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# Characterization of segregation in bidispersed granular media by linear and nonlinear acoustic methods

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**Abstract**

The segregation of size is studied by linear and nonlinear acoustic methods for an unconsolidated granular medium. By applying vertical vibrations we study the variation of the average compacity and the acoustic transfer function in order to follow the segregation process with an acoustic probing.


*Keywords:* acoustic probing; granular segregation; nonlinear elastic parameters

## 1. Introduction

The segregation of size is a process occurring in natural unconsolidated media like sand, cereals and in industrial media such as metallic powder. It is sometime referred to the so-called "brazil-nut effect" due to the observation of the segregation effect in manipulated mixed nuts containing large Brazil nuts: they always end-up at the top of the pile [1]. This effect, such as the compaction process, can be studied in a laboratory scale experiment, where a small container filled with two sizes of spherical beads made of the same material is submitted to repeated mechanical taps. Here we propose some characterization of the segregation process by acoustic means, having in mind that the acoustic properties of the granular medium should depend on the bead size. However, it is not straightforward to understand how the acoustic propagation is influence by the bead size, neither in the long wavelength limit nor in the multiple scattering regimes. One of our goals is consequently to extract the acoustic wave attenuation and velocity evolutions with bead size in three-dimensional granular media, especially when the wavelength is much larger than the bead size.

By applying vertical vibrations of the initially prepared medium where the largest beads (a half volume of the total medium) are at the bottom and smallest at the top, we study the variation of the average compacity (obtained from a measurement of the volume) and the evolution of the acoustic transfer function close to the bottom of the container. We observe that this average compacity evolution with taps is not monotonous like in mono dispersed

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media but grows to a maximum before decreasing at the end of the segregation process. Due to the differences in the acoustic properties for packing with different bead sizes, it is possible to follow the segregation process with an acoustic probing. To do so, we record the acoustic transfer function between two piezo-transducers at 60 stages along the process. Using the observed resonances, we derive the linear elastic parameters of the medium. We also apply the nonlinear resonance method in order to obtain some nonlinear elastic parameters and their evolutions along the segregation process. We show that the acoustic transfer functions are strongly sensitive to the segregation stage.

The acoustics of three-dimensional bidispersed granular packing is poorly documented in the literature. We start here with the simple case of the bidispersed chain of beads, and derive the long wavelength acoustic wave velocity. For two beads with the same mechanical characteristics [2] [3],

- beads 1, radius $R_1$, masse $m_1$,
- beads 2, radius $R_2$, masse $m_2$.

The distance of approach is written as

$$\delta_0 = \left(E^* F_0\right)^{2/3} \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{1/3}, \tag{1}$$

and the stiffness constant is

$$K = \frac{3}{2} F_0^{1/3} \left( E^{*2} \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right)^{-1/3}, \tag{2}$$

where $F_0$ is the force applied on the beads and $E^* = 3\left(1-\nu^2\right)/4E$ is the effective modulus the beads, with $E$ the Young's modulus and $\nu$ the Poisson's ratio.

We consider now a one-dimensional chain composed of two types of beads as shown in Fig.(1) where contacts between two beads are represented as springs.

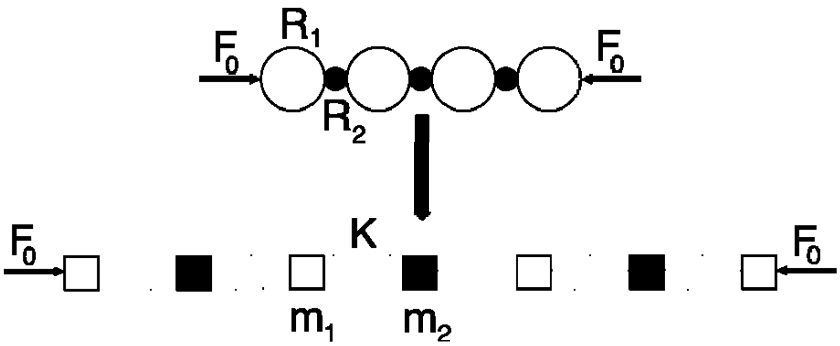


Fig.1 One-dimensional bidispersed chain

For each type of beads corresponds one equation of motion [4]:

$$m_1 \frac{\partial u_n}{\partial t^2} = K\left(u_{n+1} + u_{n-1} - 2u_n\right), \tag{3}$$

$$m_2 \frac{\partial u_{n+1}}{\partial t^2} = K\left(u_n + u_{n+2} - 2u_{n+1}\right), \tag{4}$$

