King Fahd University of Petroleum & Minerals

CISE 302
Linear Control Systems Laboratory Manual

Systems Engineering Department

Revised - September 2012

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CISE 302

Linear Control Systems

Lab Experiment 1: Using MATLAB for Control Systems

Objectives: This lab provides an introduction to MATLAB in the first part. The lab also provides tutorial of polynomials, script writing and programming aspect of MATLAB from control systems view point.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

Category Soft-Experiment

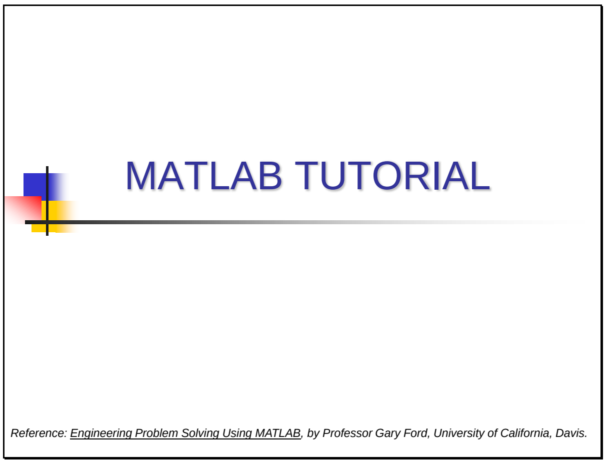
Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results should be reported properly.

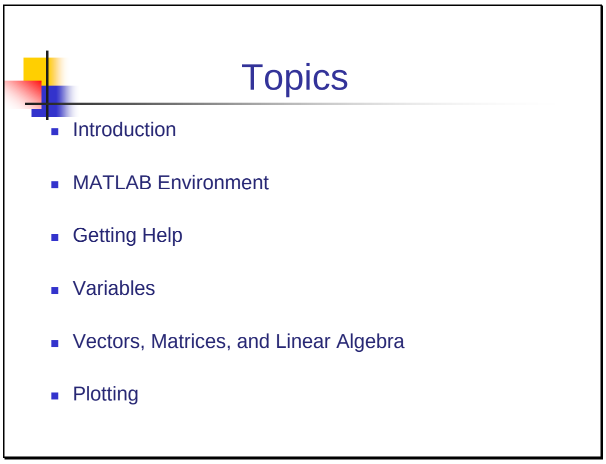
Part I: Introduction to MATLAB

Objective: The objective of this exercise will be to introduce you to the concept of mathematical programming using the software called MATLAB. We shall study how to define variables, matrices etc, see how we can plot results and write simple MATLAB codes.



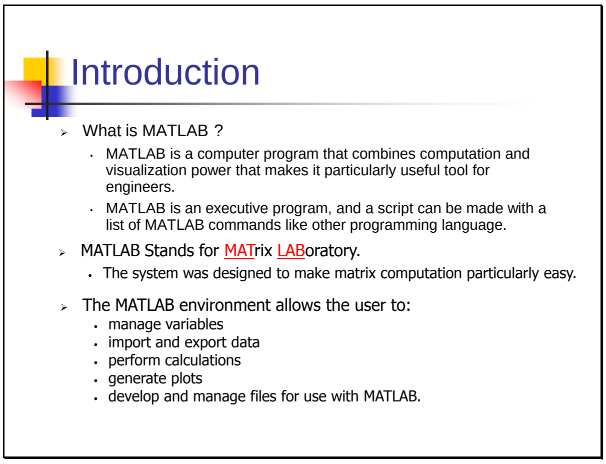
MATLAB TUTORIAL

Reference: Engineering Problem Solving Using MATLAB, by Professor Gary Ford, University of California, Davis.



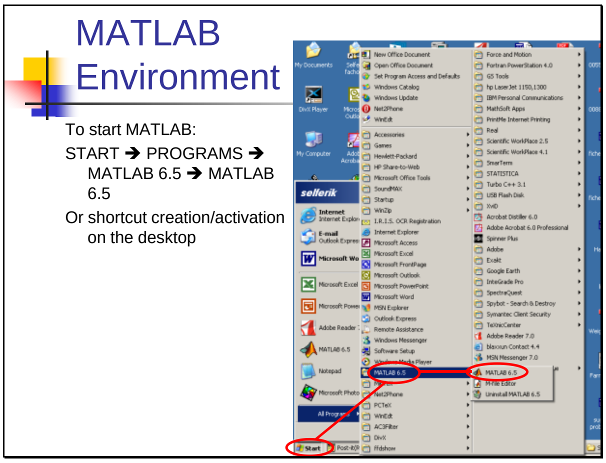
Topics

- Introduction
- MATLAB Environment
- Getting Help
- Variables
- Vectors, Matrices, and Linear Algebra
- Plotting



Introduction

- What is MATLAB ?
  - MATLAB is a computer program that combines computation and visualization power that makes it particularly useful tool for engineers.
  - MATLAB is an executive program, and a script can be made with a list of MATLAB commands like other programming language.
- MATLAB Stands for MATrix LABoratory.
  - The system was designed to make matrix computation particularly easy.
- The MATLAB environment allows the user to:
  - manage variables
  - import and export data
  - perform calculations
  - generate plots
  - develop and manage files for use with MATLAB.



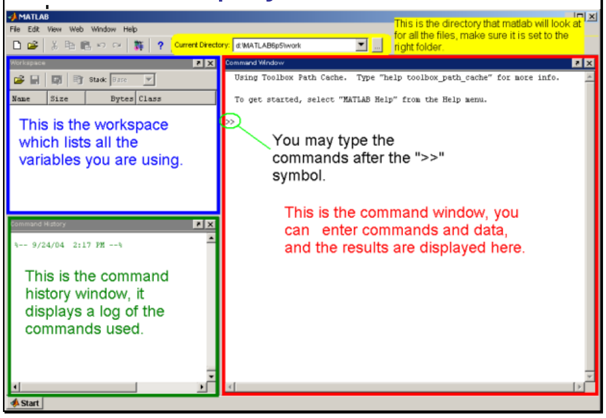
MATLAB Environment

To start MATLAB:

START ➔ PROGRAMS ➔ MATLAB 6.5 ➔ MATLAB 6.5

Or shortcut creation/activation on the desktop

Display Windows



This is the directory that matlab will look at for all the files, make sure it is set to the right folder.

```
Using Toolbox Path Cache.  Type "help toolbox_path_cache" for more info.

To get started, select "MATLAB Help" from the Help menu.

>>
```

You may type the commands after the ">>" symbol.

This is the command window, you can enter commands and data, and the results are displayed here.

This is the workspace which lists all the variables you are using.

```
%-- 9/24/04  2:17 PM --%
```

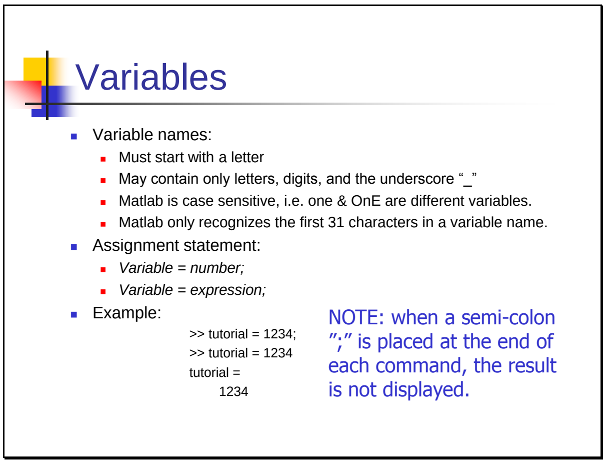
This is the command history window, it displays a log of the commands used.

Display Windows (con't…)

- Graphic (Figure) Window
  - Displays plots and graphs
  - Created in response to graphics commands.
- M-file editor/debugger window
  - Create and edit scripts of commands called M-files.

Getting Help

- type one of following commands in the command window:
  - **help** – lists all the help topic
  - **help** *topic* – provides help for the specified topic
  - **help** *command* – provides help for the specified command
    - **help help** – provides information on use of the help command
  - **helpwin** – opens a separate help window for navigation
  - **lookfor** *keyword* – Search all M-files for *keyword*

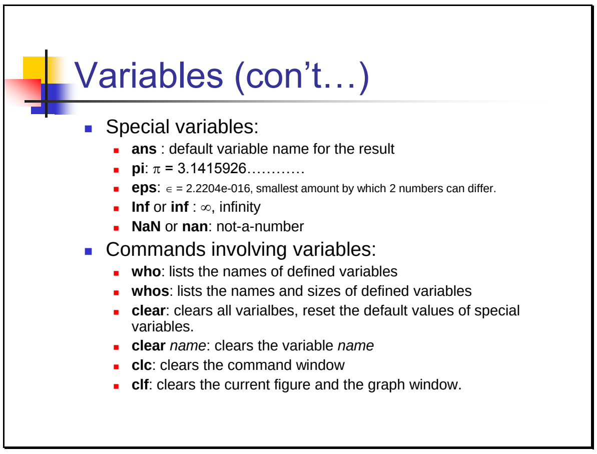


Variables

- Variable names:
  - Must start with a letter
  - May contain only letters, digits, and the underscore "_"
  - Matlab is case sensitive, i.e. one & OnE are different variables.
  - Matlab only recognizes the first 31 characters in a variable name.
- Assignment statement:
  - *Variable = number;*
  - *Variable = expression;*
- Example:

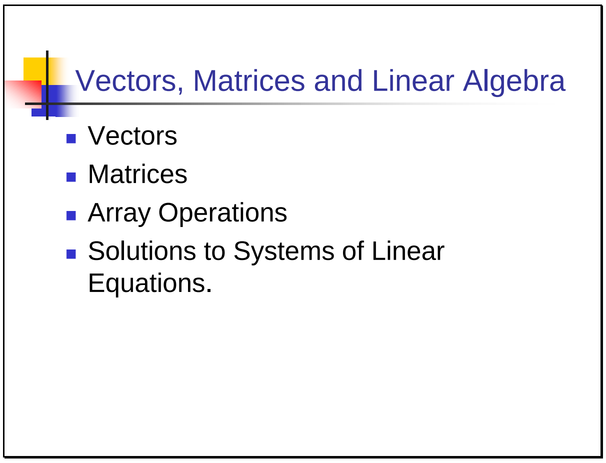
```
>> tutorial = 1234;
>> tutorial = 1234
tutorial =
     1234
```

NOTE: when a semi-colon ";" is placed at the end of each command, the result is not displayed.



