CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

MATHEMATICS

0580/02
0581/02

Paper 2

May/June 2003

1 hour 30 minutes

Candidates answer on the Question Paper.
Additional Materials: Electronic calculator
                         Geometric instruments
                         Mathematical tables (optional)
                         Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen in the spaces provided on the Question Paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown below that question.
The total of the marks for this paper is 70.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's Use

If you have been given a label, look at the details. If any details are incorrect or missing, please fill in your correct details in the space given at the top of this page.
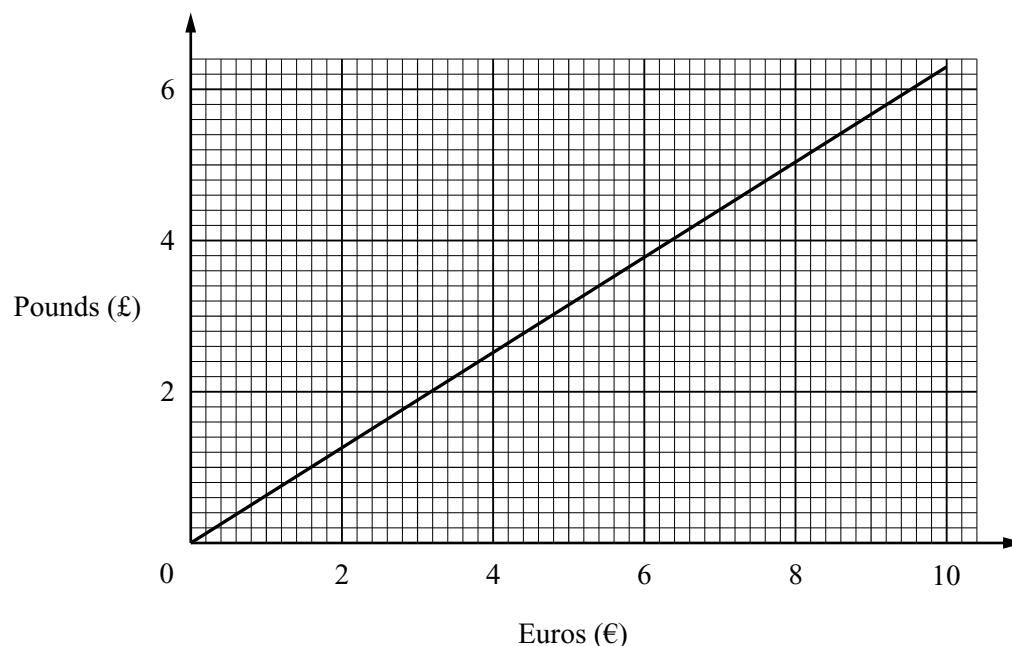
Stick your personal label here, if provided.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

[Turn over

1   Write in order of size, smallest first,

$$\frac{5}{98}, \quad 0.049, \quad 5\%.$$

Answer 0.049 .....<...... 5% .....<...... 5/98 .... [2]

2   The graph below can be used to convert between euros (€) and pounds (£).

Pounds (£)

Euros (€)

(a) Change £5 into euros.

Answer (a) € ...7.9................................. [1]

(b) Change €90 into pounds.

Answer (b) £ .56.50............................. [1]

3   The top speed of a car is 54 metres per second.
     Change this speed into kilometres per hour.

Answer ....194..............km/h [2]

4   **a** = $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and **b** = $\begin{pmatrix} 5 \\ -1 \end{pmatrix}$.   Find 3**a** – 2**b**.

3a = $\begin{pmatrix} \end{pmatrix}$ 2b = $\begin{pmatrix} \end{pmatrix}$,   3a – 2b = $\begin{pmatrix} \end{pmatrix}$ = $\begin{pmatrix} \end{pmatrix}$ $\begin{pmatrix} \end{pmatrix}$

Answer

[2]

5 The ratios of teachers : male students : female students in a school are 2 : 17 : 18.
The total number of **students** is 665.
Find the number of **teachers**.

Answer ....$\overset{38}{........}$.................................. [2]

6 A rectangular field is 18 metres long and 12 metres wide.
Both measurements are correct to the nearest metre.
Work out exactly the smallest possible area of the field.

