Circuit Theorems: Thevenin and Norton Equivalents, Maximum Power Transfer

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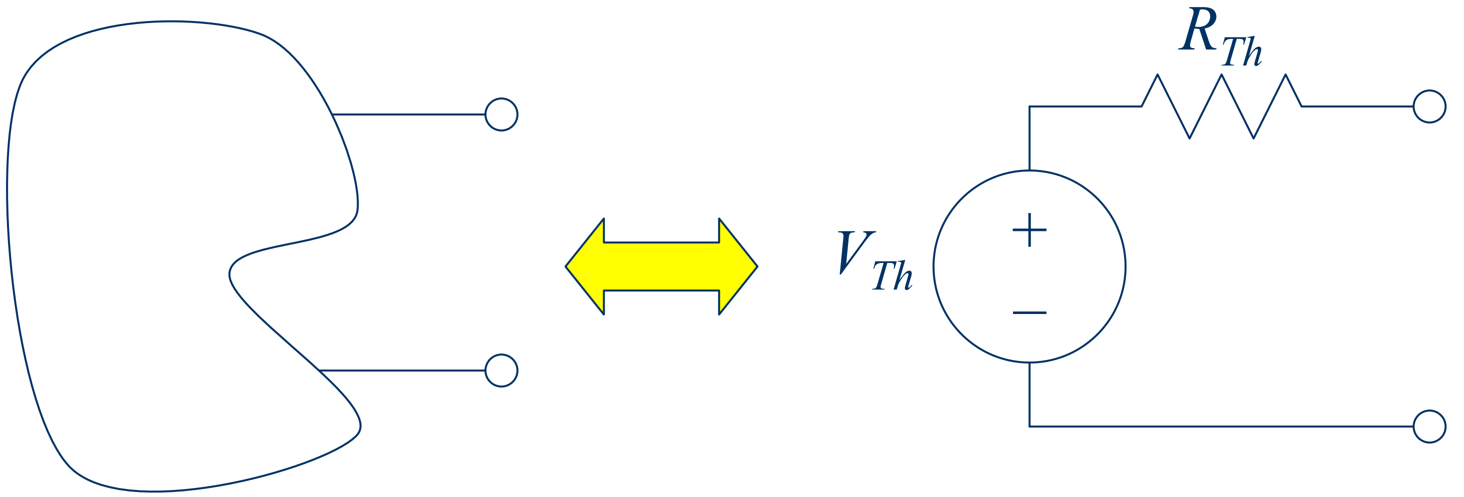
Thevenin’s Theorem

- Any circuit with sources (dependent and/or independent) and resistors can be replaced by an equivalent circuit containing a single voltage source and a single resistor.
- Thevenin’s theorem implies that we can replace arbitrarily complicated networks with simple networks for purposes of analysis.

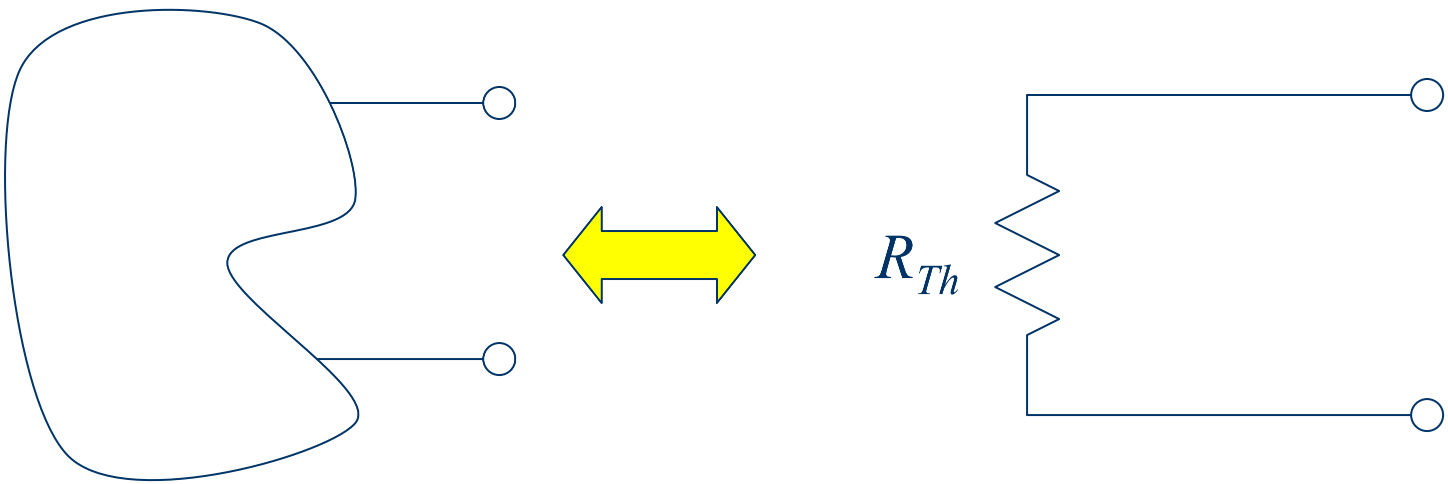
Independent Sources (Thevenin)



