

## CHAPTER

## 1

# Unit 1: Transformations, Congruence and Similarity

## Chapter Outline

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- 1.1 TRANSFORMATIONS IN THE COORDINATE PLANE
  - 1.2 RULES FOR TRANSLATIONS
  - 1.3 GRAPHS OF TRANSLATIONS
  - 1.4 RULES FOR REFLECTIONS
  - 1.5 GRAPHS OF REFLECTIONS
  - 1.6 RULES FOR DILATIONS
  - 1.7 RULES FOR ROTATIONS
  - 1.8 COMPOSITION OF TRANSFORMATIONS
  - 1.9 COMPOSITION OF TRANSFORMATIONS
  - 1.10 COMPOSITE TRANSFORMATIONS
  - 1.11 COMPLEMENTARY ANGLES
  - 1.12 VERTICAL ANGLES
  - 1.13 SIMILARITY AND CONGRUENCE
  - 1.14 CONGRUENT FIGURES
  - 1.15 SUPPLEMENTARY ANGLES
  - 1.16 VERTICAL ANGLES
  - 1.17 RECOGNIZING DILATIONS
  - 1.18 CORRESPONDING ANGLES
  - 1.19 CORRESPONDING ANGLES
  - 1.20 COMPLEMENTARY ANGLES
  - 1.21 DILATIONS
  - 1.22 DILATION
  - 1.23 RECOGNIZING SIMILARITY
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### KEY STANDARDS

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

**MCC8.G.1** Verify experimentally the properties of rotations, reflections, and translations:

- a. Lines are taken to lines, and line segments to line segments of the same length.
- b. Angles are taken to angles of the same measure.
- c. Parallel lines are taken to parallel lines.

**MCC8.G.2** Understand that a two<sup>[U+2010]</sup> dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

**MCC8.G.3** Describe the effect of dilations, translations, rotations, and reflections on two [U+2010] dimensional figures using coordinates.

**MCC8.G.4** Understand that a two [U+2010] dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two [U+2010] dimensional figures, describe a sequence that exhibits the similarity between them.

**MCC8.G.5** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle [U+2010] angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the three angles appear to form a line, and give an argument in terms of transversals why this is so.*

## 1.1 Transformations in the Coordinate Plane

### Introduction

#### *The Kings Chamber*



In one room of the museum was a Kings bedroom. The furniture in the room was large and wooden and old with great golden cloths. On the walls was a beautiful red, blue and gold pattern.

Jessica thought that the pattern was the most beautiful one that she had ever heard.

I love this, she said to Mrs. Gilman. I want to draw it, but Im not sure how.

Well, you could break it up into a coordinate grid since the pattern repeats itself and use what we have learned about transformations to draw it in.

How could I get started? Jessica asked.

Well, start by drawing the coordinate grid, then use these coordinates for one of the diamonds. See if you can figure it out from there.

In Jessicas notebook, Mrs. Gilman wrote down the following coordinates.

(4,1)      (5,2)      (5,0)      (6,1)

Jessica began to draw it in. Then she got stuck.

**This is where you come in. This lesson will teach you all about drawing transformations. Follow along closely and you can help Jessica draw in the diamonds at the end of the lesson in each quadrant.**

#### *What You Will Learn*

In this lesson you will learn how to complete the following activities and skills.

- Identify and describe transformations in the coordinate plane.
- Translate a figure in the coordinate plane using coordinate notation, and graph the resulting image.
- Reflect a figure in the coordinate plane using coordinate notation, and graph the resulting image.
- Rotate a figure in the coordinate plane using coordinate notation, and graph the resulting image.

#### *Teaching Time*

## I. Identify and Describe Transformations in the Coordinate Plane

In the last lesson you learned how to identify and perform different *transformations*. Remember that a **transformation is when we move a figure in some way, even though we don't change the figure at all**. This lesson will teach you how to identify and perform transformations in the coordinate plane.

The *coordinate plane* is a representation of two-dimensional space. It has a horizontal axis, called the *x-axis*, and a vertical axis, called the *y-axis*. We can graph and move geometric figures on the coordinate plane.

Do you remember the three types of transformations?

The first is a *translation* or slide. A translation **moves a figure up, down, to the right, to the left or diagonal without altering the figure**.

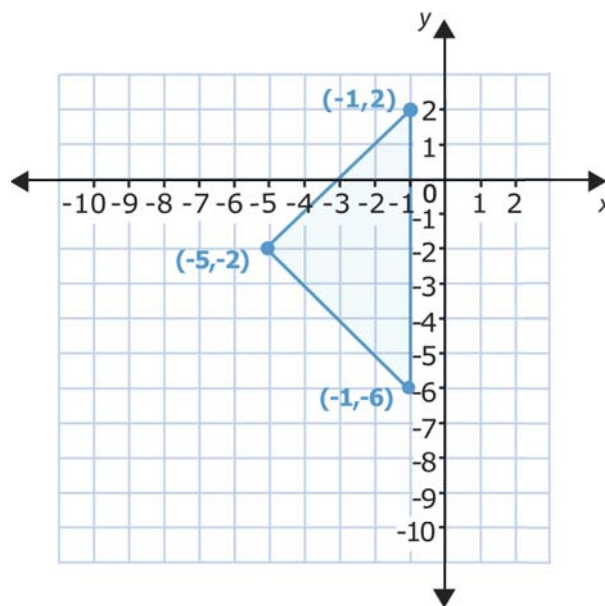
The second is a *reflection* or flip. A reflection **makes a mirror image of the figure over a line of symmetry**. The line of symmetry can be vertical or horizontal.

The third is a *rotation* or turn. A rotation **moves a figure in a circle either clockwise or counterclockwise**.

Now let's look at performing each type of transformation in the coordinate plane.

## II. Translate a Figure in the Coordinate Plane Using Coordinate Notation, and Graph the Resulting Image

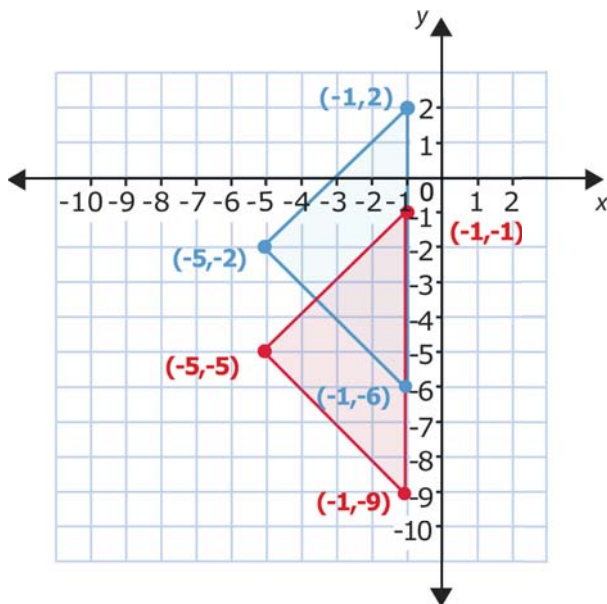
As we have said, when we perform *translations*, we slide a figure left or right, up or down. This means that **on the coordinate plane, the coordinates for the vertices of the figure will change**. Take a look at the example below.



Now let's look at performing a translation or slide of this figure.

We can choose the number places that we want to move the triangle and the direction that we wish to move it in. If we slide this triangle 3 places down, all of its vertices will shift 3 places down the *y-axis*. That means that the ordered pairs for the new vertices will change. Specifically, the *y*-coordinate in each pair will decrease by 3.

Let's see why this happens.



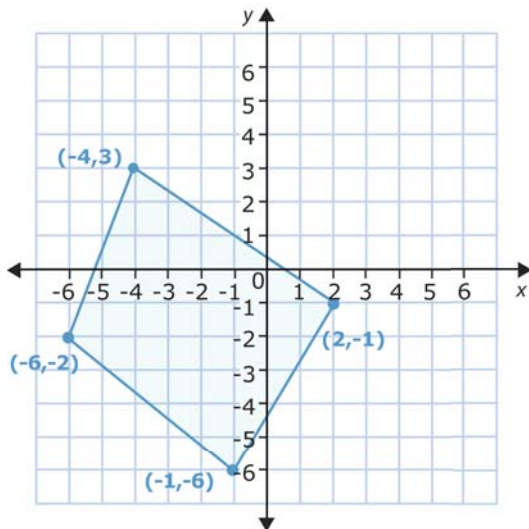
**We can see the change in all of the  $y$ -coordinates.** Compare the top points. The  $y$ -coordinate on the left is 2. The  $y$ -coordinate for the corresponding point in the triangle after it moves is -1. The  $y$ -coordinate decreased by 3. Now compare the left-hand point of each triangle. The  $y$ -coordinate originally is -2, and the  $y$ -coordinate after the translation is -5. Again, the difference shows a change of -3 in the  $y$ -coordinate. For the last point, the  $y$ -coordinate starts out as -6, and shifts to -9 after the downward slide. For each point, then, the  $y$ -coordinate decreases by 3 while the  $x$ -coordinates stay the same. **This means that we slid the triangle down 3 places.**

**We can translate figures in other ways, too.** As you might guess, we move figures right or left on the coordinate grid by their  $x$ -coordinates. We can also move figures diagonally by changing both their  $x$ - and  $y$ -coordinates. One way to recognize translations, then, is to compare their points. The  $x$ -coordinates will all change the same way, and the  $y$ -coordinates will all change the same way.

**To graph a translation, we perform the same change for each point.** Lets try graphing a translation.

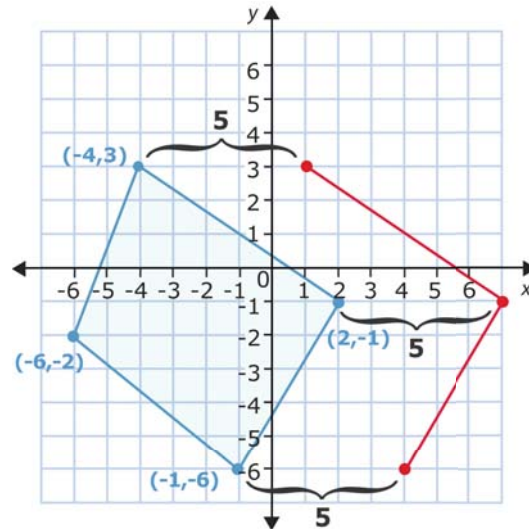
Example

Slide the following figure 5 places to the right.



**In this translation, we will move the figure to the right.** That means the  $x$ -coordinates for each point will

change but the  $y$ -coordinates will not. We simply count 5 places to the right from each point and make a new point.



Once we relocate each point 5 places to the right, we can connect them to make the new figure that shows the translation.

We can check to see if we performed the translation correctly by adding 5 to each  $x$ -coordinate (because we moved to the right) and then checking these against the ordered pairs of the figure you drew. This is called *coordinate notation*. Notice that each point is represented by coordinates.

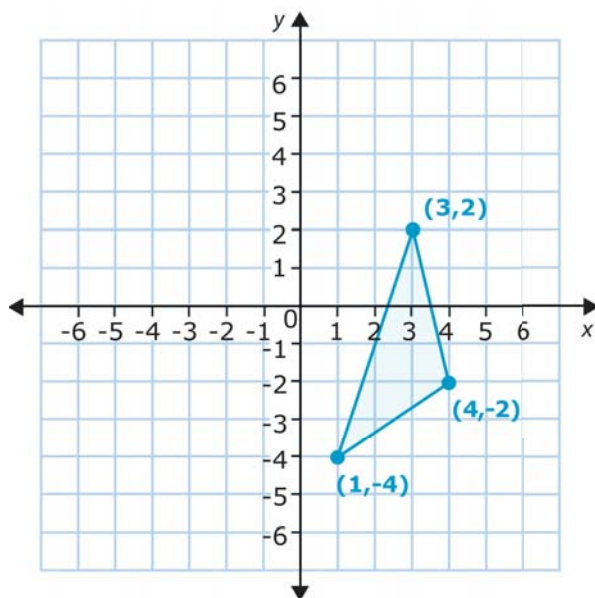
$$\begin{array}{cccc}
 (-4, 3) & (-6, -2) & (-1, -6) & (2, -1) \\
 + 5 & + 5 & + 5 & + 5 \\
 (1, 3) & (-1, -2) & (4, -6) & (7, -1)
 \end{array}$$

These are the points we graphed, so we have performed the translation correctly.

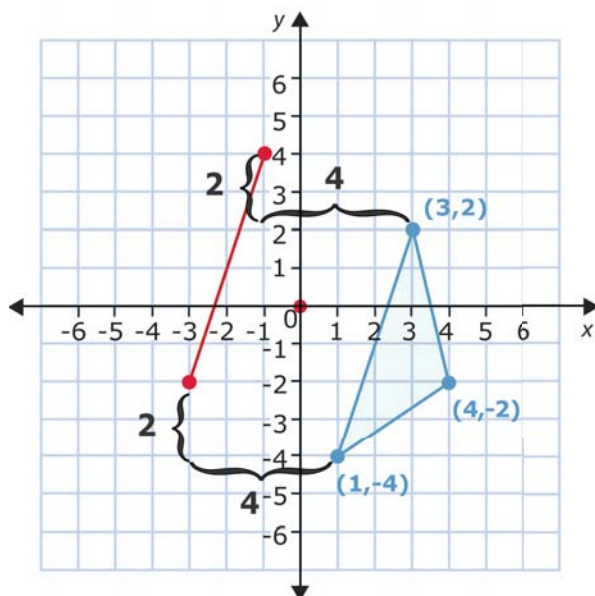
Lets try another.

Example

Slide the following figure 4 places to the left and 2 places up.



**This time we need to perform two movements, both left and up. That means we will change both the  $x$ - and  $y$ -coordinates of the ordered pairs.** We graph each point by counting 4 places to the left first, and from there 2 places up (2 places up from where you started, not 2 places up from the  $y$ -axis!). Make a mark and repeat this process for each point. Then connect the new points.



Again, we can check that we performed the translation correctly by changing the  $x$ - and  $y$ -coordinates in the ordered pairs and then comparing these to the points we graphed. This time we subtract 4 from each  $x$ -coordinate (because we moved left; imagine a number line) and add 2 to each  $y$ -coordinate. Lets see what happens.

$$\begin{array}{rcc}
 (3, 2) & (4, -2) & (1, -4) \\
 -4 + 2 & -4 + 2 & -4 + 2 \\
 (-1, 4) & (0, 0) & (-3, -2)
 \end{array}$$

**These are the points we graphed, so we performed the translation correctly.**

**80. Lesson Exercises**

Use coordinate notation to write the coordinates of each translated triangle. The vertices of the original figure have been given to you.

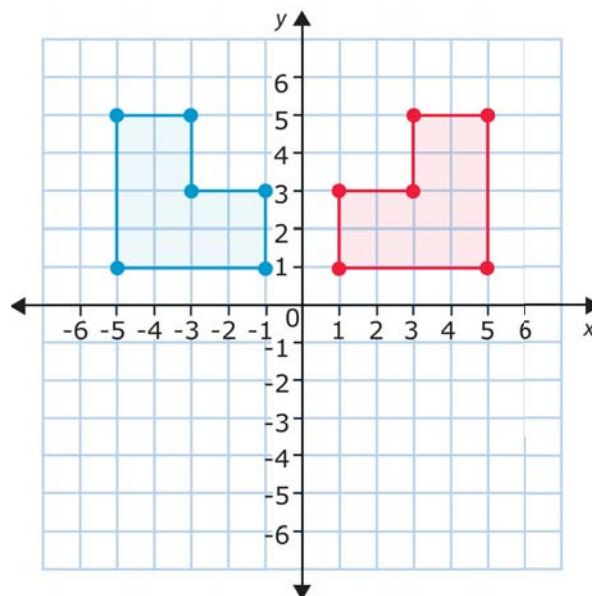
1. Triangle  $ABC$   $(0, 1)(1, 3)(4, 0)$  translate this figure up 4.
2. Triangle  $DEF$   $(-3, 2)(1, 6)(2, 1)$



Take a few minutes to check your work with a neighbor. Be sure that your answers are accurate. Correct any errors before continuing.

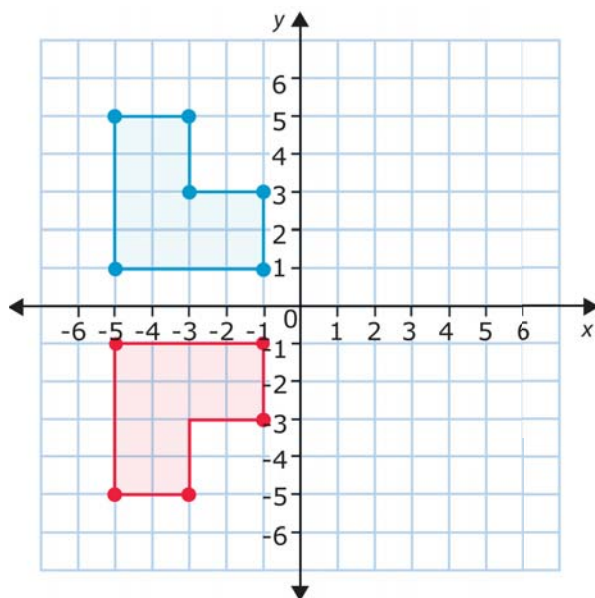
**III. Reflect a Figure in the Coordinate Plane Using Coordinate Notation, and Graph the Resulting Image**

We can also identify a reflection by the changes in its coordinates. Recall that in a *reflection*, the figure flips across a line to make a mirror image of itself. Take a look at the reflection below.



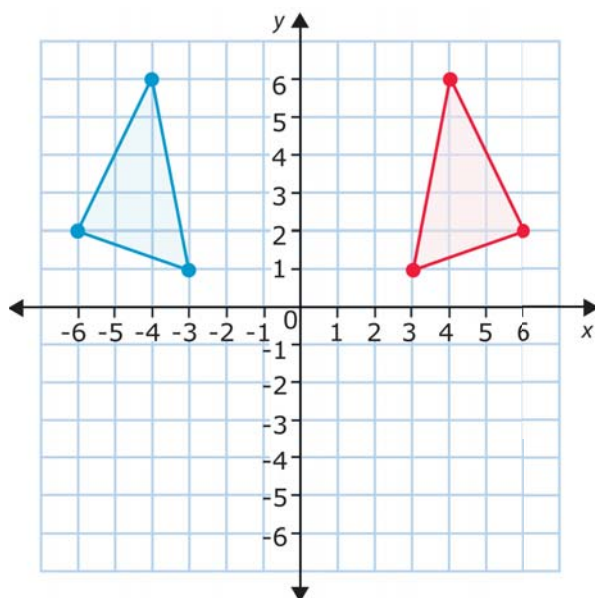
We usually reflect a figure across either the  $x$ - or the  $y$ -axis. In this case, we reflected the figure across the  $x$ -axis. If we compare the figures in the first example vertex by vertex, we see that the  $x$ -coordinates change but the  $y$ -coordinates stay the same. This is because the reflection happens from left to right across the  $x$ -axis. When we reflect across the  $y$ -axis, the  $y$ -coordinates change and the  $x$ -coordinates stay the same. Take a look at this example.





**Now lets compare some of the vertices.** In the figure above the coordinates for the upper-left vertex of the original figure are  $(-5, 5)$ . After we reflect it across the  $y$ -axis, the coordinates for the corresponding vertex are  $(-5, -5)$ . How about the lower-right vertex? It starts out at  $(-1, 1)$ , and after the flip it is at  $(-1, -1)$ . As you can see, the  $x$ -coordinates stay the same while the  $y$ -coordinates change. In fact, the  $y$ -coordinates all become the opposite integers of the original  $y$ -coordinates. **This indicates that this is a vertical (up/down) reflection or we could say a reflection over the  $x$ -axis.**

In a horizontal (left/right) reflection or a reflection over the  $y$ -axis, the  $x$ -coordinates would become integer opposites. Lets see how.



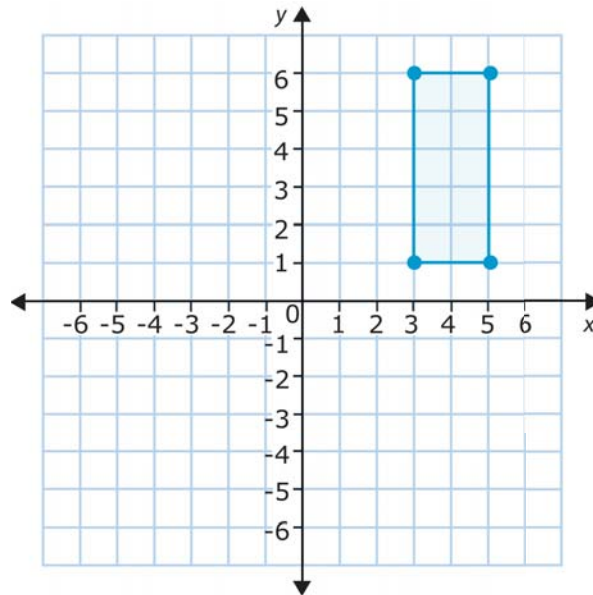
This is a reflection across the  $x$ -axis. Compare the points. Notice that the  $y$ -coordinates stay the same. The  $x$ -coordinates become the integer opposites of the original  $x$ -coordinates. Look at the top point of the triangle, for example. The coordinates of the original point are  $(-4, 6)$ , and the coordinates of the new point are  $(4, 6)$ . The  $x$ -coordinate has switched from  $-4$  to  $4$ .

We can recognize reflections by these changes to the  $x$ - and  $y$ -coordinates. If we reflect across the  $x$ -axis, the  $x$ -coordinates will become opposite. If we reflect across the  $y$ -axis, the  $y$ -coordinates will become opposite.

We can also use this information to graph reflections. To graph a reflection, we need to decide whether the reflection will be across the  $x$ -axis or the  $y$ -axis, and then change either the  $x$ - or  $y$ -coordinates. Lets give it a try.

Example

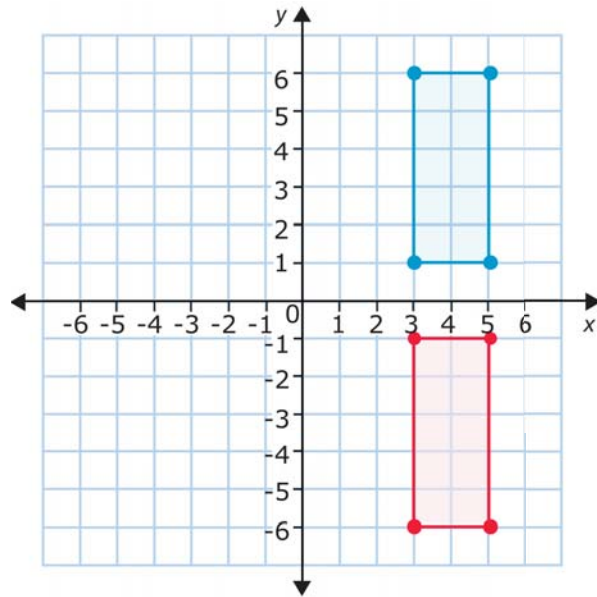
Draw a reflection of the figure below across the  $y$ -axis.



**We need to reflect the rectangle across the  $y$ -axis, so the flip will move the rectangle down. Because the reflection is across the  $y$ -axis, we'll need to change the coordinates (which determine where points are up and down). Specifically, we need to change them to their integer opposites. An integer is the same number with the opposite sign. This gives us the new points.**

$$\begin{array}{cccc} (3,6) & (5,6) & (3,1) & (5,1) \\ (3,-6) & (5,-6) & (3,-1) & (5,-1) \end{array}$$

Now we graph the new points. Remember to move right or left according to the  $x$ -coordinate and up or down according to the  $y$ -coordinate.



Here is the completed reflection. Lets practice with coordinate notation.

### 8P. Lesson Exercises

Write each set of coordinates to show a reflection in the  $y$ -axis.

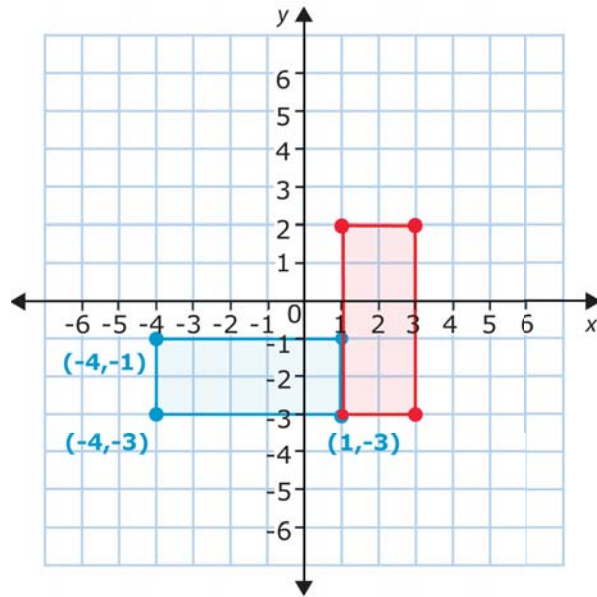
1.  $(-3, 1)$   $(0, 3)$   $(1, 2)$
2.  $(-3, 6)$   $(-2, 3)$   $(2, 3)$   $(3, 6)$



*Take a few minutes to check your work.*

### IV. Rotate a Figure in the Coordinate Plane Using Coordinate Notation, and Graph the Resulting Image

Now lets look at the third kind of transformation: rotations. A *rotation* is a transformation that turns the figure in either a clockwise or counterclockwise direction. The figure below has been rotated. What are its new coordinates?



The new coordinates of the rectangles vertices are  $(1, -3)$ ,  $(1, 2)$ ,  $(3, 2)$ , and  $(3, -3)$ . As you can see, both the  $x$ - and  $y$ -coordinates changed. **Unlike a translation or reflection, a rotation can change both of the coordinates in an ordered pair.** Now look closely. **One of the points remained exactly the same!** We say that we rotated the figure about this point. Imagine you put your finger on this corner of the rectangle and spun it. That's what happened in the rotation. **The rectangle has been rotated  $90^\circ$  clockwise.**

**How do we graph a rotation?**

**When we graph a rotation, we first need to know how much the figure will be rotated. Rotating the rectangle above  $90^\circ$  stands it up on end. Rotating it  $180^\circ$  would make it flat again. We also need to know which point we will rotate it around. This is the point that stays the same.**

Next, we need to count how many units long and wide the figure is. The figure above stretches from 1 on the  $x$ -axis to  $-4$  on the  $x$ -axis. This is a total of 5 units along the  $x$ -axis. When we rotate a figure  $90^\circ$ , the distance on the  $x$ -axis becomes the distance on the  $y$ -axis. Look at the rectangle. The long sides are horizontal at first, but after we rotate it, they become the vertical sides. This means that the  $x$ -distance of 5 will become a  $y$ -distance of 5.

Now, remember the point  $(1, -3)$  stays the same, so it is one corner of the rotated figure. **We add 5 to the  $y$ -coordinate to find the next vertex of the rectangle.  $-3 + 5 = 2$ . This puts a vertex at  $(1, 2)$ .**

To find the other points of the rotated rectangle, we need to think about its width. Find the width, or short side, of the original rectangle by counting the units between vertices along the  $y$ -axis. The rectangle covers 2 units on the  $y$ -axis. As you might guess, this becomes the  $x$ -distance in the rotated figure. In other words, we add 2 to the  $x$ -coordinate of the point that stays the same.  $1 + 2 = 3$ , so **another vertex of the rectangle will be  $(3, -3)$ . To find the fourth and final vertex, add 2 to the  $x$ -coordinate of the other ordered pair we know,  $(1, 2)$ . This puts the last vertex at  $(3, 2)$ .**

**Lets go back to the problem in the introduction and use what we have learned to figure it out!**

### Real Life Example Completed

#### *The Kings Chamber*



**Here is the original problem once again. Reread it before working on the drawing.**

In one room of the museum was a Kings bedroom. The furniture in the room was large and wooden and old with great golden cloths. On the walls was a beautiful red, blue and gold pattern.

Jessica thought that the pattern was the most beautiful one that she had ever heard.

I love this, she said to Mrs. Gilman. I want to draw it, but Im not sure how.

Well, you could break it up into a coordinate grid since the pattern repeats itself and use what we have learned about transformations to draw it in.

How could I get started? Jessica asked.

Well, start by drawing the coordinate grid, then use these coordinates for one of the diamonds. See if you can figure it out from there.

In Jessicas notebook, Mrs. Gilman wrote down the following coordinates.

(4, 1)  
 (5, 2)  
 (5, 0)  
 (6, 1)

Jessica began to draw it in. Then she got stuck.

**Now we can draw this diamond in on a coordinate grid. It belongs in Quadrant one. Now we want to draw a diamond into each of the other three quadrants. We can draw this, but we can also use mathematics to figure out the coordinates for each of the other diamonds first.**

**The diamond in the second quadrant is reflected over the  $y$ -axis. Therefore, the  $x$ -coordinate is going to change and become negative in each of the four vertices of the diamond. Here are the coordinates.**

(-4, 1)  
 (-5, 0)  
 (-5, 2)  
 (-6, 1)

**Next, we can reflect the original diamond in the first quadrant over the  $x$ -axis into the fourth quadrant. Here the  $y$ -coordinates will be negative.**

(4, -1)  
 (5, 0)  
 (5, -2)  
 (6, -1)

**Finally we can reflect this diamond over the  $y$ -axis into the third quadrant. Notice that here the  $x$  and  $y$ coordinates will both be negative.**

$(-4, -1)$   
 $(-5, 0)$   
 $(-5, -2)$   
 $(-6, -1)$

**Did you notice any patterns? Take a minute and create this pattern of diamonds in a coordinate grid. Then you will have an even deeper understanding of how a pattern like this one is created.**

**If you wanted to add in the gold  $X$  that crosses through the original pattern could you do it? Explain your thinking with a friend and then add in the  $X$  to the coordinate grid with the diamonds.**

## Vocabulary

Here are the vocabulary words found in this lesson.

### Transformation

a figure that is moved in the coordinate grid is called a transformation.

### Coordinate Plane

a representation of a two-dimensional plane using an  $x$  axis and a  $y$  axis.

### $x$ -axis

the horizontal line in a coordinate plane.

### $y$ -axis

the vertical line in a coordinate plane.

### Translation

a slide. A figure is moved up, down, left or right.

### Reflection

a flip. A figure can be flipped over the  $x$ -axis or the  $y$ -axis.

### Rotation

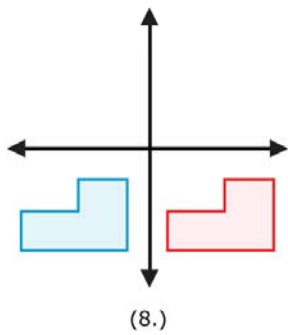
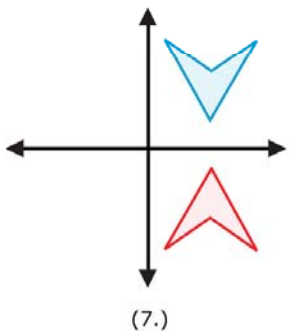
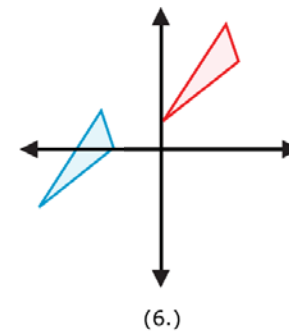
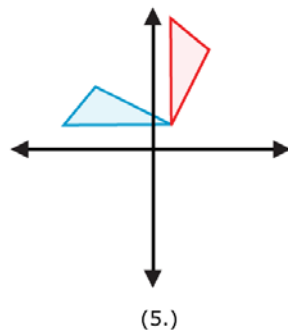
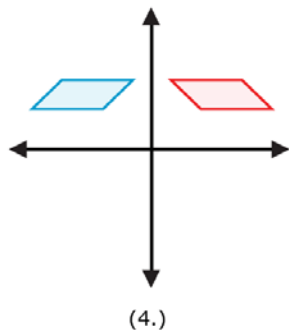
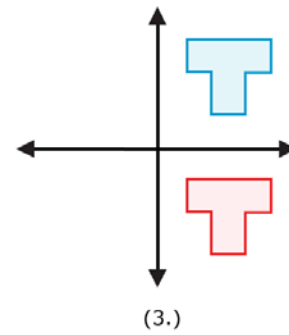
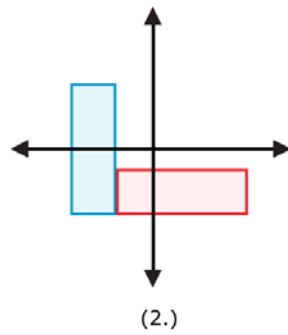
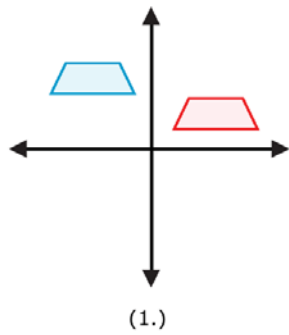
a turn. A figure can be turned clockwise or counterclockwise.

### Coordinate Notation

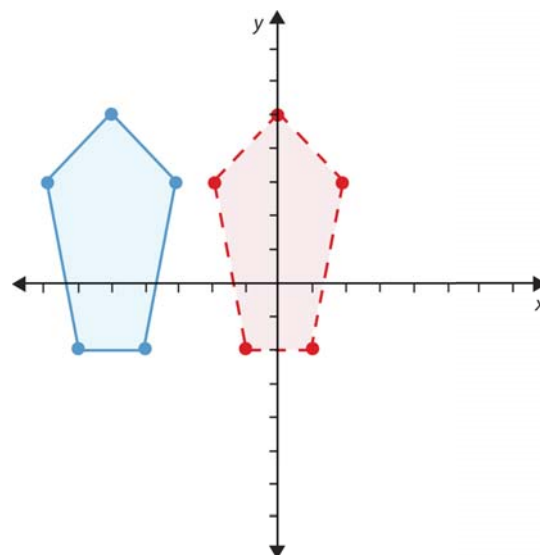
notation that shows where the figure is located in the coordinate plane. The vertices of the figure are represented using ordered pairs.

## Time to Practice

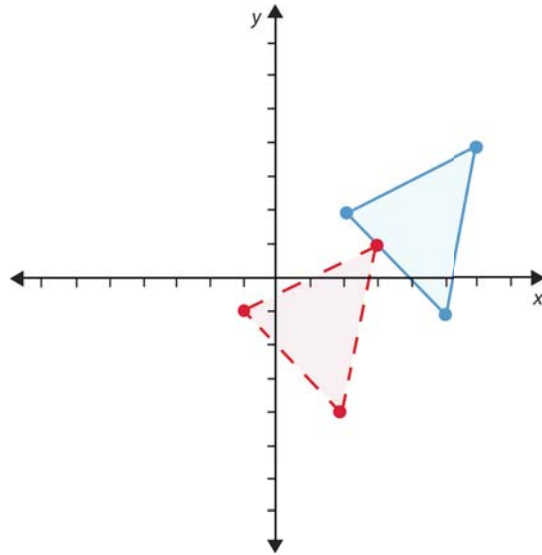
Directions: Identify the transformations shown below as a translation, reflection, or rotation.



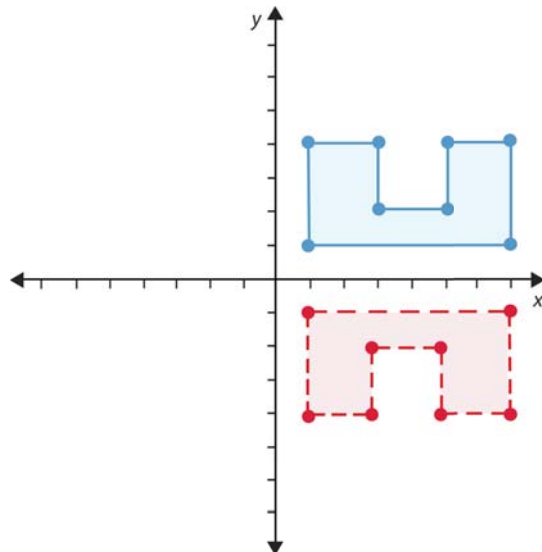
9. True or false. This figure has been translated 5 places to the right.



10. True or false. This is a picture of a reflection.

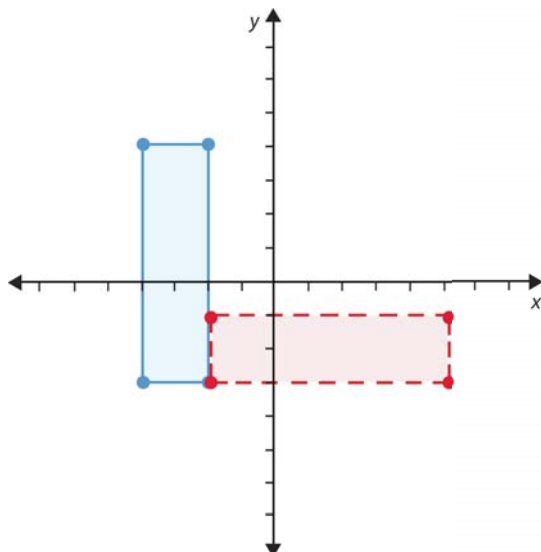


11. True or false. The figure below is an image of a reflection.



12. True or false. This figure has been rotated  $180^\circ$ .



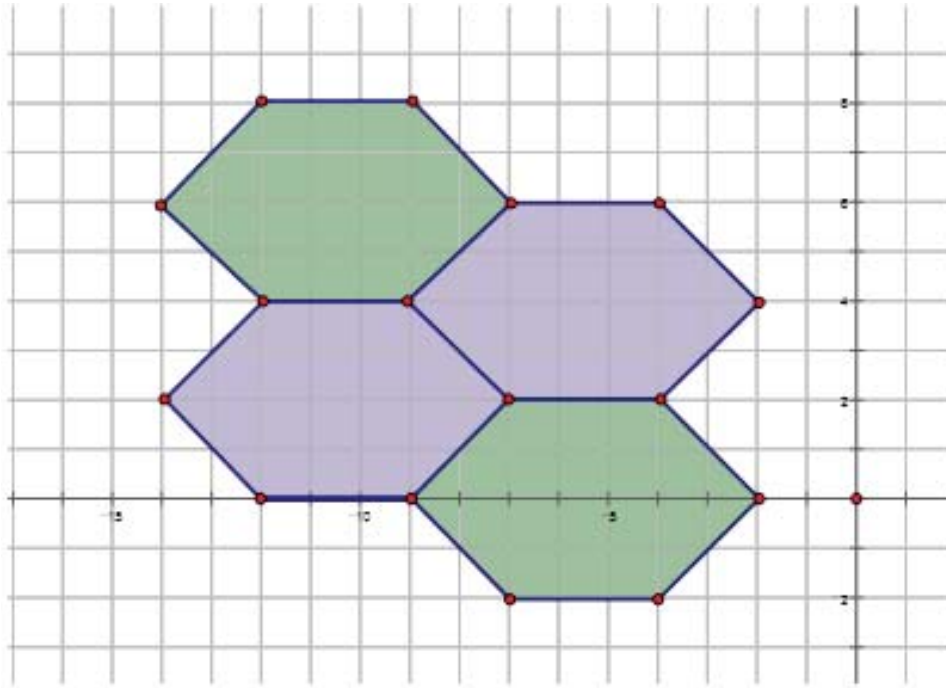


Directions: Independent work. Have the students work to draw five different figures in the coordinate plane. Then with a partner have them switch papers and ask them to create a rotation, translation and a reflection of each.

## 1.2 Rules for Translations

Here you will learn the different notation used for translations.

The figure below shows a pattern of a floor tile. Write the mapping rule for the translation of the two blue floor tiles.



### Watch This

First watch this video to learn about writing rules for translations.



MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10RulesforTranslationsA](#)

Then watch this video to see some examples.



MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10RulesforTranslationsB](#)

## Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape (called the preimage) to create a new shape (called the image). A translation is a type of transformation that moves each point in a figure the same distance in the same direction. Translations are often referred to as slides. You can describe a translation using words like "moved up 3 and over 5 to the left" or with notation. There are two types of notation to know.

1. One notation looks like  $T_{(3, 5)}$ . This notation tells you to add 3 to the  $x$  values and add 5 to the  $y$  values.
2. The second notation is a mapping rule of the form  $(x, y) \rightarrow (x - 7, y + 5)$ . This notation tells you that the  $x$  and  $y$  coordinates are translated to  $x - 7$  and  $y + 5$ .

The mapping rule notation is the most common.

### Example A

Sarah describes a translation as point  $P$  moving from  $P(-2, 2)$  to  $P'(1, -1)$ . Write the mapping rule to describe this translation for Sarah.

**Solution:** In general,  $P(x, y) \rightarrow P'(x + a, y + b)$ .

In this case,  $P(-2, 2) \rightarrow P'(-2 + a, 2 + b)$  or  $P(-2, 2) \rightarrow P'(1, -1)$

Therefore:

$$\begin{array}{rcl} -2 + a = 1 & \text{and} & 2 + b = -1 \\ a = 3 & & b = -3 \end{array}$$

The rule is:

$$(x, y) \rightarrow (x + 3, y - 3)$$

### Example B

Mikah describes a translation as point  $D$  in a diagram moving from  $D(1, -5)$  to  $D'(-3, 1)$ . Write the mapping rule to describe this translation for Mikah.

**Solution:** In general,  $P(x, y) \rightarrow P'(x + a, y + b)$ .

In this case,  $D(1, -5) \rightarrow D'(1 + a, -5 + b)$  or  $D(1, -5) \rightarrow D'(-3, 1)$

Therefore:

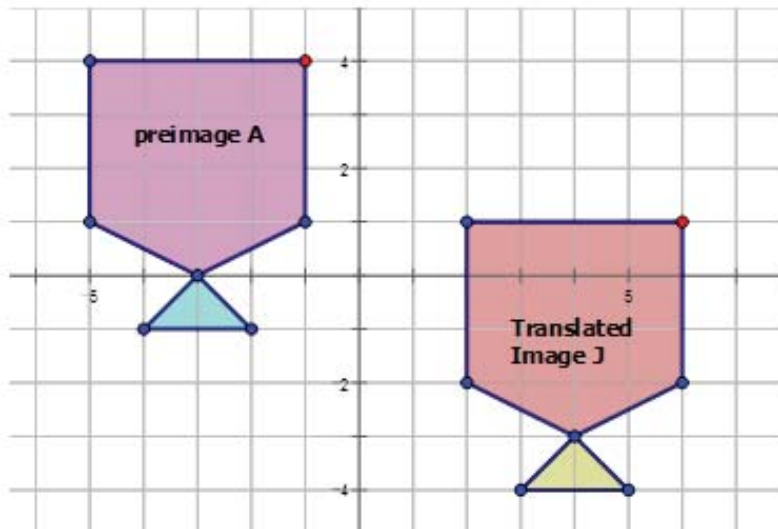
$$\begin{array}{rcl} 1 + a = -3 & \text{and} & -5 + b = 1 \\ a = -4 & & b = 6 \end{array}$$

The rule is:

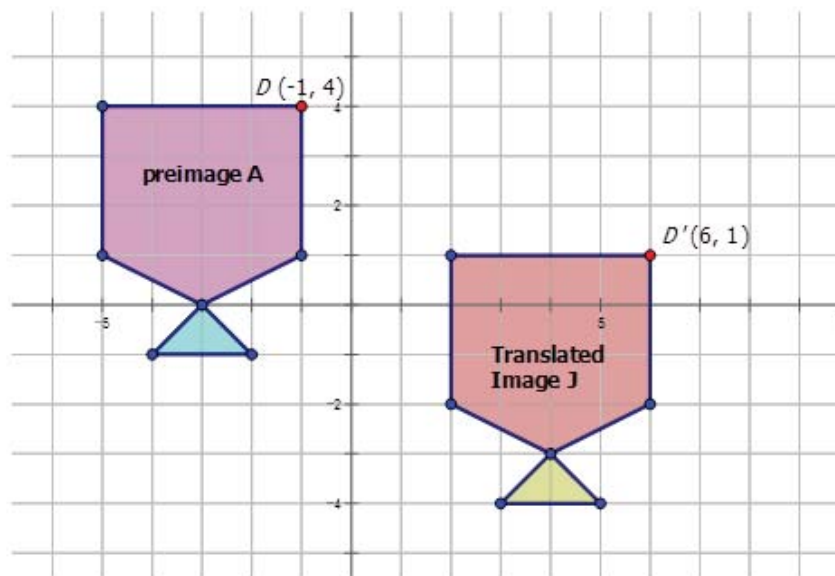
$$(x, y) \rightarrow (x - 4, y + 6)$$

### Example C

Write the mapping rule that represents the translation of the preimage  $A$  to the translated image  $J$  in the diagram below.



**Solution:** First, pick a point in the diagram to use to see how it is translated.



$$D : (-1, 4) \quad D' : (6, 1)$$

$$D(x, y) \rightarrow D'(x + a, y + b)$$

$$\text{So: } D(-1, 4) \rightarrow D'(-1 + a, 4 + b) \text{ or } D(-1, 4) \rightarrow D'(6, 1)$$

Therefore:

$$\begin{array}{l} -1 + a = 6 \quad \text{and} \quad 4 + b = 1 \\ a = 7 \quad \quad \quad b = -3 \end{array}$$

The rule is:

$$(x, y) \rightarrow (x + 7, y - 3)$$

## Vocabulary

### Mapping Rule

A **mapping rule** has the following form  $(x, y) \rightarrow (x - 7, y + 5)$  and tells you that the  $x$  and  $y$  coordinates are translated to  $x - 7$  and  $y + 5$ .

### Translation

A **translation** is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides**.

### Image

In a transformation, the final figure is called the **image**.

### Preimage

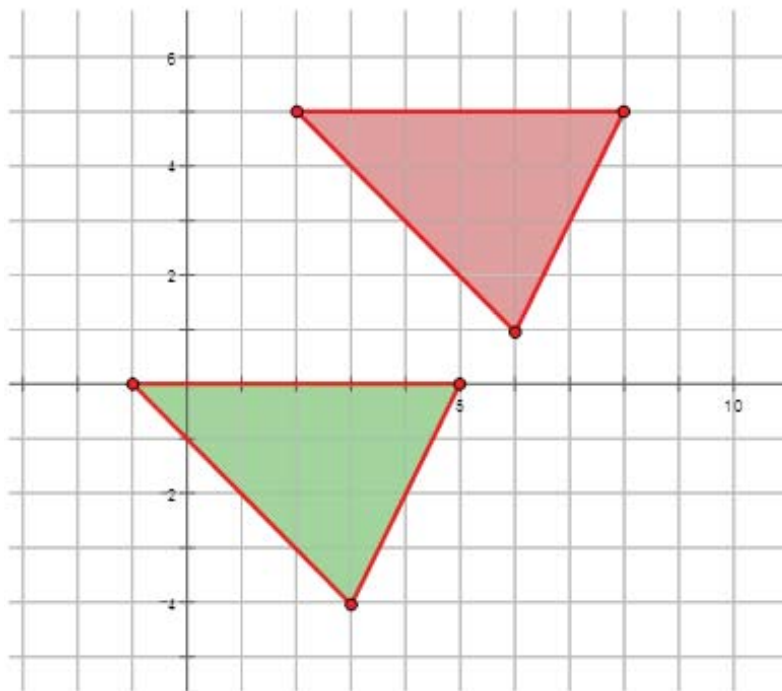
In a transformation, the original figure is called the **preimage**.

### Transformation

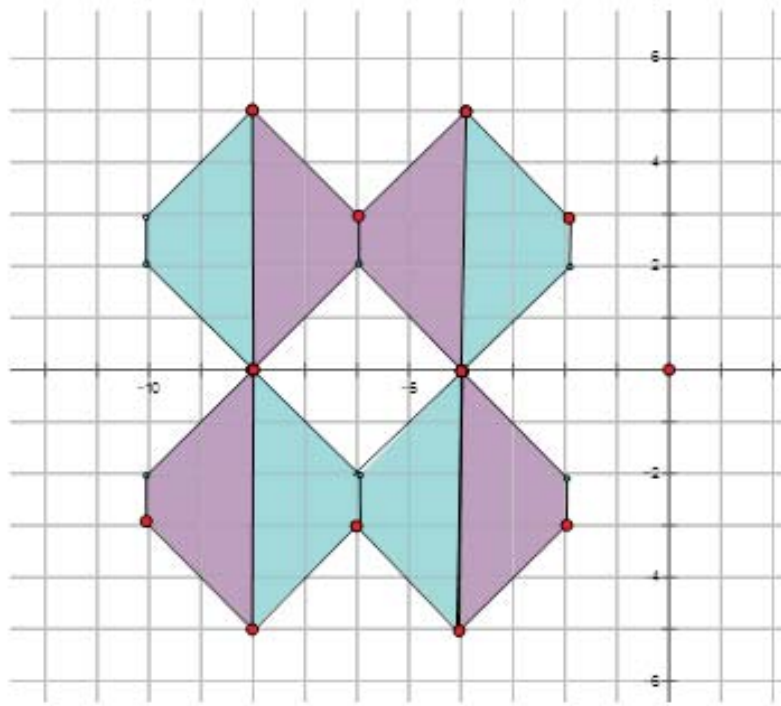
A **transformation** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

## Guided Practice

- Jack describes a translation as point  $J$  moving from  $J(-2, 6)$  to  $J'(4, 9)$ . Write the mapping rule to describe this translation for Jack.
- Write the mapping rule that represents the translation of the red triangle to the translated green triangle in the diagram below.



- The following pattern is part of wallpaper found in a hotel lobby. Write the mapping rule that represents the translation of one blue trapezoid to a translated blue trapezoid shown in the diagram below.

**Answers:**

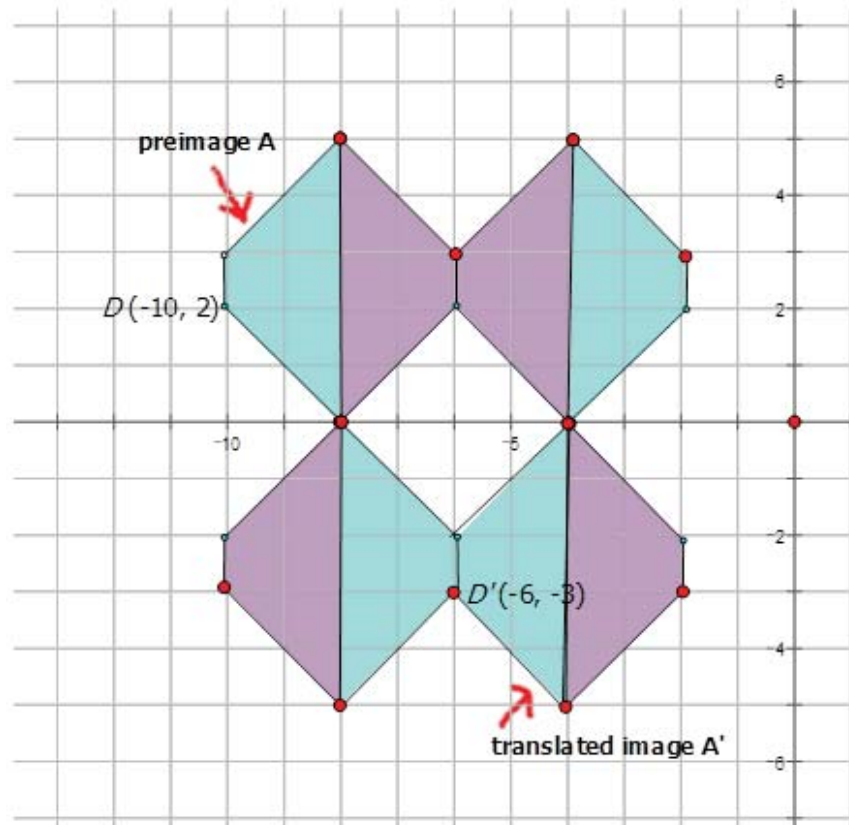
1.

$$(x, y) \rightarrow (x + 6, y + 3)$$

2.

$$(x, y) \rightarrow (x - 3, y - 5)$$

3. If you look closely at the diagram below, there two pairs of trapezoids that are translations of each other. Therefore you can choose one blue trapezoid that is a translation of the other and pick a point to find out how much the shape has moved to get to the translated position.



For those two trapezoids:

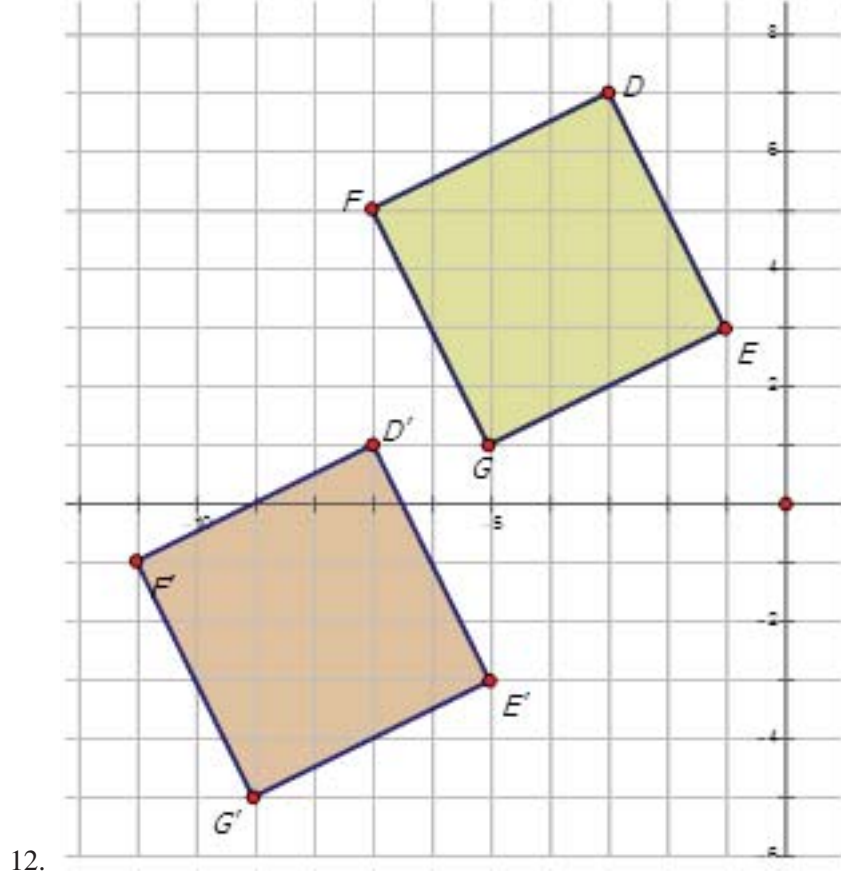
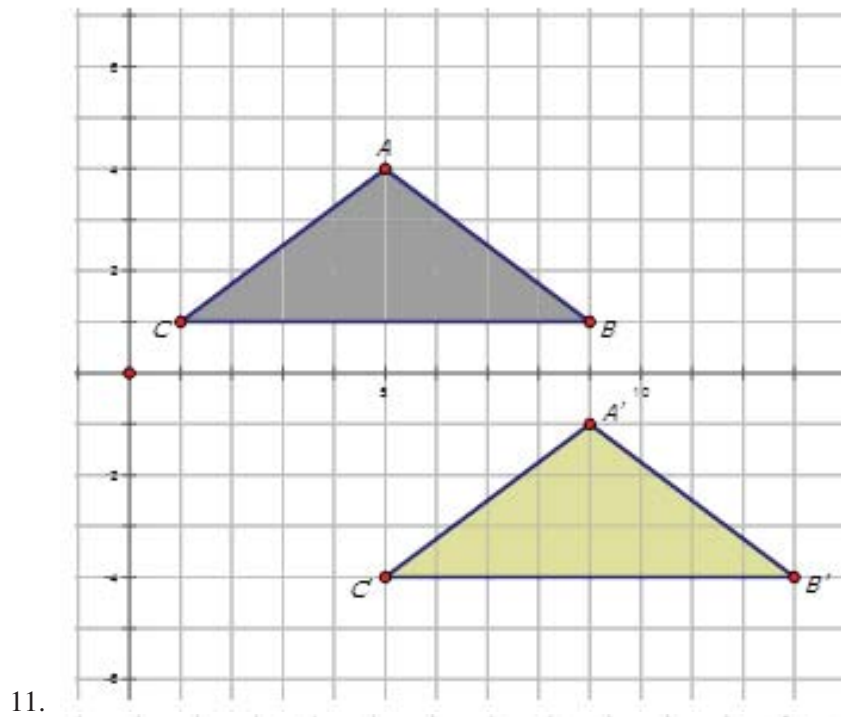
$$(x, y) \rightarrow (x + 4, y - 5)$$

### Practice

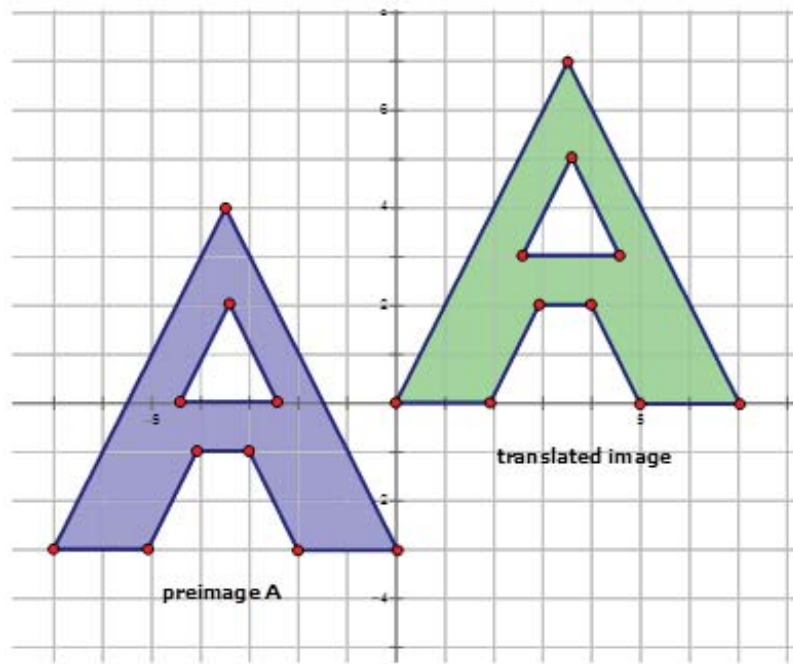
Write the mapping rule to describe the movement of the points in each of the translations below.

1.  $S(1, 5) \rightarrow S'(2, 7)$
2.  $W(-5, -1) \rightarrow W'(-3, 1)$
3.  $Q(2, -5) \rightarrow Q'(-6, 3)$
4.  $M(4, 3) \rightarrow M'(-2, 9)$
5.  $B(-4, -2) \rightarrow B'(2, -2)$
6.  $A(2, 4) \rightarrow A'(2, 6)$
7.  $C(-5, -3) \rightarrow C'(-3, 4)$
8.  $D(4, -1) \rightarrow D'(-4, 2)$
9.  $Z(7, 2) \rightarrow Z'(-3, 6)$
10.  $L(-3, -2) \rightarrow L'(4, -1)$

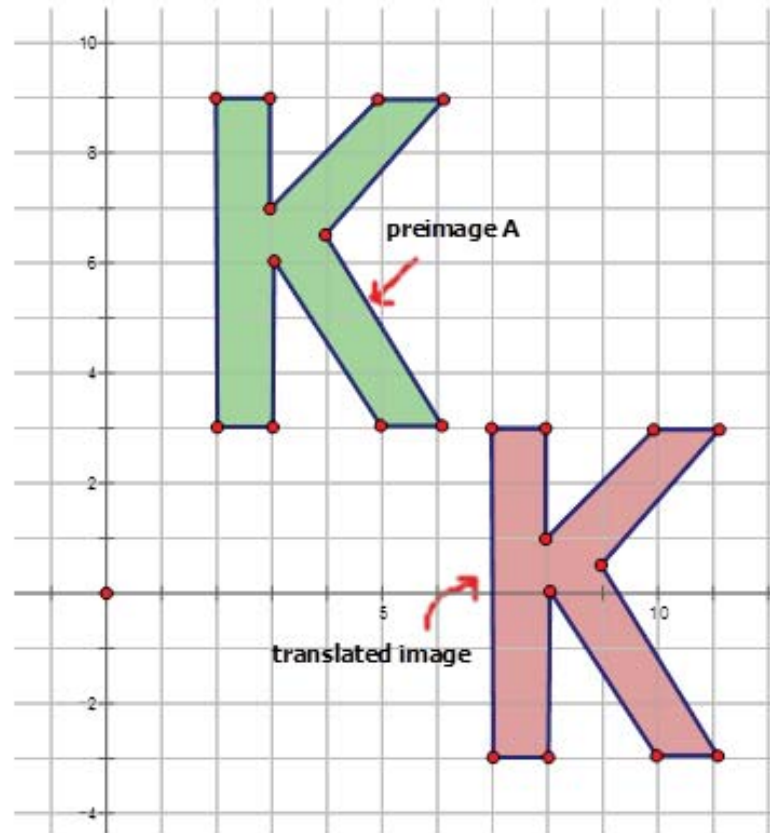
Write the mapping rule that represents the translation of the preimage to the image for each diagram below.



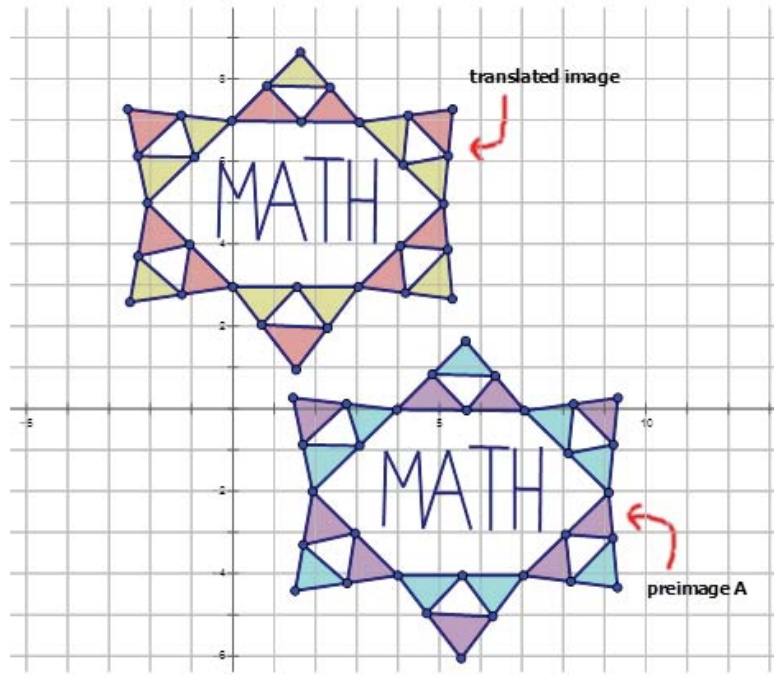




13.



14.



15.

## 1.3 Graphs of Translations

Here you will learn how to graph a translation given a description of the translation.

Triangle  $ABC$  has coordinates  $A(1, 1)$ ,  $B(8, 1)$  and  $C(5, 8)$ . Draw the triangle on the Cartesian plane. Translate the triangle up 4 units and over 2 units to the right. State the coordinates of the resulting image.

### Watch This

First watch this video to learn about graphs of translations.



MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10GraphsofTranslationsA](#)

Then watch this video to see some examples.



MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10GraphsofTranslationsB](#)

### Guidance

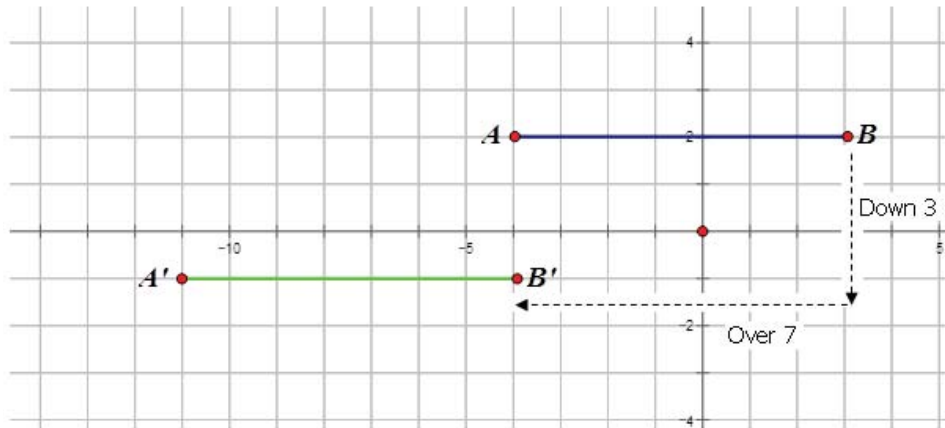
In geometry, a transformation is an operation that moves, flips, or changes a shape (called the preimage) to create a new shape (called the image). A translation is a type of transformation that moves each point in a figure the same distance in the same direction. Translations are often referred to as slides. When you perform a translation on a shape, the coordinates of that shape will change:

- translating up means you will add the translated unit to the  $y$  coordinate of the  $(x, y)$  points in the preimage
- translating down means you will subtract the translated unit from the  $y$  coordinate of the  $(x, y)$  points in the preimage
- translating right means you will add the translated unit to the  $x$  coordinate of the  $(x, y)$  points in the preimage
- translating left means you will subtract the translated unit from the  $x$  coordinate of the  $(x, y)$  points in the preimage

### Example A

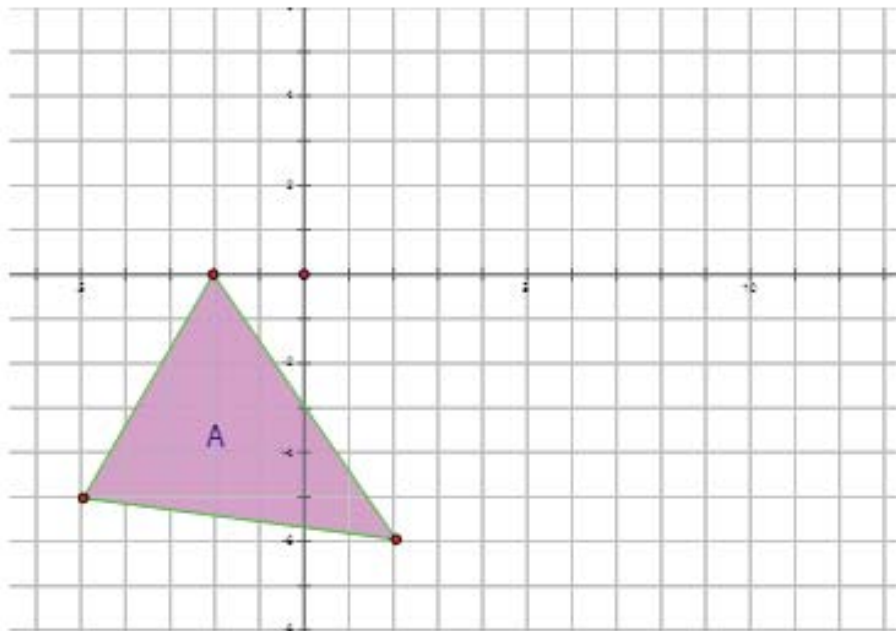
Line  $\overline{AB}$  drawn from  $(-4, 2)$  to  $(3, 2)$  has been translated 3 units down and 7 units to the left. Draw the preimage and image and properly label each.

**Solution:**

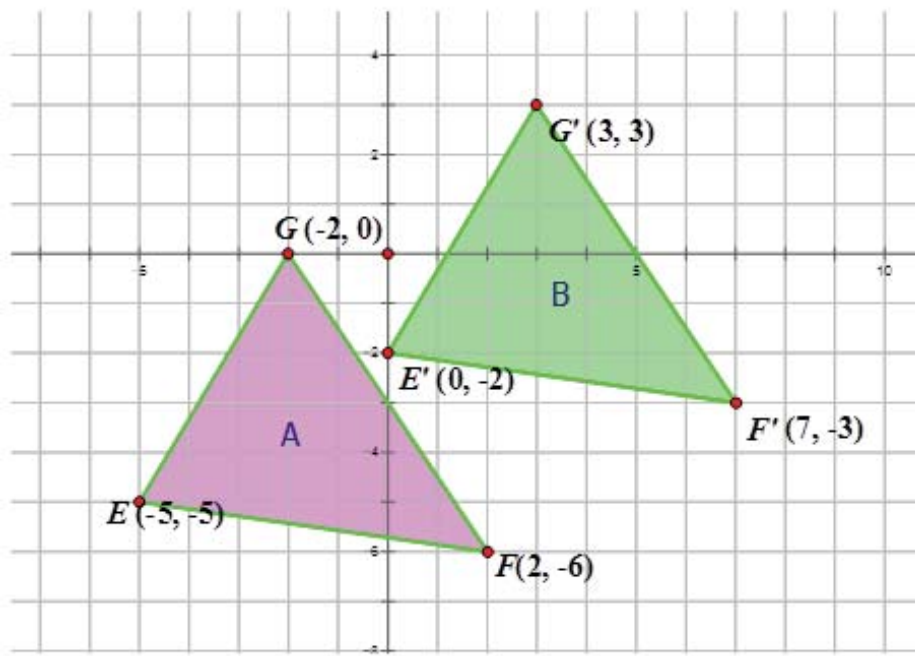


**Example B**

Triangle  $A$  is translated 3 units up and 5 units to the right to make triangle  $B$ . Find the coordinates of triangle  $B$ . On the diagram, draw and label triangle  $B$ .

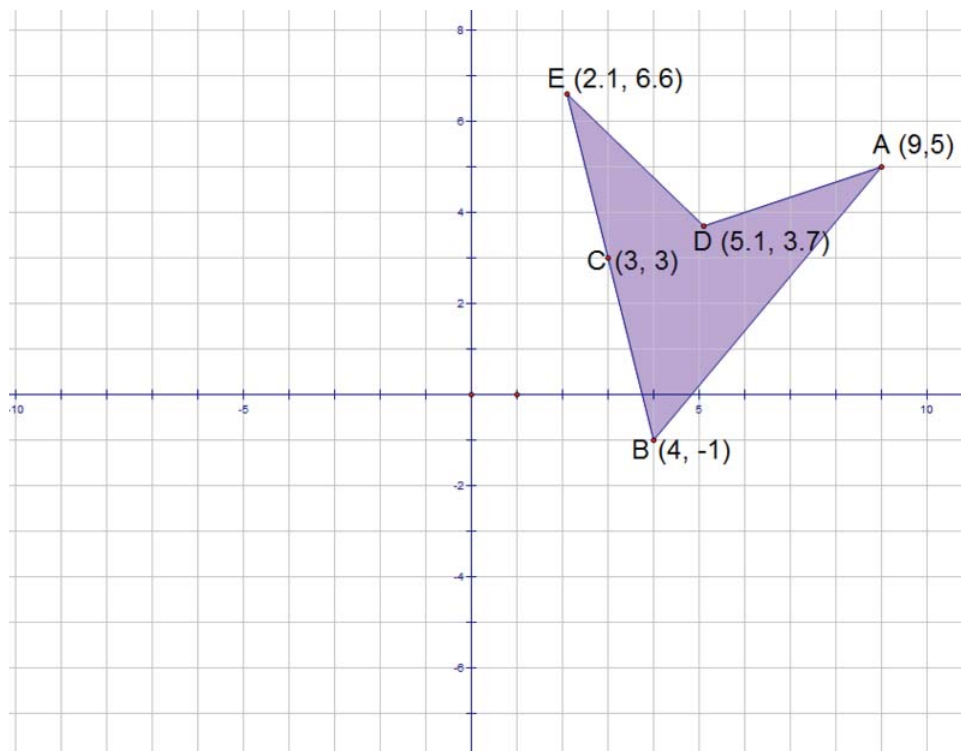


**Solution:**

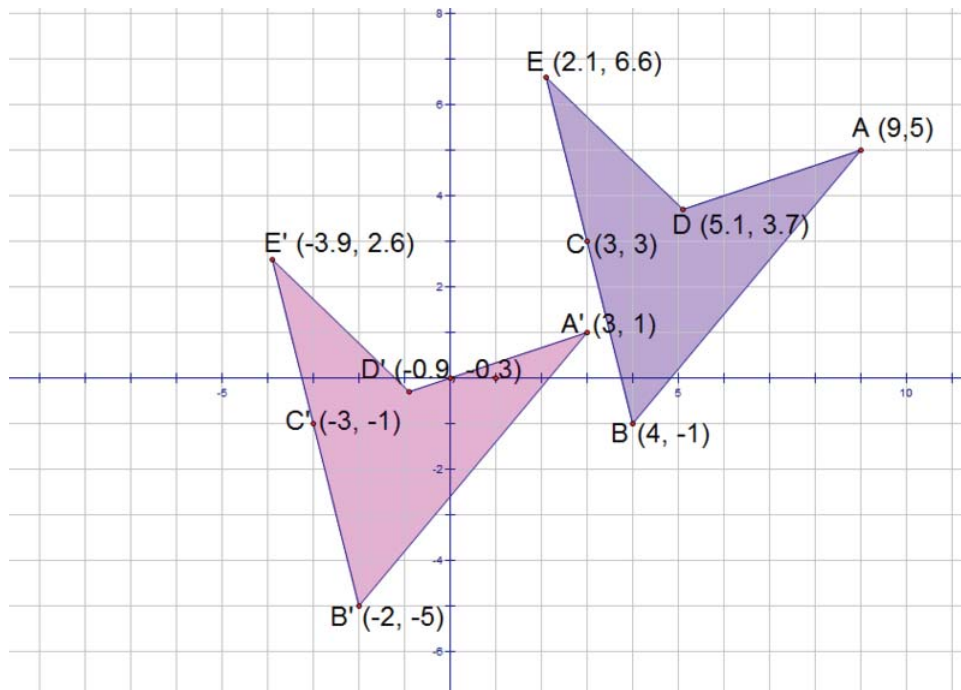


### Example C

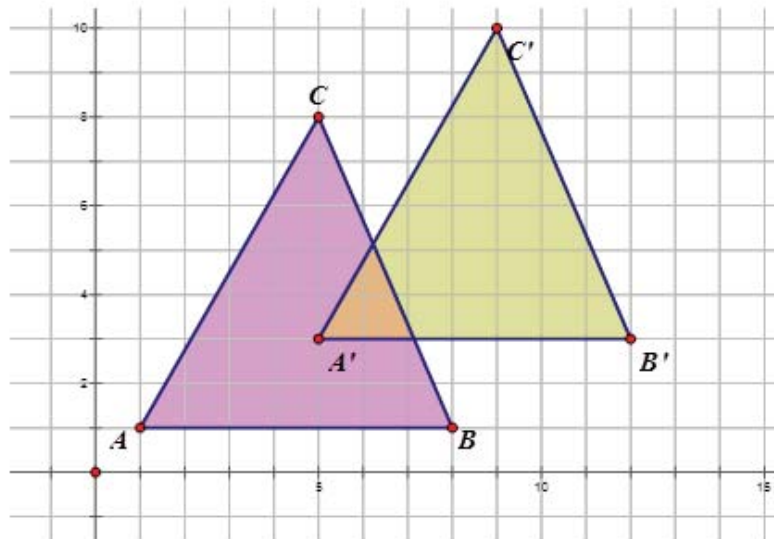
The following figure is translated 4 units down and 6 units to the left to make a translated image. Find the coordinates of the translated image. On the diagram, draw and label the image.



**Solution:**



### Concept Problem Revisited



The coordinates of the new image are  $A'(5,3)$ ,  $B'(12,3)$  and  $C'(9,10)$ .

### Vocabulary

#### Image

In a transformation, the final figure is called the *image*.

#### Preimage

In a transformation, the original figure is called the *preimage*.

**Transformation**

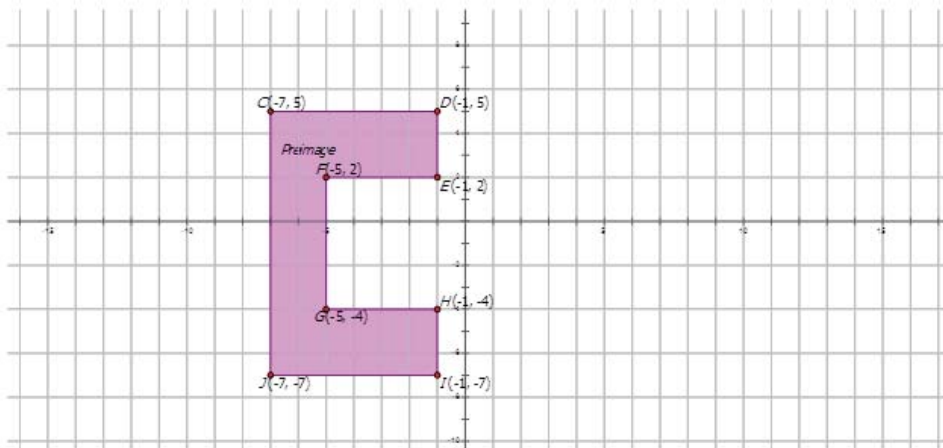
A **transformation** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

**Translation**

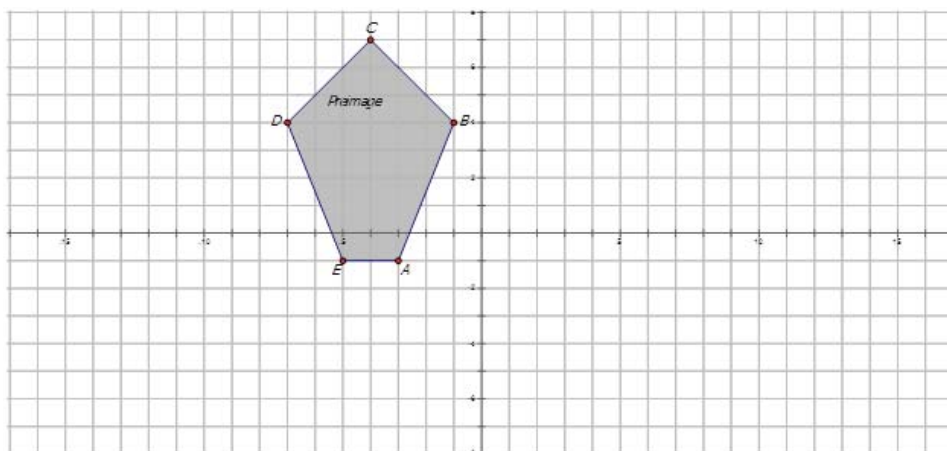
A **translation** is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides**.

**Guided Practice**

1. Line  $\overline{ST}$  drawn from  $(-3, -3)$  to  $(-3, 8)$  has been translated 4 units up and 3 units to the right. Draw the preimage and image and properly label each.
2. The polygon below has been translated 3 units down and 10 units to the right. Draw the translated image and properly label each.

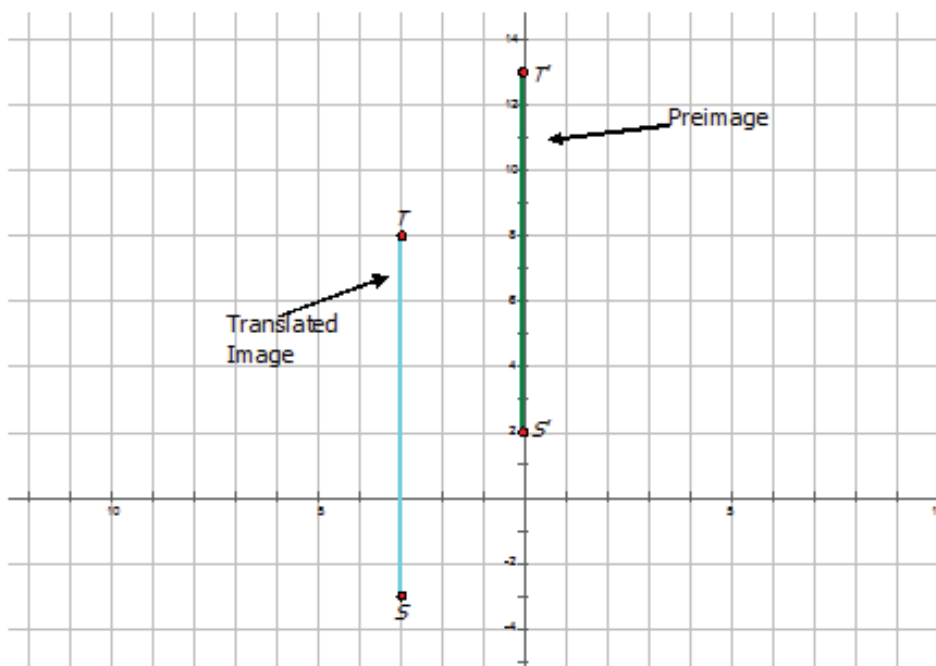


3. The purple pentagon is translated 5 units up and 8 units to the right to make the translated pentagon. Find the coordinates of the purple pentagon. On the diagram, draw and label the translated pentagon.

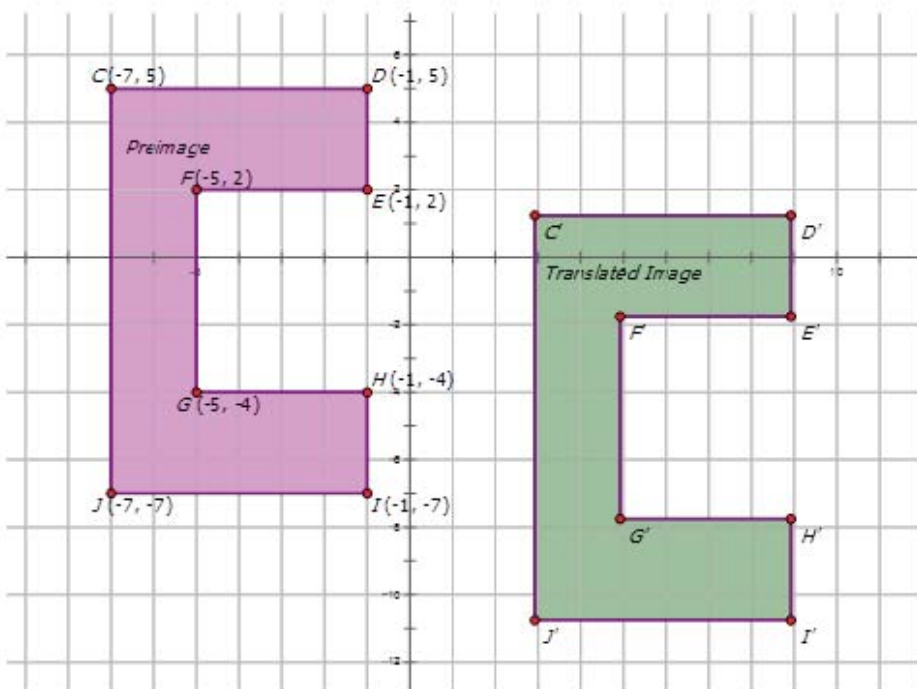


**Answers:**

- 1.

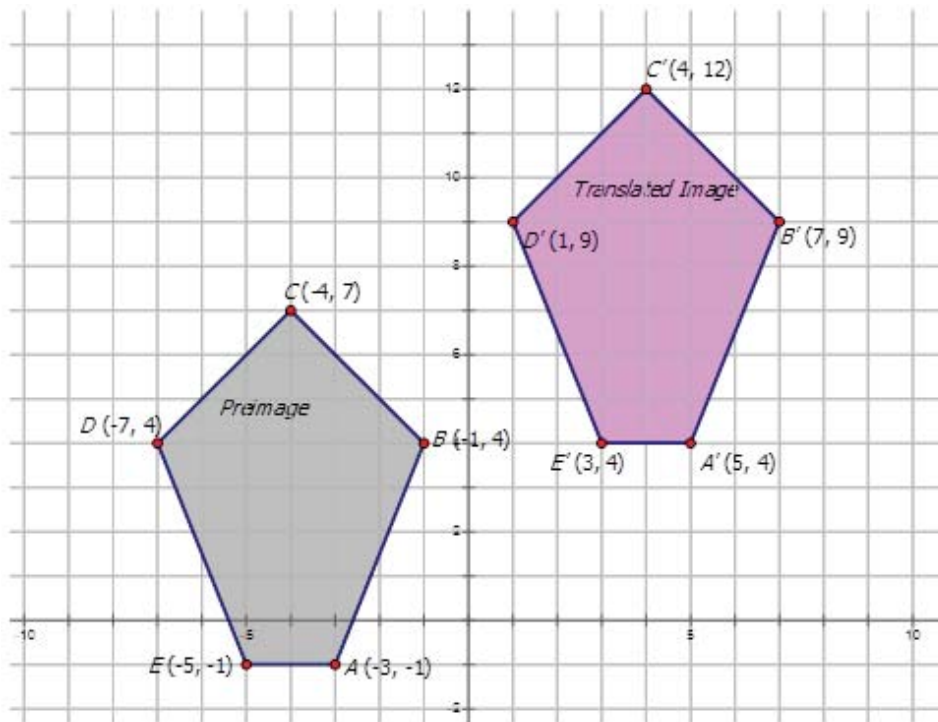


2.

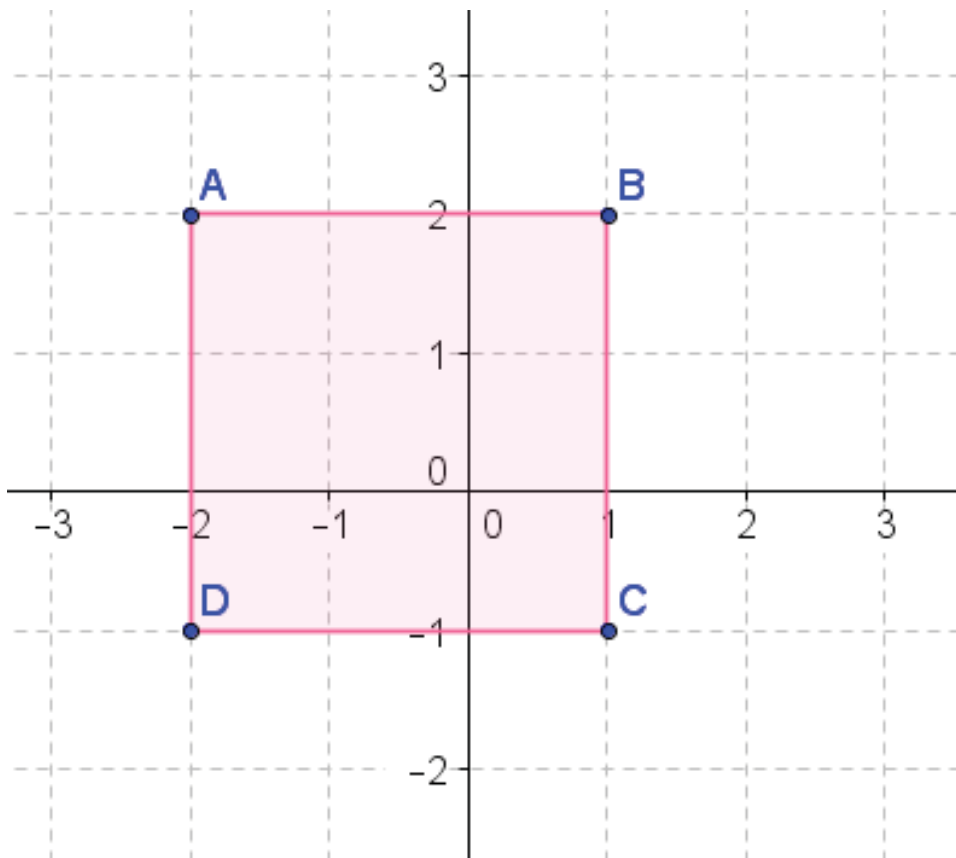


3.

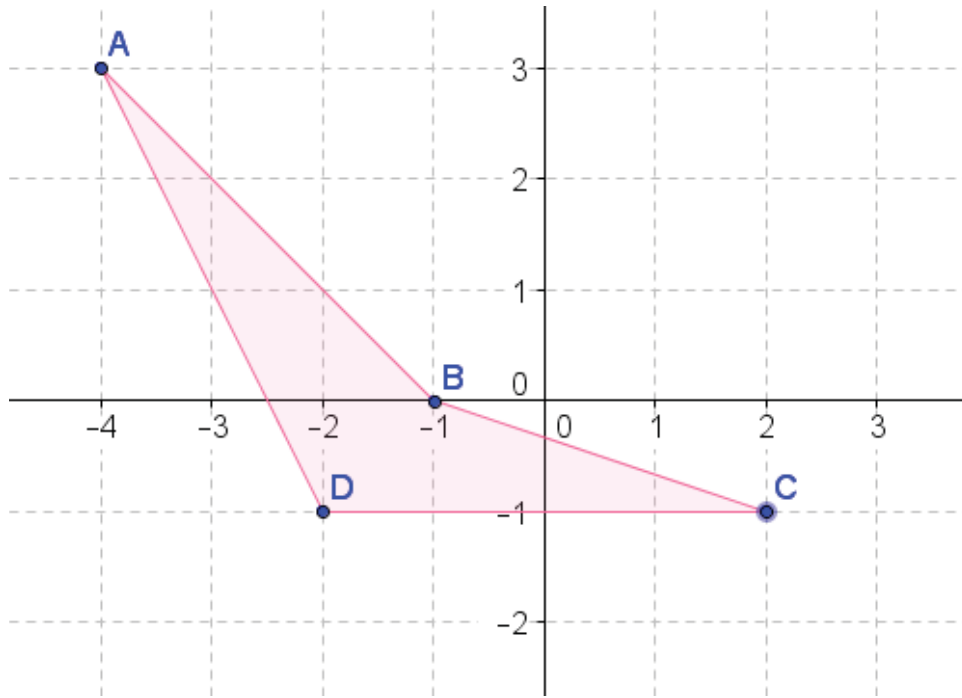




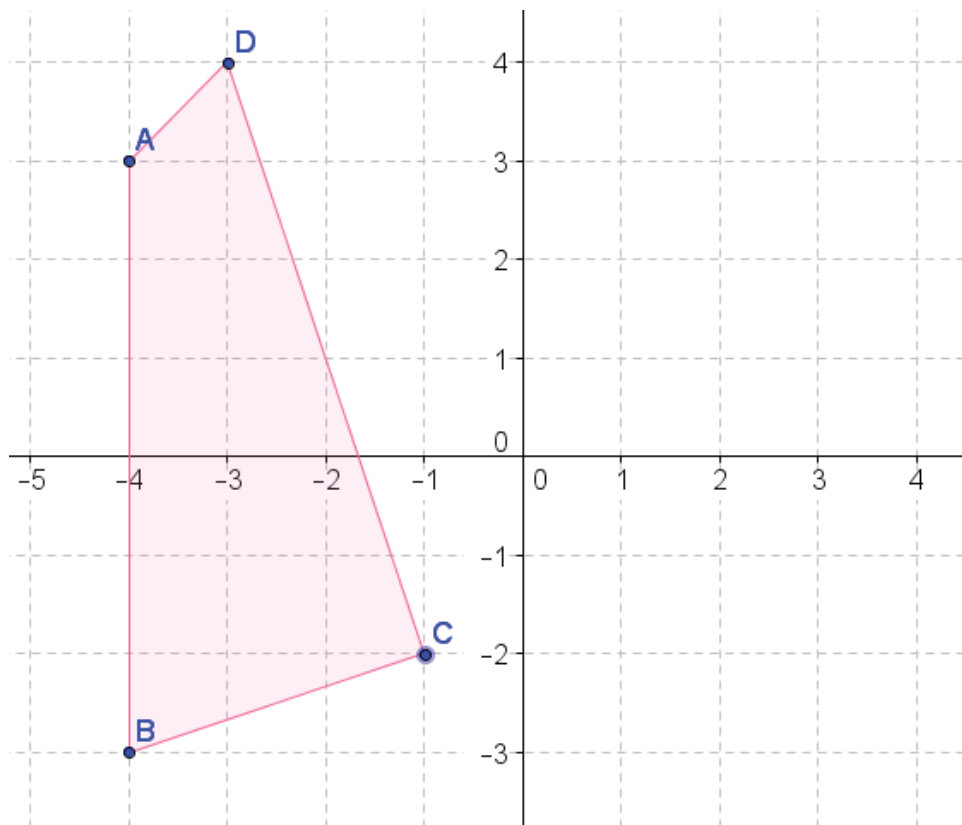
### Practice



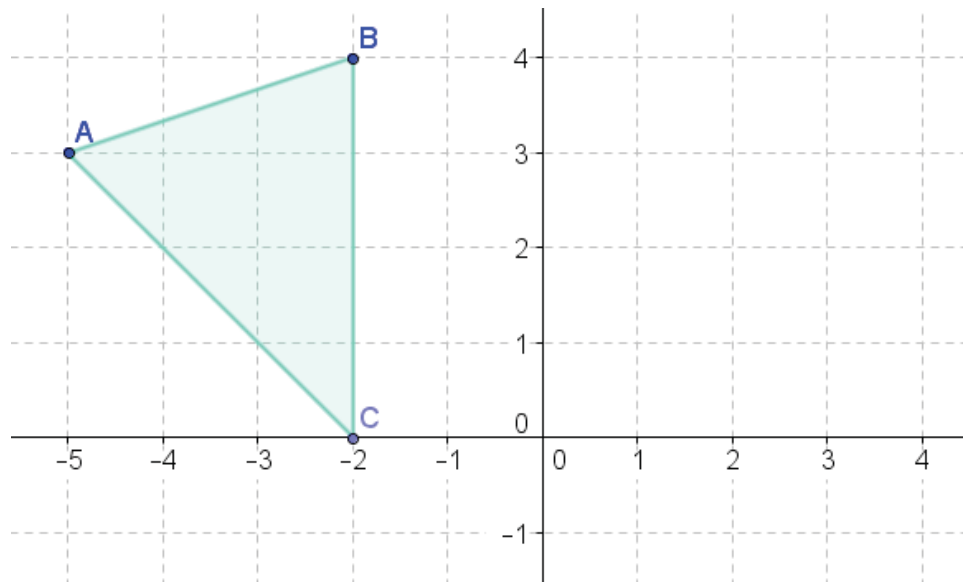
1. Translate the above figure 3 units to the right and 4 units down.
2. Translate the above figure 2 units to the left and 2 units up.



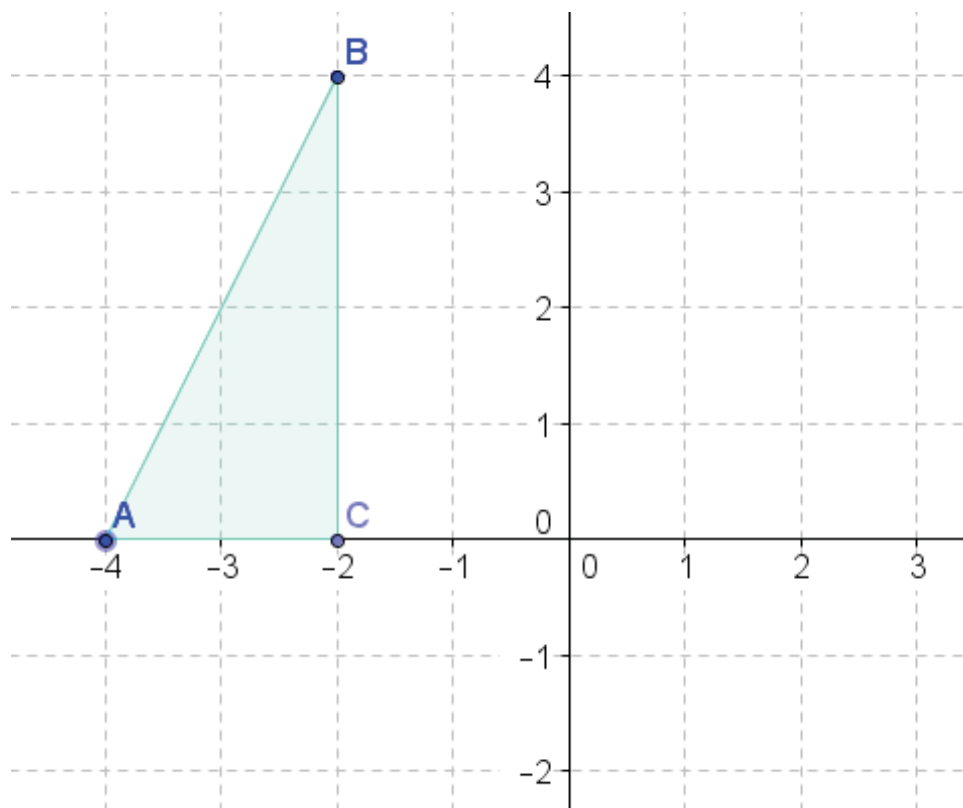
3. Translate the above figure 3 units to the right and 2 units down.
4. Translate the above figure 5 units to the left and 1 unit up.



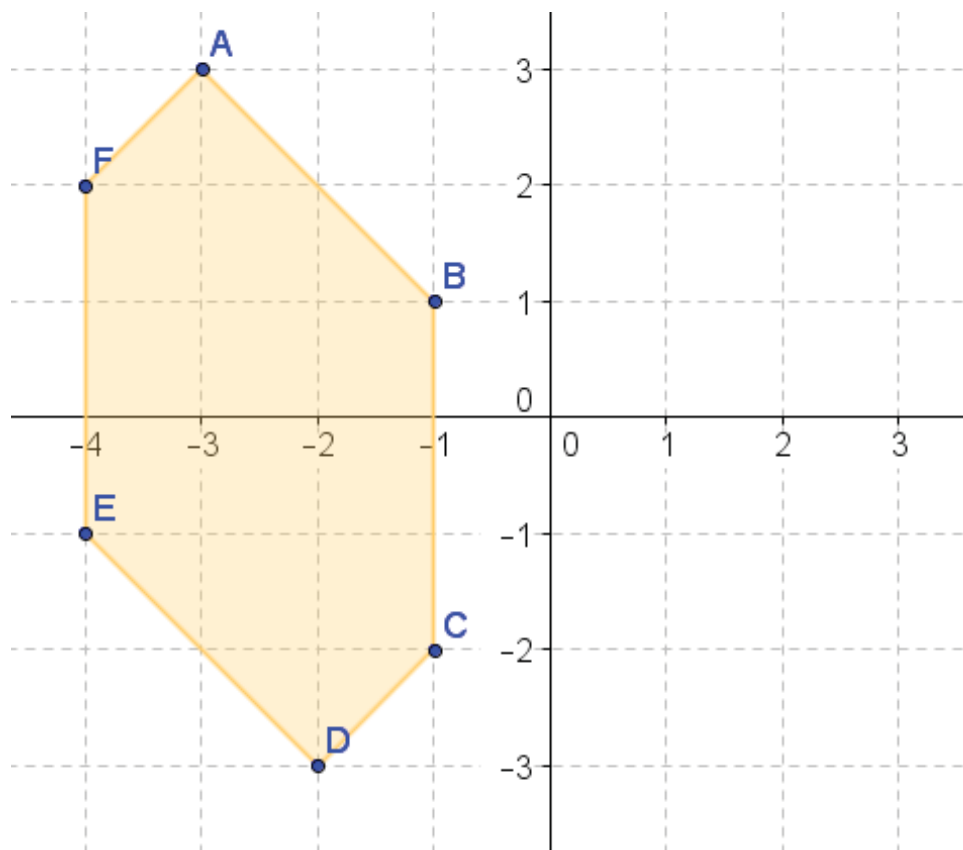
5. Translate the above figure 2 units to the right and 3 units down.
6. Translate the above figure 4 units to the left and 1 unit up.



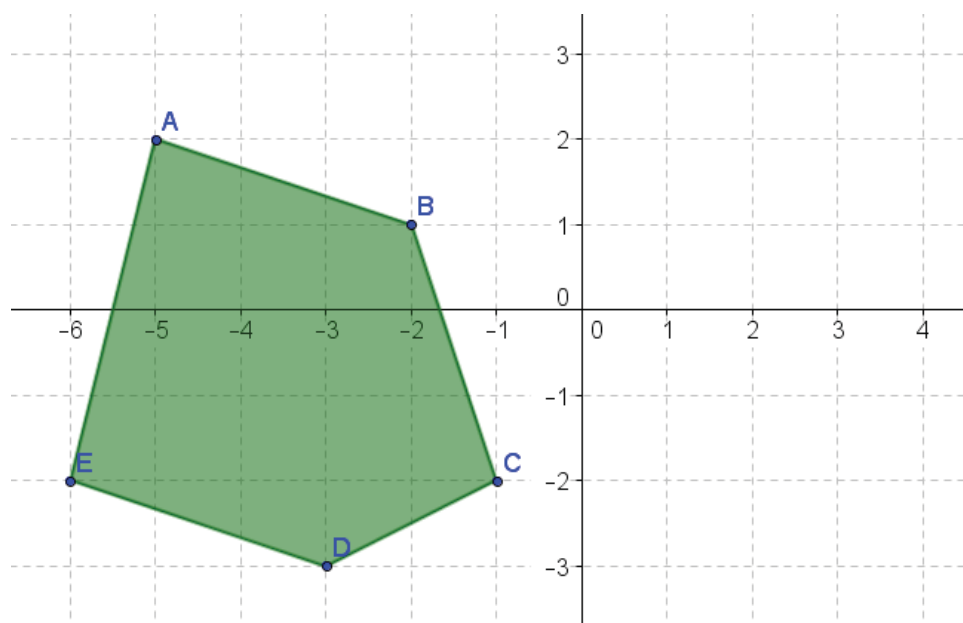
7. Translate the above figure 6 units to the right and 2 units down.
8. Translate the above figure 4 units to the left and 6 units up.



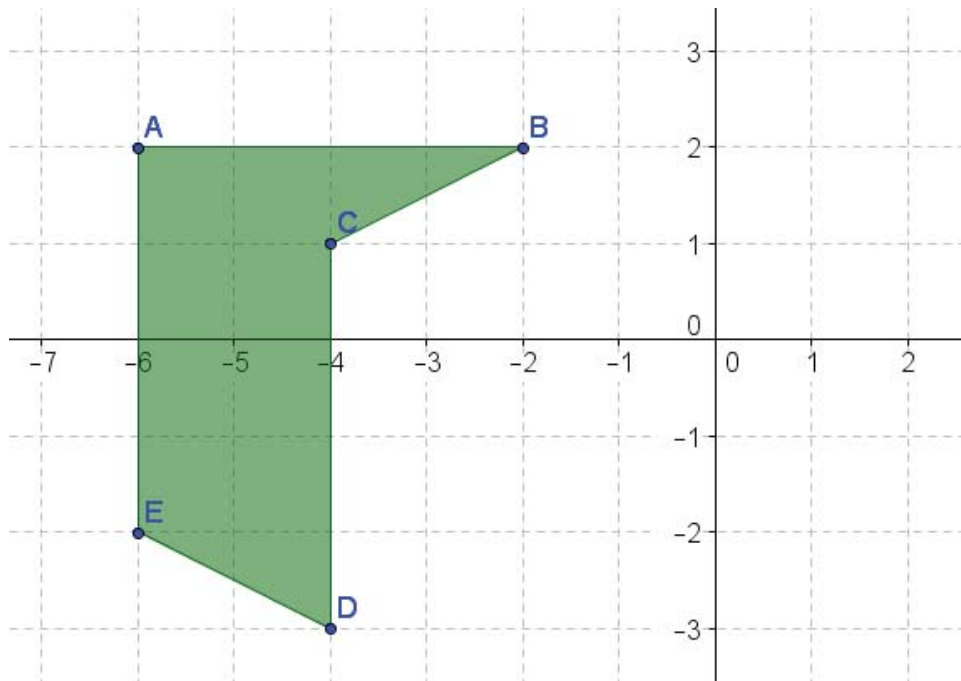
9. Translate the above figure 2 units to the right and 3 units down.
10. Translate the above figure 5 units to the left and 5 units up.



11. Translate the above figure 3 units to the right and 6 units down.
12. Translate the above figure 2 units to the left and 2 units up.



13. Translate the above figure 3 units to the right and 3 units down.
14. Translate the above figure 5 units to the left and 2 units up.

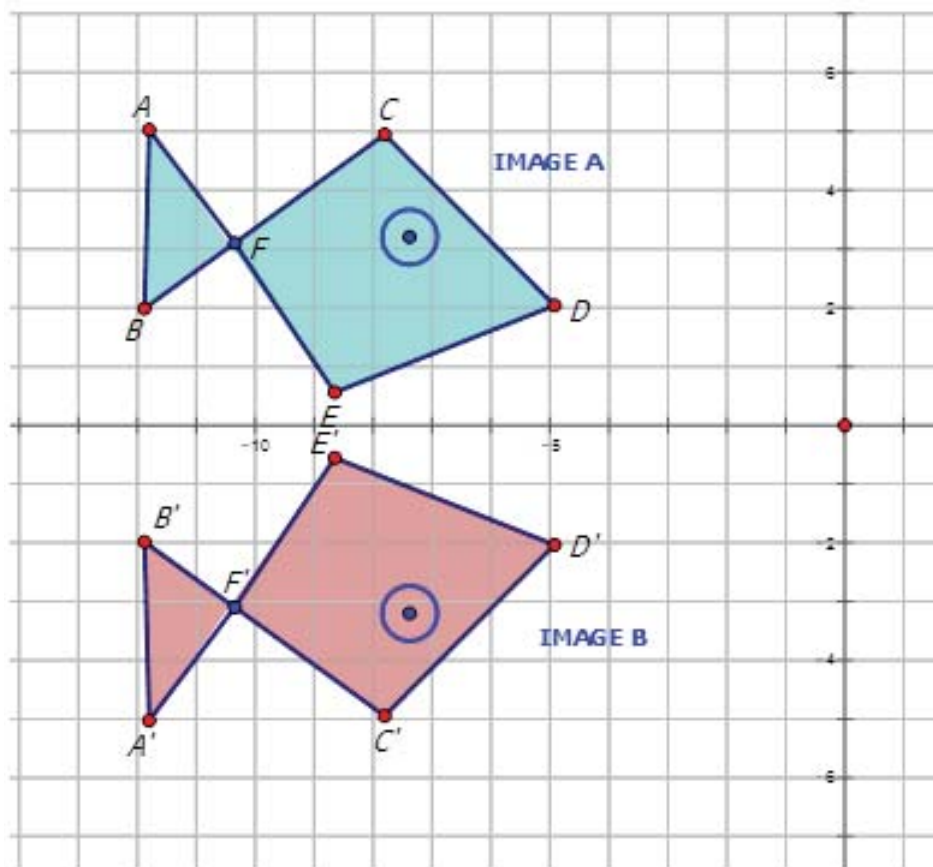


15. Translate the above figure 7 units to the right and 4 units down.
16. Translate the above figure 1 unit to the left and 2 units up.

## 1.4 Rules for Reflections

Here you will learn notation for describing a reflection with a rule.

The figure below shows a pattern of two fish. Write the mapping rule for the reflection of Image A to Image B.



### Watch This

First watch this video to learn about writing rules for reflections.



#### MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10RulesforReflectionsA](#)

Then watch this video to see some examples.

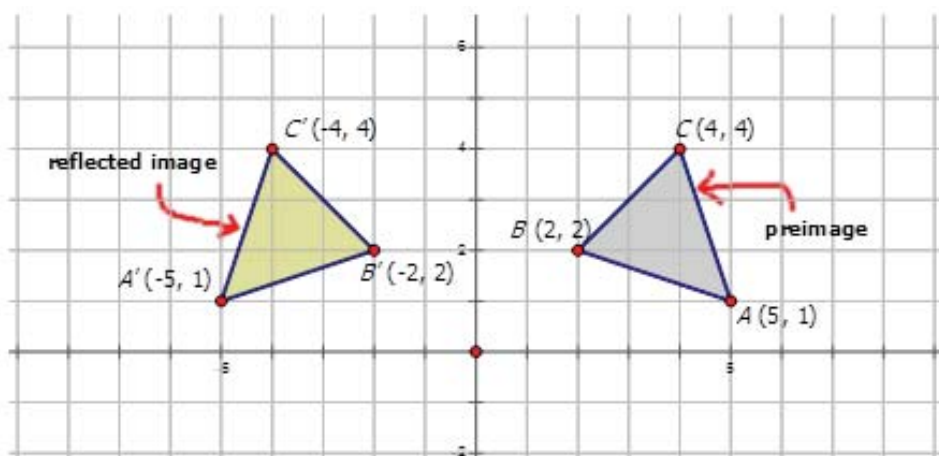

**MEDIA**

Click image to the left for more content.

## CK-12 Foundation Chapter 10 Rules for Reflections B

**Guidance**

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A reflection is an example of a transformation that takes a shape (called the preimage) and flips it across a line (called the line of reflection) to create a new shape (called the image). By examining the coordinates of the reflected image, you can determine the line of reflection. The most common lines of reflection are the  $x$ -axis, the  $y$ -axis, or the lines  $y = x$  or  $y = -x$ .



The preimage has been reflected across the  $y$ -axis. This means, all of the  $x$ -coordinates have been multiplied by  $-1$ . You can describe the reflection in words, or with the following notation:

$$r_{y\text{-axis}}(x, y) \rightarrow (-x, y)$$

Notice that the notation tells you exactly how each  $(x, y)$  point changes as a result of the transformation.

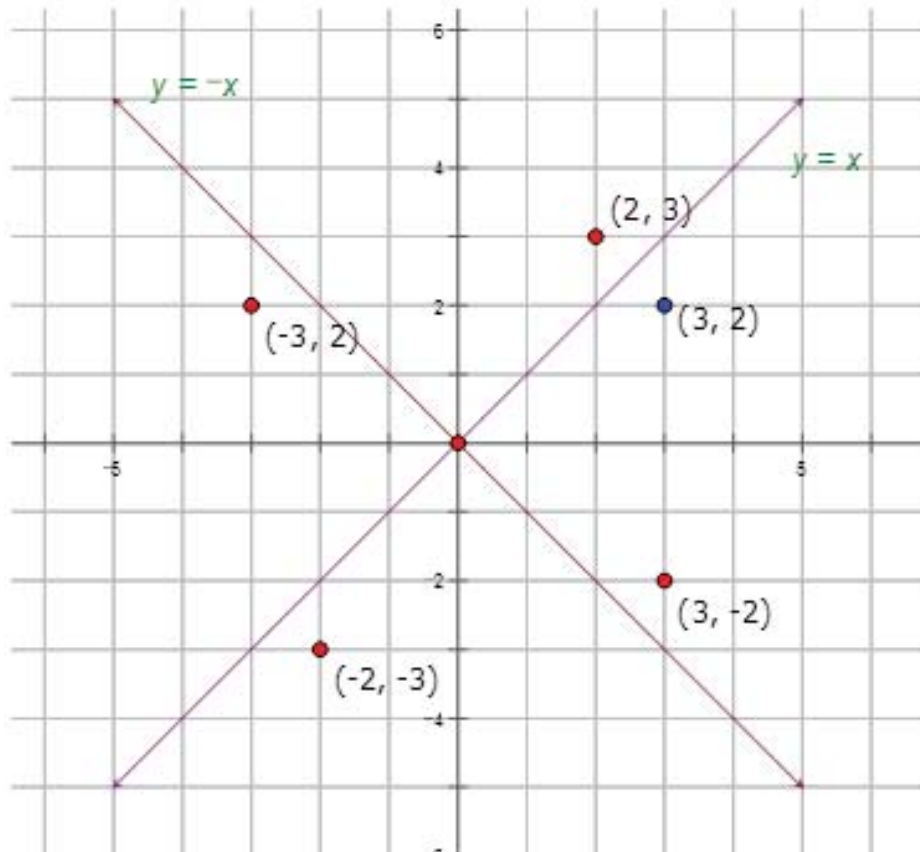
**Example A**

Find the image of the point  $(3, 2)$  that has undergone a reflection across

- the  $x$ -axis,
- the  $y$ -axis,
- the line  $y = x$ , and
- the line  $y = -x$ .

Write the notation to describe the reflection.

**Solution:**



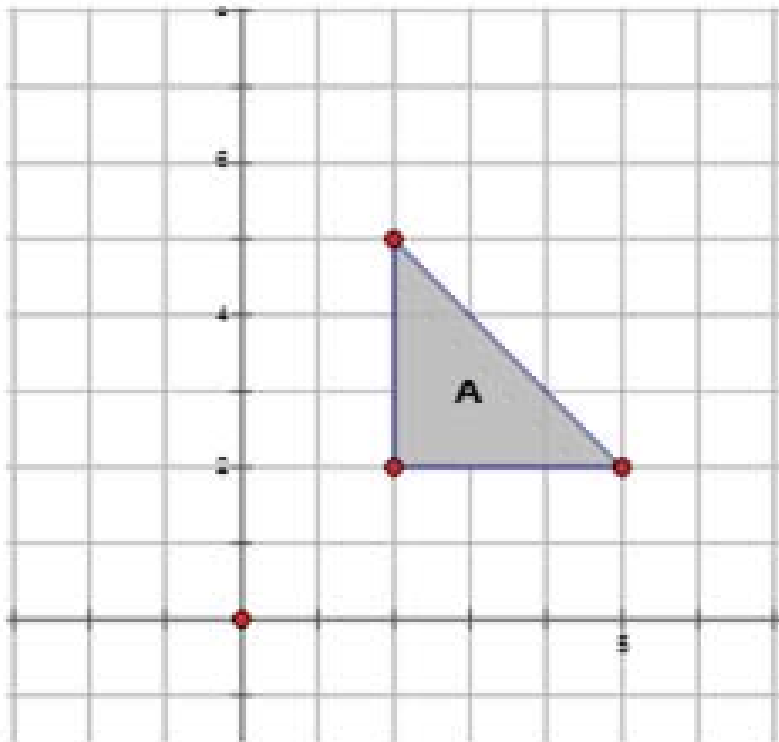
- a) Reflection across the  $x$ -axis:  $r_{x\text{-axis}}(3, 2) \rightarrow (3, -2)$
- b) Reflection across the  $y$ -axis:  $r_{y\text{-axis}}(3, 2) \rightarrow (-3, 2)$
- c) Reflection across the line  $y = x$ :  $r_{y=x}(3, 2) \rightarrow (2, 3)$
- d) Reflection across the line  $y = -x$ :  $r_{y=-x}(3, 2) \rightarrow (-2, -3)$

### Example B

Reflect Image A in the diagram below:

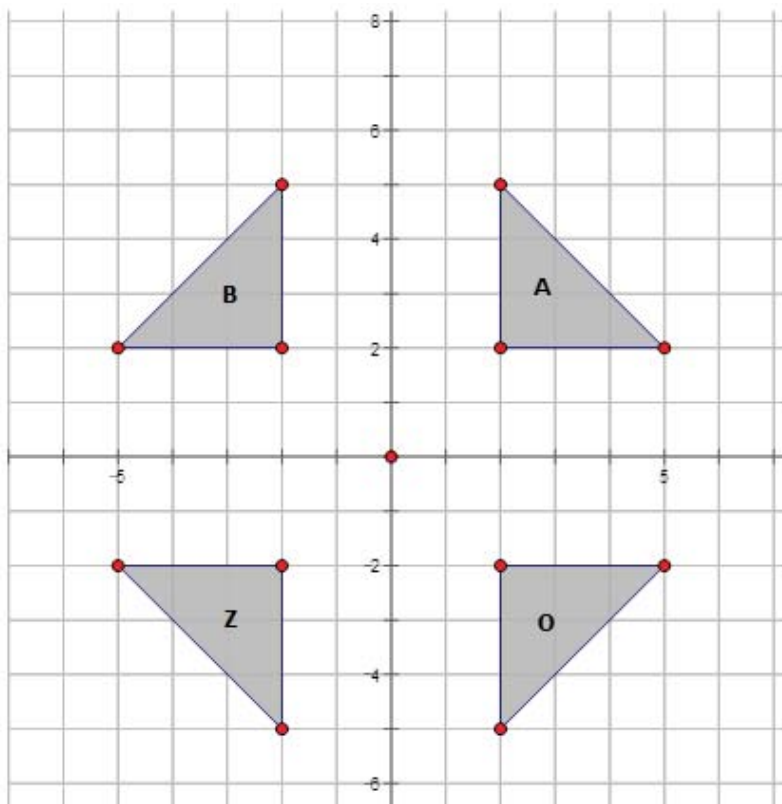
- a) Across the  $y$ -axis and label it  $B$ .
- b) Across the  $x$ -axis and label it  $O$ .
- c) Across the line  $y = -x$  and label it  $Z$ .





Write notation for each to indicate the type of reflection.

**Solution:**



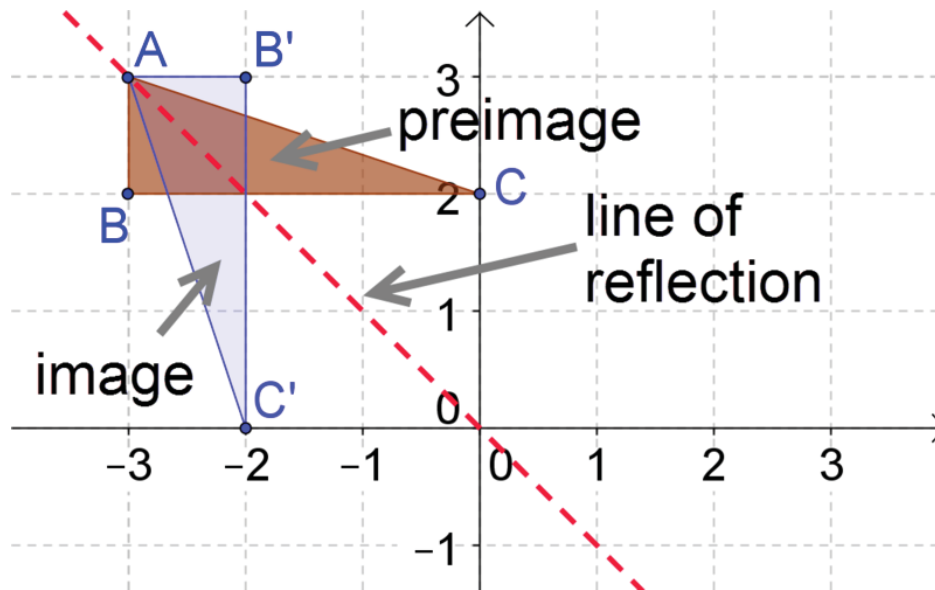
a) Reflection across the  $y$ -axis:  $r_{y\text{-axis}}A \rightarrow B = r_{y\text{-axis}}(x,y) \rightarrow (-x,y)$

b) Reflection across the  $x$ -axis:  $r_{x\text{-axis}}A \rightarrow O = r_{x\text{-axis}}(x,y) \rightarrow (x,-y)$

c) Reflection across the  $y = -x$ :  $r_{y=-x}A \rightarrow Z = r_{y=-x}(x,y) \rightarrow (-y, -x)$

### Example C

Write the notation that represents the reflection of the preimage to the image in the diagram below.

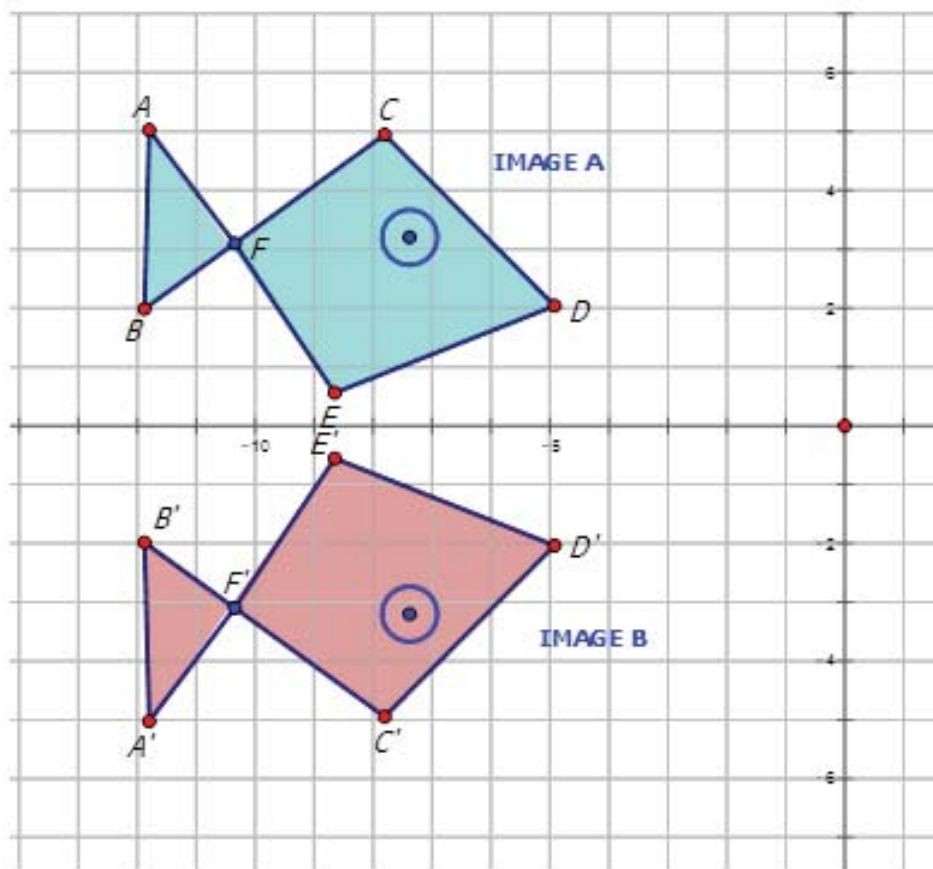


### Solution:

This is a reflection across the line  $y = -x$ . The notation is  $r_{y=-x}(x,y) \rightarrow (-y, -x)$ .

### Concept Problem Revisited

The figure below shows a pattern of two fish. Write the mapping rule for the reflection of Image A to Image B.



To answer this question, look at the coordinate points for Image A and Image B.

**TABLE 1.1:**

Image A	$A(-11.8, 5)$	$B(-11.8, 2)$	$C(-7.8, 5)$	$D(-4.9, 2)$	$E(-8.7, 0.5)$	$F(-10.4, 3.1)$
Image B	$A'(-11.8, -5)$	$B'(-11.8, -2)$	$C'(-7.8, -5)$	$D'(-4.9, -2)$	$E'(-8.7, -0.5)$	$F'(-10.4, -3.1)$

Notice that all of the  $y$ -coordinates have changed sign. Therefore Image A has reflected across the  $x$ -axis. To write a rule for this reflection you would write:  $r_{x\text{-axis}}(x, y) \rightarrow (x, -y)$ .

## Vocabulary

### Notation Rule

A **notation rule** has the following form  $r_{y\text{-axis}}A \rightarrow B = r_{y\text{-axis}}(x, y) \rightarrow (-x, y)$  and tells you that the image A has been reflected across the  $y$ -axis and the  $x$ -coordinates have been multiplied by  $-1$ .

### Reflection

A **reflection** is an example of a transformation that flips each point of a shape over the same line.

### Image

In a transformation, the final figure is called the **image**.

### Preimage

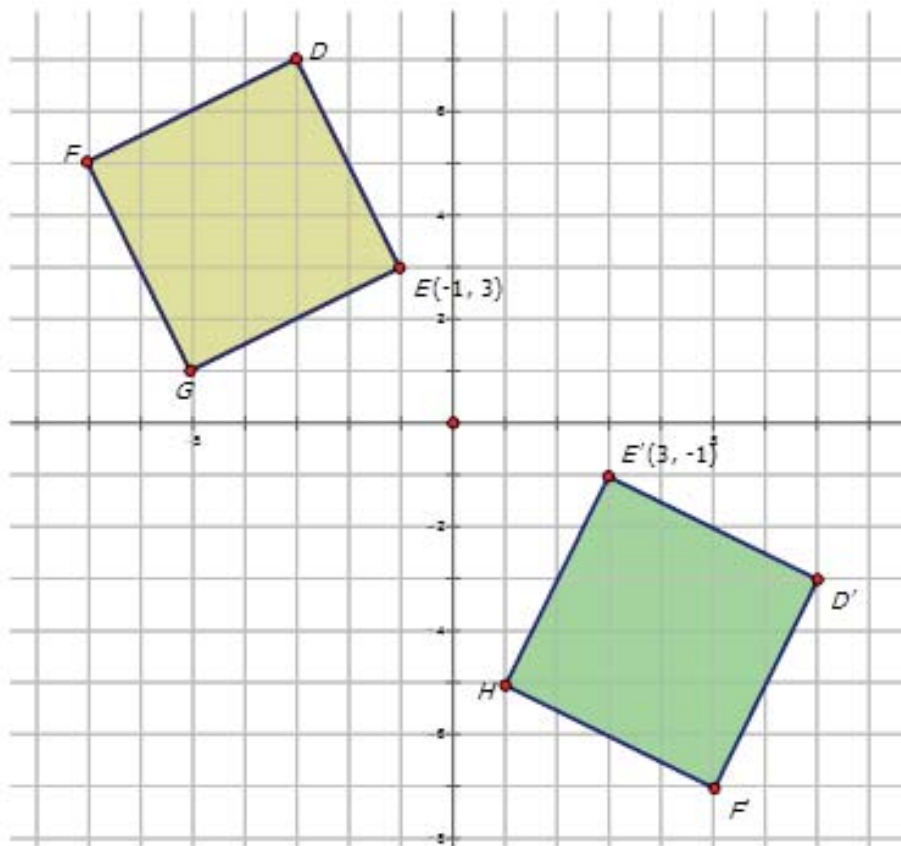
In a transformation, the original figure is called the **preimage**.

**Transformation**

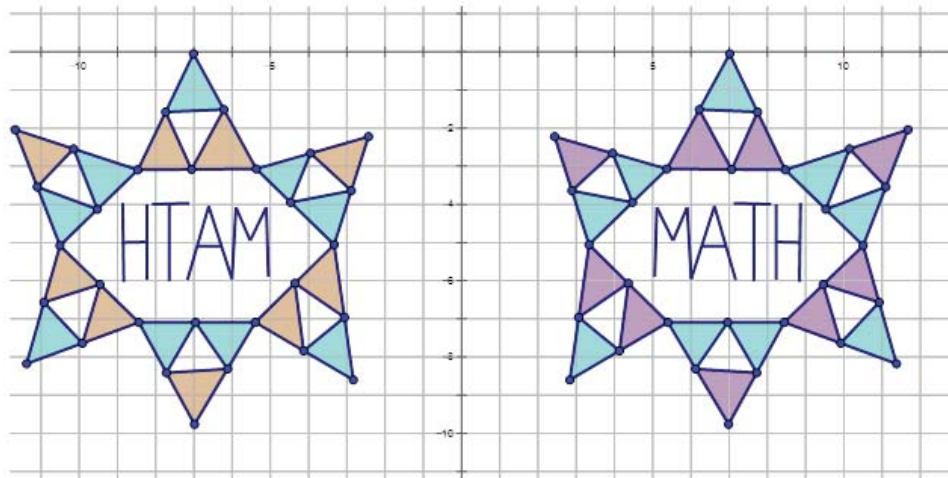
A **transformation** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

**Guided Practice**

1. Thomas describes a reflection as point  $J$  moving from  $J(-2, 6)$  to  $J'(-2, -6)$ . Write the notation to describe this reflection for Thomas.
2. Write the notation that represents the reflection of the yellow diamond to the reflected green diamond in the diagram below.



3. Karen was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.



**Answers:**

1.  $J : (-2, 6) \quad J' : (-2, -6)$

Since the  $y$ -coordinate is multiplied by  $-1$  and the  $x$ -coordinate remains the same, this is a reflection in the  $x$ -axis. The notation is:  $r_{x\text{-axis}}J \rightarrow J' = r_{x\text{-axis}}(-2, 6) \rightarrow (-2, -6)$

2. In order to write the notation to describe the reflection, choose one point on the preimage (the yellow diamond) and then the reflected point on the green diamond to see how the point has moved. Notice that point  $E$  is shown in the diagram:

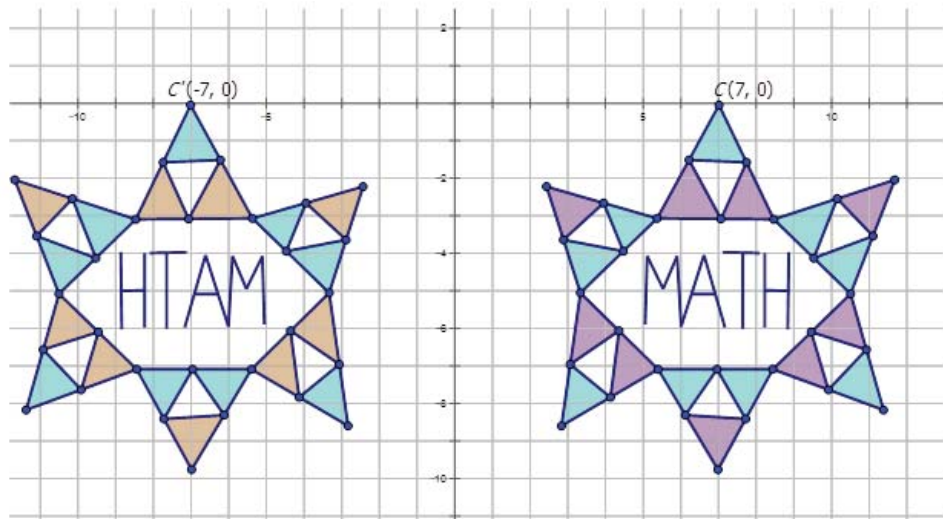
$E(-1, 3) \rightarrow E'(3, -1)$

Since both  $x$ - and  $y$ -coordinates are reversed numbers, the reflection is in the line  $y = x$ . The notation for this reflection would be:  $r_{y=x}(x, y) \rightarrow (y, x)$ .

