AQR UNIT 7

NETWORKS AND GRAPHS:

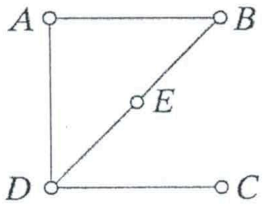
Circuits, Paths, and Graph Structures

Packet #2

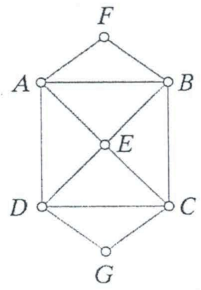
BY:_____

Attempt to reach every vertex in these pictures without raising your pencil. Record the order that you reached the vertices in.

Shape 1:

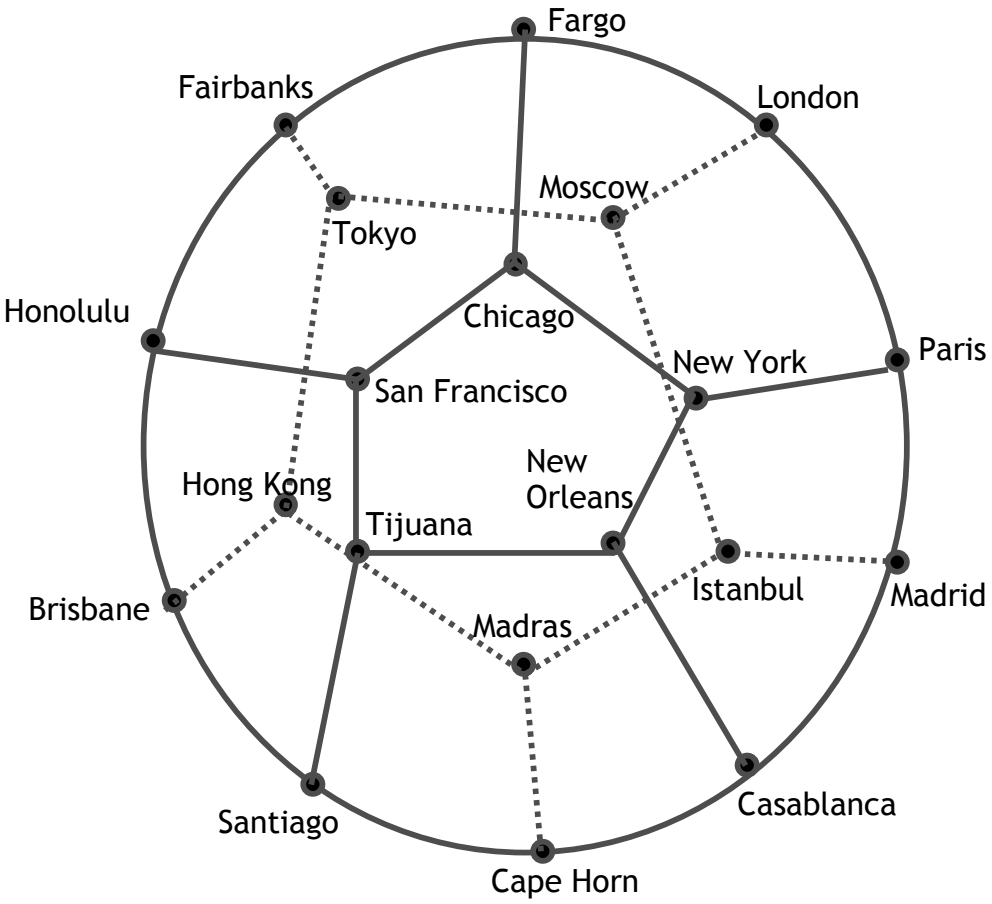


Shape 2:



A Voyage Around the World

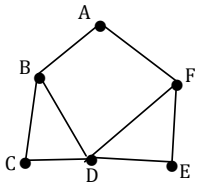
1. Plan a trip around the world by visiting each city exactly once and using only the identified routes to travel from city to city. The dashed lines represent routes on the opposite side of the globe.

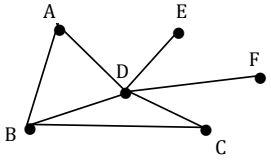


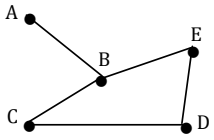
Hamiltonian Circuits:

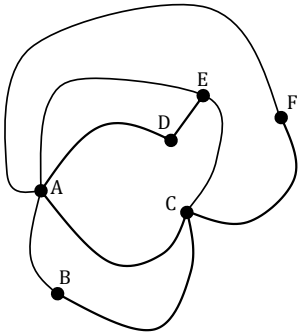
Hamilton looked at the 'graph puzzle' from a slightly different perspective than Euler. He asked if it was possible to traverse the graph and pass through each vertex only once. Again similar to Euler, the graph is consider to have a Hamilton Circuit if you can end at the same vertex you started with and a Hamilton Path if you start and end on different vertices. List out the circuit or path that can be found and identify which it is.