Circit with independent sources

Thevenin equivalent circuit

No Independent Sources

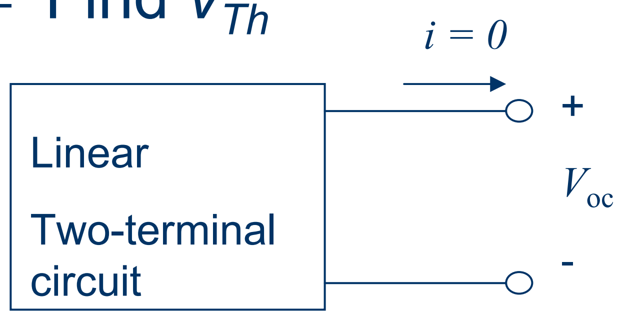


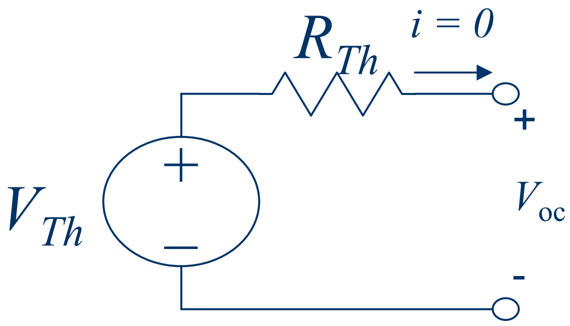
Circuit without independent sources

Thevenin equivalent circuit

Thevenin Equivalent Circuit

- Basic steps to determining Thevenin equivalent are
  - Find v_{Th}



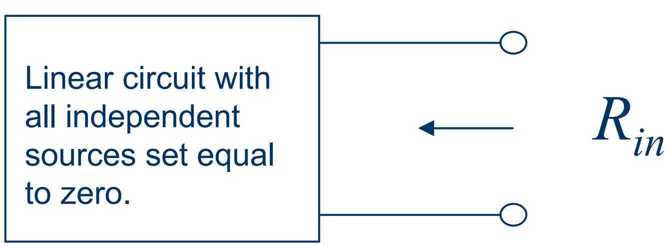


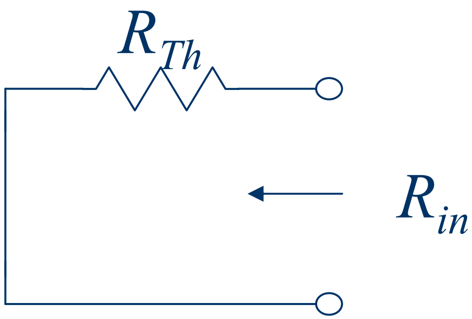
$$\boxed{V_{oc} = V_{Th}}$$

Thevenin Equivalent Circuit

- Compute the Thevenin equivalent resistance, R_{Th}

  (*a*) If there are <u>only</u> independent sources, then short circuit all the voltage sources and open circuit the current sources (just like superposition).



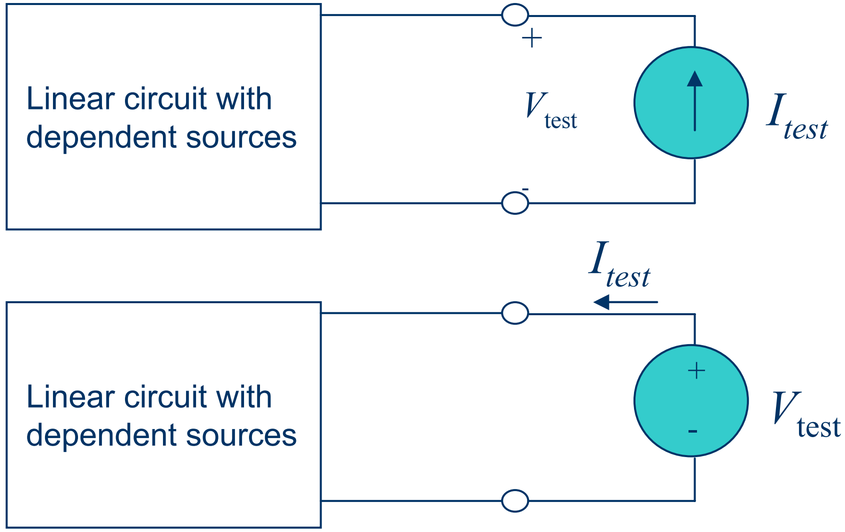


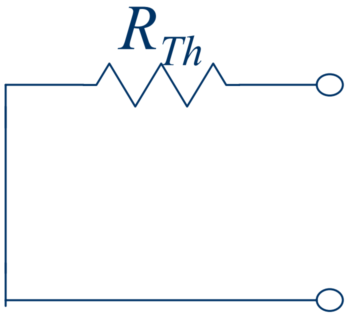
$$\boxed{R_{in} = R_{Th}}$$

Thevenin Equivalent Circuit

(*b*) If there are only dependent sources, then must use a test voltage or current source in order to calculate

$$R_{Th} = V_{Test}/I_{test}$$



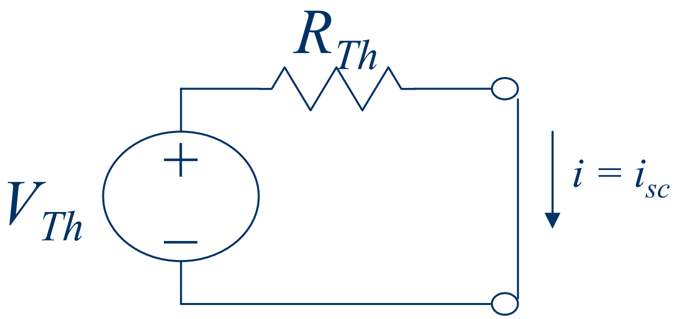


Thevenin Equivalent Circuit

(c) If there are <u>both</u> independent and dependent sources, then compute

(i) $R_{Th} = V_{Test}/I_{test}$ (all independent sources set equal to zero)