3. In order to write the notation to describe the transformation, choose one point on the preimage (purple and blue diagram) and then the transformed point on the orange and blue diagram to see how the point has moved. Notice that point  $A$  is shown in the diagram:

$C(7, 0) \rightarrow C'(-7, 0)$

Since both  $x$ -coordinates only are multiplied by  $-1$ , the transformation is a reflection in  $y$ -axis. The notation for this reflection would be:  $r_{y\text{-axis}}(x, y) \rightarrow (-x, y)$ .

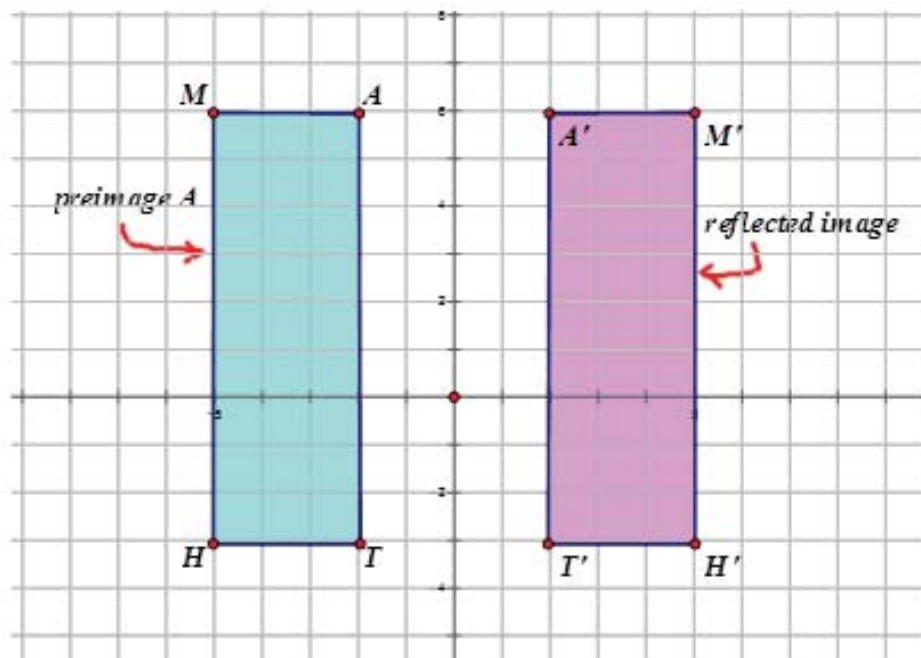


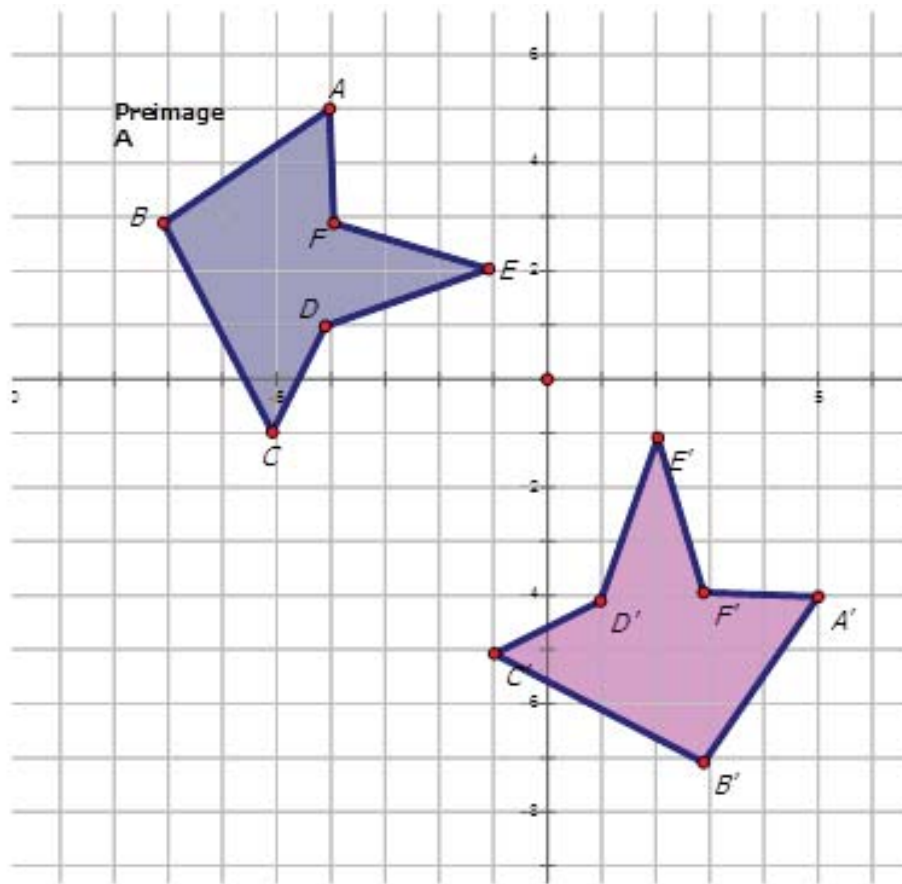
**Practice**

Write the notation to describe the movement of the points in each of the reflections below.

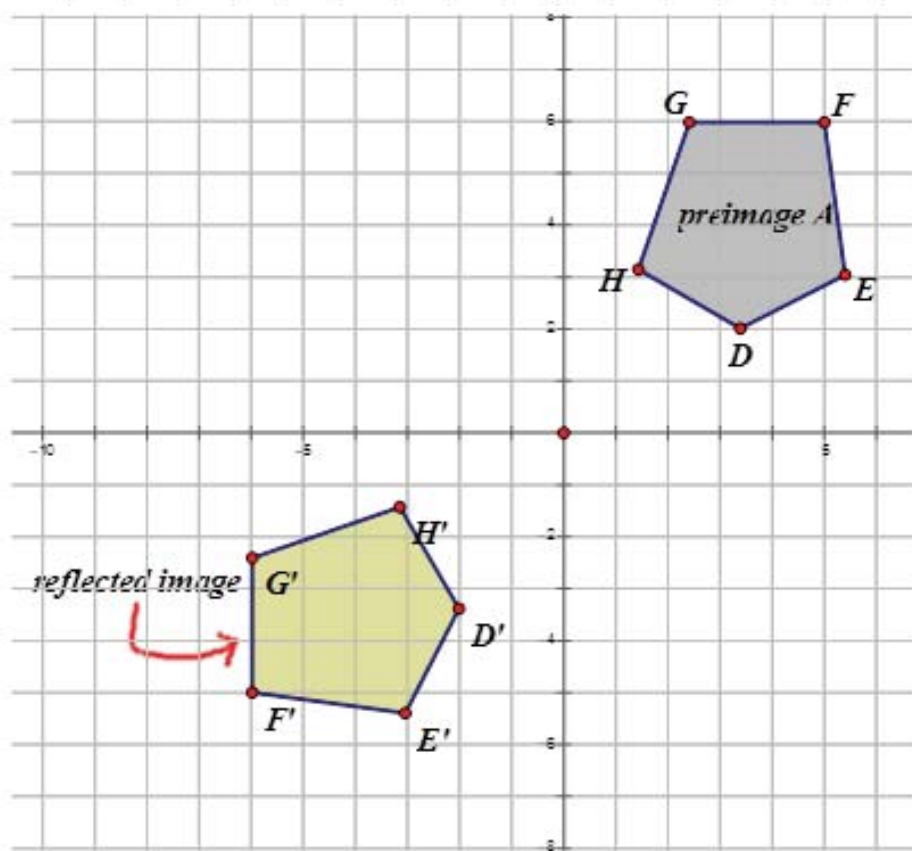
1.  $S(1,5) \rightarrow S'(-1,5)$
2.  $W(-5,-1) \rightarrow W'(5,-1)$
3.  $Q(2,-5) \rightarrow Q'(2,5)$
4.  $M(4,3) \rightarrow M'(-3,-4)$
5.  $B(-4,-2) \rightarrow B'(-2,-4)$
6.  $A(3,5) \rightarrow A'(-3,5)$
7.  $C(1,2) \rightarrow C'(2,1)$
8.  $D(2,-5) \rightarrow D'(5,-2)$
9.  $E(3,1) \rightarrow E'(-3,1)$
10.  $F(-4,2) \rightarrow F'(-4,-2)$
11.  $G(1,3) \rightarrow G'(1,-3)$

Write the notation that represents the reflection of the preimage image for each diagram below.

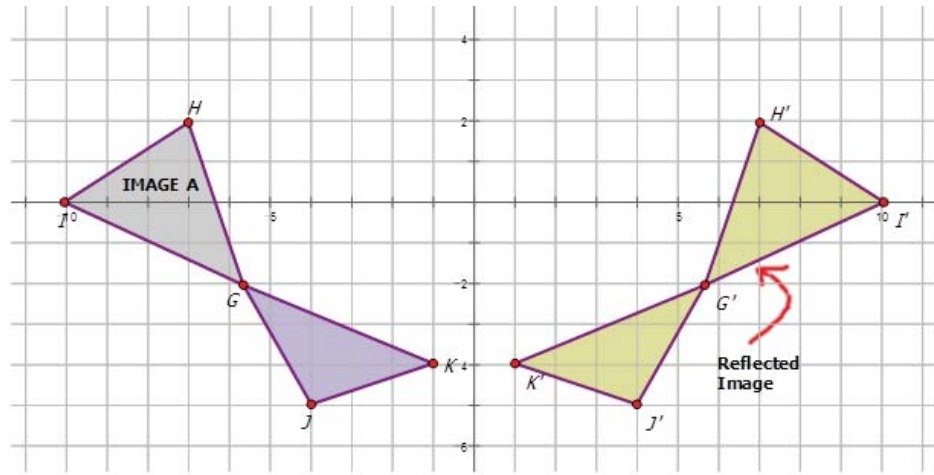




13.



14.



15.



## 1.5 Graphs of Reflections

Here you will learn how to reflect an image on a coordinate grid.

Triangle  $A$  has coordinates  $E(-5, -5)$ ,  $F(2, -6)$  and  $G(-2, 0)$ . Draw the triangle on the Cartesian plane. Reflect the image across the  $y$ -axis. State the coordinates of the resulting image.

### Watch This

First watch this video to learn about graphs of reflections.



MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10GraphsofReflectionsA](#)

Then watch this video to see some examples.



MEDIA

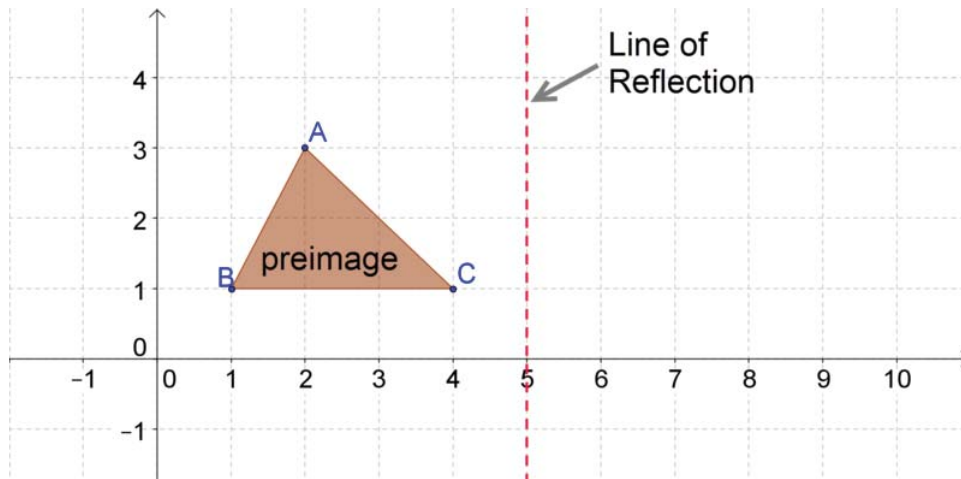
Click image to the left for more content.

[CK-12 FoundationChapter10GraphsofReflectionsB](#)

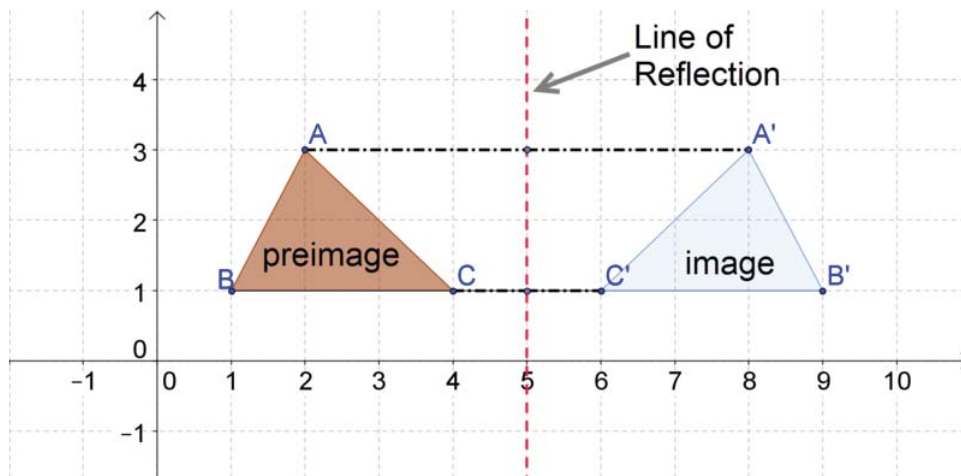
### Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A reflection is an example of a transformation that takes a shape (called the preimage) and flips it across a line (called the line of reflection) to create a new shape (called the image).

To graph a reflection, you can visualize what would happen if you flipped the shape across the line.



Each point on the preimage will be the same distance from the line of reflection as its corresponding point in the image. For example, for the pair of triangles below, both  $A$  and  $A'$  are 3 units away from the line of reflection.



For common reflections, you can also remember what happens to their coordinates:

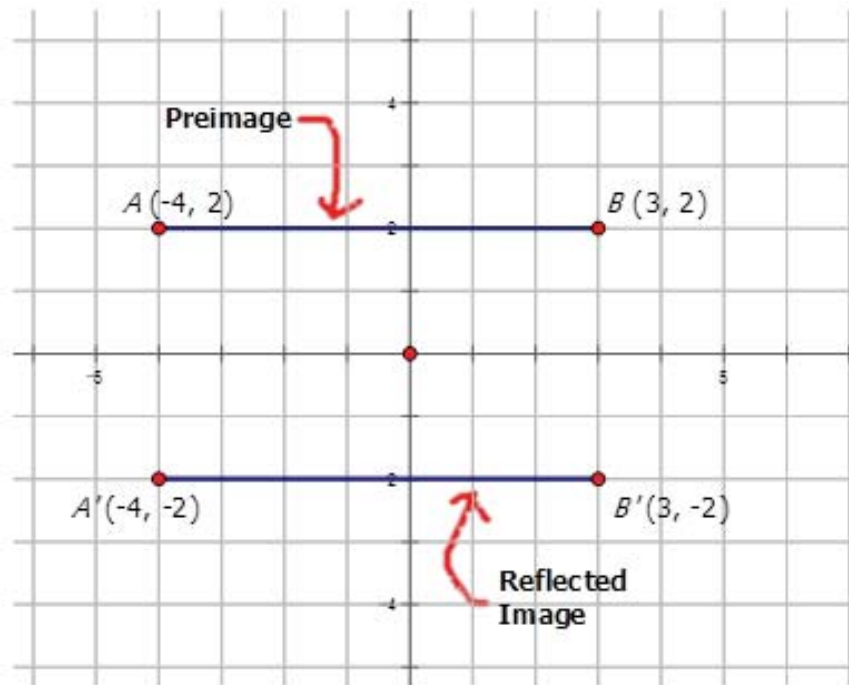
- reflections across the  $x$ -axis:  $y$  values are multiplied by  $-1$ .
- reflections across the  $y$ -axis:  $x$  values are multiplied by  $-1$ .
- reflections across the line  $y = x$ :  $x$  and  $y$  values switch places.
- reflections across the line  $y = -x$ :  $x$  and  $y$  values switch places and are multiplied by  $-1$ .

Knowing the rules above will allow you to recognize reflections even when a graph is not available.

### Example A

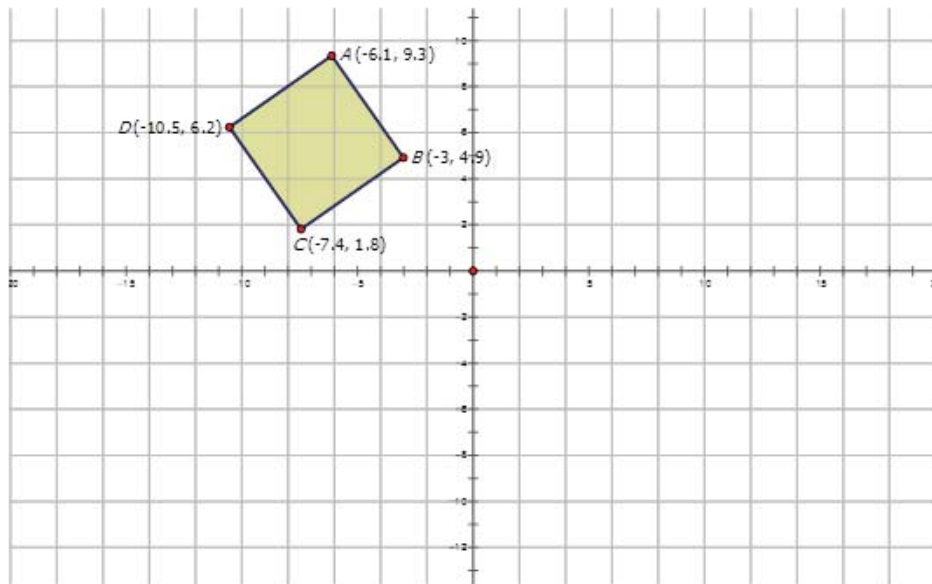
Line  $\overline{AB}$  drawn from  $(-4, 2)$  to  $(3, 2)$  has been reflected across the  $x$ -axis. Draw the preimage and image and properly label each.

**Solution:**

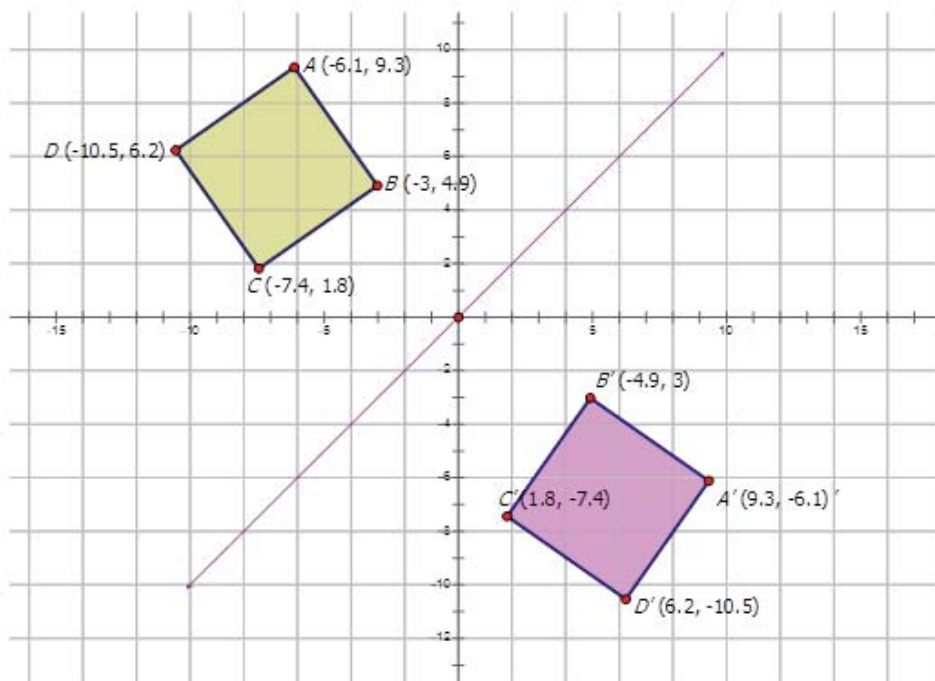


### Example B

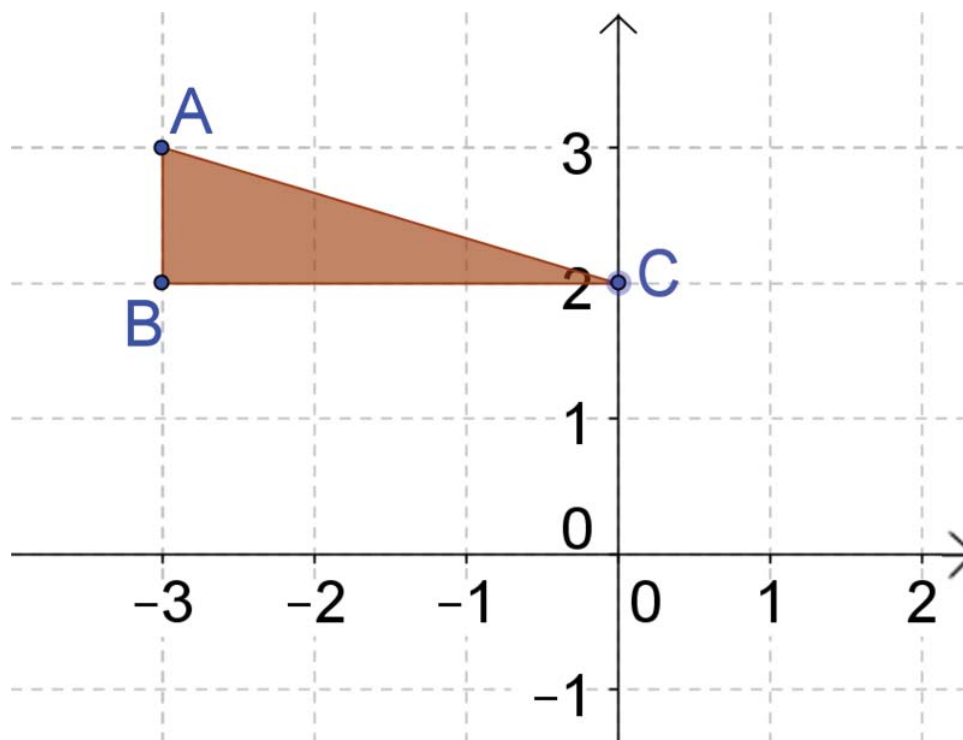
The diamond  $ABCD$  is reflected across the line  $y = x$  to form the image  $A'B'C'D'$ . Find the coordinates of the reflected image. On the diagram, draw and label the reflected image.



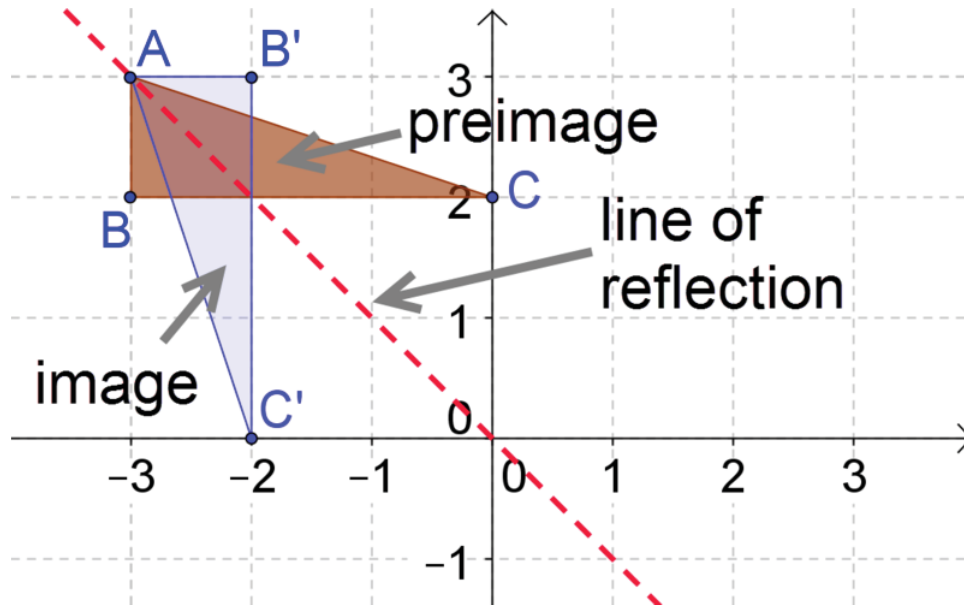
**Solution:**

**Example C**

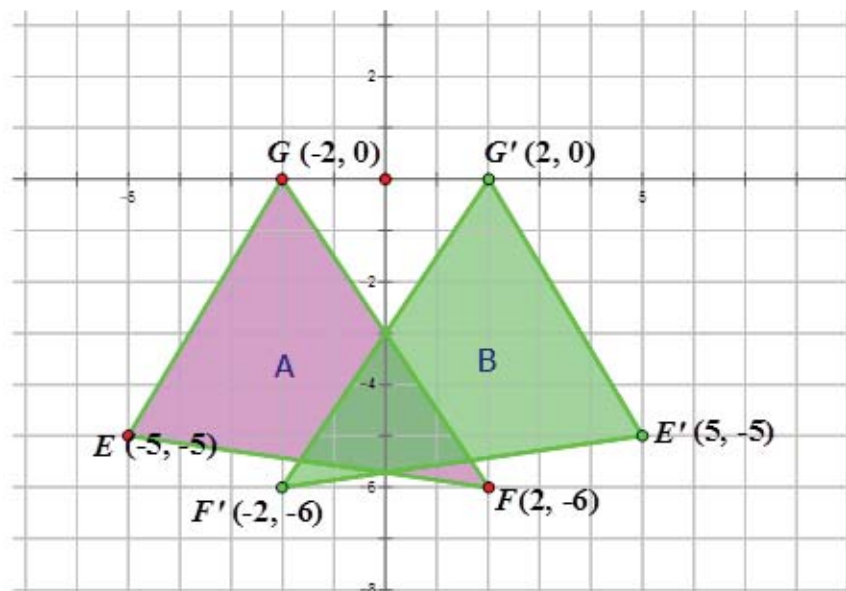
Triangle  $ABC$  is reflected across the line  $y = -x$  to form the image  $A'B'C'$ . Draw and label the reflected image.



**Solution:**



**Concept Problem Revisited**



The coordinates of the new image (*B*) are  $E'(5, -5)$ ,  $F'(2, -6)$  and  $G'(2, 0)$ .

**Vocabulary**

**Image**

In a transformation, the final figure is called the *image*.

**Preimage**

In a transformation, the original figure is called the *preimage*.

**Transformation**

A *transformation* is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

**Reflection**

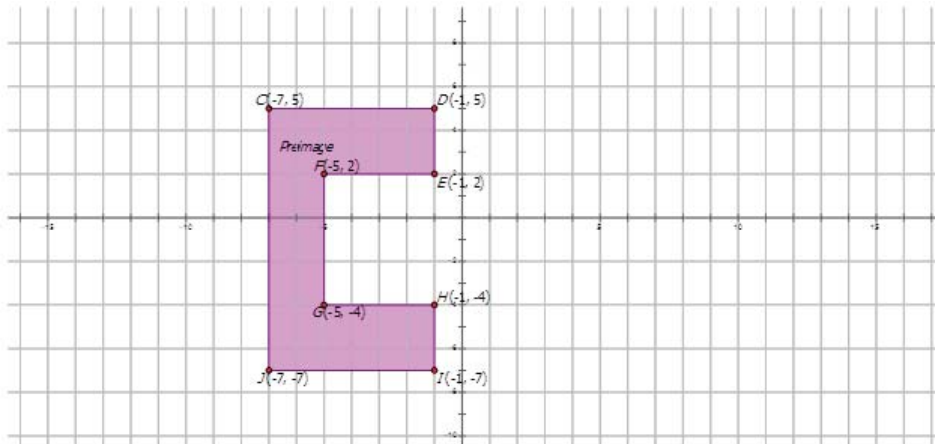
A *reflection* is an example of a transformation that flips each point of a shape over the same line.

**Line of Reflection**

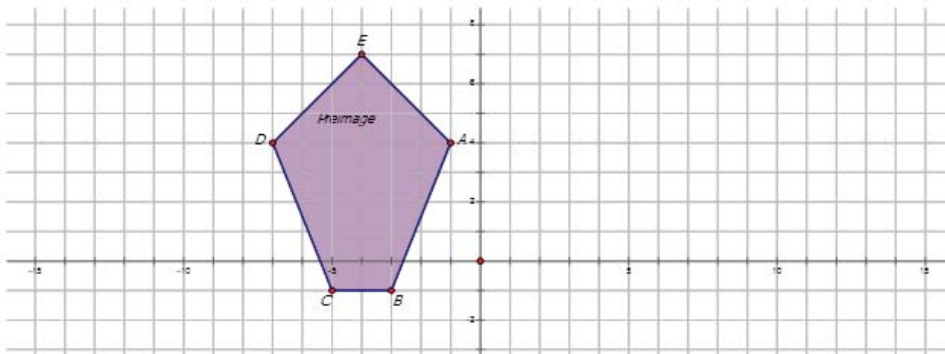
The *line of reflection* is the line that a shape reflects (flips) across when undergoing a reflection.

**Guided Practice**

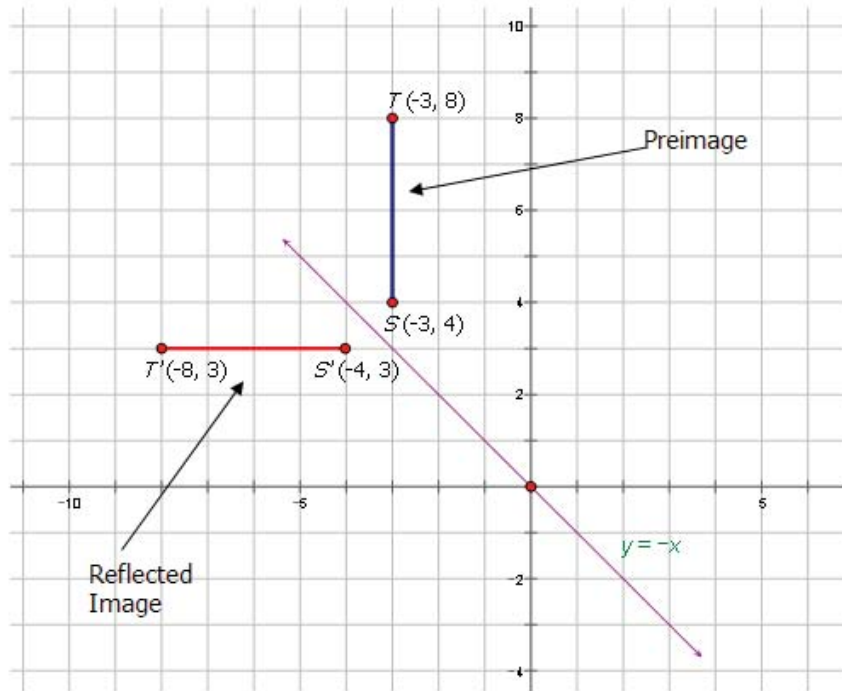
- Line  $\overline{ST}$  drawn from  $(-3, 4)$  to  $(-3, 8)$  has been reflected across the line  $y = -x$ . Draw the preimage and image and properly label each.
- The polygon below has been reflected across the  $y$ -axis. Draw the reflected image and properly label each.



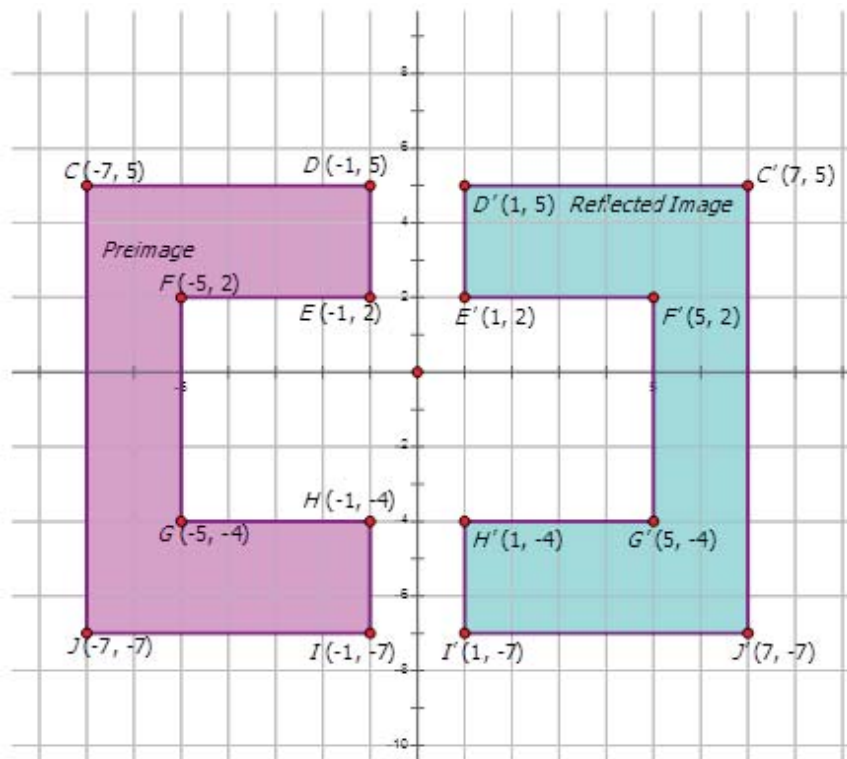
- The purple pentagon is reflected across the  $y$ -axis to make the new image. Find the coordinates of the purple pentagon. On the diagram, draw and label the reflected pentagon.

**Answers:**

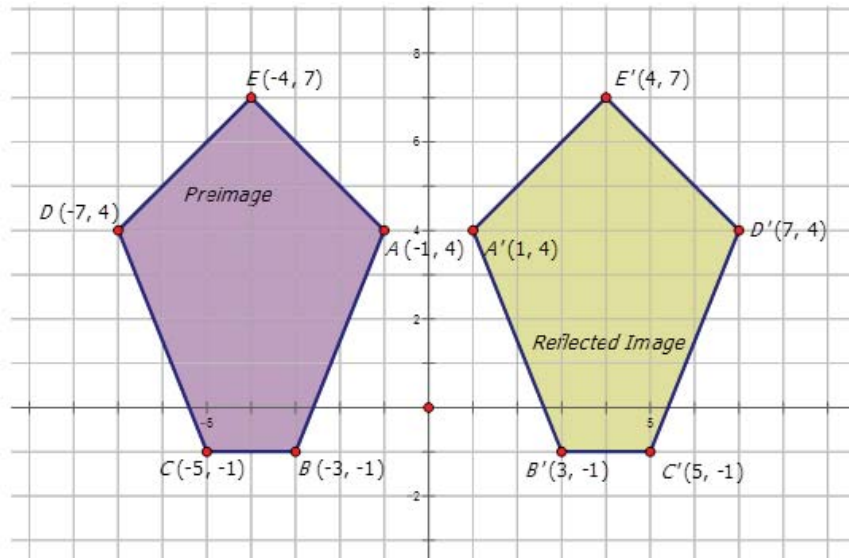
1.



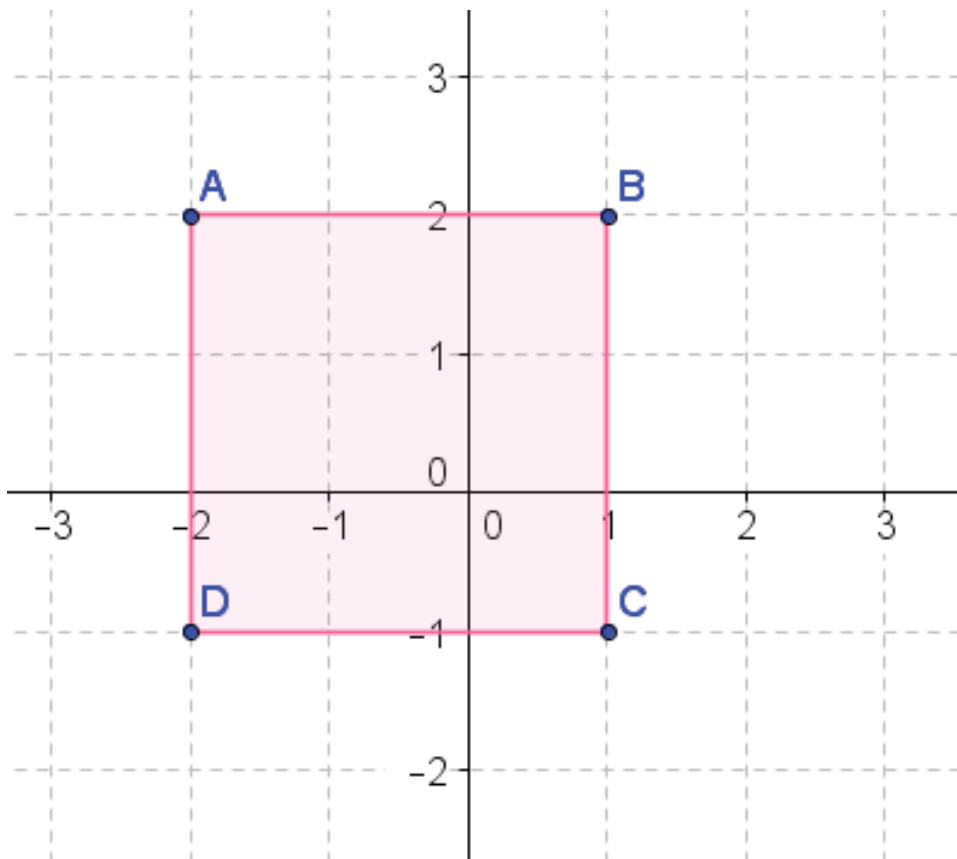
2.



3.

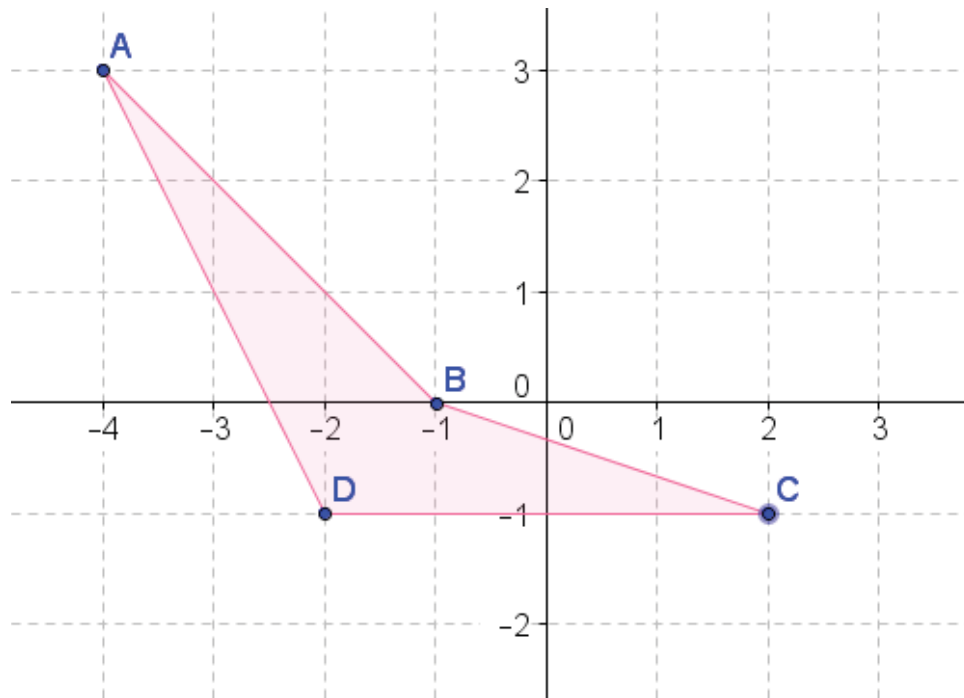


### Practice

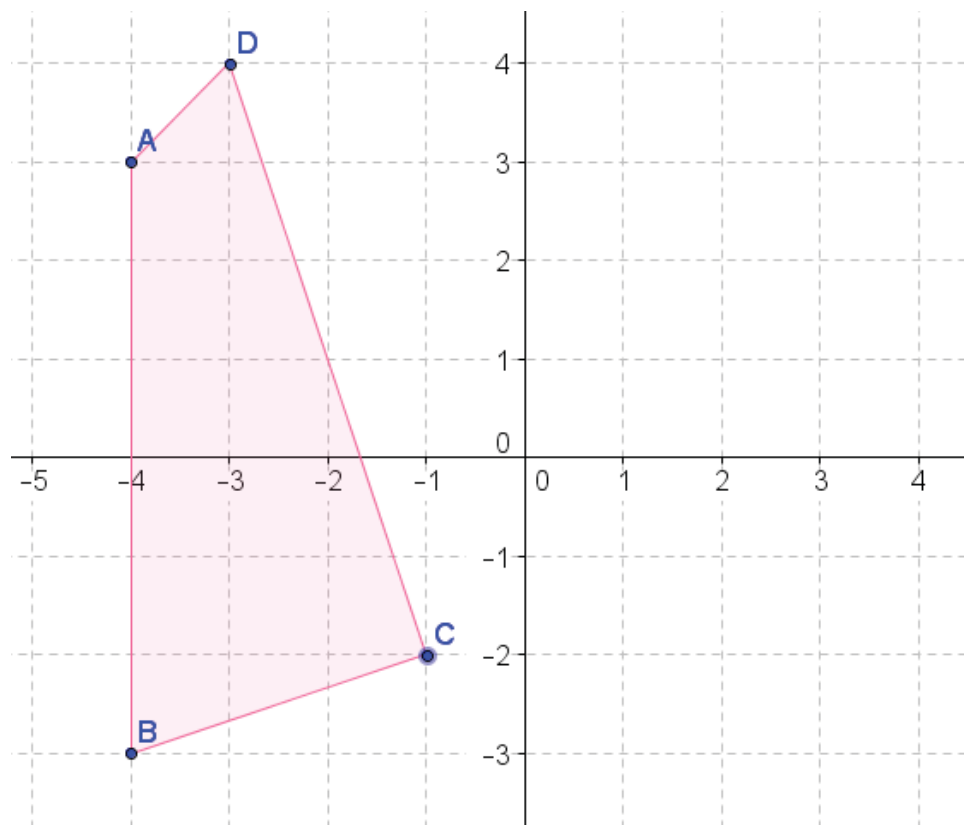


1. Reflect the above figure across the x-axis.
2. Reflect the above figure across the y-axis.
3. Reflect the above figure across the line  $y = x$ .

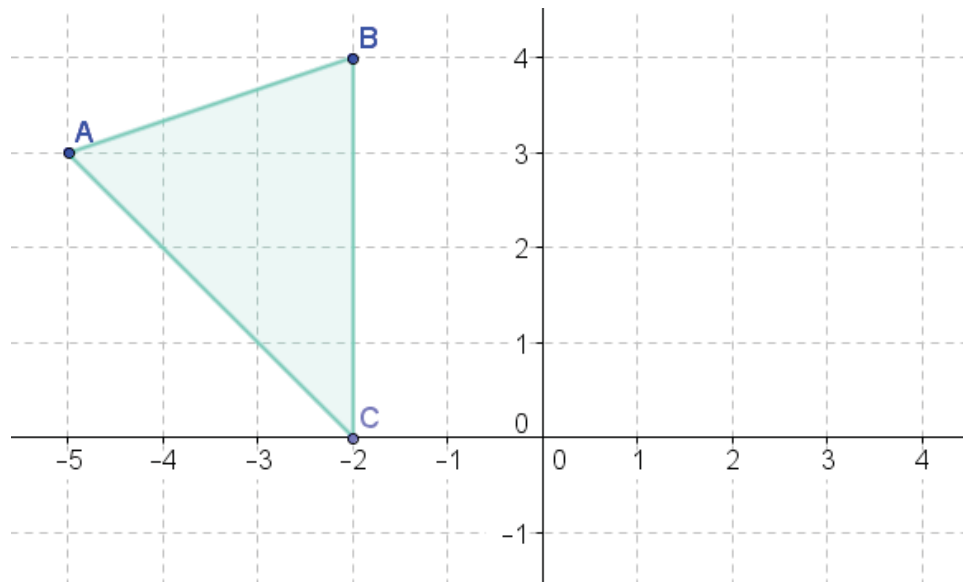




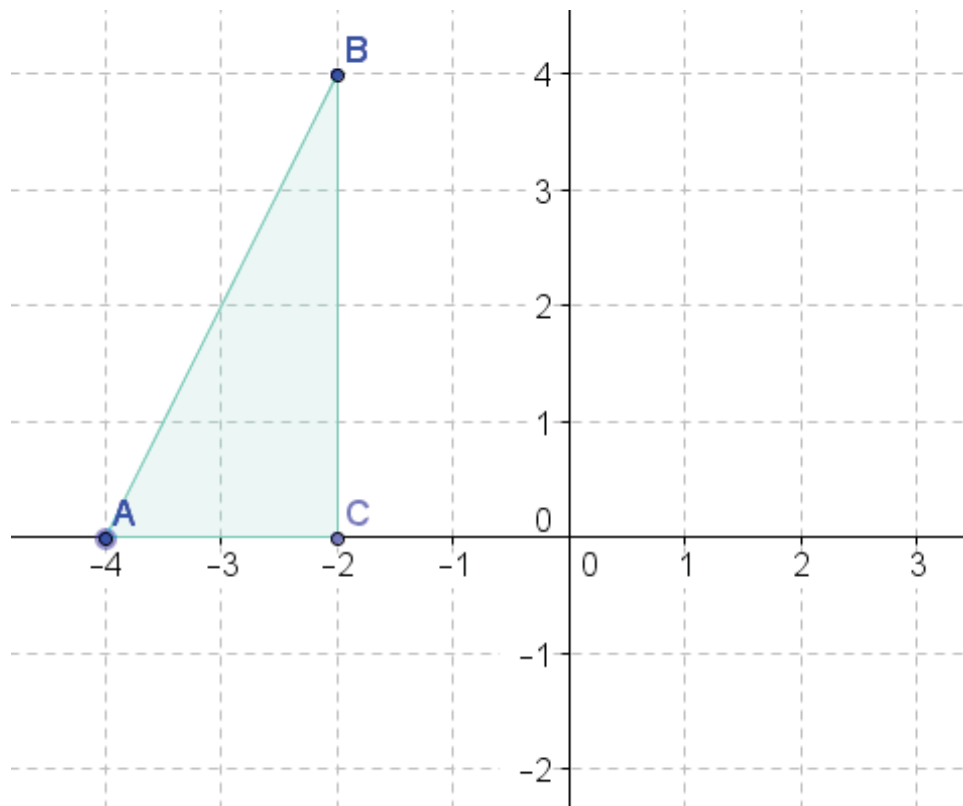
4. Reflect the above figure across the x-axis.
5. Reflect the above figure across the y-axis.
6. Reflect the above figure across the line  $y = x$ .



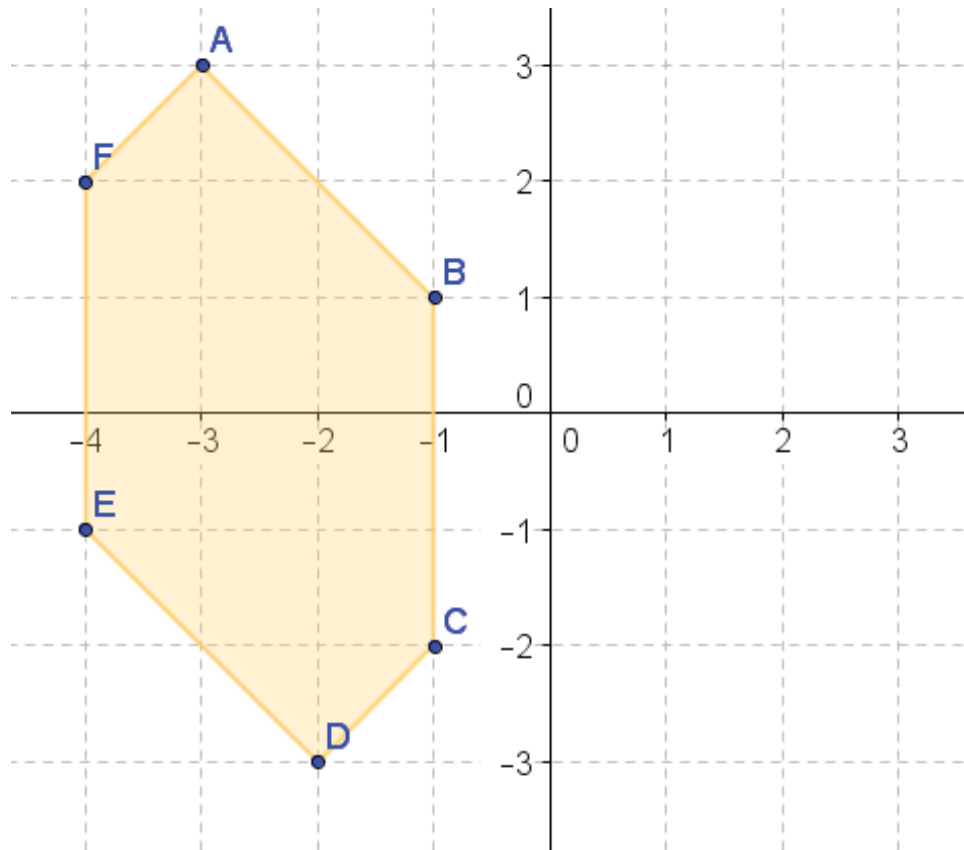
7. Reflect the above figure across the x-axis.
8. Reflect the above figure across the y-axis.
9. Reflect the above figure across the line  $y = x$ .



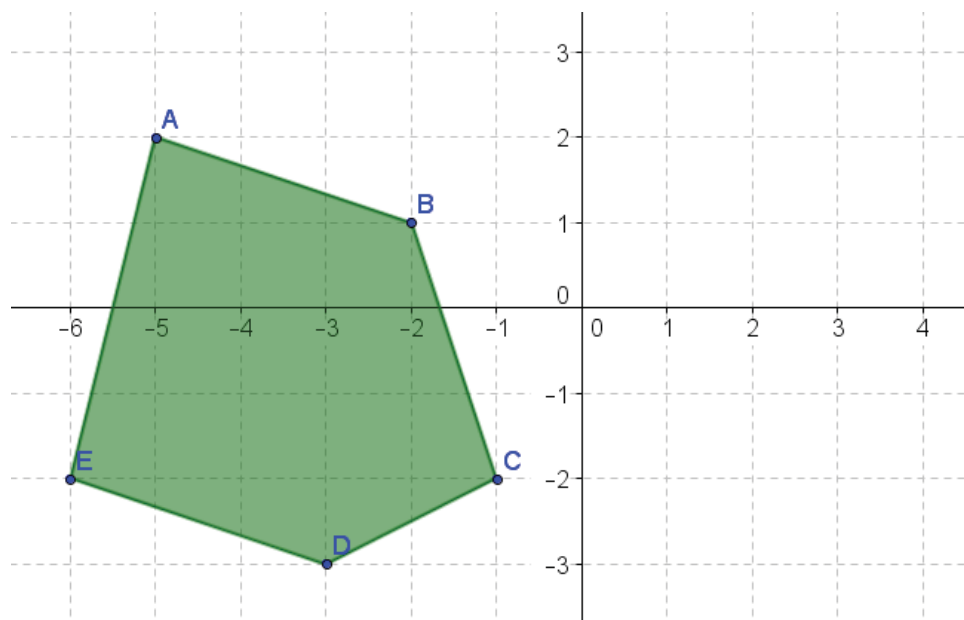
10. Reflect the above figure across the  $x$ -axis.
11. Reflect the above figure across the  $y$ -axis.
12. Reflect the above figure across the line  $y = x$ .



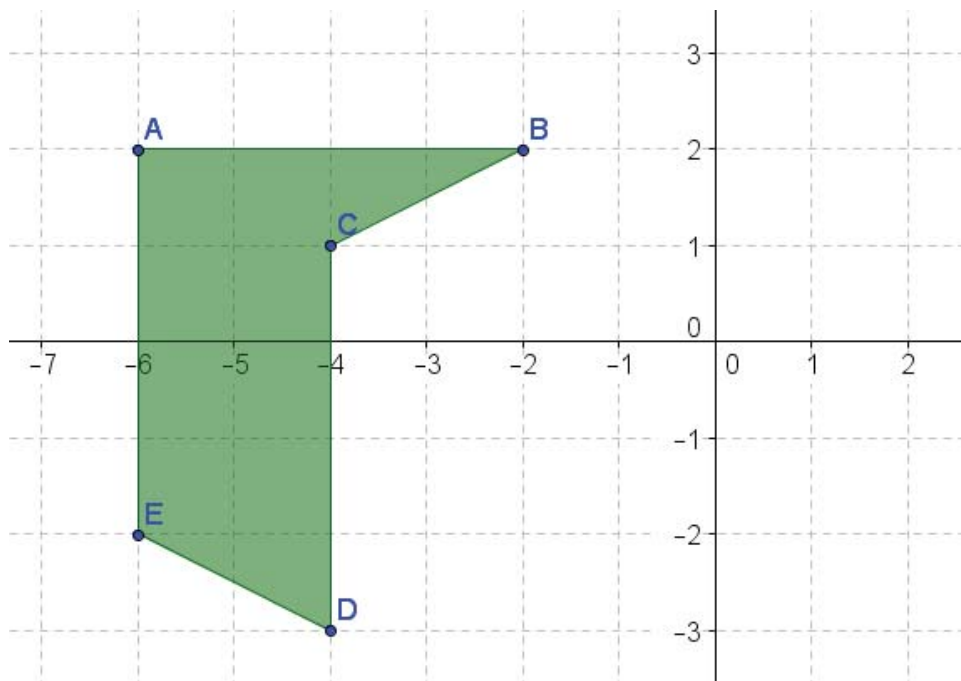
13. Reflect the above figure across the  $x$ -axis.
14. Reflect the above figure across the  $y$ -axis.
15. Reflect the above figure across the line  $y = x$ .



16. Reflect the above figure across the x-axis.
17. Reflect the above figure across the y-axis.
18. Reflect the above figure across the line  $y = x$ .



19. Reflect the above figure across the x-axis.
20. Reflect the above figure across the y-axis.
21. Reflect the above figure across the line  $y = x$ .

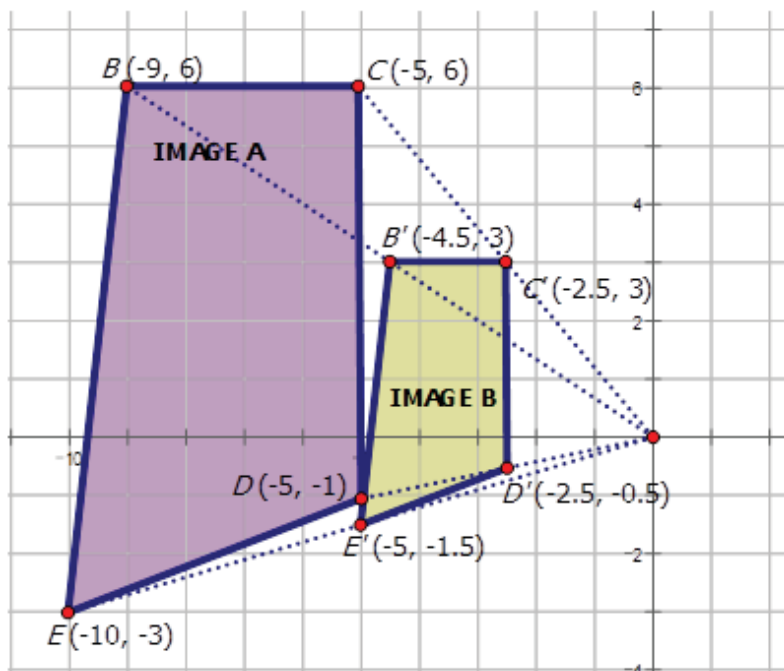


22. Reflect the above figure across the  $x$ -axis.
23. Reflect the above figure across the  $y$ -axis.
24. Reflect the above figure across the line  $y = x$ .

## 1.6 Rules for Dilations

Here you will learn the notation for describing a dilation.

The figure below shows a dilation of two trapezoids. Write the mapping rule for the dilation of Image A to Image B.



### Watch This

First watch this video to learn about writing rules for dilations.



**MEDIA**

Click image to the left for more content.

[CK-12 FoundationChapter10RulesforDilationsA](#)

Then watch this video to see some examples.



**MEDIA**

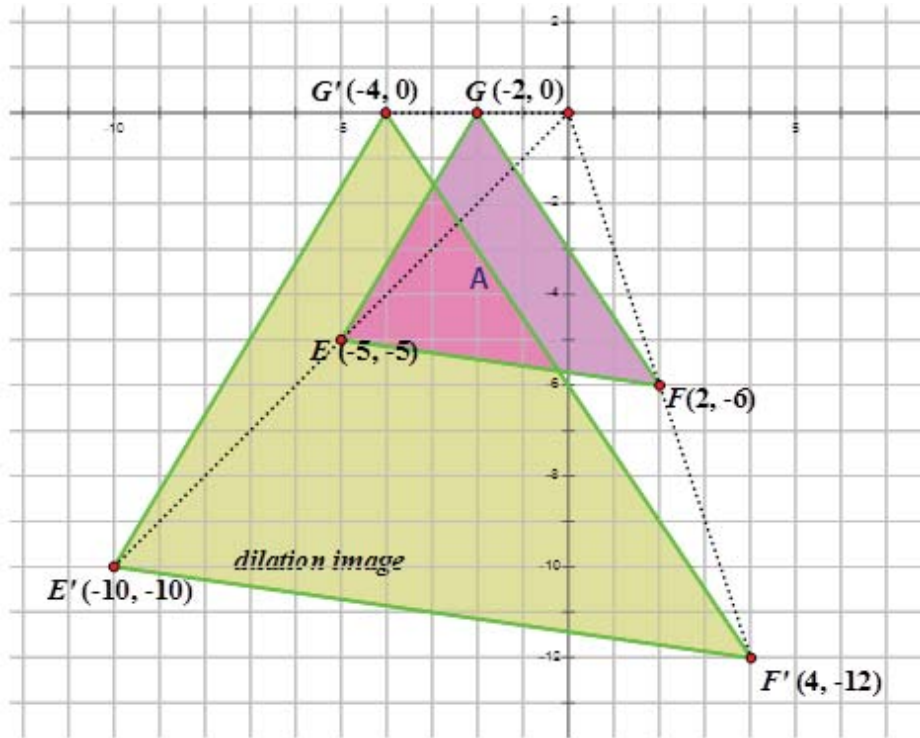
Click image to the left for more content.

[CK-12 FoundationChapter10RulesforDilationsB](#)

## Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A dilation is a type of transformation that enlarges or reduces a figure (called the preimage) to create a new figure (called the image). The scale factor,  $r$ , determines how much bigger or smaller the dilation image will be compared to the preimage.

Look at the diagram below:



The Image A has undergone a dilation about the origin with a scale factor of 2. Notice that the points in the dilation image are all double the coordinate points in the preimage. A dilation with a scale factor  $k$  about the origin can be described using the following notation:

$$D_k(x, y) = (kx, ky)$$

$k$  will always be a value that is greater than 0.

**TABLE 1.2:**

### Scale Factor, $k$

$$k > 1$$

$$0 < k < 1$$

$$k = 1$$

### Size change for preimage

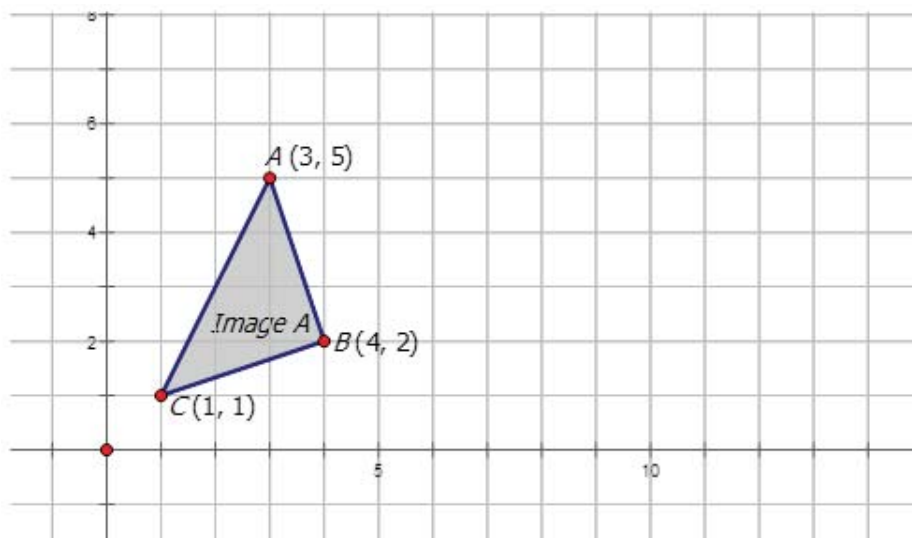
Dilation image is larger than preimage

Dilation image is smaller than preimage

Dilation image is the same size as the preimage

## Example A

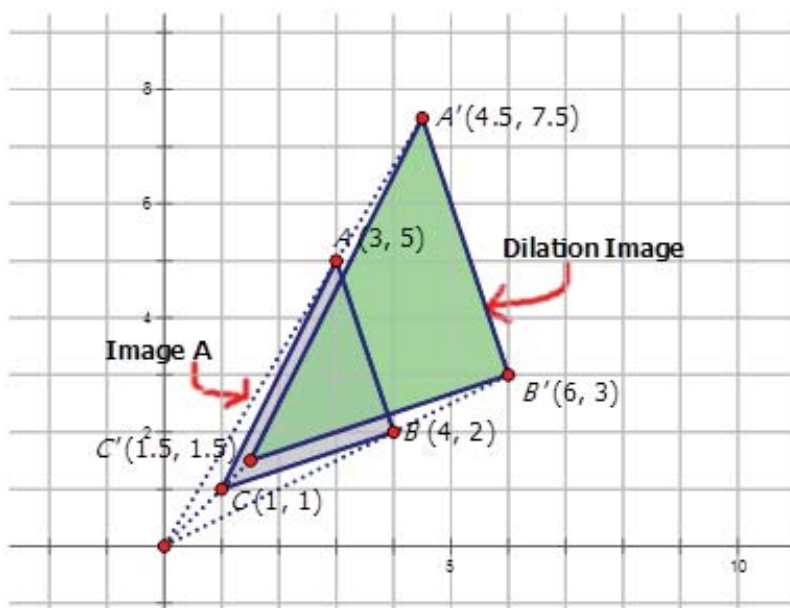
The mapping rule for the dilation applied to the triangle below is  $(x, y) \rightarrow (1.5x, 1.5y)$ . Draw the dilation image.



