Chapter Two A:
Linear Expressions and Equations

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Algebra I Equations and Their Solutions 2A A

A LOT of time is spent in Algebra learning how to solve equations and then solving them for various purposes. So, it goes without saying that we really need to understand what it means for something to "solve" an equation. First, let's make sure we understand what an equation is:

EQUATION DEFINITION

An equation is simply a statement about the **equality** of two expressions. In other words, anything that takes this form:

Expression #1 = Expression #2

Exercise 1: Decide if each of the following are *equations or expressions*. You do not need to solve the equation or evaluate the expression.

1) $-5(2x - 1) + 6x = 21$ ____________________

2) $\frac{1}{3}(6h + 15)$ ____________________

3) $5^3 - 2|6x - 4|$ ____________________

4) $\frac{8(x+3)}{6} - 4 = 9$ ____________________

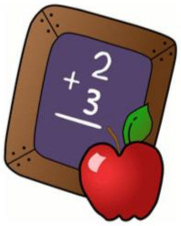
Exercise 2: Which of the following is not an equation?

(1) $3+1=4+0$

(3) $2(4x+1)$

(2) $x^2-2x=8$

(4) $1+3=6$



Equations can be either true, like (1) above, or false like (4) above, depending on whether the two expressions are equal (true) or not equal (false).

Exercise 3: Consider the equation $2x - 8 = 10 - x$

(a) Why can't you determine whether this equation is true or false?

(b) If $x = 5$, will the equation be true? How can you tell?

(c) Show that $x = 6$ makes the equation true. Remember to think carefully always about your order of operations.

SOLUTIONS TO EQUATIONS

A value for a variable is called a **solution to the equation** if, when substituted into both expressions, results in the equation being **true**.

This concept of the solution to an equation is **amazingly important**. It implies that you can always know when you have solved an equation correctly. As long as you can check the truth of the equation with arithmetic, then you will know if your value (of x often) is correct.

Exercise 4: Determine whether each of the following values for the given variable is a solution to the given equation. Show the calculations that lead to your final conclusions.

(a) $2x + 3 = 17$ and $x = 7$

(b) $\frac{x-20}{5} = -4$ and $x = 10$

(c) $2(x + 5) = 6(x - 1)$ and $x = 4$

(d) $x^2 - 1 = 2x + 2$ and $x = -1$

(e) $\frac{3(x+2)}{4} - 1 = 5$ and $x = 2$

(f) $\frac{3}{4}x - 1 = -\frac{1}{2}x + 9$ and $x = 8$

Exercise 5: Kirk was checking to see if $x = 7$ was a solution to the equation $4x - 3 = 2x + 11$. He concluded that it was not a solution based on the following work. Was he correct?

$$4x - 3 = 2x + 11$$

$$4 \cdot 7 - 3 = 2 \cdot 7 + 11$$

$$4 \cdot 4 = 2 \cdot 18$$

$$16 = 36 \quad \text{No!}$$

So there are *NO EXCUSES*! If you solve an equation, you should always be able to check to see if your solution is correct. Sometimes, mistakes happen, and it is good to be able to spot them.

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HOMEWORK:

1) Decide if each of the following are **equations** or **expressions**. You do not need to solve the equations or evaluate the expressions.

(a) $5x+13$

(b) $4x+3=12$

(c) $\frac{6(x-1)}{4}+1=5$

(d) $3(x+2)^2-(45)^3$

(e) $3^2-5|2x-15|$

(f) $3[(x+2)^2+2(x-4)]=3\sqrt{4(2x+1)}$

2) Determine whether each of the following values for the given variable is a solution to the given equation. Show the calculations that lead to your final conclusions.

(a) $x-4=12 \quad and \quad x=8$

(b) $\frac{3+x}{4}=3 \quad and \quad x=9$

(c) $(x+2)-3(x-4)=6 \quad and \quad x=4$

(d) $\frac{1}{3}(x+2)=-\frac{2}{5}(x-9) \quad and \quad x=4$

(e) $2(x+4)=3(x+6) \quad and \quad x=-2$

(f) $\frac{1}{4}(6-2x)=-5 \quad and \quad x=13$

3) A disease has three treatments, depending on the percent of the body affected by the disease. Doctors have the treatment down to three stages as follow:

Stage 1: less than 15%   Stage 2: 15-25%   Stage 3: 25-50%

For anything more than 50% there is no cure. If the disease is spreading according to the formula $P = 6d + 5$ where P is the percent of the body affected and d is the number of days, fill out the following chart and explain to a patient what you observed.