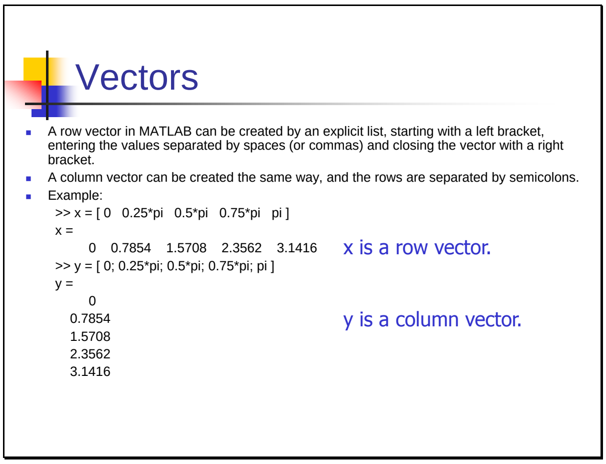
Variables (con't…)

- Special variables:
  - **ans** : default variable name for the result
  - **pi**: π = 3.1415926…………
  - **eps**: ∊ = 2.2204e-016, smallest amount by which 2 numbers can differ.
  - **Inf** or **inf** : ∞, infinity
  - **NaN** or **nan**: not-a-number
- Commands involving variables:
  - **who**: lists the names of defined variables
  - **whos**: lists the names and sizes of defined variables
  - **clear**: clears all varialbes, reset the default values of special variables.
  - **clear** *name*: clears the variable *name*
  - **clc**: clears the command window
  - **clf**: clears the current figure and the graph window.



Vectors, Matrices and Linear Algebra

- Vectors
- Matrices
- Array Operations
- Solutions to Systems of Linear Equations.



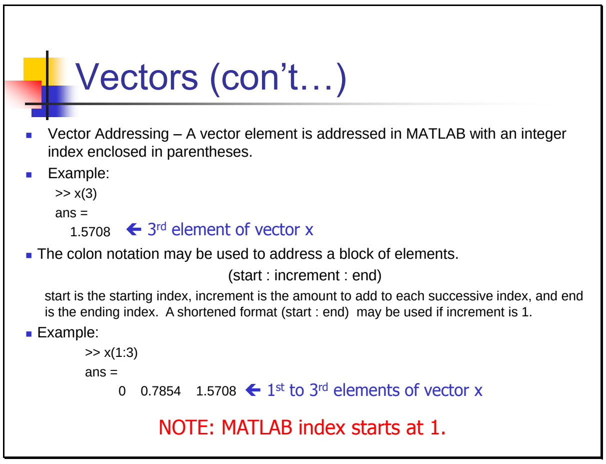
Vectors

- A row vector in MATLAB can be created by an explicit list, starting with a left bracket, entering the values separated by spaces (or commas) and closing the vector with a right bracket.
- A column vector can be created the same way, and the rows are separated by semicolons.
- Example:

```
>> x = [ 0   0.25*pi   0.5*pi   0.75*pi   pi ]
x =
       0    0.7854    1.5708    2.3562    3.1416
>> y = [ 0; 0.25*pi; 0.5*pi; 0.75*pi; pi ]
y =
       0
  0.7854
  1.5708
  2.3562
  3.1416
```

x is a row vector.

y is a column vector.



Vectors (con’t…)

- Vector Addressing – A vector element is addressed in MATLAB with an integer index enclosed in parentheses.
- Example:

```
>> x(3)
ans =
   1.5708
```

← 3rd element of vector x

- The colon notation may be used to address a block of elements.

(start : increment : end)

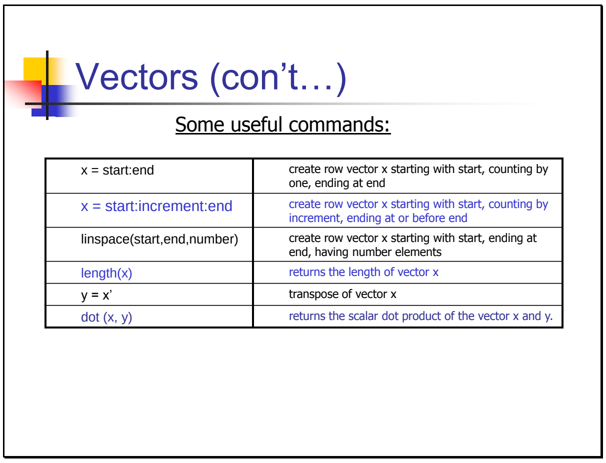
start is the starting index, increment is the amount to add to each successive index, and end is the ending index. A shortened format (start : end) may be used if increment is 1.

- Example:

```
>> x(1:3)
ans =
     0    0.7854    1.5708
```

← 1st to 3rd elements of vector x

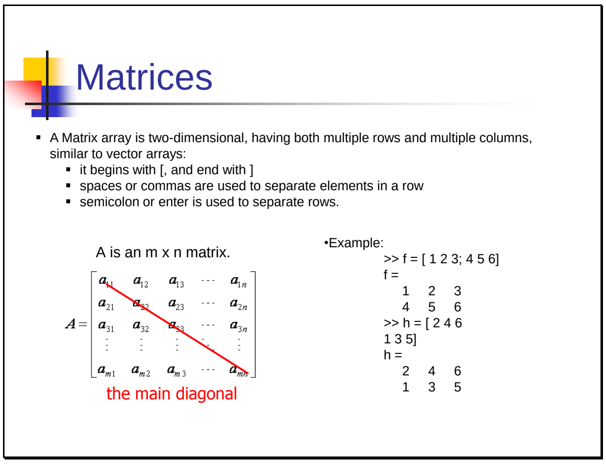
NOTE: MATLAB index starts at 1.



Vectors (con’t…)

Some useful commands:

x = start:end	create row vector x starting with start, counting by one, ending at end
x = start:increment:end	create row vector x starting with start, counting by increment, ending at or before end
linspace(start,end,number)	create row vector x starting with start, ending at end, having number elements
length(x)	returns the length of vector x
y = x’	transpose of vector x
dot (x, y)	returns the scalar dot product of the vector x and y.



Matrices

- A Matrix array is two-dimensional, having both multiple rows and multiple columns, similar to vector arrays:
  - it begins with [, and end with]
  - spaces or commas are used to separate elements in a row
  - semicolon or enter is used to separate rows.

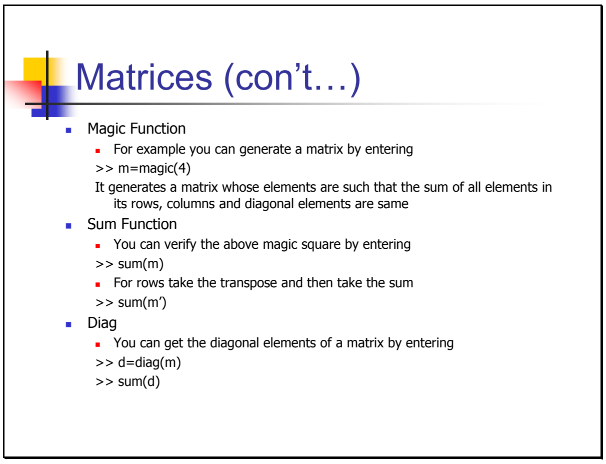
A is an m x n matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

the main diagonal

•Example:

```
>> f = [ 1 2 3; 4 5 6]
f =
    1    2    3
    4    5    6
>> h = [ 2 4 6
1 3 5]
h =
    2    4    6
    1    3    5
```



Matrices (con't…)

- Magic Function
  - For example you can generate a matrix by entering

    >> m=magic(4)

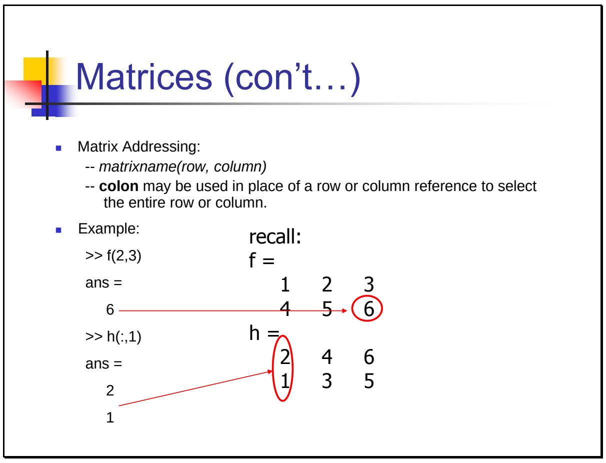
    It generates a matrix whose elements are such that the sum of all elements in its rows, columns and diagonal elements are same
- Sum Function
  - You can verify the above magic square by entering

    >> sum(m)
  - For rows take the transpose and then take the sum

    >> sum(m')
- Diag
  - You can get the diagonal elements of a matrix by entering

    >> d=diag(m)

    >> sum(d)



Matrices (con't…)

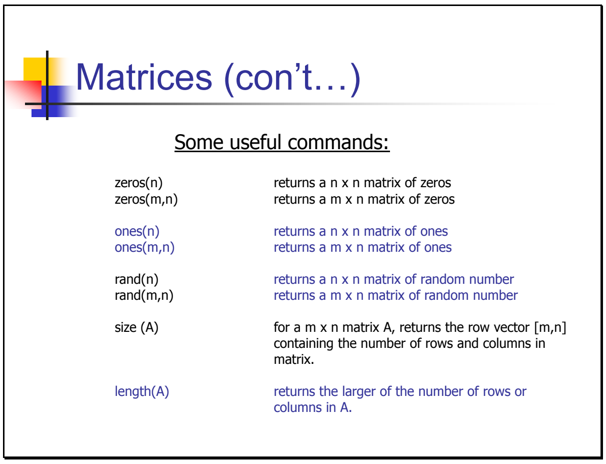
- Matrix Addressing:

  -- *matrixname(row, column)*

  -- **colon** may be used in place of a row or column reference to select the entire row or column.
- Example:

```
>> f(2,3)
ans =
    6
>> h(:,1)
ans =
    2
    1
```

```
recall:
f =
    1   2   3
    4   5   6
h =
    2   4   6
    1   3   5
```



Matrices (con’t…)

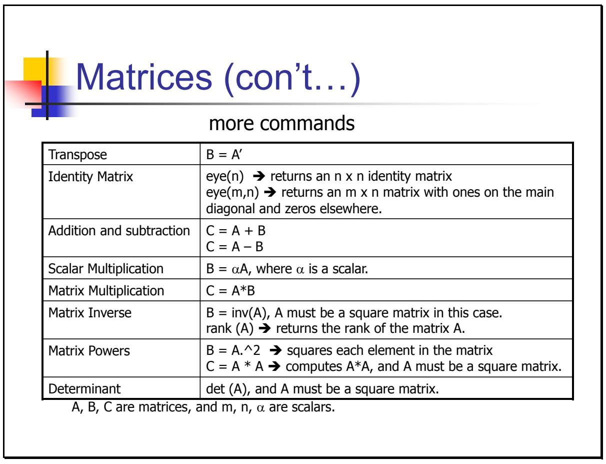
Some useful commands:

zeros(n)
zeros(m,n)
	returns a n x n matrix of zeros
	returns a m x n matrix of zeros

ones(n)
ones(m,n)
	returns a n x n matrix of ones
	returns a m x n matrix of ones

rand(n)
rand(m,n)
	returns a n x n matrix of random number
	returns a m x n matrix of random number

size (A)	for a m x n matrix A, returns the row vector [m,n] containing the number of rows and columns in matrix.
length(A)	returns the larger of the number of rows or columns in A.