Solutions of the type $u_n = u_{n-2} e^{jka}$ with $a = 2\left(R_1 + R_2\right)$ are used to find the dispersion relation,

$$\omega^2 = \omega_0^2 + \Omega_0^2 \pm \sqrt{\omega_0^4 + \Omega_0^4 + 2\omega_0^2 \Omega_0^2 \cos ka}\,, \tag{5}$$

with $\Omega_0 = \sqrt{K/m_1}$ and $\omega_0 = \sqrt{K/m_2}$. The long wavelength sound velocity for the acoustical mode is then obtained in the form:

$$C_0 \propto R_2^{-1/3} R_r^{1/6} \left(1 + R_r\right)^{5/6} \left(1 + R_r^3\right)^{-1/2}, \tag{6}$$

where $R_r = R_1 / R_2$ with $R_r < 1$. This formula shows that even for a simple one-dimensional bidispersed chain, the long-wavelength wave velocity has already a complicated dependence on the bead radius ratio. For three-dimensional bidispersed arrangements, it is expected that in addition to this possible effect of contact elasticity dependence on the bead radius, other effects could play a role as for instance the average number of contact per bead.

## 2. Description of the sample

The experimental setup shown on Fig.(2) is composed of a container with a movable plate at the bottom excited by vertical mechanical solicitations from a low-frequency shaker. These imposed "taps" produce an acceleration ($\Gamma$) of the movable plate of about $20\,G$ sufficient to induce segregation of the medium (in our case by a convection-type motion of the grains [5]). Along the segregation process, the shaker applies $10^6$ taps (one period of a sine wave at $60\,Hz$) separated by one second to allow for the mechanical relaxation of the medium. At $60$ stages of the segregation process, an acoustic probing of the medium is performed.

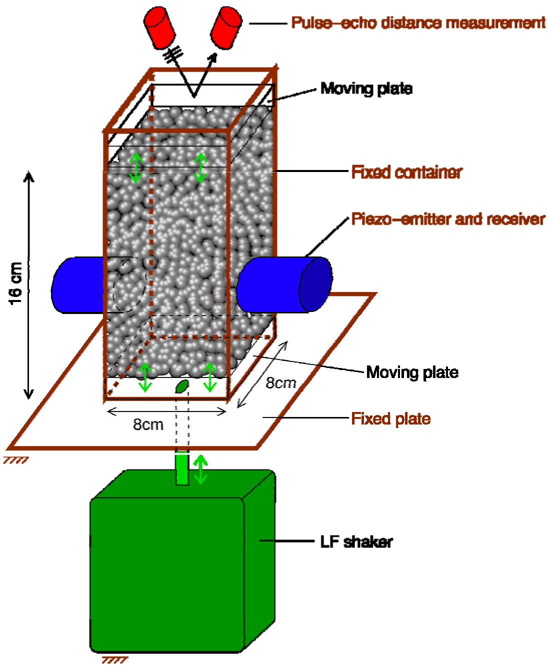


Fig.2 Schematics of the experimental setup [6].

Two piezo-transducers placed on two opposite side walls close to the bottom of the container measure the frequency response function of a granular medium slab. Acoustic resonances of this slab can be observed, starting for the lowest frequency with a half wavelength resonance because of the rigid-rigid boundary conditions imposed by the piezo-transducers. From the resonance frequency evolution with the number of taps, it is possible to assess the evolution of the longitudinal sound velocity. At a given stage of acoustic probing, it is also possible to apply the so-called nonlinear resonance method, in order to extract the nonlinear hysteretic parameters [6]. Two other piezo-transducers are used to measure the change in height of the medium along the segregation process, by a pulse-echo method.

We perform the acoustic probing of a granular sample made of glass beads with a glass density $\rho$ about $2500\ kg.m^{-3}$. The total mass of the medium is $2\ kg$ where half of it is composed of beads with a radius $R_1 = 1.10^{-3}\ m$ and the other half with beads of radius $R_2 = 4.10^{-3}\ m$. At the beginning of the experiment, the largest beads are placed at the bottom of the container. At the end of the segregation process the larger beads are at the top of the sample and the smaller ones at the bottom (where the acoustic probing is performed).