**Solution:** With a scale factor of 1.5, each coordinate point will be multiplied by 1.5.

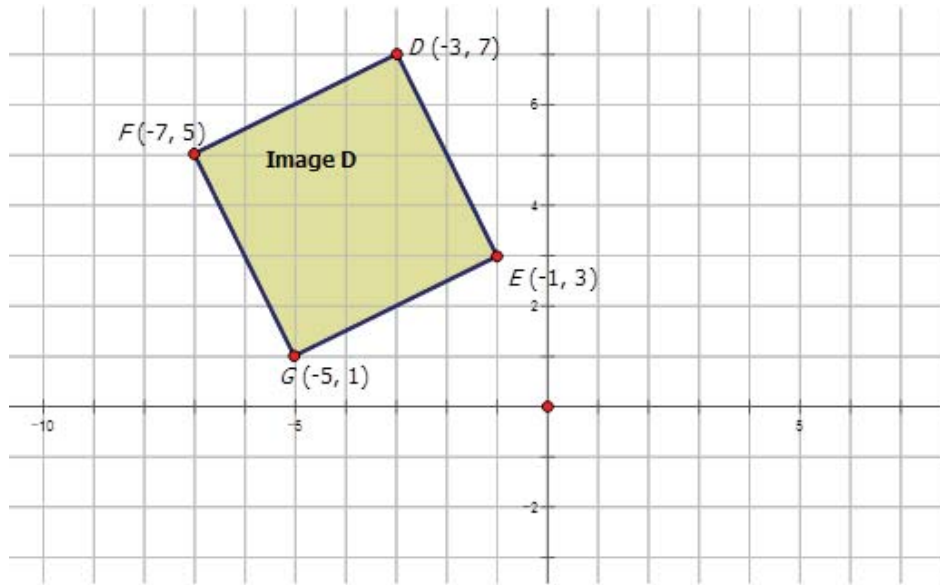
Image A	$A(3, 5)$	$B(4, 2)$	$C(1, 1)$
Dilation Image	$A'(4.5, 7.5)$	$B'(6, 3)$	$C'(1.5, 1.5)$

The dilation image looks like the following:



### Example B

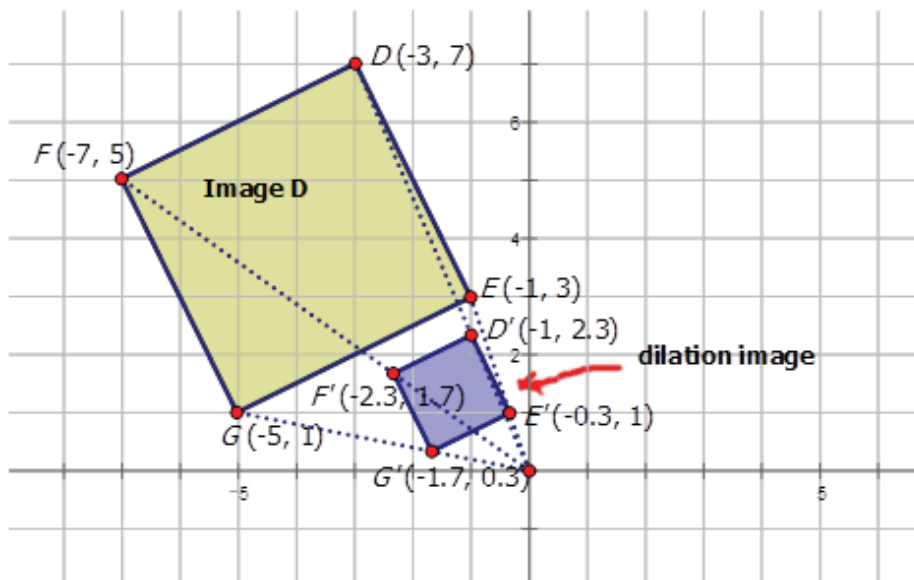
The mapping rule for the dilation applied to the diagram below is  $(x, y) \rightarrow (\frac{1}{3}x, \frac{1}{3}y)$ . Draw the dilation image.



**Solution:** With a scale factor of  $\frac{1}{3}$ , each coordinate point will be multiplied by  $\frac{1}{3}$ .

Image <i>D</i>	$D(-3, 7)$	$E(-1, 3)$	$F(-7, 5)$	$G(-5, 1)$
Dilation Image	$D'(-1, 2.3)$	$E'(-0.3, 1)$	$F'(-2.3, 1.7)$	$G'(-1.7, 0.3)$

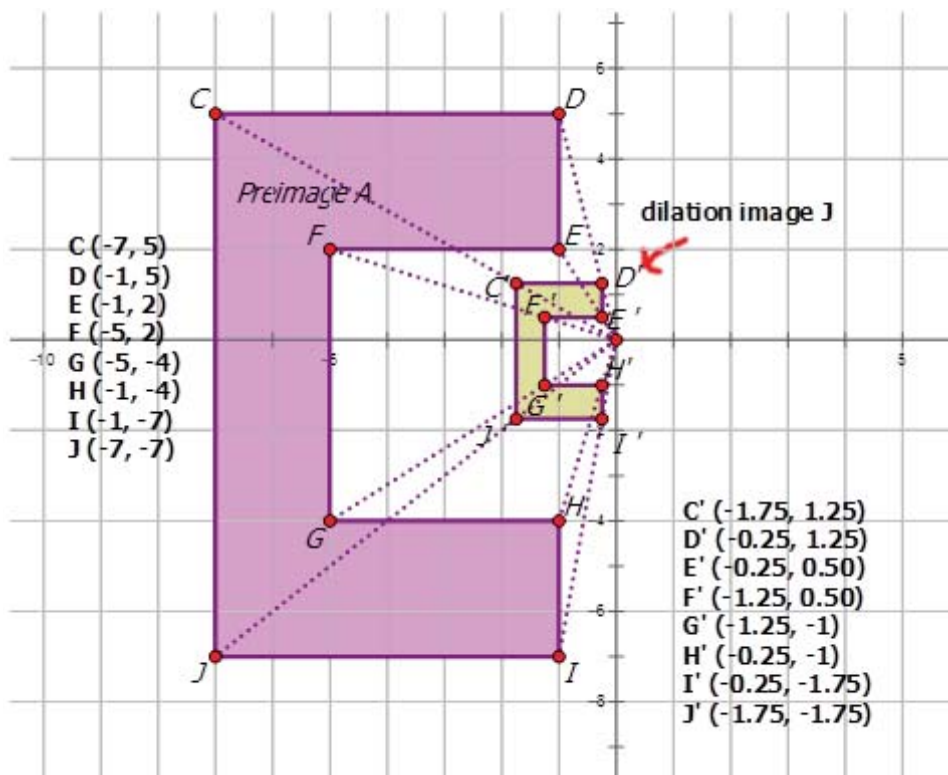
The dilation image looks like the following:



**Example C**

Write the notation that represents the dilation of the preimage A to the dilation image J in the diagram below.



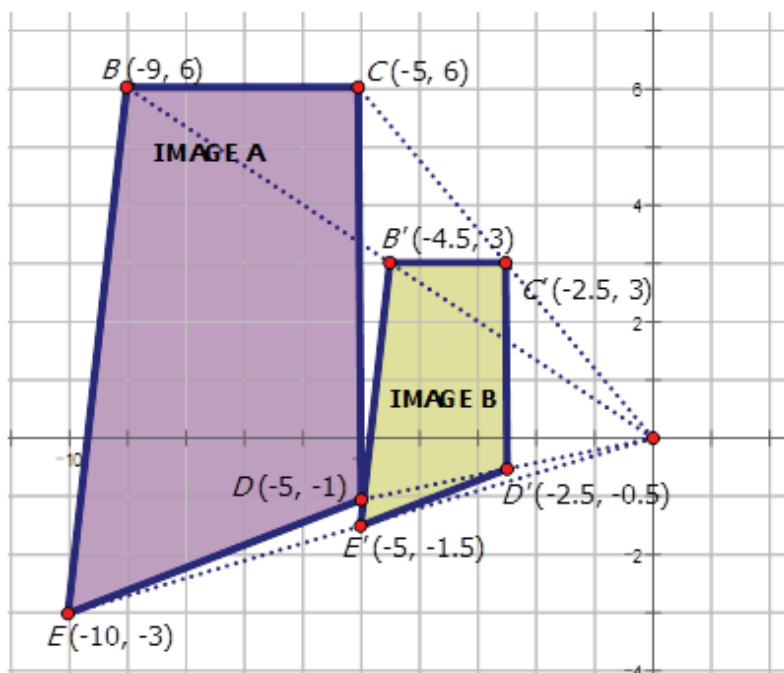


**Solution:** First, pick a point in the diagram to use to see how it has been affected by the dilation.

$$C : (-7, 5) \quad C' : (-1.75, 1.25)$$

Notice how both the  $x$ - and  $y$ -coordinates are multiplied by  $\frac{1}{4}$ . This indicates that the preimage A undergoes a dilation about the origin by a scale factor of  $\frac{1}{4}$  to form the dilation image J. Therefore the mapping notation is  $(x, y) \rightarrow (\frac{1}{4}x, \frac{1}{4}y)$ .

**Concept Problem Revisited**



Look at the points in each image:

Image A	$B(-9, 6)$	$C(-5, 6)$	$D(-5, -1)$	$E(-10, -3)$
Image B	$B'(-4.5, 3)$	$C'(-2.5, 3)$	$D'(-2.5, -0.5)$	$E'(-5, -1.5)$

Notice that the coordinate points in Image B (the dilation image) are  $\frac{1}{2}$  that found in Image A. Therefore the Image A undergoes a dilation about the origin of scale factor  $\frac{1}{2}$ . To write a mapping rule for this dilation you would write:  $(x, y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$ .

## Vocabulary

### Notation Rule

A **notation rule** has the following form  $D_k(x, y) = (kx, ky)$  and tells you that the preimage has undergone a dilation about the origin by scale factor  $k$ . If  $k$  is greater than one, the dilation image will be larger than the preimage. If  $k$  is between 0 and 1, the dilation image will be smaller than the preimage. If  $k$  is equal to 1, you will have a dilation image that is congruent to the preimage. The mapping rule corresponding to a dilation notation would be:  $(x, y) \rightarrow (kx, ky)$

### Center Point

The **center point** is the center of the dilation. You use the center point to measure the distances to the preimage and the dilation image. It is these distances that determine the scale factor.

### Dilation

A **dilation** is a transformation that enlarges or reduces the size of a figure.

### Scale Factor

The **scale factor** determines how much bigger or smaller the dilation image will be compared to the preimage. The scale factor often uses the symbol  $r$ .

### Image

In a transformation, the final figure is called the **image**.

### Preimage

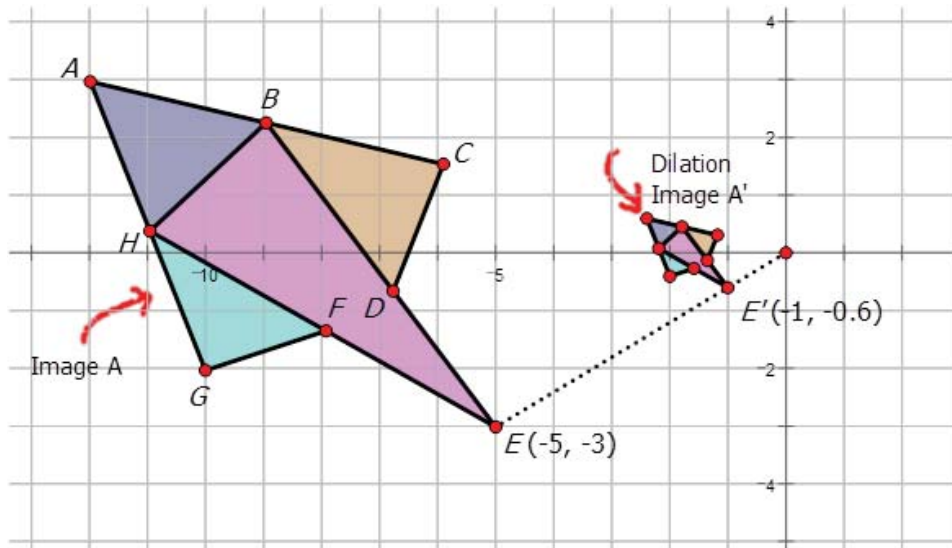
In a transformation, the original figure is called the **preimage**.

### Transformation

A **transformation** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

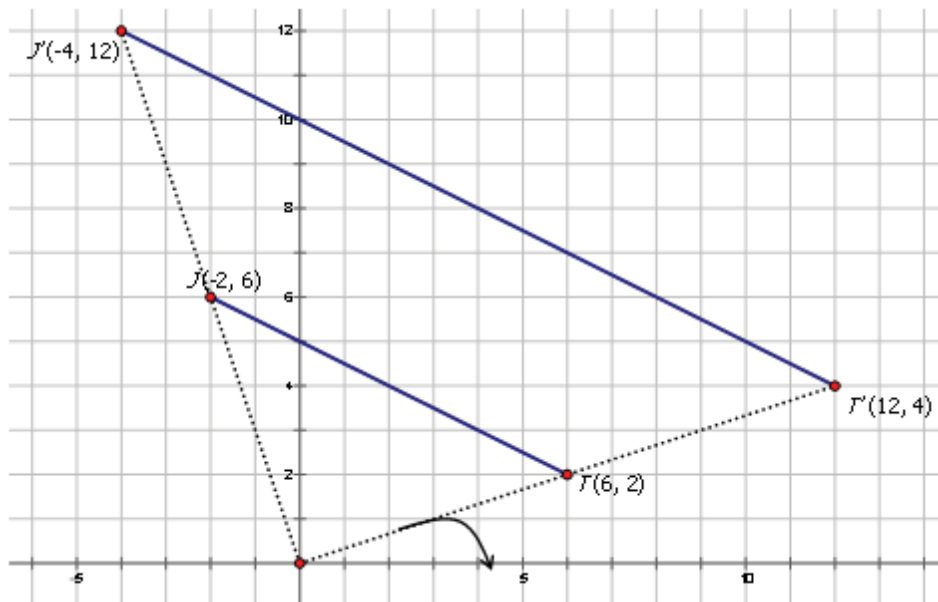
## Guided Practice

1. Thomas describes a dilation of point  $JT$  with vertices  $J(-2, 6)$  to  $T(6, 2)$  to point  $J'T'$  with vertices  $J'(-4, 12)$  and  $T'(12, 4)$ . Write the notation to describe this dilation for Thomas.
2. Given the points  $A(12, 8)$  and  $B(8, 4)$  on a line undergoing a dilation to produce  $A'(6, 4)$  and  $B'(4, 2)$ , write the notation that represents the dilation.
3. Janet was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.



**Answers:**

1.



Since the  $x$ - and  $y$ -coordinates are each multiplied by 2, the *scale factor* is 2. The mapping notation is:  $(x,y) \rightarrow (2x,2y)$

2. In order to write the notation to describe the dilation, choose one point on the preimage and then the corresponding point on the dilation image to see how the point has moved. Notice that point  $EA$  is:

$$A(12,8) \rightarrow A'(6,4)$$

Since both  $x$ - and  $y$ -coordinates are multiplied by  $\frac{1}{2}$ , the dilation is about the origin has a scale factor of  $\frac{1}{2}$ . The notation for this dilation would be:  $(x,y) \rightarrow (\frac{1}{2}x, \frac{1}{2}y)$ .

3. In order to write the notation to describe the dilation, choose one point on the preimage  $A$  and then the corresponding point on the dilation image  $A'$  to see how the point has changed. Notice that point  $E$  is shown in the diagram:

$$E(-5, -3) \rightarrow E'(-1, -0.6)$$

Since both  $x$ - and  $y$ -coordinates are multiplied by  $\frac{1}{5}$ , the dilation is about the origin has a scale factor of  $\frac{1}{5}$ . The notation for this dilation would be:  $(x, y) \rightarrow (\frac{1}{5}x, \frac{1}{5}y)$ .

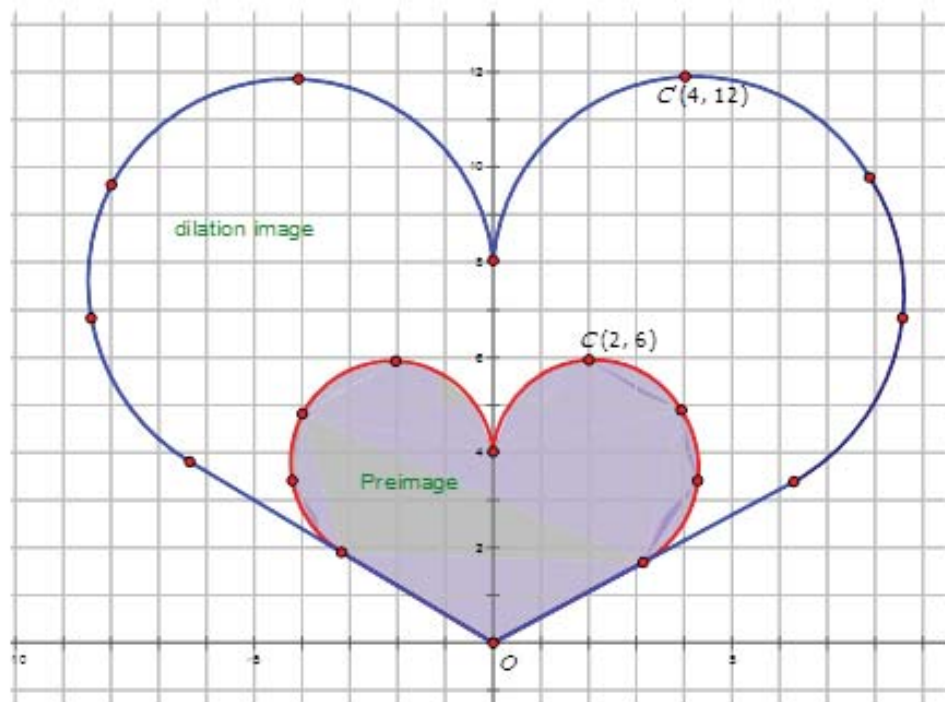
### Practice

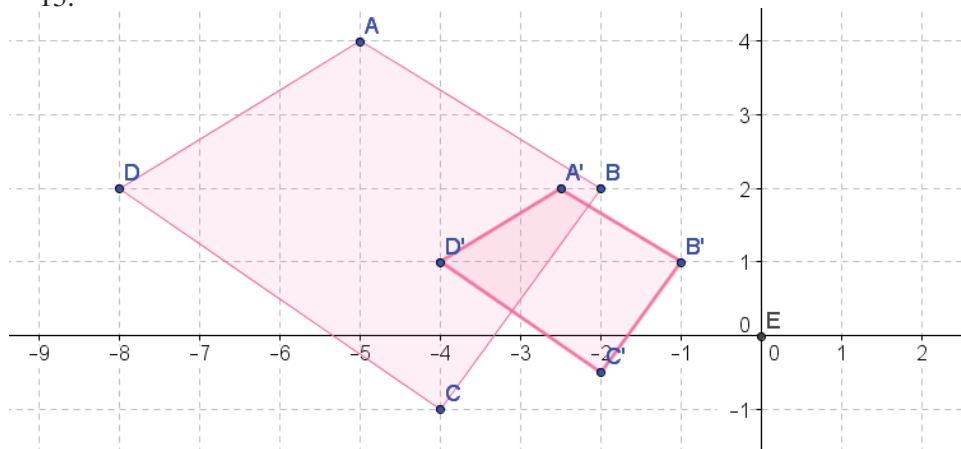
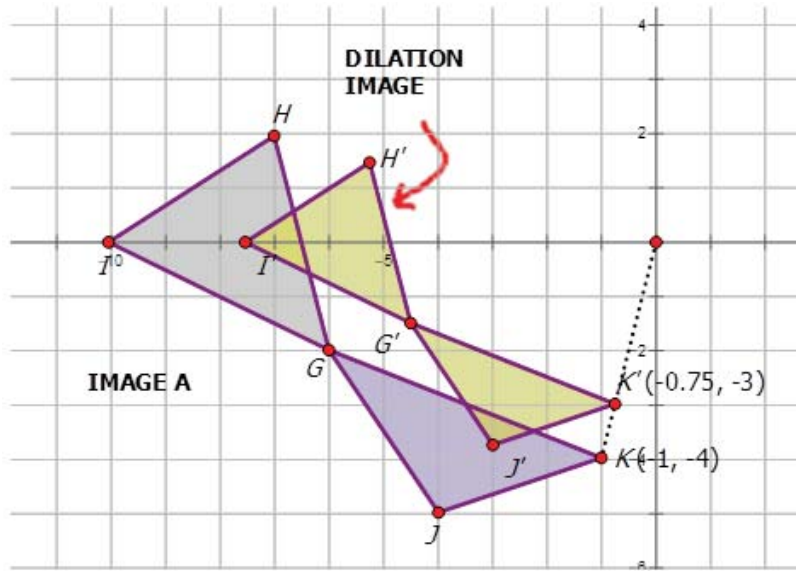
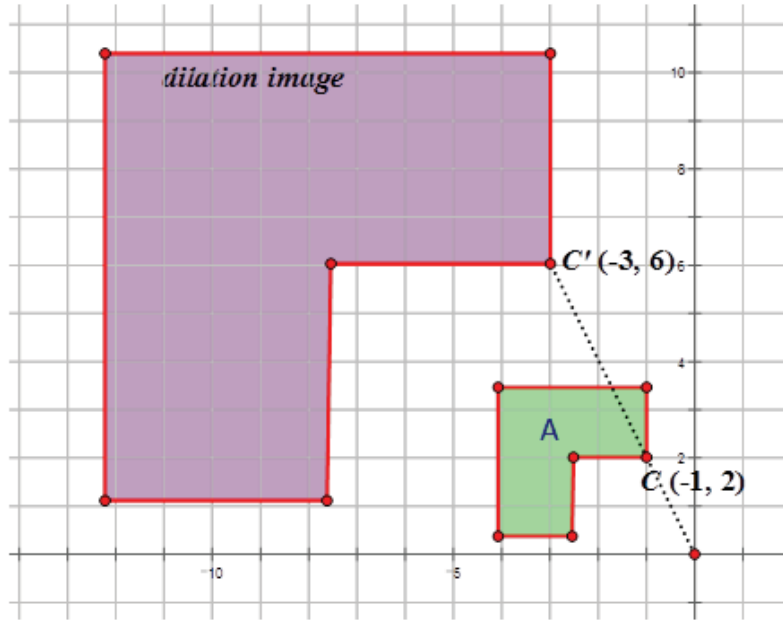
Complete the following table. Assume that the center of dilation is the origin.

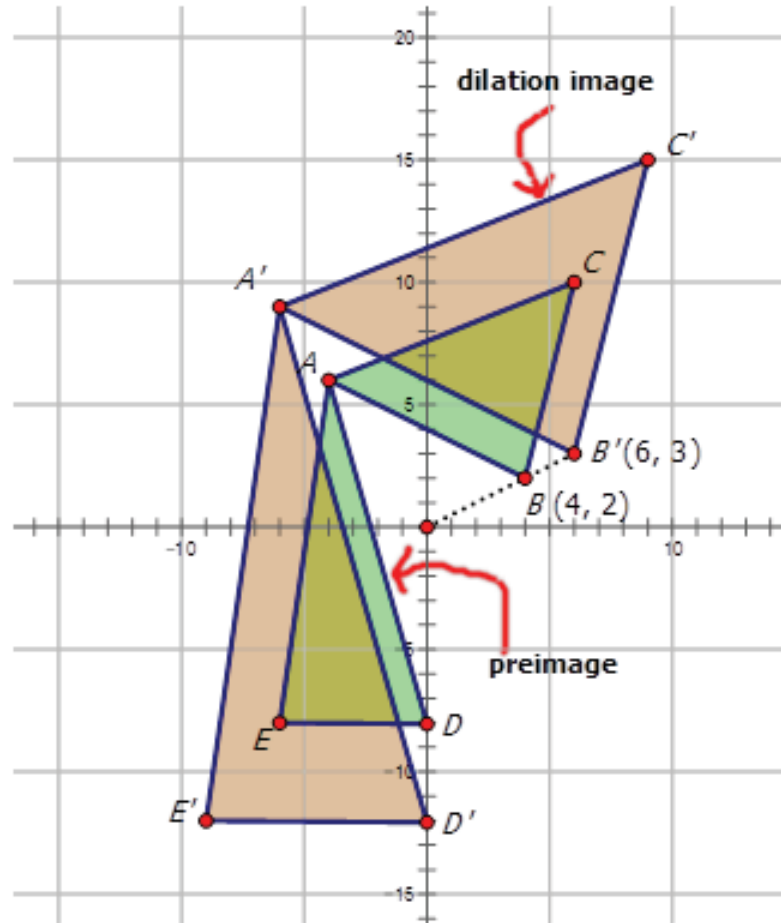
**TABLE 1.3:**

Starting Point	$D_2$	$D_5$	$D_{\frac{1}{2}}$	$D_{\frac{3}{4}}$
1. (1, 4)				
2. (4, 2)				
3. (2, 0)				
4. (-1, 2)				
5. (-2, -3)				
6. (9, 4)				
7. (-1, 3)				
8. (-5, 2)				
9. (2, 6)				
10. (-5, 7)				

Write the notation that represents the dilation of the preimage to the image for each diagram below.





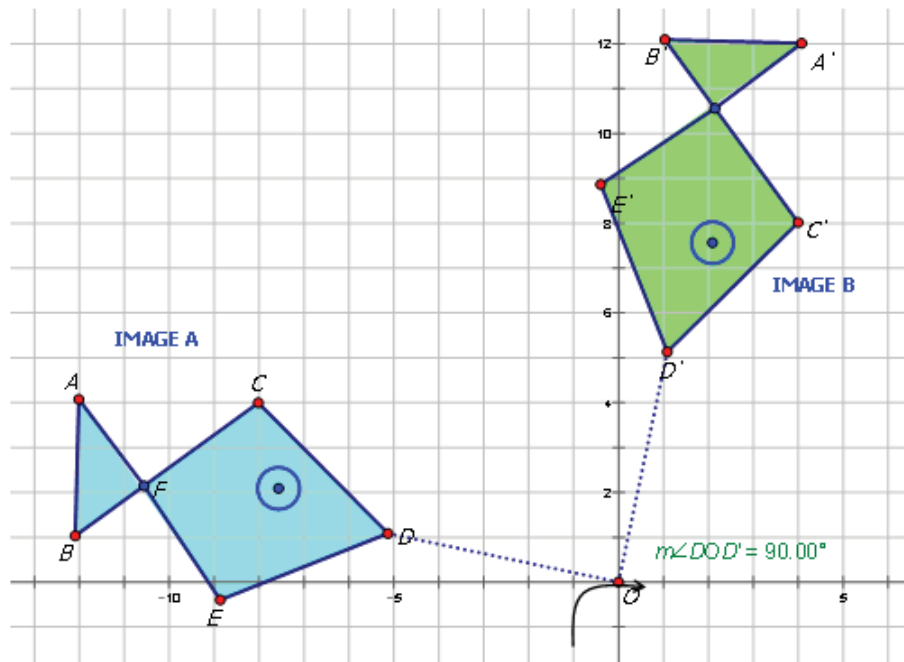


15.

## 1.7 Rules for Rotations

Here you will learn the notation used for rotations.

The figure below shows a pattern of two fish. Write the mapping rule for the rotation of Image A to Image B.



### Watch This

First watch this video to learn about writing rules for rotations.



MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10RulesforRotationsA](#)

Then watch this video to see some examples.



MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10RulesforRotationsB](#)

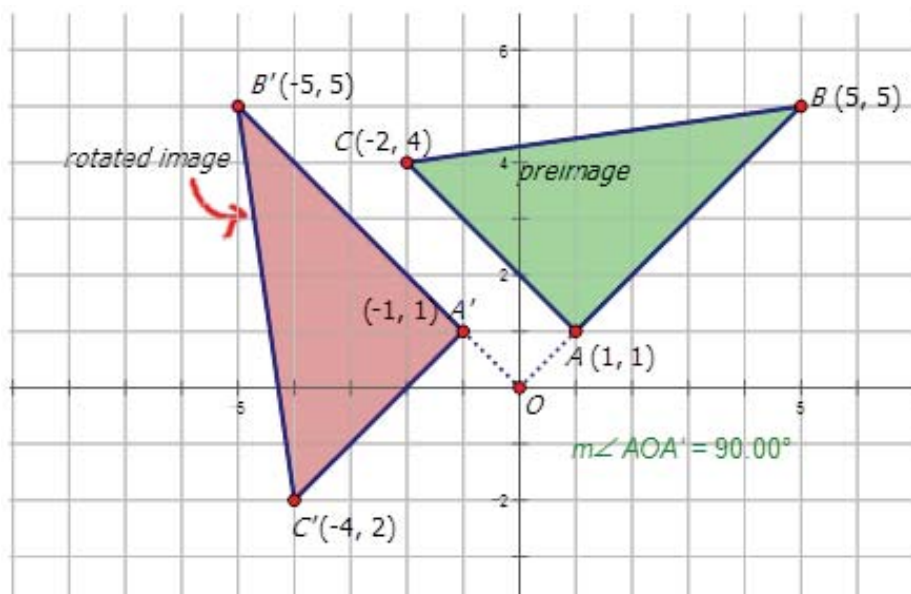
## Guidance

In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A rotation is an example of a transformation where a figure is rotated about a specific point (called the center of rotation), a certain number of degrees. Common rotations about the origin are shown below:

**TABLE 1.4:**

Center of Rotation	Angle of Rotation	Preimage (Point $P$ )	Rotated (Point $P'$ )	Image	Notation (Point $P'$ )
(0, 0)	$90^\circ$ (or $-270^\circ$ )	$(x, y)$	$(-y, x)$		$(x, y) \rightarrow (-y, x)$
(0, 0)	$180^\circ$ (or $-180^\circ$ )	$(x, y)$	$(-x, -y)$		$(x, y) \rightarrow (-x, -y)$
(0, 0)	$270^\circ$ (or $-90^\circ$ )	$(x, y)$	$(y, -x)$		$(x, y) \rightarrow (y, -x)$

You can describe rotations in words, or with notation. Consider the image below:



Notice that the preimage is rotated about the origin  $90^\circ$  CCW. If you were to describe the rotated image using notation, you would write the following:

$$R_{90^\circ}(x, y) = (-y, x)$$

### Example A

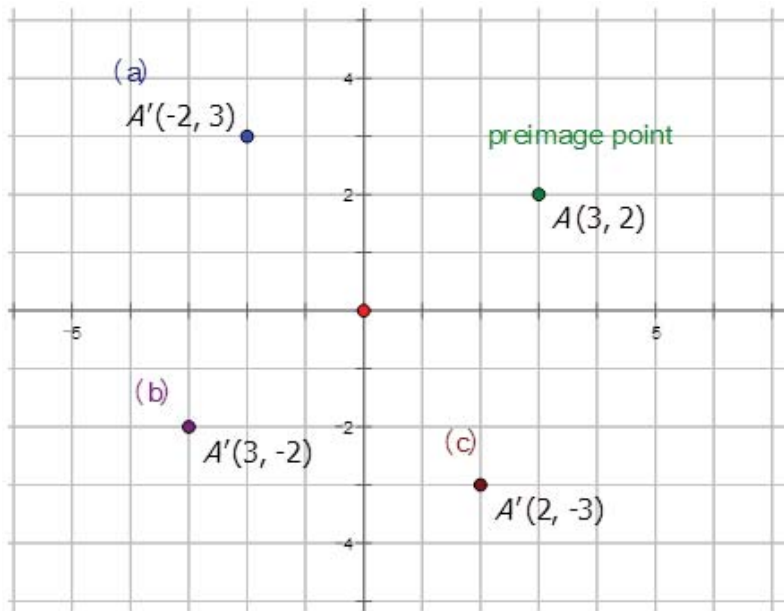
Find an image of the point  $(3, 2)$  that has undergone a clockwise rotation:

- about the origin at  $90^\circ$ ,
- about the origin at  $180^\circ$ , and
- about the origin at  $270^\circ$ .

Write the notation to describe the rotation.

**Solution:**



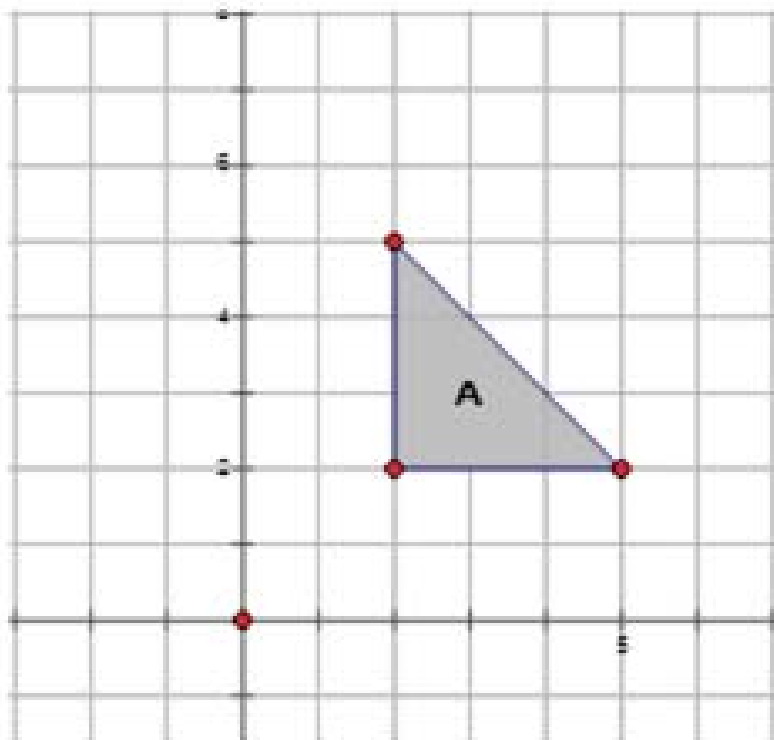


- a) Rotation about the origin at  $90^\circ$  :  $R_{90^\circ}(x, y) = (-y, x)$   
b) Rotation about the origin at  $180^\circ$  :  $R_{180^\circ}(x, y) = (-x, -y)$   
c) Rotation about the origin at  $270^\circ$  :  $R_{270^\circ}(x, y) = (y, -x)$

### Example B

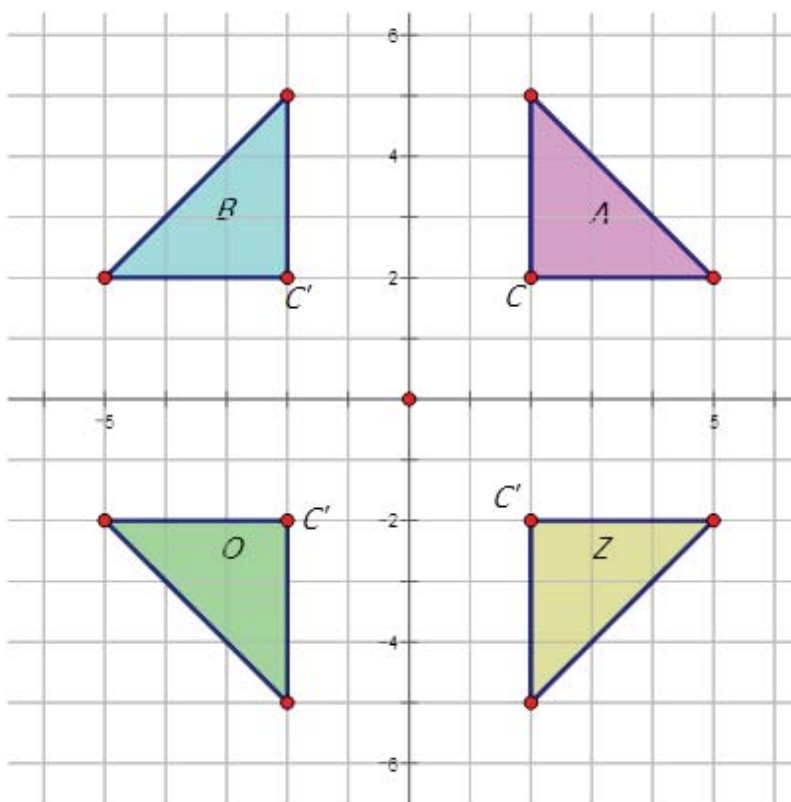
Rotate Image A in the diagram below:

- a) about the origin at  $90^\circ$ , and label it *B*.  
b) about the origin at  $180^\circ$ , and label it *O*.  
c) about the origin at  $270^\circ$ , and label it *Z*.



Write notation for each to indicate the type of rotation.

**Solution:**



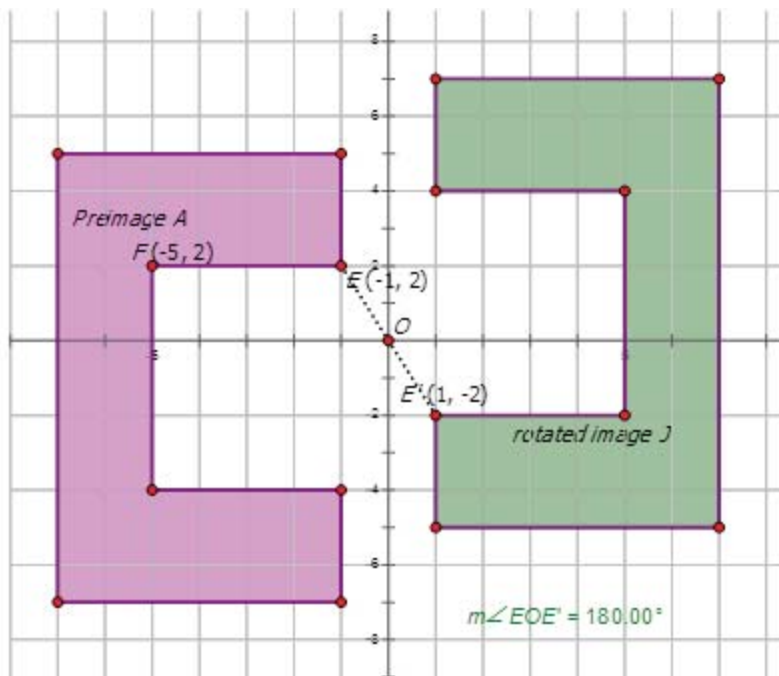
a) Rotation about the origin at  $90^\circ$ :  $R_{90^\circ}A \rightarrow B = R_{90^\circ}(x,y) \rightarrow (-y,x)$

b) Rotation about the origin at  $180^\circ$ :  $R_{180^\circ}A \rightarrow O = R_{180^\circ}(x,y) \rightarrow (-x,-y)$

c) Rotation about the origin at  $270^\circ$ :  $R_{270^\circ}A \rightarrow Z = R_{270^\circ}(x, y) \rightarrow (y, -x)$

### Example C

Write the notation that represents the rotation of the preimage A to the rotated image J in the diagram below.



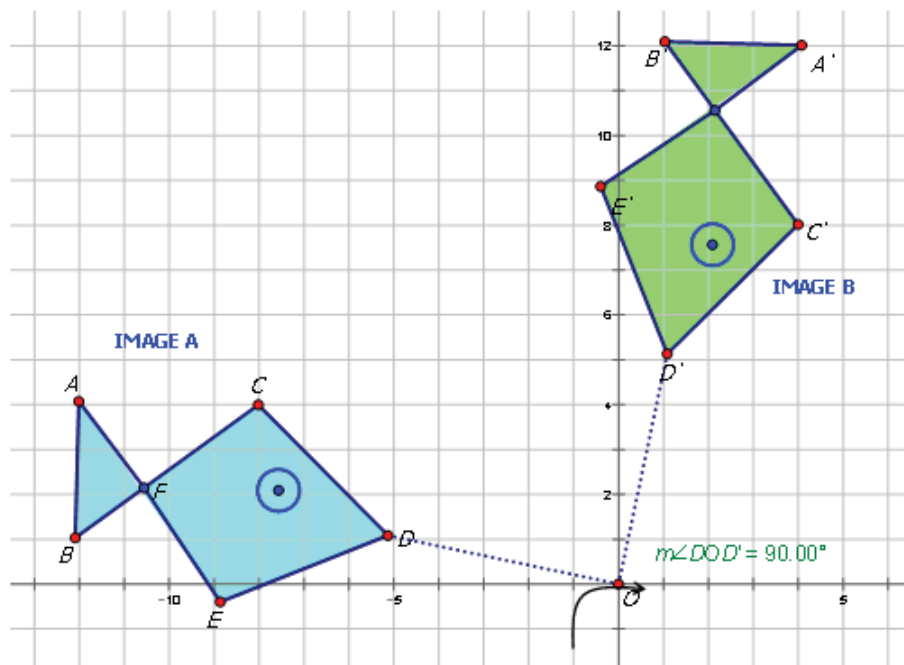
First, pick a point in the diagram to use to see how it is rotated.

$$E : (-1, 2) \quad E' : (1, -2)$$

Notice how both the  $x$ - and  $y$ -coordinates are multiplied by  $-1$ . This indicates that the preimage A is reflected about the origin by  $180^\circ$  CCW to form the rotated image J. Therefore the notation is  $R_{180^\circ}A \rightarrow J = R_{180^\circ}(x, y) \rightarrow (-x, -y)$ .

### Concept Problem Revisited

The figure below shows a pattern of two fish. Write the mapping rule for the rotation of Image A to Image B.



Notice that the angle measure is  $90^\circ$  and the direction is clockwise. Therefore the Image A has been rotated  $-90^\circ$  to form Image B. To write a rule for this rotation you would write:  $R_{270^\circ}(x,y) = (-y,x)$ .

## Vocabulary

### Notation Rule

A **notation rule** has the following form  $R_{180^\circ}A \rightarrow O = R_{180^\circ}(x,y) \rightarrow (-x,-y)$  and tells you that the image A has been rotated about the origin and both the  $x$ - and  $y$ -coordinates are multiplied by  $-1$ .

### Center of rotation

A **center of rotation** is the fixed point that a figure rotates about when undergoing a rotation.

### Rotation

A **rotation** is a transformation that rotates (turns) an image a certain amount about a certain point.

### Image

In a transformation, the final figure is called the **image**.

### Preimage

In a transformation, the original figure is called the **preimage**.

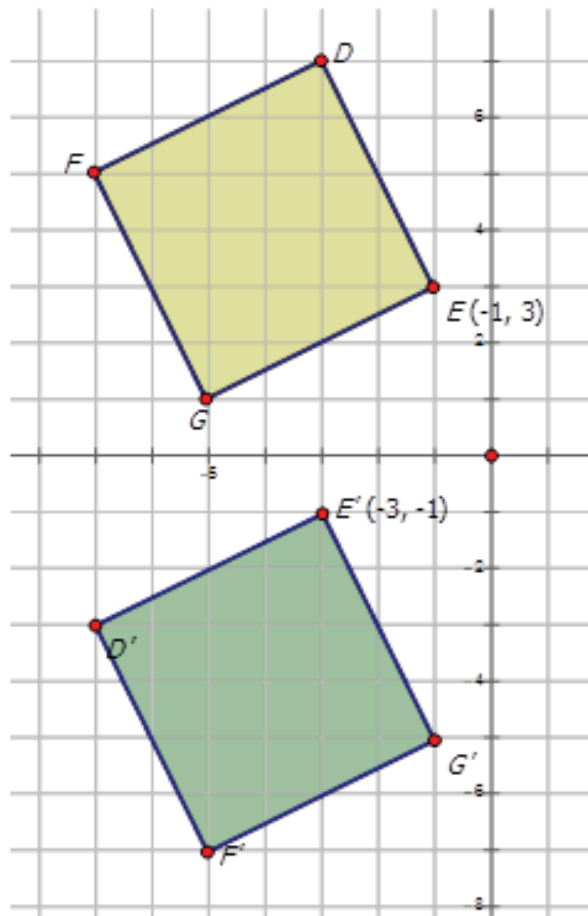
### Transformation

A **transformation** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

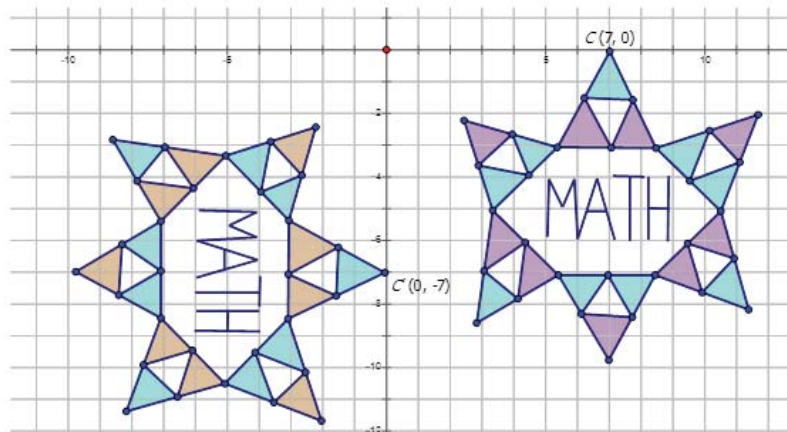
## Guided Practice

1. Thomas describes a rotation as point  $J$  moving from  $J(-2,6)$  to  $J'(6,2)$ . Write the notation to describe this rotation for Thomas.

2. Write the notation that represents the rotation of the yellow diamond to the rotated green diamond in the diagram below.

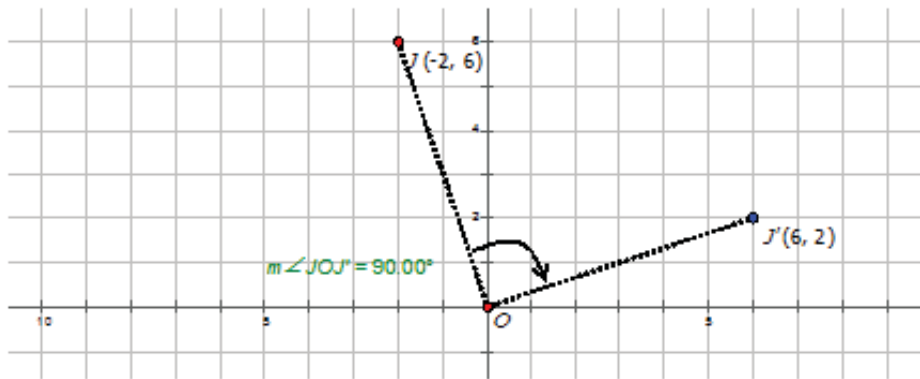


3. Karen was playing around with a drawing program on her computer. She created the following diagrams and then wanted to determine the transformations. Write the notation rule that represents the transformation of the purple and blue diagram to the orange and blue diagram.



**Answers:**

1.  $J : (-2, 6) \quad J' : (6, 2)$

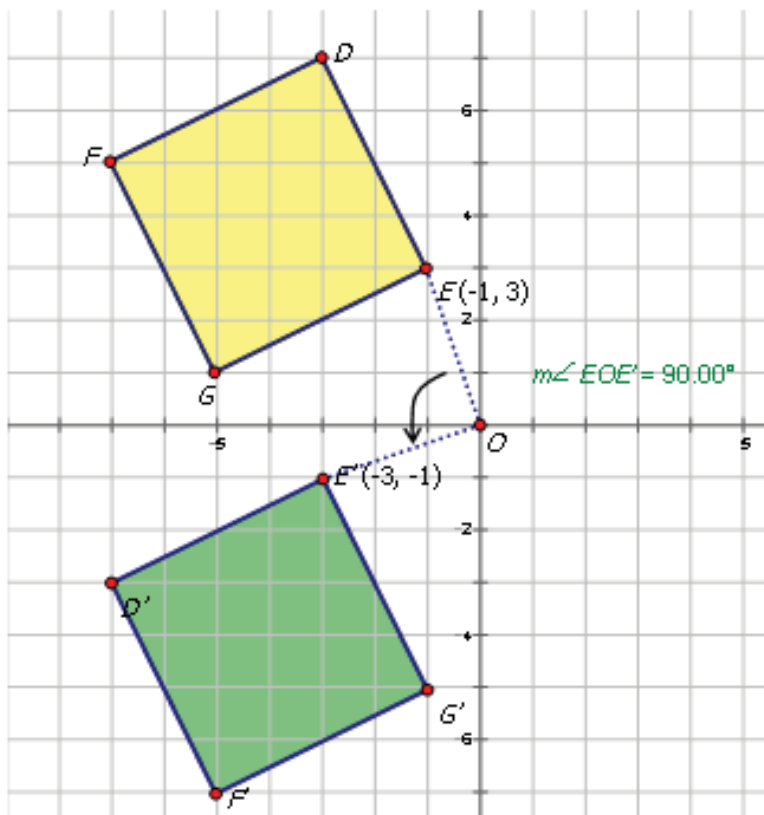


Since the  $x$ -coordinate is multiplied by  $-1$ , the  $y$ -coordinate remains the same, and finally the  $x$ - and  $y$ -coordinates change places, this is a rotation about the origin by  $270^\circ$  or  $-90^\circ$ . The notation is:  $R_{270^\circ}J \rightarrow J' = R_{270^\circ}(x,y) \rightarrow (y, -x)$

2. In order to write the notation to describe the rotation, choose one point on the preimage (the yellow diamond) and then the rotated point on the green diamond to see how the point has moved. Notice that point  $E$  is shown in the diagram:

$$E(-1, 3) \rightarrow E'(-3, -1)$$

Since both  $x$ - and  $y$ -coordinates are reversed places and the  $y$ -coordinate has been multiplied by  $-1$ , the rotation is about the origin  $90^\circ$ . The notation for this rotation would be:  $R_{90^\circ}(x,y) \rightarrow (-y,x)$ .



3. In order to write the notation to describe the transformation, choose one point on the preimage (purple and blue

diagram) and then the transformed point on the orange and blue diagram to see how the point has moved. Notice that point  $C$  is shown in the diagram:

$$C(7,0) \rightarrow C'(0,-7)$$

Since the  $x$ -coordinates only are multiplied by  $-1$ , and then  $x$ - and  $y$ -coordinates change places, the transformation is a rotation about the origin by  $270^\circ$ . The notation for this rotation would be:  $R_{270^\circ}(x,y) \rightarrow (y,-x)$ .

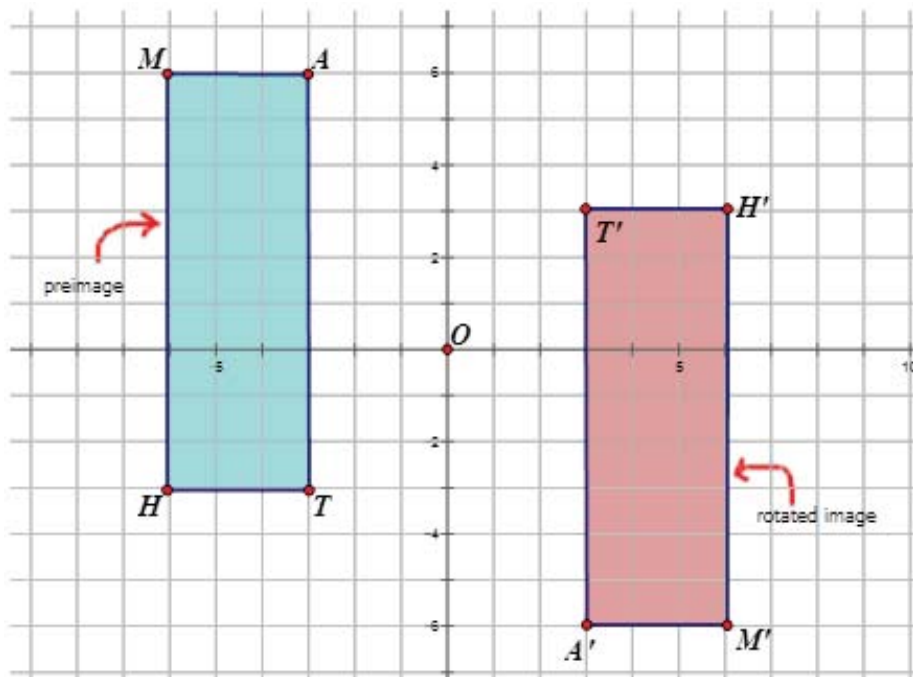
**Practice**

Complete the following table:

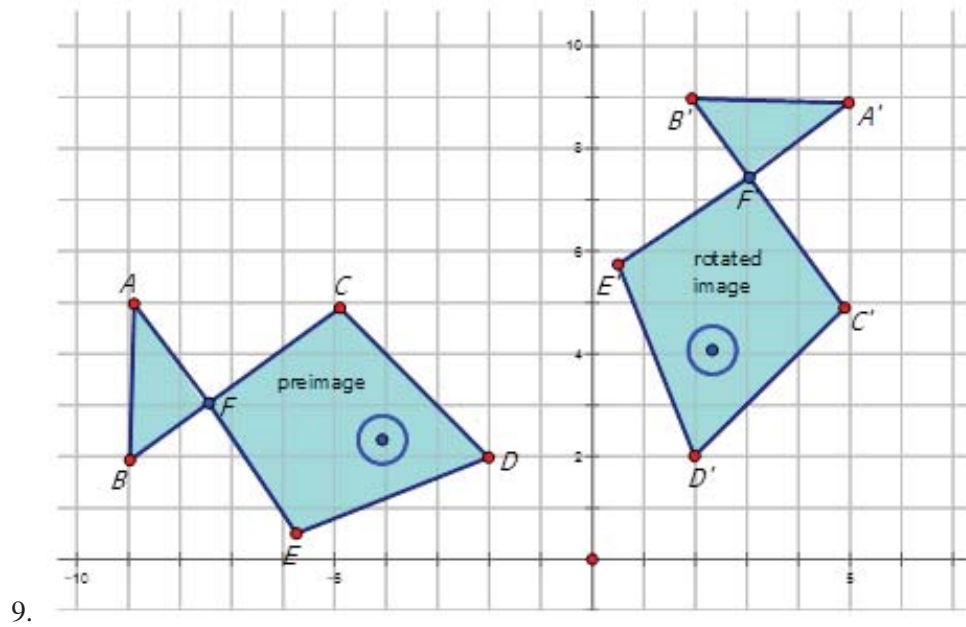
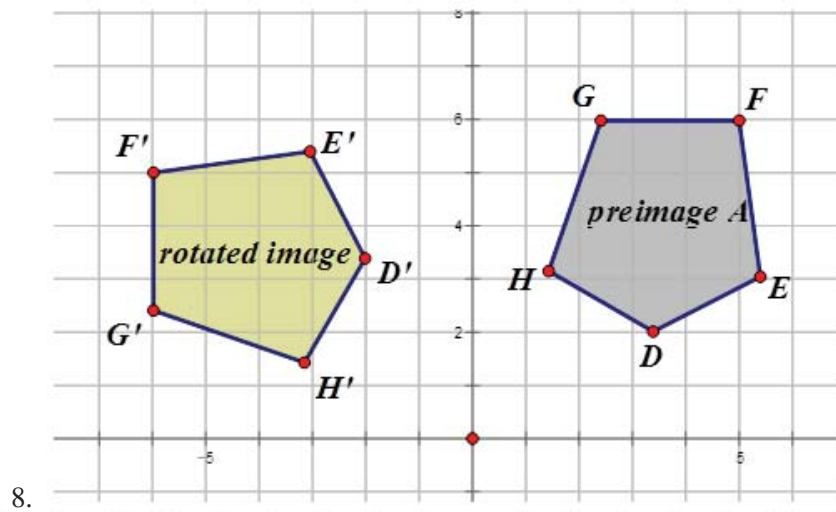
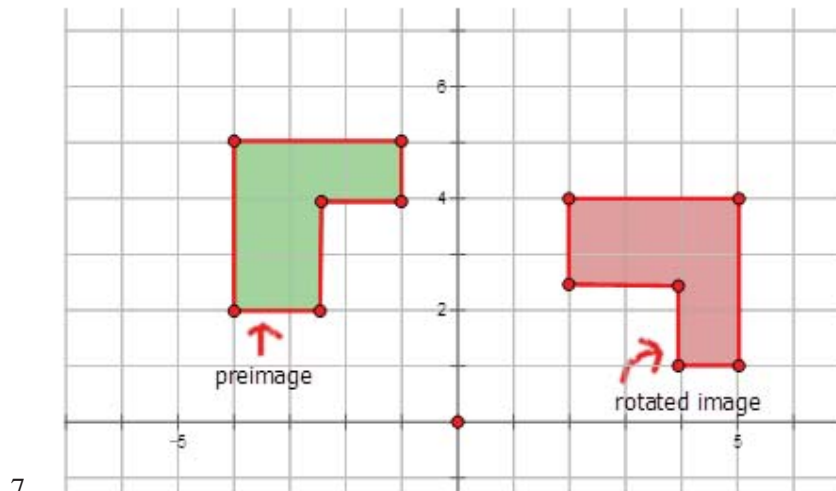
**TABLE 1.5:**

Starting Point	$90^\circ$ Rotation	$180^\circ$ Rotation	$270^\circ$ Rotation	$360^\circ$ Rotation
1. (1, 4)				
2. (4, 2)				
3. (2, 0)				
4. (-1, 2)				
5. (-2, -3)				

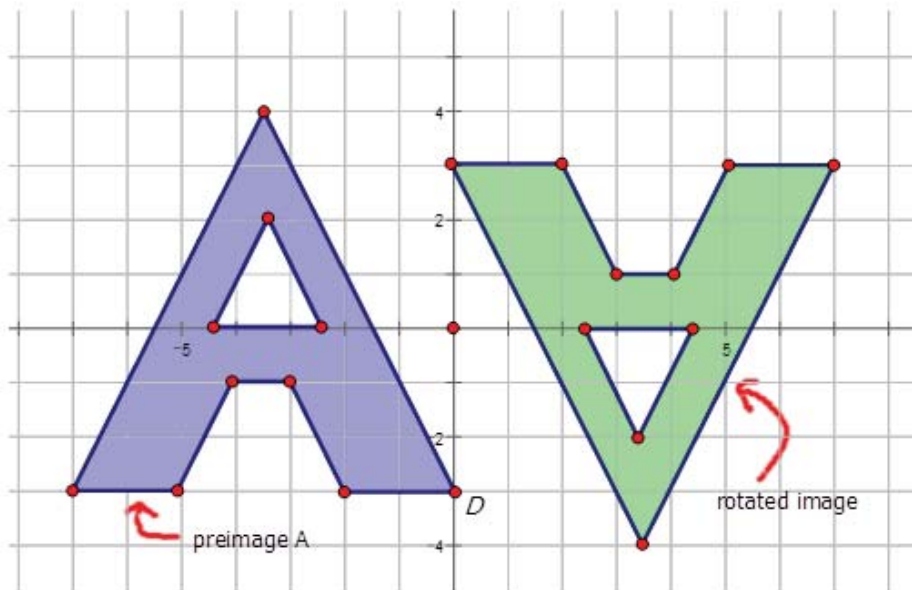
Write the notation that represents the rotation of the preimage to the image for each diagram below.



6.

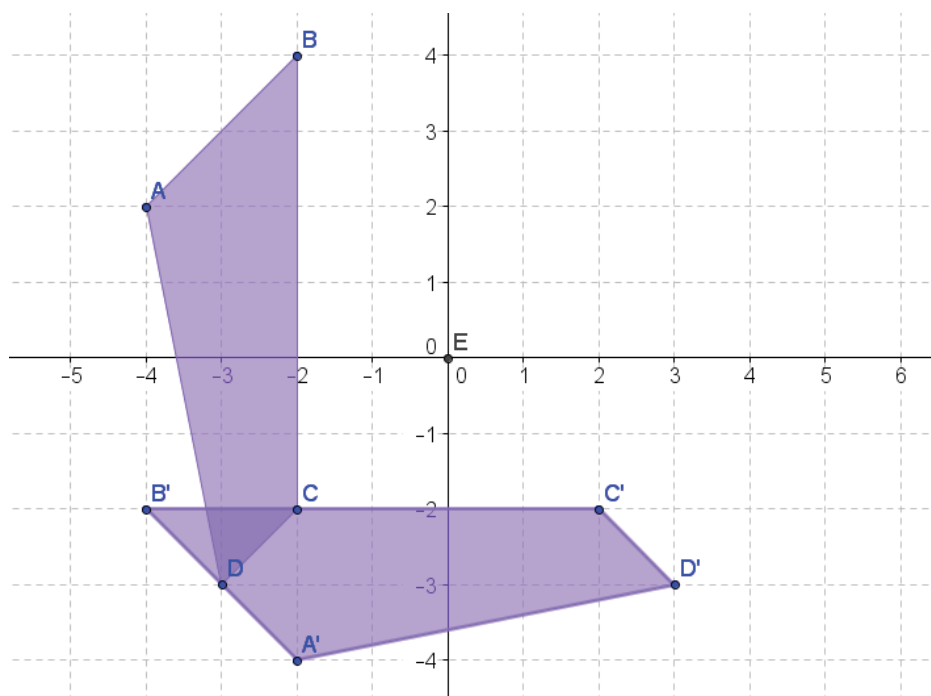




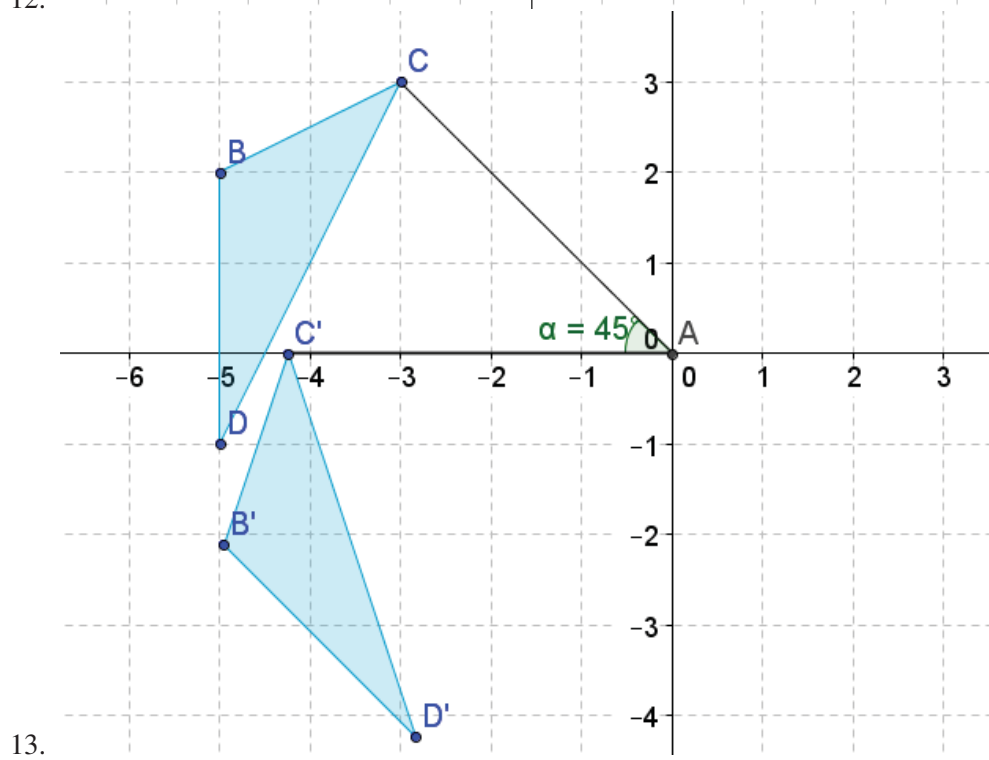
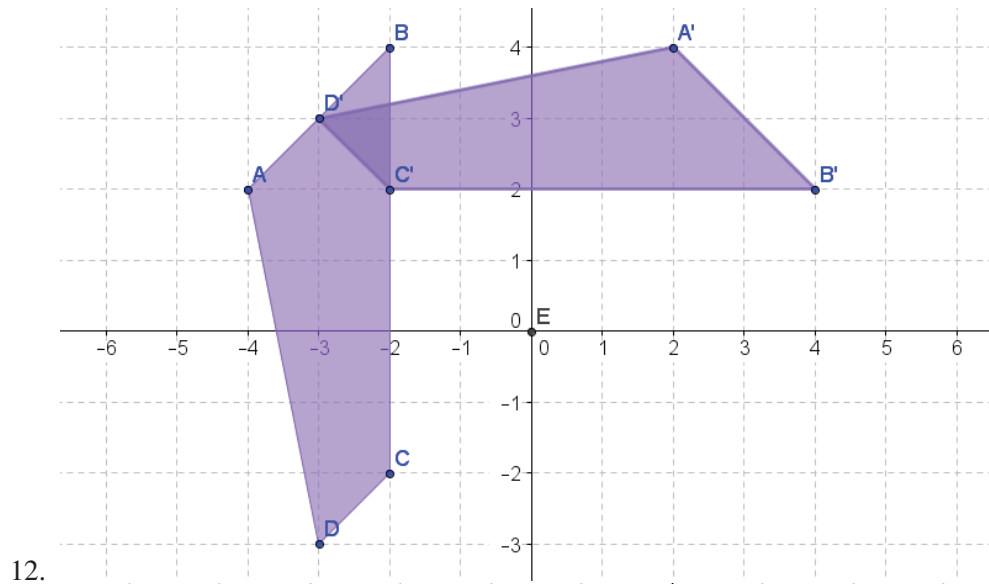


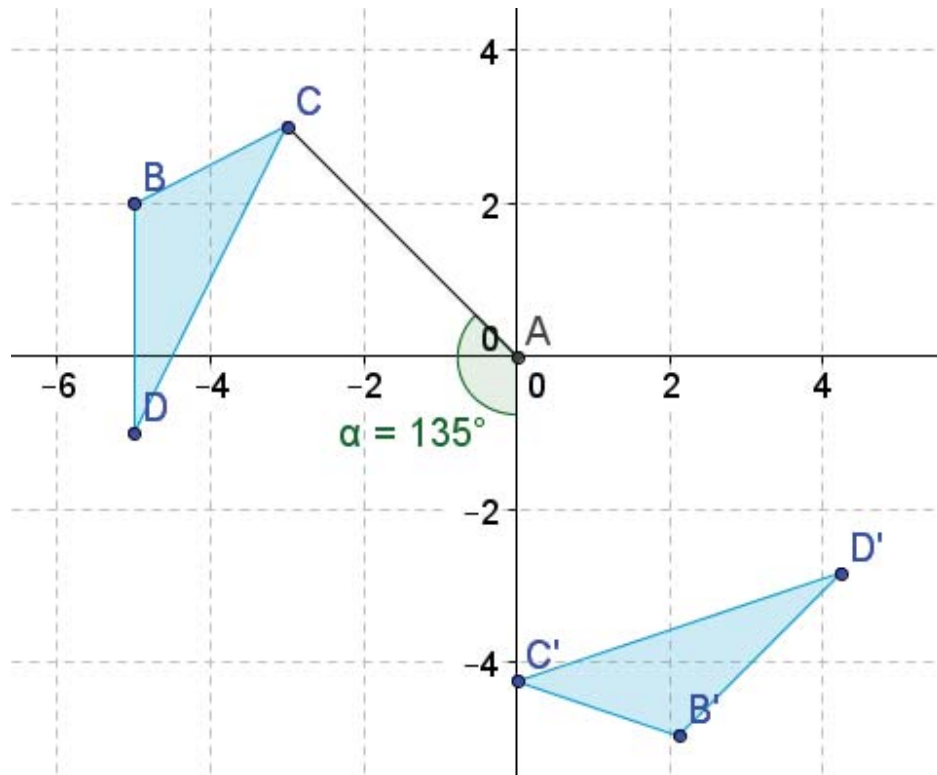
10.

