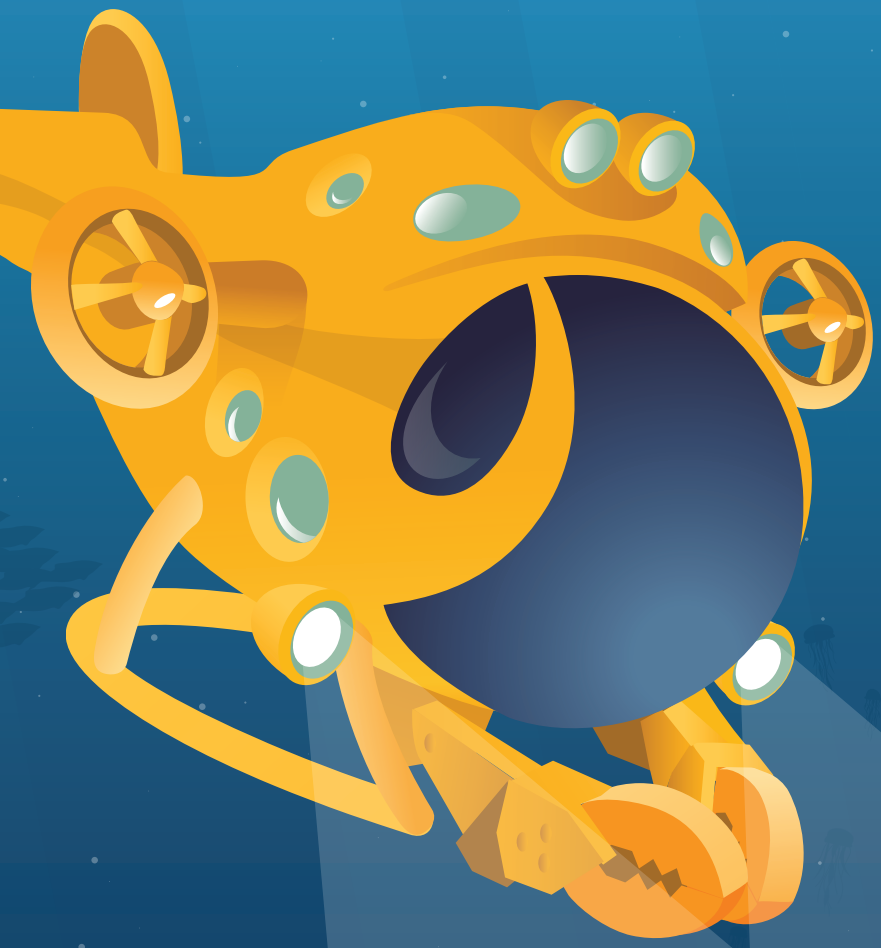
MATHCOUNTS®
COMPETITION SERIES

EST. 1983



2019-2020
School Handbook

Check out this year's math problems on

PG. 11!

2019-2020 MATHCOUNTS® SCHOOL HANDBOOK

Raytheon

2020 MATHCOUNTS
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STEMWORKS AT WestEd

WestEd has recognized MATHCOUNTS as having one of the nation's most effective STEM learning programs, listing the Math Video Challenge as an Accomplished Program in STEMworks.

APPROVED FOR LISTING NASSP 2019–2020 STUDENT CONTESTS, PROGRAMS & ACTIVITIES

The National Association of Secondary School Principals has placed all three MATHCOUNTS programs on the NASSP Advisory List of National Contests and Activities for 2019-2020.

HOW TO USE THIS SCHOOL HANDBOOK

IF YOU'RE A NEW COACH

Welcome! We're so glad you're a coach this year.
Check out the **Guide for New Coaches**
starting on the next page.

IF YOU'RE A RETURNING COACH

Welcome back! Thank you for coaching again.
Get the **2019-2020 Handbook Materials**
starting on page 8.

GUIDE FOR NEW COACHES

Welcome to the MATHCOUNTS® Competition Series! Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

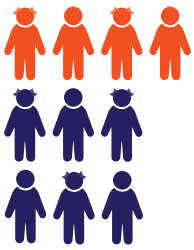
If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at info@mathcounts.org.

THE MATHCOUNTS COMPETITION SERIES IN A NUTSHELL

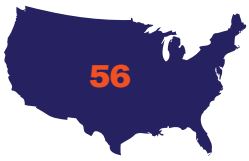
The **MATHCOUNTS Competition Series** is a national program that provides students the opportunity to compete in live, in-person math contests against and alongside their peers. Created in 1983, it is the longest-running MATHCOUNTS program and is open to all sixth-, seventh- and eighth-grade students.

HOW DOES IT WORK? The Competition Series has 4 levels of competition—school, chapter, state and national. Here's what a typical program year looks like.

Schools register in the fall and work with students during the year. Coaches administer the School Competition, usually in January. Any number of students from your school can participate in your team meetings and compete in the School Competition. MATHCOUNTS provides the School Competition to coaches in November. Many coaches use this to determine which student(s) will advance to the Chapter Competition.



Between 1 and 10 students from each school advance to the local Chapter Competition, which takes place in February. Each school can send a team of 4 students plus up to 6 individual competitors. All chapter competitors—whether they are team members or individuals—participate in the individual rounds of the competition; then just the 4 team members participate in the team round. Schools also can opt to send just a few individual competitors, rather than forming a full team. Over 500 Chapter Competitions take place across the country.



Top students from each Chapter Competition advance to their State Competition, which takes place in March. Your school's registration fees cover your students as far as they get in the Competition Series. If your students make it to one of the 56 State Competitions, no additional fees are required.

Top 4 individual competitors from each State Competition receive an all-expenses-paid trip to the National Competition, which takes place in May. These 224 students combine to form 4-person state teams, while also competing individually for the title of National Champion.

WHAT DOES THE TEST LOOK LIKE? Every MATHCOUNTS competition consists of 4 rounds—Sprint, Target, Team and Countdown Round. Altogether the rounds are designed to take about 3 hours to complete. Here's what each round looks like.

Sprint Round
40 minutes
30 problems total
no calculators used
focus on speed and accuracy

Target Round
Approx. 30 minutes
8 problems total
calculators used
focus on problem-solving and mathematical reasoning

The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.

Team Round
20 minutes
10 problems total
calculators used
focus on problem-solving and collaboration

Only the 4 students on a school's team can take this round officially.

Countdown Round
Maximum of 45 seconds per problem
no calculators used
focus on speed and accuracy

Students with highest scores on Sprint and Target Rounds compete head-to-head. This round is optional at the school, chapter and state level.

HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS? What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions is the best way to prepare to compete. Each year MATHCOUNTS provides the *School Handbook* to all coaches, plus lots of additional free resources online.

The next sections of this Guide for New Coaches will explain the layout of the *MATHCOUNTS School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

THE ROLE OF THE COMPETITION COACH

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general **every good MATHCOUNTS coach must do the following**.

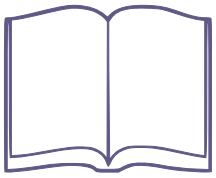
- Schedule and run an adequate number of practices for participating students.
- Help motivate and encourage students throughout the program year.
- Select the 1-10 student(s) who will represent the school at the Chapter Competition in February.
- Take students to the Chapter Competition or make arrangements with parents and volunteers to get them there.

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

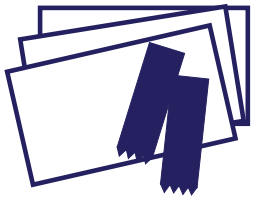
You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

MAKING THE MOST OF YOUR RESOURCES

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



2019-2020 MATHCOUNTS School Handbook
The most important resource included in the School Competition Kit. Includes 200 problems.



Student Recognition Ribbons and Certificates
10 participation certificates, 1 Champion ribbon and 1 Second Place ribbon.

You'll also get access to electronic resources including **50 more handbook problems**. The following are available to coaches at **www.mathcounts.org/coaches**. This section of the MATHCOUNTS website is restricted to coaches and you already should have received an email with login instructions. *If you have not received this email, please contact us at info@mathcounts.org to confirm we have your correct email address.*

Official 2020 MATHCOUNTS School Competition
Released in November 2019
Includes all 4 test rounds and the answer key

2019 MATHCOUNTS School, Chapter + State Competitions
Released by mid-April 2019
Each level includes all 4 test rounds and the answer key

MATHCOUNTS Problem of the Week
Released each Monday
Each multi-step problem relates to a timely event

You can use the **2020 MATHCOUNTS School Competition** to choose the students who will represent your school at the Chapter Competition. Sometimes coaches already know which students will attend the Chapter Competition. If you do not need the School Competition to determine your chapter competitors, then we recommend using it as an additional practice resource for your students.

The **2019-2020 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the 4 rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

WHAT'S IN THE HANDBOOK? There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 8 of this book, but below are the sections that you'll use the most when coaching your students.

- **Handbook Problems:** 250 math problems (200 in the book and 50 online) divided into Warm-Ups, Workouts and Stretches. These problems increase in difficulty as the students progress through the book. (pg. 11)

COACHES: log in at mathcounts.org/coaches to get the problems, answers, step-by-step solutions and problem index for problems 201-250.

- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches. (pg. 38)
- **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems. (pg. 35)
- **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards. (pg. 36)

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

Warm-Ups
14 Warm-Ups in handbook
10 questions per Warm-Up
no calculators used

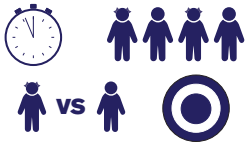
Warm-Ups prepare students particularly for the Sprint and Countdown Rounds.

Workouts
8 Workouts in handbook
10 questions per Workout
calculators used

Workouts prepare students particularly for the Target and Team Rounds.

Stretches
3 Stretches in handbook
Number of questions and use of calculators vary by Stretch

Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all 4 rounds.



IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR? On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the School and Chapter Competition approach.

