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The development of mathematical knowledge in traditional societies.**A study of Malagasy divination**

We present fieldworks and analyses on Malagasy divination dealing with the status of diviners, the rules of their divination system and the underlying mathematical knowledge embedded in it. Some psychological experiments involved in the study of this knowledge are also described. The main feature of this research is a database of notebooks written by diviners to keep particular arrangements of seeds used in divination.

Introduction

Claude Lévi-Strauss has pointed out in his famous book *The Savage Mind* the fact that traditional societies have developed efficient techniques for their practical needs in a way which is similar to our scientific approach:

“It was in neolithic times that man’s mastery of the great arts of civilization of pottery, weaving, agriculture and the domestication of animals became firmly established. No one today would any longer think of attributing these enormous advances to the fortuitous accumulation of a series of chance discoveries or believe them to have been revealed by the passive perception of certain natural phenomena. Each of these techniques assumes centuries of active and methodical observation, of bold hypotheses tested by means of endlessly repeated experiments” (Lévi-Strauss, 1962, p. 13).

Furthermore, he observed that the concrete applications of techniques developed in traditional societies are not the unique motivation of what he calls the “savage mind”. Besides the resolution of practical problems, it also addresses problems at a purely intellectual level and reveals a “thirst for objective knowledge” (Ibid., p. 3), which is similar to ours and is one of the most neglected aspects of the thought of people we call “primitive”:

“Animals and plants are not known as a result of their usefulness; they are deemed to be useful or interesting because they are first of all known. It may be objected that science of this kind can scarcely be of much practical effect. The answer to this is that its main purpose is not a practical one. It meets intellectual requirements rather than or instead of satisfying needs” (Ibid., p. 9).

In the first pages of his book he analyzes the distinction between science and magic. Lévi-Strauss points out the fact that one should not reduce magic “to a moment or stage in technical and scientific evolution”. According to him, both science and magic “require the same sort of mental operations and they differ not so much in kind as in the different types of phenomena to which they are applied” (Ibid., p. 13). Thus the human mind is logical whatever it be ‘savage’ or not:

“The false antinomy between logical and prelogical mentality was surmounted at the same time. The savage mind is logical in the same sense and the same fashion as ours, though as our own is only when it is applied to knowledge of a universe in which it recognizes physical and semantic properties simultaneously. This misunderstanding once dispelled, it remains no less true that, contrary to Lévy-Bruhl’s opinion, its thought proceeds through understanding, not affectivity, with the aid of distinctions and oppositions, not by confusion and participation” (Ibid., p. 268).

These considerations have opened the door for a study of mathematical knowledge in traditional societies. The numeral concepts have become the subject of different researches involving both cognitive and cultural aspects, in particular the relation between arithmetic and language. For instance there exist researches dealing with numerical cognition in speakers of Mundurucu, an Amazonian indigene group, who have a very small lexicon of number words (Pica *et al.*, 2004). And since mathematics cannot be reduced to the manipulation of numbers, geometry provides another subject which leads to cognitive and cross-cultural approaches (Dehaene *et al.*, 2006).

But how can we talk about ‘mathematics’ in societies lacking the support of writing for the development of their abstract knowledge? Philip Davis and Reuben Hersh have brought their insightful reflection to this question by distinguishing analytic and analog aspects of mathematics:

“In analytic mathematics, the symbolic material predominates. It is almost always hard to do. It is time consuming. It is fatiguing. It requires special training. It may require constant verification by the whole mathematical culture to assure reliability. Analytic mathematics is performed only by very few people. Analytic mathematics is elitist and self-critical. The practitioners of its higher manifestations form a ‘talentocracy’. The great virtue of analytic mathematics arises from this, that while it be may impossible to verify another’s intuitions, it is possible, though often difficult, to verify his proofs” (Davis, Hersh, 1982, p. 303).

Besides this analytic dimension of mathematics, there exist another one they called “analog”, which relies mostly on intuition and is much closer to every day life, but which is not necessarily elementary and can be related to complex ideas:

“Results [of analog mathematics] may be expressed not in words but in ‘understanding’, ‘intuition’, or ‘feeling’” (Ibid., p. 303).

When we talk about mathematics in non-literate societies, as we shall do in this paper, we assume it is this *analog part of mathematics* that is dealt with.

