MATHCOUNTS®
COMPETITION SERIES

EST. 1983

2020 - 2021
School Handbook

CHECK OUT THIS YEAR'S MATH PROBLEMS ON

PG.12!

HOW TO USE THIS SCHOOL HANDBOOK

IF YOU'RE A NEW COACH

Welcome! We're so glad you're a coach this year. Please read the **New Program Info for 2020-21** starting on the next page, then check out the **Guide for New Coaches** starting on page 4.

IF YOU'RE A RETURNING COACH

Welcome back! Thank you for coaching again. Please read the **New Program Info for 2020-21** starting on the next page, then access this year's **Handbook Materials** on page 9.

NEW PROGRAM INFO FOR 2020-21

Welcome to the MATHCOUNTS® Competition Series! The safety and wellbeing of our students, coaches, volunteers and members of the MATHCOUNTS community are our top priority, which means the Competition Series needs to be a little different this year...but still an awesome experience! If you have questions about the program details below, please feel free to contact the MATHCOUNTS national office at info@mathcounts.org.

DUE TO COVID-19, THIS YEAR'S MATHCOUNTS COMPETITION SERIES HAS BEEN MODIFIED.

Created in 1983, the **MATHCOUNTS Competition Series** is a national program that provides students in grades 6-8 the opportunity to compete in live math contests against and alongside their peers. Typically, these competitions are conducted as in-person events, but just for this year, we have decided to conduct online competitions at each level leading up to the National Competition.

HOW WILL IT WORK? The 2020-2021 Competition Series will have 4 levels of *official* competition—chapter, chapter invitational, state and national—and 4 *unofficial* online practice competitions.

Schools register in the fall and work with students during the year. Students will have the opportunity to take 4 online practice competitions beginning in October. Any number of students from a school can participate in team meetings. Practice competitions will be released October 15, November 15, December 15 and January 22. MATHCOUNTS strongly recommends schools participate in these practice competitions. It is important your students know how to use the competition platform before competition day.

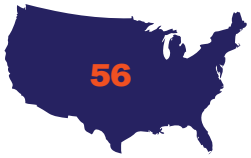
> ***Any student whose school is not participating in the program can register as a non-school competitor (NSC).*** *MATHCOUNTS encourages students to pursue all avenues to participate through their school before registering as NSCs.*

Between 1 and 15 individuals from each school participate in the 2021 Chapter Competition, which will be available online February 5 at 1:00pm ET through February 6 at 1:00pm ET. Coaches determine which students will participate in the Chapter Competition. The competition will be conducted through the Art of Problem Solving (AoPS) Contest Platform, and each student will compete solely as an individual; there will be no team competition.

> *Like school competitors,* ***NSCs*** *will take the Chapter Competition online through the AoPS Contest Platform February 5-6 and will compete as individuals.*

The top scoring student from each school and the top 20% of individuals from each chapter advance to the 2021 Chapter Invitational, taking place online February 25 at 7:00pm ET. Every competitor advancing to this level will be required to take the competition on the AoPS Contest Platform at the same time. Each student will compete solely as an individual; there will be no team competition.

> ***NSCs*** *can only advance to the Chapter Invitational by scoring in the top 20% of their chapter. Like school competitors, NSCs must take the 2021 Chapter Invitational on February 25 at 7:00pm ET on the AoPS Contest Platform.*



Top students from the Chapter Invitational advance to the 2021 State Competition, taking place online March 25 at 7:00pm ET. (State level advancement details will be announced prior to the Chapter Invitational.) Every competitor advancing to this level will be required to take the competition on the AoPS Contest Platform at the same time. Each student will compete solely as an individual; there will be no team competition.

*Advancement to the State Competition will be the same for both school competitors and **NSCs**. NSCs must take the 2021 State Competition on March 25 at 7:00pm ET on the AoPS Contest Platform.*

The top individuals from each state receive an all-expenses-paid trip to the 2021 Raytheon Technologies MATHCOUNTS National Competition, which takes place May 8-11 in Washington, DC. Up to 224 students will be invited to compete for individual and team awards. As of August 2020, this is planned as an in-person event, but subject to change, if necessary.

*Advancement to the national competition will be the same for both school competitors and **NSCs**. An NSC's state will be determined by school location.*

HOW WILL REGISTRATION WORK? Registration this year is a 2-step process:

1. Registering and paying through the MATHCOUNTS Foundation
2. Setting up online competition access through Art of Problem Solving (AoPS)

1 Coaches or school officials should register their schools online at **www.mathcounts.org/compreg**:

- **June 15 – October 1**: Early Bird Registration ($30 per student)
- **October 2 – December 1**: Regular Registration ($35 per student)
- **December 2 – January 15**: Late Registration ($40 per student)

Before registering, non-school competitors (NSCs) must confirm with school officials that their school will not be participating in the Competition Series. MATHCOUNTS (1) may contact the schools of registered NSCs to confirm that the school is not participating in the Competition Series and (2) reserves the right to cancel an NSC's registration, without refund, if their school registers for the Competition Series. NSCs must register online and pay with a credit card at **www.mathcounts.org/nscreg**:

- **August 3 – October 1**: Early Bird Registration ($60 per student)
- **October 2 – December 1**: Regular Registration ($65 per student)
- **December 2 – January 15**: Late Registration ($70 per student)

Schools and NSCs must register and pay by January 15, 2021 to participate. MATHCOUNTS cannot guarantee participation for registrants with outstanding invoices after January 15.

2 Because arrangements must be made for online competitions...

- **All coaches, school competitors and NSCs must create a free account at AoPS.** Coaches at schools will be given instructions to ensure their students are signed up with AoPS and linked to their school.
- **Schools must indicate which of their students will participate in the Chapter Competition by January 15, 2021.**

Guide For New Coaches

Welcome to the MATHCOUNTS® Competition Series! Thank you so much for serving as a coach this year. Your work truly does make a difference in the lives of the students you mentor. We've created this Guide for New Coaches to help you get acquainted with the Competition Series and understand your role as a coach in this program.

If you have questions at any point during the program year, please feel free to contact the MATHCOUNTS national office at info@mathcounts.org.

WHAT DOES THE TEST LOOK LIKE? MATHCOUNTS competition typically consists of 4 rounds—Sprint, Target, Team and Countdown Rounds. Altogether the rounds take about 3 hours to complete. **However, Team and Countdown Rounds will not be conducted officially in the 2020-2021 Competition Series until the national level.** Here's what each round looks like.

Sprint Round
40 minutes
30 problems total
no calculators used
focus on speed and accuracy

Target Round
Approx. 30 minutes
8 problems total
calculators used
focus on problem-solving and mathematical reasoning

The problems are given to students in 4 pairs. Students have 6 minutes to complete each pair.

Team Round
20 minutes
10 problems total
calculators used
focus on problem-solving and collaboration

National level only for 2020-2021. The online practice competitions will have team rounds.

Countdown Round
Maximum of 45 seconds per problem
no calculators used
focus on speed and accuracy

National level only for 2020-2021.

HOW DO I GET MY STUDENTS READY FOR THESE COMPETITIONS? What specifically you do to prepare your students will depend on your schedule as well as your students' schedules and needs. But in general, working through lots of different MATHCOUNTS problems and completing practice competitions are the best ways to prepare. Each year, MATHCOUNTS provides the *MATHCOUNTS School Handbook* to all coaches, plus lots of additional free resources online. This guide will explain the layout of the *School Handbook* and other resources, plus give you tips on structuring your team meetings and preparation schedule.

THE ROLE OF THE COMPETITION COACH

Your role as the coach is such an important one, but that doesn't mean you need to know everything, be a math expert or treat coaching like a full-time job. Every MATHCOUNTS coach has a different coaching style and you'll find the style that works best for you and your students. But in general, **every good MATHCOUNTS coach must do the following**:

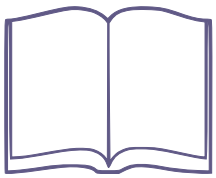
- Schedule and run an adequate number of practices for participating students (these can be online).
- Help motivate and encourage students throughout the program year.
- Select the 1-15 student(s) who will represent the school at the Chapter Competition in February.
- Ensure students can log in on the AoPS Contest Platform and are familiar with using the platform.

You don't need to know how to solve every MATHCOUNTS problem to be an effective coach. In fact, many coaches have told us that they themselves improved in mathematics through coaching. Chances are, you'll learn with and alongside your students throughout the program year.

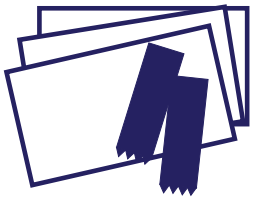
You don't need to spend your own money to be an effective coach. You can prepare your students using solely the free resources and this handbook. We give coaches numerous detailed resources and recognition materials so you can guide your Mathletes® to success even if you're new to teaching, coaching or competition math, and even if you use only the free resources MATHCOUNTS provides all competition coaches.

MAKING THE MOST OF YOUR RESOURCES

As the coach of a registered competition school, you already have received what we at MATHCOUNTS call the **School Competition Kit**. Your kit includes the following materials for coaches.



2020-2021 MATHCOUNTS School Handbook
The most important resource included in the School Competition Kit. Includes 200 problems.



Student Ribbons and Certificates
Ribbons and participation certificates for each registered student.

You'll also get access to electronic resources including **50 more handbook problems**. The following are available to coaches at **www.mathcounts.org/coaches**. This section of the MATHCOUNTS website is restricted to coaches and you already should have received an email with login instructions. *If you have not received this email, please contact us at info@mathcounts.org to confirm we have your correct email address.*

2020 MATHCOUNTS School, Chapter + State Competitions
Released by mid-April 2020
Each level includes all 4 test rounds and the answer key

MATHCOUNTS Practice Plans for Team Meetings
Pre-planned virtual mini-lessons, each 45-60 minutes, that cover a particular math topic.

MATHCOUNTS Problem of the Week
Released each Monday
Each multi-step problem relates to a timely event

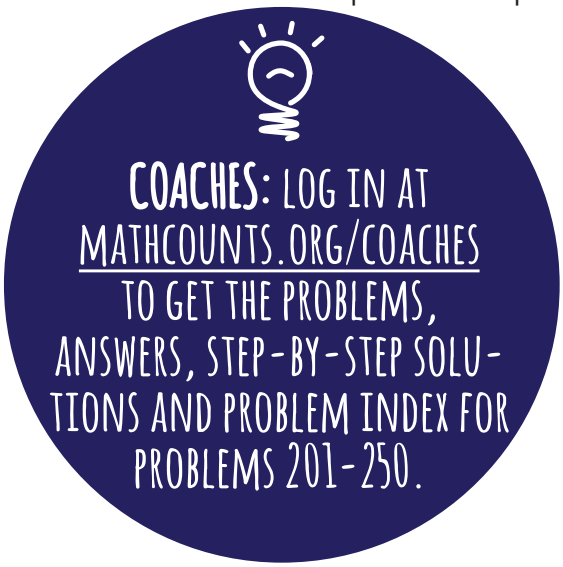
You can use any or all of these resources to choose the students who will represent your school at the Chapter Competition.

It is especially important though that your official competitors utilize the online 2020-2021 practice competitions so they are familiar with the online competition platform.

The **2020-2021 MATHCOUNTS School Handbook** will be your primary resource for the Competition Series this year. It is designed to help your students prepare for each of the rounds of the test, plus build critical thinking and problem-solving skills. This section of the Guide for New Coaches will focus on how to use this resource effectively for your team.

WHAT'S IN THE HANDBOOK? There is a lot included in the *School Handbook*, and you can find a full table of contents on pg. 9 of this book, but below are the sections that you'll use the most when coaching your students.