Mid-August – September 2019	**October 2019**	**November 2019**	**December 2019**
Warm-Ups 1, 2 + 3 Workouts 1 + 2	Warm-Ups 4 + 5 Workout 3 Ratios Stretch	Warm-Ups 6 + 7 Workout 4 Venn Diagrams Stretch	Warm-Ups 8 + 9 Workout 5 Clocks Stretch
January 2020 Warm-Ups 10 +11 Workout 6 *2020 MATHCOUNTS School Competition* *Select chapter competitors (optional at this time)*		**February 2020** Practice Competition: 2019 School Competition Practice Competition: 2019 Chapter Competition *Select chapter competitors (required by this time)* *2020 MATHCOUNTS Chapter Competition*	

FINISHED ALL THE PROBLEMS IN THE HARD-COPY HANDBOOK? FIND 50 MORE CHALLENGING PROBLEMS AT WWW.MATHCOUNTS.ORG/COACHES

You'll notice that in January or February you'll need to select the 1-10 student(s) who will represent your school at the Chapter Competition. This must be done before the start of your local Chapter Competition. You'll submit the names of your chapter competitors either online at **www.mathcounts.org/coaches** or directly to your local Chapter Coordinator.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. You and your Mathletes can sign up for free at mathcounts.nextthought.com.

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work individually on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new Problem of the Week, due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to Problem of the Week in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

WHAT SHOULD MY TEAM PRACTICES LOOK LIKE? Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Use the **Problem of the Week** (posted online every Monday)
- Practice working in groups to develop teamwork (and to prepare for the Team Round)
- Practice oral presentations to reinforce understanding

On the following page is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

MATHCOUNTS Team Practice Sample Agenda – 1 Hour

Review Problem of the Week (20 minutes)

- Have 1 student come to the board to show how s/he solved the first part of the problem.
- Discuss as a group other strategies to solve the problem (and help if student answers incorrectly).
- Have students divide into groups of 4 to discuss the solutions to the remaining parts of the problem.
- Have 2 groups share answers and explain their solutions.

Timed Practice with Warm-Up (15 minutes)

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

Play Game to Review Warm-Up Answers (25 minutes)

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 45 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 45 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.

OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN? Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
  1. posting intriguing math questions (specific to your school) and referring students to the first meeting for answers.
  2. designing a bulletin board or display case with your MATHCOUNTS poster (included in your School Competition Kit) and/or photos and awards from past years.
  3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
  4. adding information about the MATHCOUNTS team to your school's website.
  5. making a presentation at the first pep rally or student assembly.

Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at info@mathcounts.org.

COACH RESOURCES: WWW.MATHCOUNTS.ORG/COACHES

2019-2020 HANDBOOK MATERIALS

Thank you for being a coach in the MATHCOUNTS Competition Series this year!
We hope participating in the program is meaningful and enriching for you and your Mathletes.
Don't forget to log in at **www.mathcounts.org/coaches** for additional resources!

WHAT'S IN THIS YEAR'S HANDBOOK

COACHES:

FIND PROBLEMS, ANSWERS, SOLUTIONS + PROBLEM INDEX FOR #201–250 ONLINE AT WWW.MATHCOUNTS.ORG/COACHES

HIGHLIGHTED RESOURCES

ACCESS MORE RESOURCES AT WWW.MATHCOUNTS.ORG/COACHES

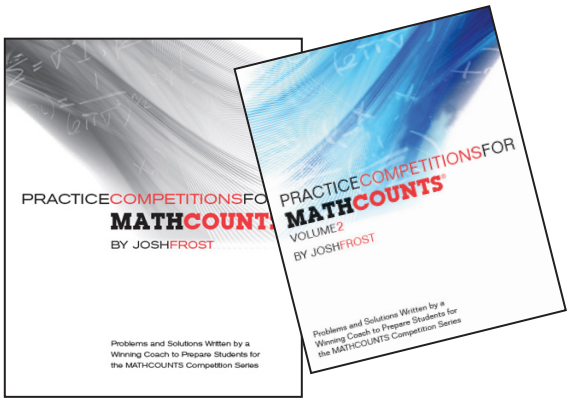
MATHCOUNTS OPLET

The Online Problem Library & Extraction Tool (OPLET) is a database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!

SAVE $25 WHEN YOU BUY YOUR SUBSCRIPTION BY OCT. 1, 2019

www.mathcounts.org/myoplet

PRACTICE COMPETITIONS FOR MATHCOUNTS, VOL. I & II



Practice books written by repeat national-level coach Josh Frost. Each volume includes 4 complete mock-competitions plus solutions.

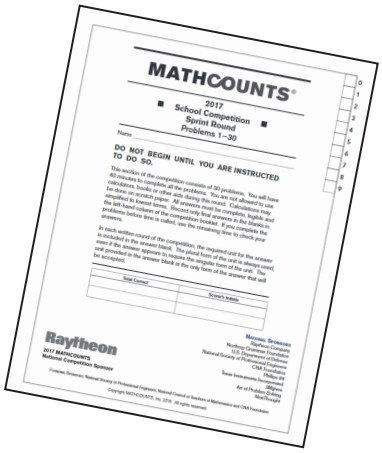
www.mathcounts.org/store

PAST COMPETITIONS

Last year's school, chapter and state competitions are **free** online! Other years' competitions can be purchased.

www.mathcounts.org/pastcompetitions

www.mathcounts.org/store



CRITICAL 2019-2020 DATES

2019

Jun. 17 – Dec. 1

Submit your school's registration to participate in the Competition Series and receive this year's School Competition Kit, which includes a hard copy of the *2019-2020 MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and December 31.

The fastest way to register is online at **www.mathcounts.org/compreg**. You also can download the MATHCOUNTS Competition Series Registration form and mail or email it with payment to:

MATHCOUNTS Foundation – Competition Series Registrations
1420 King Street, Alexandria, VA 22314
Email: reg@mathcounts.org

To add students to your school's registration, log in at **www.mathcounts.org/coaches** to access the Dashboard. **Questions?** Email the MATHCOUNTS national office at info@mathcounts.org.

Nov. 1

The 2020 School Competition will be available online. All registered coaches can log in at www.mathcounts.org/coaches to download the competition.

Oct. 1 (postmark)

Deadline to register for the Competition Series at reduced registration rates ($30 per student, $300 for full registration of 10 students). After October 1, registration rates will be $35 per student, $350 for full registration.

Dec. 1 (postmark)

Competition Series Registration Deadline
In some circumstances, late registrations might be accepted at the discretion of MATHCOUNTS and the local coordinator. *Late fees will apply. Register on-time to ensure your students' participation.*

2020

Early Jan.

If you have not been contacted with details about your upcoming competition, call your local or state coordinator. Coordinator contact information is available at www.mathcounts.org/findmycoordinator.

Late Jan.

If you have not received your School Competition Kit, contact the MATHCOUNTS national office at info@mathcounts.org.

Feb. 1-29

Chapter Competitions

March 1-31

State Competitions

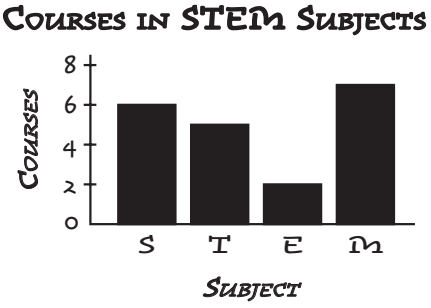
May 10-11

2020 Raytheon MATHCOUNTS National Competition in Orlando, FL

Ratios Stretch

Definition

A ***ratio*** is the comparison of two quantities by division.



This graph shows the number of courses in **S**cience, **T**echnology, **E**ngineering and **M**ath offered at a particular school. According to the graph, the ratio of Engineering to Technology courses is 2 to 5, also written E:T = 2:5. The ratio E:T is a comparison of two of the four "parts" that combine to make up the "whole" group of STEM courses. Some ratios compare a "part" to the "whole." Ratios are often written in the form of a fraction, decimal or percent. According to the graph, this school offers a total of 20 STEM courses. So, the ratio of Math courses to STEM courses is 7 to 20. In other words, Math courses account for $\frac{7}{20} = 0.35 = 35\%$ of the STEM courses at this school.

1. _____ Dave's digital library contains 25 fiction books and 15 nonfiction books. What is the ratio of nonfiction books to fiction books in Dave's digital library? Express your answer as a common fraction.

2. _____ For a 30-60-90 right triangle, what is the ratio of the length of the longer leg to the length of the hypotenuse? Express your answer as a common fraction in simplest radical form.

3. _____ A jar contains seven blue marbles and eight green marbles. Ming adds four yellow marbles and five blue marbles to the jar. What is the ratio of green marbles to non-green marbles in the jar? Express your answer as a common fraction.

Fairy Godmother has granted wishes to Aurora, Belle and Cindi in the ratio 6:8:11. Use this information to solve problems 4 through 7.

4. _____ What fraction of the wishes were granted to Belle? Express your answer as a common fraction.

5. _____ % What percent of the wishes granted by Fairy Godmother were *not* granted to Aurora?

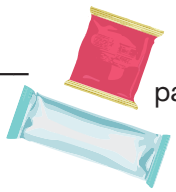
6. _____ % What is the absolute difference between the percents of wishes Fairy Godmother has granted to Aurora and to Cindi?

7. _____ wishes Fairy Godmother has granted at least 20 wishes each to Aurora, Belle and Cindi. What is the least possible number of wishes that she has granted to Cindi?

8. _____ % Jennie put 38 mL of water in a cylinder with a capacity of 60 mL. If she increases the volume of water in the cylinder by 50%, what percent of the cylinder will contain water?

9. _____ % A shop owner increased the price of a jacket by 17%. What percent of the new price is the original price of the jacket? Express your answer to the nearest tenth.

10. _____ New packaging for fruit snacks contains 10% less weight than the original packaging. If the new package costs 15% more than the original package, by what fraction did the unit price increase? Express your answer as a common fraction.



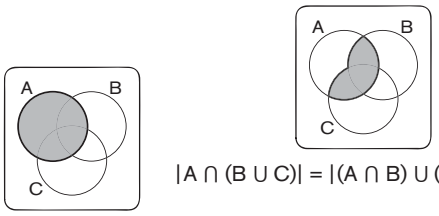
Venn Diagrams Stretch

Set Theory Review

A **set** is a collection of objects or elements, called **members**. Consider two sets A and B.