Days	% of body Affected
1	
2	
3	
4	
5	
6	
7	
8	

Explanation of what you observed:

4) Bobby wants to go on a school trip that will cost him $250. He comes up with an equation that represents how much he needs to save each week as follows:

$25w + 30 = 250$, where w is the number of weeks spent saving

(a) If he has 9 weeks to save, will he have enough money to go on the trip? Explain.

(b) He also wants to have $100 spending cash on the trip. He decides to save an extra $10 a week. To do this he changes his original equation as follows:

$25w + 30 + 10w = 250 + 100$, where w is the number of weeks spent saving

Will nine weeks be enough time now? Show your calculations and explain.

Review Section:

5) Find the **product** of $x + 5$ and $x - 2$

6) Find the product of $2x^4y$ and $8xy^8$

7) Simplify the following: $\frac{1}{2}(x + 4) - 2x(4x + 5)$

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Algebra I Seeing Structure to Solve Equations 2A B

You spent a lot of time in 8th grade Common Core Math **solving linear equations** (ones where the variable is raised to the first power only). In fact, the expectation is that you mastered solving linear equations. These types of equations are so **essential in mathematics**, though, that is pays to work with them more. In today's lesson we will be solving linear equations where the variable occurs once. We will solve these equations by seeing the structure of the expression involving x and using this structure to "undo" what has been done to it.

SOLVING EQUATIONS BY INVERSE OPERATIONS
If the **variable** you are solving for shows up only once, identify the operations that have been done on it and reverse them in the opposite order in which they occur.

Examples:

If you are:

Adding, you would *Subtract*

Multiplying, you would *Divide*, etc...

Remember to use the **"Law of Equality"**. This states that what you do to one side of the equation, you must do to the other side of the equation

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Warm Up: Solve the following for x.

(a) $5x - 10 = 15$ (b) $-3x + 1 = 13$ (c) $\frac{1}{2}x + 3 = 15$

Exercise 1: Consider the equation $5x + 3 = 23$.

(a) List the operations that have been done to the variable x on the left hand side of the equation in the order in which they occurred.

(b) Solve the equation by reversing what has been done to x. Verify that your value of x is a solution by seeing if it makes the equation true.

Exercise 2: Find the value of x that solves each equation. In each case, first identify the operations that have occurred to x and reverse them. Show each step.

(a) $\frac{x-3}{2} + 7 = 23$

What happened to x?

1. x was decreased by 3
2. The result was divided by 2.
3. The result was increased by 7.

Now reverse.

$$\frac{x-3}{2} + 7 = 23$$
$$-7 \quad -7$$
$$(2)\frac{x-3}{2} = 16(2)$$
$$x - 3 = 32$$
$$+3 \quad +3$$
$$x = 35$$

(b) $4(x+1)-2=-6$

What happened to x?

Now reverse.

Often equations can be solved in multiple ways. Let's take a look at the next problem to see an example.

Exercise 3: Solve the following equation in two different ways. In (a), reverse the operations that have been done to x. In (b), apply the distributive property first.

(a) *[Reverse the operations]*

$$-2(x-4)+8 = 2$$
$$-8 \quad -8$$
$$-2(x-4) = -6$$
$$\div(-2) \quad \div(-2)$$
$$x-4 = 3$$
$$+4 \quad +4$$
$$x = 7$$

(b) *[Distributive Property first]*

$$-2(x-4)+8 = 2$$
$$-2x+8+8 = 2$$
$$-2x+16 = 2$$
$$-16 \quad -16$$
$$-2x = -14$$
$$\div(-2) \quad \div(-2)$$
$$x = 7$$

So you can see, we get the same answer! It is just a different way to solve it!

Exercise 4: Solve the following equation in two different ways. In (a), reverse the operations that have been done to x. In (b), apply the distributive property first.

(a) $4(x+1)-2=-6$

(b) $4(x+1)-2=-6$

Exercise 5: Set up equations that translate the following verbal phrases into mathematics and then solve the equations.

(a) Ten less than five times a number results in thirty five. What is the number? Carefully set up an equation, solve it, and check your answer for reasonableness. Watch out! Subtraction is involved.

(b) When three times the sum of a number and seven is increased by ten, the result is four. What is the number? Carefully set up an equation and solve it. Check for reasonableness.

HOMEWORK:

1) Solve for x by reversing your operations.

(a) $x + 15 = 28$ (b) $15 - x = 21$ (c) $\frac{x}{4} = 5$

(d) $6 + x = 34$ (e) $9x = 45$ (f) $21 - 4x = 45$

2) In the expression $\frac{x}{5} - 3$ which is the correct order in which operations have been done to x?