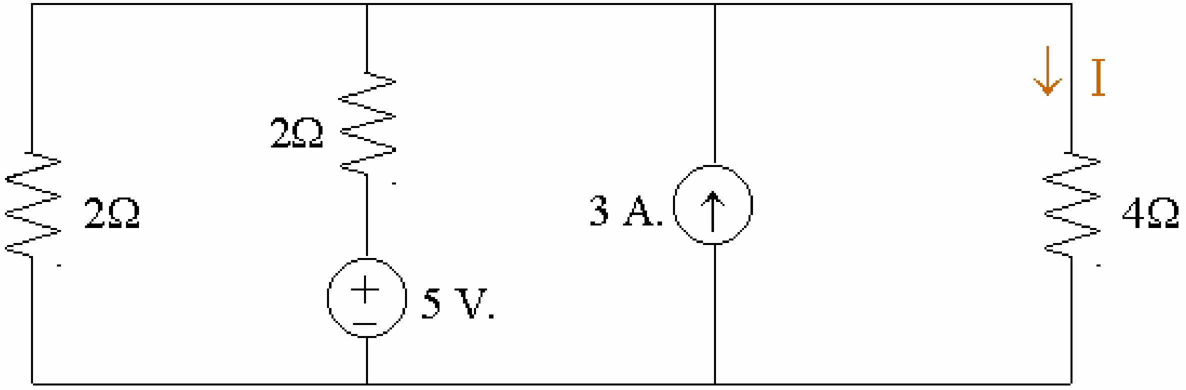
(ii) compute R_{Th} from V_{OC}/I_{SC}.



$$\boxed{i_{sc} = V_{Th} / R_{Th}}$$

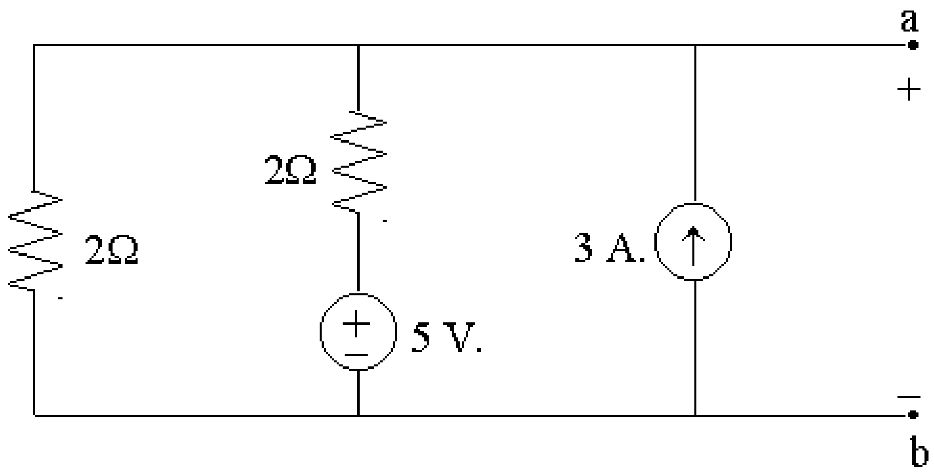
Example

Find I using Thevenin's Theorem



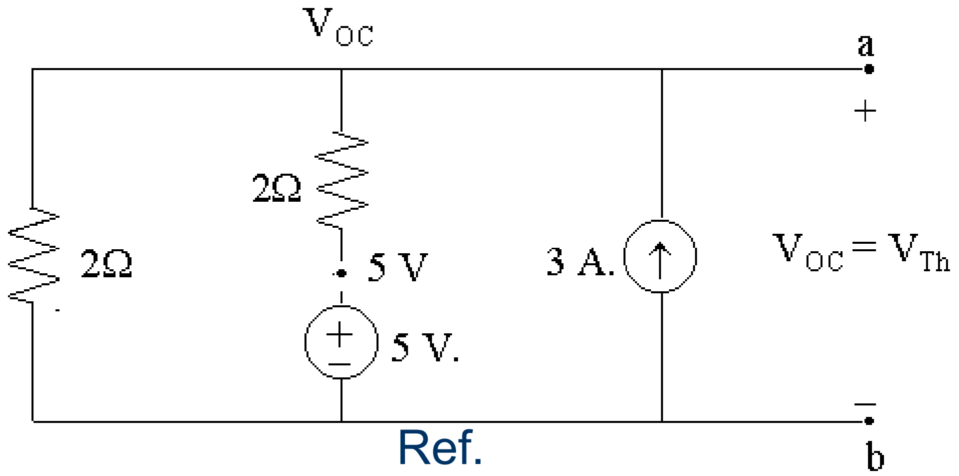
Example cont.

Step 1: Get the Thevenin Equiv. of the circuit to the left of terminals a-b



Example cont.

Step 1a: Open circuit voltage calculation

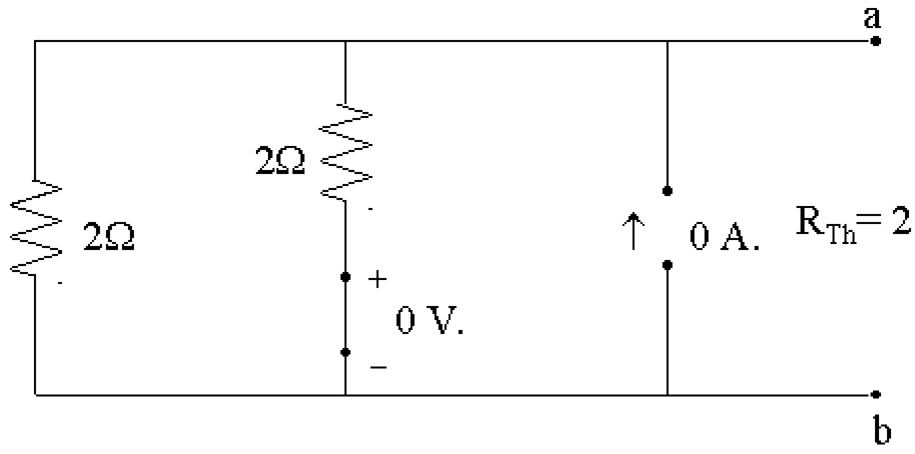


KCL at V_{oc} :

$$\frac{V_{oc}}{2}+\frac{V_{oc}-5}{2}=3 \Rightarrow V_{oc}=5.5V$$

Example cont.