Divination in Madagascar

The starting point of these works is an article by Marcia Ascher on the mathematics of Malagasy divination (Ascher, 1997). This traditional activity is very popular in the country where it has been practiced for many centuries. It has an Arabic origin (Bloch, 1968, Verin *et al.*, 1991) and one can see similar practices in other parts of Africa. The divination system used in Tchad, for instance, has been the subject of mathematical investigations in the sixties by Robert Jaulin in collaboration with various mathematicians (Jaulin, 1966). We have begun in 2001 a research on Malagasy divination conducted by a team of anthropologists, psychologists and computer scientists including Victor Randriany, Denis Jacquet, Marc Zabalía and Bruno Crémilleux with a grant from the French ‘Ministère de la Recherche’ (ACI ‘Cognitive’ and ‘Histoire des savoirs’).

In Madagascar, the diviner called *mpisikidy* or *ombiasy* is an expert with an important position in the traditional society. Whereas everybody knows the basic rules of the divination system, the professional diviner has a specific knowledge. His is a specialist in guiding people, and his position is related to three main features:

- First, he owns specific sacred horns of zebu, called *mohara*,
- Secondly, he is an expert in the knowledge of medicinal plants and woods, called *volohaza*,
- Finally, he knows elaborated properties of a system of computation with seeds, which is designed by the word *sikidy*.

The most important aspect of the system of computation with seeds used by diviners is the construction of tableaux. Each of the entries in the tableau can be one seed or two seeds. The tableau has two parts. The upper part is called *mother-sikidy*, and it has four rows and four columns. The lower part has eight columns of four entries each which are called *daughter-columns*. The daughters are deduced from the mother-*sikidy* by applying a simple mathematical rule:

$$1 + 1 = 2 + 2 = 2,$$

$$1 + 2 = 2 + 1 = 1.$$

The order of generation of daughter-columns is illustrated by the following example. The rows are read from right to left, thus the outcome at position 5 will be denoted by (2 2 1 1):

	4	3	2	1	
	5
	6
	7
	8

..
..
.
.
9	10	11	12	13	14
15	16				

- At the beginning, the diviner calculates the outcomes at position 15 by combining positions 1 and 2 which gives (2 2 2 2), and at position 13 by combining positions 3 and 4, then he computes a second generation daughter between them at position 14 by combining 13 and 15 which gives (2 2 1 1).
- The same holds for the combination of mother-*sikidy* rows in the left part of the tableau. He calculates position 11 by combining 5 and 6, and position 9 by combining 7 and 8, then he computes another second generation daughter between them at position 10 by combining 9 and 11.
- Next step is to compute the third generation daughter at position 12 by combining 10 and 14.
- The last outcome at position 16 is obtained from position 12 and position 1.

There are 16 possible outcomes appearing in the tableaux that we shall call *figures*. They are classified in different ways. First of all, diviners make a distinction between figures with an even number of seeds which are called “princes” (*mpanjaka*), for instance (2 2 1 1) with 6 seeds, and the others which are called “slaves” (*andevo*), for instance (1 1 1 2) with 5 seeds.

There also exists a classification of the 16 figures according to the 4 cardinal directions:

- North: (1 2 2 2), (2 2 1 2), (1 1 1 2), (1 2 1 2)
- West: (2 2 2 1), (1 2 1 1), (1 2 2 1), (2 1 2 1), (2 1 1 1)
- East: (2 1 2 2), (2 2 1 1), (2 1 1 2)
- South: (2 2 2 2), (1 1 1 1), (1 1 2 2), (1 1 2 1)

Some tableaux with particular positions of the outcomes are considered as strongly powerful at the symbolic level. The diviner put dust on some of the outcomes occurring in the tableau, and then, the dust is used as a strong talisman able to cure an illness.

The first type of these curative tableaux is called *toka* (or *tokan-tsikidy*) and it refers to tableaux where *one of the cardinal directions is represented only once*, by a unique outcome (among the four rows and columns of the mother-*sikidy* and the eight daughter-columns). For instance, the following tableau is a double *toka* (East and South):

	w	w	e	s					
	w				
	n				
	n				
	w				

	w	n	w	w	n	w	w	n	

The other type of tableaux used by diviners is called *fohatse*, and it refers to tableaux where *one outcome is repeated at least eight times in the tableau* (among the four rows and columns of the mother-*sikidy* and the eight daughter-columns). Our first example above shows a *fohatse* with (2 2 1 1) repeated exactly nine times (at positions 1, 2, 5, 6, 9, 10, 13, 14 and 16).