- **Handbook Problems:** 250 math problems (200 in the book and 50 online) divided into Warm-Ups, Workouts and



Stretches. These problems increase in difficulty as the students progress through the book. (pg. 12)

- **Solutions to Handbook Problems:** complete step-by-step explanations for how each problem can be solved. These detailed explanations are only available to registered coaches. (pg. 36)
- **Problem Index + Common Core State Standards Mapping:** catalog of all handbook problems organized by topic, difficulty rating and mapping to Common Core State Standards. (pg. 50)
- **Answers to Handbook Problems:** key available to the general public. Your students can access this key, but not the full solutions to the problems. (pg. 52)

There are 3 types of handbook problems to prepare students for each of the rounds of the competition. You'll want to have your students practice all of these types of problems.

Warm-Ups
14 Warm-Ups in handbook
10 questions per Warm-Up
no calculators used

Warm-Ups prepare students particularly for the Sprint Round.

Workouts
8 Workouts in handbook
10 questions per Workout
calculators used

Workouts prepare students particularly for the Target Round.

Stretches
3 Stretches in handbook
Number of questions and use of calculators vary by Stretch

Each Stretch covers a particular math topic that could be covered in any round. These help prepare students for all rounds.

IS THERE A SCHEDULE I SHOULD FOLLOW FOR THE YEAR? On average coaches meet with their students for an hour once a week at the beginning of the year, and more often as the competitions approach. Practice sessions may be held virtually or in-person, before school, during lunch, after school, on weekends or at other times, coordinating with your school's schedule and avoiding conflicts with other activities.

Designing a schedule for your practices will help ensure you're able to cover more problems and prepare your students for competitions. We've designed the *School Handbook* with this in mind. Below is a suggested schedule for the program year that mixes in Warm-Ups, Workouts and Stretches from the *School Handbook*, plus free practice competitions from last year. This schedule allows your students to tackle more difficult problems as the Chapter Competition approaches.

Mid-August – September 2020 Warm-Ups 1, 2 + 3 Workouts 1 + 2	**October 2020** Warm-Ups 4 + 5 Workout 3 Mixture Stretch Practice Competition 1	**November 2020** Warm-Ups 6 + 7 Workout 4 Statistics Stretch Practice Competition 2	**December 2020** Warm-Ups 8 + 9 Workout 5 Pascal's Triangle Stretch Practice Competition 3
January 2021 Warm-Ups 10 +11 and Workout 6 Past Competitions (2020 School + Chapter) Practice Competition 4 *Select chapter competitors (required by this time)*		**February 2021** *2021 MATHCOUNTS Chapter Competition* *2021 MATHCOUNTS Chapter Invitational*	

FINISHED ALL THE PROBLEMS IN THE HARD-COPY HANDBOOK? FIND 50 MORE CHALLENGING PROBLEMS AT WWW.MATHCOUNTS.ORG/COACHES

You'll notice that in January you'll need to select the 1-15 student(s) who will represent your school in the Chapter Competition. This must be done by **January 15, 2021**. You'll identify your official competitors on the Art of Problem Solving (AoPS) Contest Platform.

It's possible you and your students will meet more frequently than once a week and need additional resources. If that happens, don't worry! You and your Mathletes can work together using the **Interactive MATHCOUNTS Platform**, powered by NextThought. This free online platform contains numerous *MATHCOUNTS School Handbooks* and past competitions, not to mention lots of features that make it easy for students to collaborate with each other and track their progress. You and your Mathletes can sign up for free at mathcounts.nextthought.com.

And remember, just because you and your students will meet once a week doesn't mean your students can only prepare for MATHCOUNTS one day per week. Many coaches assign "homework" during the week so they can keep their students engaged in problem solving outside of team practices. Here's one example of what a 2-week span of practices in the middle of the program year could look like.

Monday	Tuesday	Wednesday (Weekly Team Practice)	Thursday	Friday
-Students continue to work individually on Workout 4, due Wednesday	-Students continue to work on Workout 4 -Coach emails team to assign new homework problem(s), due Wednesday	-Coach reviews solutions to Workout 4 -Coach gives Warm-Up 7 to students as timed practice and then reviews solutions -Students discuss solutions to homework problem(s) in groups	-Coach emails math team to assign Workout 5 as individual work, due Wednesday	-Students continue to work individually on Workout 5
-Students continue to work individually on Workout 5, due Wednesday	-Students continue to work on Workout 5 -Coach emails team to assign new homework problem(s), due Wednesday	-Coach reviews solutions to Workout 5 -Coach gives Warm-Up 8 to students as timed practice and then reviews solutions -Students discuss solutions to homework problem(s) in groups	-Coach emails math team to assign Workout 6 as group work, due Wednesday	-Students work together on Workout 6 using online Interactive Platform

WHAT SHOULD MY TEAM PRACTICES LOOK LIKE? Obviously every school, coach and group of students is different, and after a few practices you'll likely find out what works and what doesn't for your students. Here are some suggestions from veteran coaches about what makes for a productive practice.

- Encourage discussion of the problems so that students learn from each other
- Encourage a variety of methods for solving problems
- Have students write math problems for each other to solve
- Practice working in groups so students can learn from one another
- Practice oral presentations to reinforce understanding

On the following page is a sample agenda for a 1-hour practice session. There are many ways you can structure math team meetings and you will likely come up with an agenda that works better for you and your group. It also is probably a good idea to vary the structure of your meetings as the program year progresses.

MATHCOUNTS Team Practice Sample Agenda – 1 Hour

Review Assigned Homework Problem(s) (10 minutes)

- Have 1 student come to the board to show how s/he solved the problem(s).
- Have students divide into groups of 4 to discuss the solution presented and other methods for solving.
- Have 2 groups share an alternate solution.
- Discuss as a group other strategies to solve the problem (and help if any groups answered incorrectly).

Timed Practice with Warm-Up (15 minutes)

- Have students put away all calculators and have one student pass out Warm-Ups (face-down).
- Give students 12 minutes to complete as much of the Warm-Up as they can.
- After 12 minutes is up, have students hold up pencils and stop working.

Play Game to Review Warm-Up Answers (35 minutes)

- Have students divide into 5 groups (size will depend on number of students in meeting).
- Choose a group at random to start and then rotate clockwise to give each group a turn to answer a question. When it is a group's turn, ask the group one question from the Warm-Up.
- Have the group members consult their completed Warm-Ups and work with each other for a maximum of 60 seconds to choose the group's official answer.
- Award 2 points for a correct answer on questions 1-3, 3 points for questions 4-7 and 5 points for questions 8-10. The group gets 0 points if they answer incorrectly or do not answer in 60 seconds.
- Have all students check their Warm-Up answers as they play.
- Go over solutions to select Warm-Up problems that many students on the team got wrong.

OK I'M READY TO START. HOW DO I GET STUDENTS TO JOIN? Here are some tips given to us from successful competition coaches and club leaders for getting students involved in the program at the beginning of the year.

- Ask Mathletes who have participated in the past to talk to other students about participating.
- Ask teachers, parent volunteers and counselors to help you recruit.
- Reach parents through school newsletters, PTA meetings or Back-to-School-Night presentations.
- Advertise around your school by:
  1. posting intriguing math questions (specific to your school) in your school and referring students to the first meeting for answers.
  2. designing a bulletin board or display case with your MATHCOUNTS poster (included in your School Competition Kit) and/or photos and awards from past years.
  3. attending meetings of other extracurricular clubs (such as honor society) so you can invite their members to participate.
  4. adding information about the MATHCOUNTS team to the website for your class or school.
  5. making a presentation at the first pep rally or student assembly.

Good luck in the competition! If you have any questions during the year, please contact the MATHCOUNTS national office at info@mathcounts.org.

COACH RESOURCES: WWW.MATHCOUNTS.ORG/COACHES

2020-2021 HANDBOOK MATERIALS

Thank you for being a coach in the MATHCOUNTS Competition Series this year!
We hope participating in the program is meaningful and enriching for you and your Mathletes.
Don't forget to log in at **www.mathcounts.org/coaches** for additional resources!

WHAT'S IN THIS YEAR'S HANDBOOK

COACHES:

FIND PROBLEMS, ANSWERS, SOLUTIONS + PROBLEM INDEX FOR #201-250 ONLINE AT WWW.MATHCOUNTS.ORG/COACHES

Highlighted Resources

ACCESS MORE RESOURCES AT WWW.MATHCOUNTS.ORG/COACHES

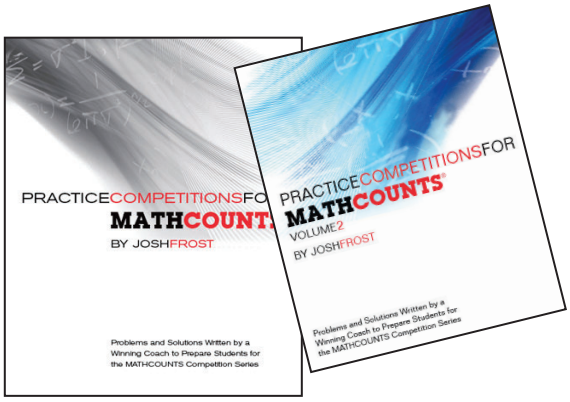
MATHCOUNTS OPLET

The Online Problem Library & Extraction Tool (OPLET) is a database of over 13,000 problems and over 5,000 step-by-step solutions. Create personalized quizzes, flash cards, worksheets and more!

Save $25 when you buy your subscription by Oct. 1, 2020

www.mathcounts.org/myoplet

PRACTICE COMPETITIONS FOR MATHCOUNTS, VOL. 1 & 2

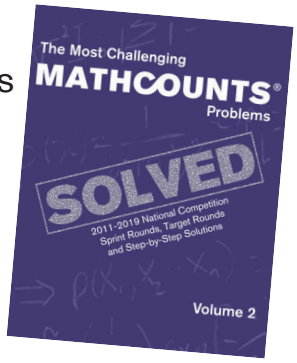


Practice books written by repeat national-level coach Josh Frost. Each volume includes 4 complete mock-competitions plus solutions.

www.mathcounts.org/store

MOST CHALLENGING MATHCOUNTS PROBLEMS SOLVED, VOL. 2

This book contains the Sprint Round and Target Round problems from the 2011 – 2019 MATHCOUNTS National Competitions plus never-before-available, step-by-step solutions!



COMING SEPTEMBER 2020!

Critical 2020-2021 Dates

Jun. 16 – Jan. 15

Submit your school's registration to participate in the Competition Series. The fastest way for schools to register is online at **www.mathcounts.org/compreg**. Schools also can download the MATHCOUNTS Competition Series Registration form and mail or email it with payment to:

MATHCOUNTS Foundation – Competition Series Registrations
1420 King Street, Alexandria, VA 22314 *Email*: reg@mathcounts.org

Parents who have confirmed their student's school is not participating in the program can register their child as a non-school competitor (NSC) this year. NSCs must register online and pay with a credit card at **www.mathcounts.org/nscreg**.

Registered schools and NSCs will receive this year's School Competition Kit, which includes a hard copy of the *2020-2021 MATHCOUNTS School Handbook*. Kits are shipped on an ongoing basis between mid-August and January 31.

Questions? Email the MATHCOUNTS national office at info@mathcounts.org.

Oct. 1 (postmark)

Early Bird Registration Deadline ($30/student for schools and $60/NSC)
After October 1, registration increases to $35/student for schools and $65/NSC.

Oct. 15

Practice Competition 1 Released, available for registered schools and NSCs at the Art of Problem Solving (AoPS) Contest Platform through January 31, 2021.

Nov. 15

Practice Competition 2 Released, available for registered schools and NSCs at the AoPS Contest Platform through January 31, 2021.

Dec. 1 (postmark)

Regular Registration Deadline ($35/student for schools and $65/NSC)
After December 1, registration will cost $40/student for schools and $70/NSC.

Dec. 15

Practice Competition 3 Released, available for registered schools and NSCs at the AoPS Contest Platform through January 31, 2021.

Early Jan.

Coaches should identify official chapter competitors on the AoPS Contest Platform at this time to ensure their students can practice using the competition platform. Identify official competitors by January 15 to ensure their participation in Practice Competition 4.

