CIRCULAR MOTION PRACTICE PROBLEMS

1.

1. In aviation, a "standard turn" for a level flight of a propeller-type plane is one in which the plane makes a complete circular turn in 2.00 minutes. If the speed of the plane is 170 m/s,

a. What is the radius of the circle?

$$v = \frac{2\pi R}{T} \qquad 170 = \frac{2\pi R}{120} \qquad R = 3247m$$

b. What is the centripetal acceleration of the plane?

$$a_c = \frac{v^2}{R} \qquad a_c = \frac{(170)^2}{3247} \qquad a_c = 8.9ms^{-2}$$

2.

A fly of mass 2.00 g is sunning itself on a phonograph turntable at a location that is 4.00 cm from the center. When the turntable is turned on and rotates at 45.0 rev/min, calculate the centripetal force needed to keep the fly from slipping?

$$F = ma_c = \frac{mv^2R}{} = \frac{4m\pi^2 R}{T} \qquad T = \frac{60\sec}{45rev} = 1.33$$

$$F = \frac{4(0.002kg)\pi^2(0.04m)}{1.33s} = 0.002N$$

3.

A 35.0 kg boy is swinging on a rope 7.00 m long. He passes through the lowest position with a speed of 3.00 m/s. What is the tension on the rope at that moment?

$$T = \frac{mv^2}{R} + mg$$

$$T = \frac{(35kg)(3m/s)^2}{7m} + (35)(9.8)$$

$$T = 388N$$

4.

4. The earth orbits the sun in 365 days. What is the tangential speed, in m/s, of the earth in orbit? The average sun-earth distance is 1.50×10^{11} m.

$$v = \frac{2\pi R}{T}$$

$$v = \frac{2\pi(1.5\times10^{11} m)}{(365d)(24hr)(3600s)} = 29886 m/s$$

5.

5. A plane comes out of a power dive, turning upward in a curve whose center of curvature is 1300 m above the plane. The plane's speed is 260 m/s.

a. Calculate the upward force of the seat cushion on the 100 kg pilot of the plane.

$$F = \frac{mv^2}{R} + mg = \frac{(100kg)(260m/s)^2}{1300m} + (100kg)(9.8ms^{-2}) = 6180N$$

b. Calculate the upward force on a 90.0 g sample of blood in the pilot's head.

$$F = \frac{mv^2}{R} + mg = \frac{(0.09kg)(260m/s)^2}{1300m} + (0.09kg)(9.8ms^{-2}) = 5.56N$$

6.

The moon's mass is 7.35×10^{22} kg, and it moves around the earth approximately in a circle or radius 3.82×10^5 km. The time required for one revolution is 27.3 days. Calculate the centripetal force that must act on the moon. How does this compare to the gravitational force that the earth exerts on the moon at that same distance?

$$F = \frac{m4\pi^2 r}{T^2} = \frac{(7.35\times10^{22}kg)(4\pi^2)(3.82\times10^8 m)}{\left((27.3d)(24hrs)(3600s)\right)^2} = 1.99\times10^{20}N$$

$$F = G\frac{mM}{R^2} = \frac{(6.67\times10^{-11}m^3kg^{-1}s^{-2})(5.98\times10^{24}kg)(7.36\times10^{22}kg)}{\left(3.82\times10^8 m\right)^2} = 2.01\times10^{20}N$$

7.

The radius of the earth is 6.37×10^6 m.

a. How fast, in m/s, is a tree at the equator moving because of the earth's rotation?

b. How fast is a polar bear moving at the north pole?

$$a.\, v = \frac{circumference}{time} = \frac{2\pi r}{T} = \frac{2\pi(6.37\times10^6)}{24h\times60m\times60s} = 463\,\frac{m}{s}$$

$$b.\, 0\,\frac{m}{s}, spinning\ in\ the\ center\ of\ rotation.$$

8.

A car of mass 1200 kg drives around a curve with a radius of 25.0 m. If the driver maintains a speed of 20.0 km/hr, what is the force of friction between the tires and the road? What is the minimum coefficient of static friction required to keep the car in this turn?

$$f_s = F_c = ma_c = m\frac{v^2}{R} = (1200)\frac{\left(20\frac{km}{hr}\times\frac{1000m}{1km}\times\frac{1hr}{3600s}\right)^2}{25m} = 1481\ N$$

$$f_s = \mu_s mg \qquad \rightarrow \mu_s = \frac{f_s}{mg} = \frac{1481}{(1200)(9.8)} = 0.126 = 0.13$$

9.

An automobile corner curve of radius ***R*** at a speed ***v***. In terms of ***R*** and ***v*** and any other required physical constants, what is the minimum coefficient of friction required for the turn?

$$v = \sqrt{\mu_s g R} \quad \rightarrow \mu_s = \frac{v^2}{gR}$$

10.