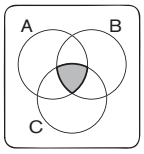
- The **intersection** of A and B, denoted $A \cap B$, is the set of elements that are in both A and B.
- The **union** of A and B, denoted $A \cup B$, is the set of elements in A or in B or in both.
- The **universal set U** is the set of all possible elements.
- The **relative complement** of A, denoted $B \setminus A$, is the set of all elements in B but not in A.
- The **complement** of A, denoted A', is the set of all elements not in A, in other words, $U \setminus A$.
- The **cardinality** of A, denoted $|A|$, is the number of elements in set A.

A **Venn diagram** is a useful tool for comparison. It helps us visualize the relationships between two or more sets. The Venn diagrams shown compare sets A, B and C.

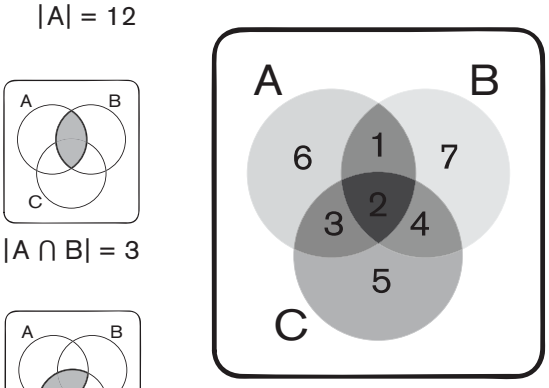


$|A \cap (B \cup C)| = |(A \cap B) \cup (A \cap C)| = 6$

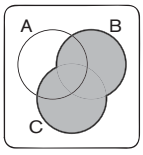
$|A| = 12$



$|A \cap B \cap C| = 2$

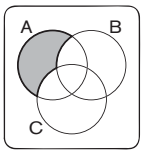


$|A \cap B| = 3$

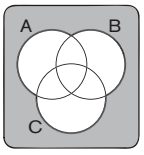


$|B \cup C| = 22$

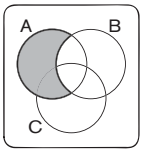
$|A \cap C| = 5$



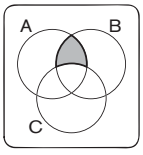
$|A \setminus B \setminus C| = 6$



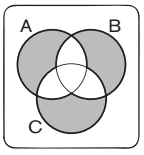
$(A \cup B \cup C)'$



$|A \setminus B| = 9$

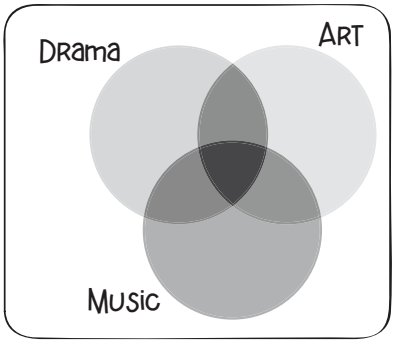


$|(A \cap B) \setminus C| = 1$



$|(A \setminus B \setminus C) \cup (B \setminus A \setminus C) \cup (C \setminus A \setminus B)| = 18$

At Mesa Performing Arts Center, 30 students take courses in one or more of the drama, music and art departments. Five students take courses in exactly one department. Of these students, twice as many take drama courses as take music courses. Five students take courses in exactly two departments. Of these students, twice as many take drama and music courses as take music and art courses. Use the provided Venn diagram to organize this information, and then answer questions 11 through 13.

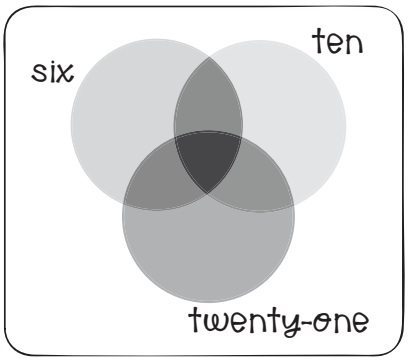


11. _____ students How many students take only art courses?

12. _____ students How many students take courses in all three departments?

13. _____ students How many students take courses in art or music but not both?

The integers from 1 to 630, inclusive, are tested for divisibility by 6, 10 and 21. Use the provided Venn diagram to help determine the cardinality of various sets that contain multiples of 6, 10 and 21, and then answer questions 14 through 16.

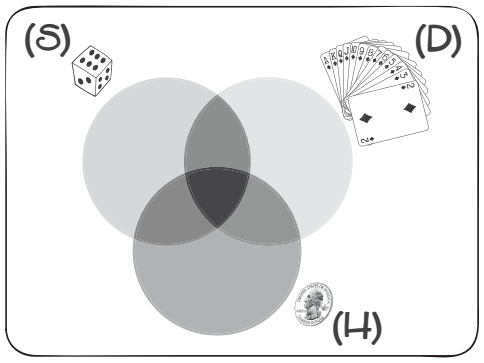


14. _____ integers How many of these integers are divisible by both 6 and 10 but not by 21?

15. _____ integers How many of these integers are divisible by 6 but not by either 10 or 21?

16. _____ integers How many of these integers are not divisible by any of 6, 10 or 21?

A fair coin is flipped, a standard six-sided die is rolled and a card is randomly selected from a standard deck of 52 playing cards. The Venn diagram shown can be used to organize the numbers of outcomes that include flipping heads (H), rolling a 6 (S) and/or selecting a diamond card (D). Use your answers to questions 17 through 19 to fill in this diagram.



17. _____ ways How many ways are there to flip heads, roll a 6 and select a diamond card?

Hint: This is the value of $|H \cap S \cap D|$.

18. _____ ways Since there are $1 \times 1 \times 52 = 52$ ways to roll a 6 and flip heads, how many ways are there to roll a 6 and flip heads but not select a diamond card?

Hint: Use $|S \cap H| = 52$ to find the value of $|(S \cap H) \setminus D|$.

19. _____ ways Since there are $1 \times 6 \times 52 = 312$ ways to flips heads and $13 \times 1 \times 6 = 78$ ways to select a diamond card and flip heads, how many ways are there to flip heads but not select a diamond card and not roll a 6?

Hint: To start, use $|D \cap H| = 78$ to find the value of $|(D \cap H) \setminus S|$. Remember that $|H| = 312$.

20. _____ What is the probability of rolling a 6 and not flipping heads ***or*** flipping heads and not selecting a diamond? Express your answer as a common fraction.

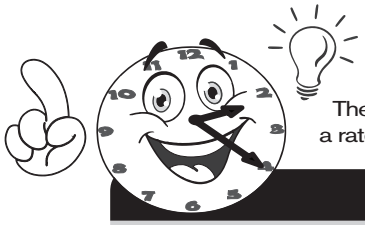
Clocks Stretch

Problems in this stretch involve 12-hour digital and analog clocks, and all time answers should be expressed to the nearest minute, unless otherwise stated.

21. ____ : ____ a.m. What time will it be 47 minutes after 7:37 a.m.?

22. ____ : ____ a.m. What time was it 43 minutes before 9:32 a.m.?

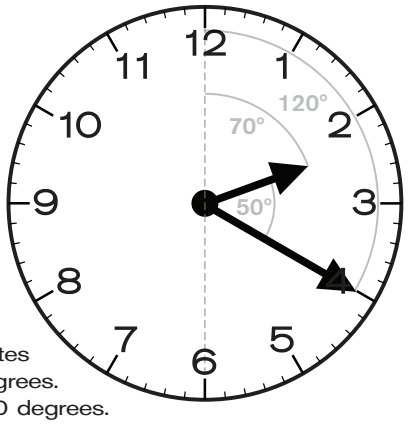
23. ____ seconds A certain clock sounds one chime at 1 o'clock, two chimes at 2 o'clock, three chimes at 3 o'clock, and so on. If this clock behaves in this manner every hour, on the hour so that each chime lasts one second and there is a one-second pause between consecutive chimes, how many seconds long are the chimes that sound at 11 o'clock?



The angle formed by the hour hand and minute hand changes (±) at a rate of $6 - \frac{1}{2} = 11\frac{1}{2} = 5\frac{1}{2}$ degrees each minute.

Rates of Rotation

Minute Hand	Hour Hand
$\frac{360 \text{ degrees}}{1 \text{ hour}} = \frac{6 \text{ degrees}}{1 \text{ minute}} = \frac{1 \text{ degree}}{\frac{1}{6} \text{ minute}}$	$\frac{30 \text{ degrees}}{1 \text{ hour}} = \frac{\frac{1}{2} \text{ degree}}{1 \text{ minute}} = \frac{1 \text{ degree}}{2 \text{ minutes}}$



In the example shown, the time is 2:20. In 2 hours 20 minutes, the hour hand rotates clockwise 70 degrees. In 20 minutes, the minute hand rotates clockwise 120 degrees. The degree measure of the acute angle formed by the clock hands is 120 − 70 = 50 degrees.

24. ____ degrees What is the degree measure of the acute angle formed by the hour and minute hands at 2:16?

25. ____ degrees What is the absolute difference in the degree measures of the smaller angles formed by the hour and minute hands at 2:08 and 8:02?

26. ____ : ____ After 5:30, when is the next time that the hour and minute hands are aligned so that the angle formed measures 0 degrees?

27. ____ minutes After 4 o'clock, how many minutes elapse between the first and second times that the hour and minute hands form a 38-degree angle? Express your answer as a mixed number.

28. ____ minutes After 3:24, how many minutes have elapsed the first time that the angle formed by the hour and minute hands is twice the measure of the angle formed by the hands at 3:24? Express your answer as a mixed number.

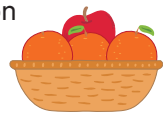
29. ____ What fraction of the times displayed on a digital clock contain the digit 5? Express your answer as a common fraction.

30. ____ times A 24-hour digital clock displays times from 00:00 to 23:59. How many of the times displayed on this clock contain the digit 2?

Warm-Up 1

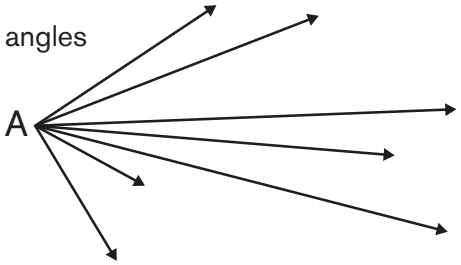
31. _____ What number is $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{8}$ of 60? Express your answer as a common fraction.

32. _____ coins Joanie bought three oranges and an apple for a total of $4.27. She paid with four $1 bills and a 50¢ coin. What is the least number of coins consisting of a combination of pennies, nickels, dimes or quarters that she could receive as change?