Answer.............................$\overset{210.25}{........}$..................m^2 [2]

7 Solve the inequality $3 < 2x - 5 < 7$.

Answer ....$\overset{4}{....}$........... $< x <$ ....$\overset{6}{....}$.............. [2]

8 Complete this table of squares and cubes.
The numbers are not in sequence.

Number	Square	Cube
3	9	27
+/− 11	121	+/− 1331
14	196	2744
−7	49	−343

(Required numbers underlined and bold.) [3]

[Turn over

9
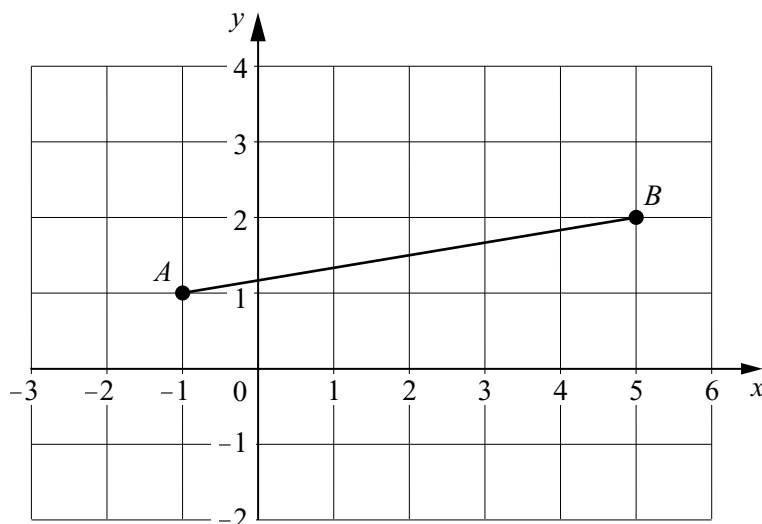
(a) Find the gradient of the line *AB*.

gradient = Vertical ÷ horizontal

= — or 0.16(...) or 0.17

Answer (a) ....0.17................ [1]

(b) Calculate the angle that *AB* makes with the *x*-axis.

Angle = arc tan(part (a))

= 9.46° or 9.5°

Answer (b) ....9.5°................ [2]

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10 Work out as a single fraction

$$\frac{2}{x-3} - \frac{1}{x+4}.$$

Answer .... [3]
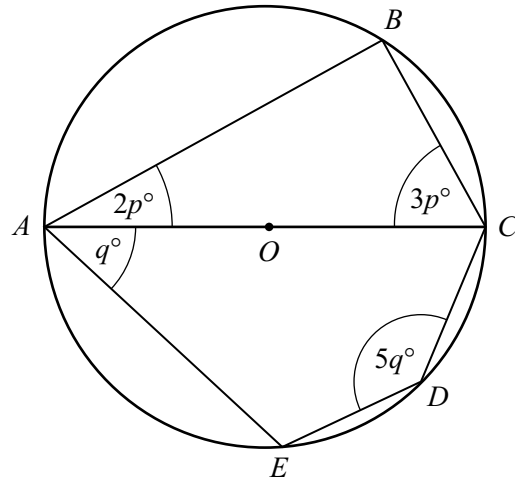
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11 Write each of these four numbers in the correct place in the Venn Diagram below.

$$2.6, \quad \frac{4}{17}, \quad \sqrt{12}, \quad \sqrt{\frac{112}{7}}$$
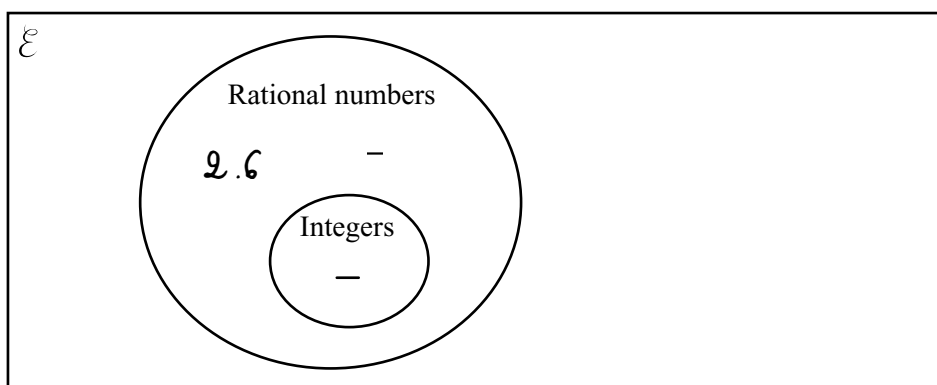
[4]