A 10.0 kg block rests on a frictionless surface and is attached to a vertical peg by a rope. What is the tension in the rope if the block is whirling in a horizontal circle of radius 2.00 m with a linear speed of 20 m/s? What would be the tension in the rope at the top and the bottom of the swing if it were whirled in a vertical circle?

$$F_c = ma_c = m\frac{v^2}{R} = (10\ kg)\frac{\left(20\frac{m}{s}\right)^2}{2m} = 2000N$$

$$top: T = \frac{mv^2}{R} - mg = 2000\ N - 10(9.8) = 1902 = 1900\ N$$

$$bottom: T = \frac{mv^2}{R} + mg = 2000\ N + 10(9.8) = 2098 = 2100\ N$$

11.

A child twirls his yo-yo about his head rather than using it properly. The yo-yo has a mass of 0.200 kg and is attached to a string 0.800 m long.

a. If the "yo yo" makes a complete revolution each second, what tension must exist in the string?

$$T = F_c = ma_c = m\frac{4\pi^2 R}{T^2} = (0.2\ kg)\frac{4\pi^2(0.8m)}{(1s)^2} = 6.32\ N$$

b. If the child increases the speed of the top so that it makes 2 complete revolutions per second, what tension must be in the string?

Doubling the speed or halfing the period will result in quadrupling the tension.

$$T = F_c = ma_c = m\frac{4\pi^2 R}{T^2} = (0.2\ kg)\frac{4\pi^2(0.8m)}{(0.5s)^2} = 25.3\ N$$

c. What is the ratio of the solutions to parts a and b?

Doubling the speed or halfing the period will result in quadrupling the tension.

12.

An athlete twirls a 8.0 kg hammer around his head at a rate of 2.00 rev/s. If his arms are 1.00 m long, compute the tension in them.

$$T = F_c = ma_c = m\frac{4\pi^2 R}{T^2} = (8\ kg)\frac{4\pi^2(1m)}{(0.5s)^2} = 1263\ N$$

13.

Two lead spheres each have a mass of 5.00×10^5 kg. The spheres are located next to one another with their centers 5.00 m apart. What gravitational force do they exert on each other?

$$F = G\frac{m_1 m_2}{d^2} = 6.67408 \times 10^{-11}\ \frac{\left(5.00 \times 10^5\right)^2}{5^2} = 0.667\text{ N}$$

14.

An old trick is to swing a pail full of water in a vertical circle. If the "swinger" is not to get wet and the radius of the circle in which she swings the pail is 0.800 m, what is the minimum tangential speed with which she can swing the pail at the top of the swing? Explain why, if she reaches this minimum speed, she not get wet?

$$v_c = \sqrt{gR} = \sqrt{(9.8)(0.8)} = 2.8\, m/s$$

The force between gravity and centripetal force balance.

15.

What is the percentage difference in a person's apparent weight at the equator and the north pole. Assume, for the purposes of this problem, that the earth is perfectly spherical.

Radius of the Earth = 6.371 x 10^6 m

$F_{normal} = F_{weight} - F_c$

$$n_1 = mg - \frac{mv^2}{R} \; at\ the\ equator$$

$$n_2 = mg \; at\ the\ North\ Pole$$

$$\frac{n_2 - n_1}{n_2} = \frac{mg - mg + mv^2}{mg} = 1 - \left(1 - \frac{v^2}{g}\right) = \frac{v^2}{g} = \frac{\frac{4\pi^2 r}{T^2}}{g} = \frac{4\pi^2 r}{T^2 g} = \frac{4\pi^2(6.371 \times 10^6)}{(24x3600)^2(9.8)} = 0.0034 = 0.34\%$$

16.

With what speed would a baseball player have to hit a baseball in order to put a 0.300 kg baseball into orbit around the earth at an altitude of 1.00 m? How fast would you have to get an elephant of mass 500 kg moving to make it orbit the earth at the same height? Explain any difference, or lack thereof, in your answers! Why would it be practically impossible to actually put either one into orbit at this height?

$$v_c = \sqrt{gR} = \sqrt{(9.8)(6.371 \times 10^6 + 1)} = 7902\,\frac{m}{s} \qquad (17384\,\frac{miles}{hour})$$

Same for the elephant, the critical speed equation is independent of the mass.

Air resistance would slow them down drastically

17.

Communications satellites are placed in what are called "geosynchronous" orbits around the earth. This means that the satellite will have an orbital period which matches the period for one earth revolution (one day). At what height above the surface of the earth must one place such a satellite in order to achieve a geosynchronous orbit? How fast will the satellite be moving? Why is it desirable for communications satellites to be placed in such orbits?

