Lecture 9

Solving Material Balances Problems Involving Non-Reactive Processes

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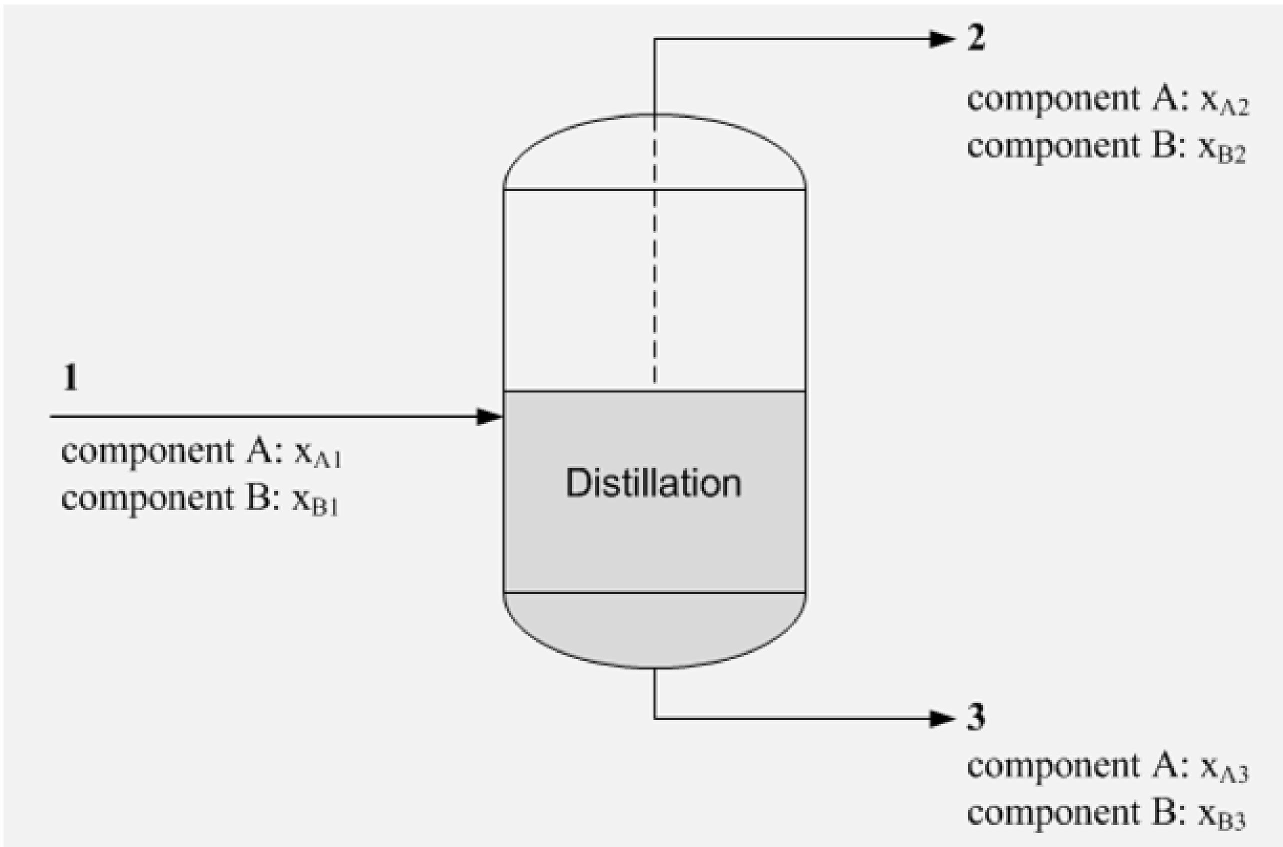
Component and Overall Material Balances

Consider a steady-state distillation process,



Component and Overall Material Balances

Since the process is at steady-state condition and no chemical reaction is involved, the material balance equation becomes

Input = Output

This balance equation can be applied to:

1. The total mass entering and leaving the process
2. Mass of individual component entering and leaving the process.

Component and Overall Material Balances

Total Mass (Overall Material Balance)

$$m_1 = m_2 + m_3$$

Component A Balance

$$m_{A1} = m_{A2} + m_{A3}$$
$$m_1 x_{A1} = m_2 x_{A2} + m_3 x_{A3}$$

Component B Balance

$$m_{B1} = m_{B2} + m_{B3}$$
$$m_1 x_{B1} = m_2 x_{B2} + m_3 x_{B3}$$

Component and Overall Material Balances

For the given process, 3 material balance equations can be written:

