Characteristic and Continuous X-rays

Jitender Singh | *www.concepts-of-physics.com*

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1 Introduction

THE X-RAYS ARE PRODUCED in a Coolidge tube when a high energy electron interacts with a heavy metal target.

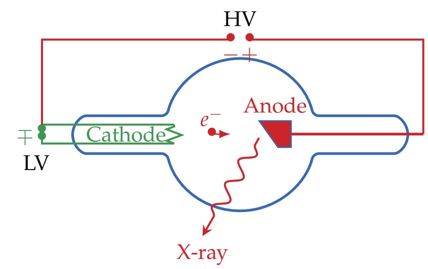

Figure 1: In Coolidge Tube, the intensity of x-rays is controlled by the filament current/voltage (LV) and the frequency of x-rays is controlled by the accelerating voltage (HV).

THE HIGH ENERGY ELECTRONS GET DECELERATED by Coulomb's interaction with the target atom. These decelerating electrons emit continuous X-rays (or bremsstrahlung). When an electron loses its entire energy in a single Coulomb's interaction then X-rays of minimum wavelength (cut-off wavelength) are emitted.

The cut-off wavelength of the continuous X-ray spectrum is related to the accelerating potential V of the Coolidge tube by

$$\lambda_{\text{cut off}} = \frac{hc}{eV} \approx \frac{12400}{V} \quad (\text{Å}/\text{V}).$$

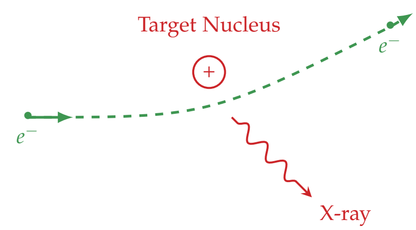

Figure 2: An electron decelerated by Coulomb's interaction of target atom emits X-rays in the continuous spectrum.

Typically, accelerating voltage is a few thousand Volts and cut off wavelength is of the order of an Angstrom. The cut-off frequency of continuous x-rays is given by $\nu_{\text{cut off}} = c/\lambda_{\text{cut off}}$. The cutoff wavelength decreases when the accelerating potential of Coolidge tube is increased. The intensity of X-rays increases with an increase in accelerating potential as shown in the figure. The intensity of X-rays depends on multiple factors.

The high energy electrons also knock out an inner shell electron of the target atom. The characteristic X-rays are emitted when an electron (in atom) makes a transition from a high energy state to a low energy state to fill the vacancy. For example, in K_α X-ray series, the electrons makes transition from the high energy state to the K shell of the atom. The frequency of characteristic X-rays is related to the atomic number of the target element by Moseley?s Law $\sqrt{\nu} = a(Z - b)$. The frequency of characteristic X-rays depends on the target material (it is independent of the accelerating potential).

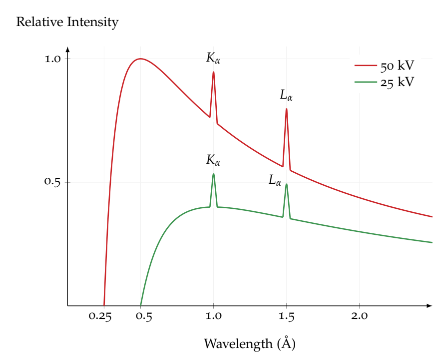

Figure 3: The spectrum shows the variation of X-rays intensity with wavelength. It consists of bremsstrahlung (continuous spectrum) and peaks (characteristic spectrum). Note the variation of cut-off wavelength with accelerating potential.

2 Solved Problems on X-rays

Problem from IIT JEE 2008

Which of the following statements is wrong in the context of X-rays generated from a X-ray tube?

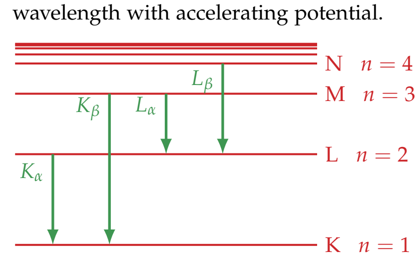

Figure 4: The characteristic X-rays are emitted when an electron in a high energy state makes a transition to a low energy state.

(*A*) Wavelength of characteristic X-rays decreases when the atomic number of the target increases.

(*B*) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.

(*C*) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube.

(*D*) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

Solution: The frequency ν of characteristic X-rays is related to atomic number Z by Moseley's law,

$$\sqrt{\nu} = a(Z - b). \tag{1}$$

which gives,

$$\lambda = \frac{c}{\nu} = \frac{c}{a^2(Z-b)^2}. \tag{2}$$

Thus, wavelength decreases with an increase in Z. The cut-off wavelength of continuous X-rays corresponds to the maximum energy of an electron in X-ray tube. It is given by,

$$hc/\lambda = eV, \tag{3}$$

where V is accelerating potential. The intensity of X-rays depends on the number of electrons striking the target per second, which in turn depends on electrical power given to the X-ray tube as energy of each electron is eV.