Jan. 15 (postmark)

Final Day to Register, Pay and Add Students ($40/student for schools and $70/NSC)
MATHCOUNTS cannot guarantee participation for schools with outstanding invoices after January 15. Schools cannot add students to their registration after January 15.

Jan. 22-23

Practice Competition 4, available from 1:00pm ET on January 22 through 1:00pm ET on January 23 on the AoPS Contest Platform. Recommended for chapter competitors to use as a run-through for the Chapter Competition.

Feb. 5-6

2021 Chapter Competition, available from 1:00pm ET on February 5 through 1:00pm ET on February 6 on the AoPS Contest Platform.

Feb. 25

2021 Chapter Invitational at 7:00pm ET on February 25 on the AoPS Contest Platform. All competitors must take this competition at the same time.

Mar. 25

2021 State Competition at 7:00pm ET on March 25 on the AoPS Contest Platform. All competitors must take this competition at the same time.

May 8-11

2021 Raytheon Technologies MATHCOUNTS National Competition in Washington, DC

Mixture Stretch

1. _____ teaspoons

Ming's recipe for sweet tea calls for 4 teaspoons of sugar. If Ming wants to make the tea 25% less sweet, how much less sugar should he use?

2. _____

Carla is mixing cherry, grape and lime candies in a bowl. Since her favorite flavor is cherry, she wants $\frac{2}{5}$ of the candies to be cherry. Since her least favorite flavor is lime, she wants $\frac{1}{4}$ of the candies to be lime. What fraction of the candies will be grape? Express your answer as a common fraction.

3. _____

A paving company makes concrete by adding water to a mix that is 1 part cement, 3 parts sand and 3 parts aggregate (stone). What fraction of this mix is aggregate? Express your answer as a common fraction.

4. _____ ounces

A beef stew recipe calls for 12 ounces of beef, 4 ounces of carrots, 7 ounces of potatoes, 4 ounces of peas and 5 ounces of beef stock. Given that there are 16 ounces in a pound, how many ounces of potatoes are needed to make 4 pounds of this stew?

5. _____ %

Jin adds 1 gallon of a water-and-bleach mixture that is 4% bleach to 2 gallons of a water-and-bleach mixture that is 10% bleach. What percent of the final mixture is bleach?

6. $ _____

Cashews cost $2.36 per pound, almonds cost $1.48 per pound and peanuts cost $0.98 per pound. To make a 20% profit, how much should Myrna charge per pound for a mixture that is 1 part cashews, 1 part almonds and 2 parts peanuts?

7. _____ gallons

Manny's cleaning supply store receives a mixture of 80% detergent and 20% water in 15-gallon buckets. Manny would like a mixture of 60% detergent and 40% water in 5-gallon buckets. To make this, he combines some 80/20 mixture with some pure water in each 5-gallon bucket. How many gallons of pure water does Manny add to each 5-gallon bucket? Express your answer as a decimal to the nearest hundredth.

8. _____ buckets

Based on the information in problem 7, how many 5-gallon buckets of 60/40 solution can Manny make from one 15-gallon bucket of 80/20 solution?

9. _____ quarts

Dara is mixing her own paint color, using 3 parts green paint to 2 parts blue to 1 part white. Given that there are 4 quarts in a gallon, if she needs 3 gallons of her paint, how many quarts of white paint should she buy?

10. _____ g/cm^3

To make a sand sculpture, Arthur used 2 cm^3 of red sand with a density of 4 g/cm^3, 7 cm^3 of yellow sand with a density of 5 g/cm^3 and 5 cm^3 of brown sand with a density of 6 g/cm^3. What is the average density of this sculpture in grams per cubic centimeter? Express your answer as a decimal to the nearest tenth.

Statistics Stretch

11. _____ A class of 28 students had a mean score of 72 on a math test. After the teacher realized that one of the questions had an alternative correct answer, he gave 4 points each to the 7 students who had given the alternative answer. What is the new mean test score?

12. _____ pages

stem	leaf
1	27 52 82
2	25 57 63 82 97
3	37 51 68 75

1|27 = 127 pages

The stem-and-leaf plot shows the number of pages in each book that Kalem read last summer. How many pages did Kalem read last summer?

13. _____ Based on the information in problem 12, what portion of the pages that Kalem read were in books having more than 275 pages? Express your answer as a common fraction.

14. _____ % A cafeteria offers apples, oranges and bananas with lunch. A student may take at most one of each fruit. Of the 61 students who got fruit with lunch, 5 students got only an apple and 7 got only an orange. Of the 16 students who got an apple and an orange, the 17 who got an orange and a banana, and the 20 who got an apple and a banana, 6 got all three fruits. What portion of the fruit taken by the 61 students were bananas? Express your answer to the nearest whole percent.

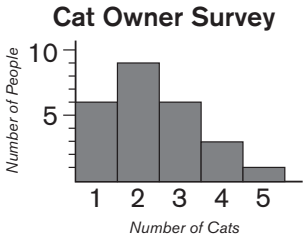
15. _____ **Chess Club Membership**

	6th	7th	8th
Beginners	1	?	5
Advanced	2	?	6

The table shows the grade and skill level of members of the chess club. If half of the members are eighth graders and one-third of the beginners are seventh-graders, what fraction of chess club members are advanced chess players? Express your answer as a common fraction.

16. _____ If A, M and R represent the arithmetic mean, median and range of the set {13, 16, 18, 23, 25, 28, 30, 31}, what is the value of $A + M - R$?

17. _____ cats This graph shows results of a survey of 25 cat owners. What is the mean number of cats per person surveyed? Express your answer as a decimal to the nearest hundredth.



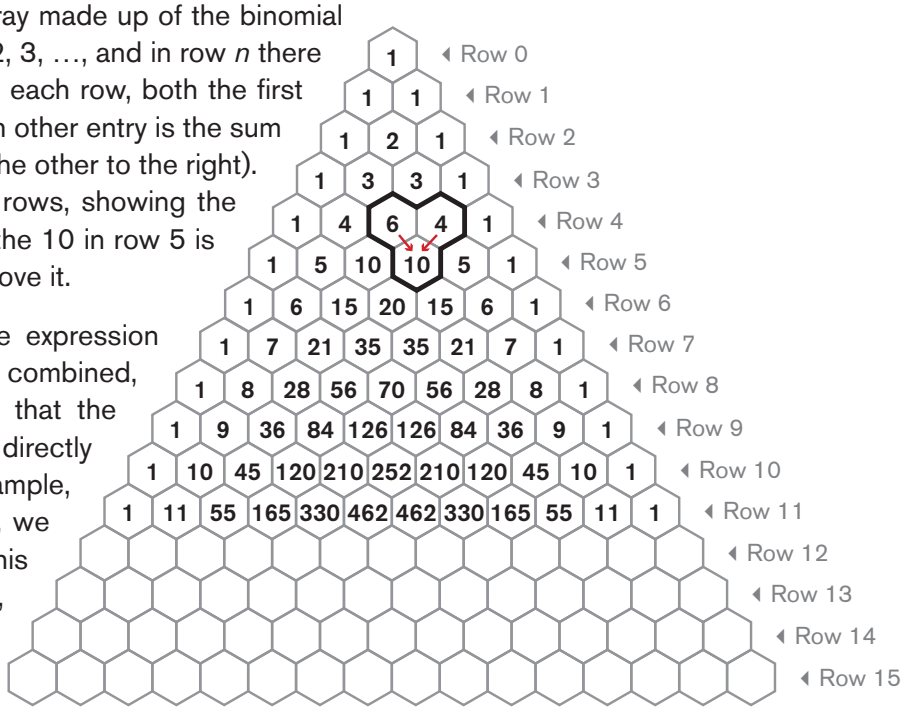
18. _____ A total of 44 Mathletes competed in a MATHCOUNTS competition. The mean score for all the competitors was 28. The mean score for all competitors *except* the 16 highest scorers was 20. What was the mean score for the 16 highest scorers?

19. _____ % Bryce orders 6 bats, 60 baseballs and 8 gloves. If each bat costs $29.95, a pack of 12 baseballs costs $39.95 and a glove costs $69.95, what portion of the total cost of this order is for the gloves? Express your answer to the nearest whole percent.

20. _____ % The graph shows the price for a popular running shoe over five months. What is the absolute difference of the percent change in price from February to April and the percent change in price from April to June? Express your answer to the nearest tenth of a percent.

Pascal's Triangle Stretch

Pascal's triangle is a famous triangular array made up of the binomial coefficients. The rows are numbered 0, 1, 2, 3, …, and in row n there are $n + 1$ entries, numbered 0, 1, …, n. In each row, both the first entry (entry 0) and the last entry are 1. Each other entry is the sum of the two entries above it (one to the left, the other to the right). Here we have a Pascal's triangle with 16 rows, showing the entries for rows 0 through 11. Notice that the 10 in row 5 is the sum of the 6 and 4 in row 4, directly above it.

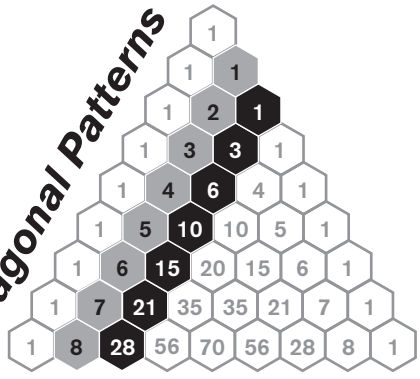


The binomial theorem says that when the expression $(x + y)^n$ is expanded and like terms are combined, the coefficient of $x^k y^{n-k}$ is $\binom{n}{k}$, meaning that the coefficients in this expansion can be read directly from row n in Pascal's triangle. For example, consider $(x + y)^4$. Expanding this binomial, we get $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$. In this case, $n = 4$. Referring to Pascal's triangle, we see that row 4 does indeed give us the coefficients of the terms in this expansion: 1, 4, 6, 4 and 1.

In general, the kth entry in row n is the binomial coefficient $\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$, the number of combinations of n objects taken k at a time. Given five objects, to find the number of ways to choose three of them, or $\binom{5}{3}$, we locate entry 3 of row 5 in Pascal's triangle and see that there are **10** ways.

Below are a few interesting properties of Pascal's triangle.

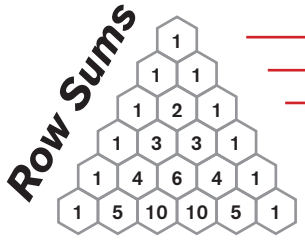
Diagonal Patterns



The gray diagonal contains the counting numbers: 1, 2, 3, 4, 5, ….

The black diagonal contains the triangular numbers: 1, 3, 6, 10, 15, …. In fact, entry 2 in row $(n + 1)$ is the nth triangular number.

Row Sums



$1 = \mathbf{1} \rightarrow 2^0$

$1 + 1 = \mathbf{2} \rightarrow 2^1$

$1 + 2 + 1 = \mathbf{4} \rightarrow 2^2$

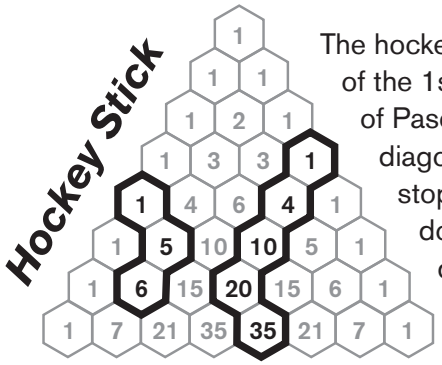
$1 + 3 + 3 + 1 = \mathbf{8} \rightarrow 2^3$

$1 + 4 + 6 + 4 + 1 = \mathbf{16} \rightarrow 2^4$

$1 + 5 + 10 + 10 + 5 + 1 = \mathbf{32} \rightarrow 2^5$

The sum of the entries in the nth row of Pascal's triangle is equal to 2^n.