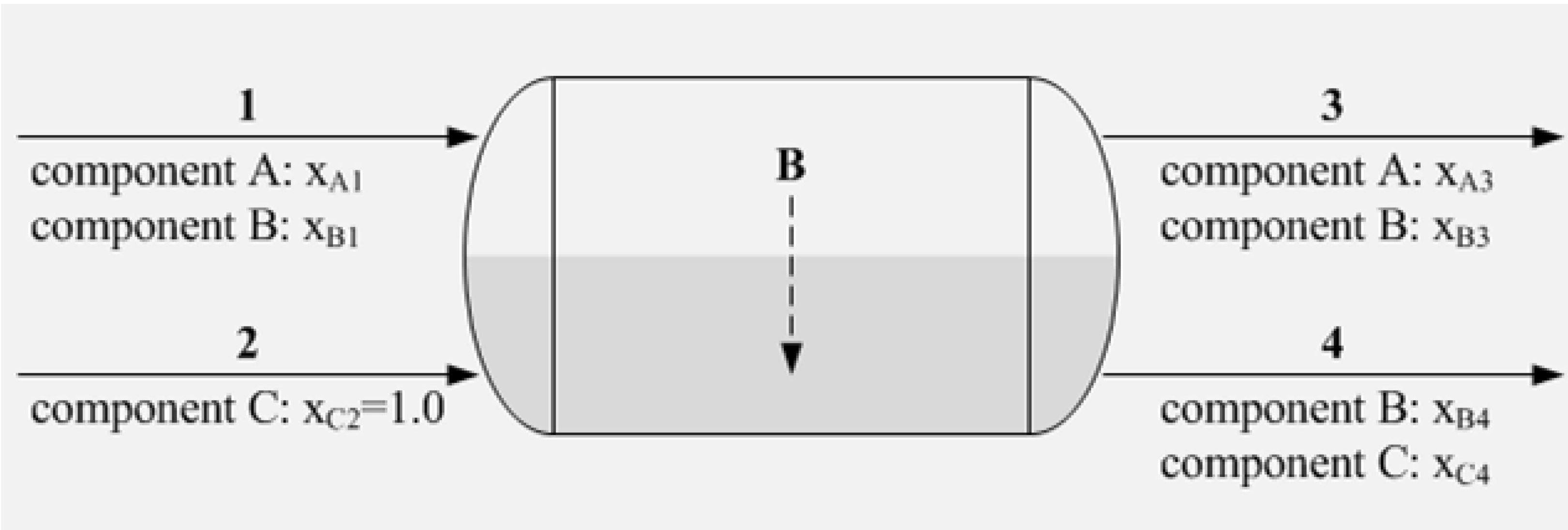
Total Balance:	$m_1 = m_2 + m_3$	**(E1)**
A-Balance:	$m_{A1} = m_{A2} + m_{A3}$	**(E2)**
B-Balance:	$m_{B1} = m_{B2} + m_{B3}$	**(E3)**

Are these material balances independent equations?

If (E1) and (E2) are added, what does the resulting equation represent?

Component and Overall Material Balances

Consider the unsteady-state extraction process:



Extraction is a physical process in which a component of a mixture is extracted using an immiscible solvent.

Component and Overall Material Balances

Writing the material balance equations:

Total Balance: $(Acc)_T = m_1 + m_2 - m_3 - m_4$

A-Balance: $(Acc)_A = m_{A1} - m_{A3}$

B-Balance: $(Acc)_B = m_{B1} - m_{B3} - m_{B4}$

C-Balance: $(Acc)_C = m_{C2} - m_{C4}$

The total balance can also be obtained by adding the last 3 equations and is no longer independent.

Number of Independent Material Balance Equations in Process without Chemical Reaction

For processes with no chemical reaction,

$$\mathbf{N_m = N_i}$$

where N_m = number of independent material balance equations.

N_i = total number of chemical species (or components) involved in the process.