12

NOT TO
SCALE

A, B, C, D and E lie on a circle, centre O. AOC is a diameter.
Find the value of

(a) p,

Angle ABC = 90°
Hence 2p + 3p = 180−90
    5p = 90°
     p = 18°

Answer (a) p = ...18 °........................... [2]

(b) q.

q + 5q = 180°
    6q = 180°
     q = 30°

Answer (b) q = ...30 °........................... [2]

[Turn over

13 A doctor's patients are grouped by age, as shown in the table and the histogram below.

Age (x years)	$0 \leqslant x < 10$	$10 \leqslant x < 30$	$30 \leqslant x < 60$	$60 \leqslant x < 100$
Number of patients	300	600	___	880

Required number underlined and bold

Frequency density

Age in years
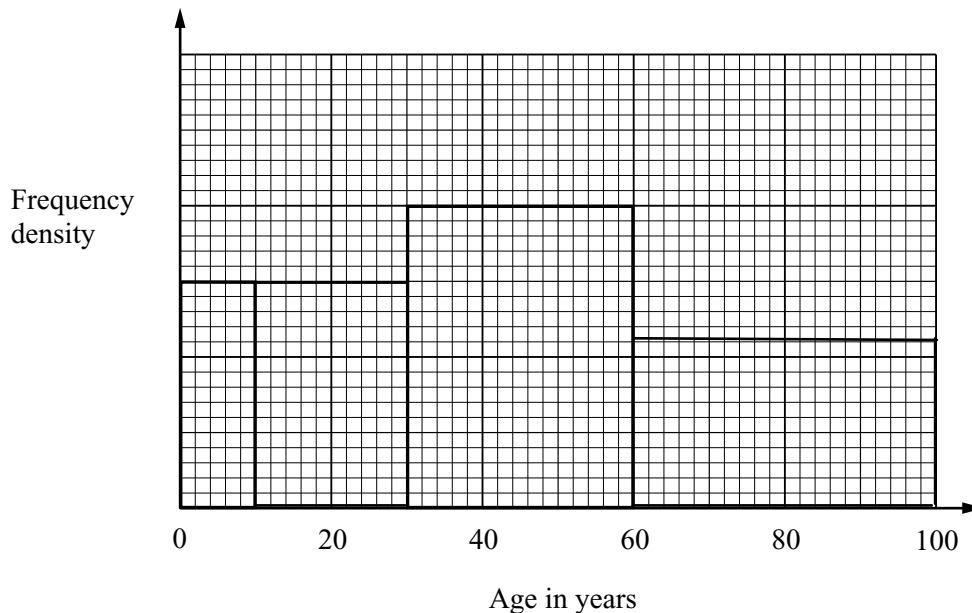
(a) Complete the following:

1 cm² represents ....100............... patients. [1]

(b) Use the histogram to fill in the blank in the table. [1]

(c) Draw the missing two rectangles to complete the histogram. [2]

14 (a) Multiply $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 & -4 \\ 0 & 3 & 6 \end{pmatrix}$.

$\begin{pmatrix} 5\times2 + 4\times0 & 5\times1 + 4\times3 & 5\times-4 + 4\times6 \\ -3\times2 + -2\times0 & -3\times1 + -2\times3 & -3\times-4 + -2\times6 \end{pmatrix}$

Answer (a) $\begin{pmatrix} 10 & 17 & 4 \\ -6 & -9 & 0 \end{pmatrix}$ [2]

(b) Find the inverse of $\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$.

The determinant is $5 \times (-2) - (-3) \times 4$

$= -10 - (-12)$

$= 2$

Answer (b) $\frac{1}{2}\begin{pmatrix} -2 & -4 \\ 3 & 5 \end{pmatrix}$ [2]

15 In 1950, the population of Switzerland was 4 714 900.
In 2000, the population was 7 087 000.

(a) Work out the percentage increase in the population from 1950 to 2000.

Population increase = 7 087 000 − 4 714 900
                    = 2 372 100
Percentage increase = $\frac{2\ 372\ 100}{4\ 714\ 900} \times 100$
                    = 50.3%

Answer (a)...........................50.3............... % [2]

(b) (i) Write the 1950 population correct to 3 significant figures.