Write the notation that represents the rotation of the preimage to the image for each diagram below.

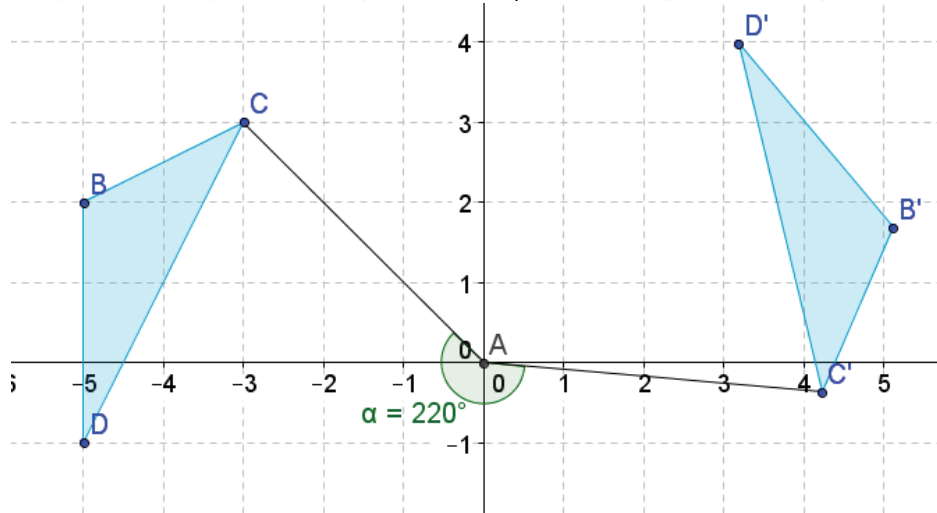


11.





14.

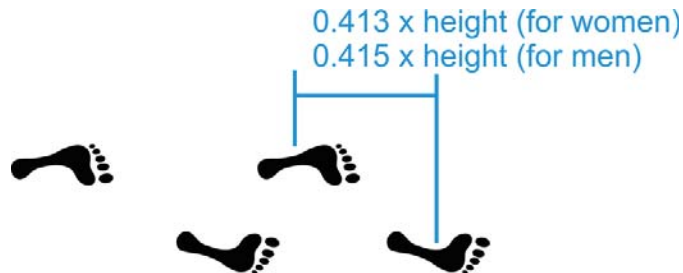


15.

## 1.8 Composition of Transformations

Here you'll learn how to perform a composition of transformations. You'll also learn some common composition of transformations.

What if you were told that your own footprint is an example of a glide reflection? The equations to find your average footprint are in the diagram below. Determine your average footprint and write the rule for one stride. You may assume your stride starts at  $(0, 0)$ . After completing this Concept, you'll be able to answer this question.



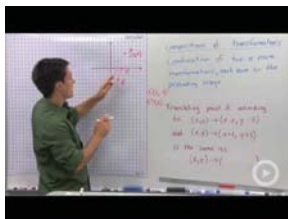
### Watch This



#### MEDIA

Click image to the left for more content.

### CK-12 Foundation: Chapter12CompositionofTranformationsA



#### MEDIA

Click image to the left for more content.

### Brightstorm:Compositions of Transformations



#### MEDIA

Click image to the left for more content.

### Brightstorm:Glide Reflections

## Guidance

### Transformations Summary

A **transformation** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A **rigid transformation** (also known as an **isometry** or **congruence transformation**) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the **image**. The original figure is called the **preimage**.

There are three rigid transformations: translations, reflections, and rotations. A **translation** is a transformation that moves every point in a figure the same distance in the same direction. A **rotation** is a transformation where a figure is turned around a fixed point to create an image. A **reflection** is a transformation that turns a figure into its mirror image by flipping it over a line.

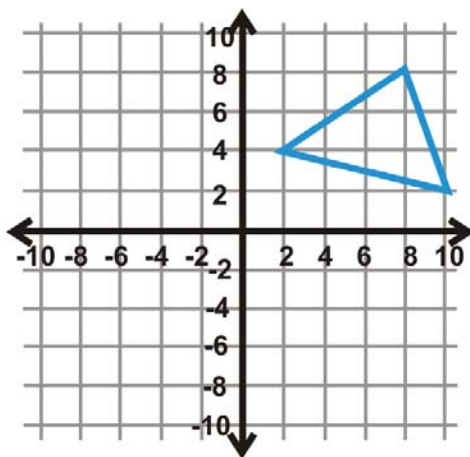
### Composition of Transformations

A **composition of transformations** is to perform more than one rigid transformation on a figure. One of the interesting things about compositions is that they can always be written as one rule. What this means is you don't necessarily have to perform one transformation followed by the next. You can write a rule and perform them at the same time. You can compose any transformations, but here are some of the most common compositions.

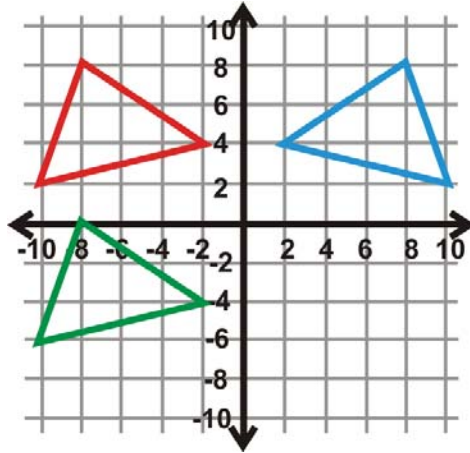
1. **Glide Reflection:** a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.
2. **Reflections over Parallel Lines Theorem:** If you compose two reflections over parallel lines that are  $h$  units apart, it is the same as a single translation of  $2h$  units. *Be careful with this theorem. Notice, it does not say which direction the translation is in. So, to apply this theorem, you would still need to visualize, or even do, the reflections to see in which direction the translation would be.*
3. **Reflection over the Axes Theorem:** If you compose two reflections over each axis, then the final image is a rotation of  $180^\circ$  of the original. *With this particular composition, order does not matter. Let's look at the angle of intersection for these lines. We know that the axes are perpendicular, which means they intersect at a  $90^\circ$  angle. The final answer was a rotation of  $180^\circ$ , which is double  $90^\circ$ . Therefore, we could say that the composition of the reflections over each axis is a rotation of double their angle of intersection.*
4. **Reflection over Intersecting Lines Theorem:** If you compose two reflections over lines that intersect at  $x^\circ$ , then the resulting image is a rotation of  $2x^\circ$ , where the center of rotation is the point of intersection.

### Example A

Reflect  $\triangle ABC$  over the  $y$ -axis and then translate the image 8 units down.



The green image below is the final answer.



$$\begin{aligned} A(8, 8) &\rightarrow A''(-8, 0) \\ B(2, 4) &\rightarrow B''(-2, -4) \\ C(10, 2) &\rightarrow C''(-10, -6) \end{aligned}$$

### Example B

Write a single rule for  $\triangle ABC$  to  $\triangle A''B''C''$  from Example A.

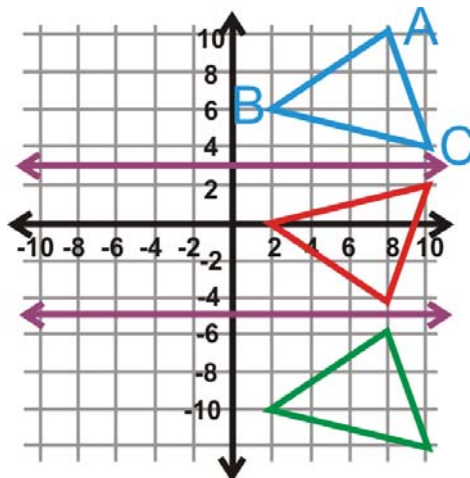
Looking at the coordinates of  $A$  to  $A''$ , the  $x$ -value is the opposite sign and the  $y$ -value is  $y - 8$ . Therefore the rule would be  $(x, y) \rightarrow (-x, y - 8)$ .

Notice that this follows the rules we have learned in previous sections about a reflection over the  $y$ -axis and translations.

### Example C

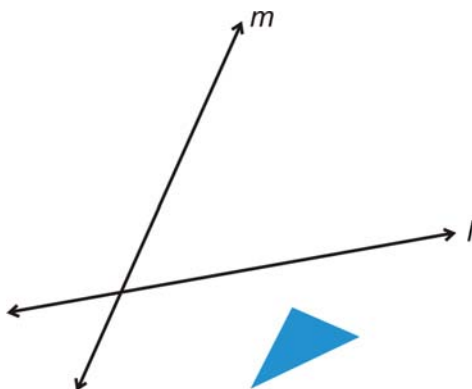
Reflect  $\triangle ABC$  over  $y = 3$  and  $y = -5$ .

Unlike a glide reflection, order matters. Therefore, you would reflect over  $y = 3$  first, followed by a reflection of this image (red triangle) over  $y = -5$ . Your answer would be the green triangle in the graph below.

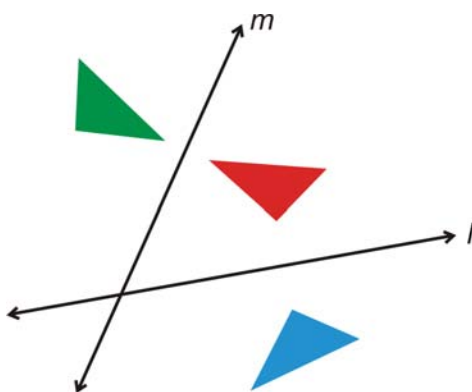


**Example D**

Copy the figure below and reflect it over  $l$ , followed by  $m$ .



The easiest way to reflect the triangle is to fold your paper on each line of reflection and draw the image. It should look like this:



The green triangle would be the final answer.

Watch this video for help with the Examples above.



**MEDIA**

Click image to the left for more content.

[CK-12 Foundation: Chapter 12 Composition of Transformations B](#)

**Concept Problem Revisited**

The average 6 foot tall man has a  $0.415 \times 6 = 2.5$  foot stride. Therefore, the transformation rule for this person would be  $(x, y) \rightarrow (-x, y + 2.5)$ .

**Vocabulary**

A **transformation** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A **rigid transformation** (also known as an **isometry** or **congruence transformation**) is a transformation that does not change

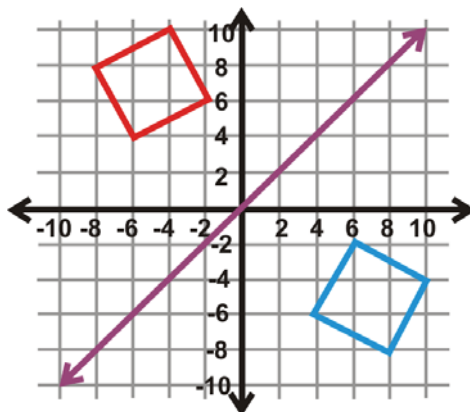
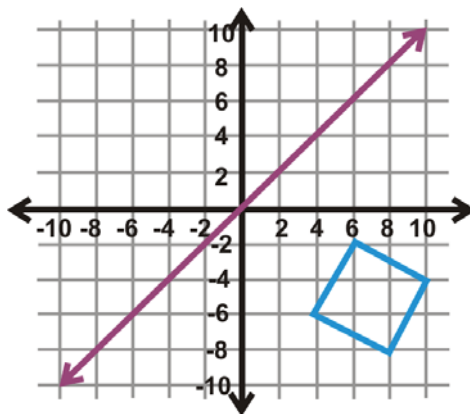
the size or shape of a figure. The new figure created by a transformation is called the *image*. The original figure is called the *preimage*.

There are three rigid transformations: translations, reflections, and rotations. A *translation* is a transformation that moves every point in a figure the same distance in the same direction. A *rotation* is a transformation where a figure is turned around a fixed point to create an image. A *reflection* is a transformation that turns a figure into its mirror image by flipping it over a line.

A *composition (of transformations)* is when more than one transformation is performed on a figure. A *glide reflection* is a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.

### Guided Practice

- $\triangle DEF$  has vertices  $D(3, -1)$ ,  $E(8, -3)$ , and  $F(6, 4)$ . Reflect  $\triangle DEF$  over  $x = -5$  and  $x = 1$ . This double reflection would be the same as which one translation?
- Reflect  $\triangle DEF$  from #1 over the  $x$ -axis, followed by the  $y$ -axis. Determine the coordinates of  $\triangle D''E''F''$  and what one transformation this double reflection would be the same as.
- Reflect the square over  $y = x$ , followed by a reflection over the  $x$ -axis.



- Determine the one rotation that is the same as the double reflection from #3.

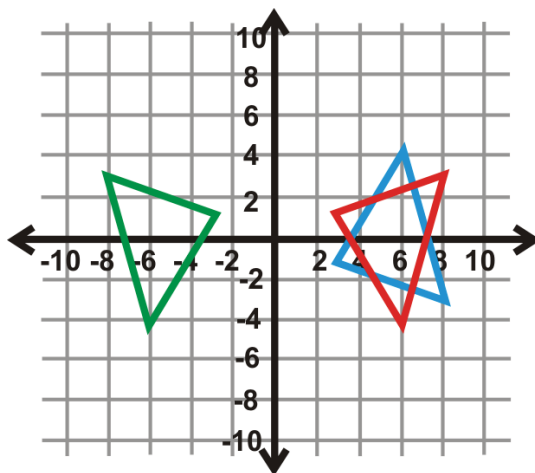
### Answers:

- From the Reflections over Parallel Lines Theorem, we know that this double reflection is going to be the same as a single translation of  $2(1 - (-5))$  or 12 units. Now, we need to determine if it is to the right or to the left. Because we first reflect over a line that is further away from  $\triangle DEF$ , to the *left*,  $\triangle D''E''F''$  will be on the *right* of  $\triangle DEF$ . So,



it would be the same as a translation of 12 units to the right. If the lines of reflection were switched and we reflected the triangle over  $x = 1$  followed by  $x = -5$ , then it would have been the same as a translation of 12 units to the *left*.

2.  $\triangle D''E''F''$  is the green triangle in the graph below. If we compare the coordinates of it to  $\triangle DEF$ , we have:



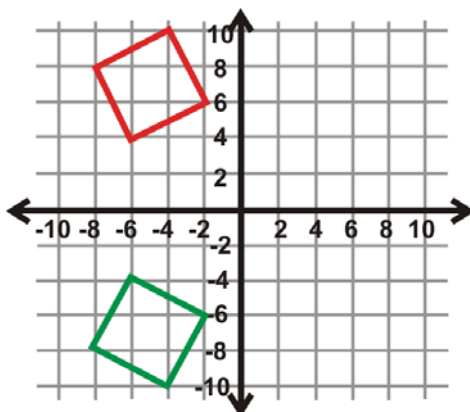
$$D(3, -1) \rightarrow D'(-3, 1)$$

$$E(8, -3) \rightarrow E'(-8, 3)$$

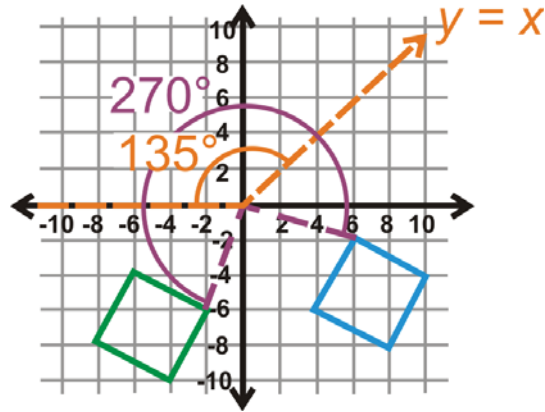
$$F(6, 4) \rightarrow F'(-6, -4)$$

If you recall the rules of rotations from the previous section, this is the same as a rotation of  $180^\circ$ .

3. First, reflect the square over  $y = x$ . The answer is the red square in the graph above. Second, reflect the red square over the  $x$ -axis. The answer is the green square below.



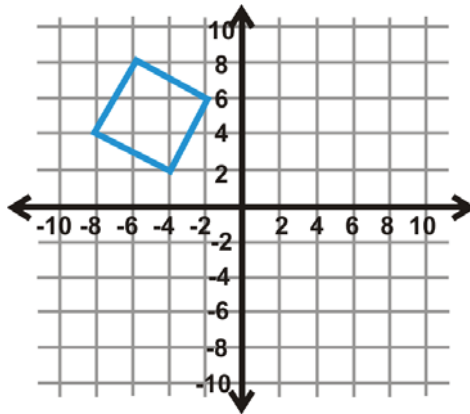
4. Let's use the theorem above. First, we need to figure out what the angle of intersection is for  $y = x$  and the  $x$ -axis.  $y = x$  is halfway between the two axes, which are perpendicular, so is  $45^\circ$  from the  $x$ -axis. Therefore, the angle of rotation is  $90^\circ$  clockwise or  $270^\circ$  counterclockwise. The correct answer is  $270^\circ$  counterclockwise because we always measure angle of rotation in the coordinate plane in a counterclockwise direction. From the diagram, we could have also said the two lines are  $135^\circ$  apart, which is supplementary to  $45^\circ$ .



### Practice

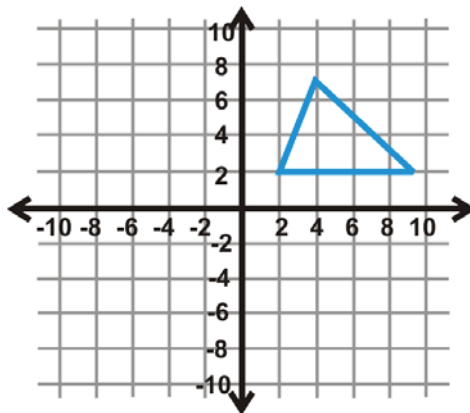
1. What one transformation is equivalent to a reflection over two parallel lines?
2. What one transformation is equivalent to a reflection over two intersecting lines?

Use the graph of the square below to answer questions 3-6.



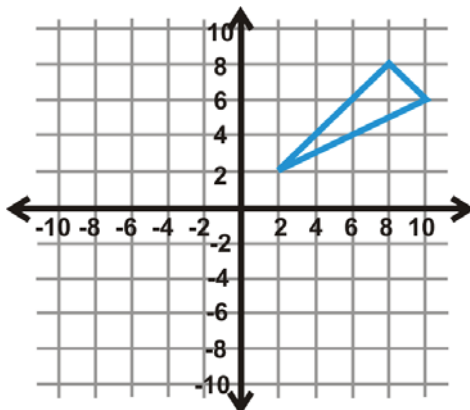
3. Perform a glide reflection over the  $x$ -axis and to the right 6 units. Write the new coordinates.
4. What is the rule for this glide reflection?
5. What glide reflection would move the image back to the preimage?
6. Start over. Would the coordinates of a glide reflection where you move the square 6 units to the right and then reflect over the  $x$ -axis be any different than #3? Why or why not?

Use the graph of the triangle below to answer questions 7-9.



7. Perform a glide reflection over the  $y$ -axis and down 5 units. Write the new coordinates.
8. What is the rule for this glide reflection?
9. What glide reflection would move the image back to the preimage?

Use the graph of the triangle below to answer questions 10-14.



10. Reflect the preimage over  $y = -1$  followed by  $y = -7$ . Write the new coordinates.
11. What one transformation is this double reflection the same as?
12. What one translation would move the image back to the preimage?
13. Start over. Reflect the preimage over  $y = -7$ , then  $y = -1$ . How is this different from #10?
14. Write the rules for #10 and #13. How do they differ?

Fill in the blanks or answer the questions below.

15. Two parallel lines are 7 units apart. If you reflect a figure over both how far apart with the preimage and final image be?
16. After a double reflection over parallel lines, a preimage and its image are 28 units apart. How far apart are the parallel lines?
17. A double reflection over the  $x$  and  $y$  axes is the same as a \_\_\_\_\_ of \_\_\_\_\_.
18. What is the center of rotation for #17?
19. Two lines intersect at an  $83^\circ$  angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
20. A preimage and its image are  $244^\circ$  apart. If the preimage was reflected over two intersected lines, at what angle did they intersect?
21. After a double reflection over parallel lines, a preimage and its image are 62 units apart. How far apart are the parallel lines?
22. A figure is to the left of  $x = a$ . If it is reflected over  $x = a$  followed by  $x = b$  and  $b > a$ , then the preimage and image are \_\_\_\_\_ units apart and the image is to the \_\_\_\_\_ of the preimage.

## 1.9 Composition of Transformations

Here you'll learn how to perform a composition of transformations. You'll also learn several theorems related to composing transformations.

What if you were given the coordinates of a quadrilateral and you were asked to reflect the quadrilateral and then translate it? What would its new coordinates be? After completing this Concept, you'll be able to perform a series of transformations on a figure like this one in the coordinate plane.

### Watch This



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Click image to the left for more content.

[Composing Transformations CK-12](#)

### Guidance

#### Transformations Summary

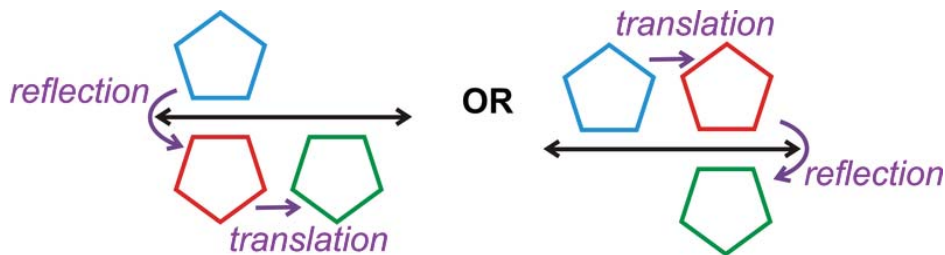
A **transformation** is an operation that moves, flips, or otherwise changes a figure to create a new figure. A **rigid transformation** (also known as an **isometry** or **congruence transformation**) is a transformation that does not change the size or shape of a figure. The new figure created by a transformation is called the **image**. The original figure is called the **preimage**.

There are three rigid transformations: translations, rotations and reflections. A **translation** is a transformation that moves every point in a figure the same distance in the same direction. A **rotation** is a transformation where a figure is turned around a fixed point to create an image. A **reflection** is a transformation that turns a figure into its mirror image by flipping it over a line.

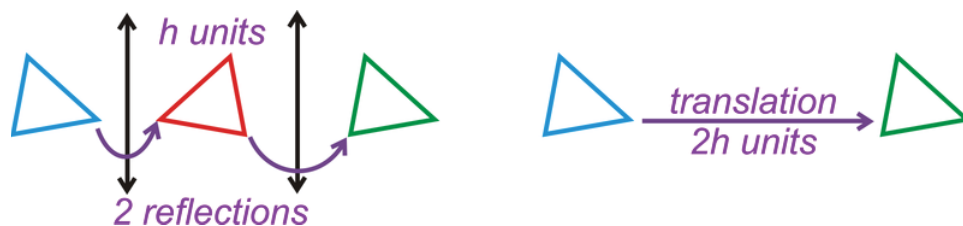
#### Composition of Transformations

A **composition (of transformations)** is when more than one transformation is performed on a figure. Compositions can always be written as one rule. You can compose any transformations, but here are some of the most common compositions:

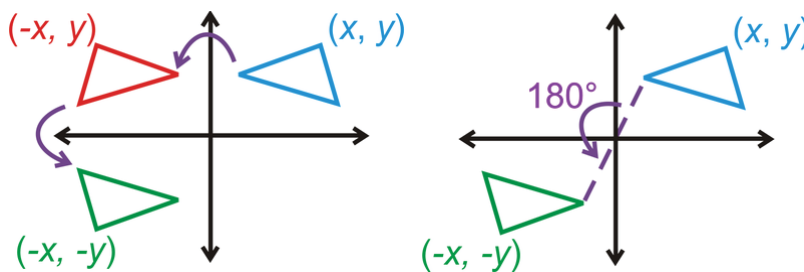
1) A **glide reflection** is a composition of a reflection and a translation. The translation is in a direction parallel to the line of reflection.



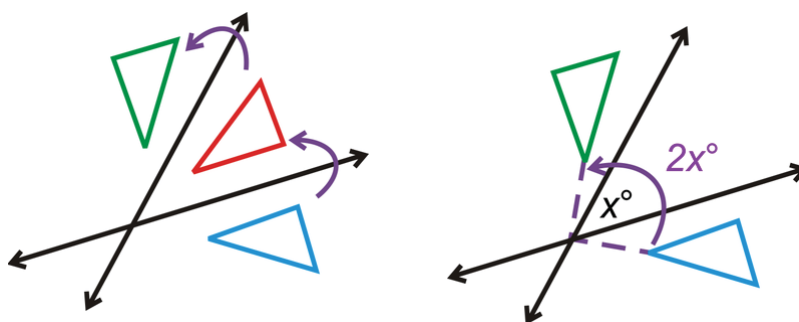
2) The composition of two reflections over parallel lines that are  $h$  units apart is the same as a translation of  $2h$  units (**Reflections over Parallel Lines Theorem**).



3) If you compose two reflections over each axis, then the final image is a rotation of  $180^\circ$  around the origin of the original (**Reflection over the Axes Theorem**).

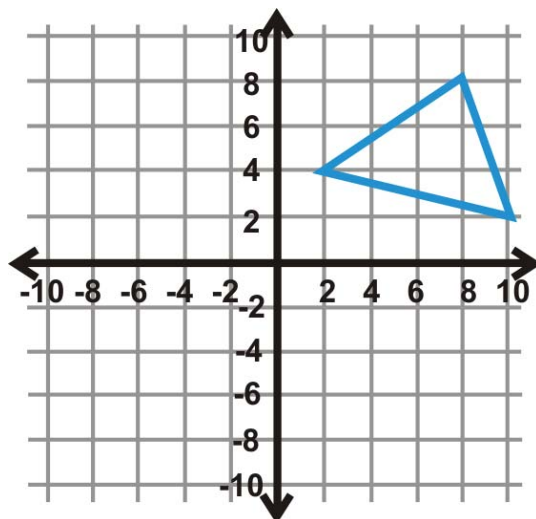


4) A composition of two reflections over lines that intersect at  $x^\circ$  is the same as a rotation of  $2x^\circ$ . The center of rotation is the point of intersection of the two lines of reflection (**Reflection over Intersecting Lines Theorem**).

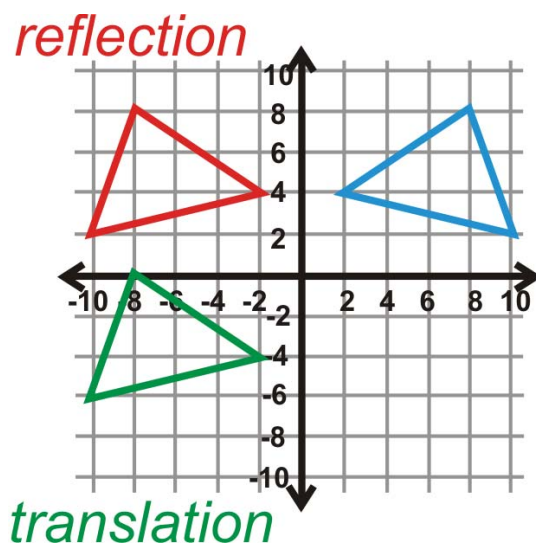


**Example A**

Reflect  $\triangle ABC$  over the  $y$ -axis and then translate the image 8 units down.



The green image to the right is the final answer.



$$\begin{aligned} A(8, 8) &\rightarrow A''(-8, 0) \\ B(2, 4) &\rightarrow B''(-2, -4) \\ C(10, 2) &\rightarrow C''(-10, -6) \end{aligned}$$

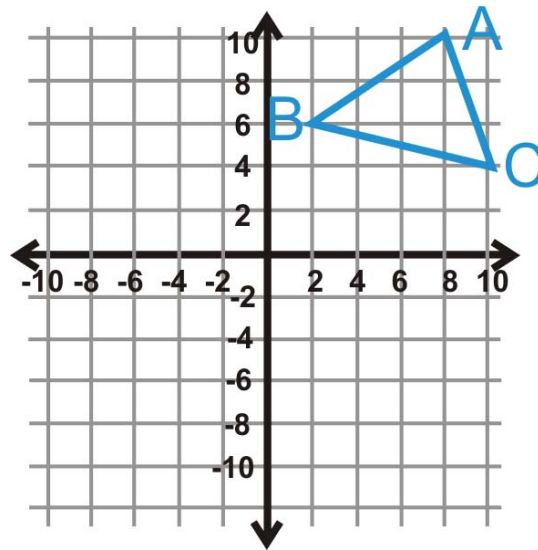
### Example B

Write a single rule for  $\triangle ABC$  to  $\triangle A''B''C''$  from Example A.

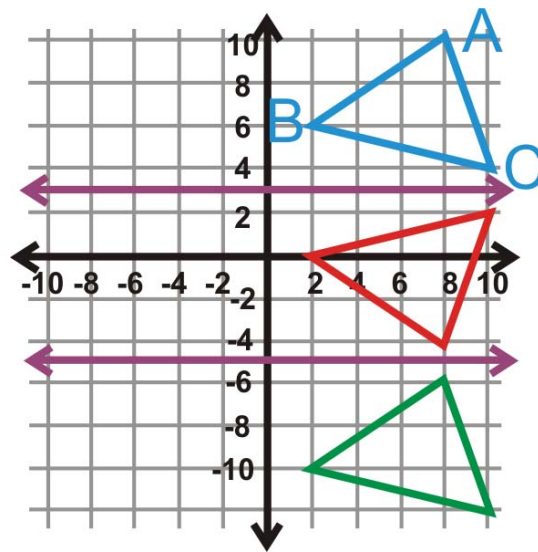
Looking at the coordinates of  $A$  to  $A''$ , the  $x$ -value is the opposite sign and the  $y$ -value is  $y - 8$ . Therefore the rule would be  $(x, y) \rightarrow (-x, y - 8)$ .

### Example C

Reflect  $\triangle ABC$  over  $y = 3$  and then reflect the image over  $y = -5$ .



Order matters, so you would reflect over  $y = 3$  first, (red triangle) then reflect it over  $y = -5$  (green triangle).



**Example D**

A square is reflected over two lines that intersect at a  $79^\circ$  angle. What one transformation will this be the same as?  
 From the Reflection over Intersecting Lines Theorem, this is the same as a rotation of  $2 \cdot 79^\circ = 178^\circ$ .



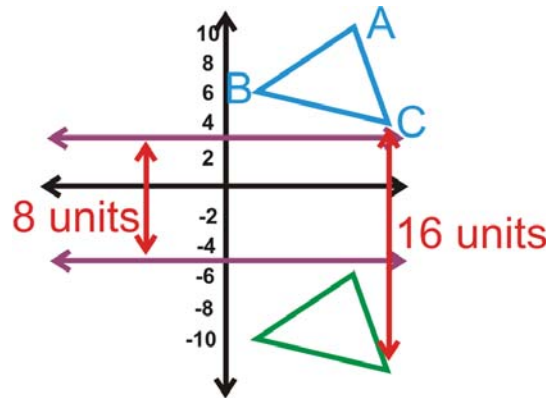
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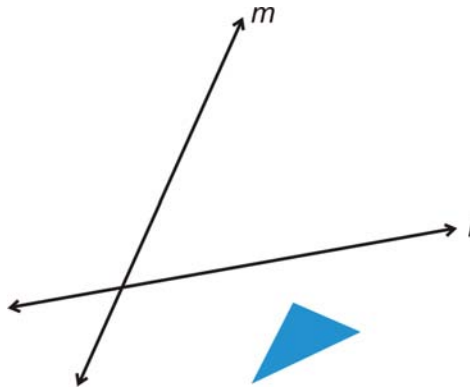
Composing Transformations CK-12

**Guided Practice**

1. Write a single rule for  $\triangle ABC$  to  $\triangle A''B''C''$  from Example C.

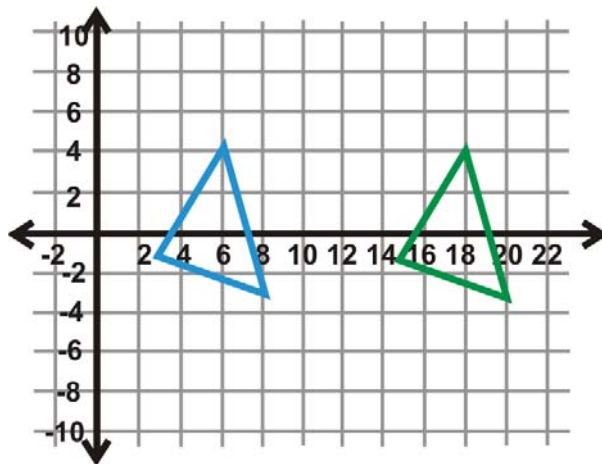


- $\triangle DEF$  has vertices  $D(3, -1)$ ,  $E(8, -3)$ , and  $F(6, 4)$ . Reflect  $\triangle DEF$  over  $x = -5$  and then  $x = 1$ . Determine which one translation this double reflection would be the same as.
- Reflect  $\triangle DEF$  from Question 2 over the  $x$ -axis, followed by the  $y$ -axis. Find the coordinates of  $\triangle D''E''F''$  and the one transformation this double reflection is the same as.
- Copy the figure below and reflect the triangle over  $l$ , followed by  $m$ .



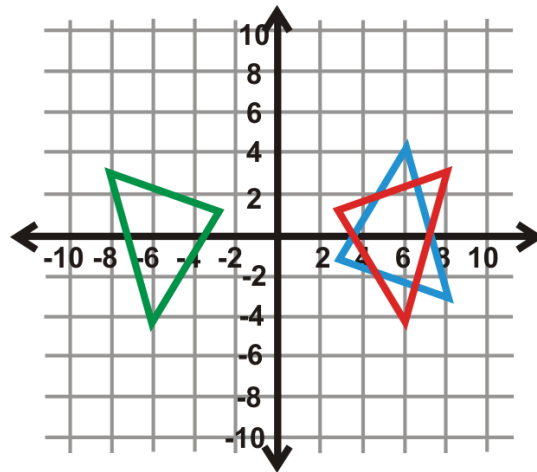
**Answers:**

- In the graph, the two lines are 8 units apart ( $3 - (-5) = 8$ ). The figures are 16 units apart. The **double** reflection is the same as a translation that is **double** the distance between the parallel lines.  $(x, y) \rightarrow (x, y - 16)$ .
- From the Reflections over Parallel Lines Theorem, we know that this double reflection is going to be the same as a single translation of  $2(1(-5))$  or 12 units.





3.  $\triangle D''E''F''$  is the green triangle in the graph to the left. If we compare the coordinates of it to  $\triangle DEF$ , we have:

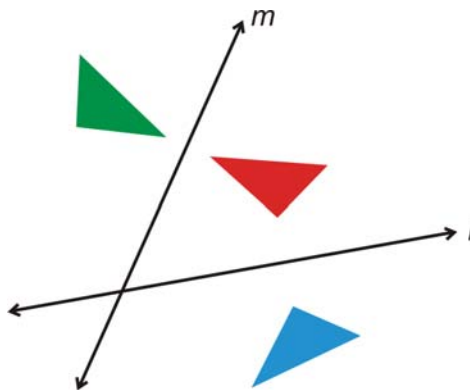


$$D(3, -1) \rightarrow D''(-3, 1)$$

$$E(8, -3) \rightarrow E''(-8, 3)$$

$$F(6, 4) \rightarrow F''(-6, -4)$$

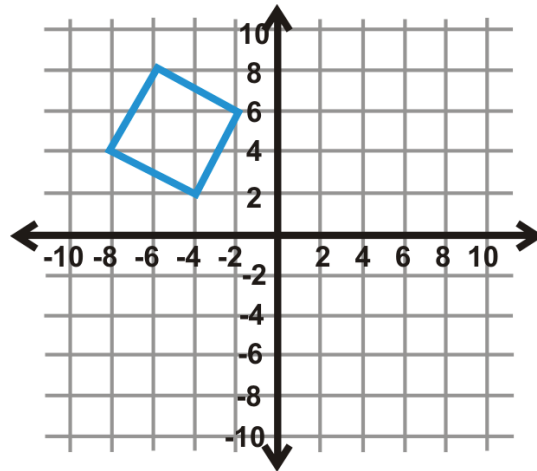
4. The easiest way to reflect the triangle is to fold your paper on each line of reflection and draw the image. The final result should look like this (the green triangle is the final answer):



### Practice

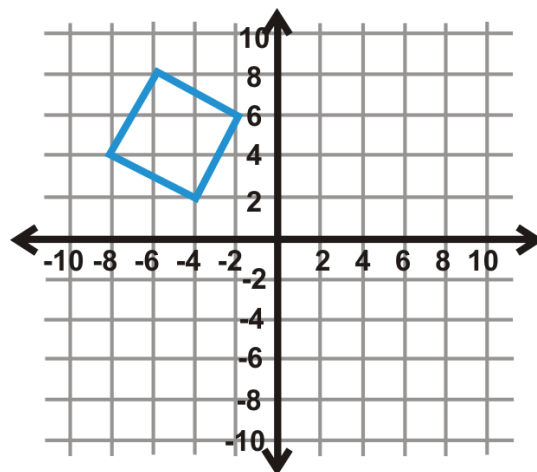
1. Explain why the composition of two or more isometries must also be an isometry.
2. What one transformation is the same as a reflection over two parallel lines?
3. What one transformation is the same as a reflection over two intersecting lines?

Use the graph of the square to the left to answer questions 4-6.



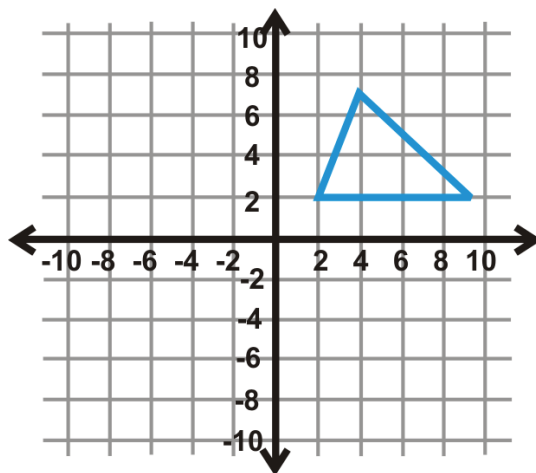
4. Perform a glide reflection over the  $x$ -axis and to the right 6 units. Write the new coordinates.
5. What is the rule for this glide reflection?
6. What glide reflection would move the image back to the preimage?

Use the graph of the square to the left to answer questions 7-9.



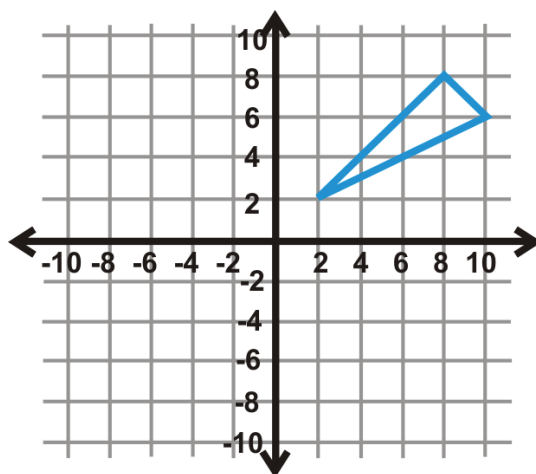
7. Perform a glide reflection to the right 6 units, then over the  $x$ -axis. Write the new coordinates.
8. What is the rule for this glide reflection?
9. Is the rule in #8 different than the rule in #5? Why or why not?

Use the graph of the triangle to the left to answer questions 10-12.



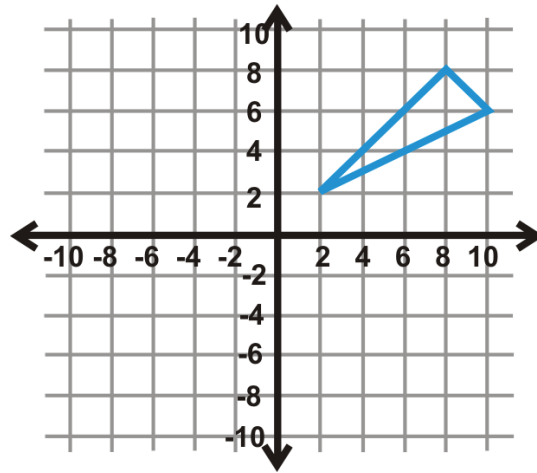
10. Perform a glide reflection over the  $y$ -axis and down 5 units. Write the new coordinates.
11. What is the rule for this glide reflection?
12. What glide reflection would move the image back to the preimage?

Use the graph of the triangle to the left to answer questions 13-15.



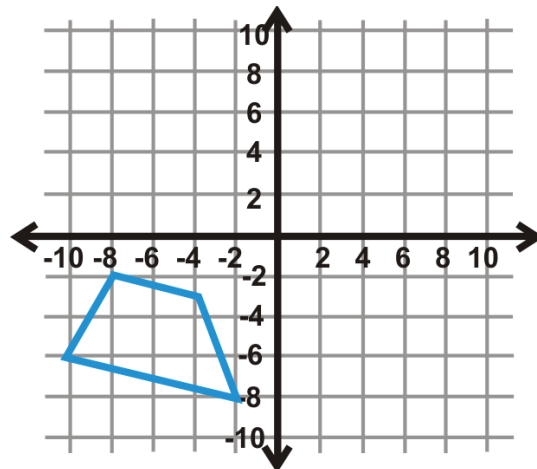
13. Reflect the preimage over  $y = -1$  followed by  $y = -7$ . Draw the new triangle.
14. What one transformation is this double reflection the same as?
15. Write the rule.

Use the graph of the triangle to the left to answer questions 16-18.



16. Reflect the preimage over  $y = -7$  followed by  $y = -1$ . Draw the new triangle.
17. What one transformation is this double reflection the same as?
18. Write the rule.
19. How do the final triangles in #13 and #16 differ?

Use the trapezoid in the graph to the left to answer questions 20-22.



20. Reflect the preimage over the  $x$ -axis then the  $y$ -axis. Draw the new trapezoid.
21. Now, start over. Reflect the trapezoid over the  $y$ -axis then the  $x$ -axis. Draw this trapezoid.
22. Are the final trapezoids from #20 and #21 different? Why do you think that is?

Answer the questions below. Be as specific as you can.

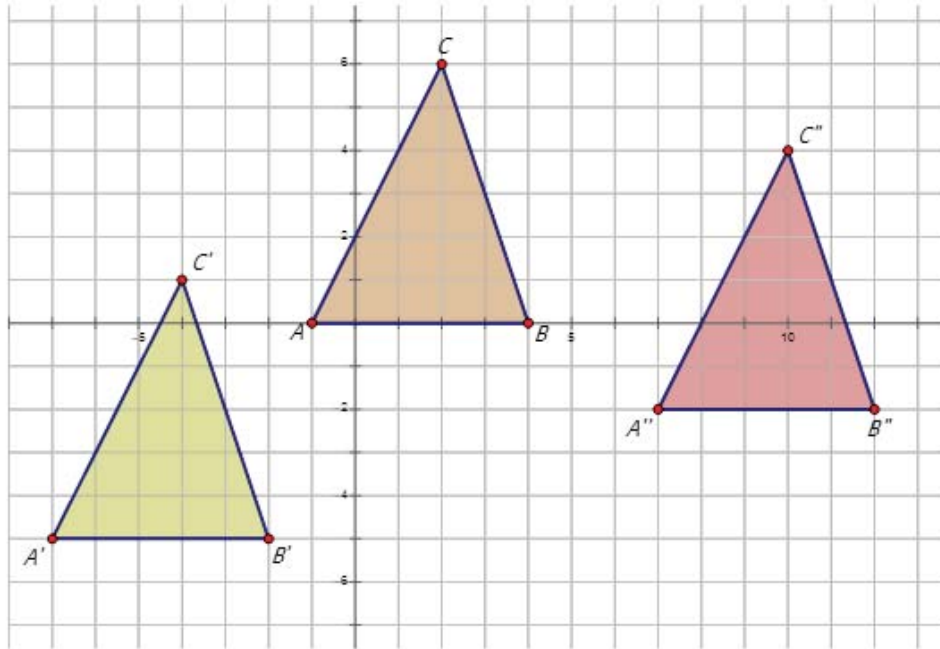
23. Two parallel lines are 7 units apart. If you reflect a figure over both how far apart with the preimage and final image be?
24. After a double reflection over parallel lines, a preimage and its image are 28 units apart. How far apart are the parallel lines?
25. Two lines intersect at a  $165^\circ$  angle. If a figure is reflected over both lines, how far apart will the preimage and image be?
26. What is the center of rotation for #25?
27. Two lines intersect at an  $83^\circ$  angle. If a figure is reflected over both lines, how far apart will the preimage and image be?

28. A preimage and its image are  $244^\circ$  apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
29. A preimage and its image are  $98^\circ$  apart. If the preimage was reflected over two intersecting lines, at what angle did they intersect?
30. After a double reflection over parallel lines, a preimage and its image are 62 units apart. How far apart are the parallel lines?

## 1.10 Composite Transformations

Here you will learn about composite transformations.

Look at the following diagram. It involves two translations. Identify the two translations of triangle  $ABC$ .



### Watch This

First watch this video to learn about composite transformations.



#### MEDIA

Click image to the left for more content.

[CK-12 FoundationChapter10CompositeTransformationsA](#)

Then watch this video to see some examples.



#### MEDIA

Click image to the left for more content.

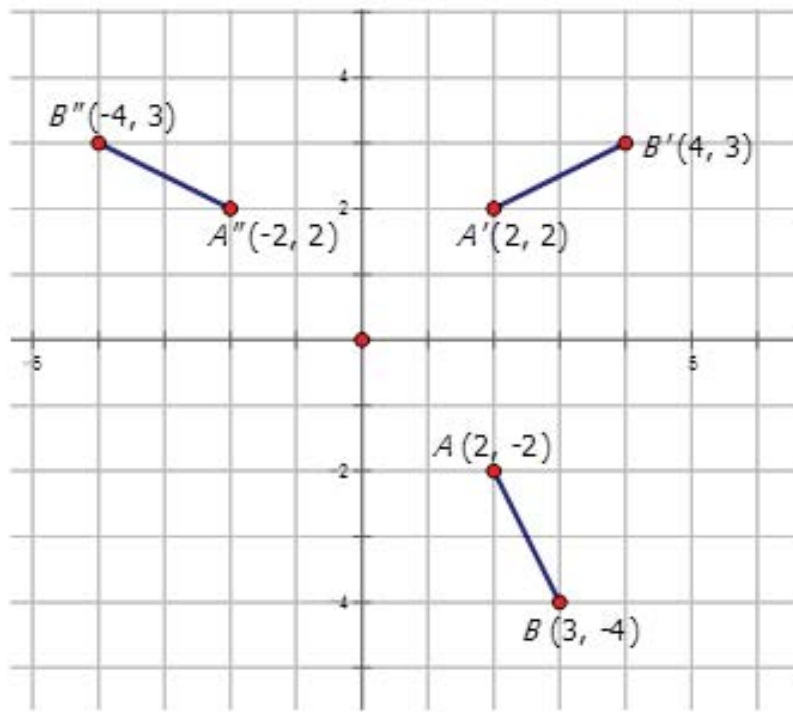
[CK-12 FoundationChapter10CompositeTransformationsB](#)

### Guidance

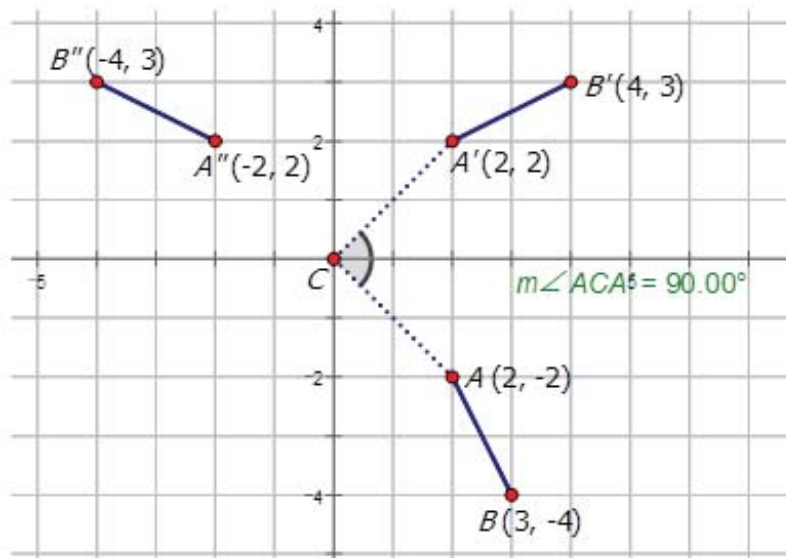
In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A composite transformation is when two or more transformations are performed on a figure (called the preimage) to produce a new figure (called the image).

### Example A

Describe the transformations in the diagram below. The transformations involve a reflection and a rotation.



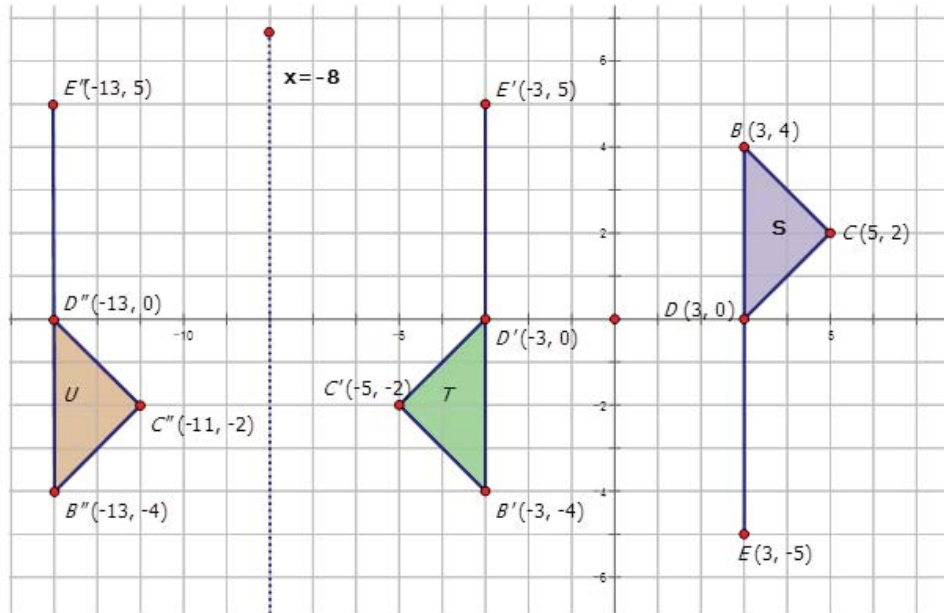
**Solution:** First line  $AB$  is rotated about the origin by  $90^\circ$  CCW.



Then the line  $A'B'$  is reflected about the  $y$ -axis to produce line  $A''B''$ .

**Example B**

Describe the transformations in the diagram below.



**Solution:** The flag in diagram S is rotated about the origin  $180^\circ$  to produce flag T. You know this because if you look at one point you notice that both  $x$ - and  $y$ -coordinate points is multiplied by  $-1$  which is consistent with a  $180^\circ$  rotation about the origin. Flag T is then reflected about the line  $x = -8$  to produce Flag U.

**Example C**

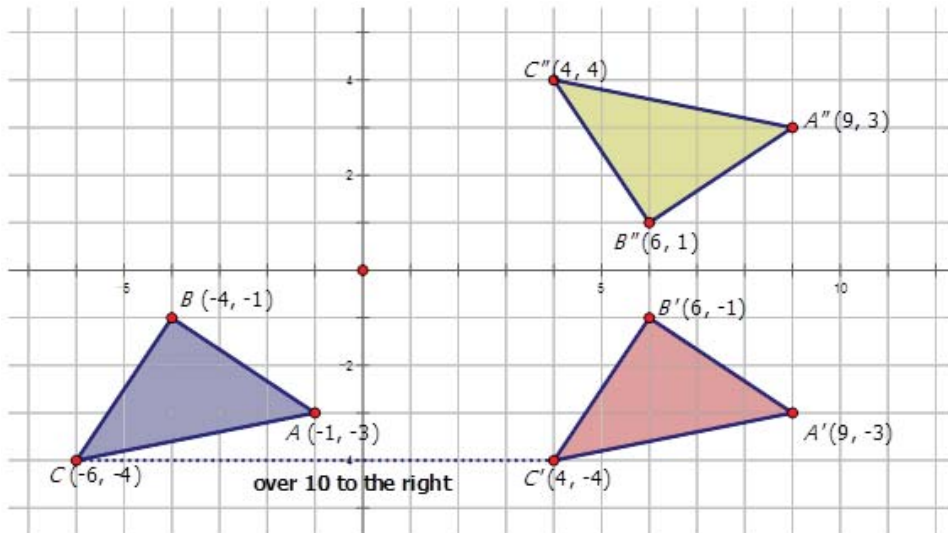
Triangle  $ABC$  where the vertices of  $\triangle ABC$  are  $A(-1, -3)$ ,  $B(-4, -1)$ , and  $C(-6, -4)$  undergoes a composition of transformations described as:

- a translation 10 units to the right, then
- a reflection in the  $x$ -axis.

Draw the diagram to represent this composition of transformations. What are the vertices of the triangle after both transformations are applied?

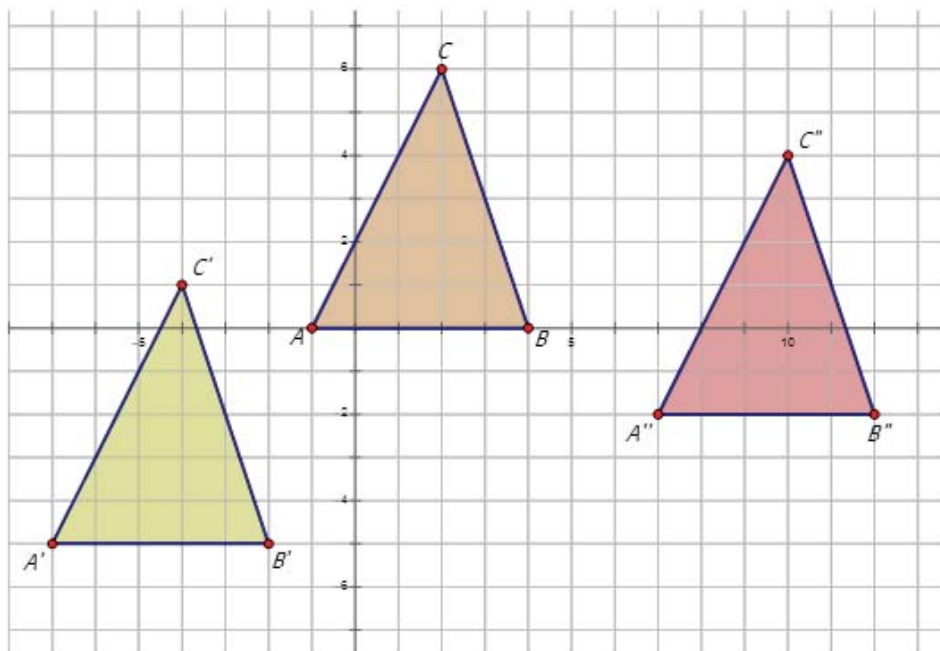
**Solution:**



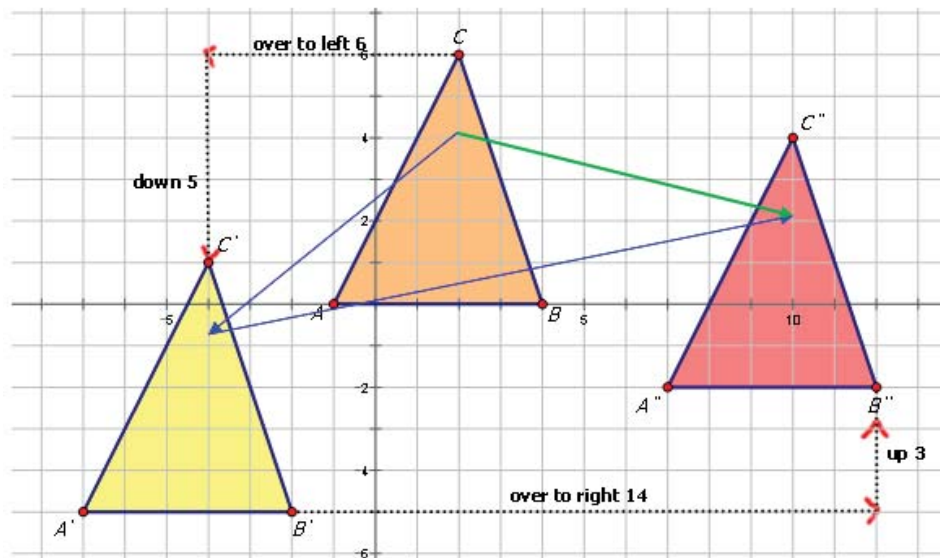


Triangle  $A''B''C''$  is the final triangle after all transformations are applied. It has vertices of  $A''(9, 3)$ ,  $B''(6, 1)$ , and  $C''(4, 4)$ .

**Concept Problem Revisited**



$\triangle ABC$  moves over 6 to the left and down 5 to produce  $\triangle A'B'C'$ . Then  $\triangle A'B'C'$  moves over 14 to the right and up 3 to produce  $\triangle A''B''C''$ . These translations are represented by the blue arrows in the diagram.



All together  $\triangle ABC$  moves over 8 to the right and down 2 to produce  $\triangle A''B''C''$ . The total translations for this movement are seen by the green arrow in the diagram above.

## Vocabulary

### Image

In a transformation, the final figure is called the *image*.

### Preimage

In a transformation, the original figure is called the *preimage*.

### Transformation

A *transformation* is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

### Dilation

A *dilation* is a transformation that enlarges or reduces the size of a figure.

### Translation

A *translation* is an example of a transformation that moves each point of a shape the same distance and in the same direction. Translations are also known as **slides**.

### Rotation

A *rotation* is a transformation that rotates (turns) an image a certain amount about a certain point.

### Reflection

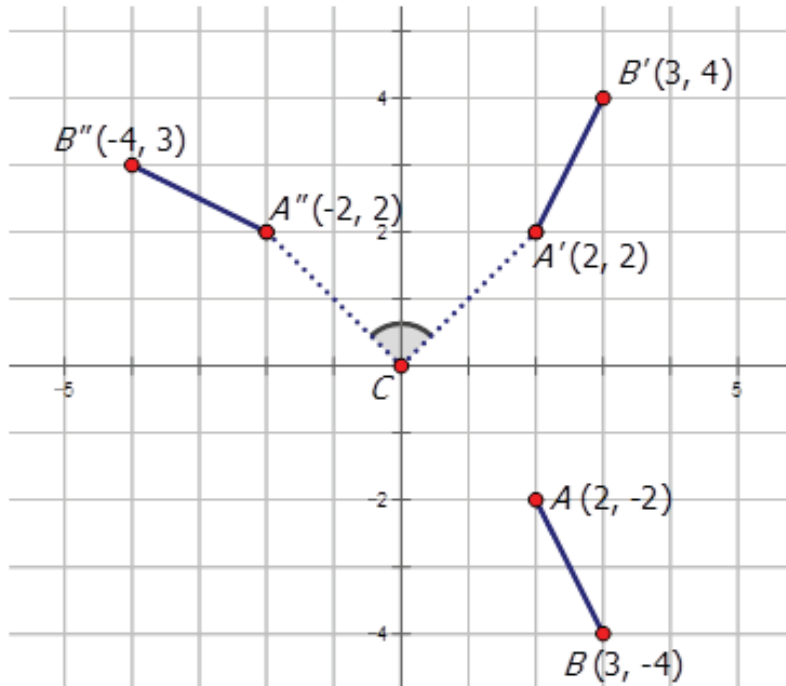
A *reflection* is an example of a transformation that flips each point of a shape over the same line.

### Composite Transformation

A *composite transformation* is when two or more transformations are combined to form a new image from the preimage.

### Guided Practice

1. Describe the transformations in the diagram below. The transformations involve a rotation and a reflection.



2. Triangle  $XYZ$  has coordinates  $X(1,2)$ ,  $Y(-3,6)$  and  $Z(4,5)$ . The triangle undergoes a rotation of 2 units to the right and 1 unit down to form triangle  $X'Y'Z'$ . Triangle  $X'Y'Z'$  is then reflected about the  $y$ -axis to form triangle  $X''Y''Z''$ . Draw the diagram of this composite transformation and determine the vertices for triangle  $X''Y''Z''$ .

3. The coordinates of the vertices of  $\triangle JAK$  are  $J(1,6)$ ,  $B(2,9)$ , and  $C(7,10)$ .

a) Draw and label  $\triangle JAK$ .

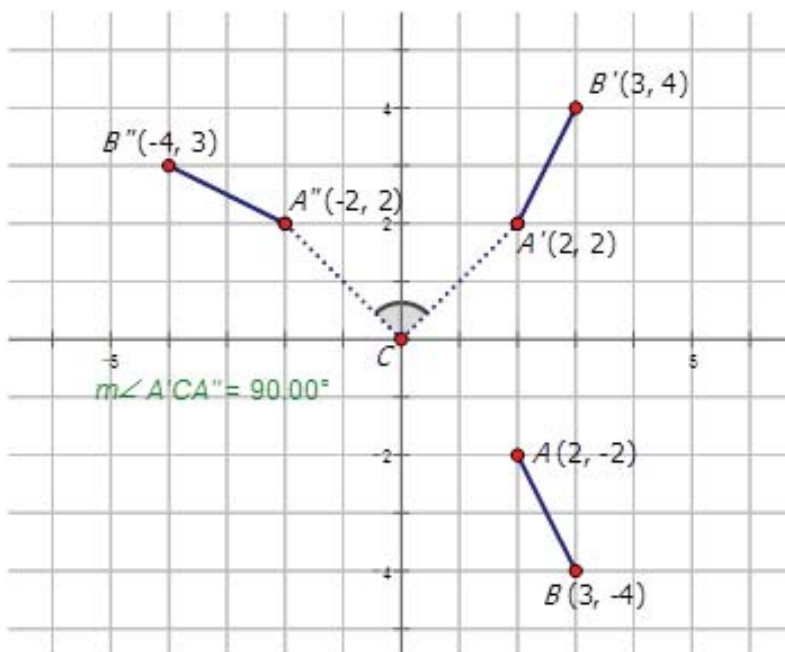
b)  $\triangle JAK$  is reflected over the line  $y = x$ . Graph and state the coordinates of  $\triangle J'A'K'$ .

c)  $\triangle J'A'K'$  is then reflected about the  $x$ -axis. Graph and state the coordinates of  $\triangle J''A''K''$ .

d)  $\triangle J''A''K''$  undergoes a translation of 5 units to the left and 3 units up. Graph and state the coordinates of  $\triangle J'''A'''K'''$ .