Hockey Stick



The hockey stick identity: Start at any of the 1s on the left or right side of Pascal's triangle. Sum entries diagonally in a straight line, and stop at any time. The next entry down diagonally in the opposite direction will equal that sum.

Solve the following problems, using what you've learned about Pascal's triangle. It may be helpful to fill in some of the missing entries in the Pascal's triangle on the previous page.

21. _____ What is the greatest entry in row 15 of Pascal's triangle?

22. _____ entries How many of the entries in row 14 of Pascal's triangle are even?

23. _____ What is the sum of the entries in row 12 of Pascal's triangle?

24. _____ choices A sports team of eight players must choose three starting players. How many different choices of three starters are there if the order in which they are chosen does not matter?

25. _____ What is the sum of the first 10 triangular numbers?

26. _____ When the expression $(x + 2)^8$ is expanded and like terms are combined, what is the coefficient of x^3?

27. _____ When the expression $(2x + y)^4$ is expanded and like terms are combined, what is the sum of the coefficients?

28. _____ When the expression $(x + 1)^{2022}$ is expanded and like terms are combined, the term with the greatest coefficient can be expressed as ax^b. What is the value of b?

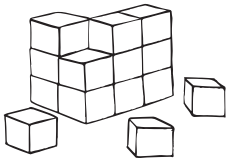
29. _____ A fair coin is flipped four times. What is the probability that it lands heads up at least as many times as it lands tails up? Express your answer as a common fraction.

30. _____ times Only the number 1 appears in Pascal's triangle more times than the number 3003 appears. How many times does 3003 appear in Pascal's triangle?

Warm-Up 1

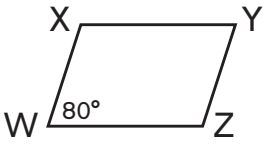
31. _____ What is the result when one hundred twenty-eight thousand is subtracted from one million?

32. _____ unit cubes How many unit cubes are required to create a larger cube with edge length 3 units?



33. _____ What is the remainder when the sum of the smallest and second-smallest prime numbers is divided by the third-smallest prime number?

34. _____ degrees



In parallelogram WXYZ, shown here, the measure of angle W is 80 degrees. What is the degree measure of angle X?

35. _____ What is the value of $6 \div 2 \times 3 + 8 \div 4 \times 2$?

36. _____ If Yasuko randomly selects a single-digit positive integer, what is the probability that it is *not* prime? Express your answer as a common fraction.

37. _____ What is the value of $0.001 \times \frac{1}{10} \times 10^5$?

38. _____ What is the value of $3(4x + 5y) - 2(7x - 3y)$ when $x = -2$ and $y = 3$?

39. _____ If $\frac{4}{18} = \frac{a}{27}$, what is the value of a?

40. _____ cups *Daifuku* is a snack made from glutinous rice flour and sweetened red bean paste. A recipe for 24 daifuku requires 3 cups of sugar. How many cups of sugar are needed to make 64 daifuku?

Warm-Up 2

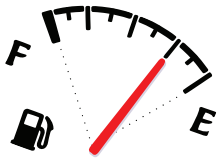
41. ____ : ____ p.m. Louisa leaves her house at 12:17 p.m., walks to the library, which takes 14 minutes, and remains there for 3 hours 27 minutes. If she walks home at the same speed as she walked to the library, at what time will she return home?

42. _____ What is the value of $7 - (3 - 4) + 11$?

43. _____ What is the least positive integer that is divisible by 4, 6 and 10?

44. _____ yards How many yards are in the perimeter of a square that measures 99 inches on each side?

45. _____ gallons When the Schwartzes left home, their gas gauge read $\frac{7}{8}$ full. When they reached their destination, their gauge read $\frac{1}{4}$ full. If their gas tank holds 16 gallons, how many gallons of gas did they use on their trip?



46. _____ What is the eighth term of the sequence that begins with 1, 3, 7, 13, 21, …?

47. _____ What is the value of x that satisfies the equation $5(x + 2) - 3(x - 8) = 16$?

48. _____ minutes

Andrew mowed one-half of a lawn, and Ben mowed one-third of the same lawn, each at a constant rate. If Andrew, continuing at the same rate, finished mowing the rest of the lawn in 12 minutes, how many minutes would it have taken him to mow the entire lawn by himself?

49. _____ If 125% of n is 30, what is 25% of n?

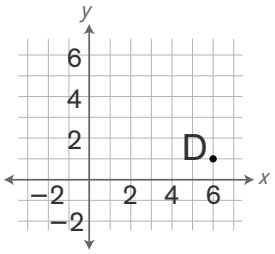
50. _____ outfits Don has four short-sleeved shirts, one each in black, white, red and gray, and two long-sleeved shirts, one each in red and gray. Don has three pairs of pants, one each in gray, black and tan. If Don chooses to wear only one black item or no black items at any one time, how many different outfits consisting of a shirt and a pair of pants can he make?

Warm-Up 3

51. _____ times The face of a clock has the numbers 1 through 12 painted on it. How many times is the digit 1 painted?

52. _____ What is the value of 5 + (−6) + 5 − (−6) + 5 − 6 + (−5 + 6)?

53. _____ What is the sum of the coordinates of point D on the coordinate grid shown?



54. _____ What is the value of $\left(1-\frac{1}{2}\right)^2\left(1-\frac{1}{3}\right)^2$? Express your answer as a common fraction.

55. _____ What is the value of 0.123 + 1.032 + 2.301 + 3.210? Express your answer as a decimal to the nearest thousandth.

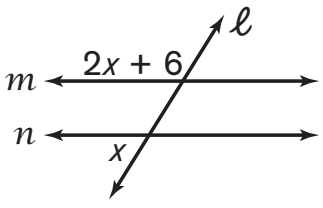
56. _____ primes Let $p(n)$ be the number of primes less than n. What is $p(50)$?

57. _____ cups

Maria wants to make a casserole to serve 12 people. She plans to use a recipe that calls for $1\frac{1}{4}$ cups of flour to serve 5 people. How many cups of flour will Maria need for her casserole?

58. _____ Let x and y represent the LCM and GCF, respectively, of 24 and 40. What is the value of $\frac{x}{y}$?

59. _____ degrees The figure shows parallel lines m and n with transversal ℓ. Based on the degree angle measures shown, what is the value of x?



60. _____ What is the greatest two-digit prime number that is one greater than a perfect square?

Warm-Up 4

61. _____ grams Each of 6 Mathletes ate a cheeseburger that contained 63 calories of fat. If there are 9 calories per gram of fat, how many total grams of fat did the Mathletes eat?

62. _____ What is the value of the 40th positive odd integer?

63. _____ units What is the distance between C(2, 1) and D(5, 5)?

64. _____

$1^2 = 1$

$11^2 = 121$

$111^2 = 12{,}321$

$1111^2 = 1{,}234{,}321$

Based on the pattern shown, what is the sum of the digits when $11{,}111^2$ is calculated?

65. _____ What is the value of $1 + 2 \times 3 - 4 + 5 \times 6 - 7 + 8 \times 9$?

66. _____ % Last school year, the Math Club had 20 members. This school year, there are 28 members. By what percent did the membership of the Math Club increase?

67. _____ in^2 What is the area of a rectangle that has width $2\frac{3}{4}$ inches and length $3\frac{2}{5}$ inches? Express your answer as a mixed number.

68. _____ When the grid shown is filled in correctly, each of the numbers 1 through 4 will appear exactly once in each row and column. The small number in one corner of a heavily outlined rectangle is the sum of the numbers that belong in that rectangle. What number must be in the shaded cell of the grid?

	7	4	3
6	3		
			7
	4		6

69. _____ What is the value of $\dfrac{1+\frac{2}{3}}{2-\frac{3}{4}}$? Express your answer as a common fraction.

70. _____ Let $x \mathbin{☼} y = \dfrac{(x+8)^2}{y}$. What is the value of $2 \mathbin{☼} (2 \mathbin{☼} 4)$?

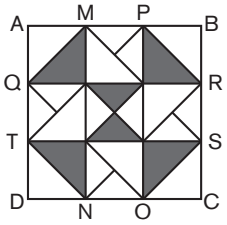
Warm-Up 5

71. _____ points In the National Sports League (NSL), a team earns 3 points for a regulation win, 2 points for an overtime win and 1 point for an overtime loss. How many total points did an NSL team with 29 regulation wins, 10 overtime wins and 4 overtime losses earn?

72. _____ What common fraction is equivalent to $48.55 - 47.37$?

73. _____ If s equals the square root of the reciprocal of 1.21, what is the value of s? Express your answer as a common fraction.

74. _____ points A group of 4 students took a math test. The mean of the numbers of points scored by the students is 95 points out of a possible 100 points. What is the minimum possible number of points scored by any student?

75. _____ 

In square ABCD, shown here, segments MN and OP trisect sides AB and DC, and segments QR and ST trisect sides AD and BC. What is the ratio of the combined area of the shaded regions to the area of square ABCD? Express your answer as a common fraction.

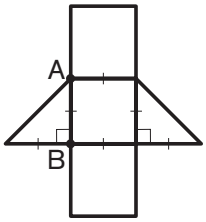
76. _____ All the students at the playground are wearing either long pants or shorts, and some are wearing a hat. Here is a table where Mossi recorded how many students are wearing various items. What is the probability that a randomly selected student on this playground is wearing shorts but no hat? Express your answer as a common fraction.

	Hat	No Hat
Pants	𝍸 I	IIII
Shorts	𝍸	𝍸

77. _____ What is the result when $-8 \times (-4) - (-8)$ is divided by the sum $-6 + (-4)$?

78. $ _____ Simon and Theo paid a total of $15 for lunch. Simon paid $\frac{2}{3}$ the amount that Theo paid. How many more dollars than Simon did Theo pay?

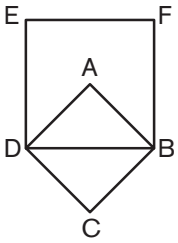
79. _____ ft^3 Molly is folding a right triangular prism out of a piece of poster board cut into the shape shown. If the distance between point A and point B is 24 inches, what will the volume of the prism be in cubic feet?



80. _____ What is the value of $202{,}020{,}202{,}020 \times 2021 - 202{,}120{,}212{,}021 \times 2020$?

Warm-Up 6

81. _____ If $A = 2^3 - 3^2 + 4(5 + 1)$ and $B = 7^2 - 2(3 + 1)^2$, what is the value of $A - B$?

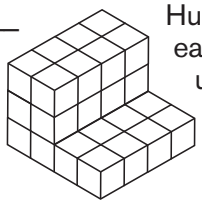
82. _____ 

What is the ratio of the area of square ABCD to that of square BDEF in the figure shown? Express your answer as a common fraction.

83. _____ What is the median of the first ten prime numbers?

84. _____ Let the value of a word equal the sum of the values of its letters. Suppose MATH has value M + A + T + H = 85, and suppose M = 8, A = 3H, T = 73 and H = 1. If S = 3M − 15 and N = 2H + S, what is the value of SAMANTHA?

85. _____ What is the remainder when the sum $10^3 + 4^2$ is divided by 8?

86. _____ 
Hugo has a solid like the one shown, composed of 32 unit cubes. He paints each face of this solid red and then separates the solid into the 32 individual unit cubes. What is the probability that a randomly selected unit cube has exactly one painted face? Express your answer as a common fraction.

87. _____ If x equals twice its reciprocal, what is the value of x^4?

88. _____ rooms A robotic vacuum cleaner vacuums one hotel room in $\frac{3}{10}$ hour. At this rate, how many rooms of the same size will it vacuum in 3 hours?

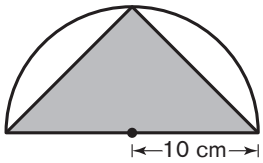
89. _____ Nya starts with an integer N and repeatedly subtracts 6. Mya starts with an integer M and repeatedly adds 8. When Nya and Mya have each performed their respective repeated operation 13 times, both have a resulting value of 25. What is the value of $N + M$?