Matrices (con’t…)

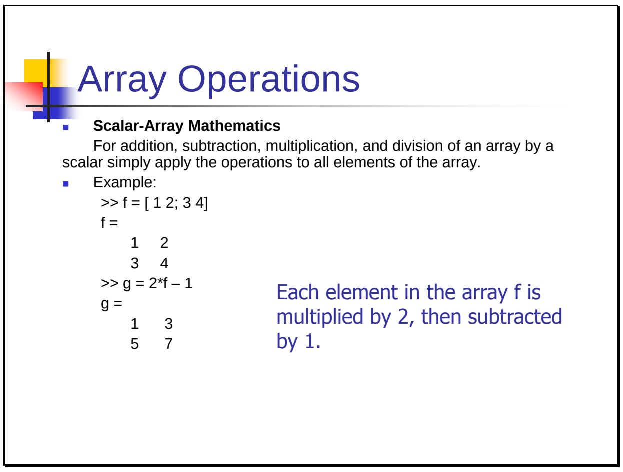
more commands

Transpose	B = A’
Identity Matrix	eye(n) ➔ returns an n x n identity matrix eye(m,n) ➔ returns an m x n matrix with ones on the main diagonal and zeros elsewhere.

Addition and subtraction	C = A + B
	C = A – B

Scalar Multiplication	B = αA, where α is a scalar.
Matrix Multiplication	C = A*B
Matrix Inverse	B = inv(A), A must be a square matrix in this case. rank (A) ➔ returns the rank of the matrix A.
Matrix Powers	B = A.^2 ➔ squares each element in the matrix C = A * A ➔ computes A*A, and A must be a square matrix.
Determinant	det (A), and A must be a square matrix.

A, B, C are matrices, and m, n, α are scalars.



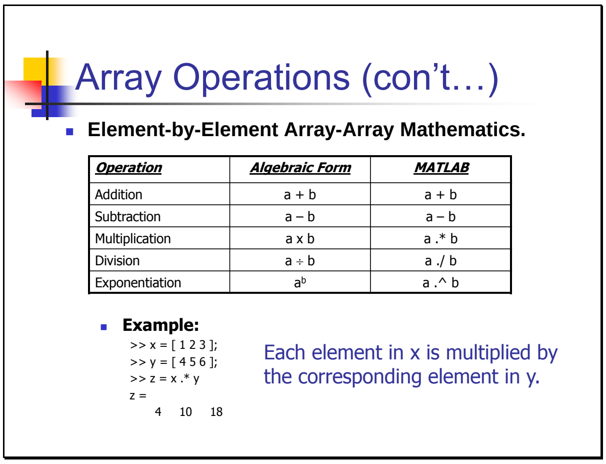
Array Operations

- **Scalar-Array Mathematics**

  For addition, subtraction, multiplication, and division of an array by a scalar simply apply the operations to all elements of the array.
- Example:

```
>> f = [ 1 2; 3 4]
f =
     1    2
     3    4
>> g = 2*f – 1
g =
     1     3
     5     7
```

Each element in the array f is multiplied by 2, then subtracted by 1.



Array Operations (con't…)

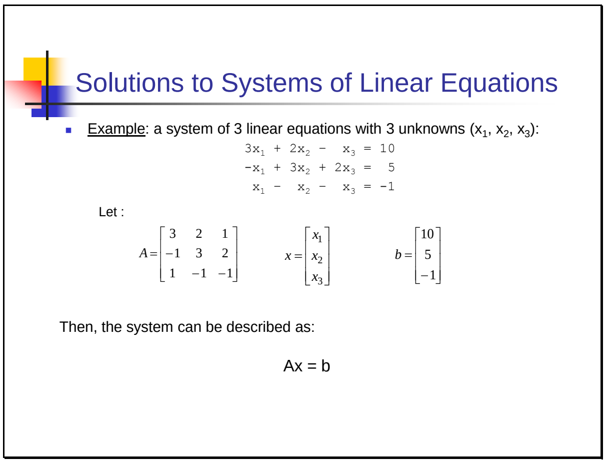
- **Element-by-Element Array-Array Mathematics.**

Operation	*Algebraic Form*	*MATLAB*
Addition	a + b	a + b
Subtraction	a – b	a – b
Multiplication	a x b	a .* b
Division	a ÷ b	a ./ b
Exponentiation	a^b	a .^ b

- **Example:**

```
>> x = [ 1 2 3 ];
>> y = [ 4 5 6 ];
>> z = x .* y
z =
      4    10    18
```

Each element in x is multiplied by the corresponding element in y.



Solutions to Systems of Linear Equations

- Example: a system of 3 linear equations with 3 unknowns (x_1, x_2, x_3):

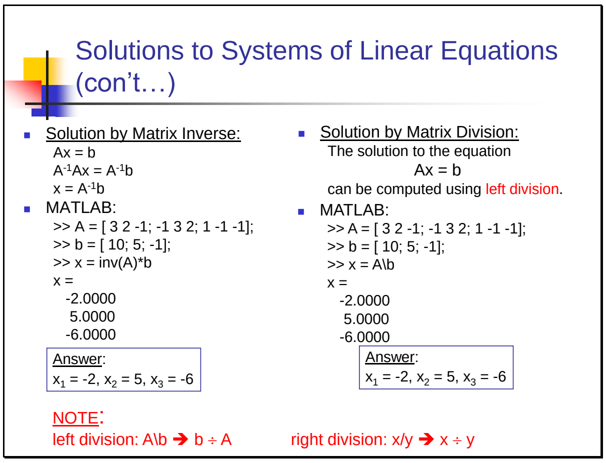
$$\begin{aligned} 3x_1 + 2x_2 - x_3 &= 10 \\ -x_1 + 3x_2 + 2x_3 &= 5 \\ x_1 - x_2 - x_3 &= -1 \end{aligned}$$

Let :

$$A = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 3 & 2 \\ 1 & -1 & -1 \end{bmatrix} \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad b = \begin{bmatrix} 10 \\ 5 \\ -1 \end{bmatrix}$$

Then, the system can be described as:

Ax = b



Solutions to Systems of Linear Equations (con't…)

- Solution by Matrix Inverse:
  Ax = b
  $A^{-1}Ax = A^{-1}b$
  $x = A^{-1}b$
- MATLAB:

```
>> A = [ 3 2 -1; -1 3 2; 1 -1 -1];
>> b = [ 10; 5; -1];
>> x = inv(A)*b
x =
   -2.0000
    5.0000
   -6.0000
```

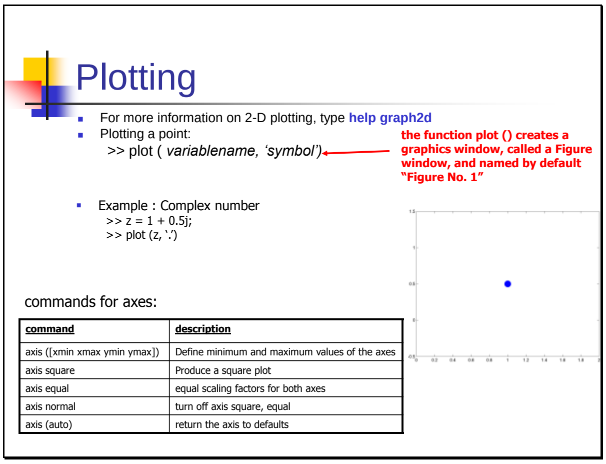
Answer:
$x_1 = -2$, $x_2 = 5$, $x_3 = -6$

- Solution by Matrix Division:
  The solution to the equation
  Ax = b
  can be computed using left division.
- MATLAB:

```
>> A = [ 3 2 -1; -1 3 2; 1 -1 -1];
>> b = [ 10; 5; -1];
>> x = A\b
x =
   -2.0000
    5.0000
   -6.0000
```

Answer:
$x_1 = -2$, $x_2 = 5$, $x_3 = -6$

NOTE:
left division: A\b ➔ b ÷ A   right division: x/y ➔ x ÷ y



Plotting

- For more information on 2-D plotting, type **help graph2d**
- Plotting a point:

```
>> plot ( variablename, 'symbol')
```

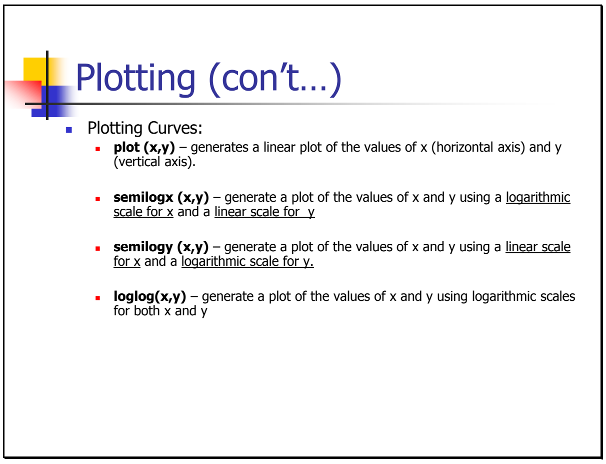
the function plot () creates a graphics window, called a Figure window, and named by default "Figure No. 1"

- Example : Complex number

```
>> z = 1 + 0.5j;
>> plot (z, '.')
```

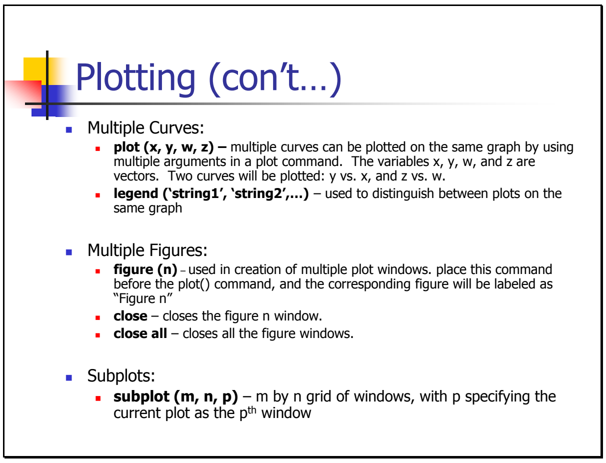
commands for axes:

command	**description**
axis ([xmin xmax ymin ymax])	Define minimum and maximum values of the axes
axis square	Produce a square plot
axis equal	equal scaling factors for both axes
axis normal	turn off axis square, equal
axis (auto)	return the axis to defaults