Example 9–1. Mixing of Methanol-Water Mixtures

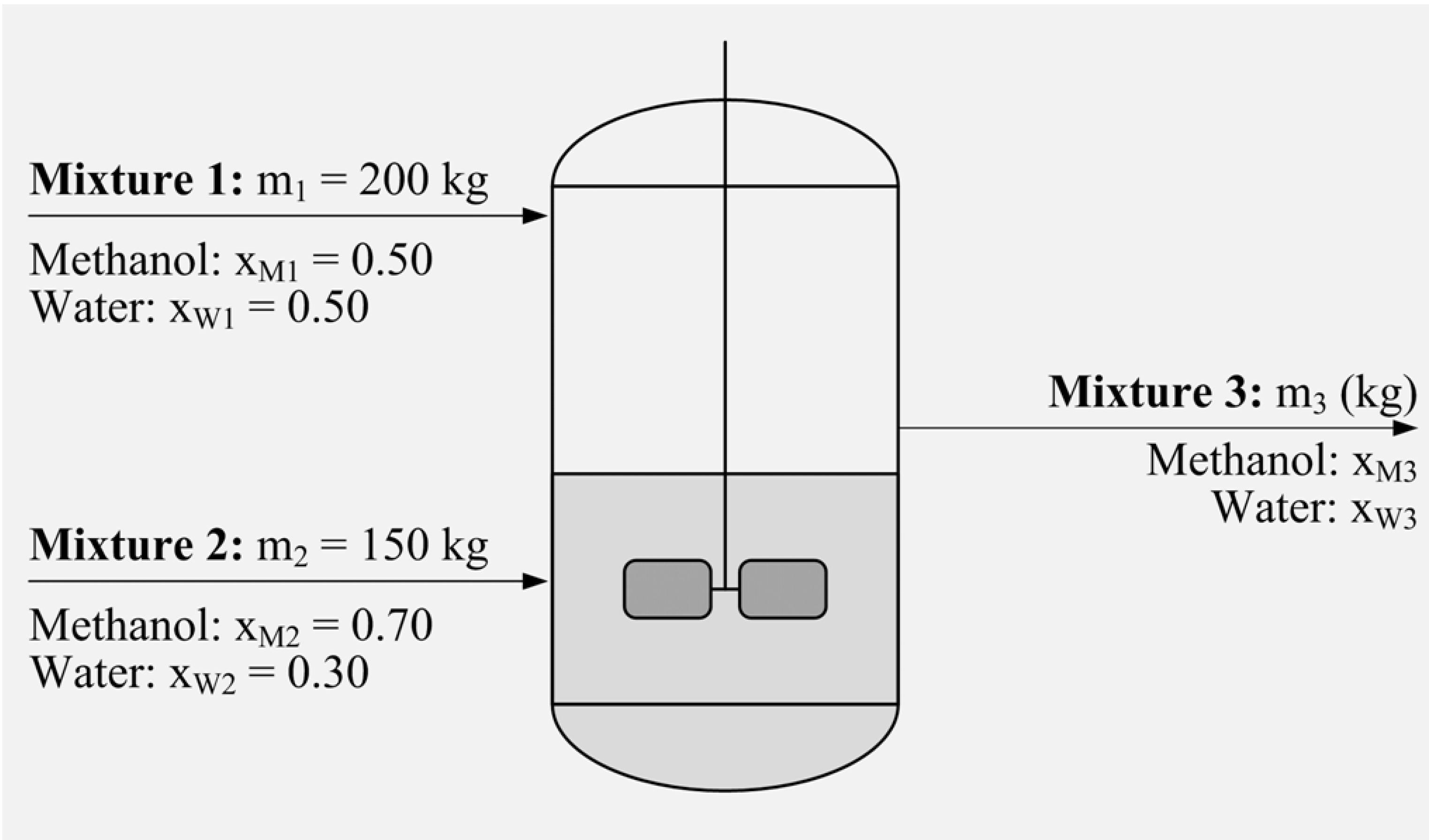
Two methanol-water mixtures are contained in separate tanks.

The first mixture contains 40.0 wt% methanol and the second contains 70.0 wt% methanol.

If 200 kg of the first mixture is combined with 150 kg of the second, what are the mass and composition of the product?