90. _____ What is the value of the product $\left(1 + \frac{1}{2}\right)\left(1 + \frac{1}{3}\right)\left(1 + \frac{1}{4}\right)\cdots\left(1 + \frac{1}{19}\right)$?

Warm-Up 7

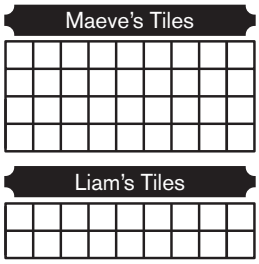
91. _____ students This school term, 25% of the students in Ms. Norton's class earned a final grade of A. If 7 students earned an A this term, how many students are in Ms. Norton's class?

92. _____ cm^2 The figure shows an isosceles triangle inscribed in a semicircle of radius 10 cm. What is the area of the triangle?



93. _____ What is the integer value of $\frac{1.4\times10^5}{7\times10^2}$?

94. _____ tiles

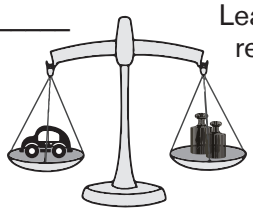


Maeve creates a design by painting $\frac{1}{3}$ of her square tiles. Liam creates a design by painting $\frac{5}{9}$ of his square tiles. What is the combined number of painted tiles in Maeve's and Liam's designs?

95. _____ If $\frac{a}{b} = \frac{2}{3}$ and $a + b = 100$, what is the value of b?

96. _____ The mean, median and unique mode of a list of five positive integers are all equal to 5. What is the greatest possible value of an integer in this list?

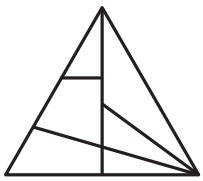
97. _____



Leah puts a toy car weighing 6 ounces on the left side of a balance. Then, reaching into a bag that contains four weights, measuring 1, 2, 4 and 5 ounces, she randomly removes two weights, without replacement. If she places the two weights on the right side of the balance, what is the probability that the balance levels? Express your answer as a common fraction.

98. _____ What is the value of $\frac{7^4-3^4}{7^2+3^2}$?

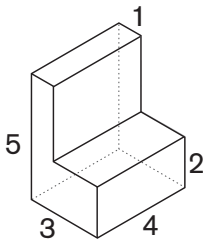
99. _____ triangles How many triangles of any size are in the figure shown?



100. _____ arrangements Sara and Ben make a playlist for a road trip. Each chooses 5 songs for the playlist, and they order the songs so that no two consecutive songs were added to the list by the same person. How many such song arrangements are possible for their playlist, assuming no song is repeated?

Warm-Up 8

101. _____ What number is one-third of two-fifths of 90?

102. _____ units^3 

What is the volume of the figure shown, in which all adjacent edges are perpendicular?

103. _____ When a number n is divided by -6 and the quotient is increased by 6, the result is 3. What is the value of n?

104. _____ What common fraction is equivalent to $1 + \dfrac{1}{1+\dfrac{1}{1+\frac{1}{3}}}$?

105. _____ What is the absolute difference between the slope of the line passing through the points given in this table and the slope of the line given by $2x - y = 4$? Express your answer as a common fraction.

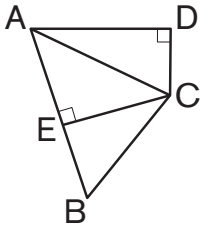
x	y
1	2
4	7
7	12

106. _____ What is the value of $2^5 \times 4^{-2}$?

107. _____ years old 
Armin's 13th birthday was on Saturday, July 4, 2020. How old will Armin be when his birthday next falls on a Saturday?

108. _____ What is the value of a in the geometric sequence $-1, 3, -9, a, -81$?

109. _____ Suppose that N is an integer such that 3^N is a factor of 10!. What is the greatest possible value of N?

110. _____ units 

In the figure shown, CD = 6 units, $m\angle CAD = 30°$, $m\angle ACE = 45°$ and $m\angle ABC = 60°$. What is the length of segment EB? Express your answer in simplest radical form.

Warm-Up 9

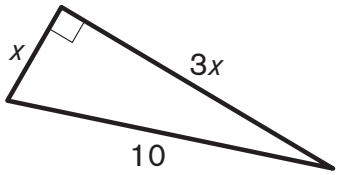
111. _____

A bag contains 12 hair bows: 5 red, 4 white and 3 blue. Jo Jo reaches into the bag and randomly pulls out two bows without replacement. What is the probability that those two bows are the same color? Express your answer as a common fraction.

112. _____ If $4\left(\frac{5}{8}x - \frac{1}{2}\right) = 3$, what is the value of x?

113. _____ A fair coin is flipped 5 times. What is the probability that no two consecutive flips have the same result? Express your answer as a common fraction.

114. _____ A right triangle has side lengths x, $3x$ and 10, as shown. What is the value of x? Express your answer in simplest radical form.



115. _____ What is the value of $4^{12} \times 125^7$? Express your answer in scientific notation.

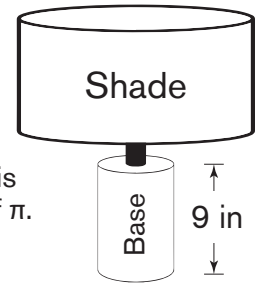
116. _____ cm Rachelle draws a rectangle of perimeter 46 cm and area 90 cm^2. Evan draws a rectangle with twice the perimeter and half the area of Rachelle's rectangle. What is the smaller dimension of Evan's rectangle?

117. _____ coins Tonya found \$2.25 in nickels and quarters in her sofa cushions. If the number of nickels Tonya found is five more than three times the number of quarters she found, what is the total number of coins Tonya found?

118. _____ A line passes through the points (−7, 1), (5, 7) and (0, b). What is the value of b? Express your answer as a common fraction.

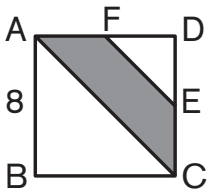
119. _____ Every positive integer can be expressed in the form $6n + k$, where $0 \leq k \leq 5$. If 1841 is expressed in this form, what is the value of $n + k$?

120. _____ in^3 A lamp's base and shade are both cylindrical as shown. The shade has circumference 18π inches, which is three times that of the lamp's base. If the lamp's base is made of solid brass and has height 9 inches, what is the volume of brass in the lamp's base? Express your answer in terms of π.



Warm-Up 10

121. _____ cm² Square ABCD, shown here, has side length 8 cm. If E and F are the midpoints of sides CD and AD, respectively, what is the area of shaded trapezoid ACEF?



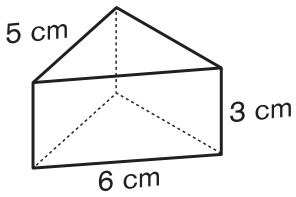
122. _____ If $f(x) = 2x^3 - 5x^2 + 9x + 4$, what is the value of $f(-2)$?

123. _____ Three numbers are selected at random without replacement from the set {2, 3, 5, 7, 11, 13}. What is the probability that the sum of the three numbers will be a multiple of 3? Express your answer as a common fraction.

124. _____  In the square shown, stripes run parallel to the sides and divide the top and right sides of the square into congruent segments. What fraction of the figure is shaded? Express your answer as a common fraction.

125. _____ What is the value of $\left[\left(\frac{2}{3}\right)^2 \div 4\right]^{\frac{1}{2}}$?

126. _____ cm² The right triangular prism shown has bases that are isosceles triangles. If each triangular base has congruent sides of length 5 cm and a third side of length 6 cm, and the prism has height 3 cm, what is the surface area of this prism?



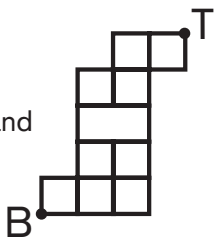
127. _____ What is the smallest five-digit number that has exactly one 4 and exactly one 6 and is a multiple of 9?

128. _____ 

In the circular spinner shown, sections A and B are congruent, each with a 90-degree central angle, and sections C, D and E are all congruent. When Allie spins this spinner, what is the probability that it lands on section A or D? Express your answer as a common fraction.

129. _____ Two numbers have a geometric mean of 4 and an arithmetic mean of 5. What is the larger of the two numbers?

130. _____ paths In the figure shown, how many paths from B to T are there that move up and right along the line segments?



Warm-Up 11

131. _____ If $f(x) = x^2$ and $g(x) = 2x - 1$, what is the value of $f(5) - g(8)$?

132. _____ A *nugget number* is a positive integer that can be obtained by adding together any combination of the numbers 6, 9 and 20. For example, 75 is a nugget number because 20 + 20 + 20 + 9 + 6 = 75, whereas, 34 is not a nugget number. What is the largest nugget number less than 200?

133. _____ What is the value of $81^{\frac{3}{4}}$?

134. _____ arrangements How many unique arrangements are there of all six letters of SQUARE?

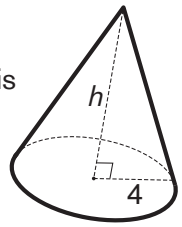
135. _____ ways In how many ways can all the numbers 1, 2, 3, 4, 5, 6 and 7 be separated into two groups, so that the sum of the numbers in both groups is the same?

136. _____ If $x + \frac{1}{x} = -2$, what is the value of $x^4 + \frac{1}{x^4}$?

137. _____ burgers

Bodacious Burgers sells burgers for $2.50 each and french fries for $0.99 per container. Four friends ate at Bodacious Burgers. The bill for the meal was $17.97 before tax. How many burgers were ordered?

138. _____ cm A right circular cone with base radius 4 cm has surface area 56π cm^2. What is the height of the cone? Express your answer in simplest radical form.



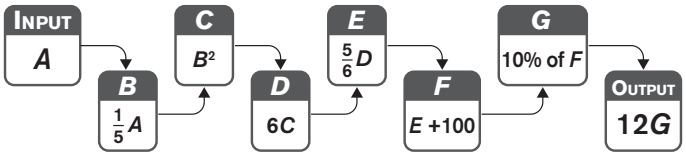
139. _____ What is the least positive two-digit integer that leaves a remainder of 3 when divided by each of the numbers 4, 5 and 6?

140. _____ units A triangle has sides of lengths 18, 24 and 30 units. What is the length of the shortest altitude of this triangle? Express your answer as a common fraction.

Workout 1

141. _____ What is the sum of the integers less than 100 that have both 6 and 9 as divisors?

142. _____ Devon wrote a program that takes a positive integer A as an input and performs a series of operations, each time assigning the result to a new variable, as shown. If the output of the program is 144, what was the value of the input A?



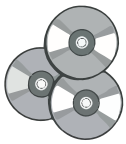
143. _____ What is the arithmetic mean of the numbers 3, 66 and 999?

144. _____ hours Grace notices interstate highway markers placed at the end of each mile and numbered consecutively: 1, 2, 3, 4, …. If an accurate speedometer says Grace is traveling 72 mi/h, how many hours will it take her to travel from mile marker 7 to mile marker 29? Express your answer as a decimal to the nearest hundredth.

145. _____ % Cliff's piano has 52 white keys and 36 black keys. What percent of the keys on his piano are white? Express your answer to the nearest whole percent.

146. _____ mL On average, in men, 39 mL per kilogram of body weight is blood plasma. In women, 40 mL per kilogram of body weight is blood plasma. Rob weighs 80 kg, and Kristen weighs 60 kg. How many more milliliters of blood plasma does Rob have than Kristen?

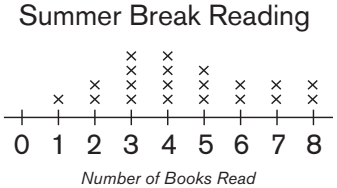
147. _____ times



A CD spins 5 times per second. How many times will the CD spin while playing a song that is 3 minutes 43 seconds long?

148. _____ What is the greatest possible product of two positive integers whose sum is 34?