### 3. Average compacity evolution along the segregation process

The compacity of the granular sample is defined by

$$\phi = \frac{V_b}{V_t}, \qquad (7)$$

where $V_b$ is the volume of the beads and $V_t$ is the total volume of the sample (beads + saturating air). Equation (7) can be rewritten with the geometrical parameters of beads and container for a bidispersed medium in the form:

$$\phi = \frac{\frac{4}{3}\sum_{i=1}^{2} n_i R_i^3}{L^2 h}, \tag{8}$$

where $R_i$ is the radius and $n_i$ the number of each types of beads, $L$ are the lateral dimensions of the container and $h$ is the height of the granular sample. As we measure the variations in the granular sample height, we have access to the total volume changes, thus to the compacity averaged over the sample volume. Fig.(3) shows the results for the evolution of the average compacity (repeated over five experiments). This non-monotonous curve shows that there are two stages during the segregation process. The first stage corresponds to the medium mixing and the second stage to the separation in two distinct regions of the beads with different sizes. The first stage is associated with an increase of the compacity and the second one to a decrease of the compacity. At the beginning of the experiment, pores between larger beads are filled by the smaller beads, which explains why the compacity increases, and also why it reaches a higher value than the random close packing limit of 0.63 for identical beads. The compacity is maximum when the medium is well mixed. When the larger beads begin to rise up the sample and to separate, the compacity decreases.

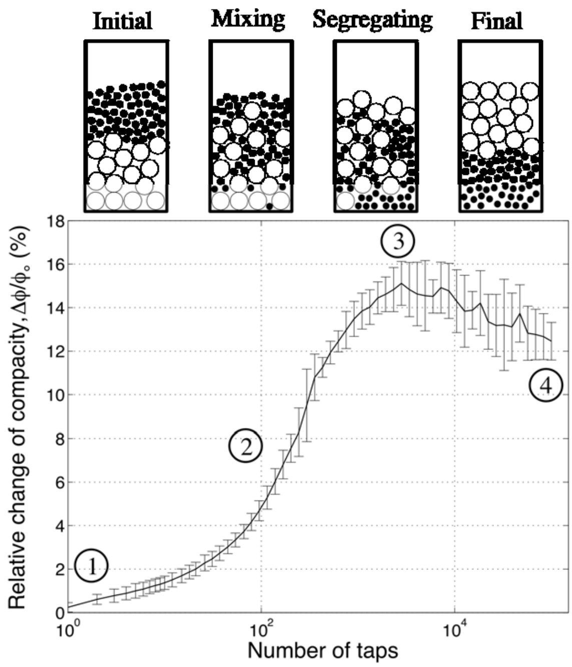


Fig.3 Stages and dynamics of the average compacity along the segregation process

We distinguish two main stages during the segregation process, that can be described using four steps as shown in Fig.(3).

1. At first, the larger beads are at the bottom of the container.
2. The medium is being mixed, the smaller beads fall into the large pores between the large beads. The average compacity increases.

3. The maximum of average compacity is reached; the first larger beads reach the top of the sample. After this point, the average compacity decreases.
4. The final configuration where the smaller beads are at the bottom and the larger at the top is reached.

## 4. Acoustic transfer function of the granular slab

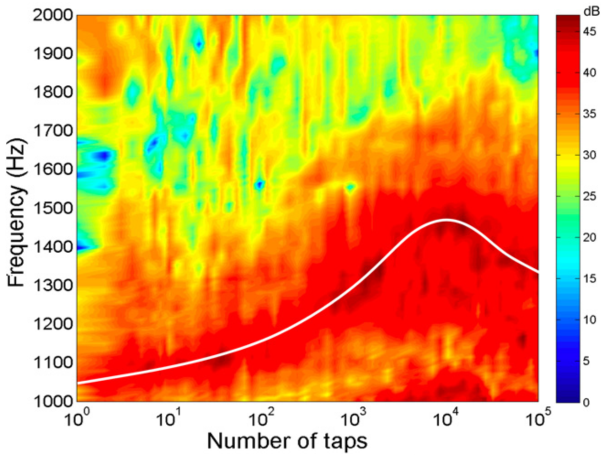


Fig.4 Evolution of acoustic tranfer function of the granular slab as a function of the numbers of taps. The white spline shows the resonance frequency

The acoustic transfer function of the granular slab is recorded at 60 stages of the segregation process. Fig.(4) shows the evolution of the transfer function amplitude in color scale from $1000$ to $2000\ Hz$, around the first resonance of the granular slab. The white line highlights the maximum of the resonance curve. The resonance frequency has a non-monotonous behavior, close to the one observed for the average compacity as a function of the number of taps.

Fig.(5) shows the comparison between the normalized variations of the frequency and of the compacity along the segregation process. The maxima are not located at the same number of taps. This difference could be explained, at least partially, by the fact that the acoustic probing is performed close to the bottom of the container and the compacity is measured in average over the sample.

A plot of the normalized variation of the resonance frequency as a function of the normalized variation of the average compacity, derived from the data plotted in Fig.(5), is presented in Fig.(6). This curve shows that for the same measured average compacity, it is possible to obtain at least two different resonance frequency values, i.e. two different wave velocities.