Besides the curative function of particular tableaux, there exists another way to use them which is related to the divining practice itself. In this case, *the mother-sikidy is chosen randomly*, by taking 16 piles of seeds and reducing them to one or two seeds by deleting them two at a time.

From a mathematical point of view, there is an algebraic group structure beneath the construction of the tableaux. In fact, the addition of figures is a group operation with neutral element (2 2 2 2) and each figure is its own inverse. There also exist different kind of *subgroups* underlying the mathematical knowledge of the diviners. For instance,

when one combines two *mpanjaka* (figure with an even number of seeds), the result is still *mpanjaka*. Thus the group operation preserves the set of *mpanjaka* figures, which means that it forms a subgroup with eight elements. One can consider the quotient group which has two elements $\{mpanjaka, andevo\}$. It also has a group structure where the combination of classes follows a general law which can be represented by a two dimension table:

	<i>mpanjaka</i>	<i>andevo</i>
<i>mpanjaka</i>	<i>mpanjaka</i>	<i>andevo</i>
<i>andevo</i>	<i>andevo</i>	<i>mpanjaka</i>

This means for instance that one has the following general rule: *mpanjaka* + *andevo* = *andevo*. We have brought evidences of the fact that Malagasy diviners are aware of such abstract combination laws (Chemillier *et al.*, 2007; Chemillier, 2007).

Does the subgroup structure apply to the distribution of the figures into four cardinal point classes? This is not the case in Malagasy classification. But we have shown in (Chemillier, 2007) that it derives from older Arabic classifications. One of them is called *système au repos* (not related to the cardinal points) and it has been studied by Robert Jaulin (1966):

- First class: (2 2 2 1), (2 1 2 2), (1 2 1 2), (1 1 1 1)
- Second class: (2 2 1 2), (2 1 1 1), (1 2 2 2), (1 1 2 1)
- Third class: (2 2 1 1), (2 1 1 2), (1 2 2 1), (1 1 2 2)
- Fourth class: (1 2 1 1), (1 1 1 2), (2 2 2 2), (2 1 2 1)

Jaulin observed that in each class, the figures can be grouped by two such that their combination gives (2 1 2 1). For instance in the first class, the combination of (2 2 2 1) and (2 1 2 2) gives (2 1 2 1). One can prove that in this case the class with four elements containing the neutral figure (2 2 2 2) is necessarily a subgroup. Indeed, the neutral element can be combined with another one of the class such that it gives (2 1 2 1). Thus (2 1 2 1) must belong to the class. Then the combination of the two other elements of the class gives (2 1 2 1), which is still in the class. This proves that the class is preserved by the group operation, and thus it forms a subgroup. There are only two nonisomorphic groups with 4 elements : (i) the cyclic group and (ii) the Klein group where every element is its own inverse. Both share Jaulin property but only the Klein group can be a subgroup of the sikidy group.

In the “système au repos” studied by Jaulin, the equivalence classes are not compatible with the group operation, which implies that they do not form a quotient group.” This means that the classes cannot be combined at an abstract level. But there exist other classifications in Arabic tradition that satisfy this property, such as the following one taken from a manuscript of the Bibliothèque Nationale de France (ms. ar. 2697 f° 5v, see Moulis 2004):

- First class: (2 1 2 2), (1 1 1 1), (2 2 2 1), (1 2 1 2)
- Second class: (1 2 2 1), (2 2 1 2), (1 1 2 2), (2 1 1 1)
- Third class: (1 2 1 1), (2 2 2 2), (1 1 1 2), (2 1 2 1)
- Fourth class: (2 1 1 2), (1 1 2 1), (2 2 1 1), (1 2 2 2)

One can verify that the third class with four elements *is a subgroup for the group operation*. Furthermore, the classes can be combined in a consistent way by applying the following rules:

	First	Second	Third	Fourth
First	Third	Fourth	First	Second
Second	Fourth	Third	Second	First
Third	First	Second	Third	Fourth
Fourth	Second	First	Fourth	Third

As an example, the table indicates that Second + First = Fourth, which means that one can take any figures of these classes and verify the rule. For instance (2 1 1 1) and (1 1 1 1) are in the second and first classes respectively, and their combination gives (1 2 2 2) which, as expected, belongs to the fourth class.