4 714 900 = 4 710 000 or 4.71 × 10⁶

Answer (b)(i) ....4 710 000........... [1]

(ii) Write the 2000 population in standard form.

Answer (b)(ii) ...7.087 × 10⁶........... [1]

16

NOT TO SCALE
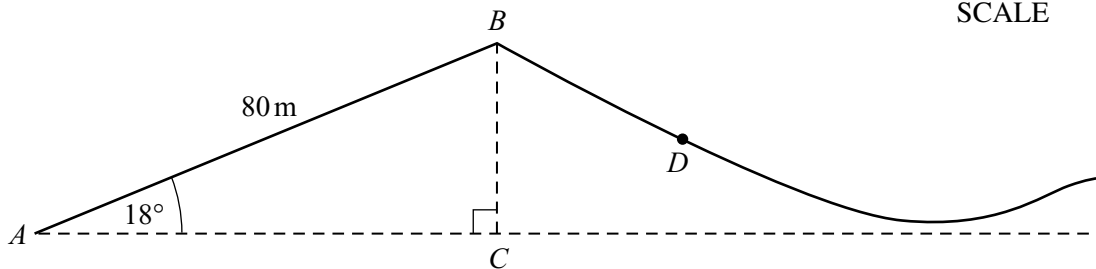
The diagram shows the start of a roller-coaster ride at a fairground.
A car rises from *A* to *B* along a straight track.

(a) *AB* = 80 metres and angle *BAC* = 18°.
Calculate the vertical height of *B* above *A*.

Sin 18° = BC/80
=> BC = 80 sin 18°
      = 24.7m

Answer (a).................24.7........... m [2]

(b) The car runs down the slope from *B* to *D*, a distance of *s* metres.
Use the formula *s* = *t*(*p* + *qt*) to find the value of *s*, given that *p* = 4, *t* = 3 and *q* = 3.8.
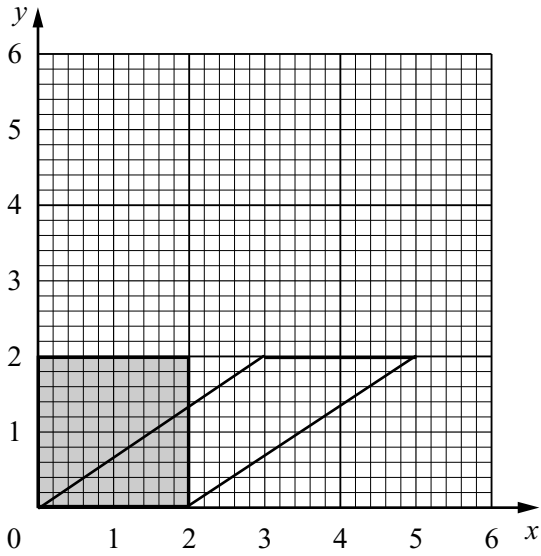
s = t(p + qt)
s = 3(4 + 3.8 × 3)
    s = 3 × 15.4
    s = 46.2

Answer (b) s = ...46.2........... [2]

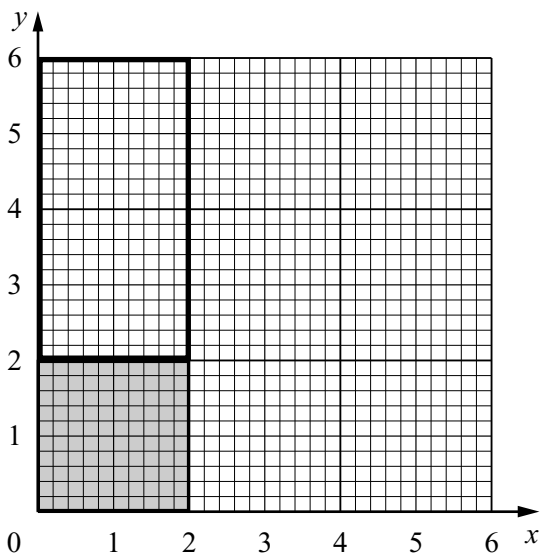**[Turn over**

17 **(a)**

Draw the shear of the shaded square with the *x*-axis invariant and the point $(0, 2)$ mapping onto the point $(3, 2)$.

