Chattahoochee High School

AP Calculus Summer Assignment

The AP Course: AP Calculus is a college level course. The course topics are listed on the College Board website, www.apcentral.collegeboard.com .

The Summer Assignment: Students need a strong foundation to be ready for the rigorous work required throughout the term. Completing the summer assignment should prepare you for the material to be taught in the course. This packet consists of material studied during Algebra II and Pre-Calculus. Students should anticipate working approximately 10 hours to complete it properly. This packet includes:

I. A "Toolkit of Functions"; you should be familiar with each of the graphs, along with their domains and ranges, and any special characteristics.
II. A formula and identities section. These are for your reference and most should already be memorized from precalculus. You should work on memorizing the ones you to not already know.
III. A unit circle template with which to practice your unit circle. You are expected to know the 6 trig values of each point on the unit circle. This needs to be quick and from memory. Be prepared for a unit circle quiz at any point during the year.
IV. A list of skills that you will need for AP Calculus.
V. 100 Calculus Prerequisite Problems. These are optional but HIGHLY ENCOURAGED. Problems will be collected the first week of school but will not be counted for a grade. Please use your own paper for the problems. Please do the problems in order. Write out the problem, show all work in a logical and organized manner, and then place a box around your final answer. Do not list only an answer.

Calculators: Students enrolled in AP Calculus will be using a graphic calculator throughout the course. A graphic calculator is required on the AP test. A list of acceptable calculators is available on the AP website.

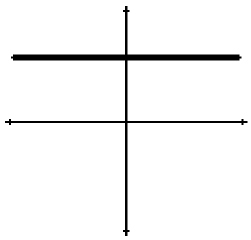
Grades earned for this material: An assessment over prerequisite material will be given within the first 2 week of school.

Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator. You should also know the domain and range of these functions.

