right) = \pi(19)^2\left(\dfrac{60}{360}\right) = \dfrac{361}{6}\pi \approx 189 \text{ mi}^2$

2. Let the distance from the top of the tower to the horizon be x mi.
$$\begin{aligned} 4000^2 + x^2 &= \left(4000 + \frac{85}{5280}\right)^2 \\ 16{,}000{,}000 + x^2 &= 16{,}000{,}128.79 \\ x^2 &= 128.79 \\ x &\approx 11.3 \text{ mi} \end{aligned}$$

3. $8.6(8.6) = 2(d - 2)$
$73.96 = 2d - 4$
$77.96 = 2d$
$d \approx 39$ in. or 3 ft 6 in.
The order of the lens is third.

MOVEABLE BRIDGES, PAGE 895

1. rotation; 14.1 ft
The rotation forms a right triangle, whose legs are each 10 ft. Thus, by Pythagorean Theorem:
$c^2 = 10^2 + 10^2$
$c = \sqrt{100 + 100} = \sqrt{200} \approx 14.1$

2. speed $= \dfrac{138 \text{ ft} - 35 \text{ ft}}{2 \text{ min}} = 51.5$ ft/min

3. time $= \dfrac{10 \text{ ft}}{51.5 \text{ ft/min}} \approx 0.194 \text{ min} \approx 12 \text{ s}$

4. time $= \dfrac{135 \text{ ft} - 49 \text{ ft}}{51.5 \text{ ft/min}} \approx 1.670 \text{ min} \approx 100 \text{ s}$

5. height $= 151 \sin 20° \approx 51.6$ ft