149. _____ books The line plot shows the number of books that each student in Ms. Coleman's homeroom reported reading over summer break. What was the mean number of books read by a student in this homeroom? Express your answer as a decimal to the nearest tenth.



150. _____ degrees Angles A and B are complementary, with $m\angle A = 5x - 6$ degrees and $m\angle B = 3x$ degrees. What is the degree measure of angle A?

Workout 2

151. _____ feet If 1 inch is equivalent to 2.54 cm, how many feet are in 50 cm? Express your answer as a decimal to the nearest hundredth.

152. _____ integers How many two-digit positive integers are there with the property that the sum of the integer's digits equals the product of those digits?

153. $ _____ Alistair has 40 coins in his pocket, including at least one penny, one nickel, one dime and one quarter. If he has no other type of coin in his pocket, what is the greatest possible total value of the coins in Alistair's pocket?

154. _____ % If Sanjay runs 80% as far as Jerome in the eighth-grade pickle-rolling race, what percent of Sanjay's distance does Jerome run?

155. _____ What is the least common multiple of the first five positive cubes?

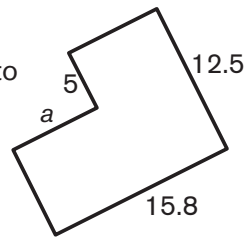
156. _____ units^2 What is the area of the triangle bounded by the line $3x + 2y = 12$ and the x- and y-axes?

157. _____ hours The table shows the hours that Gene, Doug and Pat worked canning corn one Saturday in July. What is the combined number of hours they worked canning corn? Express your answer as a decimal to the nearest hundredth.

	1st Shift		2nd Shift	
	Start	End	Start	End
GENE	7:00 a.m.	10:15 a.m.	10:30 a.m.	12:30 p.m.
DOUG	7:30 a.m.	10:15 a.m.	10:30 a.m.	12:30 p.m.
PAT	7:15 a.m.	10:15 a.m.	10:30 a.m.	11:45 a.m.

158. _____ What is the median of the integers between 1 and 1000 that are divisible by 28?

159. _____ units The figure shows a hexagon in which adjacent sides are perpendicular to each other. The hexagon with the given side lengths has an area of 160 units^2. What is the value of a? Express your answer as a decimal to the nearest tenth.



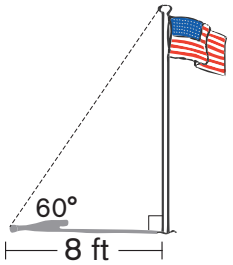
160. _____ primes Let an *optimus prime* be a prime number whose digit sum is also a prime number. How many of the first 10 primes are optimus primes?

Workout 3

161. _____ What is the median of the set $\left\{\frac{7}{3}, 2, 1.5, 1\frac{1}{4}\right\}$? Express your answer as a decimal to the nearest hundredth.

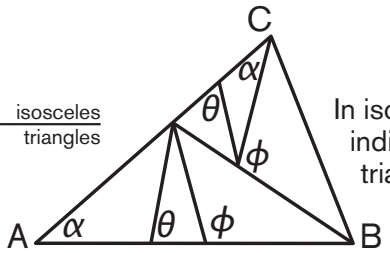
162. _____ years old Marco's age is between 20 and 60. The square of his age is a four-digit number, and the sum of the digits of his age is 8. How old is Marco if his age has two different digits and is not prime?

163. _____ feet The flagpole shown is perpendicular to the ground and casts a shadow of 8 feet. The angle of elevation to the top of the pole from the end of the shadow is 60 degrees. How tall is the pole? Express your answer as a decimal to the nearest tenth.



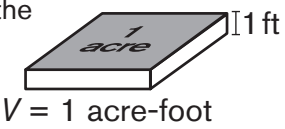
164. _____ blue-berries The number of blueberries left in a bowl is reduced by half every 2 hours. Sam filled the bowl with blueberries at 9:00 a.m. When he checked the bowl at 7:00 p.m., there were 5 blueberries left. How many blueberries did Sam originally place in the bowl at 9:00 a.m.?

165. _____ isosceles triangles In isosceles triangle ABC, shown here, AB = AC. For the angles indicated, $\alpha = 36°$, $\theta = 72°$ and $\phi = 108°$. How many isosceles triangles of any size are in this figure?



166. _____ integers How many integers between 1,000,000 and 2,000,000 are divisible by 99?

167. _____ gallons An acre-foot is defined as the volume of a rectangular prism with a base area of 1 acre and a depth of 1 foot, as shown. Given that 1 acre is defined as the area of a 66-foot × 660-foot rectangle and that 1 gallon equals 231 in^3, how many gallons are in 1 acre-foot of water? Express your answer as an integer to the nearest thousand.



V = 1 acre-foot

168. _____ % What percent of positive integers less than or equal to 100 are divisible by 3?

169. _____ students Ali's middle school has a total of 300 students on Team A and Team B. After 30 students are moved from Team A to Team B, there are twice as many students on Team A as there are on Team B. How many students were originally on Team B?

170. _____ cats The mean number of cats living in each of the 50 apartments in a particular apartment building is 0.44 cats. A total of 32 apartments in the building are cat-free. What is the mean number of cats in the apartments that have at least one cat? Express your answer to the nearest tenth.

Workout 4

171. _____ What common fraction t satisfies the equation $\frac{t}{3t+1} = \frac{4}{5}$?

172. _____ ways

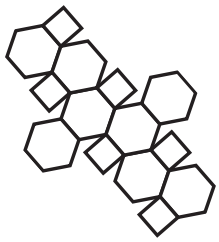
Mr. Scott has five algebra books and four geometry books. He wants to arrange them all on a single shelf. If Mr. Scott keeps all of the algebra books together and all of the geometry books together, how many ways can he arrange these books on his shelf?

173. _____ If $A + B = C + 1$, $B + C = D - 1$, $C + D = E + 1$, $D + E = F - 1$, $E + F = G + 1$, $F + G = A - 1$ and $G + A = B + 1$, what is the value of $A + B + C + D + E + F + G$?

174. _____ cm^2 What is the area of the trapezoid shown, with top base of length 10 cm and sides of lengths 10 cm and 6 cm?

175. _____ tests The mean of Danielle's test scores is 85. If Danielle's lowest test score, which is 61, were to be discarded, the mean of her remaining test scores would be 88. How many tests did Danielle take?

176. _____ vertices This figure shows the net of a three-dimensional shape called a truncated octahedron. How many vertices does a truncated octahedron have?



177. _____ cups Jamie is making pudding, using a recipe that calls for 1.5 cups of milk and 2 cups of flour. He has 7.75 cups of milk and would like to make a batch of pudding using all of the milk. How many cups of flour will he need in order to keep the ratio of ingredients constant? Express your answer as a decimal to the nearest tenth.

178. _____ years old When Shawna turned 21 years old, she was three times as old as Shelby. How old will Shawna be when she is twice as old as Shelby?

179. _____ The sum of a number x and its reciprocal equals $-\frac{17}{4}$. What is the sum of all possible values of x? Express your answer as a common fraction.

180. _____ children There were 9 adults and 11 children at the movie at 11:45 a.m. By 11:50 a.m., 7 more adults and 8 more children were at the movie. At 12:00 p.m., there were 60 adults and children, and the ratio of adults to children was the same as at 11:45 a.m. How many more children came to the movie between 11:50 a.m. and 12:00 p.m.?

Workout 5

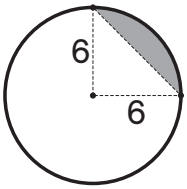
181. _____ If each of the numbers 2, 7, 3 and 8 is assigned to one of the variables a, b, c and d, what is the greatest possible value of $ab + bc + cd$?

182. _____ cm If the volume of the air in an exercise ball is 137,250 cm^3, what is the inside diameter of the ball? Express your answer to the nearest whole number.

183. _____ A square array of numbers is called an *arithmetic square* if each row and column forms an arithmetic progression. If the remaining entries in the array shown are filled in to create an arithmetic square, what is the greatest number in the array?

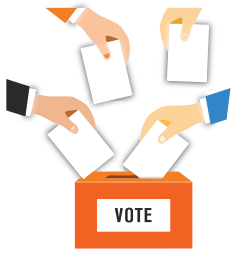
			8	
		36		
			36	
	66			

184. _____ $units^2$



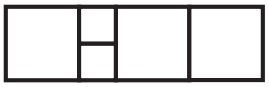
A right triangle is inside a circle, with one vertex of the triangle at the center of the circle and the other two vertices on the circle, as shown. If the circle has radius 6 units, what is the area of the shaded segment? Express your answer as a decimal to the nearest tenth.

185. _____ votes Students at Baldwin Middle School cast 432 votes for student government president. If the number of votes the losing candidate received was 60% of the number that the winner received, how many more votes than the loser did the winner receive?



186. $ _____ Michelle teaches on Tuesday and Thursday evenings. On Tuesdays she makes $112.50, and on Thursdays she makes $135. There are 31 days this month. What is the minimum amount that Michelle will earn this month?

187. _____ ways Including the example shown, how many ways are there to completely cover this 2 × 7 grid with nonoverlapping 1 × 1 and 2 × 2 tiles, if rotations and reflections of these arrangements are considered distinct?



188. _____ If $S_1 = 4$ and $S_n = S_{n-1} + (3 + n)^n$, what is the value of S_3?

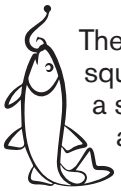
189. _____ The point P(7, 4) is reflected across the line ℓ to the point P′(2, 6). What is the slope of line ℓ? Express your answer as a common fraction.

190. _____ drops A veterinarian needs to prepare a vitamin mixture for parakeets. The ratio is 1 drop of vitamin oil to 75 drops of water. How many drops of vitamin oil are needed to prepare 1520 drops of the mixture?

Workout 6

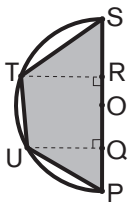
191. _____ units^2 The triangle with vertices A(0, 4), B(3, 0) and C(4, 7) is dilated by a factor of $\sqrt{2}$ about the origin. What is the area of the dilated triangle?

192. _____ prizes A raffle has prizes valued at \$1, \$3, \$15, \$60, \$120, \$360 and \$1800. If exactly \$10,000 in prizes is to be awarded, what is the least number of prizes that can be awarded, assuming that there will be at least one prize of each value awarded?

193. _____ pounds  The estimated weight of a salmon, in pounds, is the product of its length and the square of its girth, both in inches, divided by 775. What is the estimated weight of a salmon with length 28.5 inches and girth 10.25 inches? Express your answer as a decimal to the nearest tenth.

194. _____ Point Q is the reflection of P(a, 4) over the x-axis, and R is the reflection of Q over the y-axis. If the area of $\triangle PQR$ is 12 units^2 and $a > 0$, what is the value of a? Express your answer as a decimal to the nearest tenth.

195. _____ hours Trains pass through a rail yard on five separate tracks at five different frequencies: every 50 minutes, every hour, every 90 minutes, every 2 hours and every 3 hours. If trains pass through on all five tracks at 9:00 a.m., how many hours will elapse before trains next pass through on all five tracks at the same time?

196. _____ units^2 

Quadrilateral PSTU is inscribed in semicircle O, as shown, with PQ = 3 units, QR = 5 units and RS = 4 units. What is the area of quadrilateral PSTU? Express your answer as a decimal to the nearest tenth.

197. _____ units Points A, B, C, D and E on a line are positioned so that B is the midpoint of segment AD, D is three-fourths of the way from A to E, and C is one-fifth of the way from B to E. If AB = 6 units, what is the value of CE?

198. _____ % Vick pays \$50 for 33 pounds of cashews, which he separates into smaller $\frac{3}{8}$-pound bags. If Vick then sells each bag for \$0.79, what percent of this price is profit? Express your answer to the nearest tenth of a percent.