33. _____ What common fraction is equivalent to 7.5%?

34. _____ angles Seven rays intersect at point A, as shown. How many angles measuring between 0 and 180 degrees are formed?



35. _____ degrees What is the degree measure of the complement of an angle measuring 67 degrees?

36. $ _____

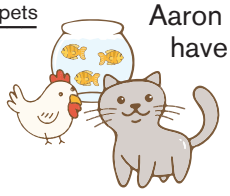
Deposit	$125.42
Withdrawal	$422.21
Deposit	$ 57.43
Withdrawal	$ 23.48

Kaleb has $310.78 in his bank account before the following sequence of withdrawals and deposits. What is the final balance of his account?

37. _____ What is the absolute difference between 999,999,999 and 234,565,432?

38. _____ What is the smallest positive integer that can be multiplied by 9.97 to give a product greater than 100?

39. _____ pets



Aaron and Jackson have a large number of pets: fish, cats and chickens. The pets have a total of 50 legs, 48 eyes and 6 wings. Assuming each cat has 4 legs, each chicken has 2 legs and 2 wings, and each pet has 2 eyes, how many pets do Aaron and Jackson have?

40. _____ What is the sum of the distinct prime factors of 156?

Warm-Up 2

41. _____ Camila's favorite number is positive, and the square of her number plus twice her number is 24. What is her favorite number?

42. _____ inches Janice is 4 inches shorter than David, and David is 2 inches shorter than Evan. If Evan is 13 inches taller than Kathleen and Kathleen is 6 inches shorter than Krysta, by how many inches is Janice taller than Krysta?

43. _____ What is the value of the expression $\left(\frac{8}{9} - \frac{7}{8}\right) \div \left(\frac{6}{7} - \frac{5}{6}\right)$? Express your answer as a common fraction.

44. _____ cm The length of a rectangle is three times its width. The area of the rectangle is 75 cm^2. What is its width?

45. _____ What is the sum of the multiples of 5 from 1 to 100, inclusive?

46. _____ inches This stem-and-leaf plot shows the heights of the basketball players at East High School. If 6 | 8 represents 68 inches, what is the average height of the basketball players at East High School?

6	8 8 9 9
7	1 2 7 7 8 8
8	0 1

47. _____ pages On the tenth day of a read-a-thon, Sarah has read 200 pages and Vivian has read 120. On each subsequent day, Sarah reads 10 pages and Vivian reads 30. Eventually, there is a day when Sarah and Vivian have each read the same number of pages. What is this number of pages?

48. _____ stickers What is the greatest number of 19¢ stickers that can be purchased with a $20 bill, assuming there is no sales tax?

49. _____ units

A single portion of a special concoction is a mixture of 3 units of ingredient A, 5 units of ingredient B and 11 units of ingredient C. If Jaden wants to make 90 portions, how many more units of ingredient C does she need than ingredient B?

50. _____ Gabe rounds 9.456 to the nearest hundredth. Then, he rounds the result to the nearest tenth. Last, he rounds that result to the nearest integer. If Angel rounds 9.456 to the nearest integer, what is the absolute difference between Gabe's and Angel's final results?

Warm-Up 3

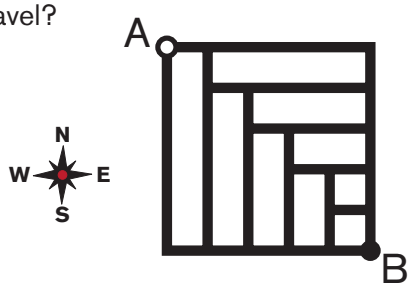
51. $_____ A new pair of shoes is on sale for $45. If this cost represents a 40% discount, what was the original cost of the shoes?

52. _____ The sum of two numbers is 7 and their product is 12. What is twice the sum of their reciprocals? Express your answer as a common fraction.

53. _____ What is the largest prime factor of $8 \times 7 \times 6 \times 5 \times 4 \times 3 - 6 \times 5 \times 4 \times 3 \times 2 \times 1$?

54. $_____ The price of a pizza is directly proportional to its area. If a 12-inch pizza costs $12.00, how much does a 14-inch pizza cost? Express your answer to the nearest dollar.

55. _____ ways A town has some strange streets that are one-way only between the hours of 7:00 a.m. and 8:00 a.m. They run either south or east during that hour. If Joe wishes to go from A to B at 7:20 a.m., how many different ways can he travel?



56. _____ If $\frac{3}{x-1} = \frac{2}{x+7}$, what is the value of x?

57. _____ Given that a, b and c are positive integers, and $abc = 210$, what is the least possible sum for $a + b + c$?

58. _____ students The students from Regent Middle School sat in 20 rows in such a way that each row after the first had 3 more students sitting in it than the previous row. If 16 students sat in the first row, how many students sat in the 19th row?

59. _____ units^2 What is the area of the polygon whose vertices are (0, 0), (3, 4), (0, 8), (5, 8) and (5, 0), connected in that order?

60. _____ Andrew and Lindsey are seated at a round table with four other people. If everyone is randomly seated, what is the probability that Andrew and Lindsey are seated next to each other? Express your answer as a common fraction.

Warm-Up 4

61. _____ What is the value of $1 - 2 \div 3 \times 4 + 5$? Express your answer as a common fraction.

62. _____ ways In how many ways can the letters A, B, C and D be arranged so that no letter is adjacent to any letter that comes immediately before it or immediately after it alphabetically?

63. _____ degrees In quadrilateral ABCD, the measure of angle A is half the sum of the measures of the other angles. What is the measure of angle A? Express your answer to the nearest integer.

64. _____ years Right now, Christina is twice as old as Mark. In five years, she will be 50% older than him. How many years old is Christina now?

65. _____ units^2 What is the area of triangle XYZ with vertices X(0, 4), Y(0, 0) and Z(3, 0)?

66. _____ What is the sum of the expression $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$? Express your answer as a common fraction.

67. _____ Ben randomly draws 2 cards, without replacement, from a standard 52-card deck of playing cards. What is the probability that Ben draws 2 cards of the same rank (two, three, …, queen, king, ace)? Express your answer as a common fraction.

68. _____ pounds  A stone is a British unit of weight equivalent to 14 pounds. After losing $1\frac{1}{2}$ stone, Jim is now 85% of his original weight. What is Jim's current weight, in pounds?

69. _____ Robin is thinking of a number. She asks, "If 29 added to 19 times my number equals 732, what is my number?"

70. _____ vertices Euler's formula states that for any convex polyhedron, there are exactly 2 fewer edges than the number of vertices and faces combined. The snub cube is a convex polyhedron with 38 faces and 60 edges. How many vertices does the snub cube have?

Warm-Up 5

71. _____ What is the value of $\left(2 + 3 + 4\right) \times \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)$? Express your answer as a common fraction.

72. _____ What is the least possible product of two-digit numbers AB and CD, where A, B, C and D represent distinct nonzero digits?

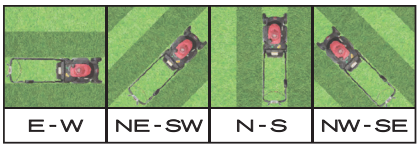
73. _____ What is the value of $\frac{2020^2 - 2012^2}{2017^2 - 2015^2}$?

74. _____ What is the absolute difference between one-half of 5 and one-third of 5? Express your answer as a common fraction.

75. _____ Let x be a positive number such that the distance between x and -3.8 on a number line is exactly two times the distance between x and 1.7. What is the value of x? Express your answer as a decimal to the nearest tenth.

76. _____ units^3 How many cubic units are in the volume of a cube with side length 12 units?

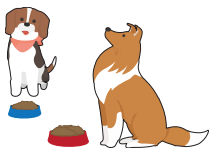
77. _____ A rectangular lawn is mowed every five days. On each of these days, the lawn is mowed in one direction, but the direction varies from one mowing to the next in the following order: East-West (E-W), Northeast-Southwest (NE-SW), North-South (N-S) and Northwest-Southeast (NW-SE). If the lawn is first mowed E-W on Day 1, on Day 201 in which direction is the lawn mowed, E-W, NE-SW, N-S or NW-SE?



78. _____ pages Alastair's favorite book series has seven books containing 223, 251, 317, 636, 766, 607 and 607 pages. Montana's favorite book series has seven books containing 309, 341, 435, 734, 870, 652 and 759 pages. What is the absolute difference between the median numbers of pages in the books of Alastair's and Montana's favorite book series?

79. _____ : _____ Sonja can write three book reviews per hour before 10:00 p.m., and she can write one and a half reviews per hour between 10:00 p.m. and midnight. If Sonja starts writing reviews at 7:30 p.m., at what time will she finish writing the tenth book review?

80. _____ cups Logan feeds her collie 3 cups of dog food each day, and she feeds her beagle 12 cups of dog food each week. If Logan buys a bag of dog food containing 75 cups and feeds both dogs for two weeks, how many cups of dog food will be left in the bag?



Warm-Up 6

81. _____ What is the absolute difference between the two values of x for which $|x - 9| = 8$?

82. _____ What is the value of the expression $\frac{8! - 7 \times 7! + 6 \times 6!}{7! - 6 \times 6! + 5 \times 5!}$? Express your answer as a common fraction.

83. _____ questions On a certain standardized test with 50 problems, 5 points were awarded for each correct answer, and 1 point was deducted for each incorrect answer. Alex answered all the questions on the test and scored a total of 184 points. How many questions did he answer correctly?

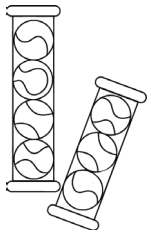
84. _____ diagonals How many diagonals can be drawn in a decagon?

85. _____ What is the value of k in the arithmetic sequence $-13, -7, -1, k, 11$?

86. _____ If $d = 5 + c \times (7 + c \times (4 + c)) - (4 + c \times (4 + c))$, what is the value of d when $c = -1$?

87. _____ degrees Convex quadrilateral WXYZ is inscribed in a circle. If $m\angle XYZ = 54$ degrees, what is the degree measure of $\angle XWZ$?

88. _____ tennis balls Sports Depot sells tennis balls in cans of either 3 or 4 tennis balls each. What is the greatest number of tennis balls that Kaycee cannot buy from Sports Depot as an exact combination of these two types of cans?