Example 9–1. Mixing of Methanol-Water Mixtures

Step 1. *Draw a flowchart to visually organize the data.*



Example 9–1. Mixing of Methanol-Water Mixtures

Step 2. *Determine the degrees of freedom (DF)*

Number of unknowns (U): **3 unknowns (m_3, x_{M3}, x_{W3})**

Number of independent equations (V):

material balances:	**2 equations**
physical constraint:	**1 equation** ($\Sigma x = 1.00$)
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total independent equations	**3 equations**

DF = U – V = 3 – 3 = 0, the problem is solvable

Example 9–1. Mixing of Methanol-Water Mixtures

Step 3. *Write down the equations*

Material Balances (Steady-State, Non-Reactive Process):

Total Balance: $m_1 + m_2 = m_3$

Methanol-Balance: $m_1x_{M1} + m_2x_{M2} = m_3x_{M3}$

Water-Balance: $m_1x_{W1} + m_2x_{W2} = m_3x_{W3}$

(*choose only 2 equations since one of them is no longer independent)*

Physical Constraint (applied to mixture 3):

$$x_{M3} + x_{W3} = 1.00$$

Example 9–1. Mixing of Methanol-Water Mixtures

Step 4. *Solve the unknowns (* $\boldsymbol{m_3}$, $\boldsymbol{x_{M3}}$, $\boldsymbol{x_{W3}}$ *)*

Always start with the equation with the least number of unknowns if possible and minimize solving equations simultaneously.

Total Balance (m_3)

↓

Methanol Balance (x_{M3})

↓

Physical Constraint (x_{W3})

Example 9-1. Mixing of Methanol-Water Mixtures

Step 4. *Solve the unknowns (m_3, x_{M3}, x_{W3})*

Total balance:

$$m_3 = (200 \text{ kg}) + (150 \text{ kg})$$
$$m_3 = 350 \text{ kg}$$

CH_3OH balance:

$$(200 \text{ kg})(0.40) + (150 \text{ kg})(0.70) = (350 \text{ g})x_{M3}$$
$$x_{M3} = 0.529$$

Physical constraint: $x_{W3} = 1.00 - x_{M3} = 1 - 0.529$

$$x_{W3} = 0.471$$

Example 9–2. Mixing of Ethanol-Water Mixtures

Three hundred gallons of a mixture containing 75.0 wt% ethanol and 25% water (mixture specific gravity = 0.877) and a quantity of a 40.0 wt% ethanol-60% water mixture (SG=0.952) are blended to produce a mixture containing 60.0 wt% ethanol.

Determine the required volume of the 40% mixture.

Ans.

Volume require = 207 gal

Example 9–3. Humidification of Air

A stream of humid air containing 1.00 mole% $H_2O_{(v)}$ and the balance dry air is to be humidified to a water content of 10.0 mole% $H_2O_{(v)}$.

For this purpose, liquid water is fed through a flow meter and evaporated into the air stream. The flow meter reading, R, is 95.

The only available calibration data for the flow meter are two points, indicating that readings of R=15 and R=50 correspond to flow rates V'=40.0 ft^3/h and V'=96.9 ft^3/h, respectively. Estimate the molar flow rate (lbmole/h) of the humidified air.

Ans. 6489 lbmole/h of humidified air

Example 9–4. Absorption of SO_2

A waste gas containing SO_2 (a precursor of acid rain) and several other species (collectively designated as A) is fed to a scrubbing tower where it contacts a solvent (B) that absorbs SO_2.

The solvent feed rate to the tower is 1000L/min. The specific gravity of the solvent is 1.30. Absorption of A and evaporation of B in the scrubber may be neglected.

The volumetric flow rate of the feed gas is determined with an orifice meter, with a differential mercury manometer and the calibration equation is found to be

$$\mathbf{V' = 13.2h^{0.515}}$$

Example 9–4. Absorption of SO_2

where V' is volumetric flow rate (in m^3/min) and h is orifice reading (in mmHg).

The mole fraction of SO_2 in the inlet and outlet streams is measured with an electrochemical detector. The reading in the detector is calibrated and the relationship is determined to be

$$y=(5.00\times10^{-4})e^{0.0600R}$$

where y = kmol SO_2/kmol and R=detector reading.

Example 9-4. Absorption of SO_2

The molar density of the feed gas may be determined from the formula

$$\rho(mol/L)=12.2P(atm)T^{-1}(K)$$

where P and T are the absolute pressure and temperature of the gas.

The following data are taken for the feed gas: T=750F, P=150 psig, h=210 mm, R=82.4 and for the outlet gas, R=11.6.