[2]

(b)

(i) Draw the one-way stretch of the shaded square with the *x*-axis invariant and the point $(0, 2)$ mapping onto the point $(0, 6)$.

[2]

(ii) Write down the matrix of this stretch.

The unit vector ( ) stays as ( )

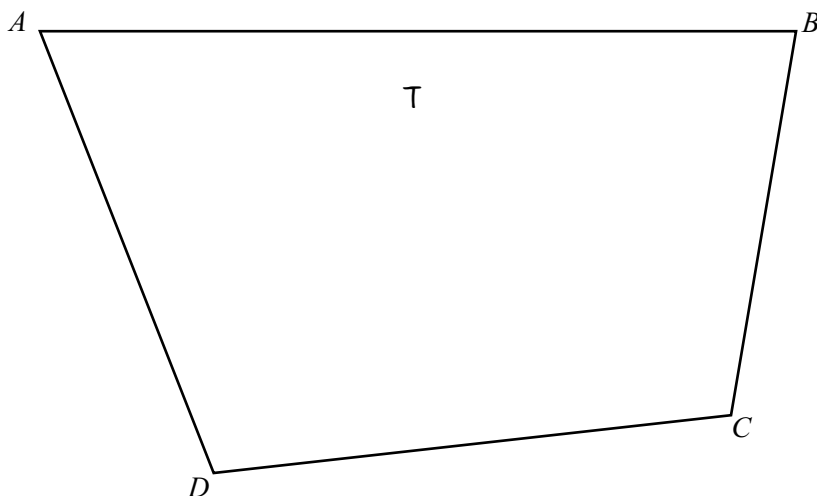and the unit vector ( ) becomes ( )   *Answer (b)*(ii) $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$ [1]

18 The diagram is a scale drawing of a field. The actual length of the side *AB* is 100 metres.

(a) Write the scale of the drawing in the form 1 : *n*, where *n* is an integer.

Length of AB on the diagram is 10 centimetres.

AB represents 100 metres, which is 10000cm.

10 : 10000 = 1 : 1000

Answer (a) 1 : ....**1000**.............. [1]

(b) In this part use a straight edge and compasses only. Leave in your construction lines.

(i) A tree in the field is equidistant from the point *A* and the point *D*. Construct the line on which the tree stands. [2]

(ii) The tree is also equidistant from the sides *BC* and *CD*. After constructing another line, mark the position of the tree and label it *T*. [3]

[Turn over

19 A ferry has a deck area of 3600 m^2 for parking cars and trucks.
Each car takes up 20 m^2 of deck area and each truck takes up 80 m^2.
On one trip, the ferry carries x cars and y trucks.

(a) Show that this information leads to the inequality $x + 4y \leqslant 180$.

x cars take up 20x of deck area.
y trucks take up 80y of deck area.
Total area taken up = 20x + 80y
20x + 80y <= 3600
x + 4y <= 180

[2]

(b) The charge for the trip is \$25 for a car and \$50 for a truck.
The total amount of money taken is \$3000.
Write down an equation to represent this information and simplify it.