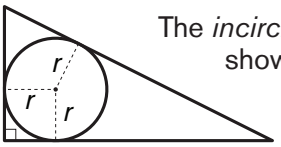


89. _____ How much more than two-thirds of 25% of 36 is 50% of one-fourth of 64?

90. _____ students There are 292 students in a math class, of whom 90 are freshmen, 80 are math majors and 145 are neither freshmen nor math majors. How many students in the class are freshman math majors?

Warm-Up 7

91. _____ What is the value of $127^2 - 14 \times 127 + 49$?

92. _____ mm 

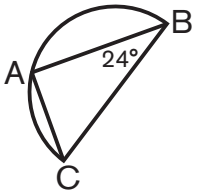
The *incircle* of a triangle is a circle tangent to all three sides of the triangle, as shown in this example. What is the radius of the incircle of a right triangle with side lengths of 12, 35 and 37 mm?

93. _____ integers How many positive integers less than 100 are a multiple of 3 but not a multiple of 5?

94. $ _____ Alta wants to put carpet down in her bedroom, which measures 18 feet by 12 feet. If she uses a carpet that costs $25 per square yard, how much will it cost to put carpet in her entire bedroom?

95. _____ If $\frac{1}{2} + \frac{1}{4} = \frac{1}{3} + \frac{1}{5} + \frac{1}{6} + \frac{1}{n}$, what is the value of n?

96. _____ degrees Triangle ABC is inscribed in a semicircle as shown. If $m\angle ABC = 24$ degrees, what is the degree measure of $\angle BCA$?



97. _____ mi/h A spacecraft traveled 5.0×10^5 miles in 2.0×10^2 hours. What was its average speed in miles per hour? Express your answer in scientific notation to two significant figures.

98. _____ ounces Two-thirds of a 12-ounce bottle is filled with cola. How many ounces of cola remain after Montez drinks one-fourth of the cola in the bottle?

99. _____ dozen If there are 15 strawberries in a bundle and 8 bundles in a crate, how many dozen strawberries are in a crate?

100. _____ mi/h Twins Miley and Riley leave home at the same time, riding their bicycles in opposite directions. Miley goes northbound traveling 18 mi/h. After 1 hour, the twins are 32 miles apart. How fast was Riley traveling in the southbound direction?

Warm-Up 8

101. _____ The greatest common factor of 42 and 24 is m. What is the least common multiple of m and 15?

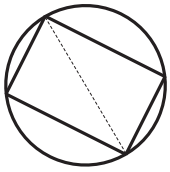
102. _____ degrees Circle P has diameter CD. Point B is on the circle such that $m\angle BPC = 30$ degrees. Point A is on the circle such that $\overline{AD}$ is parallel to $\overline{PB}$. What is the degree measure of arc ABC?

103. _____ units Points A, B, C, D and E are collinear. If AB = 6, B is the midpoint of line segment AD, D is three-fourths the way from A to E and C is one-fifth the way from B to E, what is CE?

104. _____ What integer is closest to the value of $\frac{999}{200} + \frac{898}{301} + \frac{797}{402} + \frac{696}{503}$?

105. _____ What is the value of $\frac{1000^2}{252^2 - 248^2}$?

106. _____ units^2 The figure shows a rectangle inscribed in a circle. If the rectangle has integer sides and integer diagonal lengths, what is the smallest possible area of the rectangle?



107. _____ pennies On March 1st, Kenny and Linny had jars containing the same number of pennies. One month later, Kenny has two more than three times Kenny's original number of pennies, while Linny has seven fewer than four times Linny's original number. If Linny has eight more pennies than Kenny, how many pennies did they each start with on March 1st?

108. _____ minutes A train departs D.C. at 8:15 a.m. EST and arrives in New York City at 12:05 p.m. EST on the same day. How many minutes was the train ride?

109. _____ If $(x^2y^3)^4(x^4y^5)^6 = x^ay^b$ for all real numbers x and y, what is the sum of a and b?

110. _____ What is the value of 11,111,111 – 2,222,222 + 333,333 – 44,444 + 5555 – 666 + 77 – 8?

Warm-Up 9

111. _____ years Now that it is a year after Jake joined the National Math Club, he has been a member for one-third of the time that Maggie has been a member. In how many more years will Jake have been a member for two-thirds of the time that Maggie will have been?

112. _____ cm^2 Kent draws a regular hexagon of side length 4 cm and then draws a semicircle outward along each side. The total area enclosed by Kent's drawing can be expressed in simplest radical form, in terms of π, as $a\sqrt{b} + c\pi$. What is the value of $\frac{a}{b} + c$?

113. _____ minutes Sandy rakes and bags the leaves on her front lawn in 30 minutes. Randy does the same job in 20 minutes. How many fewer minutes will it take to complete the same job if Sandy and Randy work together than if Sandy works alone?

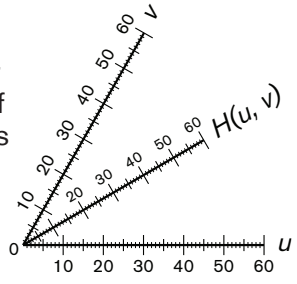
114. _____ $units^2$ What is the area of an isosceles triangle that has a base of length 12 units and base angles measuring 30 degrees? Express your answer in simplest radical form.

115. _____ If $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{n} + \frac{1}{62} + \frac{1}{124} + \frac{1}{248} = \frac{495}{496}$, what is the value of n?

116. _____ If a and b are integers such that $a^2 = b^2 + 15$, what is the greatest possible value of a?

117. _____ If $\frac{18}{x} = \frac{y}{28} = \frac{3}{7}$, what is the value of $\frac{x}{y}$? Express your answer as a common fraction.

118. _____ A *nomogram* is a graphical calculating device that relates three or more variables. The nomogram shown relates u, v and the value of the function $H(u, v)$. A straight line passing through specific values of u and v intersects the third scale at the value of the function H for those two variables. To the nearest multiple of 5, what is the value of $H(60, 36)$?



119. _____ In the array of numbers shown, each row begins and ends with the number 1. In every row after the first, each interior entry is the sum of the number immediately above it, the number above and to the left (if there is one) and the number above and to the right (if there is one). What is the value of $p + q + r$?

```
                1
            1   1   1
        1   2   3   2   1
    1   3   6   7   6   3   1
1   4  10  p   q   r  10   4   1
```

120. _____ What is the value of $\left(\frac{1}{2}\right)^{-3}$?

Warm-Up 10

121. _____ feet When the sun was at a particular place in the sky, a 40-foot tall vertical tree cast a 60-foot long shadow on the ground. Two years later, the tree had grown to 60 feet tall. When the sun was again at that particular place in the sky, how many feet longer was its shadow?

122. _____ What is the absolute difference between the median and mean of $\frac{5}{8}$, $\frac{3}{5}$ and $\frac{13}{20}$?

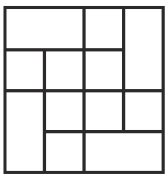
123. _____ ft^2 Spot is on a 9-foot leash that is attached to an outside corner of his doghouse. If Spot's doghouse is a regular hexagon of side length 9 feet, how many square feet is the entire area that Spot can reach outside of the doghouse? Express your answer in terms of π.

124. _____ times A path drawn in the coordinate plane begins at (1, 1), then goes up 2 units, right 2 units, up 3 units, right 3 units, up 4 units, right 4 units, and so on. Each time, the number of units up increases by 1, and the number of units right increases by 1 until the path ends at (91, 91). How many times does this path from (1, 1) to (91, 91) intersect the line $y = x$?

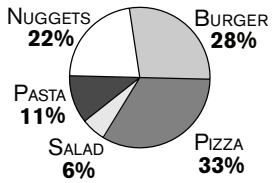
125. _____ Filip expresses a number in the form $4x + 5$. Gala expresses the same number as $6x - 7$. If Henry wants to express this number in the form $5x + d$, what would be the value of d?

126. _____ cm The length of the base of a particular triangle is 2 cm more than its height. If the triangle has area 12 cm^2, what is its height?

127. (,) The point (5, 12) is rotated 90 degrees counterclockwise about the origin. What are the coordinates of its image? Express your answer as an ordered pair.

128. _____ rectangles  How many rectangles of any size are in the figure shown?

STUDENT LUNCH ORDERS

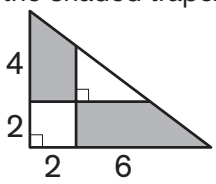


129. _____ students The circle graph shows what percent of the students who bought lunch on Friday ordered each of the five lunch options available at Lowe HS. If 132 students ordered pizza, what is the total number of students who bought lunch at Lowe HS on Friday?

130. _____ If $a \textcircled{n} b = \frac{a^n + b^n}{n}$, what is the value of $5 \textcircled{2} (1 \textcircled{3} 2)$?

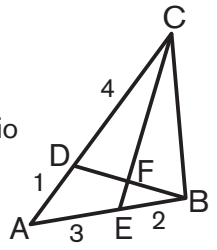
Warm-Up 11

131. _____ in^2 The right triangle shown has side lengths 6, 8 and 10 inches. In the interior of the triangle, a line is drawn parallel to each leg so that the distance between each parallel line and the corresponding leg is 2 inches. What is the combined area of the shaded trapezoidal regions? Express your answer as a common fraction.



132. _____ cm^2 What is the maximum possible area of a rectangle with a diagonal of length 7 cm? Express your answer as a common fraction.

133. _____ In triangle ABC shown, AE:EB = 3:2 and AD:DC = 1:4. What is the ratio of EF to FC? Express your answer as a common fraction.

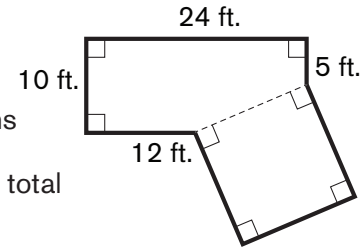


134. _____ chips Konstantin gives Adrian a drawstring bag and tells him that it contains at least one chip having each of the masses 5, 11 and 19 grams. If the total mass of the chips in the bag is 56 grams, how many chips are in the bag?

135. _____ prisms What is the number of solid 3-inch by 2-inch by 2-inch rectangular prisms that can be arranged to form a solid cube of edge length 1 foot?