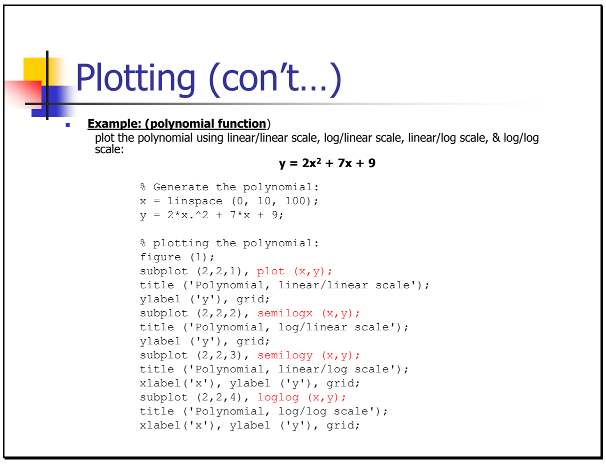
Plotting (con't…)

- Plotting Curves:
  - **plot (x,y)** – generates a linear plot of the values of x (horizontal axis) and y (vertical axis).
  - **semilogx (x,y)** – generate a plot of the values of x and y using a logarithmic scale for x and a linear scale for y
  - **semilogy (x,y)** – generate a plot of the values of x and y using a linear scale for x and a logarithmic scale for y.
  - **loglog(x,y)** – generate a plot of the values of x and y using logarithmic scales for both x and y



Plotting (con't…)

- Multiple Curves:
  - **plot (x, y, w, z) –** multiple curves can be plotted on the same graph by using multiple arguments in a plot command. The variables x, y, w, and z are vectors. Two curves will be plotted: y vs. x, and z vs. w.
  - **legend ('string1', 'string2',…)** – used to distinguish between plots on the same graph
- Multiple Figures:
  - **figure (n)** – used in creation of multiple plot windows. place this command before the plot() command, and the corresponding figure will be labeled as "Figure n"
  - **close** – closes the figure n window.
  - **close all** – closes all the figure windows.
- Subplots:
  - **subplot (m, n, p)** – m by n grid of windows, with p specifying the current plot as the pth window

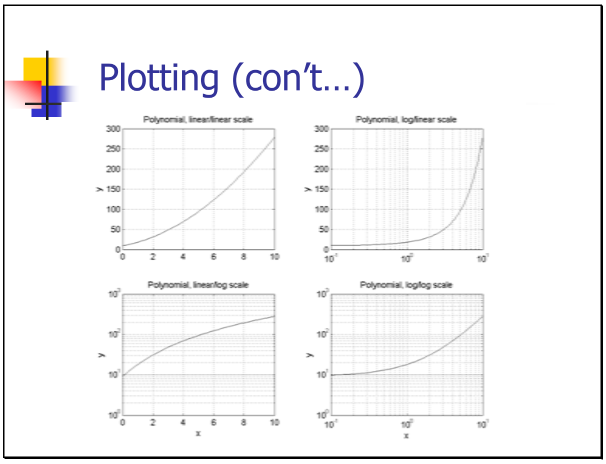


Plotting (con't…)

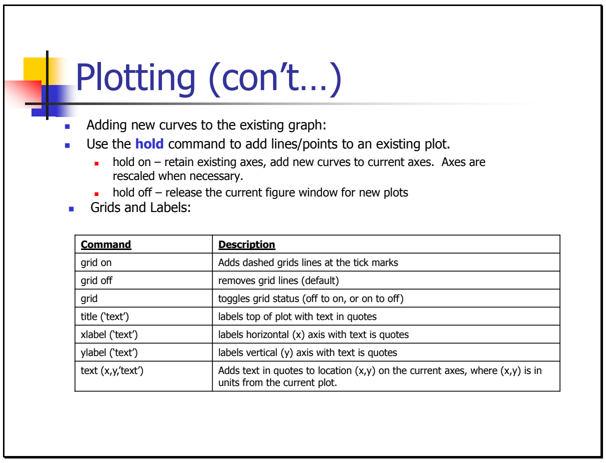
- **Example: (polynomial function)**
  plot the polynomial using linear/linear scale, log/linear scale, linear/log scale, & log/log scale:

y = 2x² + 7x + 9

```
% Generate the polynomial:
x = linspace (0, 10, 100);
y = 2*x.^2 + 7*x + 9;

% plotting the polynomial:
figure (1);
subplot (2,2,1), plot (x,y);
title ('Polynomial, linear/linear scale');
ylabel ('y'), grid;
subplot (2,2,2), semilogx (x,y);
title ('Polynomial, log/linear scale');
ylabel ('y'), grid;
subplot (2,2,3), semilogy (x,y);
title ('Polynomial, linear/log scale');
xlabel('x'), ylabel ('y'), grid;
subplot (2,2,4), loglog (x,y);
title ('Polynomial, log/log scale');
xlabel('x'), ylabel ('y'), grid;
```



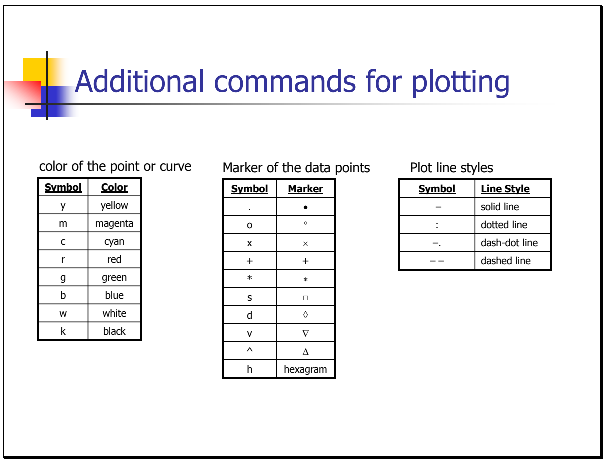
Plotting (con't…)



Plotting (con't…)

- Adding new curves to the existing graph:
- Use the **hold** command to add lines/points to an existing plot.
  - hold on – retain existing axes, add new curves to current axes. Axes are rescaled when necessary.
  - hold off – release the current figure window for new plots
- Grids and Labels:

Command	Description
grid on	Adds dashed grids lines at the tick marks
grid off	removes grid lines (default)
grid	toggles grid status (off to on, or on to off)
title ('text')	labels top of plot with text in quotes
xlabel ('text')	labels horizontal (x) axis with text is quotes
ylabel ('text')	labels vertical (y) axis with text is quotes
text (x,y,'text')	Adds text in quotes to location (x,y) on the current axes, where (x,y) is in units from the current plot.



Additional commands for plotting

color of the point or curve

Symbol	Color
y	yellow
m	magenta
c	cyan
r	red
g	green
b	blue
w	white
k	black

Marker of the data points

Symbol	Marker
.	•
o	°
x	×
+	+
*	*
s	□
d	◊
v	∇
^	Δ
h	hexagram

Plot line styles

Symbol	Line Style
–	solid line
:	dotted line
–.	dash-dot line
– –	dashed line

Exercise 1:

Use Matlab command to obtain the following

a) Extract the fourth row of the matrix generated by magic(6)

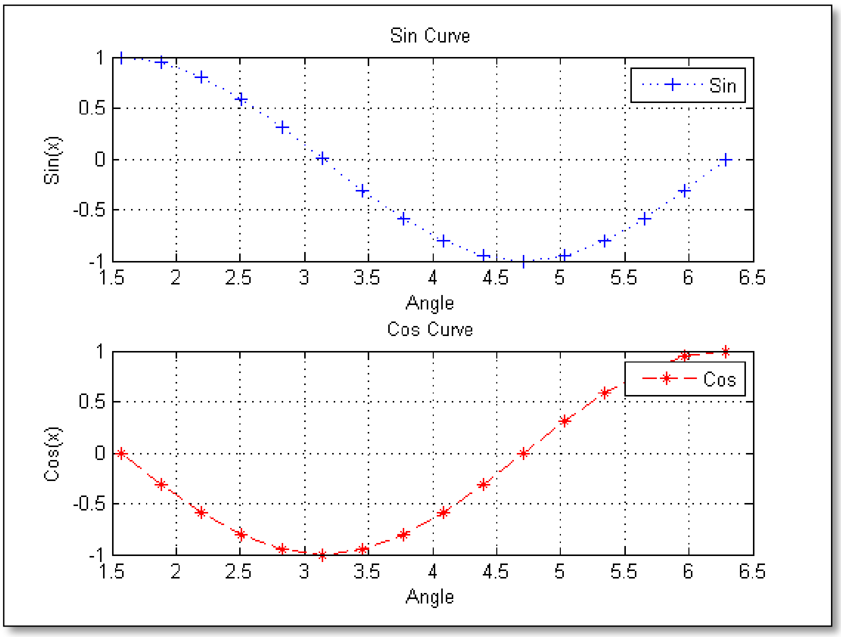
b) Show the results of ‘x’ multiply by ‘y’ and ‘y’ divides by ‘x’.
Given x = [0:0.1:1.1] and y = [10:21]

c) Generate random matrix ‘r’ of size 4 by 5 with number varying between -8 and 9

Exercise 2:

Use MATLAB commands to get exactly as the figure shown below

```
x=pi/2:pi/10:2*pi;
y=sin(x);
z=cos(x);
```



Part II: Polynomials in MATLAB

Objective: The objective of this session is to learn how to represent polynomials in MATLAB, find roots of polynomials, create polynomials when roots are known and obtain partial fractions.

Polynomial Overview:

MATLAB provides functions for standard polynomial operations, such as polynomial roots, evaluation, and differentiation. In addition, there are functions for more advanced applications, such as curve fitting and partial fraction expansion.

Polynomial Function Summary

Function	Description
Conv	Multiply polynomials
Deconv	Divide polynomials
Poly	Polynomial with specified roots
Polyder	Polynomial derivative
Polyfit	Polynomial curve fitting
Polyval	Polynomial evaluation
Polyvalm	Matrix polynomial evaluation
Residue	Partial-fraction expansion (residues)
Roots	Find polynomial roots

Symbolic Math Toolbox contains additional specialized support for polynomial operations.