(1) x was divided by 5 and the result was subtracted from 3
(2) x had 3 subtracted from it and the result was then divided by 5.
(3) x was divided by 5 and 3 was subtracted from the result
(4) 5 was divided by x and then 3 was subtracted from the result.

3) Which of the following is the solution to $6x + 1 = 4$? Show the steps or explain how you found the solution.

(1) $x = \frac{7}{6}$ (3) $x = \frac{4}{3}$

(2) $x = \frac{1}{2}$ (4) $x = \frac{5}{6}$

4) The solution to $5(x-2) - 6 = 24$ is which of the following? Show the steps in your solution process.

(1) $x = 7$ (3) $x = -3$

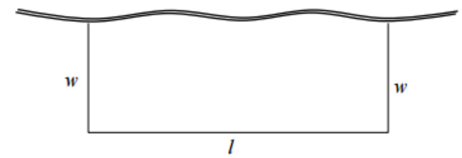
(2) $x = -12$ (4) $x = 8$

5) If a number is increased by five and the result is then divided by three, the result is seven. Write an equation that models this verbal description and solve the equation for the number described.

6) Max and his friend Zeke are comparing their ages. They figure out that if they double Max's age from 3 years ago and add it to Zeke's current age, the sum is 26. If Zeke is currently 8 years old, determine how old Max currently is.

7) A rectangular area is being fenced in along a river that serves as one side of the rectangle.

(a) Write an equation that relates the amount of fencing, F, needed as a function of the width, w, and the length, l.



(b) If $w = 12$ feet and $l = 20$ feet, what is the value of F?

(c) If we know that the amount of fencing we have available is 120 feet and we want to devote 30 feet to the length, l, then set up an equation to solve for w and find the width.

8) Consider the equation $\frac{5(2x-1)}{3} - 4 = 11$. This equation looks complicated, but we can unravel all of the operations that have been done to x to produce the output of 11.

(a) List the operations that have been done to x and the order in which they have been done.

(b) Reverse the operations from (a) to solve for x.

9) Think about the equation $4(3x + 2) = -16$.

(a) Solve this equation by reversing what has been done to x.

(b) Solve this equation by first distributing the multiplication by 4.

Review Section:

10) If the difference $(3x^2 - 2x + 5) - (x^2 + 3x - 2)$ is multiplied by $\frac{1}{2}x^2$, what is the result, written in standard form?

11) Fred is given a rectangular piece of paper. If the length of Fred's piece of paper is represented by $2x - 6$ and the width is represented by $3x - 5$, then the paper has a total area represented by

(1) $5x - 11$

(2) $6x^2 - 28x + 30$

(3) $10x - 22$

(4) $6x^2 - 6x - 11$

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Algebra I Linear Equations Solving Review 2A C

The expectation of the Common Core is that students have mastered solving all types of linear equations in 8th grade Common Core mathematics. In this lesson, we simply present a variety of linear equations for you to practice solving.

<u>Exercise 1:</u> Solve each of the following "two step" linear equations. Keep in mind, this is what we were doing in the last lesson by reversing the operations that had occurred to the variable. Some of these answers will be *non-integer rational numbers*. Simplify where possible.

(a) $\frac{x}{3} - 7 = -2$

(b) $4x + 3 = -17$

(c) $5x + 12 = 87$

(d) $\frac{x+7}{3} = 2$

(e) $-6(x - 1) = 18$

(f) $8x + 2 = -2$

(g) $\frac{3}{4}x - 5 = 4$

(h) $-\frac{5}{2}x + 6 = 1$

For most of what we do the rest of the way, you will be using the distributive property as well as others to solve the problems. Don't forget our primary technique of solving by reversing the operations that have been done to our variable. This technique is particularly useful when the **variable shows up only once!**

Exercise 2: Solve the following equation for x by identifying the operations that have been done to x and reversing them.

$$\frac{5(x-3)}{8} + 2 = 7$$

Reverse them!

Operations?

Now let's try some a little harder:

Exercise 3: Solve for following for the variable x.

(a) $4x + 3x + 5 = 26$

(b) $2(3x + 1) + 2x = 18$

Exercise 4: Consider the equation $5(x-3)+2x=4(x+3)$.

(a) By using the distributive property, write equivalent expressions for both sides of the equation. Show the work below.

(b) Solve the equation for x. Check to make sure the **original equation** has a true value for the x you find.

Exercise 5: Get more practice on these more complicated equations. Generally, use the distributive property when needed.