199. \$ _____ To make bracelets, Alice bought 12 red beads and 10 white beads for \$14, while Nevea bought 8 red beads and 15 white beads for \$13.50. What is the cost of a red bead?

200. _____ If $8 - x^2 - y^2 - 2xy = 0$, for real numbers x and y, what is the value of $|x + y|$? Express your answer in simplest radical form.

Competition Coach Toolkit

This is a collection of lists, formulas and terms that Mathletes frequently use to solve problems like those found in this handbook. There are many others we could have included, but we hope you find this collection useful.

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50
$\frac{1}{3}$	$0.\overline{3}$	$33.\overline{3}$
$\frac{1}{4}$	0.25	25
$\frac{1}{5}$	0.2	20
$\frac{1}{6}$	$0.1\overline{6}$	$16.\overline{6}$
$\frac{1}{8}$	0.125	12.5
$\frac{1}{9}$	$0.\overline{1}$	$11.\overline{1}$
$\frac{1}{10}$	0.1	10
$\frac{1}{11}$	$0.\overline{09}$	$9.\overline{09}$
$\frac{1}{12}$	$0.08\overline{3}$	$8.\overline{3}$

Common Arithmetic Series

$$1 + 2 + 3 + 4 + \cdots + n = \frac{n(n+1)}{2}$$

$$1 + 3 + 5 + 7 + \cdots + (2n-1) = n^2$$

$$2 + 4 + 6 + 8 + \cdots + 2n = n^2 + n$$

Prime Numbers

2	43
3	47
5	53
7	59
11	61
13	67
17	71
19	73
23	79
29	83
31	89
37	97
41	

Combinations & Permutations

$${}_nC_r = \frac{n!}{r!(n-r)!} \qquad {}_nP_r = \frac{n!}{(n-r)!}$$

n	n^2	n^3
1	1	1
2	4	8
3	9	27
4	16	64
5	25	125
6	36	216
7	49	343
8	64	512
9	81	729
10	100	1000
11	121	1331
12	144	1728
13	169	2197
14	196	2744
15	225	3375

Geometric Mean

$$\frac{a}{x} = \frac{x}{b} \quad \text{and} \quad x = \sqrt{ab}$$

Divisibility Rules

2: units digit is 0, 2, 4, 6 or 8

3: sum of digits is divisible by 3

4: two-digit number formed by tens and units digits is divisible by 4

5: units digit is 0 or 5

6: number is divisible by both 2 and 3

8: three-digit number formed by hundreds, tens and units digits is divisible by 8

9: sum of digits is divisible by 9

10: units digit is 0

Equation of a Line

Standard Form

$Ax + By = C$

Slope-Intercept Form

$y = mx + b$

m = slope $\qquad$ b = y-intercept

Point-Slope Form

$y - y_1 = m(x - x_1)$

m = slope $\qquad$ (x_1, y_1) = point on the line

Distance Traveled

Distance = Rate × Time

Quadratic Formula

For $ax^2 + bx + c = 0$, where $a \neq 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagorean Triples

(3, 4, 5)	(5, 12, 13)	(7, 24, 25)
(8, 15, 17)	(9, 40, 41)	(12, 35, 37)

Difference of Squares

$$a^2 - b^2 = (a + b)(a - b)$$

Sum and Difference of Cubes

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

Circles

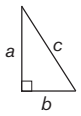
Circumference	Area
$2 \times \pi \times r = \pi \times d$	$\pi \times r^2$

For radius r

Arc Length	Sector Area
$\frac{x}{360} \times 2 \times \pi \times r$	$\frac{x}{360} \times \pi \times r^2$

For central angle of x degrees

Pythagorean Theorem



$a^2 + b^2 = c^2$

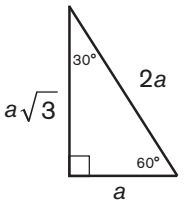
Given A(x_1, y_1) and B(x_2, y_2)

Distance from A to B = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

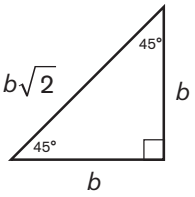
Midpoint of $\overline{AB}$ = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Slope of $\overline{AB}$ = $\frac{y_2 - y_1}{x_2 - x_1}$

Special Right Triangles



30-60-90 Right Triangle



45-45-90 Right Triangle

Area of Polygons

Square	side length s	s^2
Rectangle	length l, width w	$l \times w$
Parallelogram	base b, height h	$b \times h$
Trapezoid	bases b_1, b_2, height h	$\frac{1}{2}(b_1 + b_2) \times h$
Rhombus	diagonals d_1, d_2	$\frac{1}{2} \times d_1 \times d_2$
Triangle	base b, height h	$\frac{1}{2} \times b \times h$
Triangle	semi-perimeter s, side lengths a, b, c	$\sqrt{s(s-a)(s-b)(s-c)}$
Equilateral Triangle	side length s	$\frac{s^2\sqrt{3}}{4}$

Polygon Angles (n sides)

Sum of the interior angle measures:

$180 \times (n - 2)$

Central angle measure of a regular polygon:

$\frac{360}{n}$

Interior angle measure of a regular polygon:

$\frac{180 \times (n - 2)}{n}$ or $180 - \frac{360}{n}$

Solid	Dimensions	Surface Area	Volume
Cube	side length s	$6 \times s^2$	s^3
Rectangular Prism	length l, width w, height h	$2 \times (l \times w + w \times h + l \times h)$	$l \times w \times h$
Cylinder	circular base radius r, height h	$2 \times \pi \times r \times h + 2 \times \pi \times r^2$	$\pi \times r^2 \times h$
Cone	circular base radius r, height h	$\pi \times r^2 + \pi \times r \times \sqrt{r^2 + h^2}$	$\frac{1}{3} \times \pi \times r^2 \times h$
Sphere	radius r	$4 \times \pi \times r^2$	$\frac{4}{3} \times \pi \times r^3$
Pyramid	base area B, height h		$\frac{1}{3} \times B \times h$

Vocabulary & Terms

The following list is representative of terminology used in the problems but **should not** be viewed as all-inclusive. It is recommended that coaches review this list with their Mathletes.

absolute difference
absolute value
acute angle
additive inverse *(opposite)*
adjacent angles
apex
arithmetic mean
arithmetic sequence
base ten
binary
bisect
box-and-whisker plot
center
chord
circumscribe
coefficient
collinear
common divisor
common factor
common fraction
complementary angles
congruent
convex
coordinate plane/system
coplanar
counting numbers
counting principle
diagonal of a polygon
diagonal of a polyhedron
digit sum
dilation
direct variation
divisor
domain of a function
edge
equiangular
equidistant
expected value
exponent
exterior angle of a polygon
factor
finite
frequency distribution
frustum
function
GCF (GCD)
geometric sequence
hemisphere
image(s) of a point(s) *(under a transformation)*
improper fraction
infinite series
inscribe
integer
interior angle of a polygon
intersection
inverse variation
irrational number
isosceles
lateral edge
lateral surface area
lattice point(s)
LCM
median of a set of data
median of a triangle
mixed number
mode(s) of a set of data
multiplicative inverse *(reciprocal)*
natural number
obtuse angle
ordered pair
origin
palindrome
parallel
Pascal's Triangle
percent increase/decrease
perpendicular
planar
polyhedron
polynomial
prime factorization
principal square root
proper divisor
proper factor
proper fraction
quadrant
quadrilateral
random
range of a data set
range of a function
rate
ratio
rational number
ray
real number
reciprocal *(multiplicative inverse)*
reflection
regular polygon
relatively prime
revolution
right angle
right polyhedron
rotation
scalene triangle
scientific notation
sector
segment of a circle
segment of a line
semicircle
semiperimeter
sequence
set
significant digits
similar figures
slope
space diagonal
square root
stem-and-leaf plot
supplementary angles
system of equations/inequalities
tangent figures
tangent line
term
transformation
translation
triangular numbers
trisect
twin primes
union
unit fraction
variable
whole number
y-intercept

PROBLEM INDEX

It is very difficult to categorize many of the problems in this handbook. MATHCOUNTS problems often straddle multiple categories and cover several concepts, but in this index, we have placed each problem in exactly one category and mapped it to exactly one Common Core State Standard (CCSS). **In this index, code 9 (3) 7.SP.3 would refer to problem #9 with difficulty rating 3 mapped to CCSS 7.SP.3.** The difficulty rating and CCSS mapping are explained below.

DIFFICULTY RATING: Our scale is 1-7, with 7 being most difficult. These general ratings are only approximations:

- **1, 2 or 3**: Appropriate for students just starting the middle school curriculum; 1 concept; 1- or 2-step solution.
- **4 or 5**: Knowledge of some middle school topics necessary; 1-2 concepts; multi-step solution.
- **6 or 7**: Knowledge of advanced middle school topics and/or problem-solving strategies necessary; multiple and/or advanced concepts; multi-step solution.

COMMON CORE: We align our problems to the NCTM Standards for Grades 6-8, however we also have mapped these problems to CCSS because 42 states, D.C., 4 territories and the Dept. of Defense Education Activity (DoDEA) have voluntarily adopted it. Our CCSS codes contain (in this order):

1. **Grade level** in the K-8 Standards for Mathematical Content (SMC). Courses that are in the high school SMC instead have the first letter of the course name.
2. **Domain** within the grade level or course and then the **individual standard**.

Here are 2 examples:

- *6.RP.3 → Standard #3 in the Ratios and Proportional Relationships domain of grade 6*
- *G-SRT.6 → Standard #6 in the Similarity, Right Triangles and Trigonometry domain of Geometry*

Some math concepts are not specifically mentioned in CCSS. For problems using these concepts, we use the code of a related standard, when possible. Some of our problems are based on concepts outside the scope of CCSS or are based on concepts in the K-5 SMC but are more difficult than a grade K-5 problem. When appropriate, we coded these problems SMP for the CCSS Standards for Mathematical Practice.

STATISTICS

11	(3)	6.SP.5
12	(2)	6.SP.4
16	(3)	6.SP.2
17	(3)	6.SP.5
18	(4)	7.SP.3
74	(3)	7.SP.3
83	(2)	6.SP.2
96	(3)	6.SP.2
143	(2)	6.SP.2
149	(2)	6.SP.4
158	(2)	6.SP.5
161	(2)	7.NS.2
170	(3)	6.SP.5
175	(3)	7.SP.3

PLANE GEOMETRY

34	(2)	8.G.5
59	(3)	8.G.5
75	(3)	SMP
82	(3)	SMP
92	(3)	7.G.6
110	(5)	G-SRT.6
114	(3)	8.G.7
140	(5)	7.G.6
150	(3)	7.G.5
159	(3)	7.G.6
165	(4)	SMP
184	(5)	G-C.2
196	(6)	G-SRT.6

MEASUREMENT

44	(2)	5.MD.1
67	(3)	7.G.6
79	(4)	7.G.6
116	(4)	SMP
121	(2)	7.G.6
163	(4)	G-SRT.6
174	(3)	7.G.6
182	(3)	G-GMD.3
193	(2)	6.EE.7

PROPORTIONAL REASONING

40	(2)	6.RP.2
57	(3)	7.RP.1
88	(2)	6.RP.3
146	(2)	6.RP.1
147	(1)	7.RP.1
151	(3)	6.RP.3
167	(4)	6.RP.3
177	(3)	6.RP.3
180	(4)	SMP
190	(2)	6.RP.3