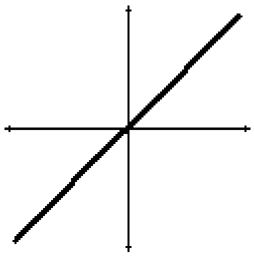
Constant

$f(x) = a$



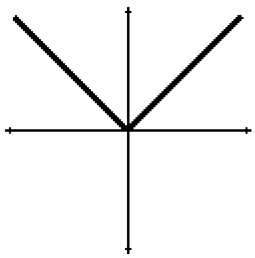
Identity

$f(x) = x$



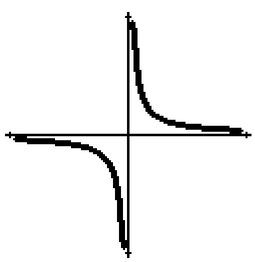
Absolute Value

$f(x) = |x|$



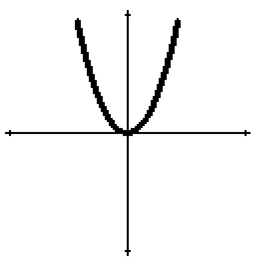
Reciprocal

$f(x) = \dfrac{1}{x}$



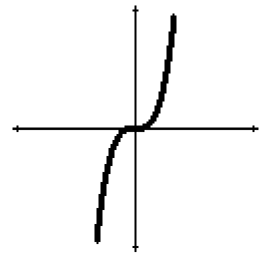
Quadratic

$f(x) = x^2$



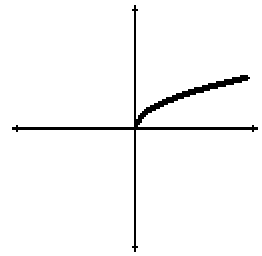
Cubic

$f(x) = x^3$



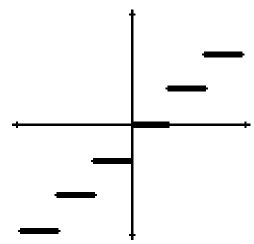
Square Root

$f(x) = \sqrt{x}$



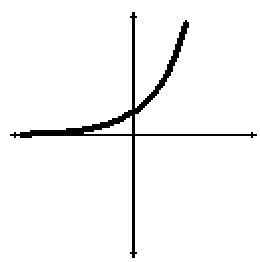
Greatest Integer

$f(x) = \lfloor x \rfloor$



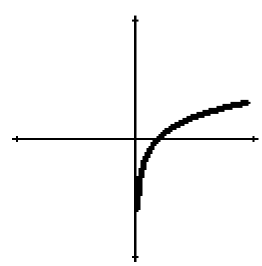
Exponential

$f(x) = a^x$



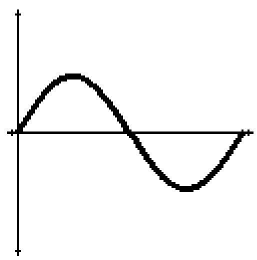
Logarithmic

$f(x) = \ln x$

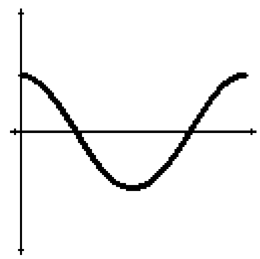


Trig Functions

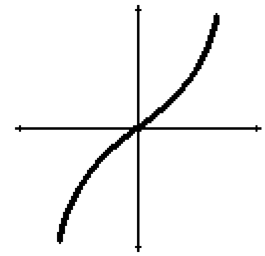
$f(x) = \sin x$



$f(x) = \cos x$

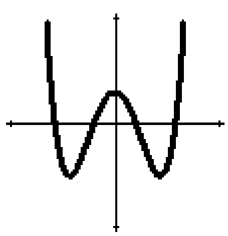
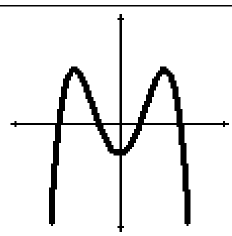


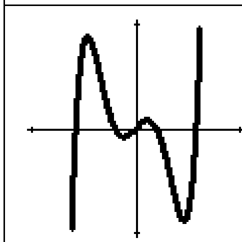
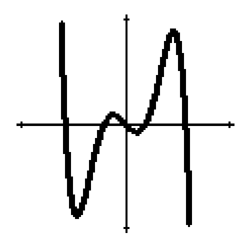
$f(x) = \tan x$



Polynomial Functions:

A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ where *n* is a nonnegative integer and the numbers $a_0,\ a_1, a_2,\ \ldots a_n$ are constants**.**

Even Degree	
Positive Leading Coefficient	Negative Leading Coefficient
	

Odd Degree	
Positive Leading Coefficient	Negative Leading Coefficient
	

- Number of roots equals the degree of the polynomial.
- Number of x intercepts is less than or equal to the degree.
- Number of “turns” is less than or equal to degree – 1.

Formulas and Identities

Trig Formulas:

Arc Length of a circle: $L = r\theta$ where θ is measured in radians.

Area of a sector of a circle: $A = \frac{1}{2}r^2\theta$ where θ is measured in radians.

Solving Triangles:

Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines: $a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

Area of a Triangle: $Area = \frac{1}{2}ab\sin C$ or $Area = \frac{1}{2}ac\sin B$ or $Area = \frac{1}{2}ab\sin C$

Heron's Formula: $Area = \sqrt{s(s-a)(s-b)(s-c)}$ where s is the semi perimeter.

Ambiguous Case:

θ is acute **Compute:** $altitude = adjacent \bullet \sin\theta$	
$opposite < altitude$	**No triangle**
$opposite = altitude$	**1 triangle**
$altitude < opposite < adjacent$	**2 triangles**
$opposite > adjacent$	**1 triangle**

θ is obtuse or right	
$opposite \le adjacent$	**No triangle**
$opposite > adjacent$	**1 triangle**

Does a triangle exist? Yes, when $(\text{difference of 2 sides}) < \text{third side} < (\text{sum of 2 sides})$

Trig Identities:

Reciprocal Identities: $\csc A = \frac{1}{\sin A}$ $\sec A = \frac{1}{\cos A}$ $\cot A = \frac{1}{\tan A}$

Quotient Identities: $\tan A = \frac{\sin A}{\cos A}$ $\cot A = \frac{\cos A}{\sin A}$

Pythagorean Identities: $\sin^2 A + \cos^2 A = 1$ $\tan^2 A + 1 = \sec^2 A$ $\cot^2 A + 1 = \csc^2 A$

Sum and Difference Identities: $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

Double Angle Identities: $\sin(2A) = 2\sin A \cos A$ $\quad$ $\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$

$$\cos(2A) = \cos^2 A - \sin^2 A \qquad \cos(2A) = 2\cos^2 A - 1 \qquad \cos(2A) = 1 - 2\sin^2 A$$