(a) $7(x-2)-3(x+3)=5(x-3)+x$

(b) $9-6(x+1)=2(x-4)+27$

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Algebra I Linear Equations Solving Review 2A C Homework

HOMEWORK:

1) Solve the following equations for x using inverse operations.

(a) $7x-15=1$

(b) $\frac{x+2}{4}=-2$

(c) $-\frac{3}{5}x+2=7$

2) Solve the equations for x. Check to make sure the original equation has a true value for the x that you find. (Check your solution by substituting it back into the original equation).

(a) $\frac{5(x+1)+4}{6}=4$

(b) $\frac{5(x-3)}{8}+2=7$

(c) $-\frac{3}{2}x+2=-4$

(d) $5(x+1)-2x=2(3+x)$

(e) $3(x-4)-2(3x+4)=4(3-x)+5x+4$

(f) $\frac{1}{2}(2-6x)-4\left(x+\frac{3}{2}\right)=-(x-3)+4$

In the real world, many scenarios may be modeled with linear equations like the ones you've seen so far. Sometimes, though, linear models may not give variable results, and we must interpret the answer we find. To see an example of this, let's look at the following.

3) A tile warehouse has Inventory at hand and can put in for a back order from a supplier of bundles of tiles. Currently they have 38 tiles of a certain kind in stock, and can only order more in groups of 12 tiles per bundle. The equation that represents this order is as follows;

The number of tiles = $12b + 38$, where b is the number of bundles ordered.

(a) If a customer needs 150 tiles, how many bundles will need to be ordered? Explain how you got your answer. Why do we need to round our answer up in this problem?

(b) If the store likes to keep 30 tiles in stock at all times, how many bundles do they need to order now, after selling the 150 tiles to the customer? Think about how many you had left over from the customer who ordered 150 tiles.

4) Look through the following work, find the mistake, and circle it. Then, to the side, show the appropriate work.

$$\frac{-2(x-3)}{5} = 4$$

$$5 \cdot \frac{-2(x-3)}{5} = 4 \cdot 5$$

$$-2(x-3) = 20$$

$$-2x - 6 = 20$$

$$-2x - 6 + 6 = 20 + 6$$

$$-2x = 26$$

$$\frac{-2x}{-2} = \frac{26}{-2}$$

$$x = -13$$

Review Section:

5) What is the value of the expression $\frac{1}{2}x^2 - 2x - 3$ when $x = 4$?

(1) -3 (2) -8 (3) 3 (4) 7

6) If $A = 2x + 4$ and $B = 5x - 7$. What is $A - B$?

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Algebra I Justifying Steps in Solving Equations 2A D

Do Now:

1) Solve the following equations for the value of x.

(a) $2(x + 3) - 5 = -7$ (b) $6x + 7 = 2x + 35$

Now that we have reviewed how to solve linear equations involving variables on both sides, it is time to take it to another level. The Common Core asks us not only to know how but also the why. Generally, we justify the steps we take in solving linear equations by using the commutative, associative, and distributive properties of real numbers along with the following two properties of equality:

PROPERTIES OF EQUALITY

(1) **ADDITIVE PROPERTY OF EQUALITY**: If $a = b$ then $a + c = b + c$ (you can add or subtract the same quantity from both sides and retain the equality).

(2) **MULTIPLICATIVE PROPERTY OF EQUALITY**: If $a = b$ then $c \cdot a = c \cdot b$ (you can multiply or divide by the same quantity on both sides and retain the equality).

$x - 4 = 6$
$+4 \quad +4$ (add 4 to each side)
$x = 10$

$x + 2 = 5$
$-2 \quad -2$ (subtract 2 from each side)
$x = 3$

$\frac{5x}{5} = \frac{10}{5}$ (divide each side by 5)
$x = 2$

$\frac{1}{2}x = 4$
$(2)\frac{1}{2}x = 4(2)$ (multiply both sides by 2)
$x = 8$

Exercise 1: Consider the equation $2x + 9 = 21$. The steps in solving the equation are shown below. Justify each step.

Step 1: $2x + 9 - 9 = 21 - 9$ Justification: ________________________________

Step 2: $\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 12$ Justification: ________________________________
$x = 6$

Exercise 2: Consider the equation $3(x + 2) - 2(x + 7) = 4x + 7$. As in the last problem, each step of the solution is shown. Justify each with either a property of equality or a property of real numbers.