Representing Polynomials

MATLAB represents polynomials as row vectors containing coefficients ordered by descending powers. For example, consider the equation

$$p(x) = x^3 - 2x - 5$$

This is the celebrated example Wallis used when he first represented Newton's method to the French Academy. To enter this polynomial into MATLAB, use

```
>>p = [1 0 -2 -5];
```

Polynomial Roots

The roots function calculates the roots of a polynomial:

```
>>r = roots(p)
```

```
r =
    2.0946
   -1.0473 +      1.1359i
   -1.0473 -     1.1359i
```

By convention, MATLAB stores roots in column vectors. The function poly returns to the polynomial coefficients:

```
>>p2 = poly(r)

p2 =
    1   8.8818e-16   -2   -5
```

poly and roots are inverse functions,

Polynomial Evaluation
The polyval function evaluates a polynomial at a specified value. To evaluate p at s = 5, use

```
>>polyval(p,5)

ans =
   110
```

It is also possible to evaluate a polynomial in a matrix sense. In this case the equation $p(x) = x^3 - 2x - 5$ becomes $p(X) = X^3 - 2X - 5I$, where X is a square matrix and I is the identity matrix.

For example, create a square matrix X and evaluate the polynomial p at X:

```
>>X = [2 4 5; -1 0 3; 7 1 5];
>>Y = polyvalm(p,X)

Y =
   377   179   439
   111    81   136
   490   253   639
```

Convolution and Deconvolution
Polynomial multiplication and division correspond to the operations convolution and deconvolution. The functions conv and deconv implement these operations. Consider the polynomials $a(s) = s^2 + 2s + 3$ and $b(s) = 4s^2 + 5s + 6$. To compute their product,

```
>>a = [1 2 3]; b = [4 5 6];
>>c = conv(a,b)

c =
    4    13    28    27    18
```

Use deconvolution to divide back out of the product:

```
>>[q,r] = deconv(c,a)

q =
    4     5     6

r =
    0     0     0     0     0
```

Polynomial Derivatives

The polyder function computes the derivative of any polynomial. To obtain the derivative of the polynomial

```
>>p= [1 0 -2 -5]
>>q = polyder(p)
```

```
q =
    3    0   -2
```

polyder also computes the derivative of the product or quotient of two polynomials. For example, create two polynomials a and b:

```
>>a = [1 3 5];
>>b = [2 4 6];
```

Calculate the derivative of the product a*b by calling polyder with a single output argument:

```
>>c = polyder(a,b)
```

```
c =
    8   30   56   38
```

Calculate the derivative of the quotient a/b by calling polyder with two output arguments:

```
>>[q,d] = polyder(a,b)
```

```
q =
   -2   -8   -2

d =
    4   16   40   48   36
```

q/d is the result of the operation.

Partial Fraction Expansion

'residue' finds the partial fraction expansion of the ratio of two polynomials. This is particularly useful for applications that represent systems in transfer function form. For polynomials b and a,

$$\frac{b(s)}{a(s)} = \frac{r_1}{s-p_1} + \frac{r_2}{s-p_2} + \cdots + \frac{r_n}{s-p_n} + k_s$$

if there are no multiple roots, where r is a column vector of residues, p is a column vector of pole locations, and k is a row vector of direct terms.

Consider the transfer function

```
>>b = [-4 8];
>>a = [1 6 8];
>>[r,p,k] = residue(b,a)
```

```
r =
   -12
     8

p =
    -4
    -2

k =
     []
```

Given three input arguments (r, p, and k), residue converts back to polynomial form:

```
>>[b2,a2] = residue(r,p,k)

b2 =
    -4     8
a2 =
     1     6     8
```

Exercise 1:

Consider the two polynomials $p(s) = s^2 + 2s + 1$ and $q(s) = s + 1$. Using MATLAB compute

a. $p(s) * q(s)$
b. Roots of $p(s)$ and $q(s)$
c. $p(-1)$ and $q(6)$

Exercise 2:

Use MATLAB command to find the partial fraction of the following

a. $\frac{B(s)}{A(s)} = \frac{2s^3+5s^2+3s+6}{s^3+6s^2+11s+6}$
b. $\frac{B(s)}{A(s)} = \frac{s^2+2s+3}{(s+1)^3}$

Part III: Scripts, Functions & Flow Control in MATLAB

Objective: The objective of this session is to introduce you to writing M-file scripts, creating MATLAB Functions and reviewing MATLAB flow control like ‘if-elseif-end’, ‘for loops’ and ‘while loops’.

Overview:

MATLAB is a powerful programming language as well as an interactive computational environment. Files that contain code in the MATLAB language are called M-files. You create M-files using a text editor, then use them as you would any other MATLAB function or command. There are two kinds of M-files:

- Scripts, which do not accept input arguments or return output arguments. They operate on data in the workspace. MATLAB provides a full programming language that enables you to write a series of MATLAB statements into a file and then execute them with a single command. You write your program in an ordinary text file, giving the file a name of ‘filename.m’. The term you use for ‘filename’ becomes the new command that MATLAB associates with the program. The file extension of .m makes this a MATLAB M-file.
- Functions, which can accept input arguments and return output arguments. Internal variables are local to the function.

If you're a new MATLAB programmer, just create the M-files that you want to try out in the current directory. As you develop more of your own M-files, you will want to organize them into other directories and personal toolboxes that you can add to your MATLAB search path. If you duplicate function names, MATLAB executes the one that occurs first in the search path.

Scripts:

When you invoke a script, MATLAB simply executes the commands found in the file. Scripts can operate on existing data in the workspace, or they can create new data on which to operate. Although scripts do not return output arguments, any variables that they create remain in the workspace, to be used in subsequent computations. In addition, scripts can produce graphical output using functions like plot. For example, create a file called ‘myprogram.m’ that contains these MATLAB commands:

```
% Create random numbers and plot these numbers
clc
clear
r = rand(1,50)
plot(r)
```

Typing the statement ‘myprogram’ at command prompt causes MATLAB to execute the commands, creating fifty random numbers and plots the result in a new window. After execution of the file is complete, the variable ‘r’ remains in the workspace.

Functions:

Functions are M-files that can accept input arguments and return output arguments. The names of the M-file and of the function should be the same. Functions operate on variables within their own workspace, separate from the workspace you access at the MATLAB command prompt. An example is provided below:

```
function f = fact(n)                          Function definition line
% Compute a factorial value.                  H1 line
% FACT(N) returns the factorial of N,         Help text
% usually denoted by N!

% Put simply, FACT(N) is PROD(1:N).           Comment
f = prod(1:n);                                Function body
```

M-File Element	Description
Function definition line (functions only)	Defines the function name, and the number and order of input and output arguments.
H1 line	A one line summary description of the program, displayed when you request help on an entire directory, or when you use 'lookfor'.
Help text	A more detailed description of the program, displayed together with the H1 line when you request help on a specific function
Function or script body	Program code that performs the actual computations and assigns values to any output arguments.
Comments	Text in the body of the program that explains the internal workings of the program.

The first line of a function M-file starts with the keyword 'function'. It gives the function name and order of arguments. In this case, there is one input arguments and one output argument. The next several lines, up to the first blank or executable line, are comment lines that provide the help text. These lines are printed when you type 'help fact'. The first line of the help text is the H1 line, which MATLAB displays when you use the 'lookfor' command or request help on a directory. The rest of the file is the executable MATLAB code defining the function.

The variable n & f introduced in the body of the function as well as the variables on the first line are all local to the function; they are separate from any variables in the MATLAB workspace. This example illustrates one aspect of MATLAB functions that is not ordinarily found in other programming languages—a variable number of arguments. Many M-files work this way. If no output argument is supplied, the result is stored in ans. If the second input argument is not supplied, the function computes a default value.

Flow Control:

Conditional Control – if, else, switch

This section covers those MATLAB functions that provide conditional program control. if, else, and elseif. The if statement evaluates a logical expression and executes a group of statements when the expression is true. The optional elseif and else keywords provide for the

execution of alternate groups of statements. An end keyword, which matches the if, terminates the last group of statements.
The groups of statements are delineated by the four keywords—no braces or brackets are involved as given below.

```
if <condition>
        <statements>;
elseif <condition>
        <statements>;
else
        <statements>;
end
```

It is important to understand how relational operators and if statements work with matrices. When you want to check for equality between two variables, you might use

```
if A == B, ...
```

This is valid MATLAB code, and does what you expect when A and B are scalars. But when A and B are matrices, A == B does not test if they are equal, it tests where they are equal; the result is another matrix of 0's and 1's showing element-by-element equality. (In fact, if A and B are not the same size, then A == B is an error.)

```
>>A = magic(4);
>>B = A;
>>B(1,1) = 0;
>>A == B
ans =
    0    1    1    1
    1    1    1    1
    1    1    1    1
    1    1    1    1
```

The proper way to check for equality between two variables is to use the isequal function:

```
if isequal(A, B), ...
```

isequal returns a scalar logical value of 1 (representing true) or 0 (false), instead of a matrix, as the expression to be evaluated by the if function.
Using the A and B matrices from above, you get

```
>>isequal(A, B)
ans =
   0
```

Here is another example to emphasize this point. If A and B are scalars, the following program will never reach the "unexpected situation". But for most pairs of matrices, including our magic squares with interchanged columns, none of the matrix conditions A > B, A < B, or A == B is true for all elements and so the else clause is executed:

```
if A > B
   'greater'
elseif A < B
   'less'
elseif A == B
   'equal'
else
   error('Unexpected situation')
end
```

Several functions are helpful for reducing the results of matrix comparisons to scalar conditions for use with if, including ‘isequal’, ‘isempty’, ‘all’, ‘any’.

Switch and Case:
The switch statement executes groups of statements based on the value of a variable or expression. The keywords case and otherwise delineate the groups. Only the first matching case is executed. The syntax is as follows

```
switch <condition or expression>
case <condition>
        <statements>;
…
case <condition>
…
otherwise
        <statements>;
end
```

There must always be an end to match the switch. An example is shown below.

```
n=5
switch rem(n,2) % to find remainder of any number ‘n’
case 0
        disp(‘Even Number’)  % if remainder is zero
case 1
        disp(‘Odd Number’)   % if remainder is one
end
```

Unlike the C language switch statement, MATLAB switch does not fall through. If the first case statement is true, the other case statements do not execute. So, break statements are not required.

For, while, break and continue:
This section covers those MATLAB functions that provide control over program loops.

for:

The ‘for’ loop, is used to repeat a group of statements for a fixed, predetermined number of times. A matching ‘end’ delineates the statements. The syntax is as follows:

```
for <index> = <starting number>:<step or increment>:<ending number>
        <statements>;
end
```

```
for n = 1:4
   r(n) = n*n;   % square of a number
end
r
```

The semicolon terminating the inner statement suppresses repeated printing, and the r after the loop displays the final result.