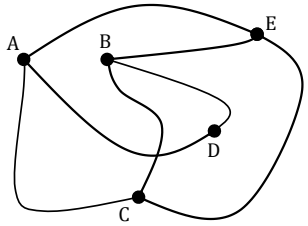
The harder question is determining what characteristics of the graphs guarantee us a Hamiltonian circuit which you should think about when you are finished; it once again deals with degree.



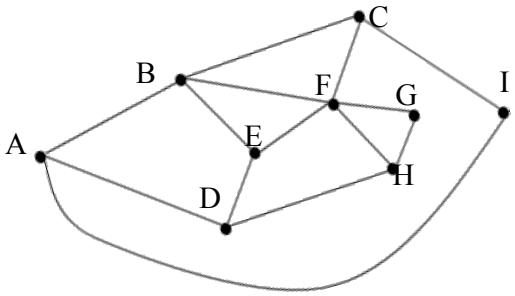




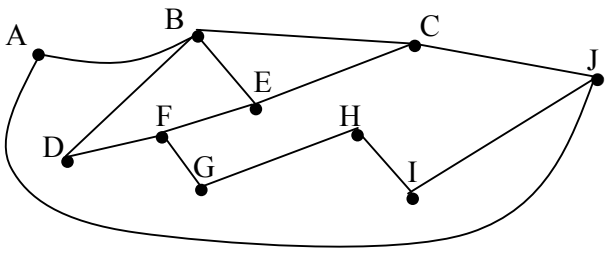




Find a **HAMILTONIAN CIRCUIT** of the graph below (Give a sequence of letters to describe the path (e.g. A, D, E, B, …….. etc.))



3. Find a **HAMILTONIAN PATH** of the graph below (Give a sequence of letters to describe the path (e.g. A, D, E, B, …….. etc.))

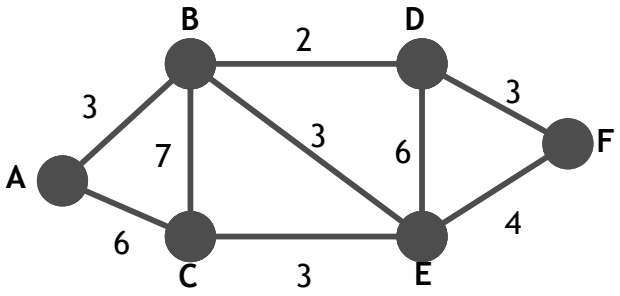


Weighted Graph Problems: Complete the following problems.

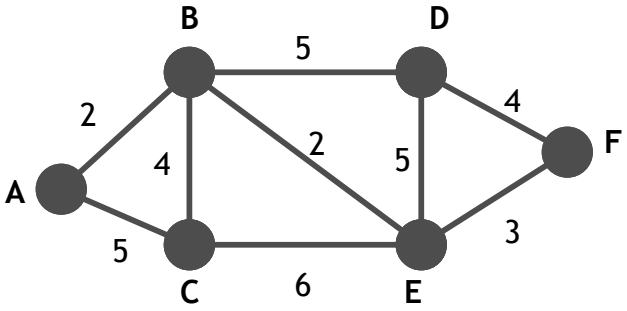
The Snowplow Problem

As the new snowplow operator, you must decide the best route through three cities. In each city, you need to plow all the roads and return to your starting place, but you must also keep from backtracking as much as possible.

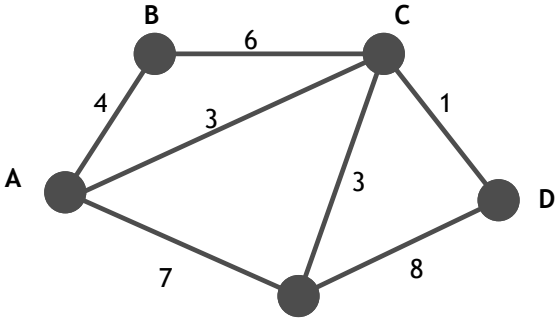
1. Construct two snowplow routes through each of the following cities and indicate the time it will take to travel each route. The time it takes to traverse each road (in hours) is indicated in the graph.



City I



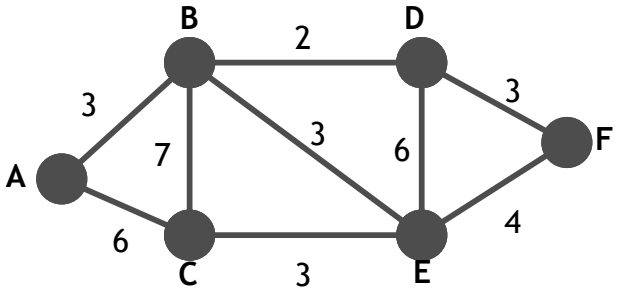
City II



City III

The Most Efficient: Using the algorithm explained below, you are able to find the most efficient route even when a Hamiltonian circuit does not exist.