Half Angle Identities: $\sin\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{2}}$ $\quad$ $\cos\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1+\cos A}{2}}$ $\quad$ $\tan\left(\frac{A}{2}\right) = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$

Polar Formulas: $x^2 + y^2 = r^2$ $\quad$ $\tan\theta = \frac{y}{x}$

$$x = r\cos\theta \qquad y = r\sin\theta$$

Geometric Formulas:

Area of a trapezoid: $A = \frac{1}{2}h(b_1 + b_2)$

Area of a triangle: $A = \frac{1}{2}bh$

Area of an equilateral Triangle: $A = \frac{\sqrt{3}}{4}s^2$

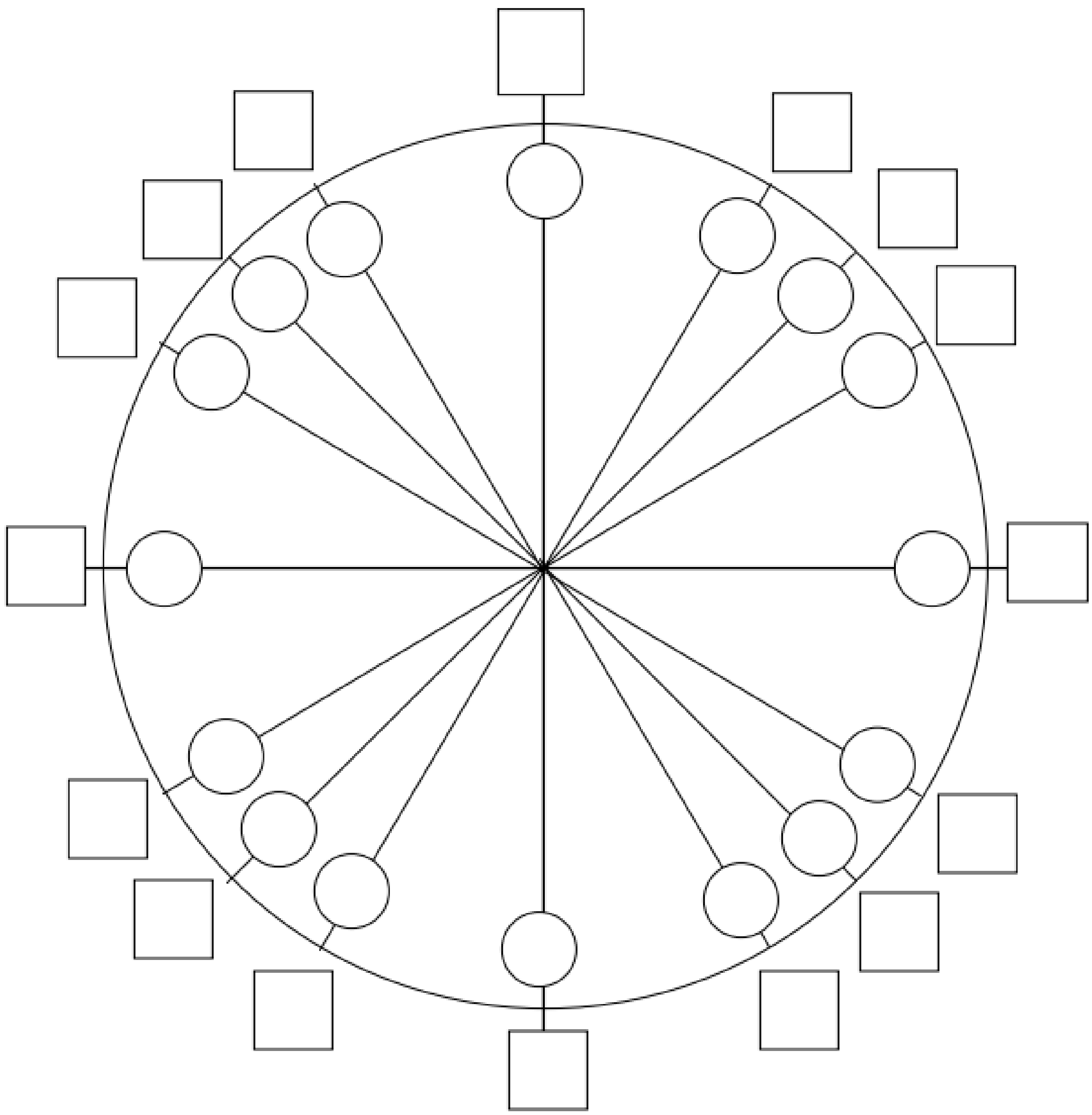
Area of a circle: $A = \pi r^2$

Circumference of a circle: $C = 2\pi r$ or $C = d\pi$

Volume of a cube: $V = s^3$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Unit Circle – Degrees and Radians



Place degree measures in the circles.