Step 1: $3x + 6 - 2x - 14 = 4x + 7$ Justification: ________________________________

Step 2: $3x + -2x + 6 + -14 = 4x + 7$ Justification: ________________________________

Step 3: $x(3 - 2) + -8 = 4x + 7$ Justification: ________________________________
$x - 8 = 4x + 7$

Step 4: $x - 8 - 4x + 8 = 4x + 7 - 4x + 8$ Justification: ________________________________

Step 5: $x - 4x - 8 + 8 = 4x - 4x + 7 + 8$ Justification: ________________________________

Step 6: $x(1 - 4) = 15$ Justification: ________________________________
$-3x = 15$

Step 7: $\frac{-3x}{-3} = \frac{15}{-3}$ Justification: ________________________________
$x = -5$

Strange things can sometimes happen when you solve an equation. Even if every step is justified, results can turn out confusing...

Exercise 3: Consider the equation $5x - 3(x + 1) = 2(x + 4)$.

(a) Fill in the missing justifications in the solution of this equation below.

Step #1: $5x - 3x - 3 = 2x + 8$	Justification: The Distributive Property
Step #2: $5x - 3x - 3 - 8 = 2x + 8 - 8$	Justification: ______________________________
Step #3: $x(5-3) - 11 = 2x$ $2x - 11 = 2x$	Justification: ______________________________
Step #4: $2x - 11 - 2x = 2x - 2x$	Justification: Additive Property of Equality
Step #5: $2x - 2x - 11 = 0$ $-11 = 0$	Justification: ______________________________

(b) The final line of this set of manipulations is a very strange statement: -11=0. Is this a true statement? Could any value of x make it a true statement?

(c) What do you think this tells you about the solutions to this equation (i.e. the values of x that make it true)?

Exercise 4: Consider the equation $7x + 2(x + 5) = 9x + 10$.

(a) Show that $x = -5$ and $x = 2$ are both solutions to this equation.

(b) Solve this equation by manipulating each side of the equation as we did before. What does its final "strange" result tell you?

(c) Test your conclusion in (b) by picking a random integer (or really any number) and showing that it is a solution to the equation.

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Algebra I Justifying Steps in Solving Equations 2A D Homework

HOMEWORK:

1) Solve the following to find the value of x.

(a) $9(3x + 6) - 6(7x - 3) = 12$

(b) $3(2x - 5) - 4x = 33$

(c) $3x - 25 = 11x - 5 + 2x$

(d) $5(2x + 14) = 2(3x + 31)$

2) Which property justifies the second line in the following solution?

$3x + 2 = 8$
$3x + 2 - 2 = 8 - 2$

(1) Multiplicative Property of Equality
(2) Associative
(3) Distributive
(4) Additive Property of Equality

3) What is the solution to the following equation? Show all work. $3(x + 2) - 2x = -2(x - 3) + 3x$

(1) No Solutions
(2) Infinite Solutions
(3) $x = 2$
(4) $x = -3$

4) Give a property of real numbers (associative, commutative, or distributive) or a property of equality (addition or multiplication) that justifies each step in the following equation:

$3x + 1 + 2x - 7 = x + 22$

(1) $3x + 2x + 1 - 7 = x + 22$ (1) ____________________________

(2) $x(3 + 2) - 6 = x + 22$
$5x - 6 = x + 22$ (2) ____________________________

(3) $5x - 6 + 6 = x + 22 + 6$
$5x = x + 28$ (3) ____________________________

(4) $5x - x = x + 28 - x$ (4) ____________________________

(5) $x(5 - 1) = 28$
$4x = 28$ (5) ____________________________

(6) $\frac{1}{4} \cdot 4x = \frac{1}{4} \cdot 28$
$x = 7$ (6) ____________________________

5) Antonio just signed up for a new phone plan and is comparing his fees to that of his friend Marcus. They both create equations so that they could compare their fees with each other.

Antonio's plan: Monthly cost $= 3(.75m+10)+2.50m-15$ where m is the number of minutes used

Marcus's Plan: Monthly cost $= 2(1.75m+12.50)-.75m+4$ where m is the number of minutes used

(a) ***By setting their monthly cost equal***, decide after how many minutes the two plans will cost the same.

(b) Antonio compares his plan to another friend, Brielle's. Given that both Antonio and Brielle will only be charged for full minutes, is there an amount of time when their ***two plans cost the same***? Explain.

Brielle's plan: Monthly cost $= 2(1.50m+12)+m-4$ where m is the number of minutes used

6) Without solving the following equations, decide where there will be ***one solution, no solutions***, or ***infinitely many solutions*** and explain why you think so.

$3x-2=3x-2$	$2x-4=2x-7$	$3x-5=6x-5$

Review Section:

7) Three times the sum of a number and four is equal to five times the number, decreased by two. If x represents the number, which equation is a correct translation of the statement?