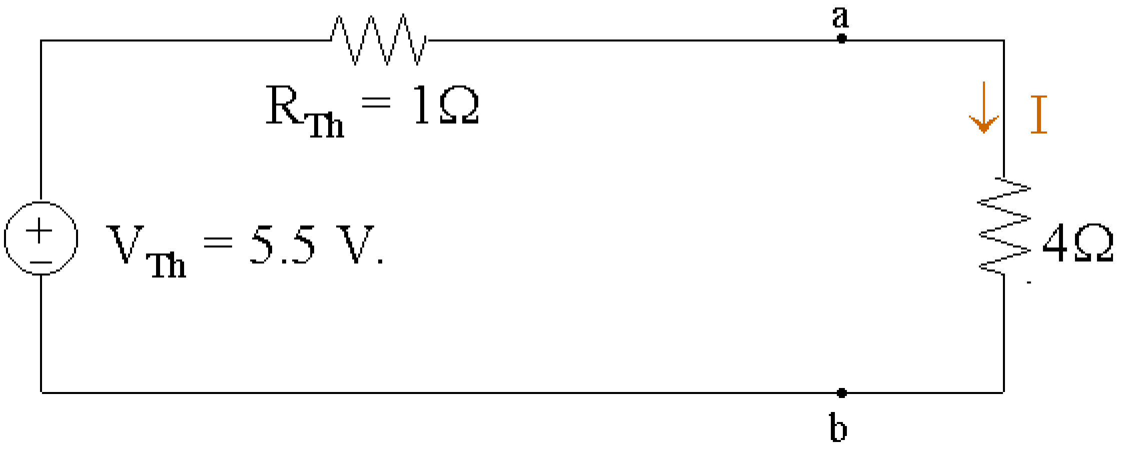
Step 1b: Determination of R_{Th}



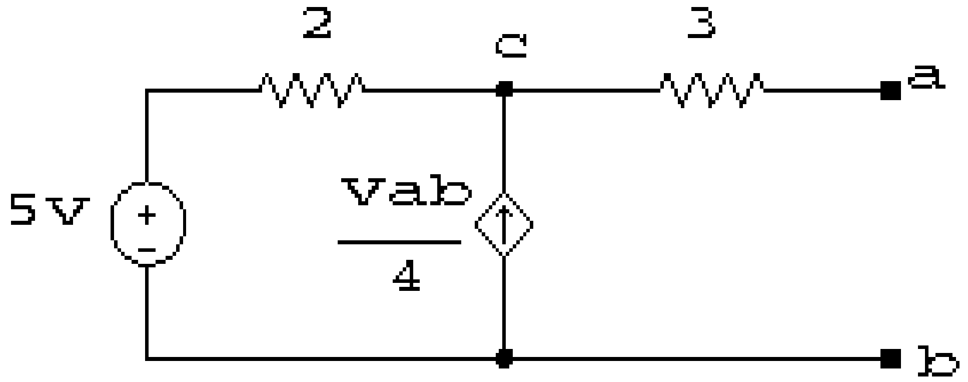
$R_{Th} = 2\Omega \,\|\, 2\Omega = 1\,\Omega$

Example cont.

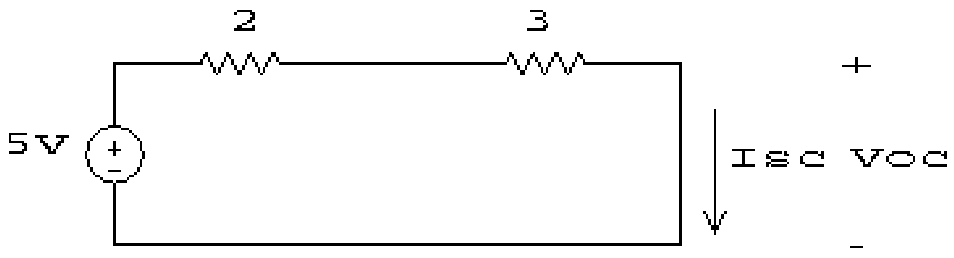
$I = 5.5\ V. / (1 + 4)\ \Omega = 1.1\ A.$
(Ohm's Law)



- **Problem:** for the following circuit , determine the Thevenin equivalent circuit.



- **Solution:**
- Step 1: In this circuit, we have a dependent source. Hence, we start by finding the open circuit voltage $V_{oc} = V_{ab}$.
- KCL at node C
- $(5 - V_{oc})/2 + V_{oc}/4 = 0$
- $V_{oc} = 10$ V

- Step 2: We obtain the short circuit current Isc by shorting nodes a-b and finding the current through it.
- 