Place radian measure in the squares.

Place $(\cos\theta, \sin\theta)$ in parenthesis outside the square.

Place $\tan\theta$ outside the parenthesis.

$\tan\theta =$ ______

$\cot\theta =$ ______

$\sec\theta =$ ______

$\csc\theta =$ ______

Skills Needed for Calculus

* A <u>solid</u> working foundation in these areas is very important.

I. Algebra:

*A. Exponents (operations with integer, fractional, and negative exponents)
*B. Factoring (GCF, trinomials, difference of squares and cubes, sum of cubes, grouping)
C. Rationalizing (numerator and denominator)
*D. Simplifying rational expressions
*E. Solving algebraic equations and inequalities (linear, quadratic, higher order using synthetic division, rational, radical, and absolute value equations)
F. Simultaneous equations

II. Graphing and Functions:

*A. Lines (intercepts, slopes, write equations using point-slope and slope intercept, parallel, perpendicular, distance and midpoint formulas)
B. Conic Sections (circle, parabola, ellipse, and hyperbola)
*C. Functions (definition, notation, domain, range, inverse, composition)
*D. Basic shapes and transformations of the following functions (absolute value, rational, root, higher order curves, log, ln, exponential, trigonometric, piece-wise, inverse functions)
E. Tests for symmetry: odd, even

III. Geometry

A. Pythagorean Theorem
B. Area Formulas (Circle, polygons, surface area of solids)
C. Volume formulas
D. Similar Triangles

*** IV. Logarithmic and Exponential Functions**

*A. Simplify Expressions (Use laws of logarithms and exponents)
*B. Solve exponential and logarithmic equations (include ln as well as log)
*C. Sketch graphs
*D. Inverses

*** V. Trigonometry**

*A. Unit Circle (definition of functions, angles in radians and degrees)
B. Use of Pythagorean Identities and formulas to simplify expressions and prove identities
*C. Solve equations
*D. Inverse Trigonometric functions
E. Right triangle trigonometry
*F. Graphs

Calculus Prerequisite Problems

Show all necessary work and place a box around your final answer.

I. Algebra

A. Exponents: 1) Simplify: $\dfrac{8x^3yz^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}\left(yz^{\frac{2}{3}}\right)^{-1}}$

B. Factor Completely:

2) $9x^2+3x-3xy-y$ (Hint: use grouping)

3) $64x^6-1$ (Hint: use differences of squares and sum/difference of cubes)

4) $42x^4+35x^2-28$

5) $15x^{\frac{5}{2}}-2x^{\frac{3}{2}}-24x^{\frac{1}{2}}$ (Hint: use GCF first)

6) $x^{-1}-3x^{-2}+2x^{-3}$ (Hint: use GCF first)

C. Rationalize denominator / numerator:

7) $\dfrac{3-x}{1-\sqrt{x-2}}$ 8) $\dfrac{\sqrt{x+1}+1}{x}$

D. Simplify the rational expression:

9) $\dfrac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$

E. Solve algebraic equations and inequalities:

For problems 10 & 11, use synthetic division to help factor. State ALL factors and roots.

10) $p(x)=x^3+4x^2+x-6$ 11) $p(x)=6x^3-17x^2-16x+7$

12) Explain why $\frac{3}{2}$ cannot be a root of $f(x)=4x^5+cx^3-dx+5$ where c and d are integers.

13) Explain why $f(x)=x^4+7x^2+x-5$ must have a root in the interval $[0,1]$. Check the graph and use signs of $f(0)$ and $f(1)$ to justify your answer.

For problems 14-21, solve the equation. You may use your graphing calculator to check solutions.

14) $(x+3)^2>4$ 15) $\dfrac{x+5}{x-3}\le 0$ 16) $3x^3-14x^2-5x\le 0$

17) $x<\dfrac{1}{x}$ 18) $\dfrac{x^2-9}{x+1}\ge 0$ 19) $\dfrac{1}{x-1}+\dfrac{4}{x-6}>0$

20) $x^2<4$ 21) $|2x+1|<\dfrac{1}{4}$

F. Solve the system

Solve the systems algebraically and then check the solution by graphing each function using your graphing calculator and find the points of intersection.