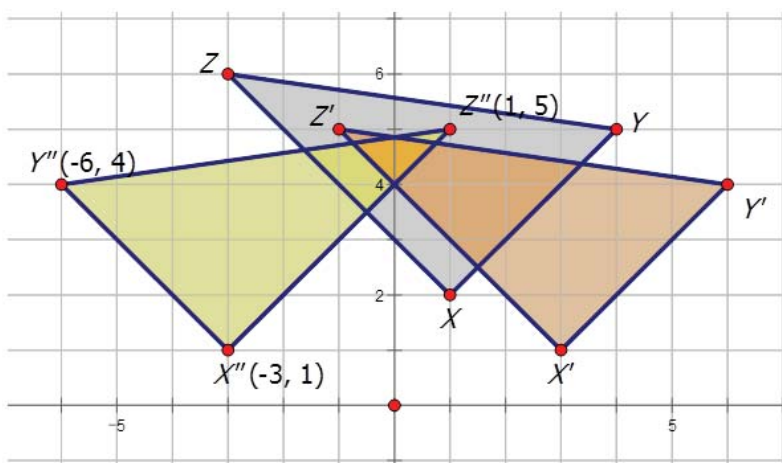
### Answers:

1. The transformations involve a reflection and a rotation. First line  $AB$  is reflected about the  $y$ -axis to produce line  $A'B'$ .

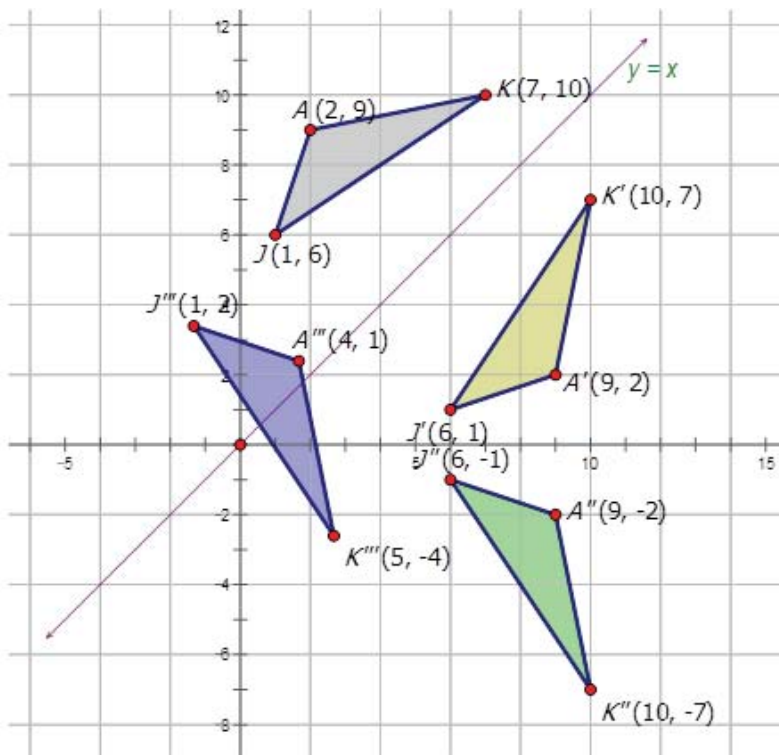


Then the line  $A'B'$  is rotated about the origin by  $90^\circ$  CCW to produce line  $A''B''$ .

2.



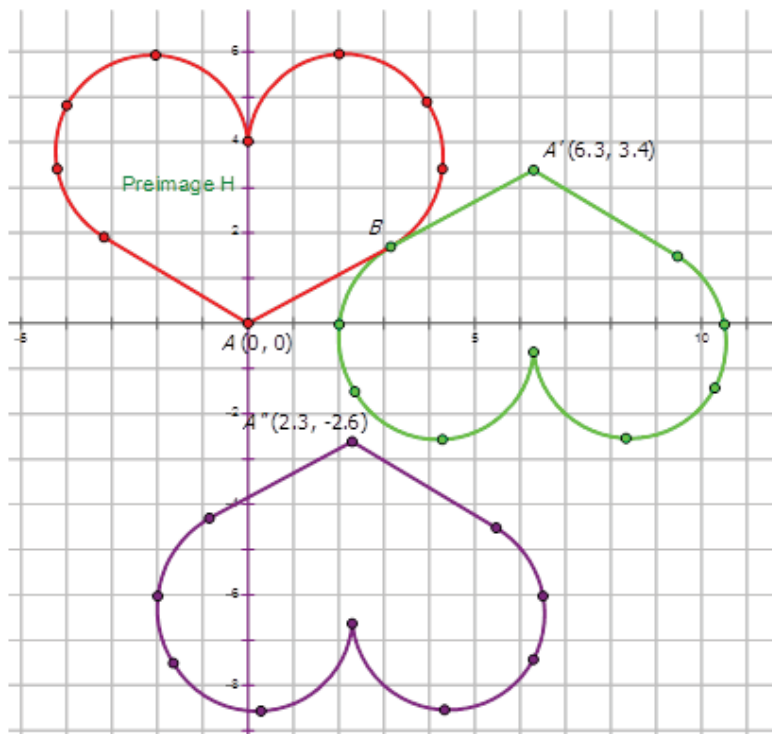
3.



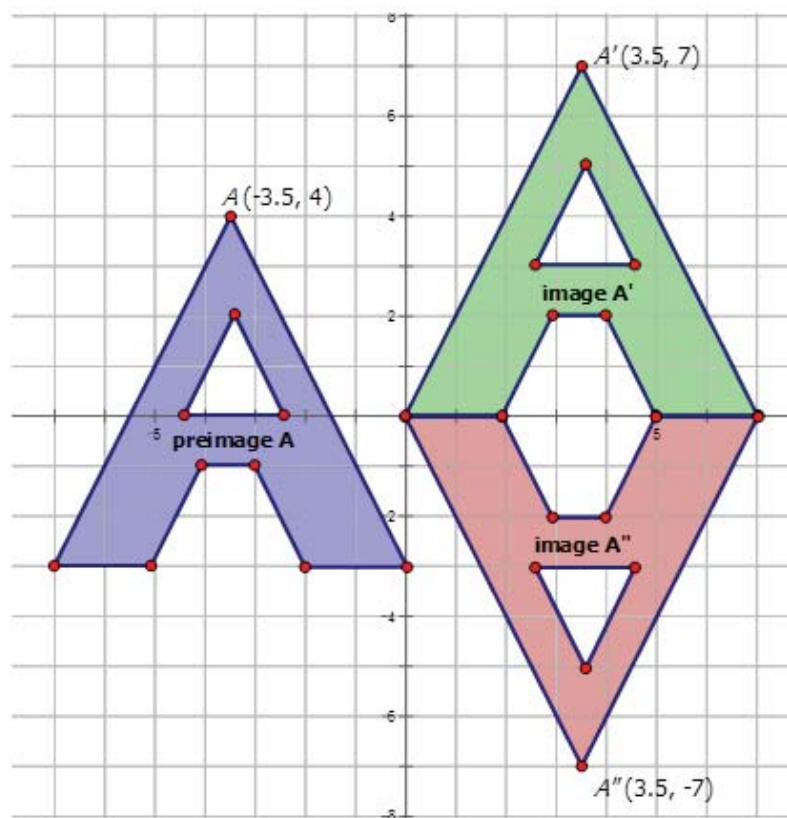
**Practice**

1. A point  $X$  has coordinates  $(-1, -8)$ . The point is reflected across the  $y$ -axis to form  $X'$ .  $X'$  is translated over 4 to the right and up 6 to form  $X''$ . What are the coordinates of  $X'$  and  $X''$ ?
2. A point  $A$  has coordinates  $(2, -3)$ . The point is translated over 3 to the left and up 5 to form  $A'$ .  $A'$  is reflected across the  $x$ -axis to form  $A''$ . What are the coordinates of  $A'$  and  $A''$ ?
3. A point  $P$  has coordinates  $(5, -6)$ . The point is reflected across the line  $y = -x$  to form  $P'$ .  $P'$  is rotated about the origin  $90^\circ$  CW to form  $P''$ . What are the coordinates of  $P'$  and  $P''$ ?
4. Line  $JT$  has coordinates  $J(-2, -5)$  and  $T(2, 3)$ . The segment is rotated about the origin  $180^\circ$  to form  $J'T'$ .  $J'T'$  is translated over 6 to the right and down 3 to form  $J''T''$ . What are the coordinates of  $J'T'$  and  $J''T''$ ?
5. Line  $SK$  has coordinates  $S(-1, -8)$  and  $K(1, 2)$ . The segment is translated over 3 to the right and up 3 to form  $S'K'$ .  $S'K'$  is rotated about the origin  $90^\circ$  CCW to form  $S''K''$ . What are the coordinates of  $S'K'$  and  $S''K''$ ?
6. A point  $K$  has coordinates  $(-1, 4)$ . The point is reflected across the line  $y = x$  to form  $K'$ .  $K'$  is rotated about the origin  $270^\circ$  CW to form  $K''$ . What are the coordinates of  $K'$  and  $K''$ ?

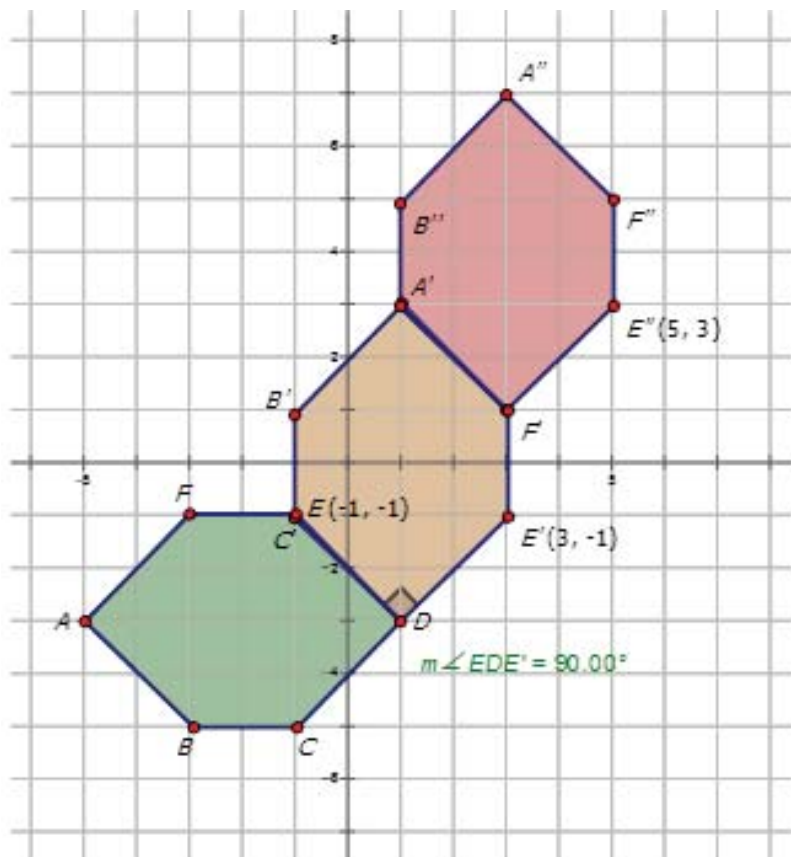
Describe the following composite transformations:



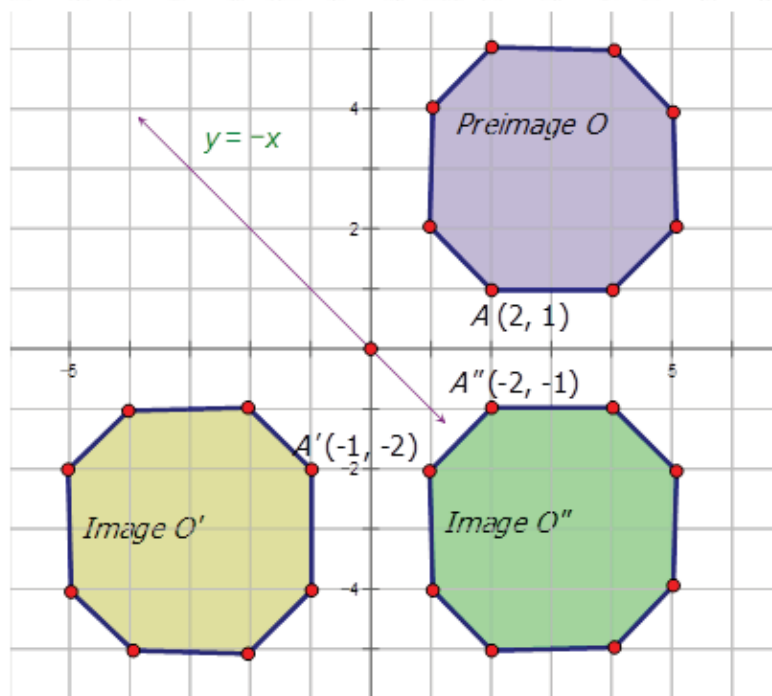
7.



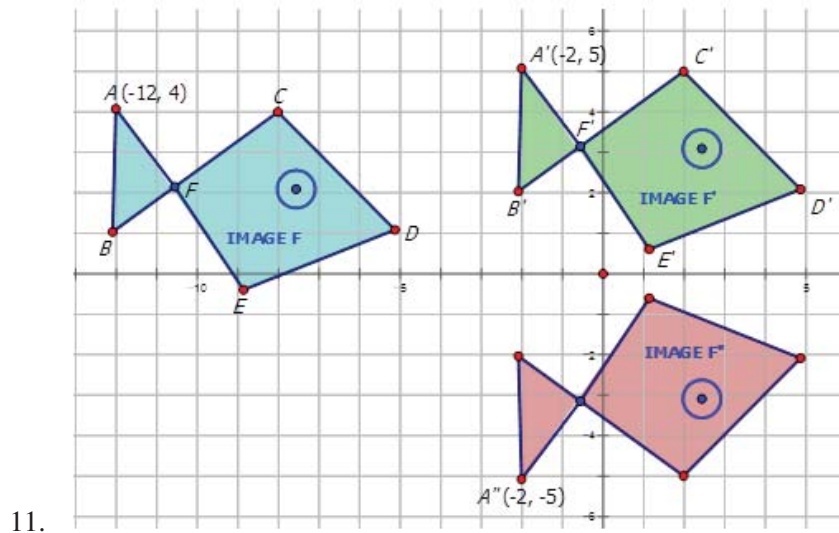
8.



9.



10.



12. Explore what happens when you reflect a shape twice, over a pair of parallel lines. What one transformation could have been performed to achieve the same result?
13. Explore what happens when you reflect a shape twice, over a pair of intersecting lines. What one transformation could have been performed to achieve the same result?
14. Explore what happens when you reflect a shape over the x-axis and then the y-axis. What one transformation could have been performed to achieve the same result?
15. A composition of a reflection and a translation is often called a glide reflection. Make up an example of a glide reflection. Why do you think it's called a **glide** reflection?



## 1.11 Complementary Angles

Here you'll learn what complementary angles are and how they can help you to solve problems.

What if you knew that two angles together made a right angle? After completing this Concept, you'll be able to use what you know about complementary angles to solve problems about these angles.

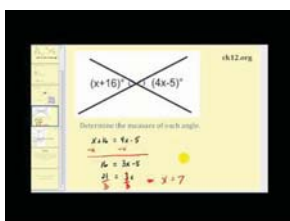
### Watch This



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CK-12 Foundation: Chapter 1 Complementary Angles A



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Click image to the left for more content.

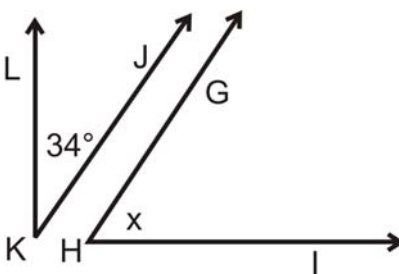
James Sousa: Complementary Angles

### Guidance

Two angles are **complementary** when they add up to  $90^\circ$ . Complementary angles do not have to be congruent to each other, nor do they have to be next to each other.

### Example A

The two angles below are complementary.  $m\angle GHI = x$ . What is  $x$ ?

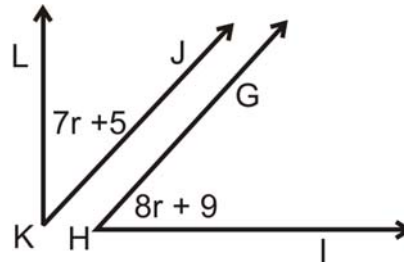


Because the two angles are complementary, they add up to  $90^\circ$ . Make an equation.

$$\begin{aligned}x + 34^\circ &= 90^\circ \\x &= 56^\circ\end{aligned}$$

### Example B

The two angles below are complementary. Find the measure of each angle.



Again, the two angles add up to  $90^\circ$ . Make an equation.

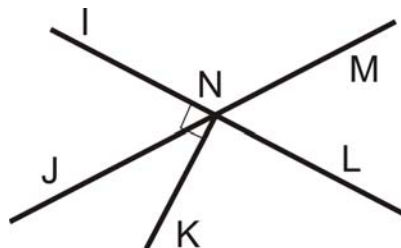
$$\begin{aligned}8r + 9^\circ + 7r + 5^\circ &= 90^\circ \\15r + 14^\circ &= 90^\circ \\15r &= 76^\circ \\r &= 5.067^\circ\end{aligned}$$

However, this is not what the question asks for. You need to plug  $r$  back into each expression to find each angle.

$$\begin{aligned}m\angle GHI &= 8(5^\circ) + 9^\circ = 49^\circ \\m\angle JKL &= 7(5^\circ) + 6^\circ = 41^\circ\end{aligned}$$

### Example C

Name one pair of complementary angles in the diagram below.



One example is  $\angle INJ$  and  $\angle JNK$ .

Watch this video for help with the Examples above.




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**MEDIA**

 Click image to the left for more content.
 

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## CK-12 Foundation: Chapter 1 Complementary Angles B

**Vocabulary**

Two angles are *complementary* when they add up to  $90^\circ$ .

**Guided Practice**

Find the measure of an angle that is complementary to  $\angle ABC$  if  $m\angle ABC$  is

1.  $45^\circ$
2.  $82^\circ$
3.  $19^\circ$
4.  $z^\circ$

**Answers:**

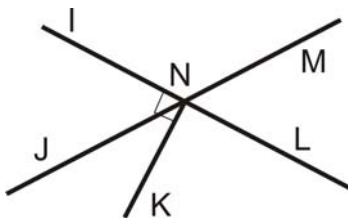
1.  $45^\circ$
2.  $8^\circ$
3.  $71^\circ$
4.  $90 - z^\circ$

**Practice**

Find the measure of an angle that is complementary to  $\angle ABC$  if  $m\angle ABC$  is:

1.  $3^\circ$
2.  $82^\circ$
3.  $51^\circ$
4.  $30^\circ$
5.  $22^\circ$
6.  $(x+y)^\circ$
7.  $x^\circ$

Use the diagram below for exercises 8-9. Note that  $\overline{NK} \perp \overleftrightarrow{IL}$ .



8. If  $m\angle INJ = 60^\circ$ , find  $m\angle KNJ$ .
9. If  $m\angle INJ = 70^\circ$ , find  $m\angle KNJ$ .

For 10-15, determine if the statement is true or false.

10. Complementary angles add up to  $180^\circ$ .
11. Complementary angles are always  $45^\circ$ .
12. Complementary angles are always next to each other.
13. Complementary angles add up to  $90^\circ$ .
14. Two angles that make a right angle are complementary.
15. The two non-right angles in a right triangle are complementary.

## 1.12 Vertical Angles

Here you'll learn about vertical angles and how they can help you to solve problems in geometry.

What if you want to know how opposite pairs of angles are related when two lines cross, forming four angles? After completing this Concept, you'll be able to apply the properties of these special angles to help you solve problems in geometry.

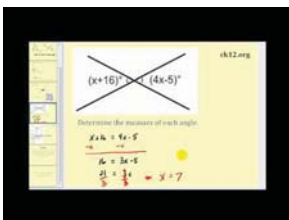
### Watch This



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CK-12 Foundation: Chapter1VerticalAnglesA



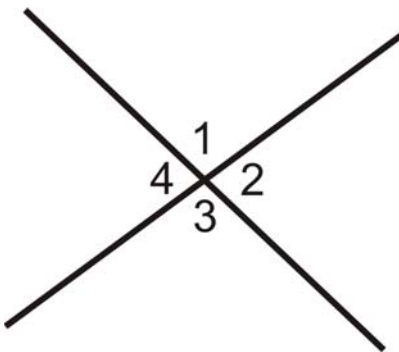
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James Sousa:Vertical Angles

### Guidance

**Vertical angles** are two non-adjacent angles formed by intersecting lines. In the picture below,  $\angle 1$  and  $\angle 3$  are vertical angles and  $\angle 2$  and  $\angle 4$  are vertical angles.



Notice that these angles are labeled with numbers. You can tell that these are labels because they do not have a degree symbol.

**Investigation: Vertical Angle Relationships**

1. Draw two intersecting lines on your paper. Label the four angles created  $\angle 1$ ,  $\angle 2$ ,  $\angle 3$ , and  $\angle 4$ . See the picture above.
2. Take your protractor and find  $m\angle 1$ .
3. What is the angle relationship between  $\angle 1$  and  $\angle 2$ ? Find  $m\angle 2$ .
4. What is the angle relationship between  $\angle 1$  and  $\angle 4$ ? Find  $m\angle 4$ .
5. What is the angle relationship between  $\angle 2$  and  $\angle 3$ ? Find  $m\angle 3$ .
6. Are any angles congruent? If so, write down the congruence statement.

From this investigation, hopefully you found out that  $\angle 1 \cong \angle 3$  and  $\angle 2 \cong \angle 4$ . This is our first theorem. That means it must be proven true in order to use it.

**Vertical Angles Theorem:** If two angles are vertical angles, then they are congruent.

We can prove the Vertical Angles Theorem using the same process we used above. However, let's not use any specific values for the angles.

From the picture above:

$\angle 1$  and  $\angle 2$  are a linear pair

$$m\angle 1 + m\angle 2 = 180^\circ$$

$\angle 2$  and  $\angle 3$  are a linear pair

$$m\angle 2 + m\angle 3 = 180^\circ$$

$\angle 3$  and  $\angle 4$  are a linear pair

$$m\angle 3 + m\angle 4 = 180^\circ$$

All of the equations =  $180^\circ$ , so set the first and second equation equal to each other and the second and third.

$$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$$

AND

$$m\angle 2 + m\angle 3 = m\angle 3 + m\angle 4$$

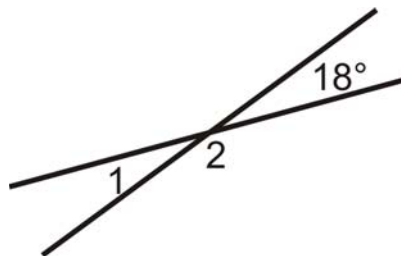
Cancel out the like terms

$$m\angle 1 = m\angle 3, m\angle 2 = m\angle 4$$

Recall that anytime the measures of two angles are equal, the angles are also congruent.

**Example A**

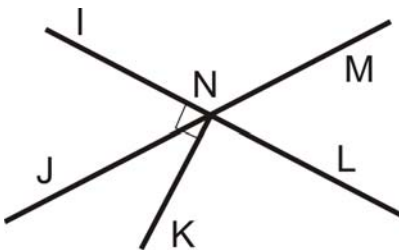
Find  $m\angle 1$  and  $m\angle 2$ .



$\angle 1$  is vertical angles with  $18^\circ$ , so  $m\angle 1 = 18^\circ$ .  $\angle 2$  is a linear pair with  $\angle 1$  or  $18^\circ$ , so  $18^\circ + m\angle 2 = 180^\circ$ .  $m\angle 2 = 180^\circ - 18^\circ = 162^\circ$ .

**Example B**

Name one pair of vertical angles in the diagram below.



One example is  $\angle INJ$  and  $\angle MNL$ .

### Example C

If  $\angle ABC$  and  $\angle DBF$  are vertical angles and  $m\angle ABC = (4x + 10)^\circ$  and  $m\angle DBF = (5x + 2)^\circ$ , what is the measure of each angle?

Vertical angles are congruent, so set the angles equal to each other and solve for  $x$ . Then go back to find the measure of each angle.

$$\begin{aligned} 4x + 10 &= 5x + 2 \\ x &= 8 \end{aligned}$$

So,  $m\angle ABC = m\angle DBF = (4(8) + 10)^\circ = 42^\circ$

Watch this video for help with the Examples above.



MEDIA

Click image to the left for more content.

CK-12 Foundation: Chapter1VerticalAnglesB

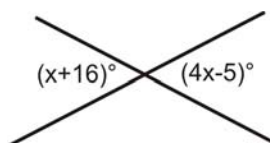
### Vocabulary

**Vertical angles** are two non-adjacent angles formed by intersecting lines. They are always congruent.

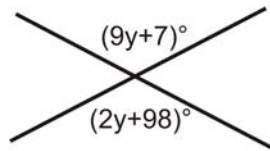
### Guided Practice

Find the value of  $x$  or  $y$ .

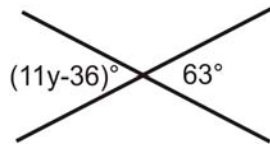
1.



2.



3.

**Answers:**

1. Vertical angles are congruent, so set the angles equal to each other and solve for  $x$ .

$$x + 16 = 4x - 5$$

$$3x = 21$$

$$x = 7^\circ$$

2. Vertical angles are congruent, so set the angles equal to each other and solve for  $y$ .

$$9y + 7 = 2y + 98$$

$$7y = 91$$

$$y = 13^\circ$$

3. Vertical angles are congruent, so set the angles equal to each other and solve for  $y$ .

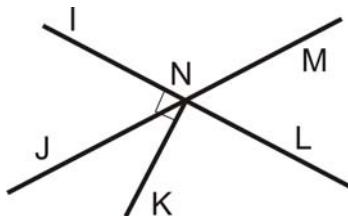
$$11y - 36 = 63$$

$$11y = 99$$

$$y = 9^\circ$$

**Practice**

Use the diagram below for exercises 1-2. Note that  $\overline{NK} \perp \overleftrightarrow{IL}$ .



1. Name one pair of vertical angles.

2. If  $m\angle INJ = 53^\circ$ , find  $m\angle MNL$ .

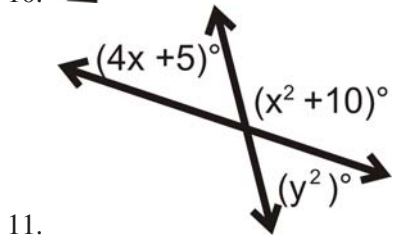
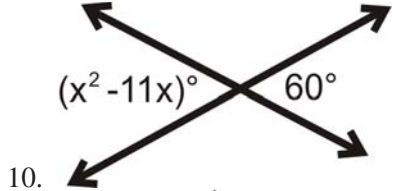
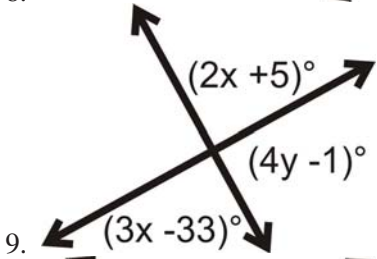
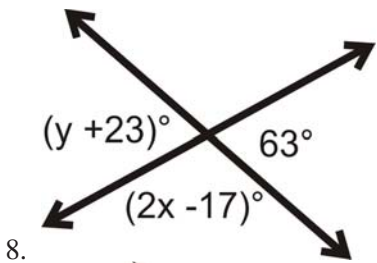
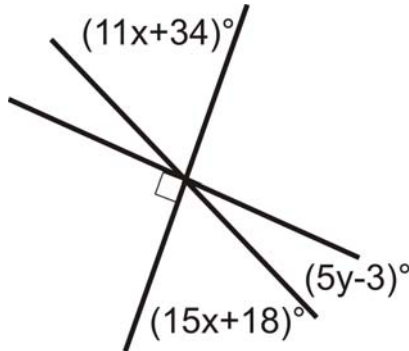


For exercises 3-5, determine if the statement is true or false.

3. Vertical angles have the same vertex.
4. Vertical angles are supplementary.
5. Intersecting lines form two pairs of vertical angles.

Solve for the variables.

6. Find  $x$ .
7. Find  $y$ .



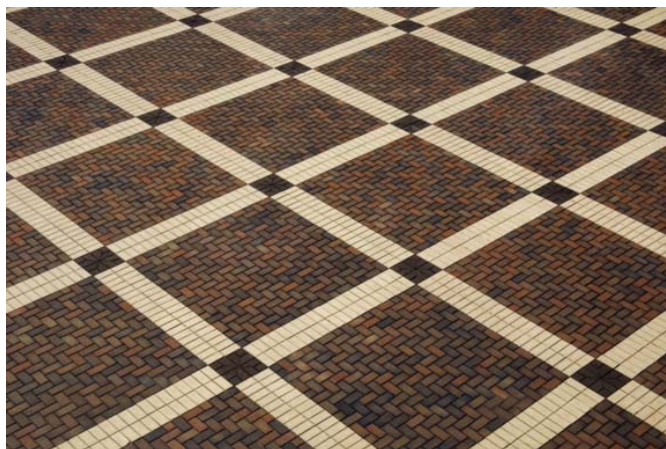
12. If  $\angle ABC$  and  $\angle DBF$  are vertical angles and  $m\angle ABC = (4x + 1)^\circ$  and  $m\angle DBF = (3x + 29)^\circ$ , what is the measure of each angle?
13. If  $\angle ABC$  and  $\angle DBF$  are vertical angles and  $m\angle ABC = (5x + 2)^\circ$  and  $m\angle DBF = (6x - 32)^\circ$ , what is the measure of each angle?

14. If  $\angle ABC$  and  $\angle DBF$  are vertical angles and  $m\angle ABC = (x + 20)^\circ$  and  $m\angle DBF = (4x + 2)^\circ$ , what is the measure of each angle?
15. If  $\angle ABC$  and  $\angle DBF$  are vertical angles and  $m\angle ABC = (2x + 10)^\circ$  and  $m\angle DBF = (4x)^\circ$ , what is the measure of each angle?

## 1.13 Similarity and Congruence

### Introduction

#### *The Mathematical Floor*



Mrs. Gilman brought a small group of students over to look at this tile floor in the hallway of the art museum.

“You see, there is even math in the floor,” she said, smiling. Mrs. Gilman is one of those teachers who loves to point out every place where math can be found.

“Okay, I get it,” Jesse started. “I see the squares.”

“There is a lot more math than just squares,” Mrs. Gilman said, walking away with a huge smile on her face.

“She frustrates me sometimes,” Kara whispered, staring at the floor. “Where is the math besides the squares?”

“I think she is talking about the size of the squares,” Hannah chimed in. “See? There are two different sizes.”

“Actually there are three different sizes, and there could be more that I haven’t found yet,” Jesse said.

“Remember when we learned about comparing shapes that are alike and aren’t alike? It has to do with proportions or something like that,” Hannah chimed in again.

All three students stopped talking and began looking at the floor again.

“Oh yeah, congruent and similar figures, but which are which?” Kara asked.

**What is the difference between congruent and similar figures? This lesson will teach you all about congruent and similar figures. When you are all finished with this lesson, you will have a chance to study the floor again and see if you can find the congruent and the similar figures.**

#### *What You Will Learn*

By the end of this lesson you will be able to demonstrate the following skills:

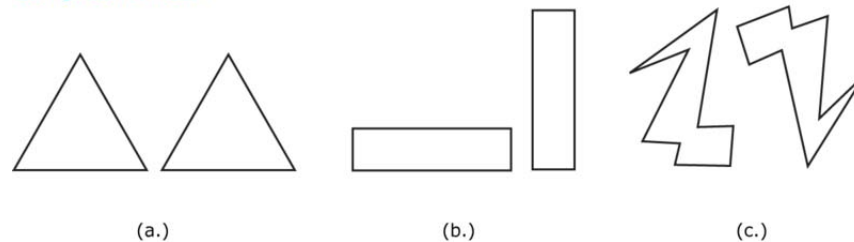
- Recognize congruence.
- Find unknown measures of congruent figures.
- Recognize similarity.
- Check for similarity between given figures

**Teaching Time****I. Recognize Congruence**

In the last lesson we began using the word “congruent.” We talked about congruent lines and congruent angles. **The word *congruent* means exactly the same.** Sometimes, you will see this symbol  $\cong$ .

**In this lesson, we are going to use the word congruent to compare figures.**

**Congruent figures have exactly the same size and shape.** They have congruent sides and congruent angles. Here are some pairs of congruent figures.

**Congruent Pairs**

Compare the figures in each pair. They are exactly the same! If you're not sure, imagine that you could cut out one figure and place it on top of the other. If they match exactly, they are congruent.

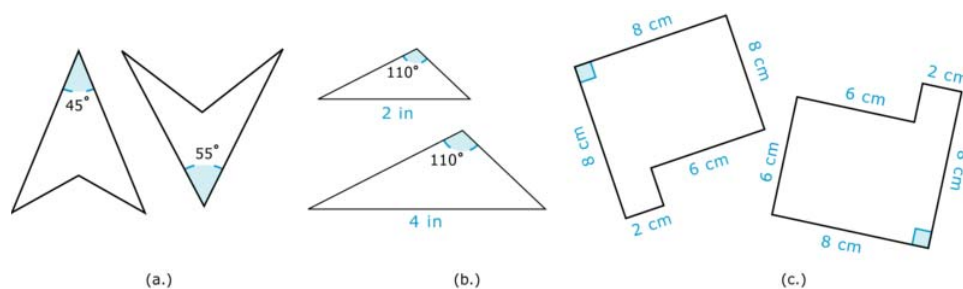
**How can we recognize congruence?**

**We test for congruency by comparing each side and angle of two figures to see if all aspects of both are the same. If the sides are the same length and the angles are equal, the figures are congruent.** Each side and angle of one figure corresponds to a side or angle in the other. We call these corresponding parts. For instance, the top point of one triangle corresponds to the top point of the other triangle in a congruent pair.

It is not always easy to see the corresponding parts of two figures. One figure may be rotated differently so that the corresponding parts appear to be in different places. If you're not sure, trace one figure and place it on top of the other to see if you can make them match. Let's see if we can recognize some congruent figures.

Example

Which pair of figures below is congruent?



**Let's analyze one pair at a time to see if we can find any corresponding angles and sides that are congruent.**

The figures in the first pair appear to be the same shape, if we rotate one  $180^\circ$  so they both point up. Now we can see all of the corresponding parts, such as the angle at the top and the two long sides on the left and right. This is not enough to go on, however. We need to compare the measures of the angles and the lengths of the sides. If any one set of corresponding parts doesn't match, the figures cannot be congruent.

We only know the measure of one angle in the first two figures. We can compare these angles if they are corresponding parts. They are, because if we rotate one figure these angles are in the same place at the top of each figure. Now

compare their measures. The angle in the first figure is  $45^\circ$ . The corresponding angle in the second figure is  $55^\circ$ . **Because the angles are different, these two figures are not congruent.** Let's look at the next pair.

The two triangles in the second pair seem to have corresponding parts: a long base and a wide angle at the top. We need to know whether any of these corresponding parts are congruent, however. We know the measure of the top angle in each figure: it is  $110^\circ$  in both. These figures might be congruent, but we need to see if the sides are congruent to be sure (as we said, similar figures also have congruent angles, but their sides are different lengths). We know the measure of each triangle's base: one is 2 inches and the other is 4 inches. **These sides are not congruent, so the triangles are not congruent. Remember, every side and every angle must be the same in order for figures to be congruent.**

That leaves the last pair. Can you find the corresponding parts? If we rotate the second figure  $180^\circ$ , we have two shapes that look like the letter *L*. Now compare the corresponding sides. The bottom side of each is 8 cm, the long left side of each is 8 cm, two sides are 6 cm, and two sides are 2 cm. All of the angles in both figures are  $90^\circ$ . **Because every side and angle in one figure corresponds to a congruent side and angle in the second, these two figures are congruent.**

### 8J. Lesson Exercises

Answer true or false for each question

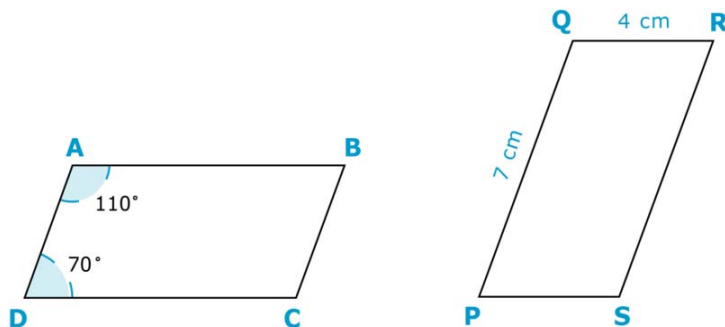
- Congruent figures have the same number of sides and angles.
- Congruent figures can have one pair of angles with the same measure, but not all angles have the same measure.
- Congruent figures can be different sizes as long as the angle measures are the same.



*Discuss your answers with a friend. Be sure you understand why each answer is true or false.*

### II. Find Unknown Measures of Congruent Figures

We know that congruent figures have exactly the same angles and sides. That means we can use the information we have about one figure in a pair of congruent figures to find the measure of a corresponding angle or side in the other figure. Let's see how this works. Take a look at the congruent figures below.



We have been told these two parallelograms are congruent.

**Can you find the corresponding parts?**

If not, trace one parallelogram and place it on top of the other. Rotate it until the parts correspond.

**Which sides and angles correspond?**

We can see that side  $AB$  corresponds to side  $PQ$ . Because they are congruent, we write.

$$AB \cong PQ.$$

**What other sides are congruent?** Let's write them out.

$$AB \cong PQ$$

$$BC \cong QR$$

$$AD \cong PS$$

$$DC \cong SR$$

**We can also write down the corresponding angles, which we know must be congruent because the figures are congruent.**

$$\angle A \cong \angle P$$

$$\angle B \cong \angle Q$$

$$\angle D \cong \angle S$$

$$\angle C \cong \angle R$$

**Now that we understand all of the corresponding relationships in the two figures, we can use what we know about one figure to find the measure of a side or angle in the second figure.**

**Can we find the length of side  $AB$ ?**

We do not know the length of  $AB$ . However, we do know that it is congruent to  $PQ$ , so if we can find the length of  $PQ$  then it will be the same for  $AB$ . Since  $PQ$  is 7 centimeters,  $AB$  must also be 7 centimeters long.

**Now let's look at the angles. Can we find the measure of  $\angle C$ ?**

It corresponds to  $\angle R$ , but we do not know the measure of  $\angle R$  either. Well, we do know the measures of two of the angles in the first parallelogram:  $70^\circ$  and  $110^\circ$ . If we had three, we could subtract from  $360^\circ$  to find the fourth, because all quadrilaterals have angles that add up to  $360^\circ$ . We do not know the measure of  $\angle B$ , but this time we do know the measure of its corresponding angle,  $\angle Q$ . These two angles are congruent, so we know that  $\angle B$  must measure  $70^\circ$ . Now we know three of the angles in the first figure, so we can subtract to find the measure of  $\angle C$ .

$$360 - (70 + 110 + 70) = \angle C$$

$$360 - 250 = \angle C$$

$$110^\circ = \angle C$$

**We were able to combine the given information from both figures because we knew that they were congruent.**

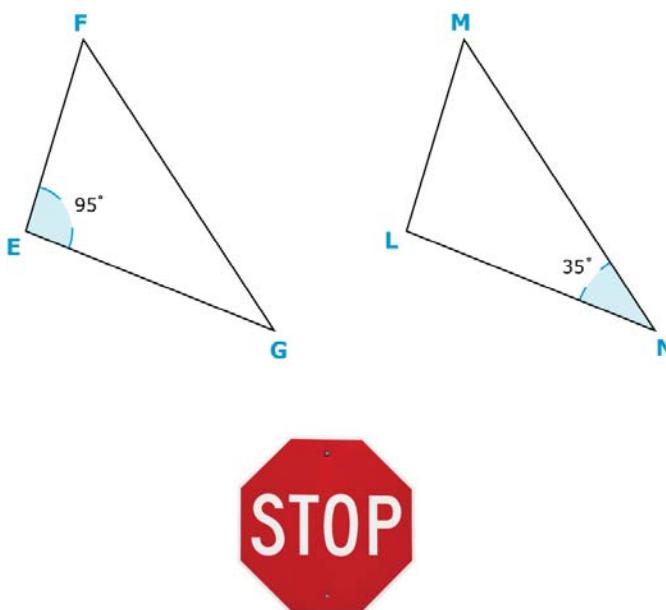


Yes and the more you work on puzzles like this one the easier they will become.

### 8K. Lesson Exercises

Answer this question.

1. What is the measure of  $\angle M$ ? (The two triangles are congruent.)



*Take a few minutes to check your answer with a friend. Correct any errors and then continue with the next section.*

### III. Recognize Similarity

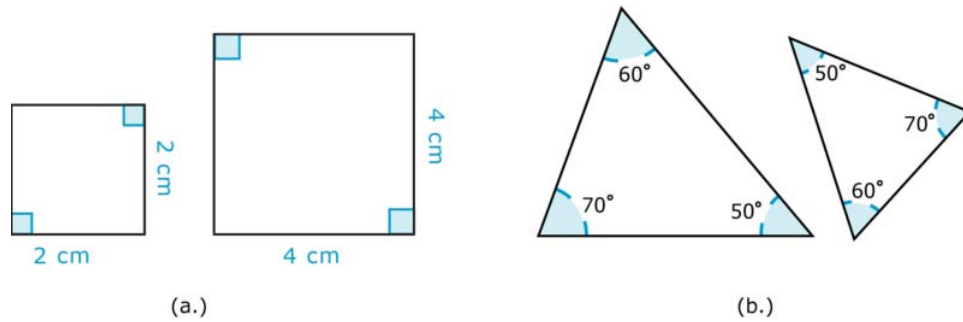
Some figures look identical except they are different sizes. The angles even look the same. When we have figures that are proportional to each other, we call these figures *similar* figures. **Similar figures have the same angle measures but different side lengths.**

**What is an example of similar figures?**

Squares are similar shapes because they always have four  $90^\circ$  angles and four equal sides, even if the lengths of their sides differ. Other shapes can be similar too, if their angles are equal.

Let's look at some pairs of similar shapes.

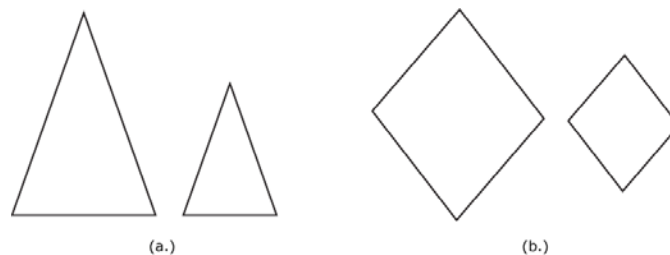
## Similar Pairs



Notice that in each pair the figures look the same, but one is smaller than the other. Since they are not the same size, they are not congruent. However, they have the same angles, so they are similar.

## IV. Check for Similarity between Given Figures

Unlike congruent figures, similar figures are not exactly the same. They do have corresponding features, but only their corresponding angles are congruent; the corresponding sides are not. **Thus when we are dealing with pairs of similar figures, we should look at the angles rather than the sides. In similar figures, the angles are congruent, even if the sides are not.**

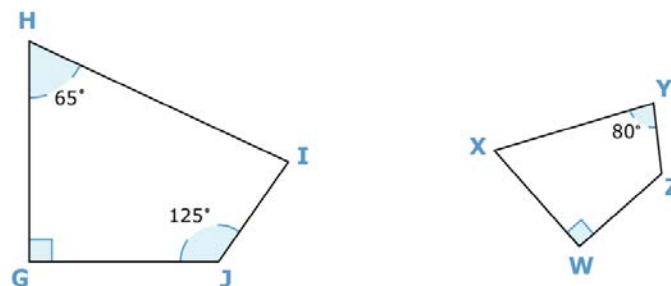


Notice that one angle in each pair of figures corresponds to an angle in the other figure. They have the same shape but not the same size. Therefore they are similar.

Let's find the corresponding angles in similar figures.

Example

List the corresponding angles in the figures below.



Angles  $G$  and  $W$  are both right angles, so they correspond to each other. Imagine you can turn the figures to line up the right angles. You might even trace the small figure so that you can place it on top of the larger one.

**How do the angles line up?**

Angles  $H$  and  $X$  correspond to each other. So do angles  $I$  and  $Y$  and angles  $J$  and  $Z$ . Now we can name these two quadrilaterals:  $GHIJ$  is similar to  $WXYZ$ .



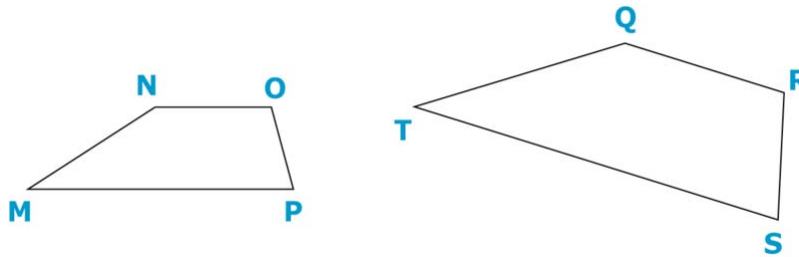
As we've said, the sides in similar figures are not congruent. They are proportional, however. Proportions have the same ratio. Look at  $GHIJ$  and  $WXYZ$  again. We can write each pair of sides as a proportion.

$$\frac{GH}{WX}, \frac{HI}{XY}, \frac{IJ}{YZ}, \frac{GJ}{WZ}$$

The sides from one figure are on the top, and the proportional sides of the other figure are on the bottom.

Example

List all of the pairs of corresponding sides in the figures below as proportions.



Try lining up the figures by their angles. It may help to trace one figure and rotate it until it matches the other.

**Which sides are proportional?**

$OP$  and  $RS$  are the shortest sides in each figure. They are proportional, so we write

$$\frac{OP}{RS}$$

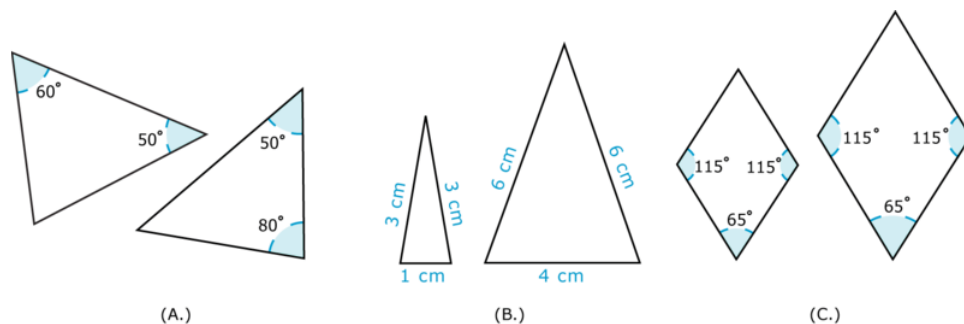
Now that we've got one pair, let's do the same for the rest.

$$\frac{NO}{QR}, \frac{MP}{TS}, \frac{MN}{TQ}$$

**Now let's use what we have learned to check for similarity between figures.**

Example

Which pair of figures below is similar?



**For figures to be similar, we know that the angles must be congruent and the sides must exist in proportional relationships to each other. Let's check each pair one at a time.**

We only know some of the angles in each triangle in the first pair. They both have a  $50^\circ$  angle, so that's a good start. All three angles must be congruent, however, so let's solve for the missing angle in each angle. Remember, the sum of the three angles is always  $180^\circ$  for a triangle.

Triangle 1

$$50 + 60 + \text{angle } 3 = 180$$

$$110 + \text{angle } 3 = 180$$

$$\text{angle } 3 = 180 - 110$$

$$\text{angle } 3 = 70^\circ$$

Triangle 2

$$50 + 80 + \text{angle } 3 = 180$$

$$130 + \text{angle } 3 = 180$$

$$\text{angle } 3 = 180 - 130$$

$$\text{angle } 3 = 50^\circ$$

**The angles in the first triangle are  $50^\circ$ ,  $60^\circ$ , and  $70^\circ$ . The angles in the second triangle are  $50^\circ$ ,  $50^\circ$ , and  $80^\circ$ . These triangles are not similar because their angle measures are different.**

Let's move on to the next pair.

This time we know side lengths, not angles. We need to check whether each set of corresponding sides is proportional. First, let's write out the pairs of proportional corresponding sides

$$\frac{6}{3}, \frac{6}{3}, \frac{4}{1}$$

The proportions show side lengths from the large triangle on the top and its corresponding side in the small triangle on the bottom. The pairs of sides must have the same proportion in order for the triangles to be similar. We can test whether the three proportions above are the same by dividing each. If the quotient is the same, the pairs of sides must exist in the same proportion to each other.

$$\frac{6}{3} = 2$$

$$\frac{6}{3} = 2$$

$$\frac{4}{1} = 4$$

**When we divide, only two pairs of sides have the same proportion (2). The third pair of sides does not exist in the same proportion as the other two, so these triangles cannot be similar.**

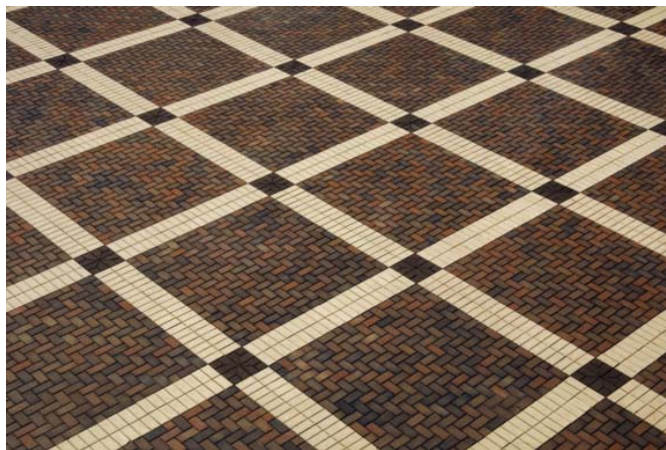
That leaves the last pair. We have been given the measures of some of the angles. If all of the corresponding angles are congruent, then these two figures are similar. We know the measure of three angles in each figure. In fact, they are all corresponding angles. Therefore the one unknown angle in the first figure corresponds to the unknown angle in the second figure.

As we know, the four angles in a quadrilateral must have a sum of  $360^\circ$ . Therefore the unknown angle in each figure must combine with the other three to have this sum. Because the three known angles are the same for both figures, we don't even need to solve for the fourth to know that it will be the same in both figures. **These two figures are similar because their angle measures are all congruent.**

**Now let's use what we have learned to solve the problem in the introduction.**

### Real Life Example Completed

#### *The Mathematical Floor*



**Here is the original problem once again. Reread it and then answer the questions at the end of this passage.**

Mrs. Gilman brought a small group of students over to look at this tile floor in the hallway of the art museum.

“You see, there is even math in the floor,” she said, smiling. Mrs. Gilman is one of those teachers who loves to point out every place where math can be found.

“Okay, I get it,” Jesse started. “I see the squares.”

“There is a lot more math than just squares,” Mrs. Gilman said, walking away with a huge smile on her face.

“She frustrates me sometimes,” Kara whispered, staring at the floor. “Where is the math besides the squares?”

“I think she is talking about the size of the squares,” Hannah chimed in. “See? There are two different sizes.”

“Actually there are three different sizes, and there could be more that I haven’t found yet,” Jesse said.

“Remember when we learned about comparing shapes that are alike and aren’t alike? It has to do with proportions or something like that,” Hannah chimed in again.

All three students stopped talking and began looking at the floor again.

“Oh yeah, congruent and similar figures, but which are which?” Kara asked.

**The students are working on which figures in the floor pattern are congruent and which ones are similar.**

**The congruent figures are exactly the same. We can say that the small dark brown squares are congruent because they are just like each other. They have the same side lengths. What is one other pair of congruent squares?**

**The similar figures compare squares of different sizes. You can see that the figures are squares, so they all have 90 degree angles. The side lengths are different, but because the angles are congruent, we can say that they have the same shape, but not the same size. This makes them similar figures.**

**The small dark brown square is similar to the large dark brown square. The small dark brown square is also similar to the square created by the ivory colored tile. There is a relationship between the different squares. Are there any more comparisons? Make a few notes in your notebook.**

## Vocabulary

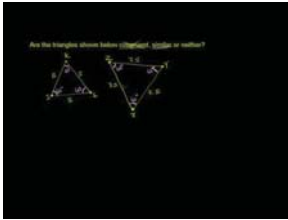
### Congruent

having exactly the same shape and size. All side lengths and angle measures are the same.

### Similar

having the same shape but not the same size. All angle measures are the same, but side lengths are not.

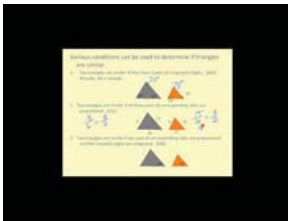
**Technology Integration**



**MEDIA**

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KhanAcademyCongruent and Similar Triangles



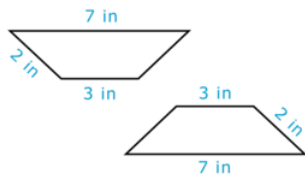
**MEDIA**

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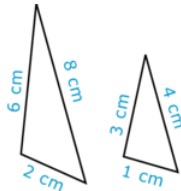
James Sousa, Congruent and Similar Triangles

**Time to Practice**

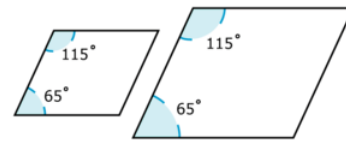
Directions: Tell whether the pairs of figures below are congruent, similar, or neither.



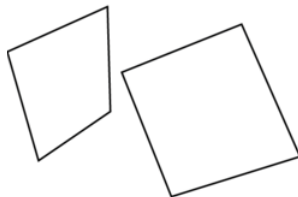
(1.)



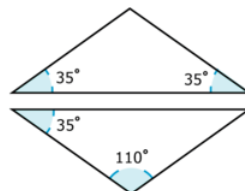
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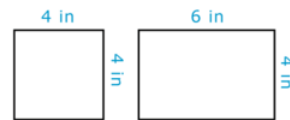
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(4.)

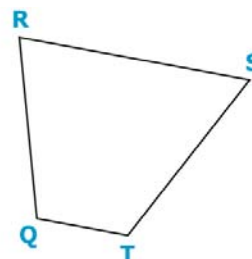
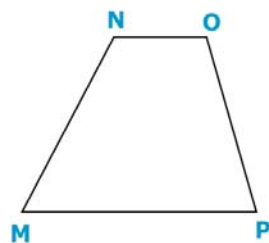


(5.)



(6.)

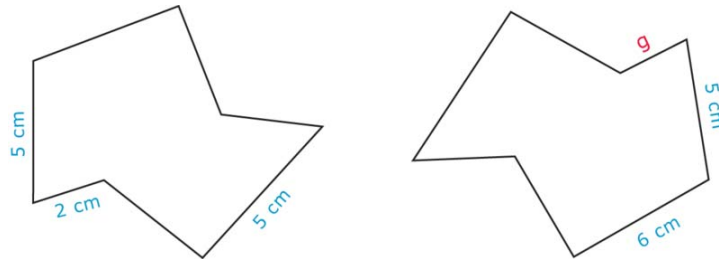
Directions: Name the corresponding parts to those given below.



7.  $\angle R$ 8.  $MN$ 9.  $\angle O$ 

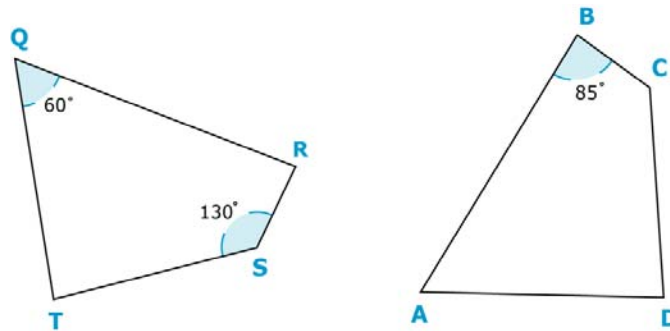
Directions: Use the relationships between congruent figures to find the measure of  $g$ . Show your work.

10.



Directions: Use the relationships between congruent figures to find the measure of  $\angle T$ . Show your work.

11.



Directions: Answer each of the following questions.

12. Triangles  $ABC$  and  $DEF$  are congruent. If the measure of angle  $A$  is  $58^\circ$ , what is the measure of angle  $D$  if it corresponds to angle  $A$ ?

13. True or false. If triangles  $DEF$  and  $GHI$  are similar, then the side lengths may be different but the angle measures are the same.

14. True or false. Similar figures have exactly the same size and shape.

15. True or false. Congruent figures are exactly the same in every way.

16. Triangles  $LMN$  and  $HIJ$  are similar. If this is true, then the side lengths are the same, true or false.

17. What is a proportion?

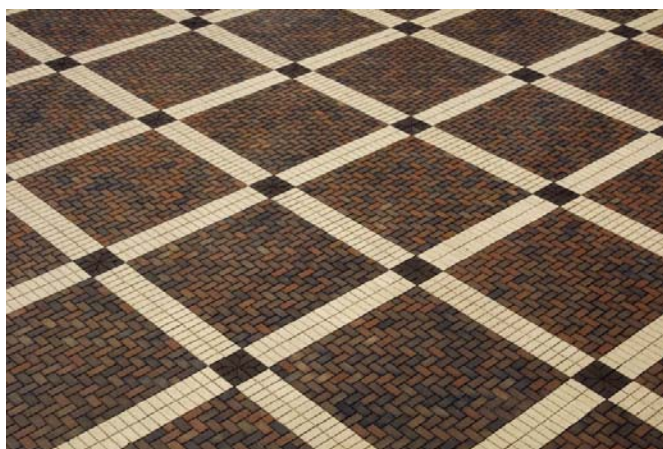
18. True or false. To figure out if two figures are similar, see if their side lengths form a proportion.

19. Define similar figures

20. Define congruent figures.

## 1.14 Congruent Figures

Here you'll recognize congruence and find unknown measures.



Mrs. Gilman brought a small group of students over to look at this tile floor in the hallway of the art museum.

“You see, there is even math in the floor,” she said smiling. Mrs. Gilman is one of those teachers who loves to point out every place where math can be found.

“Okay, I get it,” Jesse started. “I see the squares.”

“There is a lot more math than just squares,” Mrs. Gilman said walking away with a huge smile on her face.

“She frustrates me sometimes,” Kara whispered staring at the floor. “Where is the math besides the squares?”

“I think she is talking about the size of the squares,” Hannah chimed in. “See there are two different sizes.”

“Actually there are three different sizes and there could be more that I haven’t found yet,” Jesse said.

“Remember when we learned about comparing shapes that are alike and aren’t alike, it has to do with proportions or something like that,” Hannah chimed in again.

All three students stopped talking and began looking at the floor again.

“Oh yeah, congruent and similar figures, but which are which?” Kara asked.

“Let’s start with the congruent ones,” Hannah said.

**What are congruent figures? This Concept will teach you all about congruent figures. When you are all finished with this Concept, you will have a chance to study the floor again and see if you can find the congruent figures.**

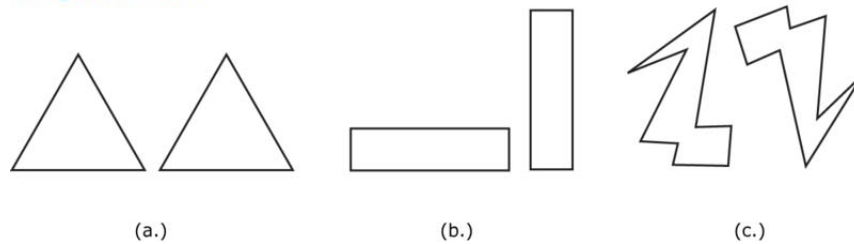
### Guidance

**The word *congruent* means exactly the same.** Sometimes, you will see this symbol  $\cong$ .

**In this Concept, we are going to use the word congruent to compare figures.**

**Congruent figures have exactly the same size and shape.** They have congruent sides and congruent angles. Here are some pairs of congruent figures.

## Congruent Pairs



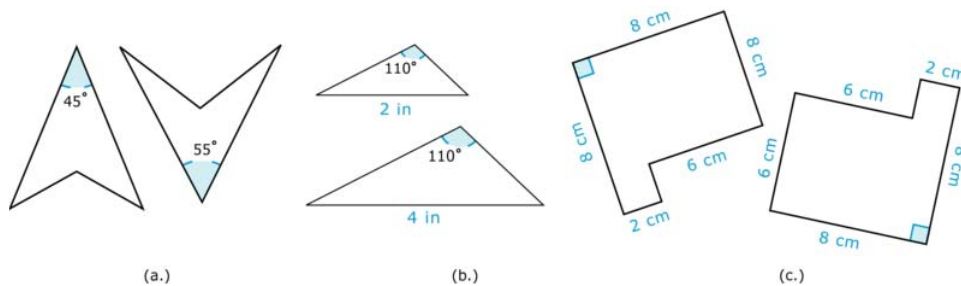
Compare the figures in each pair. They are exactly the same! If you're not sure, imagine that you could cut out one figure and place it on top of the other. If they match exactly, they are congruent.

**How can we recognize congruence?**

**We test for congruency by comparing each side and angle of two figures to see if all aspects of both are the same. If the sides are the same length and the angles are equal, the figures are congruent.** Each side and angle of one figure corresponds to a side or angle in the other. We call these corresponding parts. For instance, the top point of one triangle corresponds to the top point of the other triangle in a congruent pair.

It is not always easy to see the corresponding parts of two figures. One figure may be rotated differently so that the corresponding parts appear to be in different places. If you're not sure, trace one figure and place it on top of the other to see if you can make them match. Let's see if we can recognize some congruent figures.

Which pair of figures below is congruent?

**Let's analyze one pair at a time to see if we can find any corresponding angles and sides that are congruent.**

The figures in the first pair appear to be the same shape, if we rotate one  $180^\circ$  so they both point up. Now we can see all of the corresponding parts, such as the angle at the top and the two long sides on the left and right. This is not enough to go on, however. We need to compare the measures of the angles and the lengths of the sides. If any one set of corresponding parts doesn't match, the figures cannot be congruent.

We only know the measure of one angle in the first two figures. We can compare these angles if they are corresponding parts. They are, because if we rotate one figure these angles are in the same place at the top of each figure. Now compare their measures. The angle in the first figure is  $45^\circ$ . The corresponding angle in the second figure is  $55^\circ$ . **Because the angles are different, these two figures are not congruent.** Let's look at the next pair.

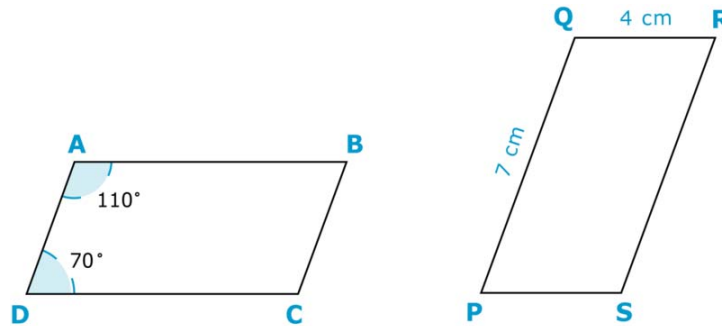
The two triangles in the second pair seem to have corresponding parts: a long base and a wide angle at the top. We need to know whether any of these corresponding parts are congruent, however. We know the measure of the top angle in each figure: it is  $110^\circ$  in both. These figures might be congruent, but we need to see if the sides are congruent to be sure (as we said, similar figures also have congruent angles, but their sides are different lengths). We know the measure of each triangle's base: one is 2 inches and the other is 4 inches. **These sides are not congruent, so the triangles are not congruent. Remember, every side and every angle must be the same in order for figures to be congruent.**

That leaves the last pair. Can you find the corresponding parts? If we rotate the second figure  $180^\circ$ , we have two shapes that look like the letter *L*. Now compare the corresponding sides. The bottom side of each is 8 cm, the long

left side of each is 8 cm, two sides are 6 cm, and two sides are 2 cm. All of the angles in both figures are  $90^\circ$ . **Because every side and angle in one figure corresponds to a congruent side and angle in the second, these two figures are congruent.**

**What about angle measures of congruent figures?**

We know that congruent figures have exactly the same angles and sides. That means we can use the information we have about one figure in a pair of congruent figures to find the measure of a corresponding angle or side in the other figure. Let's see how this works. Take a look at the congruent figures below.



We have been told these two parallelograms are congruent.

**Can you find the corresponding parts?**

If not, trace one parallelogram and place it on top of the other. Rotate it until the parts correspond.