Determine the (a) mass fraction of SO_2 in the liquid effluent stream and (b) the rate at which SO_2 is absorbed in the liquid effluent stream.

Example 9-4. Absorption of SO_2

Ans.

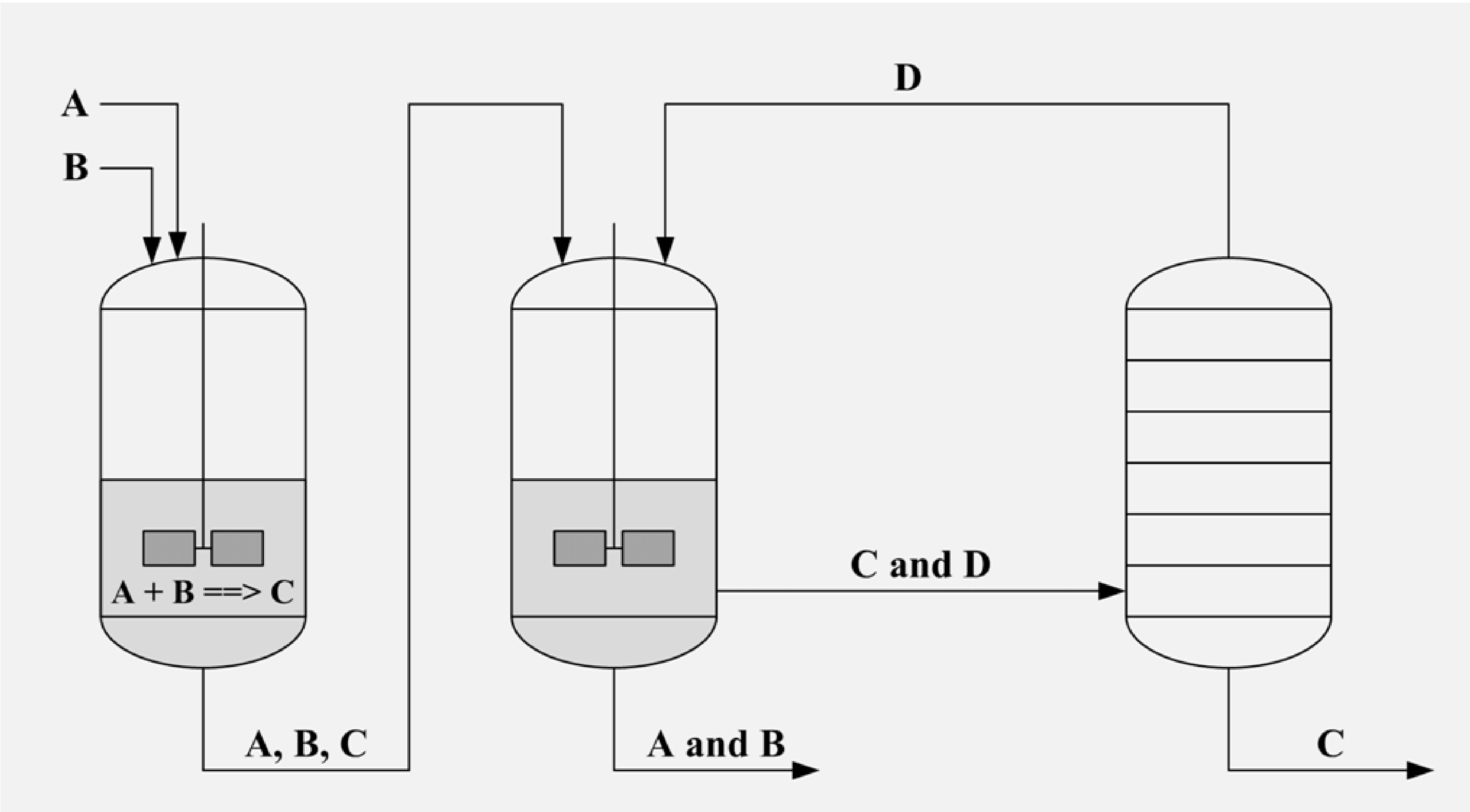
a. mass fraction of SO_2 in the liquid effluent stream.

0.245 kg SO_2 absorbed/kg stream

b. rate at which SO_2 is absorbed.