22) $\begin{array}{l} x-y+1=0 \\ y-x^2=-5 \end{array}$ 23) $\begin{array}{l} x^2-4x+3=y \\ -x^2+6x-9=y \end{array}$

II. Graphing and Functions

A. Linear Graphs

Write the equation of the line describe below.

24) Passes through the point $(2,-1)$ and has a slope of $-\frac{1}{3}$

25) Passes through the point $(4,-3)$ and is perpendicular to $3x+2y=4$

26) Passes through the point $(-1,-2)$ and is parallel to $y=\frac{3}{5}x-1$

B. Conic Sections

Identify the conic section and write the equation in standard form.

27) $x=4y^2+8x-3$ 28) $4x^2-16x+3y^2+24y+52=0$

C. Functions

For questions 29-34, Identify the domain and range. To help in finding domain, remember the following domain restrictions: denominator cannot equal zero, the argument of log or ln must be greater than zero, the radicand of an even index must be greater than or equal to zero. To determine range, use reasoning, and if all else fails, use a graphing calculator to look at the graph.

29) $y=\frac{3}{x-2}$ 30) $y=\log(x-3)$ 31) $y=x^4+x^2+2$

32) $y=\sqrt{2x-3}$ 33) $y=|x-5|$ 34) Domain only: $y=\frac{\sqrt{x+1}}{x^2-1}$

35) Given $f(x)=\begin{cases} x & x\geq 0 \\ 1 & -1\leq x<0 \\ x-2 & x<-1 \end{cases}$ graph over the domain $[-3,3]$ and determine the range.

For questions 36 - 40, find the indicated composition or inverse. Let $f(x)=x^2+3x-2$, $g(x)=4x-3$, $h(x)=\ln x$ *and,* $w(x)=\sqrt{x-4}$

36) $g^{-1}(x)$ 37) $h^{-1}(x)$ 38) $w^{-1}(x)$, for $x\geq 4$

39) $f(g(x))$ 40) $h(g(f(1)))$

41) Does $y=3x^2-9$ have an inverse function? Explain your answer.

For questions 42 & 43, find the indicated composition or inverse. Let $f(x)=2x$, $g(x)=-x$, *and* $h(x)=4$.

42) $(f \circ g)(x)$ 43) $(f \circ g \circ h)(x)$

44) Find the domain and range of $(s \circ t)(x)$ if $s(t) = \sqrt{4-x}$ and $t(x) = x^2$.

D. Basic Shapes of Curves

Sketch the graphs. You may use your graphing calculator to verify your graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

45) $y = \sqrt{x}$ 46) $y = \ln x$ 47) $y = \frac{1}{x}$ 48) $y = |x-2|$

49) $y = \frac{1}{x-2}$ 50) $y = \frac{x}{x^2-4}$ 51) $y = 2^{-x}$ 52) $y = 3\sin 2\left(x - \frac{\pi}{6}\right)$

53) $f(x) = \begin{cases} \sqrt{25-x^2} & x < 0 \\ \frac{x^2-25}{x-5} & x \geq 0, x \neq 5 \\ 0 & x = 5 \end{cases}$

E. Even, Odd, Tests for Symmetry

For questions 54-60, Identify the function as even, odd, or neither and justify your answer. To justify your answer, you must show substituting in -x! It is not enough to simply check a number. Remember that in an even function $f(-x) = f(x)$ *and in an odd function* $f(-x) = -f(x)$.

54) $f(x) = x^3 + 3x$ 55) $f(x) = x^4 - 6x^2 + 3$ 56) $f(x) = \frac{x^3 - x}{x^2}$

57) $f(x) = \sin(2x)$ 58) $f(x) = x^2 + x$ 59) $f(x) = x(x^2 - 1)$

60) $f(x) = \frac{1+|x|}{x^2}$

61) What type of function (even or odd) results from the product of two even functions? Odd functions?

For questions 62-66, test for symmetry. Show substitution with variables to justify your answers.

Replace x with -x, and if the relation stays the same, it is symmetric to y-axis

Replace y with -y, and if the relation stays the same, it is symmetric to x-axis

Replace x with -x & y with -y, and if the relation is equivalent, it is symmetric to the origin.