City I has two vertices of an odd degree.



a) Find these vertices and the shortest path between them
b)For each edge in this shortest path, put in a second copy of the edge (create a multiple edge)
c) At this point, your graph should have no vertices of an odd degree. Find a Euler circuit and compare this path with the paths you found for City I.

Follow the procedure outlined above to find a solution to the Snowplow problem for City III

Follow-up Questions:

1)

In Euler paths and Euler circuits, the goal was to find paths or circuits that include every _____ of the graph once and only once.

In **Hamilton paths and Hamilton circuits**, the goal is to find paths and circuits that include every _____ of the graph once and only once.

A Hamilton Path is

_____

_____

A Hamilton Circuit is

_____

_____

2) REFLECTION: How would you solve the Snowplow problem for a graph that has no vertices of an odd degree?

3) Describe in your own words the algorithm for the most efficient weighted graph.

4) Revisiting the snowplow problem above, what type of graph, walk, path, or circuit do you need if a town wants to model an emergency route where every house can be evacuated. Also, what type of graph, walk, path, or circuit would model a town that ideally wants every street plowed.

Quiz Review: Complete the following problems.

1)

Let's use the airline example that we have seen before.

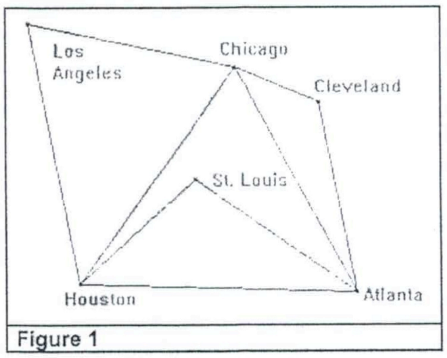


Figure 1

We already know that this graph has an Euler circuit. Does is have a Hamilton circuit as well? If so, write an example of it here.

_____

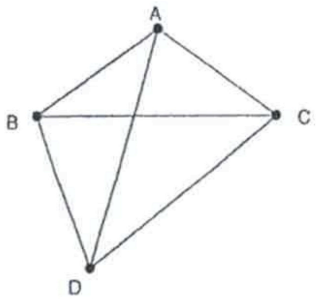
_____

_____

Could the graph also have a Hamilton path? Write it here.

_____

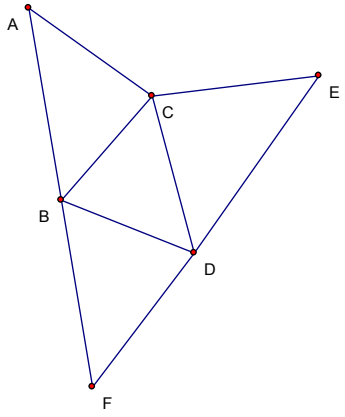
2) Determine if the following graph has an Euler circuit and/or a Hamiltonian circuit. If one exists, write the route you could take. If one does not exist, explain why.



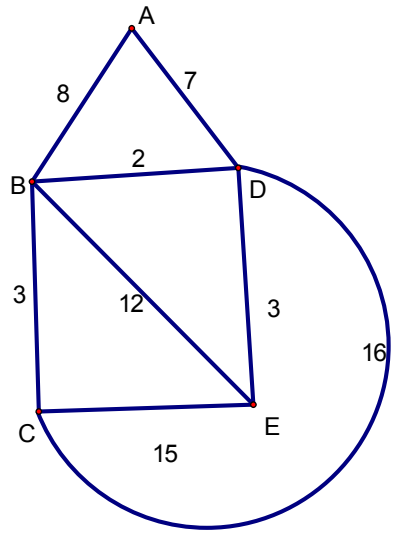
Euler Circuit:

Hamilton Circuit:

3)

	1. Identify the degree of each vertex	A.	B.	C.
	2. Does the graph contain an Euler Path? Yes No	3. Does the graph contain an Euler Circuit? Yes No	4. Does the graph contain a Hamilton Path? Yes No	5. Does the graph contain a Hamilton Circuit? Yes No

4)



6. Identify the degree of each vertex	A.	B.	C.
7. Does the graph contain an Euler Path? Yes No	8. Does the graph contain an Euler Circuit? Yes No	9. Does the graph contain a Hamilton Path? Yes No	10. Does the graph contain a Hamilton Circuit? Yes No

5) In the above problem, find the shortest snowplow route through each of the cities represented by the letters A, B, C, D, and E. Remember that you must return to the starting point. The time it takes to traverse each road is indicated in the graph.

6) Draw an example of a graph that is both Euler circuit and a Hamiltonian circuit.

7) Suppose the following graph represents the houses (vertices) and streets (edges) connecting a neighborhood. The garbage person must pick up trash from each house and begin and end at the garbage facility (Vertex A). Is this possible to do efficiently? If so, what is the route? If not, what is a route that is mostly efficient?