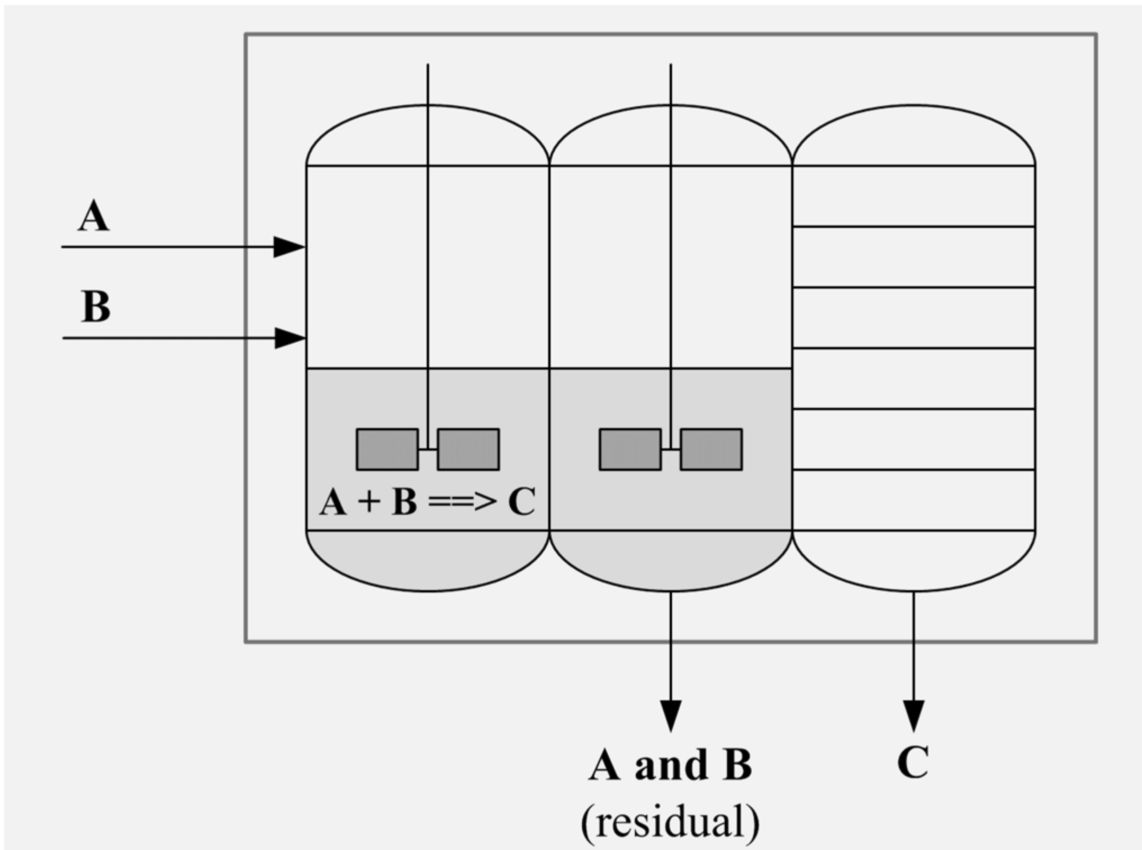
422 kg SO_2/min

Material Balances on Multiple Unit Processes



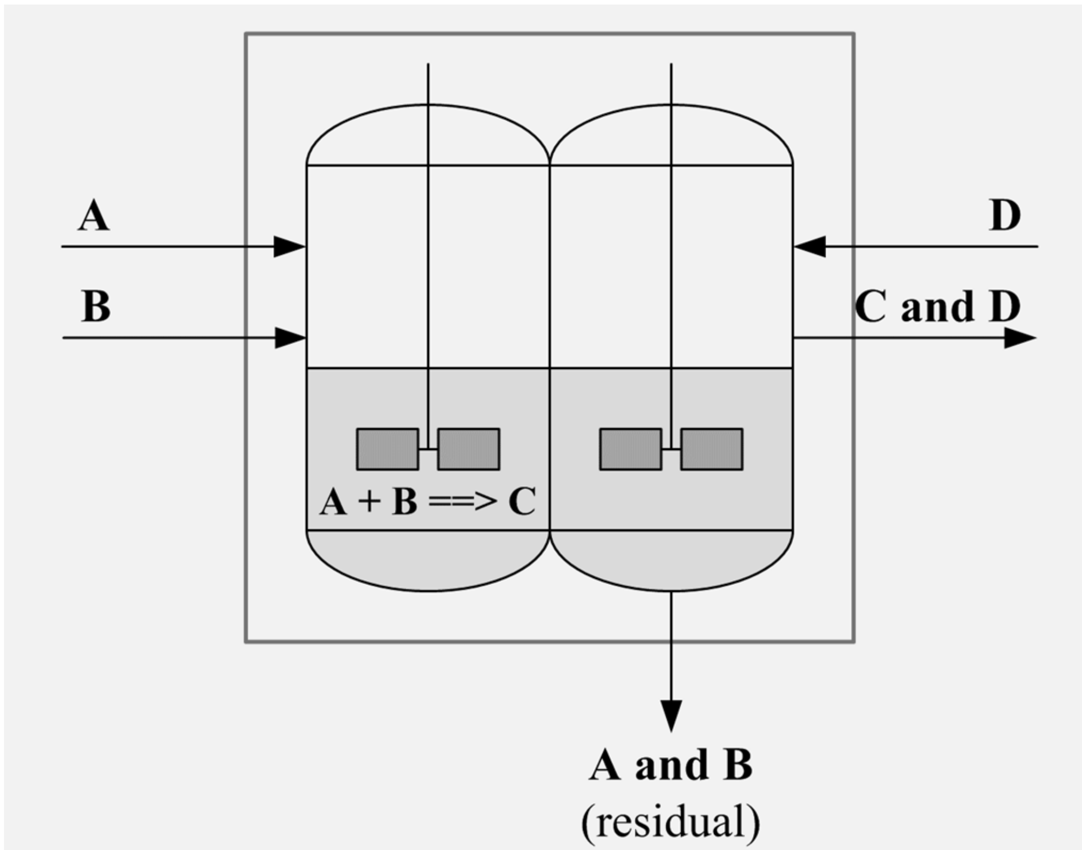
Material Balances on Multiple Unit Processes

The entire process can be analyzed as 1 unit. . .



Material Balances on Multiple Unit Processes

. . . or portions of the entire process is analyzed.



Example 9-5. Distillation of Benzene, Toluene, and Xylene

A liquid mixture containing 30.0 mole% benzene (B), 25.0% toluene (T), and the balance xylene (X) is fed to a distillation column.

The bottom product contains 98.0 mole% X and no B, and 96.0% of the X in the feed is recovered in this stream. The overhead product is fed to a second distillation column.

The overhead product from the second column contains 97.0% of the B in the feed to this column. The composition of this stream is 94.0 mole% B and the balance T.

Example 9-5. Distillation of Benzene, Toluene, and Xylene

Calculate

a. the percentage of the benzene in the process feed (i.e. the feed to the first column) that emerges in the overhead product from the second column.

   Ans. 97% recovery of benzene

b. the percentage of toluene in the process feed that emerges in the bottom product from the second column.

   Ans. 89% recovery of toluene

Example 9-6. Separation of Benzene and Toluene by Distillation