62) $y = x^4 + x^2$ 63) $y = \sin x$ 64) $y = \cos x$

65) $x = y^2 + 1$ 66) $y = \frac{|x|}{x^2+1}$

III. Logarithmic and Exponential Functions

A. Simplify Expressions

67) $\log_4 \frac{1}{16}$ 68) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3 \frac{1}{27}$ 69) $\log_9 27$ 70) $\log_{125} \frac{1}{5}$

71) $\log_w w^{45}$ 72) $\ln e$ 73) $\ln 1$ 74) $\ln e^2$

B. Solve Equations

75) $\log_6(x+3)+\log_6(x+4)=1$ 76) $\log x^2-\log 100=\log 1$ 77) $3^{x+1}=15$

IV. Trigonometry

A. Unit Circle

*You need to **KNOW** the unit circle in radians and degree measures.*

78) State the domain, range, and fundamental period for each function:

a) $y=\sin x$ b) $y=\cos x$ c) $y=\tan x$

B. Identities

79) Simplify: $\dfrac{(\tan^2 x)(\csc^2 x)-1}{(\csc x)(\tan^2 x)(\sin x)}$ 80) Simplify: $1-\cos^2 x$

81) Simplify: $\sec^2 x-\tan^2 x$ 82) Verify: $(1-\sin^2 x)(1+\tan^2 x)=1$

C. Solve the Equations

Solve the following on $[0,2\pi)$.

83) $\cos^2 x=\cos x+2$ 84) $2\sin(2x)=\sqrt{3}$

85) $\cos^2 x+\sin x+1=0$

D. Inverse Trig Functions

For questions 86 – 89, evaluate the expression. Note: $\sin^{-1}x=\text{Arcsin}\,x$.

86) Arcsin1 87) $\text{Arcsin}\left(\dfrac{\sqrt{2}}{2}\right)$ 88) $\text{Arcsin}\left(\dfrac{\sqrt{3}}{2}\right)$ 89) $\sin\left(\text{Arccos}\left(\dfrac{\sqrt{3}}{2}\right)\right)$

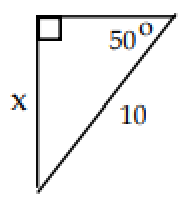
90) State the domain and range for each function:

a) $y=\text{Arcsin}\,x$ b) $y=\text{Arccos}\,x$ c) $y=\text{Arctan}\,x$

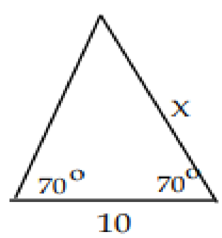
E. Right Triangle Trig

For questions 91 & 92, find the value of x. Note: Degrees!

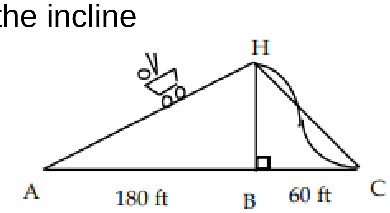
91)



92)



93) The roller coaster car shown in the diagram above takes 23.5 sec. to go up the 23 degree incline segment AH and only 2.8 seconds to go down the drop from H to C. The car covers horizontal distances of 180 feet on the incline and 60 feet on the drop. Decimals in answer may vary.



a. How high is the roller coaster above point B?
b. Find the distances AH and HC.
c. How fast (in ft/sec) does the car go up the incline?
d. What is the approximate average speed of the car as it goes down the drop?
e. Assume the car travels along HC. Is your approximate answer too big or

too small?

(<u>Advanced Mathematics</u>, Richard G. Brown, Houghton Mifflin,1994, pg 336)

F. Graphs

Identify the amplitude, period, horizontal, and vertical shifts of the functions.

94) $y = -\sin(2x)$ 95) $y = -\pi\cos\left(\frac{\pi}{2}x + \pi\right)$

V. Graphing Calculator Skills

You need to be familiar with the CALC *commands: value, root, minimum, maximum, intersect. You need to be comfortable using the window and zoom commands to zoom in or out on areas of your graph to find the information you need. When giving a decimal answer in Calculus, your answer should ALWAYS be accurate to 3 decimal places.*

For questions 96 – 99, sketch the graph and find the requested information for $f(x) = 2x^4 - 11x^3 - x^2 + 30x$.

96) Find all roots.

97) Find all local maxima.

98) Find all local minima

99) Find the following values: $f(-1)$; $f(2)$; $f(0)$; $f(0.125)$

100) Graph $f(x) = x^3 + 5x^2 - 7x + 2$ and $g(x) = 0.2x^2 + 10$. Find their points of intersection using the intersect command on your calculator.