Problem from IIT JEE 2000

Electrons with energy 80 keV are incident on the tungsten target of an X-ray tube. K-shell electrons of tungsten have 72.5 keV energy. X-rays emitted by the tube contains only,

(*A*) a continuous X-ray spectrum (Bremsstrahlung) with a minimum wavelength of ≈ 0.155 Angstrom.

(*B*) a continuous X-ray spectrum with all wavelengths.

(*C*) the characteristic X-ray spectrum of tungsten.

(*D*) a continuous X-ray spectrum with a minimum wavelength of ≈ 0.155 Angstrom and the characteristic X-ray spectrum of tungsten.

Solution: The energy of incident electron ($E_{\text{in}} = 80$ keV) is sufficient to knock out K-shell electrons (72.5 keV) thereby emitting characteristic X-rays. The minimum wavelength of continuous spectrum corresponds to E_{in} and is given by

$$\lambda = \frac{hc}{E_{\text{in}}} = \frac{(6.63 \times 10^{-34})\,(3 \times 10^{8})}{(80 \times 10^{3})\,(1.602 \times 10^{-19})} = 0.155 \text{ Angstrom.}$$

Problem from IIT JEE 1988

The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,

(A) the intensity increases.

(B) the minimum wavelength increases.

(C) the intensity remains unchanged.

(D) the minimum wavelength decreases.

Solution: The minimum wavelength of X-ray is related to the applied potential by

$$\lambda_{\text{min}} = \frac{hc}{eV} = \frac{12420}{V}\ \text{Angstrom.}$$

Thus, the minimum wavelength decreases when V is increased. The intensity is the rate of energy flow per unit area. The energy of photons increases when V is increased. Thus, intensity also increases with an increase in the applied potential.

However, intensity of X-rays is generally controlled by the filament current. An increase in the filament current increases its temperature which in turn increases the rate of electron emissions (thermionic emissions). Thus, more number of electrons strikes the target leading to increase in number of X-ray photons.

Problem from IIT JEE 2008

Which of the following statements is wrong in the context of X-rays generated from a X-ray tube?

(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases.

(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target.

(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube.

(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube.

Solution: The frequency ν of characteristic X-rays is related to atomic number Z by Moseley's law,

$$\sqrt{\nu} = a(Z - b),$$

which gives

$$\lambda = \frac{c}{\nu} = \frac{c}{a^2(Z-b)^2}.$$

Thus, the wavelength of emitted X-rays decreases with increase in Z. The cut-off wavelength of continuous X-rays corresponds to maximum energy of electron in X-ray tube. It is given by

$$hc/\lambda = eV,$$

where V is the accelerating potential. The intensity of X-rays depends on the number of electrons striking the target per second, which, in turn, depends on the electrical power given to the X-ray tube as energy of each electron is eV.

3 Questions on X-rays

Question 1

Question 1 (IIT JEE 1998): X-rays are produced in an X-ray tube operating at a given accelerating voltage. The wavelength of the continuous X-rays has values from

(A) 0 to ∞.

(B) λ_{min} to ∞, where $\lambda_{\text{min}} > 0$.

(C) 0 to λ_{max}, where $\lambda_{\text{max}} < \infty$.

(D) λ_{min} to λ_{max} where $0 < \lambda_{\text{min}} < \lambda_{\text{max}} < \infty$.

Answer: (B) The wavelength in continuous X-ray spectrum has a lower limit given by hc/eV. There is no upper limit on wavelength.

Question 2

The shortest wavelength of X-ray emitted from an X-ray tube depends on

(A) the current in the tube.

(B) the voltage applied to the tube.

(C) the nature of the gas in tube.

(D) the atomic number of the target material.

Answer: (B) The shortest wavelength of X-ray emitted by X-ray tube depends on the applied voltage V.

Question 3

When the number of electrons striking the anode of an X-ray tube is increased the .... of the emitted X-rays increases while when the speeds of the electrons striking the anode are increased the cut-off wavelength of the emitted X-rays ....

(*A*) cut off wavelength, decreases

(*B*) cut-off wavelength, decreases

(*C*) intensity, increases

(*D*) intensity, decreases

Answer: (D) The intensity of X-ray increases when the number of electrons striking the anode increases. The cut-off voltage decreases when the energy (speed) of the electrons striking the anode increases.

Question 4

Consider a photon of a continuous X-ray coming from a Coolidge tube. Its energy comes from

(*A*) the kinetic energy of the striking electron

(*B*) the kinetic energy of the free electrons of the target

(*C*) the kinetic energy of the ions of the target

(*D*) an atomic transition in the target

Answer: (A) The kinetic energy of the striking electron. For characteristic X-rays, the answer is D.

References

[1] IIT JEE Physics by Jitender Singh and Shraddhesh Chaturvedi

[2] Concepts of Physics Part 2 by HC Verma (Link to Amazon)

[3] Characteristic and Continuous X-rays, Hyperphysics

[4] Article on Coolidge Tube, Oak Ridge

[5] Properties of X-rays (pdf), Stanford

[6] Features of X-rays spectrum, Miniphysics