COORDINATE GEOMETRY

53	(1)	5.G.4
63	(1)	8.G.8
118	(3)	8.F.4
156	(4)	7.G.6
189	(4)	G-GPE.5
191	(5)	8.G.3
194	(4)	7.G.6

SOLID GEOMETRY

32	(1)	7.G.3
102	(2)	7.G.6
120	(4)	8.G.9
126	(5)	7.G.6
138	(4)	8.G.9
176	(5)	7.G.6

PERCENTS & FRACTIONS

1	(1)	6.RP.3
2	(2)	6.RP.3
3	(1)	6.RP.3
4	(3)	6.RP.3
5	(4)	6.RP.3
6	(4)	6.RP.3
7	(5)	6.RP.3
8	(4)	6.RP.3
9	(3)	6.RP.3
10	(4)	7.RP.1
13	(3)	6.RP.1
15	(4)	7.NS.3
19	(3)	6.RP.3
20	(5)	7.RP.3
23	(2)	SMP
45	(1)	6.NS.1
49	(2)	7.RP.3
66	(2)	7.RP.3
69	(3)	6.NS.1
78	(3)	7.RP.3
91	(1)	6.RP.3
94	(1)	6.NS.1
124	(3)	SMP
145	(2)	6.RP.3
154	(3)	6.RP.3
185	(4)	6.RP.3
197	(3)	7.NS.2
198	(4)	7.NS.3

NUMBER THEORY

33	(1)	6.NS.2
43	(2)	6.NS.4
54	(2)	6.EE.1
56	(2)	SMP
58	(3)	6.NS.4
60	(4)	SMP
77	(1)	7.NS.2
80	(4)	SMP
90	(3)	7.NS.3
93	(2)	8.EE.4
98	(4)	A-SEE.2
104	(3)	8.NS.1
106	(2)	6.EE.1
109	(4)	SMP
115	(4)	N-RN.2
125	(3)	N-RN.2
132	(3)	SMP
133	(5)	N-RN.2
139	(4)	SMP
141	(2)	6.NS.4
148	(2)	7.NS.1
152	(3)	6.EE.9
155	(3)	6.NS.4
160	(3)	6.NS.4
166	(3)	6.NS.4
168	(1)	6.NS.4
192	(4)	SMP
195	(4)	6.RP.3

ALGEBRAIC EXPRESSIONS & EQUATIONS

26	(4)	A-APR.5
27	(4)	A-APR.5
28	(5)	A-APR.5
38	(2)	6.EE.2
39	(2)	6.EE.6
47	(2)	6.EE.6
70	(3)	6.EE.9
73	(2)	7.NS.2
81	(2)	6.EE.1
84	(3)	7.EE.4
87	(3)	6.EE.2
95	(3)	8.EE.8
103	(1)	6.EE.2
105	(3)	8.F.2
112	(2)	8.EE.7
119	(2)	SMP
122	(3)	8.F.1
129	(4)	A-REI.4
131	(3)	8.F.1
136	(4)	8.EE.2
142	(1)	SMP
164	(2)	7.NS.3
171	(3)	6.EE.6
173	(4)	8.EE.8
178	(3)	7.EE.4
179	(4)	A-REI.4
199	(4)	7.EE.4
200	(5)	A-REI.2

PROBABILITY, COUNTING & COMBINATORICS

24	(3)	S-CP.9
29	(4)	7.SP.8
36	(1)	7.SP.7
50	(3)	S-CP.9
76	(2)	7.SP.6
97	(4)	S-CP.6
100	(4)	S-CP.9
111	(4)	S-CP.6
113	(4)	7.SP.8
123	(3)	7.SP.8
128	(4)	S-CP.7
130	(4)	SMP
134	(3)	S-CP.9
172	(3)	S-CP.9
187	(5)	S-CP.9

PROBLEM SOLVING (MISCELLANEOUS)

21	(3)	SMP
22	(2)	SMP
25	(3)	SMP
30	(7)	S-CP.9
41	(1)	SMP
48	(3)	6.NS.1
51	(1)	SMP
86	(3)	SMP
89	(3)	7.EE.4
107	(3)	SMP
117	(4)	6.EE.7
127	(3)	6.NS.4
135	(3)	SMP
137	(3)	SMP
144	(2)	7.RP.1
153	(2)	SMP
157	(2)	7.NS.3
169	(4)	7.NS.3

GENERAL MATH

31	(1)	6.NS.3
35	(1)	3.OA.2
37	(1)	5.NBT.2
42	(2)	7.NS.1
52	(1)	7.NS.1
55	(1)	6.NS.3
61	(1)	7.RP.1
65	(2)	3.OA.8
71	(2)	SMP
72	(1)	6.NS.3
85	(2)	8.EE.1
101	(2)	6.NS.1

LOGIC

14	(5)	SMP
64	(2)	F-BF.2
68	(3)	SMP
99	(4)	SMP
162	(1)	SMP
181	(4)	SMP
186	(4)	SMP

SEQUENCES, SERIES & PATTERNS

46	(2)	F-IF.3
62	(1)	F-IF.3
108	(3)	F-BF.2
183	(3)	F-BF.2
188	(3)	F-IF.3

COACHES:
FIND PROBLEMS, ANSWERS, SOLUTIONS + PROBLEM INDEX FOR PROBLEMS 201–250 at www.mathcounts.org/coaches!

ANSWERS

MIXTURE STRETCH

1.	1*	(1)
2.	7/20	(2)
3.	3/7	(1)
4.	14	(3)
5.	8	(4)
6.	1.74	(4)
7.	1.25	(5)
8.	4	(4)
9.	2	(3)
10.	5.2	(4)

STATISTICS STRETCH

11.	73	(3)
12.	3216	(2)
13.	5/8	(3)
14.	36	(5)
15.	13/22	(4)
16.	29	(3)
17.	2.36	(3)
18.	42	(4)
19.	60	(3)
20.	18.3	(5)

PASCAL'S TRIANGLE STRETCH

21.	6435	(3)
22.	7	(2)
23.	4096	(2)
24.	56	(3)
25.	220	(3)
26.	1792	(4)
27.	81	(4)
28.	1011	(5)
29.	11/16	(4)
30.	8	(7)

WARM-UP 1

31.	872,000	(1)
32.	27	(1)
33.	0	(1)
34.	100	(2)
35.	13	(1)
36.	5/9	(1)
37.	10	(1)
38.	67	(2)
39.	6	(2)
40.	8	(2)

WARM-UP 2

41.	4:12	(1)
42.	19	(2)
43.	60	(2)
44.	11	(2)
45.	10	(1)
46.	57	(2)
47.	−9	(2)
48.	72	(3)
49.	6	(2)
50.	17	(3)

WARM-UP 3

51.	5	(1)
52.	10	(1)
53.	7	(1)
54.	1/9	(2)
55.	6.666	(1)
56.	15	(2)
57.	3	(3)
58.	15	(3)
59.	58	(3)
60.	37	(4)

WARM-UP 4

61.	42	(1)
62.	79	(1)
63.	5	(1)
64.	25	(2)
65.	98	(2)
66.	40	(2)
67.	$9\frac{7}{20}$	(3)
68.	4	(3)
69.	4/3	(3)
70.	4	(3)

WARM-UP 5

71.	111	(2)
72.	59/50	(1)
73.	10/11	(2)
74.	80	(3)
75.	5/18	(3)
76.	1/4	(2)
77.	−4	(1)
78.	3 or 3.00	(3)
79.	4	(4)
80.	0	(4)

WARM-UP 6

81.	6	(2)
82.	1/2	(3)
83.	12	(2)
84.	111	(3)
85.	0	(2)
86.	3/16	(3)
87.	4	(3)
88.	10	(2)
89.	24	(3)
90.	10	(3)

WARM-UP 7

91.	28	(1)
92.	100	(3)
93.	200	(2)
94.	22	(1)
95.	60	(3)
96.	12	(3)
97.	1/3	(4)
98.	40	(4)
99.	12	(4)
100.	28,800	(4)

WARM-UP 8

101.	12	(2)
102.	36	(2)
103.	18	(1)
104.	11/7	(3)
105.	1/3	(3)
106.	2	(2)
107.	19	(3)
108.	27	(3)
109.	4	(4)
110.	$2\sqrt{6}$	(5)

WARM-UP 9

111.	19/66	(4)
112.	2	(2)
113.	1/16	(4)
114.	$\sqrt{10}$	(3)
115.	8×10^{21}	(4)
116.	1	(4)
117.	25	(4)
118.	9/2	(3)
119.	311	(2)
120.	81π	(4)

WARM-UP 10

121.	24	(2)
122.	−50	(3)
123.	7/20	(3)
124.	3/5	(3)
125.	1/3	(3)
126.	72	(5)
127.	10,467	(3)
128.	5/12	(4)
129.	8	(4)
130.	34	(4)

WARM-UP 11

131.	10	(3)
132.	199	(3)
133.	27	(5)
134.	720	(3)
135.	4	(3)
136.	2	(4)
137.	6	(3)
138.	$2\sqrt{21}$	(4)
139.	63	(4)
140.	72/5	(5)

WORKOUT 1

141.	270	(2)
142.	10	(1)
143.	356	(2)
144.	0.31	(2)
145.	59	(2)
146.	720	(2)
147.	1115	(1)
148.	289	(2)
149.	4.5	(2)
150.	54	(3)

WORKOUT 2

151.	1.64	(3)
152.	1*	(3)
153.	9.41	(2)
154.	125	(3)
155.	216,000	(3)
156.	12	(4)
157.	14.25	(2)
158.	504	(2)
159.	7.5	(3)
160.	7	(3)

WORKOUT 3

161.	1.75	(2)
162.	35	(1)
163.	13.9	(4)
164.	160	(2)
165.	12	(4)
166.	10,101	(3)
167.	326,000	(4)
168.	33	(1)
169.	70	(4)
170.	1.2	(3)

WORKOUT 4

171.	−4/7	(3)
172.	5760	(3)
173.	1	(4)
174.	84	(3)
175.	9	(3)
176.	24	(5)
177.	10.3	(3)
178.	28	(3)
179.	−17/4	(4)
180.	14	(4)

WORKOUT 5

181.	94	(4)
182.	64	(3)
183.	79	(3)
184.	10.3	(5)
185.	108	(4)
186.	990 or 990.00	(4)
187.	21	(5)
188.	245	(3)
189.	5/2	(4)
190.	20	(2)

WORKOUT 6

191.	25	(5)
192.	16	(4)
193.	3.9	(2)
194.	1.5	(4)
195.	30	(4)
196.	46.2	(6)
197.	8	(3)
198.	28.1	(4)
199.	0.75	(4)
200.	$2\sqrt{2}$	(5)

** The plural form of the units is always provided in the answer blank, even if the answer appears to require the singular form of the units.*

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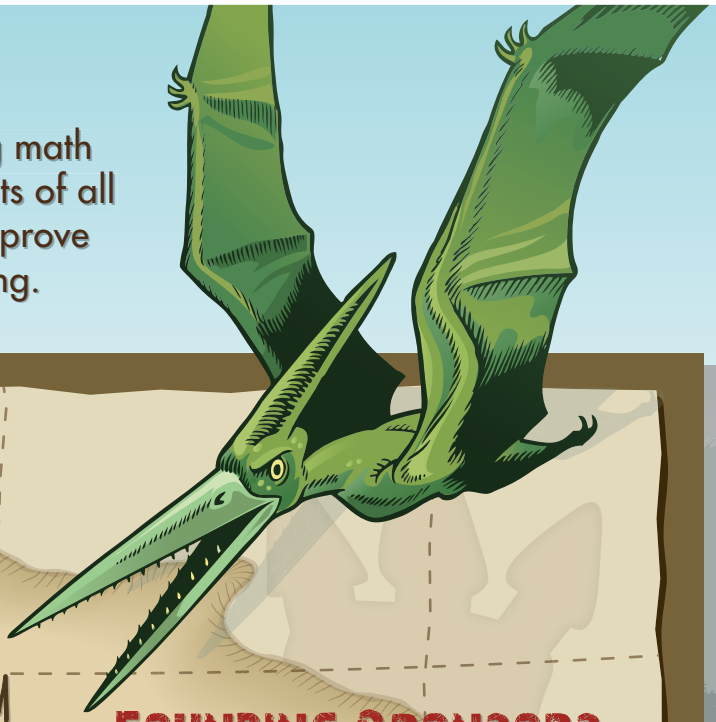

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