To geosynchronous, $F_c = F_G$

$\frac{m_1 v^2}{r} = G\frac{m_1 m_2}{r^2}$ m_1 cancels and r cancels with r^2

$v^2 = G\frac{m_2}{r}$ $\rightarrow$ $\left(\frac{2\pi r}{T}\right)^2 = G\frac{m_2}{r}$ $\rightarrow$ $r^3 = \frac{G m_2 T^2}{4\pi^2}$ $\rightarrow$

$r = \sqrt[3]{\frac{G m_2 T^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(5.98\times10^{24})(24\times3600)^2}{4\pi^2}}$ = 42250474.31 m = 4.23×10^7m

Subtract Earth's radius to get height.

4.23×10^7m – $6.371 \times 10^6 = 3.58 \times 10^7$ m

17.

How fast will the satellite be moving?

$$v = \frac{2\pi r}{T} = \frac{2\pi(4.23\times10^7)}{(24\times3600)} = 3076\,\frac{m}{s}$$

As it is at greater height, it covers larger geographical area. Hence only 3 satellites are required to cover the entire Earth.
➡Satellites are visible for 24 hours continuously from single fixed location on the Earth.
➡It is ideal for broadcasting and multi-point distribution applications.
➡Ground station tracking is not required as it is continuously visible from earth all the time from fixed location.
➡Inter-satellite handoff is not needed.
➡Less number of satellites are needed to cover the entire earth. Total three satellites are sufficient for the job.
➡Almost there is no doppler shift and hence less complex receivers can be used for the satellite communication.

18.

If a roller coaster is to successfully execute a 20.0 m radius vertical loop, what speed must each car on a roller coaster be traveling at the top of the loop. Does the speed required depend on the mass of the roller coaster or that of the occupants? Most rides have a minimum height requirement. Why are there no minimum or maximum weight restrictions?

$$v_c = \sqrt{gR} = \sqrt{(9.8)(20)} = 14\frac{m}{s}$$

NOPE, no need to consider mass because the critical velocity is independent of mass.

19.

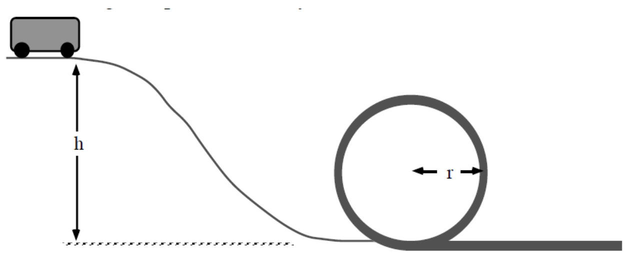
Refer to the diagram of a vertical loop roller coaster shown to the right. What is the minimum height **h** that a roller coaster of mass **m** must begin if it is to successfully maneuver a loop of radius **r**?

$$mgh = mg2r + \frac{1}{2}mv^2$$

$$mgh = mg2r + \frac{1}{2}m(\sqrt{gr})^2$$

$$mgh = mg2r + \frac{1}{2}mgr \qquad cancel\ the\ mg's$$

$$h = 2r + \frac{1}{2}r = 2.5\,r$$



20.

Banked curves are often used on racetracks to enable cars to safely execute turns at high speeds. Assuming no friction to help the car stay in a circular curve, what is the maximum speed that a 2000 kg car can travel around a curve of radius 50.0 m if the angle at which the curve is banked is 25° above the horizontal? With no friction, what will happen to a car if it goes slower than this speed? What if it goes faster? Explain!

$$tan\theta = \frac{v^2}{gR} \qquad v = \sqrt{tan\theta gR} = \sqrt{tan25^o(9.8)(50)} = 15.1\frac{m}{s}$$

It would slide down the bank of the curve.

Slide up the curve.

21.

21. What is the minimum banking angle required for an automobile to make a turn of radius ***R*** at a speed ***v*** if there is no friction between the roadway and the tires. Express your answer in terms of ***R*** and ***v*** and any other required physical constants, What will happen if the car goes slower than this speed? What if it goes faster?

$$tan\theta = \frac{v^2}{gR}$$

$$\theta = tan^{-1}\left(\frac{v^2}{gR}\right)$$

Slower = car sliding down the bank because of the decrease in the force.

Faster = car sliding up the back because it is requiring a larger radius for that speed.

22.

22. An object of mass *m* is moving at a speed v_o when it is at the bottom of a vertical circle as shown in the diagram to the right. When the object reaches point P, it "falls" out of the circle. If the speed of the block at the bottom of the circle is described by the equation $v_o = \sqrt{\frac{9}{2}gL}$, what is the angle θ, at which the object quits moving in the circle?