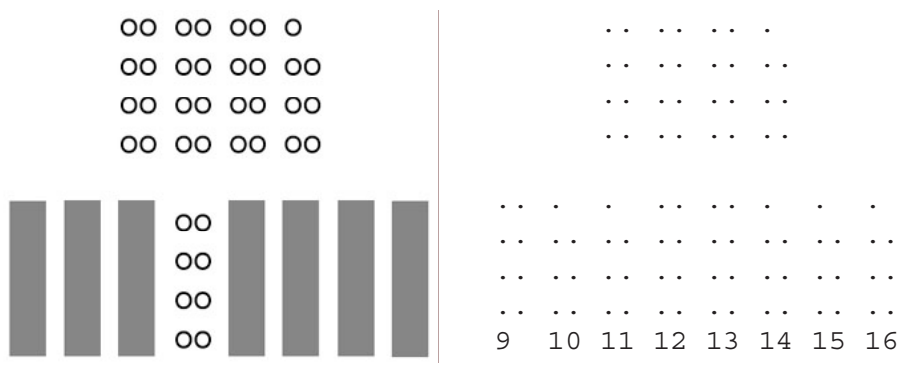
Cognitive experiments about mental calculations

Generally speaking, the mathematical aspects of activities such as divination are not associated with spoken descriptions from people doing them. It is thus difficult to analyze the way they conceive mathematical notions such as the algebraic structures described above. Dan Sperber has pointed out the fact that psychology and anthropology could be “highly relevant to one another in answering some of their respective traditional questions, and in formulating new common questions” (Sperber, 1996). Paul Dixon has addressed the conditions of the development of mathematical knowledge both on a neurophysiological and a cultural basis (Dixon, 2002). When studying logical aspects of diviner’s mathematical knowledge, the language seems to be unable to access the real mental representations of the native experts. Every notion in their mind is related to many other aspects at the semantic level. There is no formal logic *stricto sensu* according to their point of view. In such cases, psychology offers interesting methods to overcome this problem by means of computerized experiments.

One of the subjects that we have studied is the fact that diviners are able to calculate all the daughters of a tableau *without manipulating the seeds*, just by mentally calculating them from the mother-*sikidy*. Indeed, we have observed that some diviners do not compute the daughters in the order implied by their definition (daughters of the first generation *before* those of the second generation, and so on). For instance, one of them always computes the daughters from right to left, which means that he first calculates

It appears that the daughter at position 14 is calculated before the one at position 13, whereas it depends on it since $14 = 13 + 15$. In the same manner, the daughter at position 10 is calculated before the one at position 9 whereas $10 = 9 + 11$. This result proves that the diviner is able to calculate the second generation daughters without the first generations ones, directly from the mother-*sikidy*.

In this second example, the calculation goes much faster. In fact, the mother-*sikidy* is symmetrical because each row is equal to the corresponding column (from right to left). This implies that the daughter at position 12 must be equal to (2 2 2 2). The diviners are aware of this mathematical property, and in such cases they are able to answer instantly.



A database of notebooks used by diviners

Divination is not a purely oral activity because it appears that during fieldworks we have made on the subject in Madagascar, we discovered that diviners use notebooks to keep particular tableaux of seeds considered as powerful (mostly *toka* or *fohatse*).

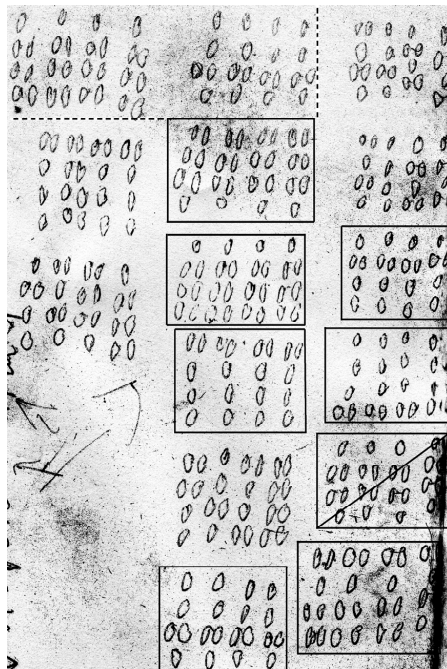
In his analysis of the effect of writing on “modes of thoughts”, Jack Goody has pointed out two main functions of writing:

“One is the storage function, that permits communication over time and space, and provides man with a marking mnemonic and recording device. Clearly this function could also be carried out by other means of storage such as the tape-recording of messages. However, the use of aural reproduction would not permit the second function of writing, which shifts language from the aural to the visual domain, and makes possible a different kind of inspection, the re-ordering and refining not only of sentences, but of individual words” (Goody, 1977, p. 78).