It is a good idea to indent the loops for readability, especially when they are nested:

```
for i = 1:m
   for j = 1:n
      H(i,j) = 1/(i+j);
   end
end
```

while:

The ‘while’ loop, repeats a group of statements indefinite number of times under control of a logical condition. So a while loop executes atleast once before it checks the condition to stop the execution of statements. A matching ‘end’ delineates the statements. The syntax of the ‘while’ loop is as follows:

```
while <condition>
        <statements>;
end
```

Here is a complete program, illustrating while, if, else, and end, that uses interval bisection to find a zero of a polynomial:

```
a = 0; fa = -Inf;
b = 3; fb = Inf;
while b-a > eps*b
   x = (a+b)/2;
   fx = x^3-2*x-5;
   if sign(fx) == sign(fa)
      a = x; fa = fx;
   else
      b = x; fb = fx;
   end
end
x
```

The result is a root of the polynomial $x^3 - 2x - 5$, namely x = 2.0945. The cautions involving matrix comparisons that are discussed in the section on the if statement also apply to the while statement.

break:

The break statement lets you exit early from a ‘for’ loop or ‘while’ loop. In nested loops, break exits from the innermost loop only. Above is an improvement on the example from the previous section. Why is this use of break a good idea?

```
a = 0; fa = -Inf;
b = 3; fb = Inf;
while b-a > eps*b
   x = (a+b)/2;
   fx = x^3-2*x-5;
   if fx == 0
      break
   elseif sign(fx) == sign(fa)
      a = x; fa = fx;
   else
      b = x; fb = fx;
   end
end
```

continue:
The continue statement passes control to the next iteration of the for loop or while loop in which it appears, skipping any remaining statements in the body of the loop. The same holds true for continue statements in nested loops. That is, execution continues at the beginning of the loop in which the continue statement was encountered.

Exersice 1: MATLAB M-file Script

Use MATLAB to generate the first 100 terms in the sequence **a(n)** define recursively by

$$\boldsymbol{a(n+1) = p * a(n) * (1 - a(n))}$$

with p=2.9 and a(1) = 0.5.

After you obtain the sequence, plot the sequence.

Exersice 2: MATLAB M-file Function

Consider the following equation

$$\boldsymbol{y(t) = \frac{y0}{\sqrt{1-\zeta}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} * t + \theta)}$$

a) Write a MATLAB M-file function to obtain numerical values of y(t). Your function must take y(0), ζ, ω_n, t and θ as function inputs and y(t) as output argument.

b) Write a second script m-file to obtain the plot for y(t) for 0<t<10 with an increment of 0.1, by considering the following two cases

Case 1: y0=0.15 m, $\omega_n = \sqrt{2}$ rad/sec, $\zeta = 3/(2\sqrt{2})$ and θ = 0;
Case 2: y0=0.15 m, $\omega_n = \sqrt{2}$ rad/sec, $\zeta = 1/(2\sqrt{2})$ and θ = 0;

Hint: When you write the function you would require element-by-element operator

Exersice 3: MATLAB Flow Control

Use ‘for’ or ‘while’ loop to convert degrees Fahrenheit (T_f) to degrees Celsius using the following equation $\boldsymbol{T_f = \frac{9}{5} * T_c + 32}$**.** Use any starting temperature, increment and ending temperature (example: starting temperature=0, increment=10, ending temperature = 200).

Please submit the exercises (m-files and results) in the next lab.

CISE 302
Linear Control Systems

Laboratory Experiment 2: Mathematical Modeling of Physical Systems

Objectives: The objective of this exercise is to grasp the important role mathematical models of physical systems in the design and analysis of control systems. We will learn how MATLAB helps in solving such models.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

Category Soft-Experiment

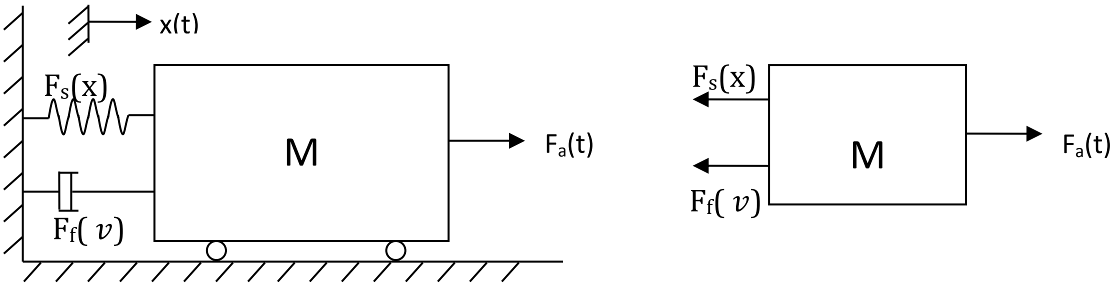
Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results for all the assignments and exercises should be properly reported.

Mass-Spring System Model

Consider the following Mass-Spring system shown in the figure. Where $F_s(x)$ is the spring force, $F_f(\dot{x})$ is the friction coefficient, $x(t)$ is the displacement and $F_a(t)$ is the applied force:



Where

$$a = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} \text{ is the acceleration,}$$
$$v = \frac{dx(t)}{dt} \text{ is the speed,}$$

and

$$x(t) \text{ is the displacement.}$$

According to the laws of physics

$$Ma + F_f(v) + F_s(x) = F_a(t) \tag{1}$$

In the case where:

$$F_f(v) = Bv = B\frac{dx(t)}{dt}$$

$$F_s(x) = Kx(t)$$

The differential equation for the above Mass-Spring system can then be written as follows

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) = F_a(t) \tag{2}$$

B is called the friction coefficient and K is called the spring constant.

The linear differential equation of second order (2) describes the relationship between the displacement and the applied force. The differential equation can then be used to study the time behavior of x(t) under various changes of the applied force. In reality, the spring force and/or the friction force can have a more complicated expression or could be represented by a graph or data table. For instance, a nonlinear spring can be designed (see figure 4.2) such that

$$F_s(x) = Kx^r(t) \quad \text{Where r > 1.}$$

Figure 4.2: MAG nonlinear spring (www.tokyo-model.com.hk/ecshop/goods.php?id=2241)
In such case, (1) becomes

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx^r(t) = F_a(t) \tag{3}$$

Equation (3) represents another possible model that describes the dynamic behavior of the mass-damper system under external force. Model (2) is said to be a linear model whereas (3) is said to be nonlinear. To decide if a system is linear or nonlinear two properties have to be verified homogeneity and superposition.

Assignment: use homogeneity and superposition properties to show that model (1) is linear whereas model (3) is nonlinear.

Solving the differential equation using MATLAB:
The objectives behind modeling the mass-damper system can be many and may include

- Understanding the dynamics of such system

- Studying the effect of each parameter on the system such as mass M, the friction coefficient B, and the elastic characteristic Fs(x).
- Designing a new component such as damper or spring.
- Reproducing a problem in order to suggest a solution.

The solution of the difference equations (1), (2), or (3) leads to finding x(t) subject to certain initial conditions.

MATLAB can help solve linear or nonlinear ordinary differential equations (ODE). To show how you can solve ODE using MATLAB we will proceed in two steps. We first see how can we solve first order ODE and second how can we solve equation (2) or (3).

Speed Cruise Control example:

Assume the spring force $F_s(x) = 0$ which means that K=0. Equation (2) becomes

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} = F_a(t) \tag{4}$$

Or

$$M\frac{dv(t)}{dt} + Bv = F_a(t) \tag{5}$$

Equation (5) is a first order linear ODE.

Using MATLAB solver ode45 we can write do the following:

1_ create a MATLAB-function cruise_speed.m

```
function dvdt=cruise_speed(t, v)

%flow rate

M=750; %(Kg)

B=30; %( Nsec/m)

Fa=300; %N

% dv/dt=Fa/M-B/M v

dvdt=Fa/M-B/M*v;
```

2_ create a new MATLAB m-file and write

```
v0= 0; %(initial speed)

[t,v]=ode45('cruise_speed', [0 125],v0);

plot(t,v); grid on;

title('cruise speed time response to a constant traction force Fa(t) ')
```

There are many other MATLAB ODE solvers such as ode23, ode45, ode113, ode15s, etc… The function dsolve will result in a symbolic solution. Do 'doc dsolve' to know more. In MATLAB write

```
>>dsolve('Dv=Fa/M-B/M*v', 'v(0)=0')
```

Note that using MATLAB ODE solvers are able to solve linear or nonlinear ODE's. We will see in part II of this experiment another approach to solve a linear ODE differently. Higher order systems can also be solved similarly.

Mass-Spring System Example:

Assume the spring force $F_s(x) = Kx^r(t)$. The mass-spring damper is now equivalent to

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx^r(t) = F_a(t)$$

The second order differential equation has to be decomposed in a set of first order differential equations as follows

Variables	New variable	Differential equation
x(t)	X_1	$\frac{dX_1}{dt} = X_2$
dx(t)/dt	X_2	$\frac{dX_2}{dt} = -\frac{B}{M}X_2 - \frac{K}{M}{X_1}^r(t) + \frac{F_a(t)}{M}$

In vector form, let $X = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$; $\frac{dX}{dt} = \begin{bmatrix} \frac{dX_1}{dt} \\ \frac{dX_2}{dt} \end{bmatrix}$ then the system can be written as

$$\frac{dX}{dt} = \begin{bmatrix} X_2 \\ -\frac{B}{M}X_2 - \frac{K}{M}{X_1}^r(t) + \frac{F_a(t)}{M} \end{bmatrix}$$

The ode45 solver can be now be used:

1_ create a MATLAB-function mass_spring.m

```
Function dXdt=mass_spring(t, X)

%flow rate

M=750; %(Kg)

B=30; %( Nsec/m)

Fa=300; %N

K=15; %(N/m)

r=1;

% dX/dt
```

```
dXdt(1,1)=X(2);

dXdt(2,1)=-B/M*X(2)-K/M*X(1)^r+Fa/M;
```

2_ in MATLAB write

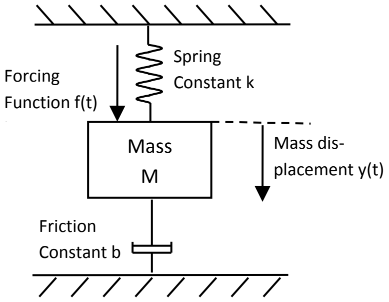
```
>> X0=[0; 0]; %(initial speed and position)

>> options = odeset('RelTol',[1e-4 1e-4],'AbsTol',[1e-5 1e-5],'Stats','on');

>>[t,X]=ode45('mass_spring', [0 200],X0);
```

Exercise 1

1. Plot the position and the speed in separate graphs.
2. Change the value of r to 2 and 3.
3. Superpose the results and compare with the linear case r=1 and plot all three cases in the same plot window. Please use different figures for velocity and displacement.

Exercise 2

Consider the mechanical system depicted in the figure. The input is given by $f(t)$, and the output is given by $y(t)$. Determine the differential equation governing the system and using MATLAB, write a m-file and plot the system response such that forcing function f(t)=1. Let $m = 10$, $k = 1$ and $b = 0.5$. Show that the peak amplitude of the output is about 1.8.