Charge for x cars is 25x
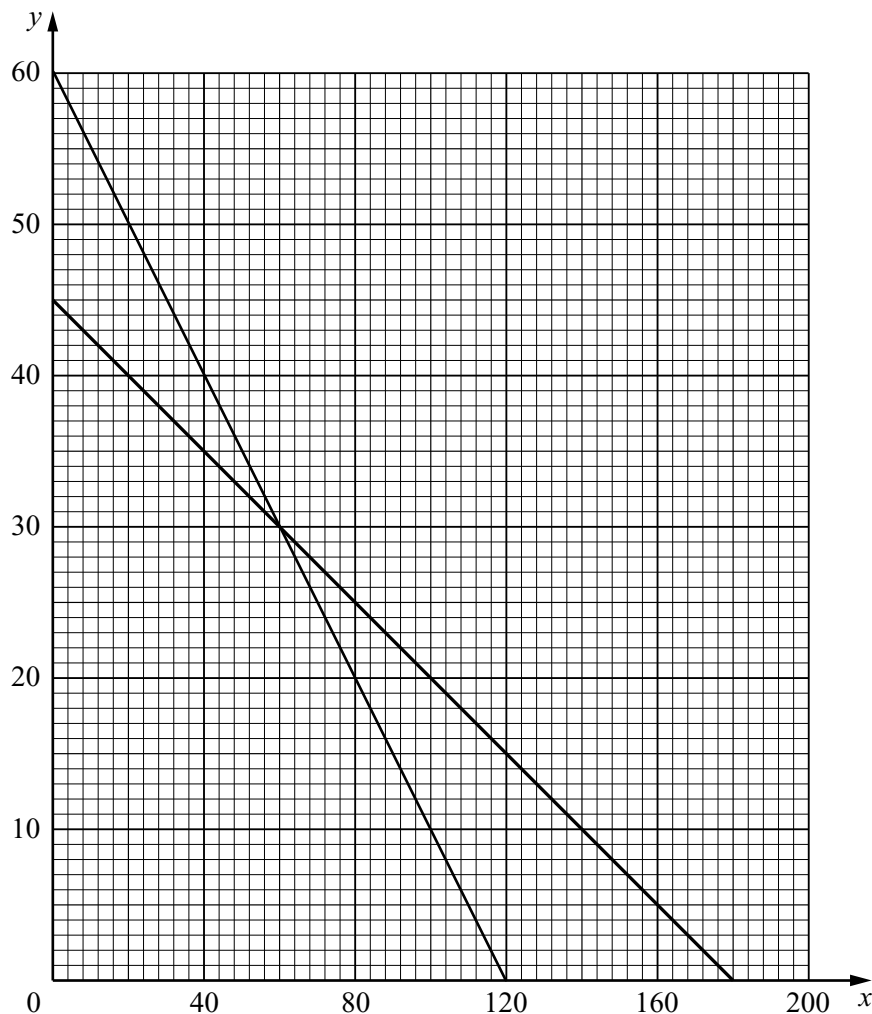Charge for y trucks is 50y
Total charge is 25x + 50y
25x + 50y = 3000
x + 2y = 120

Answer (b) .... x + 2y = 120 .... [2]

(c) The line $x + 4y = 180$ is drawn on the grid below.

    (i) Draw, on the grid, the graph of your equation in part **(b)**.

[1]

    (ii) Write down a possible number of cars and a possible number of trucks on the trip, which together satisfy both conditions.

In the equation $x + 2y = 120$, when $x = 0$, $y = 60$

When $y = 0$, $x = 120$.

Theses points are plotted and joined with a ruled, straight line.

Answer (c)(ii) ....120.... cars,........0...... trucks   [1]

     [Turn over

20 (a) Complete the table of values for $y = 3^x$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	0.1	0.2	0.3	0.6	1	1.7	3	5.2	9

[3]

(b) Use your table to complete the graph of $y = 3^x$ for $-2 \leqslant x \leqslant 2$.
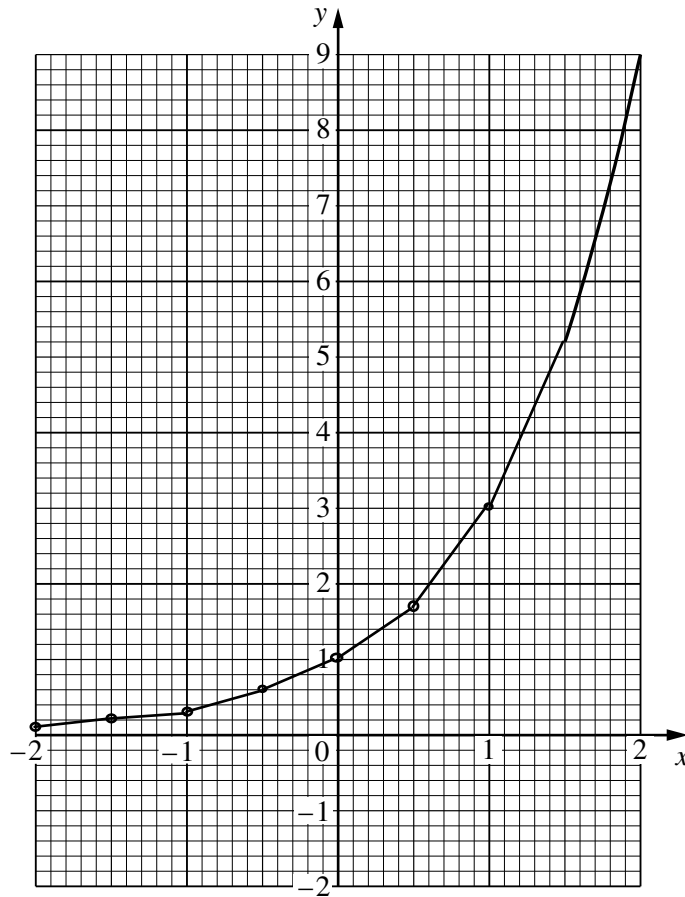
[2]