136. _____ What is the remainder when the absolute difference between 4 × 199 and 7× 219 is divided by 3?

137. _____ ft^2 The figure is the floor plan of a seven-sided room. Side lengths and right angles of the room are as shown. The dashed line separates the floor into a pentagon and a square. What is the total area of the floor in the room?



138. _____ If $\frac{x}{y} + \frac{4}{3} + \frac{9}{2} = \frac{x}{y} \times \frac{4}{3} \times \frac{9}{2}$, for positive integers x and y, what is the least possible value of $x + y$?

139. _____ There is a prime number p with $200 < p < 220$. What is the value of p?

140. _____ When the three-digit number ABC is multiplied by 9, the result is the four-digit number 1ABC. What is the three-digit number ABC?

Workout 1

141. _____ The movies in the *Revengers* series have been coming out every three years. The sum of the years in which the first six movies came out is 12,057. In what year is the seventh movie expected to come out?

142. _____ % Claire purchased 36 pounds of cashews for $56.25. She divides the cashews into smaller bags, each containing $\frac{3}{8}$ pound. If she sells each small bag of cashews for 79 cents, what percent of the money that she collects will be profit? Express your answer to the nearest tenth.

143. _____ minutes Jane is traveling on an interstate highway where signs that are numbered consecutively starting with 1 are posted at every mile. If Jane is traveling at a speed of 72 mi/h, how many minutes will it take her to travel from mile marker 7 to mile marker 29? Express your answer as a decimal to the nearest tenth.

144. _____ Jersey cows One-third of Farmer Gene's cows are Guernseys, two-fifths are Holsteins and the rest are Jerseys. How many Jersey cows are in his herd of 90 cows?

145. _____ What is 77% of 90? Express your answer as a decimal to the nearest tenth.

146. _____ feet For his dog, Ramon builds a rectangular pen with dimensions 10 feet by x feet. Heidi builds a circular pen of radius 5 feet for her dog. If the two pens have the same area, what is the value of x? Express your answer as a decimal to the nearest tenth.

147. _____ sixth graders A MATHCOUNTS club has a total of 27 sixth- and seventh-grade members. If there are 25% more seventh graders than sixth graders, how many sixth graders are in the club?

148. _____ gallons Ana's doctor advises her to drink eight 8-ounce glasses of water each day. If a gallon contains 128 ounces, how many gallons of water should Ana drink in four weeks?

149. _____ hours How many hours are in 210 minutes? Express your answer as a decimal to the nearest tenth.

150. _____ What is the sum of the unique prime factors of 9991?

Workout 2

151. _____ What is the sum of two consecutive positive integers whose squares differ by 45?

152. _____ seconds If light from the sun travels at 186,282 mi/s, how many seconds does it take that light to reach Earth from the sun when they are 93.141 million miles apart?

153. _____ minutes

Balloon Altitude (feet)

Time	Balloon #1	Balloon #2
12:00	3600	4000
12:02	3630	3980
12:04	3660	3960
12:06	3690	3940

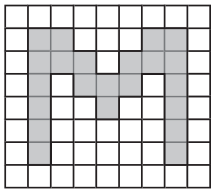
At noon, one hot-air balloon is rising as another is falling, each at a constant rate. The table shows the altitude of each balloon at various times. If the balloons continue to rise and fall at these constant rates, how many minutes after noon will the two balloons be at the same altitude?

154. _____ What is the value of $\frac{7}{10}$ of 7%? Express your answer as a decimal to the nearest thousandth.

155. _____ The sum of three distinct primes is 40. What is their product?

156. _____ meters A caterpillar travels 4.8 mm in one second. At this rate, how many meters will it travel in one hour? Express your answer as a decimal to the nearest tenth.

157. _____ In the 9 × 8 grid of unit squares shown, what is the ratio of the shaded area to the unshaded area? Express your answer as a decimal to the nearest hundredth.



158. _____ Kyle subtracts a number from 27 and then multiplies the result by 8. If the product equals the number Kyle first subtracted, what is that number?

159. _____ students $1 \diamond 2 \diamond 3 \diamond 4$ How many unique values result when each $\diamond$ is replaced by either + or × in the expression shown?

160. _____ The fourth term of an arithmetic sequence is 3, and the ninth term is 23. What is the first term of this sequence?

Workout 3

161. _____ What is the probability of getting exactly 6 correct answers on a 10-question true-false test by randomly guessing? Express your answer as a common fraction.

162. _____ inches Bradlee draws a rectangle of width 160 inches and height 70 inches. Bianca draws a rectangle whose width is 50 inches larger than Bradlee's, and whose area is 50% larger than Bradlee's. What is the height of Bianca's rectangle?

163. _____ A *Pythagorean Triple* is an ordered triple of positive integers (a, b, c) satisfying the equation $a^2 + b^2 = c^2$. If (x, 28, 53), (13, y, 85) and (z, 20, 29) are Pythagorean Triples, what is the greatest prime factor of $x + y + z$?

164. _____ ft^2 Farmer John owns a circular plot of land with a radius of 300 feet. He then fences off a square region that has vertices on the boundary of the circular plot. How many square feet of land lie inside the circular plot but outside the square region? Express your answer to the nearest thousand.

165. _____ The *odds* of success are a ratio of the form A:B, where A is the probability of a successful outcome and B is the probability of an unsuccessful outcome. If the odds of winning the game Galaxy Quest are 3:4, what is the probability of losing this game? Express your answer as a decimal to the nearest hundredth.

166. _____ % In what percent of the positive two-digit integers is the tens digit at least five more than the units digit? Express your answer to the nearest tenth.

167. _____ In November, Tiffany proofread books at a rate of 10 pages per hour. She charged $24 per hour for this service. By January, she was able to proofread an additional 2 pages per hour, so she increased her hourly rate by $3 per hour. If Tiffany earned x dollars per page proofread in November and y dollars per page in January, what is the ratio of x to y? Express your answer as a common fraction.

168. _____ pounds Ellie has two dogs, a bulldog and a golden retriever. The bulldog's weight is 60% of the golden retriever's weight. If the golden retriever weighs 28 pounds more than the bulldog, what is the combined weight of Ellie's two dogs?

169. _____ sides If the sum of the measures of the interior angles of a polygon is 1620 degrees, how many sides does the polygon have?

170. _____ pants Tanner has four times as many shirts as pants, and he can use them to make 100 different shirt-and-pants outfits. How many pants does Tanner have?

Workout 4

171. _____ bats There are 270 bats living in a cave on the first day of June. Every day in June, some number of bats arrive to join the bats already there. At the end of June 30, there are 1000 bats in the cave. On average, how many bats arrived each day in June? Express your answer as a decimal to the nearest tenth.

172. _____ % When first cut from a log, a wooden 2-by-4 measures 2 inches by 4 inches by 8 feet. This "rough-cut" 2-by-4 is run through a planer to make it smooth. By the time it has completed this "milling process," it measures 1.5 inches by 3.5 inches by 8 feet. At that point, what percentage of the wood is remaining? Express your answer to the nearest whole number.

173. _____ inches A 9-inch-diameter cherry pie is cut into six congruent slices. What is the perimeter of one slice? Express your answer as a decimal to the nearest tenth of an inch.

174. _____ °F Absolute zero is the lowest possible temperature. It is equivalent to −459.67 °F on the Fahrenheit temperature scale. In degrees Fahrenheit, how many degrees is absolute zero below the freezing point of water, 32 °F? Express your answer as a decimal to the nearest hundredth.

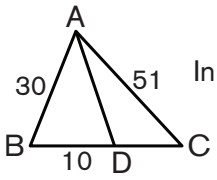
175. $ _____ The price of a pair of shoes is reduced by 15%. Then its price is raised by 25%. The new price is $10 higher than the original price. What was the original price?

176. $ _____ If 4 oranges cost $1.08, how much will 11 oranges cost?

177. _____ minutes Bob and Sam each are going to drive from Lincoln to Denver, a driving distance of 488.5 miles. Bob leaves at 6:00 a.m., traveling at an average speed of 75 mi/h. Sam leaves at 8:30 a.m., traveling at an average speed of 60 mi/h. How many minutes after Bob arrives in Denver will Sam arrive? Express your answer as a decimal to the nearest tenth.

178. _____ Let $A = \frac{3}{4} \times 50\% \times 200$, $B = 0.125 \times \frac{7}{5} \times 80$ and $C = 0.1\overline{6} \times 240 \times 25\%$. What is the value of $A - B - C$?

179. _____ units



In triangle ABC, shown here, segment AD bisects $\angle A$, AB = 30, BD = 10 and AC = 51. How many units long is side BC?

180. _____ The *strain* in a material is the quotient when the change in its length is divided by its original length. If hanging a weight from a 1-foot length of rope stretches the rope to a length of 1 foot 3 inches, what is the strain in the rope? Express your answer as a decimal to the nearest hundredth.

Workout 5

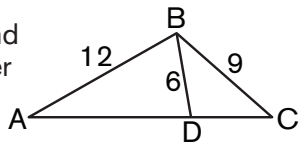
181. _____ seats It is the policy of Sky Airlines to cancel any flight that has fewer than 80% of its seats filled. A Sky Airlines plane has 24 rows with 6 seats per row. What is the maximum number of empty seats on a flight that is not canceled?

182. _____ Using only the eight digits 0, 3, 4, 6, 6, 7, 8 and 8, Erika forms two four-digit numbers whose product is as large as possible. What is the sum of Erika's numbers?

183. _____ inches Kathleen has a pattern to knit a scarf that requires 441 yards of yarn. For this particular pattern, each stitch uses 0.9 inch of yarn, each row contains 41 stitches and each full row of knitting contributes $\frac{1}{5}$ inch of length to the scarf. How many inches long will her scarf be when completed? Express your answer to the nearest whole number.

184. _____ What is the sum of the base-ten numbers from 5 to 50, inclusive, that are palindromes when written in base two?

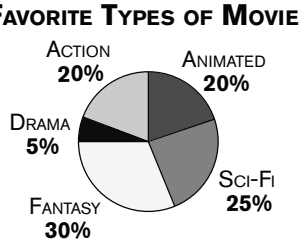
185. _____ units The figure shows triangle ABC with AB = 12, BC = 9, BD = 6 and AD = 2DC. How many units long is side AC? Express your answer as a decimal to the nearest tenth.