CISE 302

Linear Control Systems

Laboratory Experiment 3: Modeling of Physical Systems using SIMULINK

Objectives: The objective of this exercise is to use graphical user interface diagrams to model the physical systems for the purpose of design and analysis of control systems. We will learn how MATLAB/SIMULINK helps in solving such models.

List of Equipment/Software

Following equipment/software is required:

- MATLAB/SIMULINK

Category Soft-Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts, SIMULINK diagrams and their results for all the assignments and exercises should be properly reported.

Overview:

This lab introduces powerful graphical user interface (GUI), **Simulink** of Matlab. This software is used for solving the modeling equations and obtaining the response of a system to different inputs. Both linear and nonlinear differential equations can be solved numerically with high precision and speed, allowing system responses to be calculated and displayed for many input functions. To provide an interface between a system's modeling equations and the digital computer, block diagrams drawn from the system's differential equations are used. A block diagram is an interconnection of blocks representing basic mathematical operations in such a way that the overall diagram is equivalent to the system's mathematical model. The lines interconnecting the blocks represent the variables describing the system behavior. These may be inputs, outputs, state variables, or other related variables. The blocks represent operations or functions that use one or more of these variables to calculate other variables. Block diagrams can represent modeling equations in both input-output and state variable form.

We use MATLAB with its companion package **Simulink**, which provides a graphical user interface (GUI) for building system models and executing the simulation. These models are constructed by drawing block diagrams representing the algebraic and differential equations that describe the system behavior. The operations that we generally use in block diagrams are summation, gain, and integration. Other blocks, including nonlinear elements such as multiplication, square root, exponential, logarithmic, and other functions, are available. Provisions are also included for supplying input functions, using a signal generator block, constants etc and for displaying results, using a scope block.

An important feature of a numerical simulation is the ease with which parameters can be varied and the results observed directly. MATLAB is used in a supporting role to initialize parameter values and to produce plots of the system response. Also MATLAB is used for multiple runs for varying system parameters. Only a small subset of the functions of MATLAB will be considered during these labs.

SIMULINK

Simulink provides access to an extensive set of blocks that accomplish a wide range of functions useful for the simulation and analysis of dynamic systems. The blocks are grouped into libraries, by general classes of functions.

- Mathematical functions such as summers and gains are in the Math library.
- Integrators are in the Continuous library.
- Constants, common input functions, and clock can all be found in the Sources library.
- Scope, To Workspace blocks can be found in the Sinks library.

Simulink is a graphical interface that allows the user to create programs that are actually run in MATLAB. When these programs run, they create arrays of the variables defined in Simulink that can be made available to MATLAB for analysis and/or plotting. The variables to be used in MATLAB must be identified by Simulink using a "To Workspace" block, which is found in the Sinks library. (When using this block, open its dialog box and specify that the save format should be Matrix, rather than the default, which is called Structure.) The Sinks library also contains a Scope, which allows variables to be displayed as the simulated system responds to an input. This is most useful when studying responses to repetitive inputs.

Simulink uses blocks to write a program. Blocks are arranged in various libraries according to their functions. Properties of the blocks and the values can be changed in the associated dialog boxes. Some of the blocks are given below.

SUM (Math library)

A dialog box obtained by double-clicking on the SUM block performs the configuration of the SUM block, allowing any number of inputs and the sign of each. The sum block can be represented in two ways in Simulink, by a circle or by a rectangle. Both choices are shown

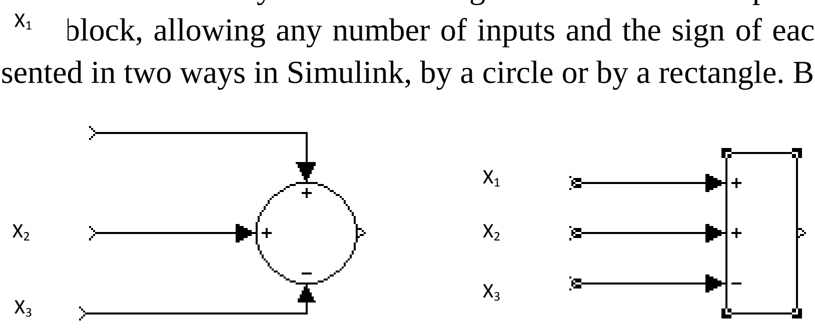


Figure 1: Two Simulink blocks for a summer representing y = x 1+ x2 – x3

GAIN (Math library)

A gain block is shown by a triangular symbol, with the gain expression written inside if it will fit. If not, the symbol - k - is used. The value used in each gain block is established in a dialog box that appears if the user double-clicks on its block.

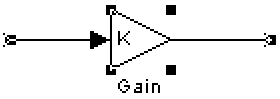


Figure 2: Simulink block for a gain of K.

INTEGRATOR (Continuous library)

The block for an integrator as shown below looks unusual. The quantity 1/s comes from the Laplace transform expression for integration. When double-clicked on the symbol for an integrator, a dialog box appears allowing the initial condition for that integrator to be specified. It may be implicit, and not shown on the block, as in Figure (a). Alternatively, a second input to the block can be displayed to supply the initial condition explicitly, as in part (b) of Figure 3. Initial conditions may be specific numerical values, literal variables, or algebraic expressions.

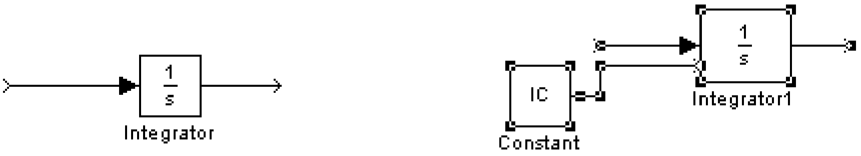


Figure3: Two forms of the Simulink block for an integrator.

(a) Implicit initial condition. (b) Explicit initial condition.

CONSTANTS (Source library)

Constants are created by the Constant block, which closely resembles Figure 4. Double-clicking on the symbol opens a dialog box to establish the constant’s value. It can be a number or an algebraic expression using constants whose values are defined in the workspace and are therefore known to MATLAB.

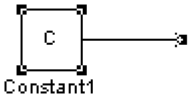


Figure 4: A constant block

STEP (Source library)

A Simulink block is provided for a Step input, a signal that changes (usually from zero) to a specified new, constant level at a specified time. These levels and time can be specified through the dialog box, obtained by double-clicking on the Step block.

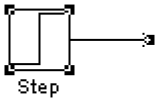


Figure 5: A step block

SIGNAL GENERATOR (Source library)

One source of repetitive signals in Simulink is called the Signal Generator. Double-clicking on the Signal Generator block opens a dialog box, where a sine wave, a square wave, a ramp (sawtooth), or a random waveform can be chosen. In addition, the amplitude and frequency of the signal may be specified. The signals produced have a mean value of zero. The repetition frequency can be given in Hertz (Hz), which is the same as cycles per second, or in radians/second.

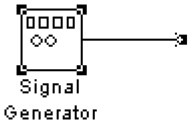


Figure 6: A signal generator block

SCOPE (Sinks library)

The system response can be examined graphically, as the simulation runs, using the Scope block in the sinks library. This name is derived from the electronic instrument, oscilloscope, which performs a similar function with electronic signals. Any of the variables in a Simulink diagram can be connected to the Scope block, and when the simulation is started, that variable is displayed. It is possible to include several Scope blocks. Also it is possible to display several signals in the same scope block using a MTJX block in the signals & systems library. The Scope normally chooses its scales automatically to best display the data.

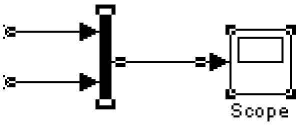


Figure 7: A scope block with MUX block

Two additional blocks will be needed if we wish to use MATLAB to plot the responses versus time. These are the Clock and the To Workspace blocks.

CLOCK (Sources library)

The clock produces the variable "time" that is associated with the integrators as MATLAB calculates a numerical (digital) solution to a model of a continuous system. The result is a string of sample values of each of the output variables. These samples are not necessarily at uniform time increments, so it is necessary to have the variable "time" that contains the time corresponding to each sample point. Then MATLAB can make plots versus "time." The clock output could be given any arbitrary name; we use "t" in most of the cases.

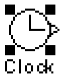


Figure 8: A clock block

To Workspace (Sinks library)

The To Workspace block is used to return the results of a simulation to the MATLAB workspace, where they can be analyzed and/or plotted. Any variable in a Simulink diagram can be connected to a ToWorkspace block. In our exercises, all of the state variables and the input variables are usually returned to the workspace. In addition, the result of any output equation that may be simulated would usually be sent to the workspace. In the block parameters drop down window, change the save format to 'array'.

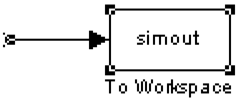


Figure 9: A To Workspace block

In the Simulink diagram, the appearance of a block can be changed by changing the foreground or background colours, or by drop shadow or other options available in the format drop down menu. The available options can be reached in the Simulink window by highlighting the block, then clicking the right mouse button. The Show Drop Shadow option is on the format drop-down menu.

Simulink provides scores of other blocks with different functions.

You are encouraged to **browse the Simulink libraries and consult the online Help facility provided with MATLAB.**

GENERAL INSTRUCTIONS FOR WRITING A SIMULINK PROGRAM

To create a simulation in Simulink, follow the steps:

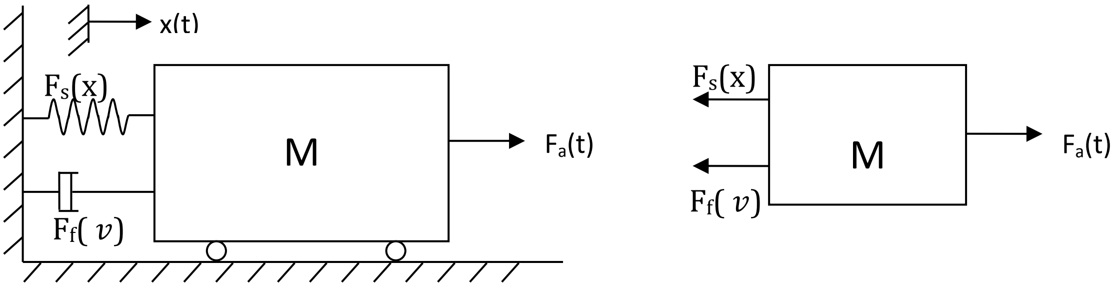
- Start MATLAB.
- Start Simulink.