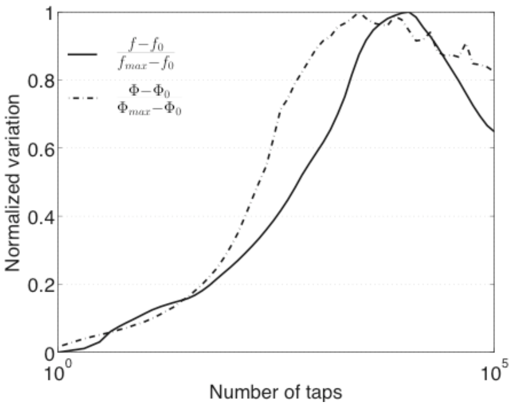


Fig.5 Normalized shift of the resonance frequency along the segregation process compared to the normalized variation of the compacity.

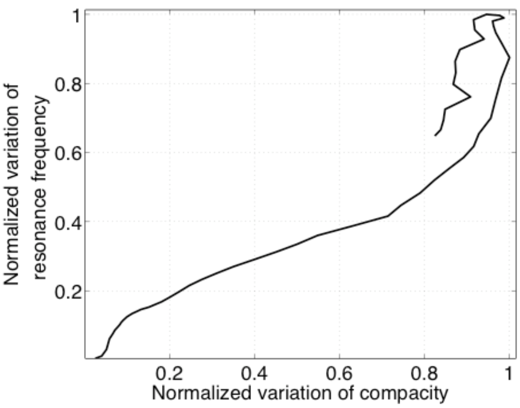


Fig.6 Normalized resonance frequency variation as a function of the normalized compacity.

In conclusion, the measured resonance frequencies, directly connected to the acoustic wave velocities in the medium, are the lowest in the initial state (larger beads), tend to increase up to their maximum value for the different bead size mixture, and then decrease close to the end of the segregation process when the larger beads escape from the probed region (the smaller bead packing is probed). The same wave velocity can thus be observed for different bead mixtures.

## 5. Nonlinear resonances

It is worth mentioning here that in our configuration where a resonance of the granular slab is studied, it is possible to increase the excitation amplitude in order to observe nonlinear effects on the resonance curves [6]. For each tap, we perform a transfer function measurement for nine amplitudes of excitation. We observe a so-called "softening" effect: the resonance frequency decreases with an excitation amplitude increase. Fig.(7) shows the softening effect for the beginning and the end of the segregation experiment. An estimation of nonlinear parameters (hysteretic elastic and dissipative parameters) along the segregation process, i.e. for different bead sizes and for mixtures of sizes, could in principle be performed as in [6] for the compaction process. Currently, few problems such as the low quality factor of the resonance curves and the complicated and unexplained physics of the wave propagation through the large bead packing (scattering, coupling with the saturating air...) do not allow for a precise estimation of these parameters.

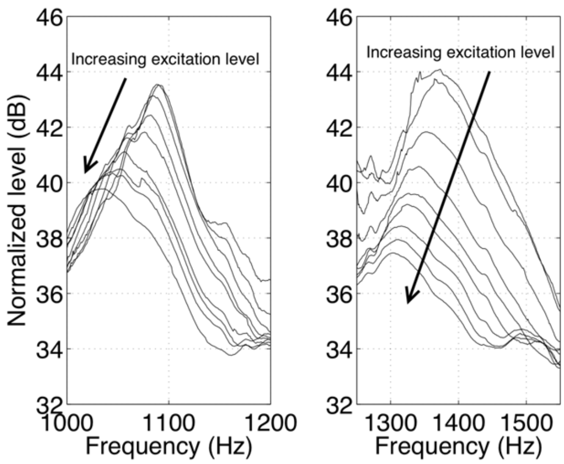


Fig.7 Typical (normalized by the excitation amplitude) resonance curves of the granular slab for increasing excitation amplitude: a. at the beginning, b. at the end of the segregation process

## 6. Conclusions

In this work, the segregation of size in unconsolidated granular media has been studied with acoustic methods. Correlation between the average compacity and the first resonance frequency of the granular slab shows that a linear acoustic probing is sensitive to the presence of larger or smaller beads in the probed region, and to a mixture of small and large beads. A larger wave velocity is observed in the mixture of bead sizes than for the smaller or larger bead packing themselves. This could be due to the larger number of contacts in the polydispersed packings than in the monodispersed packings. Work is in progress in order to study precisely the nonlinear properties of the packing along the segregation process.

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