1) $3(x+4)=5x-2$
2) $3(x+4)=5(x-2)$
3) $3x+4=5x-2$
4) $3x+4=5(x-2)$

8) The expression $(-2a^2b^3)(4ab^5)(6a^3b^2)$ is equivalent to

1) $8a^6b^{30}$
2) $48a^5b^{10}$
3) $-48a^6b^{10}$
4) $-48a^5b^{10}$

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Algebra I Linear Word Problems 2A E

Although word problems can often be some of the most challenging for students, they give us great opportunities to refine our understanding of the relationships between quantities and how to manipulate expressions to solve equations. When you solve any real world problems in mathematics you are modeling a physical situation with mathematical tools, such as equations, diagrams, tables, as well as many others.

As we work through these problems, try to make sure to always do the following:

MODELING AND SOLVING LINEAR WORD PROBLEMS

1. Clearly define the quantities involved with common sense variables and **let statements**.
2. Use your **let statements** to write out expressions for **quantities that you are interested in**.
3. Carefully translate the information you are told into an equation.
4. Solve the equation – remember to mentally note the justification for each step.
5. Check the reasonableness of your answer! This could be the most important, and neglected, step in the modeling/problem solving method.

Let's start off with a reasonably easy example:

Exercise 1: The sum of a number and five more than the number is 17. What is the number?

(a) First experiment with some numbers. This will help you when going to the abstract with variables.

(b) Now, let's carefully set up let statements and an equation that relates the quantities of interest. Solve the equation for the number.

Exercise 2: The difference between twice a number and a number that is 5 more than it is 3. Which of the following equations could be used to find the value of the number, n? Explain how you arrived at your answer?

(1) $2n - n + 5 = 3$ (3) $n + 5 - 2n = 3$

(2) $n - (2n + 5) = 3$ (4) $2n - (n + 5) = 3$

Let's try a harder one:

Exercise 3: Three numbers have the sum of 99. The 2^{nd} number is 3 more than double the first. The 3^{rd} number is 3 more than the second. Find all three numbers.

Exercise 4: Sara has three sisters. Lea is 4 less than 3 times the age of Sarah. Rachel is 3 years less than one-half Sarah's age. Ruth is 1 year older than twice the age of Sarah. If the sum of the ages of the four sisters is 50 years, how old is each sister?

Exercise 5: The difference of 2 numbers is 25. The smaller is 5 more than half the larger. Find both numbers.

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Algebra I Linear Word Problems 2A E Homework

HOMEWORK:

1) The sum of three times a number and 2 less than 4 times that same number is 15. Which of the following equations could be used to find the value of the number, n? Explain how you arrived at your choice.

(1) $3n+4n-2=15$ (3) $4n+3(n-2)=15$

(2) $3n+4(n-2)=15$ (4) $3n-4(n-2)=15$

2) Create let statements for the following examples. Be sure to carefully read the question and figure out exactly what you are looking for. Then, set up an equation that summarizes the information in the problem and solve the equation and check for reasonableness.

(a) The sum of 3 less than 5 times a number and the number increased by 9 is 24. What is the number?

(b) Tom is 4 more than twice Andrew's age. Sara is 8 less than 5 times Andrews age. If Tom and Sara are ***twins***, how old is Andrew? *Think: What does it mean to be twins in regards to your ages?)

(c) A wireless phone plan costs Eric $35 for a month of service during which he sent 450 text messages. If he was charged a fixed fee of $12.50, how much did he pay per text?

3) There is a competition at the local movie theater for free movie tickets. You must guess all four employees' ages given a few clues. The first clue is that when added together, their ages total 106 years. Kirk is twice ten years less than the manager's age. Brian is 12 years younger than twice the manager's age. Matt is 7 years older than half the manager's age. What are all four of their ages? It may help to set up four let statements, one for each employee (including the manager).

In some cases, the answers you will get won't make physical sense or need a bit of interpreting. Look at the next example and be careful when you interpret your final solution.

4) Tanisha and Rebecca are signing up for new cellphone plans that only charge for the number of minutes and everything else is included in a monthly fee. Their plans are as follows:

Tanisha's plan: \$0.15 per minute used talking and a \$25 monthly fee.
Rebecca's Plan: \$0.10 per minute used talking and a \$18.50 monthly fee.

(a) Figure out how many minutes the two plans will charge the ***same amount***.

(b) Interpret your answer. It may help to read their two plans again and think about which one you would rather pay.

<u>Review Section:</u>

5)Express the product of $2x^2 + 7x - 10$ and $x + 5$ in standard form.