(c) Use the graph to find the solution of the equation

$$3^x = 6.$$

Answer (c) x = 1.6 ⟨= x ⟨ 1.65 [1]

Summary of Comments on IGCSE Mathematics Paper 2 June 2003

Page: 2

Q1
1 mark can be gained for seeing the figures 51 or for 2 of the 3 in the correct order.
In this question all three have to be in the same form for comparison. Decimals are the easiest but percentage would work well. Putting into fractions would probably cause confusion with different denominators.

Q2(a)
The allowance for reading from the graph is 7.85 to 8. Care is needed to read the graph the correct way round.

Q2(b)
The allowance is £56.25 to £57.5(0). Although it is normal to always show 2 figures after the decimal point for UK currency the lack of the zero is not penalised in International exams.
Care is needed here in realising that this part is not just reading from the graph and again read the graph the correct way round.

Q3
Seconds to hour multiply by 3600 (60 $\times$ 60).
Metres to kilometres divide by 1000. Both are needed for the method mark. 3-figure accuracy is adequate here but more figures are accepted.
In this case the exact answer is 194.4. Note that if extra beyond three figures is given it must be correct figures.

Q4
The multiplier must multiply both components of the vector. Care must be taken with the negative values here and writing down the working in stages should help to avoid the problems of combining negative numbers. Subtracting a negative value is the same as adding the positive value. The common error is to give –9 – (–2) as equal to –11.

Q5   Not the usual problem of adding all the ratio parts. If that error is made the division does not give a whole number, which indicates an error has been made.
At extended level expect the ratio question not to be quite as straightforward as total divided by sum of ratios.

Q6   For the lowest value when given to the nearest metre, take half a metre below the given value.
Although the question needed 201.25 to be seen there was no loss of mark if 201 or 201.3 were then given. This question emphasises the need to read the question carefully and not miss applying the word 'exactly'.

Q7   If the inequality is not split the common error of adding 5 only to the right hand side is easily made.
1 mark was awarded if M1 was not gained for correctly finding either $x > 4$ or $x < 6$.

Q8   In the second row need to take the square root of 121 and then cube it. In the third and fourth rows the cube root is needed before squaring. The calculator will cope with the negative values but cube root of a negative is negative and the square of a negative is positive should be known.

Q9(a)   Same scale is on both axes so simply count squares for the horizontal value. Take care not to give the answer 6.

Q9(b)   Gradient = tangent of the angle between the line and the horizontal.
Other methods such as using Pythagoras to find AB can lead to errors. Never attempt to measure angles if 'calculate' is stated.

Q10   Following the rules for addition of numerical fractions questions the common denominator is $(x - 3)(x + 4)$. The first solution line being shown should help to avoid the errors, which often occur when multiplying out brackets. A common error of not multiplying the two negatives would result in a numerator of $x + 5$.
Avoid the temptation to cancel terms after the correct answer. In algebra questions the final answer only is marked.

Q11   All rational numbers can be expressed as fractions. This includes recurring decimals.
Integers are whole numbers.
Irrational numbers cannot be expressed as fractions.

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Q12(a)   Since ABC is a diameter, angle ABC is a right angle, 90°. Angle in a semi-circle is a right angle. This should not be confused with opposite angles of a cyclic quadrilateral adding up to 180°.

Q12(b)   ACDE is a cyclic quadrilateral with opposite angles adding up to 180°.
Take care not to think that part of ABCDE, a pentagon, does not have properties as a cyclic quadrilateral.

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Q13(a)   In histograms the area represents the frequency. 300 patients are represented by 3sq.cm. so 1sq.cm. represents 100 patients. Take care that the small squares are not counted, as this would give 300 divided by 75 giving 4 patients.