**Which sides and angles correspond?**

We can see that side  $AB$  corresponds to side  $PQ$ . Because they are congruent, we write.

$$AB \cong PQ.$$

**What other sides are congruent?** Let's write them out.

$$AB \cong PQ$$

$$BC \cong QR$$

$$AD \cong PS$$

$$DC \cong SR$$

We can also write down the corresponding angles, which we know must be congruent because the figures are congruent.

$$\angle A \cong \angle P$$

$$\angle B \cong \angle Q$$

$$\angle D \cong \angle S$$

$$\angle C \cong \angle R$$

**Now that we understand all of the corresponding relationships in the two figures, we can use what we know about one figure to find the measure of a side or angle in the second figure.**

**Can we find the length of side  $AB$ ?**

We do not know the length of  $AB$ . However, we do know that it is congruent to  $PQ$ , so if we can find the length of  $PQ$  then it will be the same for  $AB$ .  $PQ$  is 7 centimeters. Therefore  $AB$  must also be 7 centimeters long.

**Now let's look at the angles. Can we find the measure of  $\angle C$ ?**



It corresponds to  $\angle R$ , but we do not know the measure of  $\angle R$  either. Well, we do know the measures of two of the angles in the first parallelogram:  $70^\circ$  and  $110^\circ$ . If we had three, we could subtract from  $360^\circ$  to find the fourth, because all quadrilaterals have angles that add up to  $360^\circ$ . We do not know the measure of  $\angle B$ , but this time we do know the measure of its corresponding angle,  $\angle Q$ . These two angles are congruent, so we know that  $\angle B$  must measure  $70^\circ$ . Now we know three of the angles in the first figure, so we can subtract to find the measure of  $\angle C$ .

$$360 - (70 + 110 + 70) = \angle C$$

$$360 - 250 = \angle C$$

$$110^\circ = \angle C$$

**We were able to combine the given information from both figures because we knew that they were congruent.**



**Yes and the more you work on puzzles like this one the easier they will become.**

Now it's time for you to try a few on your own.

### Example A

True or false. Congruent figures have the same number of sides and angles.

**Solution: True**

### Example B

True or false. Congruent figures can have one pair of angles with the same measure, but not all angles have the same measure.

**Solution: False**

### Example C

True or false. Congruent figures can be different sizes as long as the angle measures are the same.

**Solution: False**

Now back to the congruent figures at the art museum.



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“You see, there is even math in the floor,” she said smiling. Mrs. Gilman is one of those teachers who loves to point out every place where math can be found.

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All three students stopped talking and began looking at the floor again.

“Oh yeah, congruent and similar figures, but which are which?” Kara asked.

“Let’s start with the congruent ones,” Hannah said.

**The congruent figures are exactly the same. We can say that the small dark brown squares are congruent because they are just like each other. They have the same side lengths. What is one other pair of congruent squares?**

Make a note of the congruent figures that you can find and then share your findings with a friend. Compare answers and continue with the Concept.

### Vocabulary

Here are the vocabulary words in this Concept.

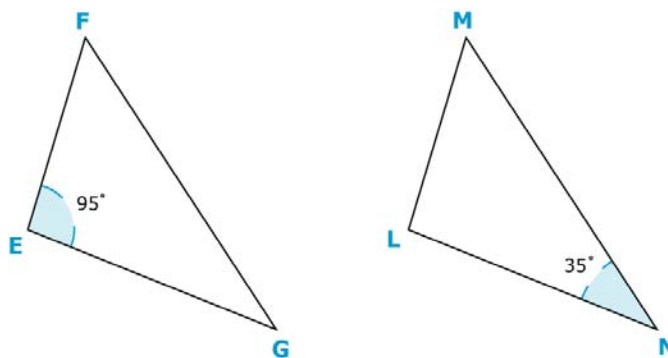
#### Congruent

having exactly the same shape and size. All side lengths and angle measures are the same.

### Guided Practice

Here is one for you to try on your own.

What is the measure of  $\angle M$ ?

**Answer**

We can use reasoning to figure out this problem. First, we know that the two triangles are congruent. We also know two of the three angle measures of the triangles.

Let's write an equation.

$$95 + 35 + x = 180$$

Now we can solve for the missing angle measure.

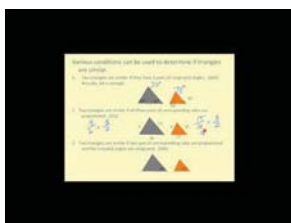
$$130 + x = 180$$

$$x = 50$$

**The measure of the missing angle is  $50^\circ$ .**

**Video Review**

Here is a video for review.



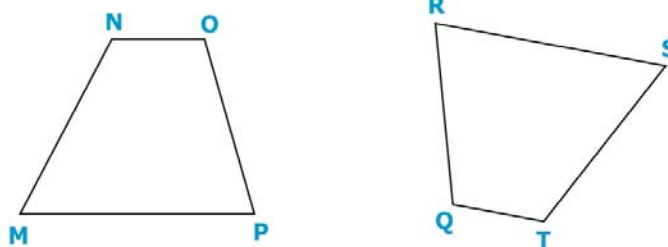
**MEDIA**

Click image to the left for more content.

- This is a James Sousa video on similar and congruent triangles.

**Practice**

Directions: Name the corresponding parts to those given below.



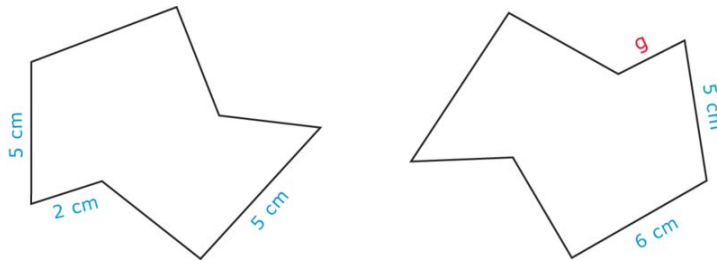
1.  $\angle R$

2.  $MN$

3.  $\angle O$

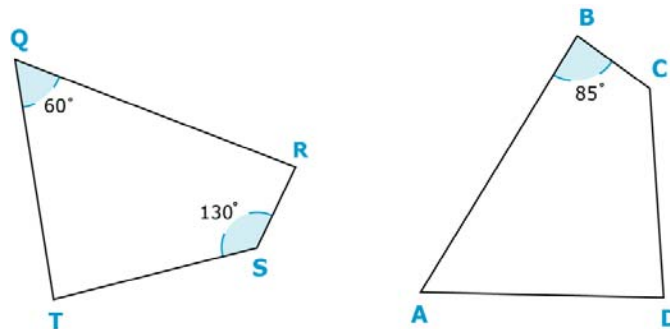
Directions: Use the relationships between congruent figures to find the measure of  $g$ . Show your work.

4.



Directions: Use the relationships between congruent figures to find the measure of  $\angle T$ . Show your work.

5.



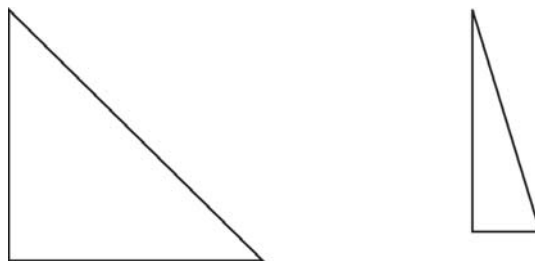
Directions: Answer each of the following questions.

6. Triangles  $ABC$  and  $DEF$  are congruent. If the measure of angle  $A$  is  $58^\circ$ , what is the measure of angle  $D$  if it corresponds to angle  $A$ ?

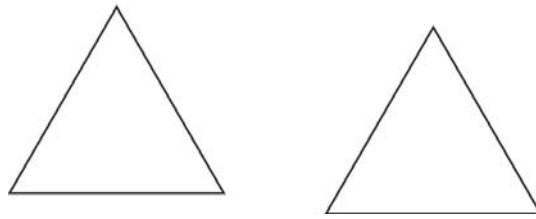
7. True or false. Congruent figures are exactly the same in every way.

Directions: Identify the given triangles as visually congruent or not.

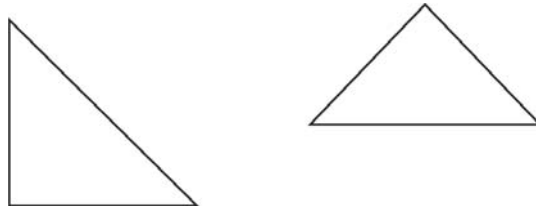
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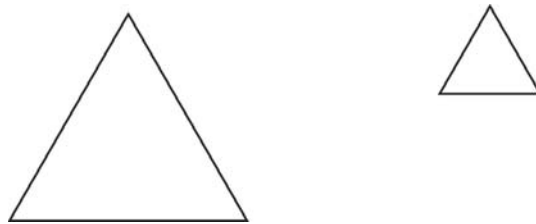
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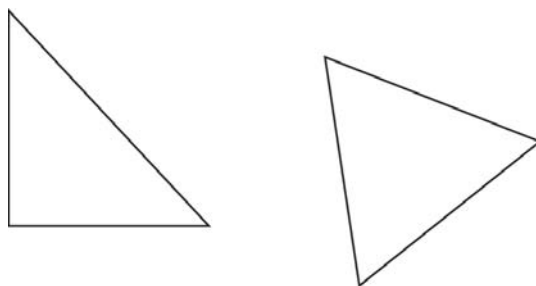
10.



11.



12.



Directions: Answer each of the following questions.

13. Triangles  $ABC$  and  $DEF$  are congruent. Does this mean that their angle measures are the same? Why?

14. Define Congruent.

15. True or false. If two figures are congruent, then they have the same length sides but not the same angle measures.

## 1.15 Supplementary Angles

Here you'll learn what supplementary angles are and how to solve supplementary angle problems.

What if you were given two angles of unknown size and were told they are supplementary? How would you determine their angle measures? After completing this Concept, you'll be able to use the definition of supplementary angles to solve problems like this one.

### Watch This

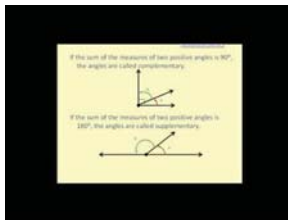


#### MEDIA

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### CK-12 Basic Geometric Definitions

Watch this video beginning at around the 3:20 mark.

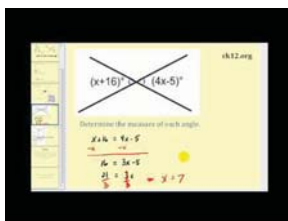


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### James Sousa: Angle Basics—Supplementary Angles

Then watch the second part of this video.



#### MEDIA

Click image to the left for more content.

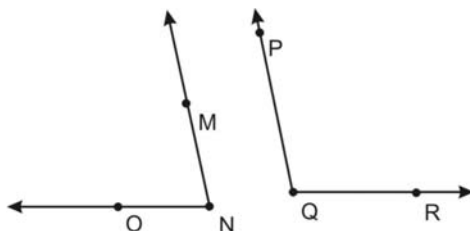
### James Sousa: Supplementary Angles

### Guidance

Two angles are **supplementary** if they add up to  $180^\circ$ . Supplementary angles **do not** have to be *congruent* or *next to each other*.

**Example A**

The two angles below are supplementary. If  $m\angle MNO = 78^\circ$  what is  $m\angle PQR$ ?



Set up an equation. However, instead of equating  $90^\circ$ , now the sum is  $180^\circ$ .

$$\begin{aligned} 78^\circ + m\angle PQR &= 180^\circ \\ m\angle PQR &= 102^\circ \end{aligned}$$

**Example B**

What are the measures of two congruent, supplementary angles?

Supplementary angles add up to  $180^\circ$ . Congruent angles have the same measure. So,  $180^\circ \div 2 = 90^\circ$ , which means two congruent, supplementary angles are right angles, or  $90^\circ$ .

**Example C**

Find the measure of an angle that is supplementary to  $\angle MRS$  if  $m\angle MRS$  is  $70^\circ$ .

Because supplementary angles have to add up to  $180^\circ$ , the other angle must be  $180^\circ - 70^\circ = 110^\circ$ .

**Guided Practice**

Find the measure of an angle that is supplementary to  $\angle ABC$  if  $m\angle ABC$  is:

1.  $45^\circ$
2.  $118^\circ$
3.  $32^\circ$
4.  $2^\circ$

**Answers:**

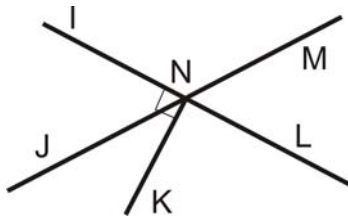
1. Because supplementary angles have to add up to  $180^\circ$ , the other angle must be  $180^\circ - 45^\circ = 135^\circ$ .
2. Because supplementary angles have to add up to  $180^\circ$ , the other angle must be  $180^\circ - 118^\circ = 62^\circ$ .
3. Because supplementary angles have to add up to  $180^\circ$ , the other angle must be  $180^\circ - 32^\circ = 148^\circ$ .
4. Because supplementary angles have to add up to  $180^\circ$ , the other angle must be  $180^\circ - 2^\circ = 178^\circ$ .

**Practice**

Find the measure of an angle that is supplementary to  $\angle ABC$  if  $m\angle ABC$  is:

1.  $114^\circ$
2.  $11^\circ$
3.  $91^\circ$
4.  $84^\circ$
5.  $57^\circ$
6.  $x^\circ$
7.  $(x+y)^\circ$

Use the diagram below for exercises 8-9. Note that  $\overline{NK} \perp \overleftrightarrow{IL}$ .

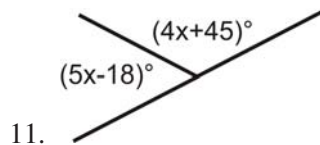


8. Name two supplementary angles.
9. If  $m\angle INJ = 63^\circ$ , find  $m\angle JNL$ .

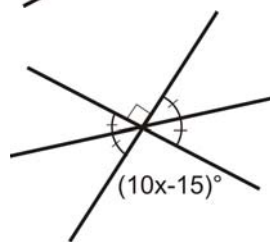
For exercise 10, determine if the statement is true or false.

10. Supplementary angles add up to  $180^\circ$

For 11-12, find the value of  $x$ .



11.



12.



## 1.16 Vertical Angles

Here you'll learn to identify adjacent and vertical angles formed by intersecting lines and then find missing angle measures.



When the students arrived at the art museum, Mrs. Gilson pointed out some of the art work in the courtyard of the museum. One of the paintings was considered “street art” and was immediately noticed by Tania and her friend Yalisha. The two girls walked all around the painting which was about five feet by five feet and stretched across an entire wall.

“This is really cool,” Tania commented. “I love the way the lines intersect. I think that this is a painting all about lines.”

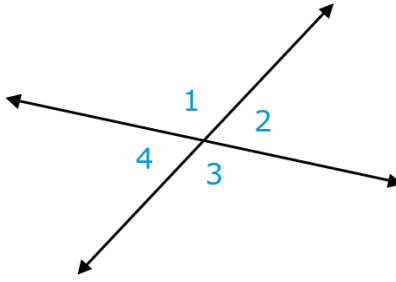
“Me too,” Yalisha agreed. “However, there are angles here too. If you look, you can see that when the lines intersect they form different angles. For example, look at the small dark purple triangle and the light purple quadrilateral. The angles formed by those lines are exactly the same. Did you know that?”

“How can that be? One shape is so much larger than the other?” Tania asked puzzled.

**How can it be? What it is about the relationship between these lines that makes the angles the same or not the same? This Concept is all about angles and special pairs of angles. Keep Tania’s question in mind as you work through this Concept. At the end, see if you can figure out why Yalisha says that the angles are the same.**

### Guidance

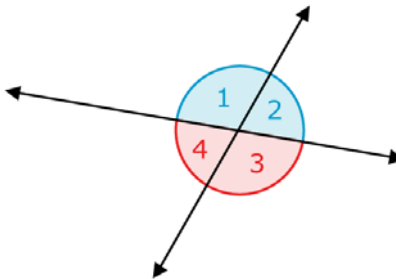
**In this Concept, we will look at the relationships among angles formed by intersecting lines.** Some lines never intersect. Others do, and when they do, they form angles. Take a look at the intersecting lines below.



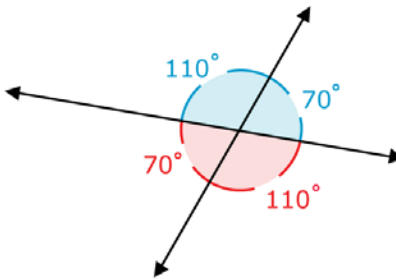
The angles formed by the two intersecting lines are numbered 1 through 4. In this lesson, we will learn how to find the measure of these angles, given the measure of any one of them.

**Let's look at the relationships formed between the angles created when two lines intersect.**

**Adjacent angles** are angles that share the same vertex and one common side. If they combine to make a straight line, adjacent angles must add up to  $180^\circ$ . Below, angles 1 and 2 are adjacent. Angles 3 and 4 are also adjacent. Adjacent angles can also be thought of as “next to” each other.



Can you see that angles 1 and 2, whatever their measurements are, will add up to  $180^\circ$ ? This is true for angles 3 and 4, because they also form a line. But that's not all. Angles 1 and 3 also form a line. So do angles 2 and 4. These are also pairs of adjacent angles. Let's see how this works with angle measurements.



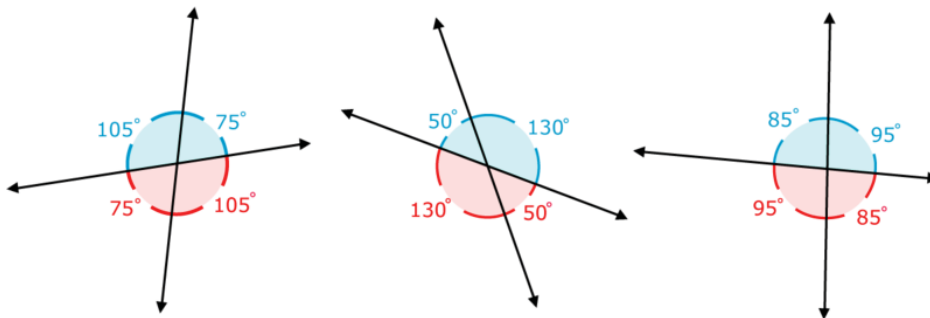
**The sum of each angle pair is  $180^\circ$ . Using the vocabulary from the last section, you can also see that these angle pairs are *supplementary*.**

This pattern of adjacent angles forms whenever two lines intersect. Notice that the two angles measuring  $110^\circ$  are diagonal from each other, and the two angles measuring  $70^\circ$  are diagonal from each other. This is the other special relationship among pairs of angles formed by intersecting lines.

**What are angles on the diagonal called?**

These angle pairs are called **vertical angles**. **Vertical angles are always equal.** Angles 1 and 4 above are vertical angles, and angles 2 and 3 are vertical angles.

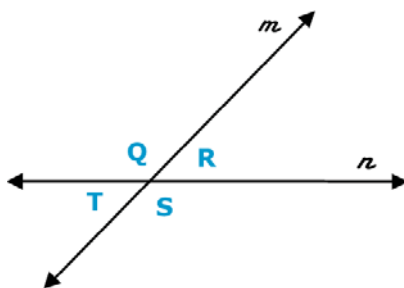
These relationships always exist whenever any two lines intersect. Look carefully at the figures below. Understanding the four angles formed by intersecting lines is a very important concept in geometry.



In each figure, there are pairs of adjacent angles that add up to  $180^\circ$  and pairs of vertical angles that are equal and opposite each other.

Now let's practice recognizing adjacent and vertical angle pairs.

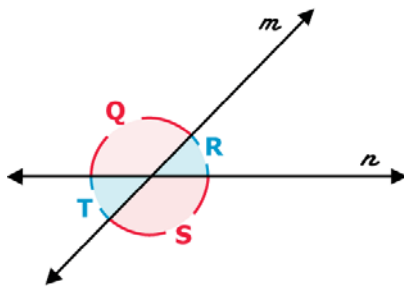
Identify all of the pairs of adjacent angles and the two pairs of vertical angles in the figure below.

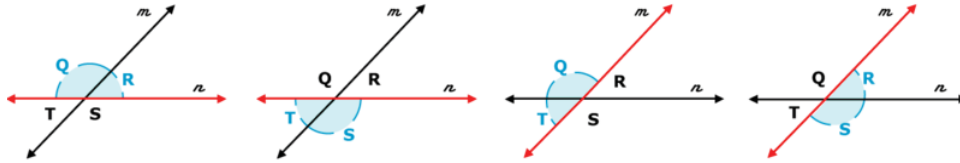


Hey, there aren't any numbers! How are we supposed to know the measures of the four angles? Well, we actually don't need to know them to answer the question. As we have said, adjacent and vertical relationships never change, no matter what the measures of the angles are. The pairs of adjacent angles will always form a straight line, and the pairs of vertical angles will always be opposite each other.

With this in mind, let's look for the adjacent angles. Adjacent angles share a side and, in the case of intersecting lines, will together form a straight line. Which adjacent angles form line  $n$ ? Angles  $Q$  and  $R$  are next to each other and together make a straight angle along line  $n$ . What about  $T$  and  $S$ ? They also sit together along line  $n$ . Both are adjacent pairs. Now let's look at line  $m$ . Which pairs of angles together make a straight angle along line  $m$ ? Angles  $Q$  and  $T$  do, and so do angles  $S$  and  $R$ . All four of these pairs are adjacent.

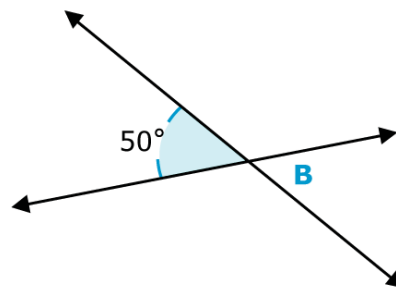
Now let's look for the vertical angles. Remember, vertical angles are equal and opposite each other. Which angles are across from each other? Angles  $Q$  and  $S$  are, and we know that these have the same measure, whatever the measure is. Angles  $T$  and  $R$ , the small angles, are also opposite each other. Therefore they are the other pair of vertical angles.





We can use what we have learned about adjacent and vertical angles to find the measure of an unknown angle formed by intersecting lines. We know that adjacent angles add up to  $180^\circ$  and that vertical angles are equal. Therefore if we are given the measure of one angle, we can use its relationship to another angle to find the measure of the second angle.

Find the measure of angle  $B$  below.



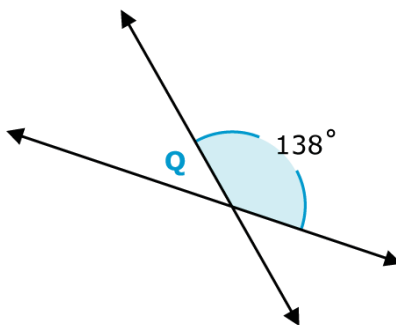
We know that one angle measures  $50$ , and we want to find the measure of angle  $B$ .

First we need to determine how these two angles are related. Is angle  $B$  adjacent or vertical to the known angle? It is opposite, so these two angles are vertical angles. And we already know that vertical angles are always equal, so angle  $B$  must also be  $50^\circ$ .



Here is another one.

Find the measure of  $\angle Q$  below.



Again, we need to find how the known angle and the unknown angle are related. This time angle  $Q$  is not opposite the known angle. It is adjacent, because together they form a straight line. What do we know about adjacent angles? They add up to  $180^\circ$ . Therefore we can use the measure of the known angle to solve for angle  $Q$ .

$$\begin{aligned}138^\circ + \angle Q &= 180 \\ \angle Q &= 180 - 138 \\ \angle Q &= 42^\circ\end{aligned}$$

Angle  $Q$  must be  $42^\circ$ .

Now answer these questions about vertical and adjacent angles.

### Example A

What does the word "adjacent" mean?

**Solution: Next to**

### Example B

True or false. Adjacent angles have the same measure?

**Solution: False**

### Example C

True or false. Vertical angles have the same measure?

**Solution: True**

Here is the original problem once again.



Have you been thinking about Tania’s question? Reread this situation and then write down your answer to her question. Are these two angles the same or aren’t they? Why or why not?

When the students arrived at the art museum, Mrs. Gilson pointed out some of the art work in the courtyard of the museum. One of the paintings was considered “street art” and was immediately noticed by Tania and her friend Yalisha. The two girls walked all around the painting which was about five feet by five feet and stretched across an entire wall.

“This is really cool,” Tania commented. “I love the way the lines intersect. I think that this is a painting all about lines.”

“Me too,” Yalisha agreed. “However, there are angles here too. If you look, you can see that when the lines intersect they form different angles. For example, look at the small dark purple triangle and the light purple quadrilateral. The angles formed by those lines are exactly the same. Did you know that?”

“How can that be? One shape is so much larger than the other?” Tania asked puzzled.

**Let’s think about Tania’s question. The size of the angles isn’t a function of whether or not the shapes are large or not. It has to do with the intersection of the lines. Remember how Tania commented that she thought that the painting was all about lines, well, here is where her point is valid.**

**First, think about what type of angles are formed by the two intersecting lines. We have vertical angles. Any time two lines intersect, the opposing angles formed by the intersection of those two lines is considered vertical angles. Vertical angles are congruent. Therefore, Yalisha’s comment is accurate.**

**Now look at the painting again. Can you find a pair of adjacent angles? Can you find another pair of vertical angles? Make a few notes in your notebook.**

## Vocabulary

Here are the vocabulary words in this Concept.

### Adjacent Angles

angles formed by intersecting lines that are supplementary and are next to each other. The sum of their angles is  $180^\circ$ .

### Vertical Angles

angles formed by intersecting lines that are on the diagonals. They have the same measure.

**Supplementary**

having a sum of  $180^\circ$ .

**Intersecting Lines**

lines that cross at one point.

**Parallel Lines**

lines that will never cross.

**Perpendicular Lines**

lines that intersect at a right angle.

**Corresponding Angles**

Angles that are in the same place in each intersection when a line crosses two parallel lines.

**Guided Practice**

Here is one for you to try on your own.

Two lines intersect. One vertical angle has a measure of  $45^\circ$ . What is the measure of the angle adjacent to it?

**Answer**

To figure this out, we can use what we know about vertical and adjacent angles.

Two lines are intersecting, the number of degrees in a straight line is  $180^\circ$ .

The known angle is  $45^\circ$ . Therefore, we can subtract this angle from 180 to find the number of degrees in the angle next to the vertical angles.

$$180 - 45 = 145$$

**The adjacent angle has a measure of  $145^\circ$ .**

**Video Review**

Here is a video for review.

**MEDIA**

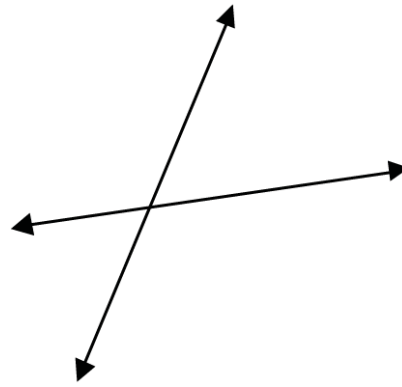
Click image to the left for more content.

- This is a James Sousa video on complementary and supplementary angles.

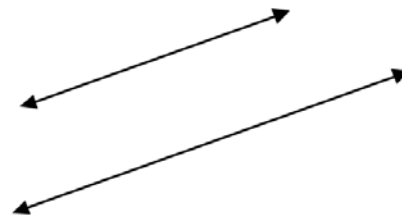
**Practice**

Directions: Identify whether the lines below are parallel, perpendicular, or just intersecting.

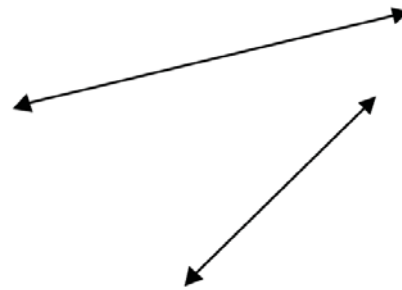
1.



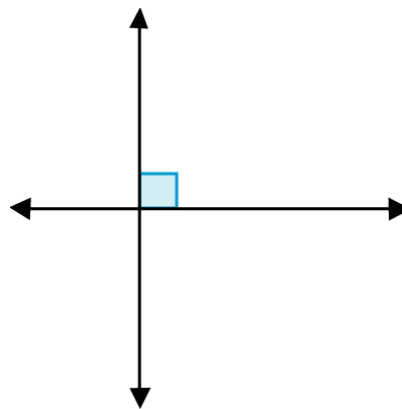
2.



3.



4.



5. Lines that will never intersect.

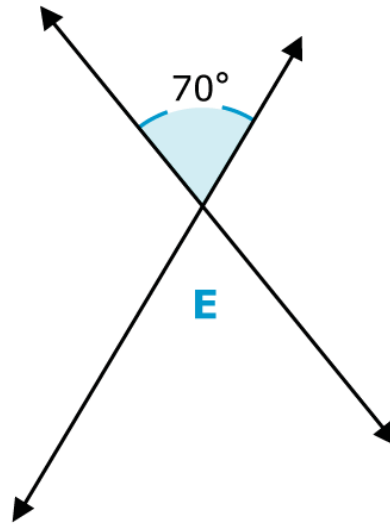
6. Lines that intersect at a  $90^\circ$  angle.

7. Lines that cross at one point.

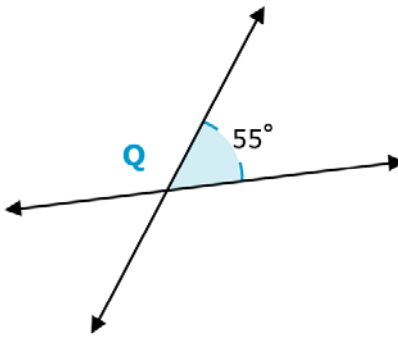
Directions: Tell whether the pairs of angles are adjacent or vertical.

8.

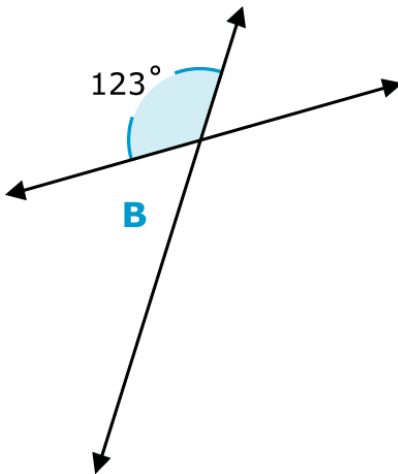




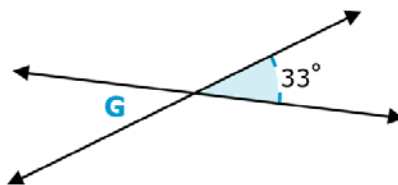
9.



10.



11.



12. Two angles with the same measure.

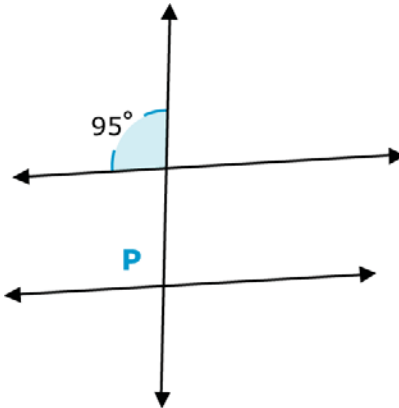
13. An angle next to another angle.

14. An angle that is congruent to another angle.

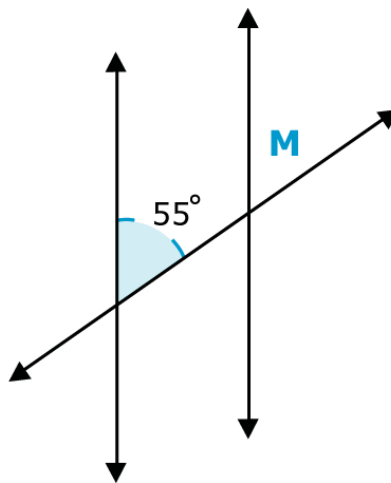
15. Two angles with different measures whose sum is  $180^\circ$ .

Directions: Find the measure of the unknown angle.

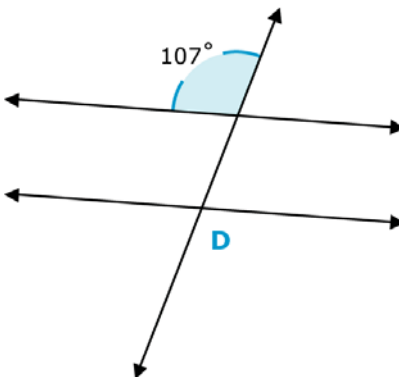
16.



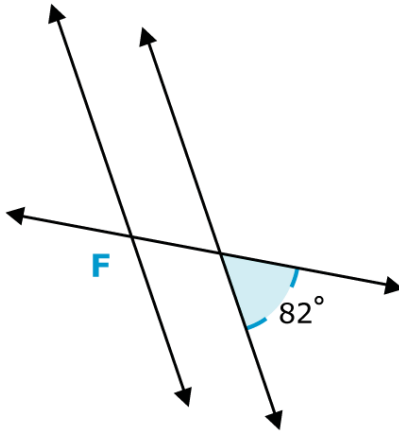
17.



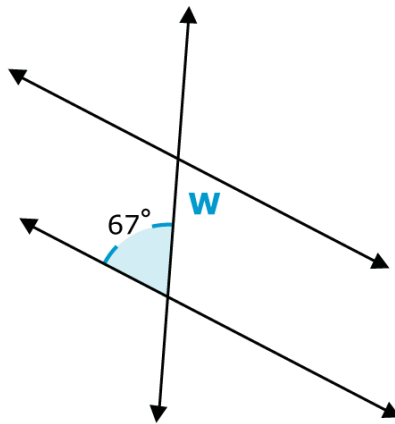
18.



19.



20.



## 1.17 Recognizing Dilations

Here you'll recognize dilations and use scale factor to determine measurements.

Have you ever thought about a log cabin? Take a look at this dilemma about building log cabins?



Sherri decided to do her project on a log cabin. Mrs. Patterson suggested that she choose a time period to work with because log cabins have been being built for a long time. Years ago, they were quite small, but today, log cabins can be designer homes too.

“Mrs. Patterson, I am going to focus on a log cabin from the 1800’s,” Sherri said taking out a book that she found at the library on log cabins.

“That is a great idea. What was the average size of a log cabin in 1800?” Mrs. Patterson asked.

“What do you mean?”

“I mean the square footage. How many feet wide was the average house and how many feet long?” Mrs. Patterson explained.

“Oh, I get it. The average house was  $20 \times 40$  feet. So the average was 800 square feet,” Sherri said.

“Terrific, now make sure that your plan shows that,” Mrs. Patterson said walking away.

Sherri is puzzled. She knows that the shape of the log cabin is a rectangle given the length and width. She knows the area of the house. To create a plan, she will need to create a dilation. Sherri decides that she will use a scale factor of  $\frac{1}{16}$ . Given this information, what will the dimensions be of her house plan?

**Use this Concept to learn about dilations and figure out the dimensions of the house by the end of it.**

### Guidance

There are all kinds of transformations. We can flip or reflect a figure, translate or slide a figure and rotate a figure. We can also stretch or shrink a figure to create a new one. This is called a **dilation**.

**A dilation is a transformation created by a scale factor.**

We can create a dilation that is smaller or larger than the original figure. Either way, a similar figure is created through a dilation.

Let's think about scale factors for a minute.

**The *scale factor* is the ratio that determines the proportional relationship between the sides of similar figures.**

For the pairs of sides to be proportional to each other, they must have the same scale factor. In other words, similar figures have congruent angles and sides with the same scale factor. A scale factor of two means that each side of the larger figure is exactly twice as long as the corresponding side in the smaller figure.

When we compare the corresponding sides of a figure, we can figure out the scale factor of that figure.

**A figure has a side length of 3 feet. What would be the corresponding side length of the next figure if the scale factor was 4?**

Let's think about this. We know the length of one of the sides of the first figure and we know the scale factor. To figure out the new length, we can multiply the scale factor times the first length.

$$3 \times 4 = 12$$

**The length of the corresponding side of the second figure is 12 feet.**

**When we have a figure that is larger than the original, we have a scale factor that is greater than one. If we have a figure that is smaller than the original, then we have a scale factor that is less than one or a fraction.**

**A figure has a side length of 5 meters. What would be the corresponding side length of the new figure if the scale factor is  $\frac{1}{2}$ ?**

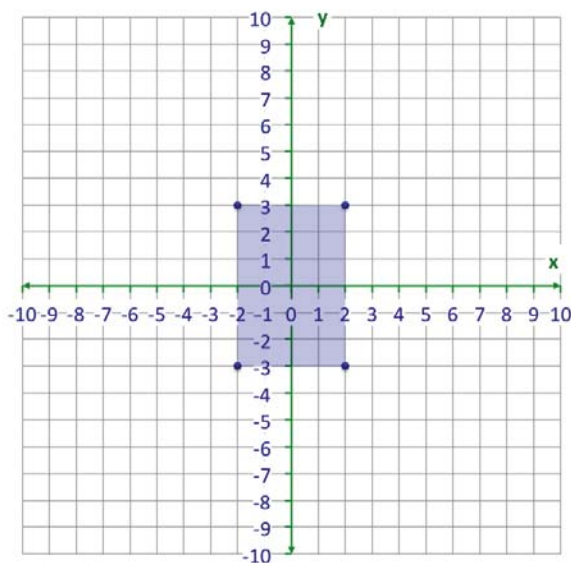
To figure this out, we have to take the given length of the first figure and divide it in half. This will give us the corresponding length of the second figure.

$$5 \left(\frac{1}{2}\right) = 2.5$$

**The length of the corresponding side will be 2.5 meters.**

Now that you understand dilations, we can look at how to work with them on the coordinate plane. Once again, we will be using coordinate notation to describe the different dilations that are created on the coordinate plane.

Let's look at this figure and then see how we can graph the dilation of it.

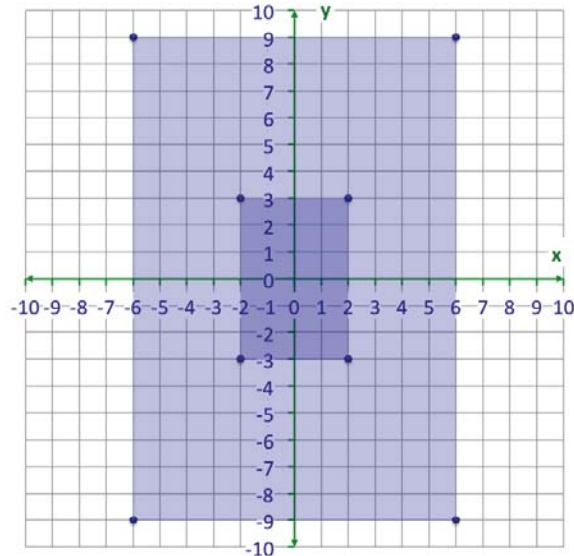


**Graphing dilations of geometric figures is actually fairly easy to do when we know the scale factor. We simply multiply both coordinates for each vertex by the scale factor to produce new coordinates.**

Suppose we want to make an enlargement of the rectangle above using a scale factor of 3. We need to multiply each coordinate by 3.

$$\begin{array}{l} (-2, -3) \quad (-2, 3) \quad (2, 3) \quad (2, -3) \\ \times 3 \\ (-6, -9) \quad (-6, 9) \quad (6, 9) \quad (6, -9) \end{array}$$

Now we can graph it on the coordinate plane.



We can create a reduction too. We create a reduction by dividing each coordinate by the scale factor. This will give us the new measurements of the figure.

Find each new measurement given the scale factor.

A quadrilateral with side measures of 6, 15, 27, 30.

### Example A

A scale factor of  $\frac{1}{3}$ .

**Solution: 2, 5, 9, 10**

### Example B

A scale factor of  $\frac{1}{2}$ .

**Solution: 3, 7.5, 13.5, 15**

### Example C

A scale factor of 2.

**Solution: 12, 30, 54, 60**

Now let's go back to the dilemma from the beginning of the Concept.

To solve this problem, we begin with the actual dimensions of the log cabin. The log cabin has real – world dimensions of  $20 \times 40$  feet.

Sherri is using a scale factor of  $\frac{1}{16}$ . That means that the dilation will be a reduction. We divide both dimensions by 16.

$$20 \div 16 = 1.25 \text{ ft.}$$

$$40 \div 16 = 2.5 \text{ ft}$$

The dimensions of Sherri’s plan will be  $1.25 \text{ ft wide} \times 2.5 \text{ ft long}$ .

## Vocabulary

### Dilation

to reduce or enlarge a figure according to a scale factor.

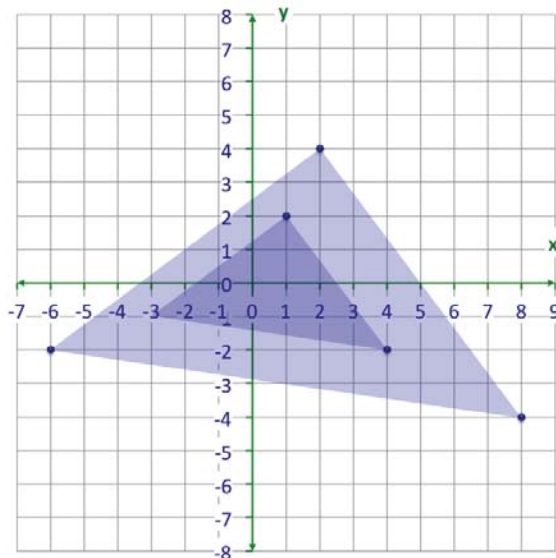
### Scale Factor

the ratio that compares the lengths of corresponding sides to each other. That comparison is the scale factor.

## Guided Practice

Here is one for you to try on your own.

Graph a reduction of the following figure if the scale factor is  $\frac{1}{2}$ .



## Solution

Notice that each of the original coordinates were divided by two to create the coordinates of the reduction.

$$(2, 4) \div 2 = (1, 2)$$

$$(8, -4) \div 2 = (4, -2)$$

$$(-6, -2) \div 2 = (-3, -1)$$

**Video Review****MEDIA**

Click image to the left for more content.

**Practice**

Directions: Use each scale factor to determine the new dimensions of each figure.

1. A triangle with side measures of 4, 5, 9 and a scale factor of 2.
2. A triangle with side measures of 4, 5, 9 and a scale factor of 3.
3. A triangle with side measures of 4, 5, 9 and a scale factor of 4.
4. A triangle with side measures of 8, 10, 14 and a scale factor of 2.
5. A triangle with side measures of 8, 10, 14 and a scale factor of 4.
6. A triangle with side measures of 2, 4, 6 and a scale factor of 2.
7. A quadrilateral with side measures of 4, 6, 8, 10 and a scale factor of  $\frac{1}{2}$ .
8. A quadrilateral with side measures of 12, 16, 20, 24 and a scale factor of  $\frac{1}{4}$ .
9. A quadrilateral with side measures of 4, 6, 8, 10 and a scale factor of 2.
10. A quadrilateral with side measures of 4, 6, 8, 10 and a scale factor of 3.
11. A quadrilateral with side measures of 4, 6, 8, 10 and a scale factor of 4.
12. A quadrilateral with side measures of 9, 12, 18, 24 and a scale factor of  $\frac{1}{3}$ .
13. A quadrilateral with side measures of 9, 12, 18, 24 and a scale factor of 2.
14. A quadrilateral with side measures of 9, 12, 18, 24 and a scale factor of 3.
15. A quadrilateral with side measures of 8, 12, 16, 24 and a scale factor of  $\frac{1}{4}$ .
16. A quadrilateral with side measures of 9, 12, 18, 24 and a scale factor of  $\frac{1}{2}$ .



## 1.18 Corresponding Angles

Here you'll learn what corresponding angles are and their relationship with parallel lines.

What if you were presented with two angles that are in the same place with respect to the transversal but on different lines? How would you describe these angles and what could you conclude about their measures? After completing this Concept, you'll be able to answer these questions using your knowledge of corresponding angles.

### Watch This

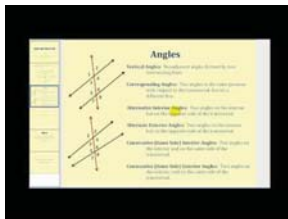


#### MEDIA

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### CK-12 Foundation: Chapter3CorrespondingAnglesA

Watch the portions of this video dealing with corresponding angles.

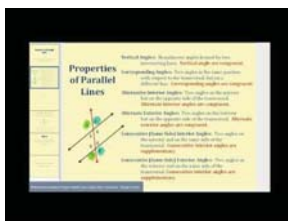


#### MEDIA

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### James Sousa:Angles and Transversals

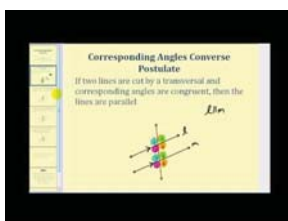
Watch this video beginning at the 4:50 mark.



#### MEDIA

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### James Sousa:Corresponding Angles Postulate



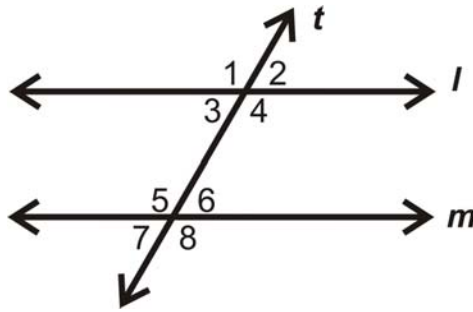
#### MEDIA

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## James Sousa:Corresponding Angles Converse

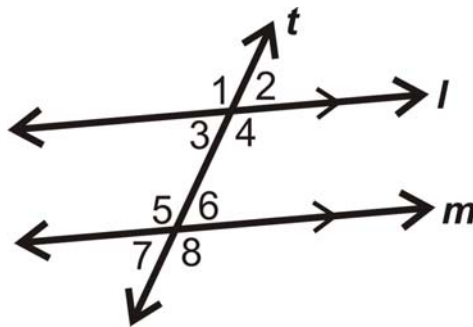
**Guidance**

**Corresponding Angles** are two angles that are in the “same place” with respect to the transversal, but on different lines. Imagine sliding the four angles formed with line  $l$  down to line  $m$ . The angles which match up are corresponding.  $\angle 2$  and  $\angle 6$  are corresponding angles.



**Corresponding Angles Postulate:** If two parallel lines are cut by a transversal, then the corresponding angles are congruent.

If  $l \parallel m$  and both are cut by  $t$ , then  $\angle 1 \cong \angle 5$ ,  $\angle 2 \cong \angle 6$ ,  $\angle 3 \cong \angle 7$ , and  $\angle 4 \cong \angle 8$ .

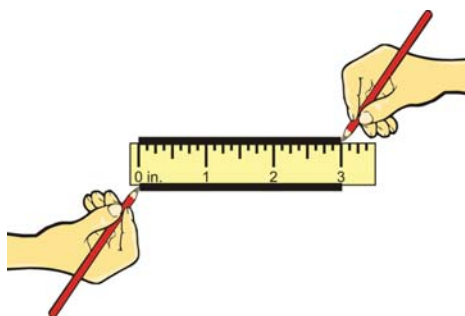


**Converse of Corresponding Angles Postulate:** If corresponding angles are congruent when two lines are cut by a transversal, then the lines are parallel.

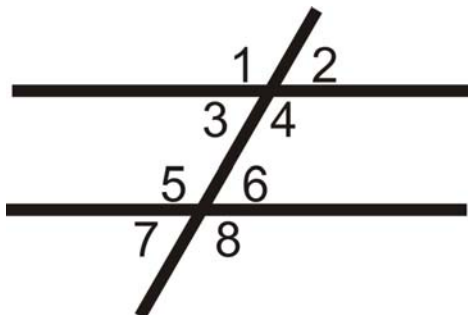
**Investigation: Corresponding Angles Exploration**

You will need: paper, ruler, protractor

1. Place your ruler on the paper. On either side of the ruler, draw lines, 3 inches long. This is the easiest way to ensure that the lines are parallel.



- Remove the ruler and draw a transversal. Label the eight angles as shown.

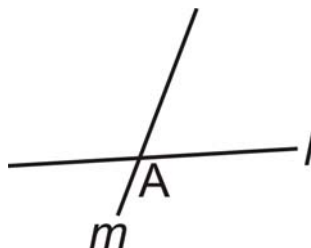


- Using your protractor, measure all of the angles. What do you notice?

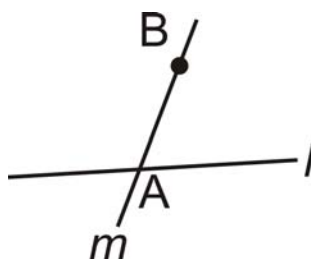
In this investigation, you should see that  $m\angle 1 = m\angle 4 = m\angle 5 = m\angle 8$  and  $m\angle 2 = m\angle 3 = m\angle 6 = m\angle 7$ .  $\angle 1 \cong \angle 4$ ,  $\angle 5 \cong \angle 8$  by the Vertical Angles Theorem. By the Corresponding Angles Postulate, we can say  $\angle 1 \cong \angle 5$  and therefore  $\angle 1 \cong \angle 8$  by the Transitive Property.

### Investigation: Creating Parallel Lines using Corresponding Angles

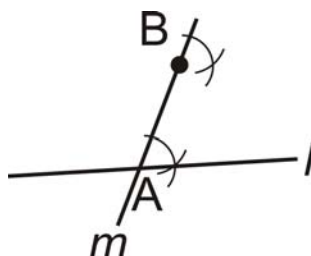
- Draw two intersecting lines. Make sure they are not perpendicular. Label them  $l$  and  $m$ , and the point of intersection,  $A$ , as shown.



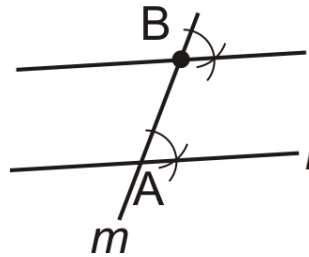
- Create a point,  $B$ , on line  $m$ , above  $A$ .



- Copy the acute angle at  $A$  (the angle to the right of  $m$ ) at point  $B$ . See Investigation 2-2 in Chapter 2 for the directions on how to copy an angle.



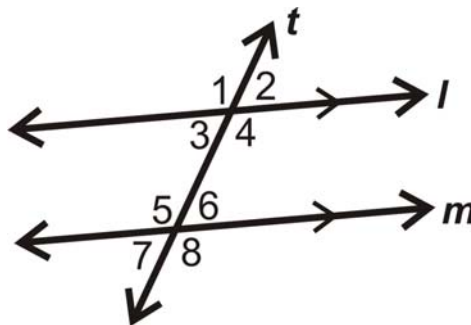
- Draw the line from the arc intersections to point  $B$ .



From this construction, we can see that the lines are parallel.

### Example A

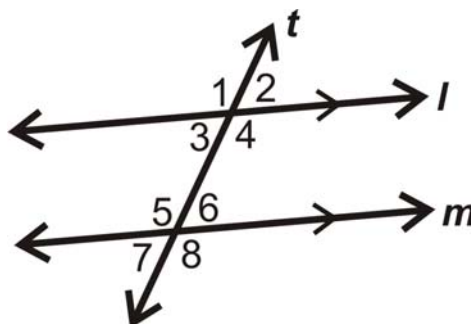
If  $m\angle 8 = 110^\circ$  and  $m\angle 4 = 110^\circ$ , then what do we know about lines  $l$  and  $m$ ?



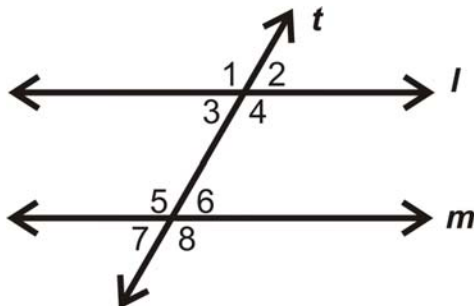
$\angle 8$  and  $\angle 4$  are corresponding angles. Since  $m\angle 8 = m\angle 4$ , we can conclude that  $l \parallel m$ .

### Example B

If  $m\angle 2 = 76^\circ$ , what is  $m\angle 6$ ?



$\angle 2$  and  $\angle 6$  are corresponding angles and  $l \parallel m$ , from the markings in the picture. By the Corresponding Angles Postulate the two angles are equal, so  $m\angle 6 = 76^\circ$ .

**Example C**

Using the picture above, list pairs of corresponding angles.

Corresponding Angles:  $\angle 3$  and  $\angle 7$ ,  $\angle 1$  and  $\angle 5$ ,  $\angle 4$  and  $\angle 8$

Watch this video for help with the Examples above.



**MEDIA**

Click image to the left for more content.

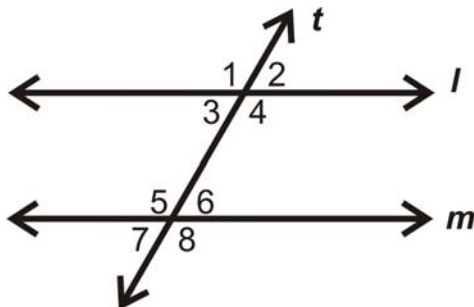
[CK-12 Foundation: Chapter3CorrespondingAnglesB](#)

**Vocabulary**

**Corresponding Angles** are two angles that are in the “same place” with respect to the transversal, but on different lines.

**Guided Practice**

Lines  $l$  and  $m$  are parallel:



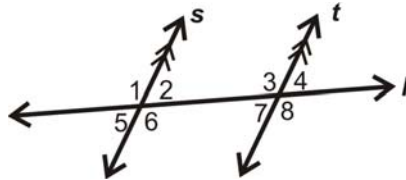
1. If  $\angle 1 = 3x + 1$  and  $\angle 5 = 4x - 3$ , solve for  $x$ .
2. If  $\angle 2 = 5x + 2$  and  $\angle 6 = 3x + 10$ , solve for  $x$ .
3. If  $\angle 7 = 5x + 6$  and  $\angle 3 = 8x - 10$ , solve for  $x$ .

**Answers:**

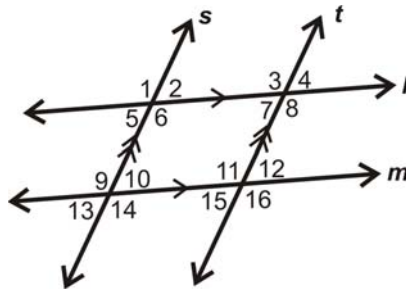
1. Since they are corresponding angles and the lines are parallel, they must be congruent. Set the expressions equal to each other and solve for  $x$ .  $3x + 1 = 4x - 3$  so  $x = 4$ .
2. Since they are corresponding angles and the lines are parallel, they must be congruent. Set the expressions equal to each other and solve for  $x$ .  $5x + 2 = 3x + 10$  so  $x = 4$ .
3. Since they are corresponding angles and the lines are parallel, they must be congruent. Set the expressions equal to each other and solve for  $x$ .  $5x + 5 = 8x - 10$  so  $x = 5$ .

### Practice

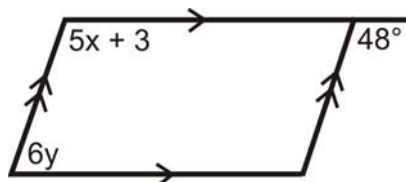
1. Determine if the angle pair  $\angle 4$  and  $\angle 2$  is congruent, supplementary or neither:



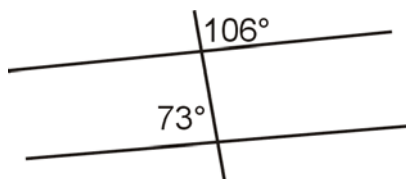
2. Give two examples of corresponding angles in the diagram:



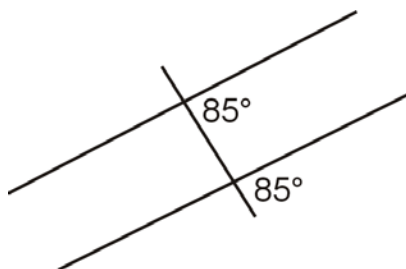
3. Find the value of  $x$ :



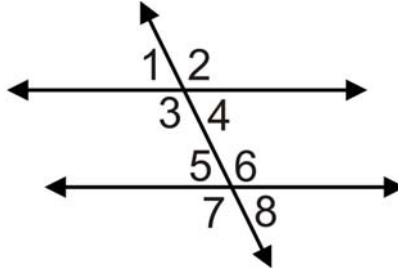
4. Are the lines parallel? Why or why not?



5. Are the lines parallel? Justify your answer.

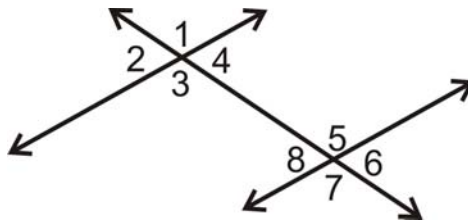


For 6-10, what does the value of  $x$  have to be to make the lines parallel?



6. If  $m\angle 1 = (6x - 5)^\circ$  and  $m\angle 5 = (5x + 7)^\circ$ .
7. If  $m\angle 2 = (3x - 4)^\circ$  and  $m\angle 6 = (4x - 10)^\circ$ .
8. If  $m\angle 3 = (7x - 5)^\circ$  and  $m\angle 7 = (5x + 11)^\circ$ .
9. If  $m\angle 4 = (5x - 5)^\circ$  and  $m\angle 8 = (3x + 15)^\circ$ .
10. If  $m\angle 2 = (2x + 4)^\circ$  and  $m\angle 6 = (5x - 2)^\circ$ .

For questions 11-15, use the picture below.



11. What is the corresponding angle to  $\angle 4$ ?
12. What is the corresponding angle to  $\angle 1$ ?
13. What is the corresponding angle to  $\angle 2$ ?
14. What is the corresponding angle to  $\angle 3$ ?
15. Are the two lines parallel? Explain.

## 1.19 Corresponding Angles

Here you'll learn to identify corresponding angles formed when a line intersects two other lines.

Do you know how to identify corresponding angles?

Jonas found a sculpture made of wire. In the sculpture there were two straight pieces of firm wire that formed parallel lines and one straight piece of wire that intersected the parallel wires.

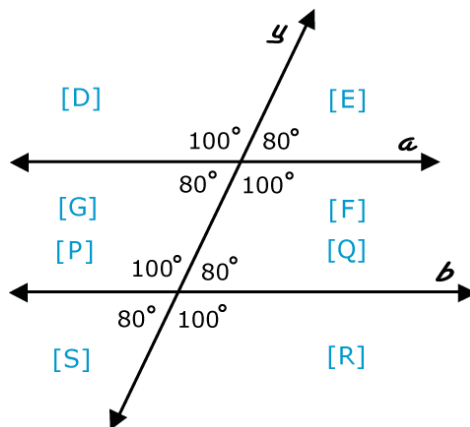
Jonas counted 7 angles, but he isn't sure this is correct.

Do you know how many angles are formed when a straight line intersects two parallel lines?

**Use this Concept to figure out the solution to Jonas' dilemma.**

### Guidance

We have seen how intersecting lines form four angles that share certain relationships with each other. Now let's take this idea one step further. When a line intersects with two lines that are parallel, it forms the same angles of intersection with the first parallel line and the second. Let's see what this looks like.

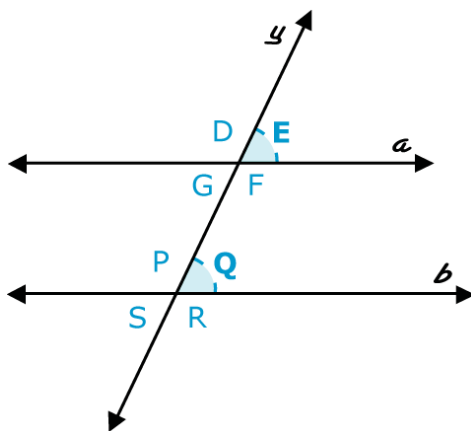


**Notice that new angle relationships are formed. We can divide the line which is  $180^\circ$  into new angles. Notice the measures of each angle that has been formed.**

**When line  $y$  intersects with line  $a$ , it forms  $100^\circ$  angles and  $80^\circ$  angles. When it intersects with line  $b$ , it also forms  $100^\circ$  and  $80^\circ$  angles! This is because lines  $a$  and  $b$  are parallel. Any line will intersect with them the same way.**

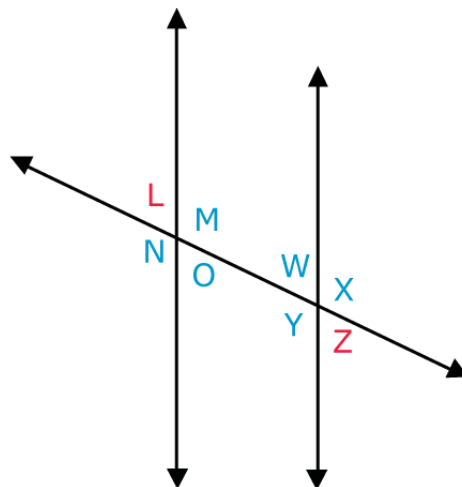
In this situation, we have another angle relationship that will help us find the measure of the angles formed at either point of intersection. Every angle at the first intersection (between lines  $y$  and  $a$ ) corresponds to an angle at the second intersection (between lines  $y$  and  $b$ ). It occurs in the same place and has the same measure. Take a look at the figure below.





**Angle  $E$  in the first intersection is in the same place as angle  $Q$  in the second intersection. We call these angles *corresponding angles*. They are in the same place in each intersection, and they have the same measure.** Angles  $D$  and  $P$  are corresponding angles, angles  $G$  and  $S$  are corresponding, and angles  $F$  and  $R$  are corresponding. These relationships always exist when a line intersects with parallel lines. Let's practice identifying corresponding angles.

What angle corresponds to angle  $Z$ ? To angle  $L$ ?



**This time the parallel lines are vertical, but the relationships stay the same. Imagine you could place one intersection on top of each other. They would be exactly the same, and the corresponding angles would be in the same place.**

We need to find the angle that corresponds to angle  $Z$ . Angle  $Z$  is the bottom right angle formed at the second intersection. Its corresponding angle will be the bottom right angle formed at the first intersection. Which angle is this?

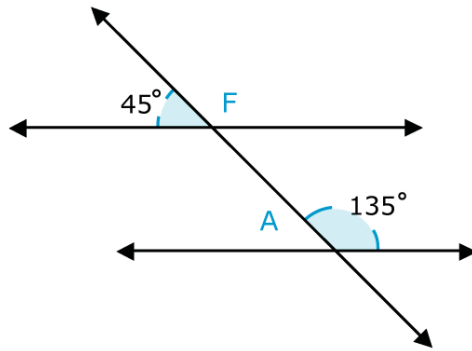
**Angle  $O$  occurs at the same place in the first intersection, so it is the corresponding angle to angle  $Z$ .**

**Angle  $L$  is the top left angle formed at the first intersection. Its corresponding angle will be the top left angle formed at the second intersection. This is angle  $W$ , so angles  $L$  and  $W$  are corresponding angles.**

Now that we understand corresponding relationships, we can use the angles at one intersection to help us find the measure of angles in the other intersection.

As we have said, **corresponding angles are exactly the same, so they have the same measure.**

Therefore **if we know the measure of an angle at one intersection, we also know the measure of its corresponding angle at the second intersection.**



In the figure above, the  $45^\circ$  angle and angle  $A$  are corresponding angles. What must the measure of angle  $A$  be? You guessed it:  $45^\circ$ . What about angle  $F$ ? It corresponds to the  $135^\circ$  angle in the second intersection, so it too must be  $135^\circ$ .



*Working in this way is a lot like figuring out a puzzle! You can figure out any missing angles with just a few clues.*

Answer the following questions and figure out the missing angle measures.

### Example A

If one vertical angle is  $45^\circ$ , what is the other vertical angle?

**Solution:**  $45^\circ$

### Example B

True or false. Corresponding angles are matching angles.

**Solution:** True.

### Example C

True or false. When a line intersects two parallel lines, then eight angles are formed.

**Solution:** True.

Here is the original problem once again.

Jonas found a sculpture made of wire. In the sculpture there were two straight pieces of firm wire that formed parallel lines and one straight piece of wire that intersected the parallel wires.

Jonas counted 7 angles, but he isn't sure this is correct.

Do you know how many angles are formed when a straight line intersects two parallel lines?

Jonas isn't correct. When a straight line intersects two parallel lines, then there are 8 angles formed.

## Vocabulary

Here are the vocabulary words in this Concept.

### Adjacent Angles

angles formed by intersecting lines that are supplementary and are next to each other. The sum of their angles is  $180^\circ$ .

### Vertical Angles

angles formed by intersecting lines that are on the diagonals. They have the same measure.

### Supplementary

having a sum of  $180^\circ$ .

### Intersecting Lines

lines that cross at one point.