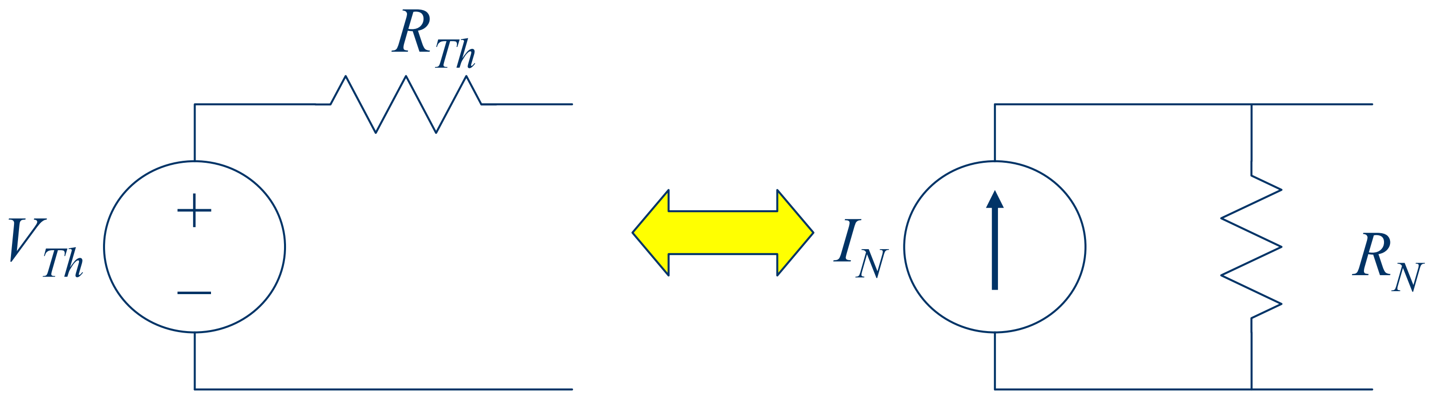
- 5 = 2 Isc + 3 Isc => Isc = 5/5
- Isc = 1 A

- Step 3: Find the equivalent Thevenin Voltage and Resistance
- $V_{th} = V_{oc} = V_{ab} = 10V$
- $R_{th} = V_{oc}/I_{sc}$ => $R_{th} = 10/1\ \Omega$
- **V_{th} = 10V**
- **R_{th} = 10 Ω**

Norton Equivalent Circuit

- Any Thevenin equivalent circuit is in turn equivalent to a current source in parallel with a resistor [source transformation].
- A current source in parallel with a resistor is called a Norton equivalent circuit.

Norton Equivalent Circuit



$$V_{Th} = R_N I_N \qquad I_N = \frac{V_{Th}}{R_{Th}} \qquad R_{Th} = R_N$$

- Finding a Norton equivalent circuit requires essentially the same process as finding a Thevenin equivalent circuit.

Thevenin/Norton Analysis

1. Pick a good breaking point in the circuit (cannot split a dependent source and its control variable).

2. **Thevenin**: Compute the open circuit voltage, V_{OC}.
   Norton: Compute the short circuit current, I_{SC}.

If there is not any independent source then both V_{OC}=0 and I_{SC}=0 [so skip step 2]