This applies very well to different types of written objects which are not necessarily words but also, for instance, graphical representations of *sikidy* tableaux where circles are used to denote seeds. The notation of tableaux on the page of a notebook allows the diviner to make “different kind of inspection” by re-ordering them according to some specific properties. As an example, we show a page from a diviner’s notebook (only the mother-*sikidy* are written).

One can see that many mother-*sikidy* on this page share the remarkable property that their four mother-columns are equal. For instance the first mother-*sikidy* at the top left corner of the page has the same figure (1 2 2 2) in its four columns. The fact that these tableaux are gathered on the same page by a diviner is a clear evidence of the existence of a mental process which has reported this shared property (on the picture, the rectangles have been added to underline the mother-*sikidy*, but they are not on the original diviner’s page).

Furthermore, one can proceed to a few mathematical deductions about these tableaux. For instance, is it possible for a tableau with four equal mother-columns to be *toka*? The answer is yes, but with a restrictive condition. The repeated column of the mother-*sikidy* must be a slave (*andevò*), that is to say a figure with an odd number of seeds. We leave the proof of this assertion to the reader (Chemillier 2007). But the question is whether or not such deductions are made by the diviners themselves. We have no answer yet, but the analysis of their notebooks will certainly provide other types of shared properties and other possible deductions about them.



We have collected some of these notebooks and they are stored in a database which is available online. The database was developed by J r my Hienne and St phane Goselin, and the Web site has been designed by Seheno Raonizanany and Ravaka Rasolo-manana: <http://ehess.modelisationsavoirs.fr/sikidy>

Every page is presented as a fac-simile on the left side, and the tableaux are displayed in full form with the daughter-columns and the cardinal points indicated by letters (n, o, e, s). A small window allows the user to make a request by selecting properties such as *toka* or *fohatse* and choosing particular figures or positions. The results are displayed with colors added to the tableaux satisfying the desired property. For instance, the picture shows the same page as the one reproduced previously, and the tableaux which have equal mother-columns and at the same time are *toka* have been colored. The user can test some aspects of the search engine on a small sample of a notebook made available without restriction in this Web site, but the access to the full database is restricted to researchers involved in academic projects. There also is a possibility to export the data in the format of the Weka software in order to search for regularities in the notebooks using data mining algorithms.

If we now return to Goody's reflections on the function of writing, we can assert that diviners notebooks clearly have a recording function. Marcia Ascher noticed that the tableaux which are *toka* are "sought by the *ombiasy* for themselves, that is, in addition to simply encountering them in the course of divinatory consultations, finding beginning data which lead to such tableaux is an intellectual problem in and for itself. Knowing as many as possible leads to an increase in prestige" (Ascher, 1997, p. 390). That is the reason why they are recorded in notebooks. But it must be stressed out that these notebooks *do not have any communication function*. Moreover, they are kept confidential by the *ombiasy* who do not want their knowledge to be borrowed by someone else. This is probably one of the most important differences in the use of writing between science and divination. It is a means of communication in the first case but not in the second one. Goody has addressed this problem by studying how "modes of thought" can be affected by change in the means of communication:

"In suggesting that some of the arguments concerning myths and history, the development of mathematical operations, the growth of individualism and the rise of bureaucracy were closely connected with the long and changing process of introducing graphic symbols for speech, of the shift from utterance to text, I do not mean to imply that pre-literate societies are without history, mathematics, individuals or administrative organizations. Rather I am interested in the further developments in these various facets of social life that seems to be associated with changes in the means and modes of communication" (Goody, 1977, p. 19).

Malagasy divination provides a rich subject in order to explore the origin of mathematical concept in the human mind. By studying mathematical structures in such traditional activities, the main difficulties are to establish a link between formal properties studied *in abstracto*, and mental representations of native people as they can be observed during fieldworks. We have described some of these difficulties in different situations, and we have presented the results of fieldworks done in Madagascar where mathematical mental representations of native people have been discovered, and we have analyzed how these result could contribute to a general reflection on evolutionary knowledge in mathematics.

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