### Parallel Lines

lines that will never cross.

### Perpendicular Lines

lines that intersect at a right angle.

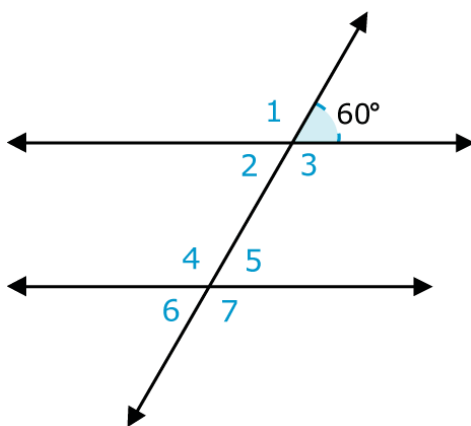
### Corresponding Angles

Angles that are in the same place in each intersection when a line crosses two parallel lines.

## Guided Practice

Here is one for you to try on your own.

Fill in the figure below with the angle measure for all of the angles shown.



**Answer**

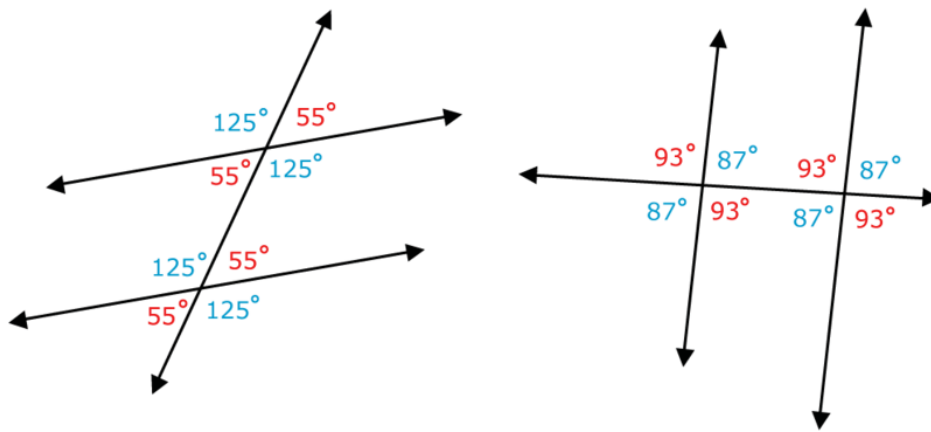
Wow, we only have one angle to go on. Not to worry though! We know how to find the measure of its adjacent angles, its vertical angle, and its corresponding angle. That's all we need to know.

Let's put in its adjacent angles first. If the known angle is  $60^\circ$ , then the adjacent angles are  $180 - 60 = 120^\circ$ . Angle 1 is adjacent to the  $60^\circ$  angle along one line, and angle 3 is adjacent to it along the other line.

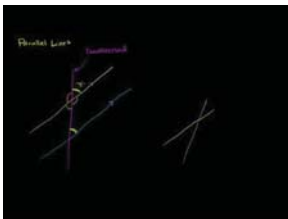
Now let's find the measure of angle 2. It is vertical to the known angle, so we know that these two angles have the same measure. Therefore angle 2 is also  $60^\circ$ . Now we know all of the angles at the first intersection!

Because these lines are parallel, all of the angles at the second intersection correspond to angles at the first intersection. Which angle corresponds to the given  $60^\circ$  angle? Angle 5 does, so it is also  $60^\circ$ . From here, we can either use angle 5 to find the remaining angles (which are either adjacent or vertical to it), or we can use the angles in the first intersection to fill in the corresponding angles. Either way, we can find that angle 4 is  $120^\circ$ , angle 6 is  $60^\circ$ , and angle 7 is  $120^\circ$ . We found all of the angles!

Take a look at your completed drawing. Four angles are  $60^\circ$  and four are  $120^\circ$ . We can change the angle measure to two different numbers, and those numbers will appear exactly the same way.

**Video Review**

Here is a video for review.

**MEDIA**

Click image to the left for more content.

- This Khan Academy video is on angles and parallel lines.

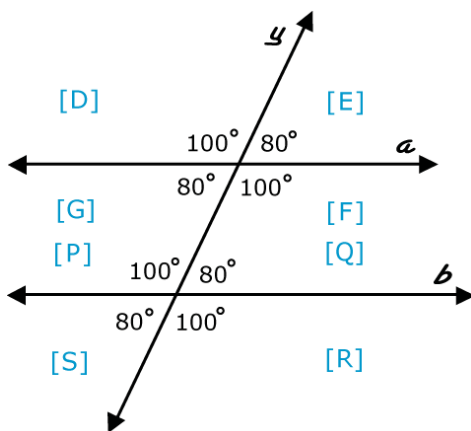
**Practice**

Directions: Define each term.

1. Adjacent Angles
2. Vertical Angles

3. Parallel lines
4. Perpendicular lines
5. Supplementary angles
6. Complementary angles
7. Corresponding angles.

Directions: Use this diagram to answer the following questions.



8. Are angles D and F vertical angles are corresponding angles?
9. One angle that corresponds to angle D is ?
10. An angle that corresponds to angle E is?
11. True or false. Angle E and angle Q are corresponding angles?
12. True or false. Angle E and angle S are corresponding angles?
13. True or false. Angle E and angle Q are adjacent angles?
14. How many pairs of vertical angles are there in this diagram?
15. Can you find corresponding angles if the intersected lines are not parallel?

## 1.20 Complementary Angles

Here you'll learn what complementary angles are and how to solve complementary angle problems.

What if you were given two angles of unknown size and were told they are complementary? How would you determine their angle measures? After completing this Concept, you'll be able to use the definition of complementary angles to solve problems like this one.

### Watch This

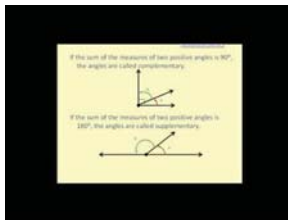


#### MEDIA

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### CK-12 Complementary Angles

Watch this video beginning at around the 3:20 mark.

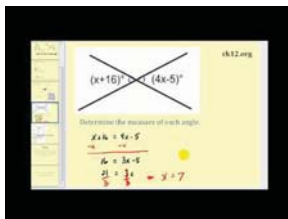


#### MEDIA

Click image to the left for more content.

### James Sousa:Angle Basics–ComplementaryAngles

Then watch the first part of this video.



#### MEDIA

Click image to the left for more content.

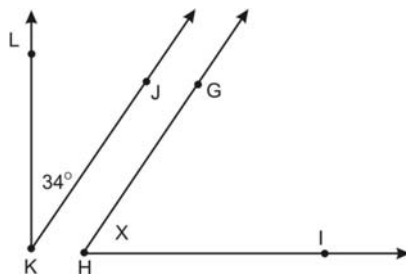
### James Sousa:Complementary Angles

### Guidance

Two angles are **complementary** if they add up to  $90^\circ$ . Complementary angles **do not** have to be *congruent* or *next to each other*.

**Example A**

The two angles below are complementary.  $m\angle GHI = x$ . What is  $x$ ?

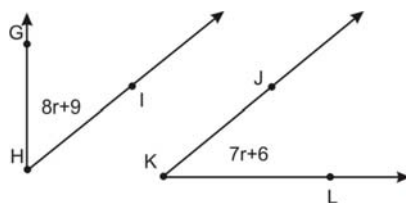


Because the two angles are complementary, they add up to  $90^\circ$ . Make an equation.

$$\begin{aligned}x + 34^\circ &= 90^\circ \\x &= 56^\circ\end{aligned}$$

**Example B**

The two angles below are complementary. Find the measure of each angle.



The two angles add up to  $90^\circ$ . Make an equation.

$$\begin{aligned}(8r + 9) + (7r + 6) &= 90 \\(15r + 15) &= 90 \\15r &= 75 \\r &= 5\end{aligned}$$

However, you need to find each angle. Plug  $r$  back into each expression.

$$\begin{aligned}m\angle GHI &= 8(5^\circ) + 9^\circ = 49^\circ \\m\angle JKL &= 7(5^\circ) + 6^\circ = 41^\circ\end{aligned}$$

**Example C**

Find the measure of an angle that is complementary to  $\angle MRS$  if  $m\angle MRS$  is  $70^\circ$ .

Because complementary angles have to add up to  $90^\circ$ , the other angle must be  $90^\circ - 70^\circ = 20^\circ$ .




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## CK-12 Complementary Angles

### Guided Practice

Find the measure of an angle that is complementary to  $\angle ABC$  if  $m\angle ABC$  is:

1.  $45^\circ$
2.  $82^\circ$
3.  $19^\circ$
4.  $12^\circ$

### Answers:

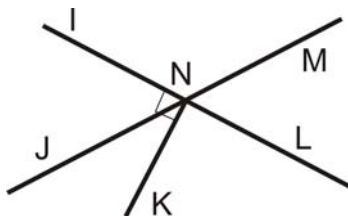
1. Because complementary angles have to add up to  $90^\circ$ , the other angle must be  $90^\circ - 45^\circ = 45^\circ$ .
2. Because complementary angles have to add up to  $90^\circ$ , the other angle must be  $90^\circ - 82^\circ = 8^\circ$ .
3. Because complementary angles have to add up to  $90^\circ$ , the other angle must be  $90^\circ - 19^\circ = 71^\circ$ .
4. Because complementary angles have to add up to  $90^\circ$ , the other angle must be  $90^\circ - 12^\circ = 78^\circ$ .

### Practice

Find the measure of an angle that is complementary to  $\angle ABC$  if  $m\angle ABC$  is:

1.  $4^\circ$
2.  $89^\circ$
3.  $54^\circ$
4.  $32^\circ$
5.  $27^\circ$
6.  $(x+y)^\circ$
7.  $z^\circ$

Use the diagram below for exercises 8-9. Note that  $\overline{NK} \perp \overleftrightarrow{IL}$ .



8. Name two complementary angles.
9. If  $m\angle INJ = 63^\circ$ , find  $m\angle KNJ$ .



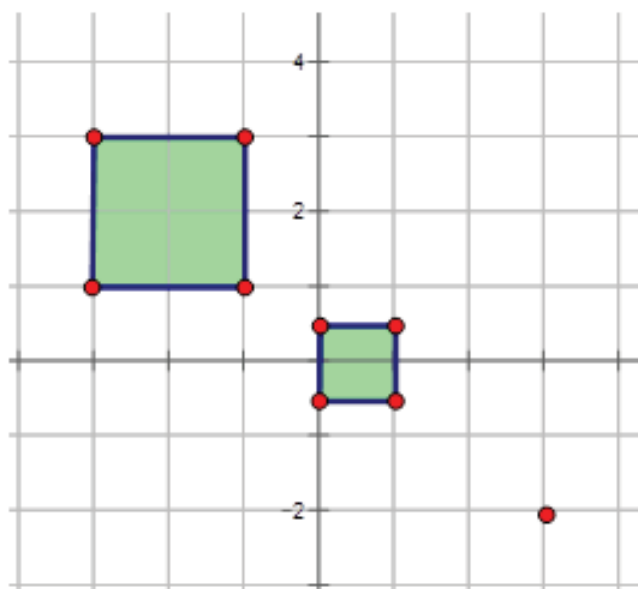
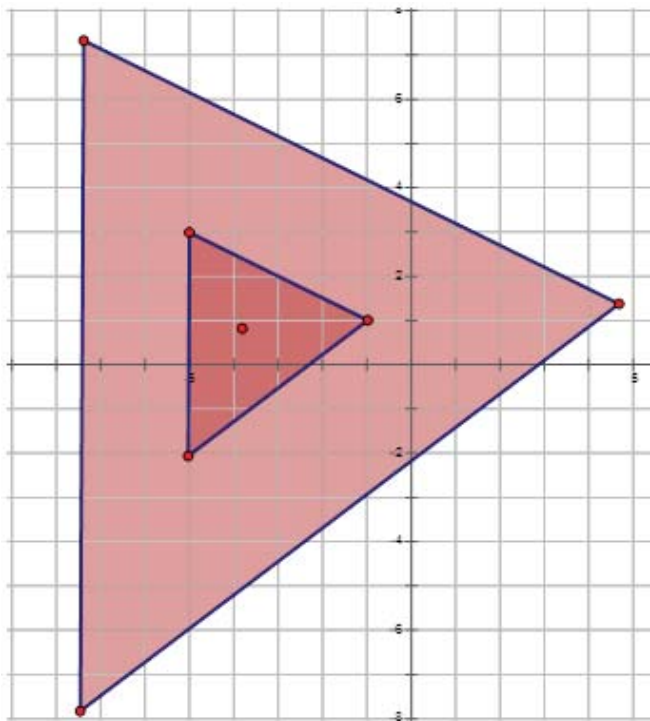
For 10-11, determine if the statement is true or false.

10. Complementary angles add up to  $180^\circ$ .
11. Complementary angles are always  $45^\circ$ .

## 1.21 Dilations

Here you will learn about geometric dilations.

Which one of the following figures represents a dilation? Explain.



**Watch This**

First watch this video to learn about dilations.

**MEDIA**

Click image to the left for more content.

[CK-12 FoundationChapter10DilationsA](#)

Then watch this video to see some examples.

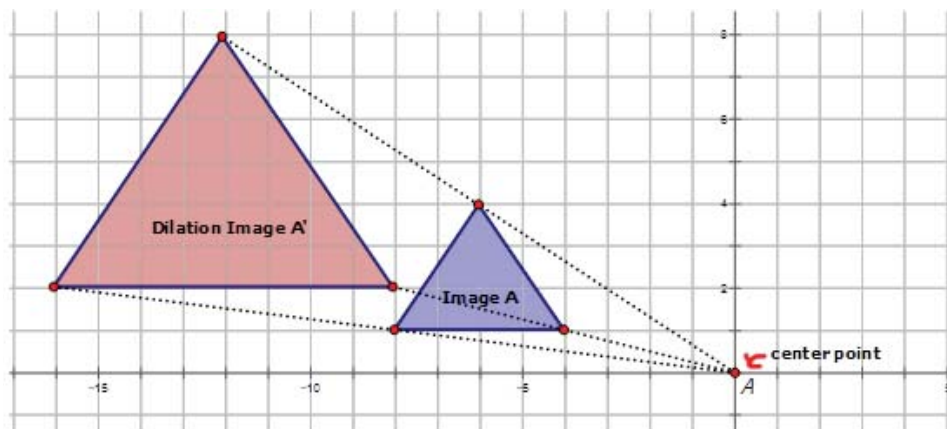
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[CK-12 FoundationChapter10DilationsB](#)

**Guidance**

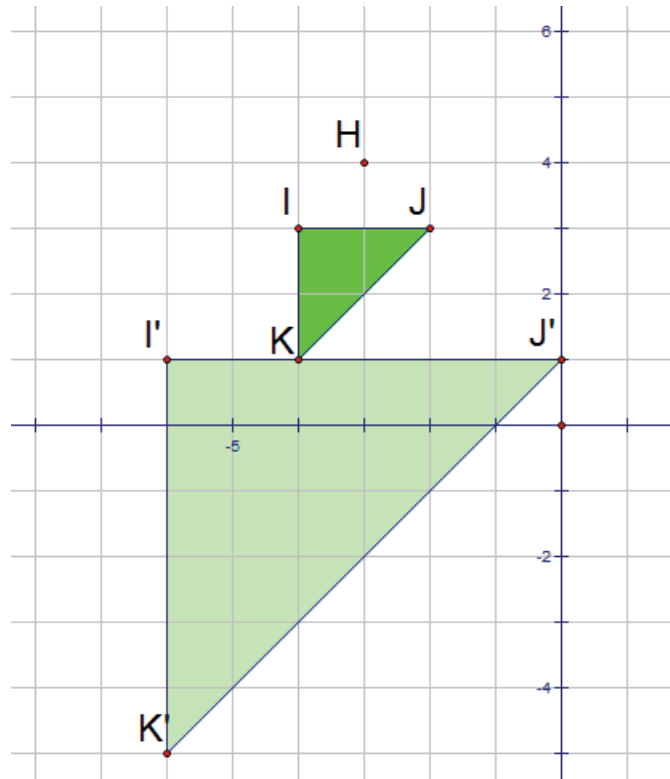
In geometry, a transformation is an operation that moves, flips, or changes a shape to create a new shape. A dilation is a type of transformation that enlarges or reduces a figure (called the preimage) to create a new figure (called the image). The scale factor,  $r$ , determines how much bigger or smaller the dilation image will be compared to the preimage. The figure below shows that the image  $A'$  is a dilation by a scale factor of 2.



Dilations also need a center point. The center point is the center of the dilation. You use the center point to measure the distances to the preimage and the dilation image. It is these distances that determine the scale factor.

**Example A**

Describe the dilation in the diagram below. The center of dilation is point  $H$ .

**Solution:**

Compare the lengths of corresponding sides to determine the scale factor.  $\overline{IJ}$  is 2 units long and  $\overline{I'J'}$  is 6 units long.  $\frac{6}{2} = 3$ , so the scale factor is 3. Therefore, the center point H is used to dilate  $\triangle IJK$  to  $\triangle I'J'K'$  by a factor of 3.

**Example B**

Using the measurement below and the scale factor, determine the measure of the dilated image.

$$m\overline{AB} = 15 \text{ cm}$$

$$r = \frac{1}{3}$$

**Solution:** You need to multiply the scale factor by the measure of  $AB$  in order to find the measurement of the dilated image  $A'B'$ .

$$m\overline{A'B'} = (r)m\overline{AB}$$

$$m\overline{A'B'} = \frac{1}{3}(15)$$

$$m\overline{A'B'} = 5 \text{ cm}$$

**Example C**

Using the measurement below and the scale factor, determine the measure of the preimage.

$$m\overline{H'I'} = 24 \text{ cm}$$

$$r = 3$$

**Solution:** Here, you need to divide the scale factor by the measurement of  $H'I'$  in order to find the measurement of the preimage  $HI$ .

$$m\overline{H'I'} = (r)m\overline{HI}$$

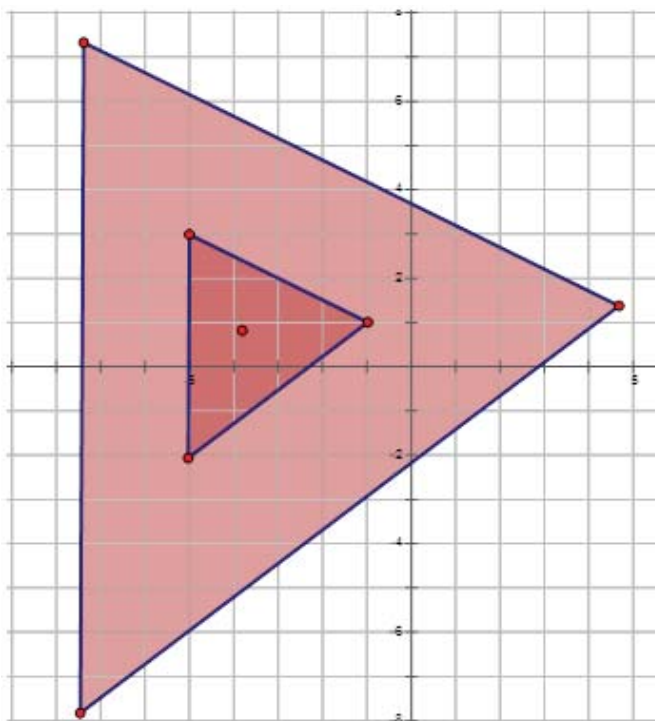
$$24 = 3m\overline{HI}$$

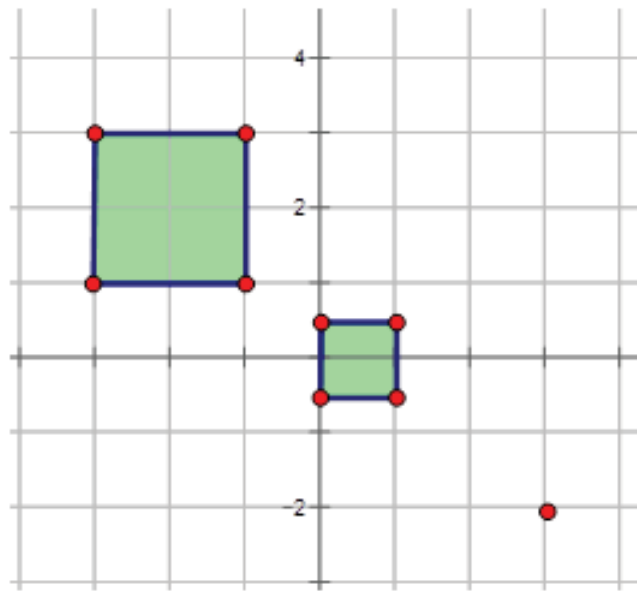
$$m\overline{HI} = \frac{24}{3}$$

$$m\overline{HI} = 8 \text{ cm}$$

### Concept Problem Revisited

Which one of the following figures represents a dilation? Explain.





You know that a dilation is a transformation that produces an image of the same shape but larger or smaller. Both of the figures above represent objects that involve dilations. In the figure with the triangles, the scale factor is 3.

The second figure with the squares also represents a dilation. In this figure, the center point  $(3, -2)$  is used to dilate the small square by a factor of 2.

## Vocabulary

### Center Point

The **center point** is the center of the dilation. You use the center point to measure the distances to the preimage and the dilation image. It is these distances that determine the scale factor.

### Dilation

A **dilation** is a transformation that enlarges or reduces the size of a figure.

### Scale Factor

The **scale factor** determines how much bigger or smaller the dilation image will be compared to the preimage. The scale factor often uses the symbol  $r$ .

### Image

In a transformation, the final figure is called the **image**.

### Preimage

In a transformation, the original figure is called the **preimage**.

### Transformation

A **transformation** is an operation that is performed on a shape that moves or changes it in some way. There are four types of transformations: translations, reflections, dilations and rotations.

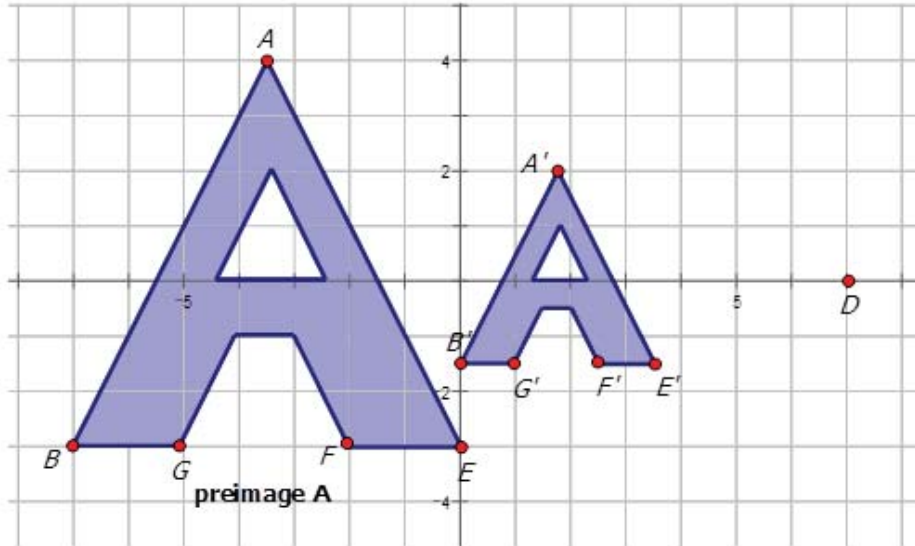
## Guided Practice

- Using the measurement below and the scale factor, determine the measure of the preimage.

$$m\overline{T'U'} = 12 \text{ cm}$$

$$r = 4 \text{ cm}$$

2. Describe the dilation in the diagram below.



3. Quadrilateral  $STUV$  has vertices  $S(-1,3)$ ,  $T(2,0)$ ,  $U(-2,-1)$ , and  $V(-3,1)$ . The quadrilateral undergoes a dilation about the origin with a scale factor of  $\frac{8}{5}$ . Sketch the preimage and the dilation image.

**Answers:**

1. Here, you need to divide the scale factor by the measurement of  $H'I'$  in order to find the measurement of the preimage  $HI$ .

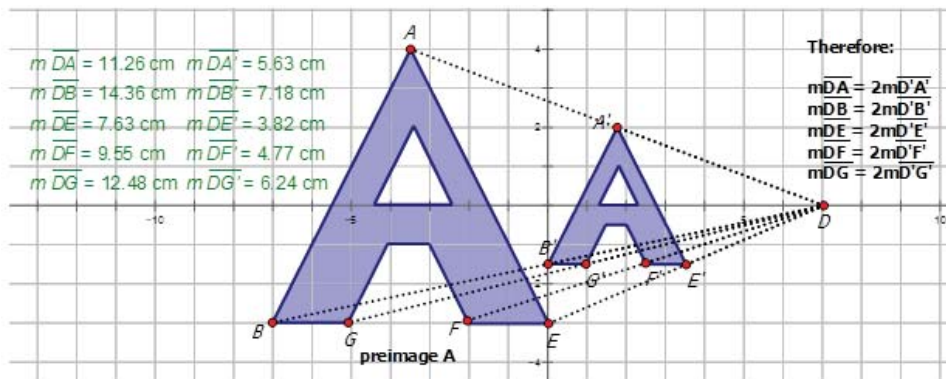
$$m\overline{T'U'} = |r|m\overline{TU}$$

$$12 = 2m\overline{TU}$$

$$m\overline{TU} = \frac{12}{2}$$

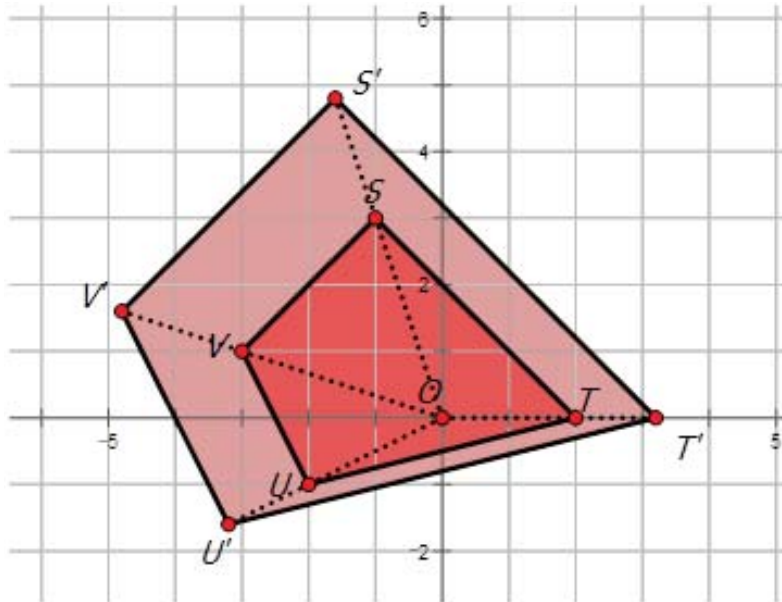
$$m\overline{TU} = 6 \text{ cm}$$

2. Look at the diagram below:



In the figure, the center point  $D$  is used to dilate the  $A$  by a factor of  $\frac{1}{2}$ .

3. Look at the diagram below:



### Practice

Find the measure of the dilation image given the following information:

1.

$$m\overline{AB} = 12 \text{ cm}$$

$$r = 2$$

2.

$$m\overline{CD} = 25 \text{ cm}$$

$$r = \frac{1}{5}$$

3.

$$m\overline{EF} = 18 \text{ cm}$$

$$r = \frac{2}{3}$$

4.

$$m\overline{GH} = 18 \text{ cm}$$

$$r = 3$$

5.

$$m\overline{IJ} = 100 \text{ cm}$$

$$r = \frac{1}{10}$$



Find the measure of the preimage given the following information:

6.

$$\begin{aligned}m\overline{K'L'} &= 48 \text{ cm} \\ r &= 4\end{aligned}$$

7.

$$\begin{aligned}m\overline{M'N'} &= 32 \text{ cm} \\ r &= 4\end{aligned}$$

8.

$$\begin{aligned}m\overline{O'P'} &= 36 \text{ cm} \\ r &= 6\end{aligned}$$

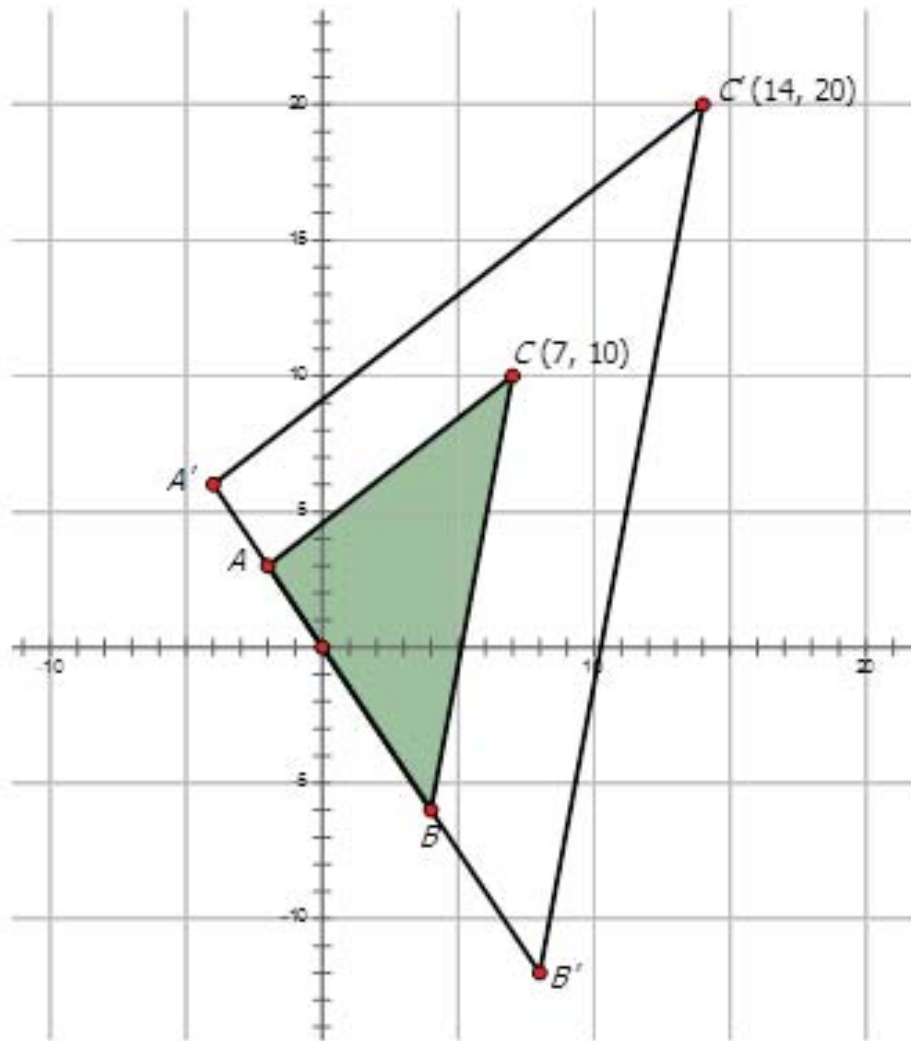
9.

$$\begin{aligned}m\overline{Q'R'} &= 20 \text{ cm} \\ r &= \frac{1}{4}\end{aligned}$$

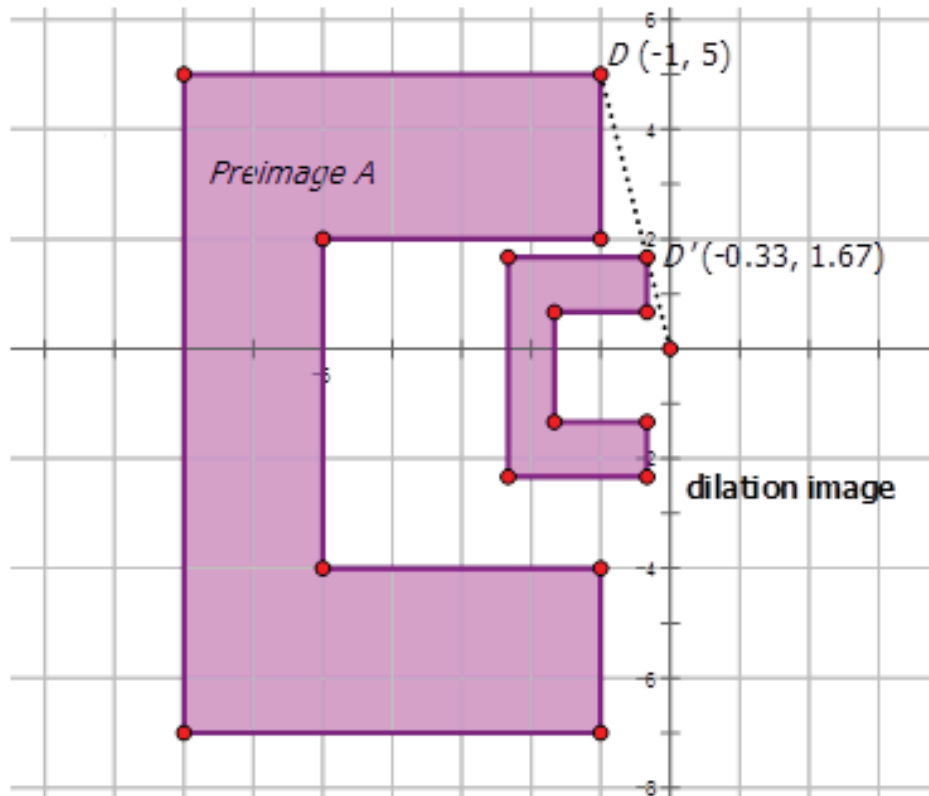
10.

$$\begin{aligned}m\overline{S'T'} &= 40 \text{ cm} \\ r &= \frac{4}{5}\end{aligned}$$

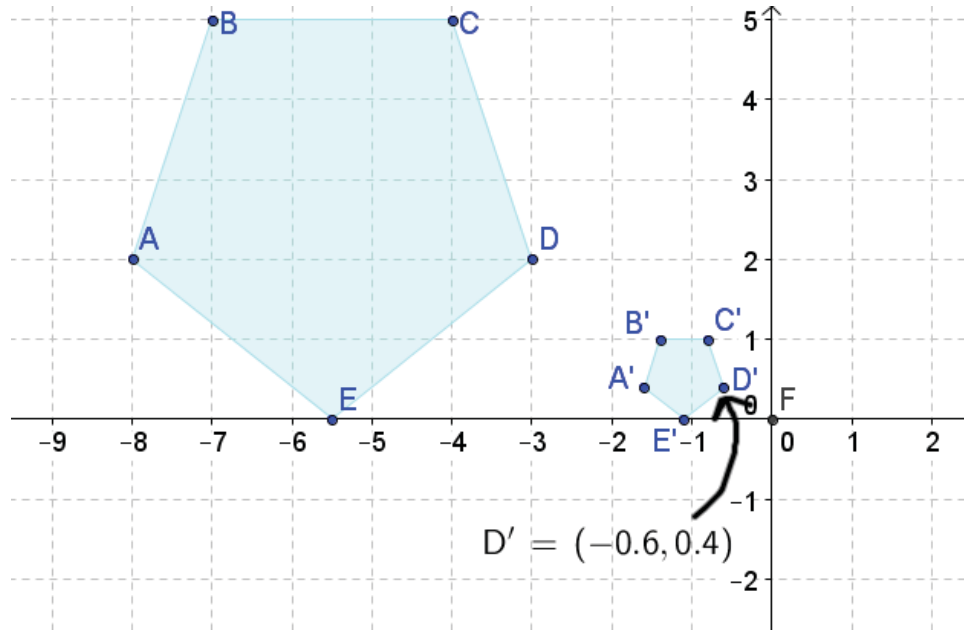
Describe the following dilations:



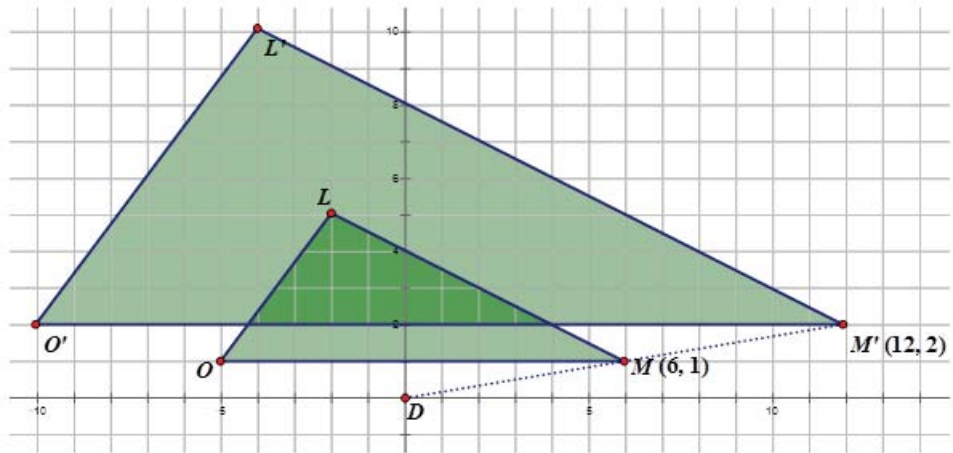
11.



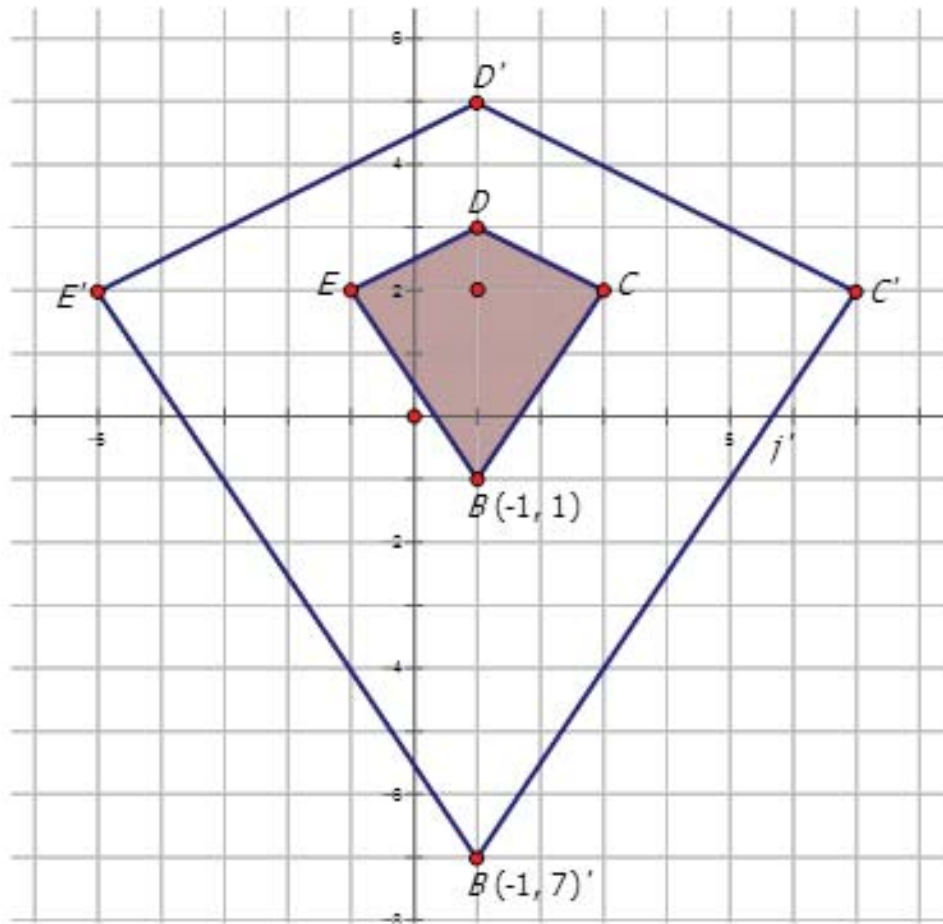
12.



13.



14.



15.

## 1.22 Dilation

Here you'll learn what a dilation is, how to dilate a figure, and how to find the scale factor by which the figure is dilated.

What if you enlarged or reduced a triangle without changing its shape? How could you find the scale factor by which the triangle was stretched or shrunk? After completing this Concept, you'll be able to use the corresponding sides of dilated figures to solve problems like this one.

### Watch This



MEDIA

Click image to the left for more content.

CK-12 Foundation: Chapter7DilationA



MEDIA

Click image to the left for more content.

Brightstorm:Dilations

### Guidance

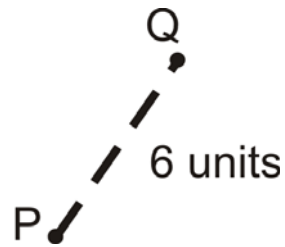
A **transformation** is an operation that moves, flips, or changes a figure to create a new figure. Transformations that preserve size are **rigid** and ones that do not are **non-rigid**. A **dilation** makes a figure larger or smaller, but has the same shape as the original. In other words, the dilation is similar to the original. All dilations have a **center** and a **scale factor**. The center is the point of reference for the dilation (like the vanishing point in a perspective drawing) and scale factor tells us how much the figure stretches or shrinks. A scale factor is typically labeled  $k$  and is always greater than zero. Also, if the original figure is labeled  $\triangle ABC$ , for example, the dilation would be  $\triangle A'B'C'$ . The ' indicates that it is a copy. This tic mark is said "prime," so  $A'$  is read "A prime." A second dilation would be  $A''$ , read "A double-prime."

*If the dilated image is smaller than the original, then the scale factor is  $0 < k < 1$ .*

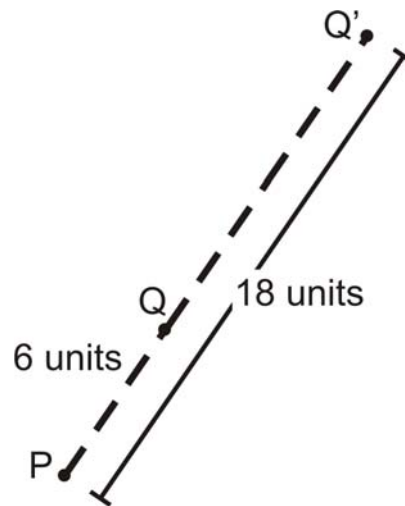
*If the dilated image is larger than the original, then the scale factor is  $k > 1$ .*

### Example A

The center of dilation is  $P$  and the scale factor is 3. Find  $Q'$ .

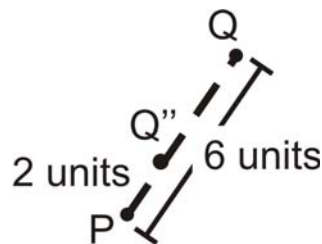


If the scale factor is 3 and  $Q$  is 6 units away from  $P$ , then  $Q'$  is going to be  $6 \times 3 = 18$  units away from  $P$ . Because we are only dilating a point, the dilation will be collinear with the original and center.



### Example B

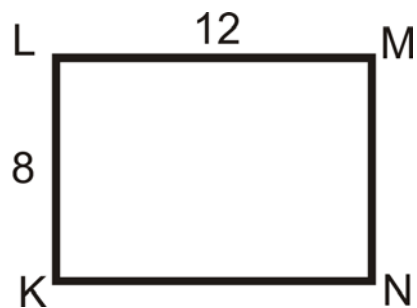
Using the picture above, change the scale factor to  $\frac{1}{3}$ . Find  $Q''$ .



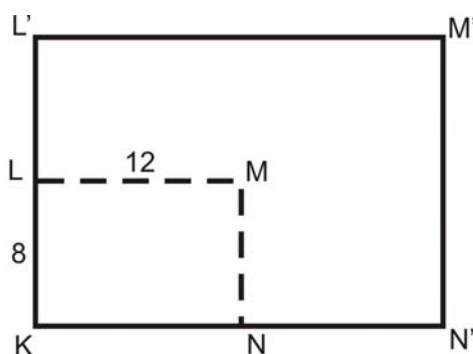
Now the scale factor is  $\frac{1}{3}$ , so  $Q''$  is going to be  $\frac{1}{3}$  the distance away from  $P$  as  $Q$  is. In other words,  $Q''$  is going to be  $6 \times \frac{1}{3} = 2$  units away from  $P$ .  $Q''$  will also be collinear with  $Q$  and center.

### Example C

$KLMN$  is a rectangle with length 12 and width 8. If the center of dilation is  $K$  with a scale factor of 2, draw  $K'L'M'N'$ .



If  $K$  is the center of dilation, then  $K$  and  $K'$  will be the same point. From there,  $L'$  will be 8 units above  $L$  and  $N'$  will be 12 units to the right of  $N$ .



Watch this video for help with the Examples above.



MEDIA

Click image to the left for more content.

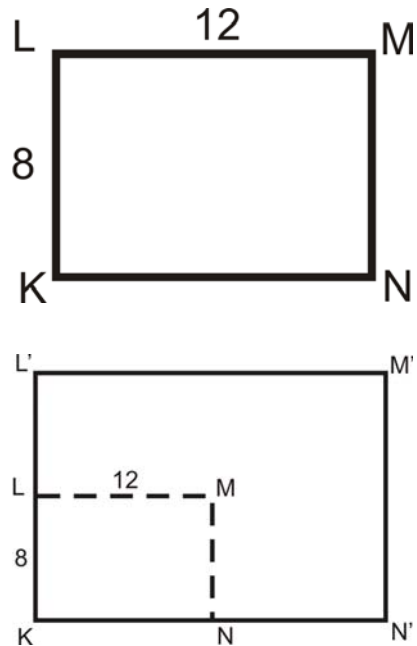
CK-12 Foundation: Chapter7DilationB

### Vocabulary

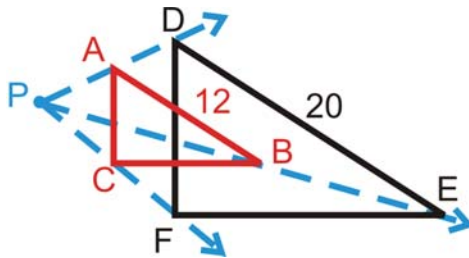
A **dilation** is an enlargement or reduction of a figure that preserves shape but not size. All dilations are similar to the original figure. **Similar** figures are the same shape but not necessarily the same size. The **center of a dilation** is the point of reference for the dilation and the **scale factor** for a dilation tells us how much the figure stretches or shrinks.

### Guided Practice

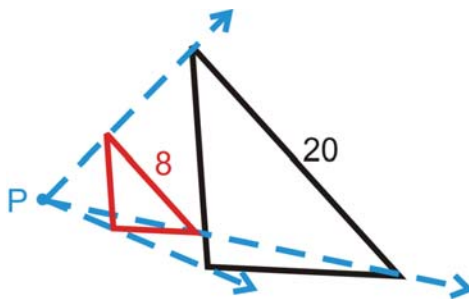
1. Find the perimeters of  $KLMN$  and  $K'L'M'N'$ . Compare this ratio to the scale factor.



2.  $\triangle ABC$  is a dilation of  $\triangle DEF$ . If  $P$  is the center of dilation, what is the scale factor?



3. Find the scale factor, given the corresponding sides. In the diagram, the **black** figure is the original and  $P$  is the center of dilation.



### Answers:

1. The perimeter of  $KLMN = 12 + 8 + 12 + 8 = 40$ . The perimeter of  $K'L'M'N' = 24 + 16 + 24 + 16 = 80$ . The ratio is  $80:40$ , which reduces to  $2:1$ , which is the same as the scale factor.

2. Because  $\triangle ABC$  is a dilation of  $\triangle DEF$ , then  $\triangle ABC \sim \triangle DEF$ . The scale factor is the ratio of the sides. Since  $\triangle ABC$  is smaller than the original,  $\triangle DEF$ , the scale factor is going to be less than one,  $\frac{12}{20} = \frac{3}{5}$ .

If  $\triangle DEF$  was the dilated image, the scale factor would have been  $\frac{5}{3}$ .

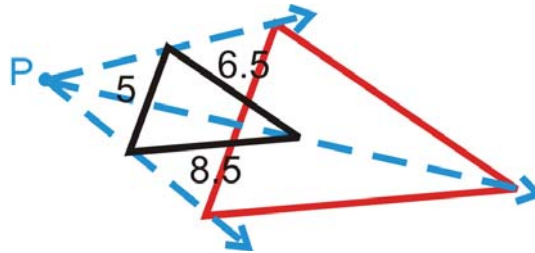
3. Since the dilation is smaller than the original, the scale factor is going to be less than one.  $\frac{8}{20} = \frac{2}{5}$



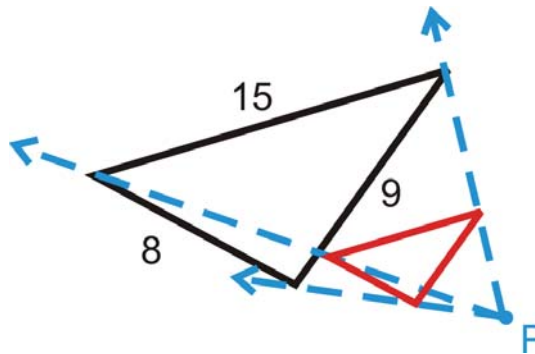
**Practice**

In the two questions below, you are told the scale factor. Determine the dimensions of the dilation. In each diagram, the **black** figure is the original and *P* is the center of dilation.

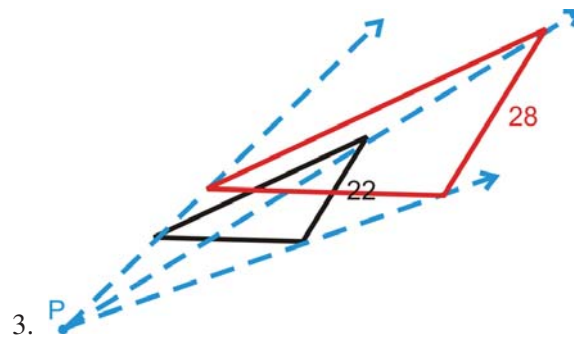
- $k = 4$



- $k = \frac{1}{3}$

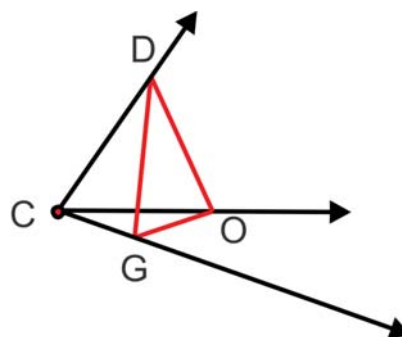


In the question below, find the scale factor, given the corresponding sides. In the diagram, the **black** figure is the original and *P* is the center of dilation.



- Find the perimeter of both triangles in #1. What is the ratio of the perimeters?
- Writing** What happens if  $k = 1$ ?

**Construction** We can use a compass and straight edge to construct a dilation as well. Copy the diagram below.



6. Set your compass to be  $CG$  and use this setting to mark off a point 3 times as far from  $C$  as  $G$  is. Label this point  $G'$ . Repeat this process for  $CO$  and  $CD$  to find  $O'$  and  $D'$ .
  7. Connect  $G', O'$  and  $D'$  to make  $\triangle D'O'G'$ . Find the ratios,  $\frac{D'O'}{DO}$ ,  $\frac{O'G'}{OG}$  and  $\frac{G'D'}{GD}$ .
  8. What is the scale factor of this dilation?
  9. Describe how you would dilate the figure by a scale factor of 4.
  10. Describe how you would dilate the figure by a scale factor of  $\frac{1}{2}$ .
- 
11. The scale factor between two shapes is 1.5. What is the ratio of their perimeters?
  12. The scale factor between two shapes is 1.5. What is the ratio of their areas? *Hint: Draw an example and calculate what happens.*
  13. Suppose you dilate a triangle with side lengths 3, 7, and 9 by a scale factor of 3. What are the side lengths of the image?
  14. Suppose you dilate a rectangle with a width of 10 and a length of 12 by a scale factor of  $\frac{1}{2}$ . What are the dimensions of the image?
  15. Find the areas of the rectangles in #14. What is the ratio of their areas?

## 1.23 Recognizing Similarity

Here you'll recognize similarity and use similar figures with indirect measurement.

Have you ever thought about shadows? Take a look at this dilemma.

A person is five feet tall and casts a shadow of 2 feet. A tower casts a shadow that is 10 feet long. What is the height of the tower?

Do you know how to figure this out?

**Pay attention and you will learn how to accomplish this task in this Concept.**

### Guidance

**Congruent means exactly the same, having the same size and shape.**

Sometimes, a figure will have the same shape, but not the same size. It will be either smaller or larger than the original figure. When this happens, we say that the two figures are 'similar.

**Similar figures have the same shape, but not the same size.**

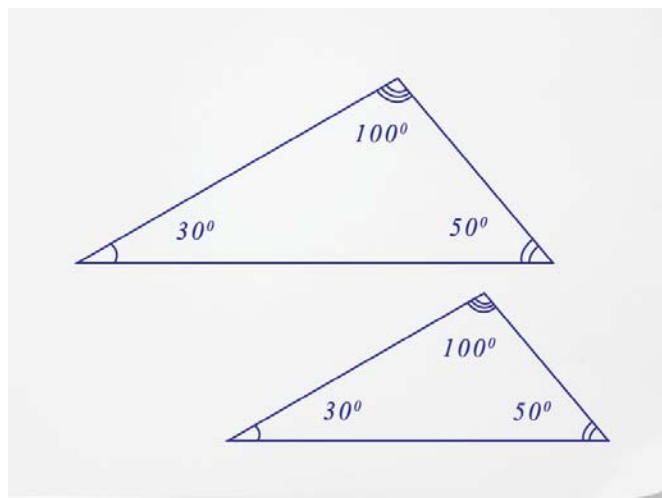
Think about this for a minute, if a figure has the same shape, but not the same size, then there is still a relationship between the two figures. The relationship is created based on the shape being the same.



**That is a good question.**

**Let's start by thinking about angles. With similar figures, each angle of one figure in a similar pair corresponds and is congruent to an angle in the other.**

For instance, the top point of one triangle corresponds to the top point of the other triangle in a similar pair. We call these corresponding parts.



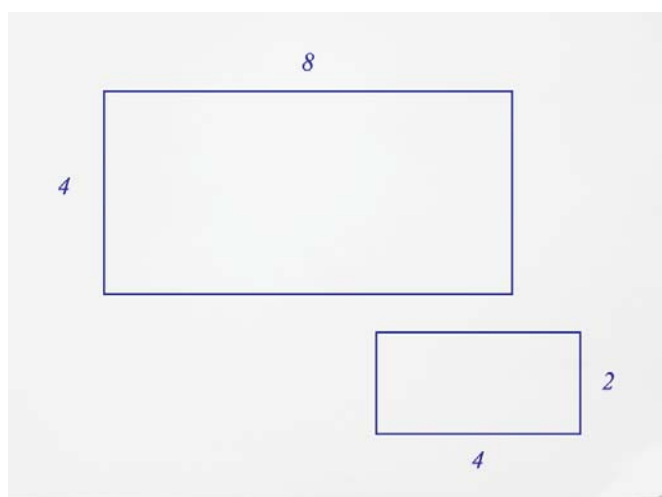
Notice that the angles match in these two triangles. The shape of the triangles is the same, and you can see that the angles do match.

*With similar triangles, the angles are congruent.*

*The side lengths form a proportion or are proportional to each other*

**What about the side lengths?**

**The sides in similar pairs also correspond to each other (such as the base of each triangle), but they are not congruent; they are proportional.** We can determine whether figures are similar to each other by comparing their corresponding parts. Corresponding parts are especially helpful when one figure is rotated so that it is not clear which angles and sides correspond to which in the other figure.



**Now let's look at the corresponding side lengths.** In the first rectangle, the short side is 4 and the long side is 8. We know that opposite sides of a rectangle are congruent, so we don't need to worry about writing measurements on the other two sides. We can compare the measurements in the first rectangle with the ones in the second rectangle. In the second rectangle, the short side is 2 and the long side is 4.

Let's write a proportion to compare the corresponding sides.

$$\frac{\text{short side}}{\text{short side}} = \frac{\text{long side}}{\text{long side}}$$
$$\frac{4}{2} = \frac{8}{4}$$

You can see that these two ratios form a proportion. You can use this information to prove whether or not two figures are similar as well. Remember, the angle measures must be the same, and the side lengths must be proportional.

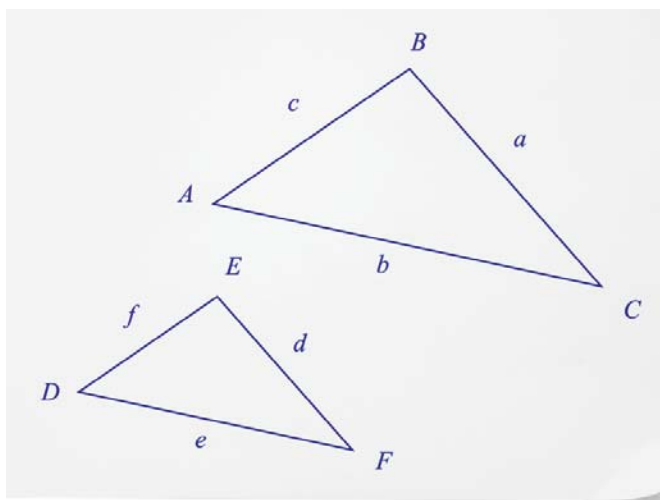


*Write these notes on similar figures down in your notebooks.*

Now that you understand how to identify whether or not two figures are similar, we can look similar triangles. Similar triangles are very useful because we can use them to figure out measurements. Many years ago, this is how people used to figure out the measurements for things that were too high or big to measure. They used *indirect measurement*. Indirect measurement uses similar triangles and proportions to figure out lengths or distances.

But first, let's think about similar triangles.

Similar triangles have the same properties as other similar figures. The angle measures are the same and the corresponding side lengths are proportional. Let's look at this diagram to understand this.



Now we can compare the angles and corresponding side lengths. Let's begin with the angles.

$$\angle A \cong \angle D$$

$$\angle B \cong \angle E$$

$$\angle C \cong \angle F$$

Next, we can look at the corresponding side lengths. In the diagram, we haven't been given any measurements, but we can use the lowercase letters to show which sides correspond.

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

This shows that the side lengths form a ratio and that each of these is proportional to the other.

**We can use this information when problem solving for missing side lengths.**



**That is a good question. First, we would have to know some of the side lengths. Let's assign some lengths to the sides in the diagram above.**

$$a = 12$$

$$b = ?$$

$$c = 3$$

$$d = 4$$

$$e = 3$$

$$f = 1$$

**Now we can take these given measures and substitute them into the proportion that we wrote earlier. Notice that we don't have the measure of side  $b$ , so we will need to solve for that missing measurement.**

$$\frac{12}{4} = \frac{b}{3} = \frac{3}{1}$$

Next, we can use two of the three ratios to solve the proportions. We have three ratios, but we don't need all three because two equal ratios form a proportion. This means that we only need to work with two ratios to solve for the value of  $b$ .

$$\frac{12}{4} = \frac{b}{3}$$

Now we can cross - multiply and solve the proportion.

$$4b = 36$$

$$b = 9$$

The value of  $b$  is 9.

The key to working with indirect measurement is to always be clear about what is being compared. You write your ratios and then form a proportion and solve for the missing length or distance.



*Write some notes about indirect measurement down in your notebook.*

Solve each for the missing value.

### Example A

$$\frac{2}{3} = \frac{4}{6} = \frac{x}{18}$$

**Solution:**  $x = 12$

### Example B

$$\frac{4}{5} = \frac{12}{15} = \frac{x}{30}$$

**Solution:**  $x = 24$

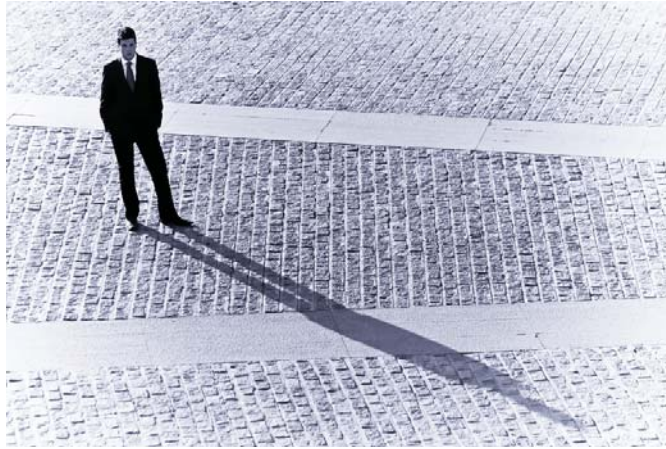
### Example C

$$\frac{8}{9} = \frac{16}{x} = \frac{32}{36}$$

**Solution:**  $x = 18$

Now let's go back to the dilemma from the beginning of the Concept.

This may seem like a very challenging problem to solve, however if you think about people and shadows as they are related to triangles it becomes much easier. Look at this picture.



Notice that the person and the shadow form two sides of a triangle and we can draw an imaginary line from the head of the person to the tip of the shadow. Shadows are a way of working with triangles and indirect measurement. In fact, you will often hear these types of problems referred to as shadow problems.

**To solve this one, let's figure out how to use similar triangles to figure out the height of the tower.** First, think about what is being compared. We are comparing the height of the person with the length of the shadow. That is the first ratio.

$$\frac{\text{person}}{\text{shadow}} = \frac{5 \text{ ft}}{2 \text{ ft}}$$

Next, we look at the tower. We don't know the height of the tower that is our variable. We do know the length of the shadow. Here is our second ratio.

$$\frac{\text{tower}}{\text{shadow}} = \frac{x}{10 \text{ ft}}$$

We can say that these two triangles are similar and that similar triangles are proportional. Therefore, these two ratios form a proportion. Let's write them as a proportion.

$$\frac{5 \text{ ft}}{2 \text{ ft}} = \frac{x}{10 \text{ ft}}$$

Now we can cross - multiply and solve the proportion.

$$2x = 50 \text{ ft}$$

$$x = 25 \text{ ft}$$

**The tower is 25 feet tall.**

## Vocabulary

### Congruent

having the same size, shape and measure.

### Similar

having the same shape, but not the same size. Angle measures are the same and side lengths are proportional.

### Proportional

the side lengths create ratios that form a proportion.

### Indirect Measurement

using similar triangles to figure out challenging distances or lengths.



### Guided Practice

Here is one for you to try on your own.

$$\frac{1}{4} = \frac{2}{8} = \frac{24}{x}$$

#### Solution

First, we can look at the relationship between the numerators.

One was multiplied by two to get two. Then two was multiplied by 12 to get 24.

Now let's look at the denominators.

Four was multiplied by two to get 8. Then we need to multiply 8 by 12 to find the missing denominator.

**Our answer is**  $x = 96$ .

### Practice

Directions: Identify whether or not each pair of triangles is similar based on the ratios of their sides.

- Triangle  $A$  has side lengths of 2, 4, and 6. Triangle  $B$  has side lengths of 6, 12 and 24. Are these triangles similar?
- Triangle  $C$  has side lengths of 4, 5, and 10. Triangle  $B$  has side lengths of 2, 2.5 and 5. Are these two triangles similar?
- Triangle  $D$  has side lengths of 5, 8, and 12. Triangle  $B$  has side lengths of 10, 16 and 24. Are these two triangles similar?
- Triangle  $A$  has side lengths of 10, 12, and 14. Triangle  $B$  has side lengths of 5, 7 and 9. Are these two triangles similar?
- Triangle  $B$  has side lengths of 8, 14, and 20. Triangle  $C$  has side lengths of 4, 7 and 10. Are these two triangles similar?
- Triangle  $E$  has side lengths of 20, 11 and 8. Triangle  $F$  has side lengths of 10, 5.5 and 5. Are these two triangles similar?
- Triangle  $G$  has side lengths of 6, 8 and 12. Triangle  $H$  has side lengths of 18, 24 and 36. Are these two triangles similar?
- Triangle  $I$  has side lengths of 8, 12, and 16. Triangle  $J$  has side lengths of 4, 8 and 10. Are these two triangles similar?

Directions: Find the missing length by looking at each series of ratios. The top value represents the side lengths of the first similar triangle. The bottom value represents the side lengths of the second similar triangle.

- $\frac{1}{2} = \frac{3}{6} = \frac{9}{x}$
- $\frac{3}{6} = \frac{6}{12} = \frac{10}{x}$
- $\frac{4}{2} = \frac{10}{x} = \frac{12}{6}$
- $\frac{6}{2} = \frac{9}{x} = \frac{12}{4}$
- $\frac{5}{10} = \frac{10}{20} = \frac{15}{x}$
- $\frac{12}{6} = \frac{20}{10} = \frac{15}{x}$
- $\frac{16}{x} = \frac{20}{5} = \frac{24}{6}$