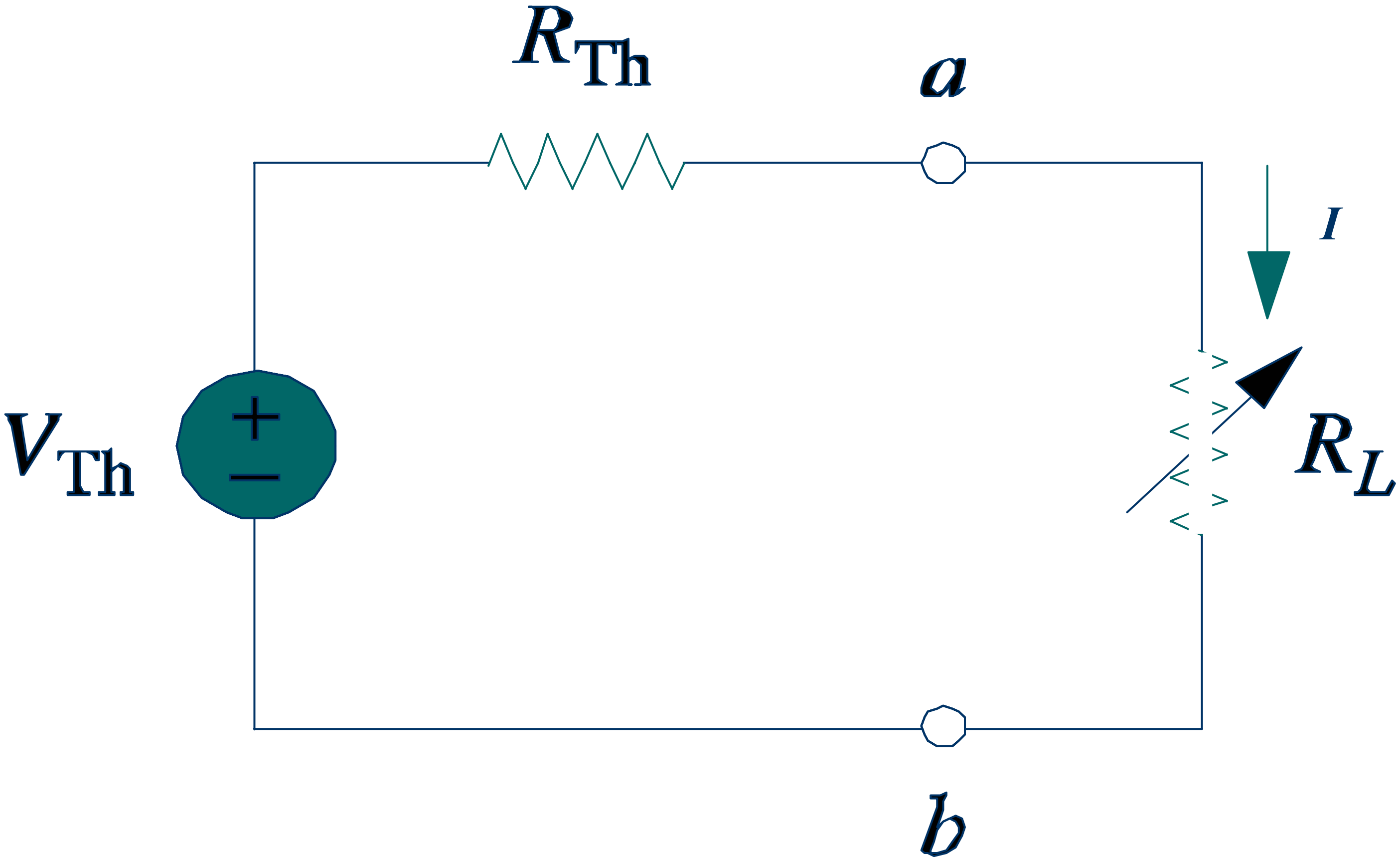
Thevenin/Norton Analysis

3. Calculate $R_{Th}(R_N) = V_{oc} / I_{sc}$
4. **Thevenin**: Replace circuit with V_{OC} in series with R_{Th}
   Norton: Replace circuit with I_{SC} in parallel with R_{Th}

Note: for circuits containing no independent sources the equivalent network is merely R_{Th}, that is, no voltage (or current) source.

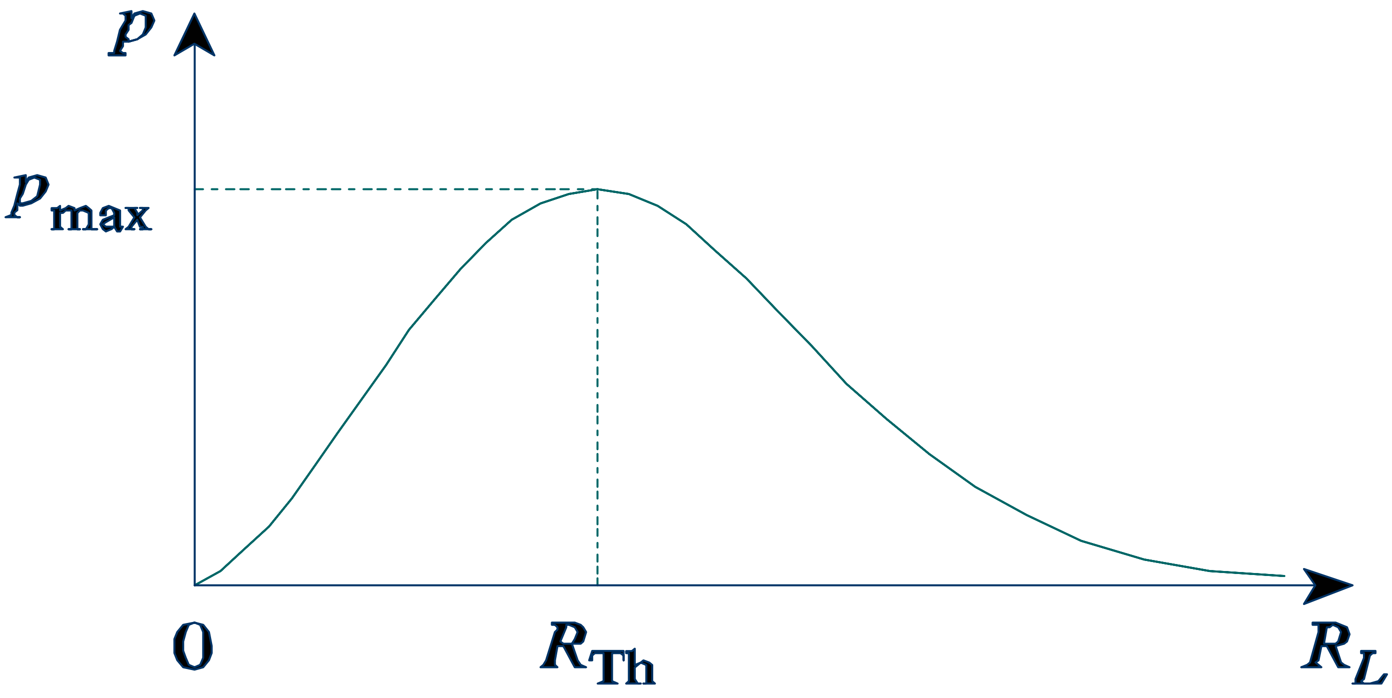
Only steps 2 & 4 differ from Thevenin & Norton!

Maximum Power Transfer



Maximum Power Transfer

Power delivered to the load as a function of R_L.



Maximum Power Transfer

$$I = \frac{v_{Th}}{R_{Th} + R_L}$$

$$P_{R_L} = I^2 R_L = \frac{V^2_{Th}}{(R_{Th} + R_L)^2} . R_L$$

To find the maxima

$$\frac{d\,p_{R_L}}{d\,R_L} = 0$$

$$\frac{d}{d\,R_L}\left[\frac{V^2_{Th} R_L}{(R_{Th} + R_L)^2}\right] = 0$$

Note : $d\left(\frac{u}{v}\right) = \frac{u'v - uv'}{v^2}$

Maximum Power Transfer

$$V^2{}_{Th}\left(\frac{(R_{Th}+R_L)^2.1-R_L.(2(R_{Th}+R_L))}{\{(R_{Th}+R_L)^2\}^2}\right)=0$$