186. _____ degrees If the degree measure of an angle's supplement is three times that of its complement, what is the degree measure of the angle?

187. _____ % Mindy and Sunny are hunting for two dozen hidden prizes. Mindy has found one-third of the prizes and Sunny has found 9 fewer than twice the number of prizes that Mindy has found. What percent of the prizes remain hidden? Express your answer as a decimal to the nearest tenth.

188. _____ students The circle graph shows the results of a survey, in which Ms. Prather asked each of her homeroom students to select her or his favorite type of movie from the five genres shown. If all 20 of Ms. Prather's homeroom students responded to the survey, how many students' favorite type of movie is fantasy?



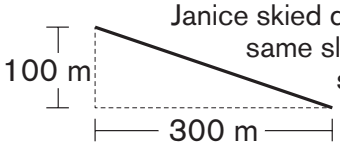
189. _____ ways In how many ways can five people of different heights line up single-file so that there is exactly one person standing immediately behind someone who is taller?

190. _____ Syd doubles the 3-digit number TWO to get the 4-digit number FOUR. Each letter represents a distinct digit, and O represents the digit 7. What is the 3-digit number TWO?

Workout 6

191. _____ If p, q and $p + q$ are all prime numbers and $p^3q^2 + p^2q^3 = 700$, what is the value of pq?

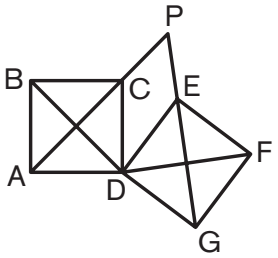
192. _____ m/s Janice skied down the slope shown in 12 seconds. Tanya skied down the same slope 1.3 seconds faster than Janice did. What was Tanya's speed? Express your answer to the nearest whole number.



193. _____ cm What is the greatest distance between any two vertices of a rectangular prism with dimensions 8 cm by 9 cm by 12 cm?

194. _____ Peien rolls four standard six-sided dice. What is the probability that the sum of the numbers rolled on one pair of dice is 4 and the sum of the numbers rolled on the other pair is 10? Express your answer as a common fraction.

195. _____ degrees In the figure shown, ABCD and DEFG are congruent squares, and the measure of angle CDE is 33 degrees. Diagonals AC and GE are extended beyond C and E, respectively, and intersect at P. What is the degree measure of angle CPE?



196. _____ If x is a positive number such that $\frac{x}{18} = \sqrt[3]{x}$, what is the value of x? Express your answer as a decimal to the nearest tenth.

197. _____ hours Donny has two part-time jobs. He earns \$12 per hour tutoring, and he earns \$20 per hour baby-sitting. Donny would like to work a combination of hours at both jobs to earn exactly \$360. If he works at least one hour at each job, with no partial hours worked at either job, what is the difference between the maximum and minimum total hours that Donny can work to earn that amount?

198. _____ Nico's quiz average of 76 is the arithmetic mean of his scores on five quizzes. If Nico's teacher removes his lowest quiz score of 54, what is his adjusted quiz average? Express your answer as a decimal to the nearest tenth.

199. _____ % Three companies each order promotional pens, key rings and cups from the same vendor. Company A orders 372 promotional items, of which one-third are key rings. Company B orders 524 items, of which the combined number of pens and cups is three times the number of key rings. Company C orders 225 items, 72 of which are key rings. What percent of the promotional items ordered by these three companies are key rings? Express your answer to the nearest whole number.

200. _____ Tom randomly selects three different numbers from the set {1, 2, 3, 4}. Jerry randomly selects one number from the set {2, 4, 6, 8, 10, 12}. What is the probability that Jerry's number is greater than the sum of Tom's numbers? Express your answer as a common fraction.

COMPETITION COACH TOOLKIT

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection useful.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}$
$\frac{1}{4}$	0.25	25
$\frac{1}{5}$	0.2	20
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$
$\frac{1}{8}$	0.125	12.5
$\frac{1}{9}$	$0.\overline{1}$	$11.\overline{1}$
$\frac{1}{10}$	0.1	10
$\frac{1}{11}$	$0.\overline{09}$	$9.\overline{09}$
$\frac{1}{12}$	$0.08\overline{3}$	$8.\overline{3}$

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Divisibility Rules

2: units digit is 0, 2, 4, 6 or 8

3: sum of digits is divisible by 3

4: two-digit number formed by tens and units digits is divisible by 4

5: units digit is 0 or 5

6: number is divisible by both 2 and 3

8: three-digit number formed by hundreds, tens and units digits is divisible by 8

9: sum of digits is divisible by 9

10: units digit is 0

Equation of a Line

Standard Form

$Ax + By = C$

Slope-Intercept Form

$y = mx + b$

m = slope $\qquad$ b = y-intercept

Point-Slope Form

$y - y_1 = m(x - x_1)$

m = slope $\qquad$ (x_1, y_1) = point on the line

Distance Traveled

Distance = Rate × Time

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagorean Triples

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(12, 35, 37)

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Sum and Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Circles

Circumference	Area
$2 \times \pi \times r = \pi \times d$	$\pi \times r^2$

For radius r

Arc Length	Sector Area
$\frac{x}{360} \times 2 \times \pi \times r$	$\frac{x}{360} \times \pi \times r^2$

For central angle of x degrees

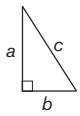
Given A(x_1, y_1) and B(x_2, y_2)

Distance from A to B $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of $\overline{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

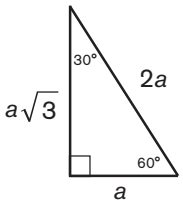
Slope of $\overline{AB} = \frac{y_2 - y_1}{x_2 - x_1}$

Pythagorean Theorem

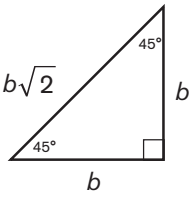


$a^2 + b^2 = c^2$

Special Right Triangles



30-60-90 Right Triangle



45-45-90 Right Triangle

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	$l \times w$
Parallelogram	base b, height h	$b \times h$
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2) \times h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2} \times d_1 \times d_2$
Triangle	base b, height h	$\frac{1}{2} \times b \times h$
Triangle	semi-perimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (n sides)

Sum of the interior angle measures:

$180 \times (n - 2)$

Central angle measure of a regular polygon:

$\frac{360}{n}$

Interior angle measure of a regular polygon:

$\frac{180 \times (n - 2)}{n}$ or $180 - \frac{360}{n}$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6 \times s^2$	s^3
Rectangular Prism	length l, width w, height h	$2 \times (l \times w + w \times h + l \times h)$	$l \times w \times h$
Circular Cylinder	base radius r, height h	$2 \times \pi \times r \times h + 2 \times \pi \times r^2$	$\pi \times r^2 \times h$
Circular Cone	base radius r, height h	$\pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$	$\frac{1}{3} \times \pi \times r^2 \times h$
Sphere	radius r	$4 \times \pi \times r^2$	$\frac{4}{3} \times \pi \times r^3$
Pyramid	base area B, height h		$\frac{1}{3} \times B \times h$

Vocabulary & Terms

The following list is representative of terminology used in the problems but **should not** be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
apex
arithmetic mean
arithmetic sequence
base ten
binary
bisect
box-and-whisker plot
center
chord
circumscribe
coefficient
collinear
common divisor
common factor
common fraction
complementary angles
congruent
convex
coordinate plane/system
coplanar
counting numbers
counting principle
diagonal of a polygon
diagonal of a polyhedron
digit-sum
dilation
direct variation
divisor
domain of a function
edge
equiangular
equidistant
expected value
exponent
exterior angle of a polygon
factor
finite
frequency distribution
frustum
function
GCF (GCD)
geometric sequence
hemisphere
image(s) of a point(s) *(under a transformation)*
improper fraction
infinite series
inscribe
integer
interior angle of a polygon
intersection
inverse variation
irrational number
isosceles
lateral edge
lateral surface area
lattice point(s)
LCM
median of a set of data
median of a triangle
mixed number
mode(s) of a set of data
multiplicative inverse *(reciprocal)*
natural number
obtuse angle
ordered pair
origin
palindrome
parallel
Pascal's Triangle
percent increase/decrease
perpendicular
planar
polyhedron
polynomial
prime factorization
principal square root
proper divisor
proper factor
proper fraction
quadrant
quadrilateral
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
reflection
regular polygon
relatively prime
revolution
right angle
right polyhedron
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
slope
space diagonal
square root
stem-and-leaf plot
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
transformation
translation
triangular numbers
trisect
twin primes
union
unit fraction
variable
whole number
y-intercept

ANSWERS

RATIOS STRETCH
1. 3/5
2. $\sqrt{3}/2$
3. 1/2
4. 8/25
5. 76
6. 20
7. 44
8. 95
9. 85.5
10. 5/18

VENN DIAGRAMS STRETCH
11. 2
12. 20
13. 7
14. 18
15. 72
16. 468
17. 13
18. 39
19. 195
20. 11/24

CLOCKS STRETCH
21. 8:24
22. 8:49
23. 21
24. 28
25. 115
26. 6:33
27. $13\frac{9}{11}$
28. $7\frac{7}{11}$
29. 5/16
30. 630

WARM-UP 1
31. 15/16
32. 5
33. 3/40
34. 21
35. 23
36. 47.94
37. 765,434,567
38. 11
39. 24
40. 18

WARM-UP 2
41. 4
42. 1*
43. 7/12
44. 5
45. 1050
46. 74
47. 240
48. 105
49. 540
50. 1

WARM-UP 3
51. 75 or 75.00
52. 7/6
53. 5
54. 16 or 16.00
55. 10
56. −23
57. 18
58. 70
59. 28
60. 2/5

WARM-UP 4
61. 10/3
62. 2
63. 120
64. 10
65. 6
66. 15/16
67. 1/17
68. 119
69. 37
70. 24

WARM-UP 5
71. 39/4
72. 312
73. 4
74. 5/6
75. 7.2
76. 1728
77. E-W or East-West
78. 45
79. 11:40
80. 9

WARM-UP 6
81. 16
82. 78/11
83. 39
84. 35
85. 5
86. 0
87. 126
88. 5
89. 2
90. 23

WARM-UP 7
91. 14,400
92. 5
93. 27
94. 600 or 600.00
95. 20
96. 66
97. 2.5×10^3
98. 6
99. 10
100. 14