Q13(b)   There are 12 one centimetre squares in the section $30 \leq x < 60$, which must then be multiplied by the answer to part (a)

Q13(c)   Alternatively if frequency densities are found (frequency /class width) the first one is 30, which as shown is represented by 3cm of height. So every 10 of frequency density is represented by 10mm of height.
The second bar is 600/20 = 30 so again 30mm high and the 4th bar is 880/40 = 22 which is 22mm high. The 3rd bar has height 40mm so frequency = frequency density $\times$ class width which is 40×30.

Q14(a)   The solution shows the rules for the multiplication of matrices.
1 mark was awarded for 4 or 5 correct final terms.
Many will not write out the method, which can often lead to errors, in particular, it is very easy to get –24 for the last term.

Q14(b)   1 mark for 1/2, 1 mark for $k \begin{pmatrix} -2 & -4 \\ 3 & 5 \end{pmatrix}$

The rule for the inverse matrix is as follows: -

Inverse of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $\dfrac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

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Q15(a)   Percentage Change = $\dfrac{\text{Change}}{\text{Original}} \times 100$

Make sure that the original quantity and not the final quantity is used for the denominator.

Alternatively $\dfrac{7\ 087\ 000}{4\ 714\ 900} = 150.3$

$150.3 - 100 = 50.3$

Take care not to approximate too soon. Do not do $150.3 \approx 150$

$150 - 100 = 50$

Q15(b)   (i) Cut off after 3 figures. Observe the fourth figure which, if 5 or more causes the 3rd figure to increase by 1.

(ii) Standard Form is $a \times 10^{n}$ where $1 \leq a < 10$ and n is an integer. (Positive for numbers greater than 10 and negative for numbers less than 1)

Q16(a)   Make sure the calculator is in degree mode. Trigonometry ratios must be known. The triangle must be right-angled for this ratio to be used.

Q16(b)   While this can be done without showing working, mistakes are often made. It is a good idea to write down the calculation that is to be done on the calculator, making clear that the method is understood. Also the method mark is then gained even if a slip is made when using the calculator.

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Q17(a)   Points on the invariant axis remain the same. The square is transformed into a parallelogram, the point (2, 2) also moving 3 units to the right.

Q17(b)   (i) Again the points on the x-axis stay the same and (2, 2) moves to (2, 6). The square is transformed into a rectangle.

(ii) Alternatively, think in terms of coordinates of the points.
$(1, 0) \Rightarrow (1, 0)$ and $(0,1) \Rightarrow (0, 3)$
This gives the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

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Q18(a)   For a ratio the units must be the same and then cancelled to the simplest form.

Q18(b)   (i) From the mid-point of AD a perpendicular line is to be constructed.
From A and D draw arcs both sides of the line AD. Join the intersection points of the arcs.
(ii) Angle BCD has to be bisected.
    From C draw arcs on BC and DC. From these arcs draw intersecting arcs and join to the point C.
    T is marked at the intersection of the two constructed lines.

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Q19(a)   Need to show the expression for the total area taken up by cars and trucks and needs quoting as less than or equal to 3600 as shown. Then the required inequality must be stated to obtain both marks.

Q19(b)   In similar fashion form an expression for the total charge. Put this equal to 3000 and cancel by dividing by 25.

Q19(c)   Make sure that lines on graphs are ruled, and clearly seen. (HB pencil advised)

Q20(a)   Using the calculator is the easiest way to find these values, which need not be given to more than 1 decimal place as they are to be plotted on the graph. They should however be correct to 1 d.p. For example when $x = -0.5$, $y = 0.57735...$needs to be 0.6 and not 0.5.
To work out a negative value, for example $x = -1.5$, on the calculator, the sequence is as follows: -
$[3][y^x][+/-][1.5][=]$

Q20(b)   When drawing the graph of $y = 3^x$ take care that the points are plotted correctly with reference to the scale.
The *x*-axis small squares are each 0.1 and the *y*-axis small squares are 0.2.
The curve should go through all the points; no gap should be seen between a point and the curve.
No section of the curve should be ruled, so not a series of joined straight lines.
The curve should be as smooth as possible, with no double lines appearing in any section. Work in pencil and if it does not appear satisfactory erase it and try again.

Q20(c)   Answer should be correct according to the candidate's graph.