$$R^2{}_{Th}+R^2{}_L+2R_{Th}R_L-2R_{Th}R_L-2R^2{}_L=0$$

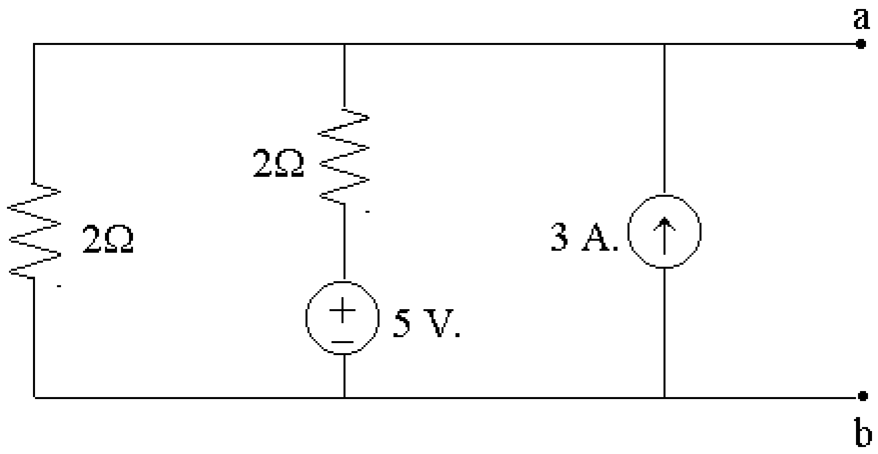
$$R^2{}_{Th}-R^2{}_L=0$$

$$R_{Th}=R_L \leftarrow \mathit{Maximum\ Power\ Transfer}$$

$$P_{\max}=\left(\frac{V^2{}_{Th}}{(2R_L)^2}\right).R_L=\left(\frac{V^2{}_{Th}}{4R_L}\right)\ \mathit{watts}$$

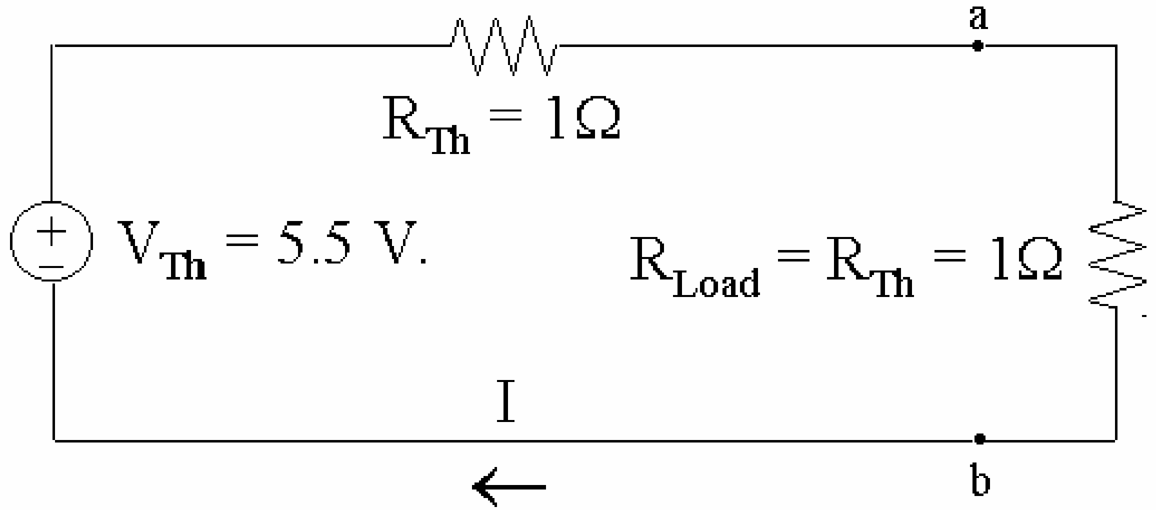
Example

What's the maximum power that can be extracted from terminals a-b?



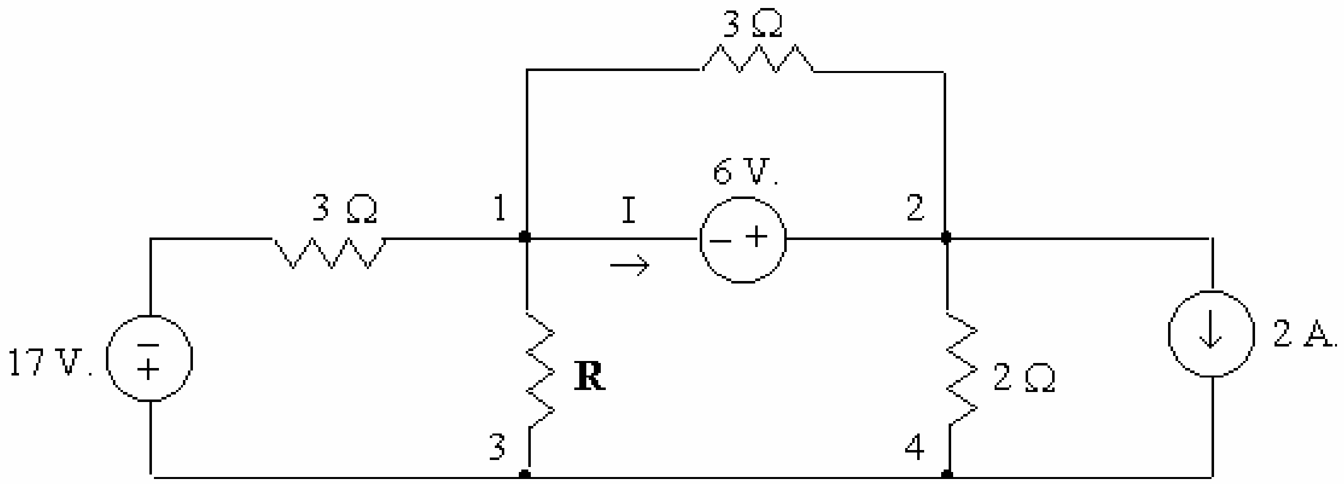
Example cont.