WARM-UP 8
101. 30
102. 60
103. 8
104. 11
105. 500
106. 12
107. 17
108. 230
109. 74
110. 9,182,736

WARM-UP 9
111. 3
112. 20
113. 18
114. $12\sqrt{3}$
115. 31
116. 8
117. 7/2
118. 45
119. 51
120. 8

WARM-UP 10
121. 30
122. 0
123. 54π
124. 13
125. −1
126. 4
127. (−12, 5)
128. 42
129. 400
130. 17

WARM-UP 11
131. 95/6
132. 49/2
133. 1/10
134. 6
135. 144
136. 2
137. 379
138. 13
139. 211
140. 125

WORKOUT 1
141. 2020
142. 25.8
143. 18.3
144. 24
145. 69.3
146. 7.9
147. 12
148. 14
149. 3.5
150. 200

WORKOUT 2
151. 45
152. 500
153. 16
154. 0.049
155. 434
156. 17.3
157. 0.44
158. 24
159. 7
160. −9

WORKOUT 3
161. 105/512
162. 80
163. 5
164. 103,000
165. 0.57
166. 16.7
167. 16/15
168. 112
169. 11
170. 5

WORKOUT 4
171. 24.3
172. 66
173. 13.7
174. 491.67
175. 160 or 160.00
176. 2.97
177. 247.7
178. 51
179. 27
180. 0.25

WORKOUT 5
181. 28
182. 17,403
183. 86
184. 210
185. 17.2
186. 45
187. 37.5
188. 6
189. 26
190. 867

WORKOUT 6
191. 10
192. 30
193. 17
194. 1/24
195. 57
196. 76.4
197. 8
198. 81.5
199. 29
200. 5/12

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

COACHES: FIND PROBLEMS, ANSWERS, SOLUTIONS + PROBLEM INDEX FOR PROBLEMS 201-250 AT WWW.MATHCOUNTS.ORG/COACHES!

PROBLEM INDEX

It is very difficult to categorize many of the problems in this handbook. MATHCOUNTS problems often straddle multiple categories and cover several concepts, but in this index, we have placed each problem in exactly one category and mapped it to exactly one Common Core State Standard (CCSS). **In this index, code 9 (3) 7.SP.3 would refer to problem #9 with difficulty rating 3 mapped to CCSS 7.SP.3.** The difficulty rating and CCSS mapping are explained below.

DIFFICULTY RATING: Our scale is 1-7, with 7 being most difficult. These general ratings are only approximations:

- **1, 2 or 3**: Appropriate for students just starting the middle school curriculum; 1 concept; 1- or 2-step solution.
- **4 or 5**: Knowledge of some middle school topics necessary; 1-2 concepts; multi-step solution.
- **6 or 7**: Knowledge of advanced middle school topics and/or problem-solving strategies necessary; multiple and/or advanced concepts; multi-step solution.

COMMON CORE: We align our problems to the NCTM Standards for Grades 6-8, however we also have mapped these problems to CCSS because 42 states, D.C., 4 territories and the Dept. of Defense Education Activity (DoDEA) have voluntarily adopted it. Our CCSS codes contain (in this order):

1. **Grade level** in the K-8 Standards for Mathematical Content (SMC). Courses that are in the high school SMC instead have the first letter of the course name.
2. **Domain** within the grade level or course and then the **individual standard**.

Here are 2 examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts are not specifically mentioned in CCSS. For problems using these concepts, we use the code of a related standard, when possible. Some of our problems are based on concepts outside the scope of CCSS or are based on concepts in the K-5 SMC but are more difficult than a grade K-5 problem. When appropriate, we coded these problems SMP for the CCSS Standards for Mathematical Practice.

ALGEBRAIC EXPRESSIONS & EQUATIONS

10	(4)	7.RP.1
28	(4)	SMP
52	(3)	SMP
56	(3)	A-REI.2
57	(4)	SMP
64	(3)	7.EE.4
73	(4)	A-SSE.2
75	(3)	6.NS.5
81	(3)	A-REI.11
86	(2)	6.EE.2
91	(3)	A-REI.4
95	(3)	7.EE.4
100	(3)	SMP
107	(4)	6.EE.7
109	(3)	8.EE.1
111	(2)	7.EE.2
115	(3)	7.EE.4
116	(4)	A-SSE.2
125	(4)	6.EE.4
130	(3)	8.F.1
138	(5)	5.NF.2
153	(3)	6.RP.3
167	(3)	6.RP.3
191	(4)	8.EE.2
196	(4)	A-REI.2
197	(4)	A-CED.1

COORDINATE GEOMETRY

59	(4)	6.G.1
65	(4)	7.G.6
124	(3)	5.G.4
127	(4)	8.G.3

PROBABILITY, COUNTING & COMBINATORICS

17	(3)	7.SP.8
18	(3)	7.SP.8
19	(3)	7.SP.8
20	(5)	7.SP.8
29	(4)	SMP
30	(4)	SMP
34	(3)	SMP
55	(2)	SMP
60	(4)	7.SP.7
62	(3)	S-CP.9
67	(3)	7.SP.7
161	(5)	S-CP.9
165	(3)	S-CP.2
166	(3)	SMP
170	(3)	6.EE.7
189	(6)	S-CP.9
194	(5)	7.SP.8
200	(4)	S-CP.8

LOGIC

11	(3)	SMP
12	(3)	SMP
13	(4)	SMP
39	(2)	SMP
42	(3)	SMP
83	(3)	SMP
128	(4)	SMP
134	(3)	SMP
159	(4)	SMP
190	(4)	SMP

MEASUREMENT

8	(4)	7.EE.2
21	(1)	4.MD.2
22	(1)	4.MD.2
35	(3)	7.G.5
44	(2)	7.G.6
103	(3)	G-GPE.6
108	(2)	4.MD.2
118	(2)	SMP
148	(3)	6.RP.3
149	(2)	5.MD.1
152	(1)	7.NS.2
156	(2)	6.RP.3
173	(4)	7.G.4
180	(3)	7.RP.3

NUMBER THEORY

7	(3)	7.EE.4
14	(4)	SMP
15	(4)	SMP
16	(5)	SMP
37	(2)	4.OA.3
38	(3)	7.NS.2
40	(4)	SMP
41	(2)	A-REI.4
45	(3)	4.OA.4
47	(2)	SMP
50	(2)	SMP
53	(3)	7.EE.2
58	(3)	F-BF.2
72	(3)	SMP
82	(4)	7.EE.2
88	(3)	SMP
93	(2)	4.OA.4
97	(2)	8.EE.4
101	(2)	6.NS.4
104	(3)	7.NS.1
105	(4)	A-SSE.2
120	(2)	N-RN.1
136	(2)	7.EE.2
139	(2)	SMP
140	(3)	6.NS.3
150	(4)	6.NS.4
151	(4)	A-CED.1
155	(4)	SMP
157	(2)	6.RP.1
158	(4)	6.EE.6
174	(2)	6.NS.5
182	(4)	SMP
184	(4)	SMP

GENERAL MATH

36	(3)	7.NS.3
43	(3)	5.NF.3
61	(1)	3.NBT.2
71	(2)	5.NF.4
80	(2)	7.NS.3
110	(3)	7.EE.2

PLANE GEOMETRY

2	(3)	G-SRT.6
24	(3)	7.G.5
25	(4)	7.G.5
26	(4)	SMP
27	(5)	SMP
63	(3)	8.G.5
84	(4)	SMP
87	(4)	G-C.2
90	(4)	SMP
92	(4)	G-C.2
96	(4)	G-CO.10
102	(4)	G-C.2
106	(4)	7.G.6
112	(4)	7.G.6
114	(4)	G-SRT.6
121	(3)	7.G.1
123	(4)	8.G.5
126	(4)	7.G.6
131	(5)	7.G.1
132	(4)	7.G.6
133	(7)	6.RP.1
137	(5)	8.G.7
146	(4)	7.G.6
163	(4)	6.EE.6
164	(4)	G-SRT.6
169	(4)	8.G.5
179	(6)	G-CO.10
185	(5)	8.EE.8
186	(3)	7.G.5
195	(4)	G-CO.10

STATISTICS

46	(5)	6.SP.4
78	(3)	7.SP.3
122	(3)	6.SP.2
129	(3)	6.RP.3
171	(2)	6.SP.2
188	(2)	6.RP.3
198	(3)	6.SP.5

PERCENTS & FRACTIONS

31	(2)	6.NS.1
33	(1)	6.RP.3
66	(2)	5.NF.1
74	(2)	5.NF.4
89	(2)	6.RP.3
144	(2)	5.NF.6
145	(2)	6.RP.3
154	(2)	7.NS.2
172	(3)	7.RP.3
175	(4)	7.RP.3
178	(3)	7.NS.2
181	(2)	6.RP.3
187	(3)	6.RP.3
199	(4)	7.RP.3

SEQUENCES, SERIES & PATTERNS

23	(2)	4.MD.2
77	(2)	SMP
85	(2)	F-BF.2
119	(2)	4.OA.5
141	(4)	F-BF.2
160	(4)	F-BF.2

SOLID GEOMETRY

70	(3)	7.EE.4
76	(2)	7.G.6
135	(3)	SMP
193	(4)	8.G.7

PROPORTIONAL REASONING

1	(1)	6.RP.3
3	(2)	6.RP.1
4	(2)	6.RP.2
5	(2)	6.RP.3
6	(3)	6.RP.3
9	(3)	7.RP.3
49	(3)	7.RP.1
51	(2)	7.RP.3
54	(3)	6.RP.1
68	(3)	6.RP.3
98	(2)	SMP
99	(2)	6.RP.3
113	(4)	6.RP.3
117	(3)	7.RP.2
147	(4)	7.RP.3
162	(3)	7.G.6
168	(3)	6.RP.1
176	(3)	6.RP.1
183	(3)	6.RP.1
192	(4)	8.G.7

PROBLEM SOLVING (MISCELLANEOUS)

32	(2)	SMP
48	(2)	6.RP.2
69	(2)	SMP
79	(4)	7.RP.3
94	(3)	7.RP.3
142	(3)	7.NS.2
143	(4)	6.RP.3
177	(4)	SMP

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