6) When solving the equation $4(3x^2 + 2) - 9 = 8x^2 + 7$, Emily wrote $4(3x^2 + 2) = 8x^2 + 16$ as her first step. Which property justifies Emily's first step?

(1) addition property of equality

(2) commutative property of addition

(3) multiplication property of equality

(4) distributive property of multiplication over addition

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Algebra I Consecutive Integers Word Problems 2A F

One of the ways we can practice our ability to work with algebraic expressions and equations is to play around with problems that involve **consecutive integers**. Make sure you know what the integers are:

THE INTEGERS AND CONSECUTIVE INTEGERS

The **integers** are the subset of the **real numbers**: $\{\ldots -4, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ (so positive and negative whole numbers).

Consecutive integers are any list of integers (however long) that are separated by only 1 unit. Such as:

$1, 2, 3$ or $5, 6, 7, 8$ or $-4, -3, -2$ or $-10, -9, -8, -7, -6$

Consecutive Evens	Consecutive Odds
$4, 6, 8$ or $-8, -6, -4, -2$ or $14, 16$	$7, 9, 11$ or $-5, -3, -1, 1$ or $-9, -7, -5$

Rules to follow for word problems:

1) Unknown starting point means that the first number is always equal to x.
2) CONSECUTIVE integers increase by (+1)
3) EVEN integers increase by (+2)
   ODD integers increase by (+2)

** This means that the let column will look the same for both even and odd consecutive integers!!!**

Let Statements:

Consecutive:	*Consecutive Even:*	*Consecutive Odd:*
n	n	n
n+1	n+2	n+2
n+2	n+4	n+4
n+3	n+6	n+6

Let's try one!

Exercise 1: Let's work with just two consecutive integers first. Say we have two consecutive integers whose sum is eleven less than three times the smaller integer.

(a) It is important to play around with this problem numerically. So, try a variety of combinations and see if you can find the correct pair of consecutive integers. Be sure to show your calculations.

(b) Now, carefully set up let statements that give expressions for our two consecutive integers. Using these expressions, set up an equation that allows you to find them and solve the equation.

Exercise 2: I'm thinking of three consecutive odd integers. When I add the larger two the result is nine less than three times the smallest of them. What are the three consecutive odd integers?

Exercise 3: Three consecutive even integers have the property that when the difference between the first and twice the second is found, the result is eight more than the third. Find the three consecutive even integers.

Exercise 4: The sum of four consecutive integers is -18. What are the four integers?

HOMEWORK:

1) Set up let statements for appropriate expressions and using these expressions, set up an equation that allows you to find each number described. Be sure to find EACH integer you are looking for.

(a) Find 4 consecutive even integers such that the sum of the 2^{nd} and 4^{th} is -132.

(b) Find two consecutive integers such that ten more than twice the smaller is seven less than three times the larger.

(c) Find two consecutive even integers such that their sum is equal to the difference of three times the larger and two times the smaller.

(d) Find three consecutive integers such that three times the largest increased by two is equal to five times the smallest increased by three times the middle integer.

(e) Find three consecutive off integers such that the sum of the smaller two is three times the largest increased by seven.

2) In an opera theater, sections of seating consisting of three rows are being laid out. It is planned so each row will be two more seats than the one before it and 90 people must be seated in each section. How many people will be in the third row?

3) Instead of finding even or odd consecutive integers, we could also look for integers that differ by a number other than 2. Find three numbers that each differ by 3 such that 5 times the largest integer is equal to three times the smallest increased by 5 times the middle. *(Hint: First is n, second is n+3, third is n+6)*

4) What do you think every other even integer means? Set up a let statement that would show this. *(Hint: List some numbers that would consist of every other even integer)*

5) Find three every other even integers such that the sum of all three is equal to three times the largest decreased by the other two numbers.

Review Section:

6) What is the value of $\frac{x^2-4y}{2}$, if $x = 4$ and $y = -3$?

7) Solve algebraically for x:

$$3(x+1) - 5x = 12 - (6x - 7)$$

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Algebra I

Solving Linear Equations with Unspecified Constants

2A G

At this point we should feel very competent in solving linear equations. In many situations, we might even solve equations when there are no actual numbers given. Let's take a look at what we means in Exercise 1.

Exercise 1: Solve each of the following problems for the value of x. In (b), write your answer in terms of the unspecified constants a, b, and c.

(a) $5x + 3 = 33$

(b) $ax + b = c$

The rules for solving linear equations (all equations) don't depend on whether the constants in the problem are specified or not. The biggest difference in #1 between (a) and (b) is that in (b) you have to leave the results of the intermediate calculation undone.