The circuit's Thevenin equivalent loaded with R_{Th} at terminals a-b yields:
$I = 5.5\ V./(1 + 1)\Omega = 2.75\ A.$ so the (maximum) load power is: $P_{max.} = I^2R = (2.75\ A.)^2 \times 1\ \Omega = 7.5625\ W.$



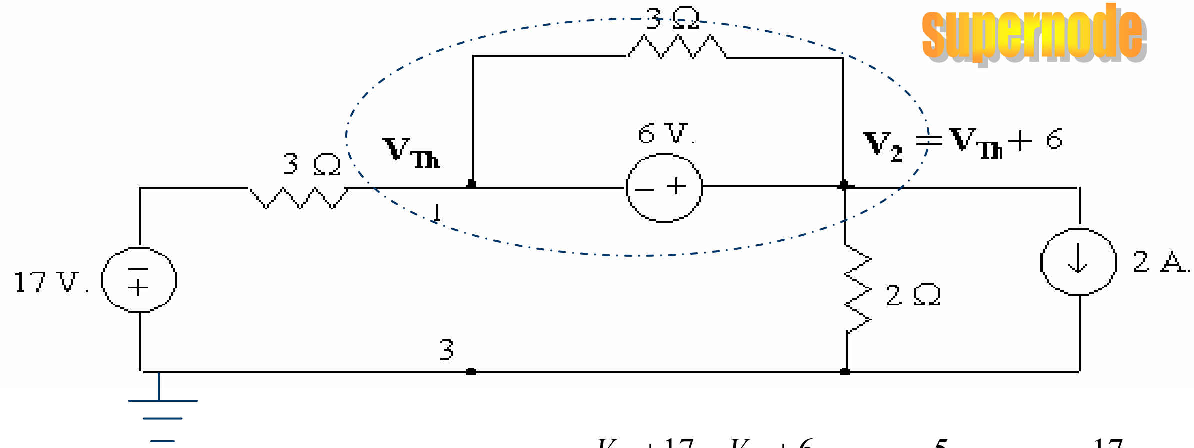
Example

Determine the value of R in the circuit which will draw maximum power and calculate the corresponding maximum power.



Example cont.

First find V_{Th} = open-circuit voltage across terminals 1 & 3



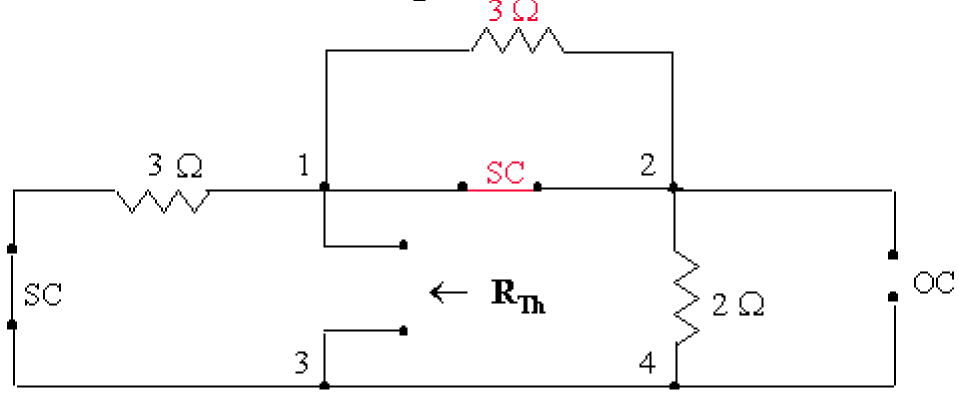
KCL at the supernode:

$$\frac{V_{Th}+17}{3}+\frac{V_{Th}+6}{2}=-2 \Rightarrow \frac{5}{6}V_{Th}=-2-\frac{17}{3}-3$$

$$V_{Th}=\frac{6}{5}\left(\frac{-32}{3}\right)=-\frac{64}{5}=-12.8V$$

Example cont.

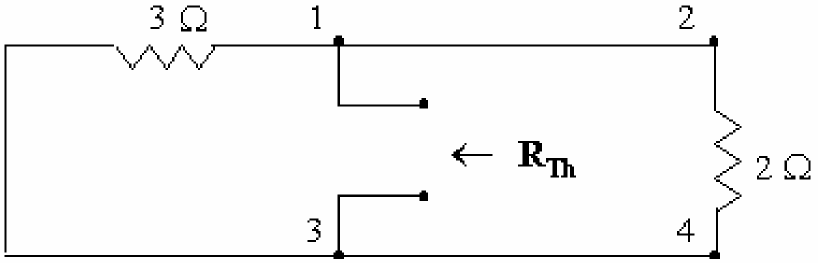
R_{Th} = Resistance across (open-circuited) terminals 1 & 3 with the independent sources deactivated



The parallel combination of the 0 Ω SC and the 3 Ω resistor is 0 Ω (another SC) so the circuit becomes (next slide) ...

Example cont.

R_{Th} = Resistance across (open-circuited) terminals 1 & 3 with the independent sources deactivated



$$R_{Th} = 3\ \Omega \parallel 2\ \Omega = 1.2\ \Omega$$

Example cont.

The circuit's Thevenin equivalent loaded with $R = R_{Th}$ draws a current of:

$$I = V_{Th}/(R_{Th} + R) = (-12.8\ \text{V.})/(1.2\ \Omega + 1.2\ \Omega) = -\ 5^1/_3 \text{A.}$$

and the corresponding maximum power is

$$P_{max.} = I^2R = (-\ 5^1/_3 \text{A.})^2 \times 1.2\ \Omega = 34^2/_{15}\ \text{W.} \approx 34.13\ \text{W.}$$