An equimolar liquid mixture of benzene and toluene is separated into two product streams by distillation.

The vapor stream leaving at the top of the column, which contains 97 mole% benzene, is fed to a condenser to undergo complete condensation.

The condensed liquid is split into two equal fractions: one is taken off as the final overhead product stream and the other (the reflux) is recycled to the top of the column.

The final overhead product contains 89.2% of the benzene fed to the column.

Example 9-6. Separation of Benzene and Toluene by Distillation

The liquid leaving the bottom of the column is fed to a partial reboiler in which 45% of it is vaporized.

The vapor generated in the reboiler (the boilup) is recycled to the column, and the residual reboiler liquid is taken as the final bottom product stream.

The compositions of the streams leaving the reboiler are governed by the relation:

$$\frac{y_B/(1-y_B)}{x_B/(1-x_B)} = 2.25$$

Example 9-6. Separation of Benzene and Toluene by Distillation

where y_B and x_B are the mole fractions of benzene in the vapor and liquid streams, respectively.

Using 100 mol of feed as a basis, calculate:

a. the molar amounts of the overhead and bottoms products.
   Ans. Overhead = 45.98 mol ; Bottoms = 54.02 mol

b. the mole fraction of benzene in the bottoms product
   Ans. 0.100 mol B/mol

c. the percentage recovery of toluene in the bottoms product. **Ans. 97% recovery**