Exercise 2: Solve for y, in in terms of x: $3y + x = 15$

Exercise 3: Solve the following two equations. In letter (b), leave your answer in terms of the constants a, b, c, and d.

(a) $\frac{x+5}{2} - 7 = 3$

(b) $\frac{x+a}{b} - c = d$

Of course, we can have numbers with known (specified constants) thrown into the mix. The most important thing is to know when we can combine and produce a result and when we can't.

Exercise 4: When $2(x - h) + k = 8$ is solved for x in terms of h and k, its solution is which of the following? Show the algebraic manipulations you used to get your answer.

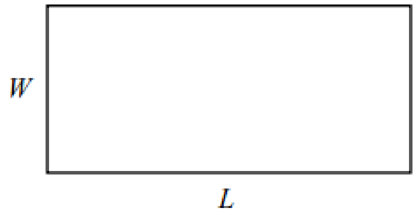
(1) $4 + h - k$ (3) $k - \frac{h}{2} + 8$

(2) $h + 4 - \frac{k}{2}$ (4) $4 - h + k$

Many times this technique is used when we want to rearrange a formula to solve for a quantity of interest.

Exercise 5: For a rectangle, the perimeter, P, can be found if the two dimensions of length, L, and width, W, are known.

(a) If a rectangle has a length of 12 inches and a width of 5 inches, what is the value of its perimeter? Include units.



(b) Write a formula for the perimeter, P, in terms of L and W.

(c) Rearrange this formula so that it "solves" for the length, L. Determine the value of L when P=20 and W=4.

There is one last complication we need to look at that is often challenging for students at all levels. Let's take a look at this in the next problem.

Exercise 6: Consider the equation $ax + b = cx + d$. We'd like to solve this equation for x. Let's start with the situation where we know the values of a, b, c, and d.

(a) Solve: $8x + 1 = 5x + 22$ (b) Now solve: $ax + b = cx + d$

Exercise 7: Which of the following solves the equation $ax - k = 3(x + h)$ for x in terms of a, k, and h. Show the manipulations to find your answer.

(1) $\frac{3h + k}{a - 3}$ (3) $\frac{k + 3h}{a + 3}$

(2) $\frac{3a + k}{h - 1}$ (4) $\frac{h + 3}{a + k}$

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Algebra I Solving Linear Equations with Unspecified Constants

HOMEWORK:

1) If $2a + 3r = 6b + r$, then what is the value of a in terms of b and r be expressed as?

2) The members of the senior class are planning a dance. They use the equation $r = pn$ to determine the total receipts. What is n expressed in terms of r and p?

3) If $3d = 7v + 5$, then what is the value of v in terms of d?

____ 4) Which of the following is equivalent to solving for a, using $7a - 8b = 10x$

(1) $a = \frac{18xb}{7}$ (2) $a = \frac{10x+8b}{7}$ (3) $a = \frac{10x-8b}{7}$

5) When $\frac{3(x-k)}{w} = 4$ is solved for x in terms of w and k, its solution is which of the following? Show the algebraic manipulations you used to get your answer.

(1) $\frac{4}{3}w+k$ (3) $k-\frac{4}{3}w$

(2) $k-\frac{3w}{4}$ (4) $\frac{4}{3}+w-k$

6) Solve the following equations for x. It may help to make up an equation with numbers and solve it to the side to make sure you are not making any mistakes.

(a) $a(x+b)-c=d$ (b) $\frac{e(x+c)}{b} = 2$

7) If $14a = 10 - 2h$, then what is the value of h in terms of a?

8) In physics the following formula relates your distance above the ground, d, relative to how long, t, and object has been in the air:

$$d = v_0 t + \frac{1}{2}at^2$$

(a) Solve the formula for a, the acceleration due to gravity.

(b) Using your manipulated equation, find the value of a if $d = 80$, $v_o = 50$, and $t = 8$.
Note: an acceleration towards the ground is negative.

9) When traveling abroad, many of the units used are different. One of the most common is the unit of temperature namely Fahrenheit verses Celsius. The conversion between the 2 temperatures is as follows.

$$C = \frac{5}{9}(F-32)$$

(a) Using the formula above, convert 50°F to Celsius.

(b) This conversion formula is very useful if you are given Fahrenheit, but less useful if you know a Celsius temperature. Solve the above equation for Fahrenheit, F, and then convert 50°C into Fahrenheit. Is there a large difference in Fahrenheit and Celsius?

Review Section:

10) Find four consecutive even integers that have a sum of 940.

11) Write three examples that are expressions and three examples that are equations.

Expressions: *Equations:*