- Open the libraries that contain the blocks you will need. These usually will include the Sources, Sinks, Math and Continuous libraries, and possibly others.
- Open a new Simulink window.
- Drag the needed blocks from their library folders to that window. The Math library, for example, contains the Gain and Sum blocks.
- Arrange these blocks in an orderly way corresponding to the equations to be solved.
- Interconnect the blocks by dragging the cursor from the output of one block to the input of another block. Interconnecting branches can be made by right-clicking on an existing branch.
- Double-click on any block having parameters that must be established, and set these parameters. For example, the gain of all Gain blocks must be set. The number and signs of the inputs to a Sum block must be established. The parameters of any source blocks should also be set in this way.
- It is necessary to specify a stop time for the solution. This is done by clicking on the Simulation > Parameters entry on the Simulink toolbar.

At the Simulation > Parameters entry, several parameters can be selected in this dialog box, but the default values of all of them should be adequate for almost all of the exercises. If the response before time zero is needed, it can be obtained by setting the Start time to a negative value. It may be necessary in some problems to reduce the maximum integration step size used by the numerical algorithm. If the plots of the results of a simulation appear “choppy” or composed of straight-line segments when they should be smooth, reducing the max step size permitted can solve this problem.

Mass-Spring System Model

Consider the Mass-Spring system used in the previous exercise as shown in the figure. Where $F_s(x)$ is the spring force, $F_f(\dot{x})$ is the friction coefficient, x(t) is the displacement and $F_a(t)$ is the applied force:



The differential equation for the above Mass-Spring system can then be written as follows

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) = F_a(t) \qquad (1)$$

For Non-linear such case, (1) becomes

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx^r(t) = F_a(t) \qquad (2)$$

Exercise 1: Modeling of a second order system

Construct a Simulink diagram to calculate the response of the Mass-Spring system. The input force increases from 0 to 8 N at t = 1 s. The parameter values are M = 2 kg, K= 16 N/m, and B =4 N.s/m.

Steps:

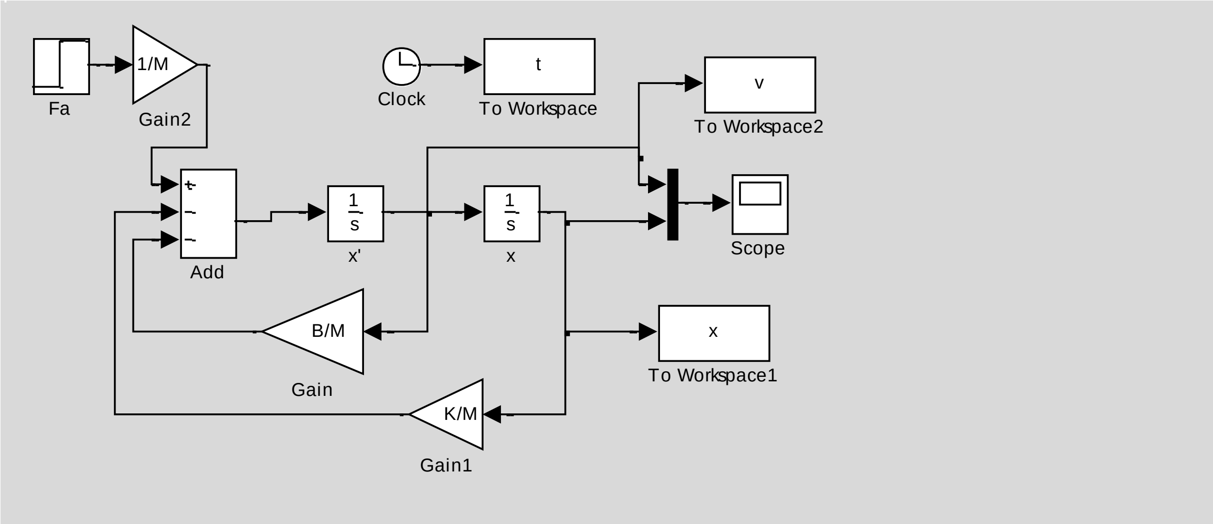
- Draw the free body diagram.
- Write the modeling equation from the free body diagram
- Solve the equations for the highest derivative of the output.
- Draw a block diagram to represent this equation.
- Draw the corresponding Simulink diagram.
- Use Step block to provide the input fa(t).
- In the Step block, set the initial and final values and the time at which the step occurs.
- Use the "To Workspace" blocks for t, fa(t), x, and v in order to allow MATLAB to plot the desired responses. Set the save format to array in block parameters.
- Select the duration of the simulation to be 10 seconds from the Simulation > Parameters entry on the toolbar

Given below is a file that will set up the MATLAB workspace by establishing the values of the parameters needed for the Simulink simulation of the given model.

M-file for parameter values

```
% This file is named exl_parameter.m.
% Everything after a % sign on a line is a comment that
% is ignored by M This file establishes the
% parameter values for exl_model.mdl.
%
M2;             %kg
K= 16;          %N/m
B=4;            % Ns/m
```

Simulink block diagram



Plotting the outputs in MATLAB:

The file to create the plots of the output is given below. Create the file and save it by the name given below.

M-file to produce the plot

```
% This file is named exl_plot.m.
% It makes a plot of the data produced by exl_model.mdl.
plot(t,x); grid           % Plots x for the case with B=4.
xlabel(’Time (s)’);
ylabel (‘Displacement (m) ')
```

A semicolon in a physical line ends the logical line, and anything after it is treated as if it were on a new physical line. A semicolon at the end of a line that generates output to the command window suppresses the printing of that output.

Program Execution:

Follow the following steps to execute these files:

- Enter the command exl_parameter in the command window. This will load the parameter values of the model.
- Open the Simulink model exl_model.mdl and start the simulation by clicking on the toolbar entry Simulation> Start.
- Enter the command exl_plot in the command window to make the plot.

Making Subplots in MATLAB:

When two or more variables are being studied simultaneously, it is frequently desirable to plot them one above the other on separate axes, as can be done for displacement and velocity in. This is accomplished with the subplot command. The following M-file uses this command to produce both plots of displacement and velocity.

M-file to make subplots

```
% This file is named exl_plot2.m.
% It makes both plots, displacement and velocity.
% Execute exlparameter.m first.
subplot(2,l,1);
plot(t,x); grid % Plots x for the case with B=4. xlabel (‘Time (s) ‘) ;
ylabel (‘Displacement (m) ‘); subplot(2,1,2);
plot(t,v); grid % Plots v for the case with B=4. xlabel(’Time (s)’);
ylabel(’Velocity (m per s)’);
```

Exercise 2: Simulation with system parameter variation

The effect of changing B is to alter the amount of overshoot or undershoot. These are related to a term called the damping ratio. Simulate and compare the results of the variations in B in exercise 1. Take values of B = 4, 8, 12, 25 N-s/m.

Steps:

Perform the following steps. Use the same input force as in Exercise 1.

- Begin the simulation with B = 4 N-s/m, but with the input applied at t = 0
- Plot the result.
- Rerun it with B = 8 N.s/m.
- Hold the first plot active, by the command hold on
- Reissue the plot command plot(t,x), the second plot will superimpose on the first.
- Repeat for B = 12 N-s/m and for B = 25 N-s/m
- Release the plot by the command hold off
- Show your result.

Running SIMULINK from MATLAB command prompt

If a complex plot is desired, in which several runs are needed with different parameters, this can using the command called “sim”. “sim” command will run the Simulink model file from the Matlab command prompt. For multiple runs with several plot it can be accomplished by executing ex1_model (to load parameters) followed by given M-file. Entering the command ex1_plots in the command window results in multiple runs with varying values if B and will plot the results.

M-file to use “sim” function and produce multiple runs and their plots

```
% This file is named ex2_plots.m.
% It plots the data produced by exl_model.mdl for
% several values of B. Execute exl_parameter.m first.
sim(’exl_model’)       % Has the same effect as clicking on
                       % Start on the toolbar.
plot(t,x)              % Plots the initial run with B=4
hold on                % Plots later results on the same axes % as the first.
B = 8;                 % New value of B; other parameter values % stay the same.
sim(‘exl_model’)       % Rerun the simulation with new B value.
plot(t,x)              % Plots new x on original axes.
B 12; sim(’exl_model’);plot(t,x)
B = 25; sim(’exl_model’ ) ;plot(t,x)
hold off
```

Exercise 3: System response from the stored energy with zero input

Find the response of the above system when there is no input for t ≥0, but when the initial value of the **displacement x(0) is zero and the initial velocity v(0) is 1 m/s.**

Steps:

In the previous program

- Set the size of the input step to zero
- Set the initial condition on Integrator for velocity to 1.0.
- Plot the results by running m-files.

Exercise 4: Cruise System

As we know in the cruise system, the spring force $F_s(x) = 0$ which means that K=0. Equation (2) becomes

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} = F_a(t) \qquad (3)$$

Or

$$M\frac{dv(t)}{dt} + Bv = F_a(t) \qquad (4)$$

Find the velocity response of the above system by constructing a **Simulink block diagram** and calling the block diagram from Matlab m-file. Use M=750, B=30 and a constant force Fa = 300. **Plot the response of the system such that it runs for 125 seconds.**

CISE 302

Linear Control Systems

Laboratory Experiment 4: Linear Time-invariant Systems and Representation

Objectives: This experiment has following two objectives:

1. Continued with the learning of Mathematical Modeling from previous experiment, we now start focusing the linear systems. We will learn commands in MATLAB that would be used to represent such systems in terms of transfer function or pole-zero-gain representations.
2. We will also learn how to make preliminary analysis of such systems using plots of poles and zeros locations as well as time response due to impulse, step and arbitrary inputs.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

Category Soft-Experiment

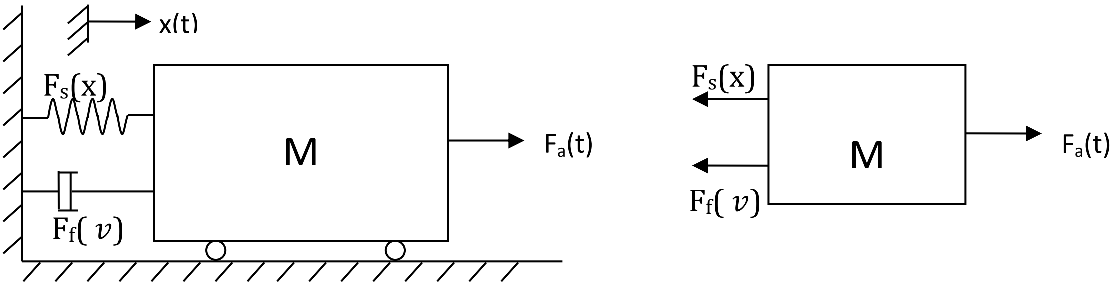
Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results should be reported properly.

Mass-Spring System Model

The spring force is assumed to be either linear or can be approximated by a linear function $F_s(x) = Kx$, B is the friction coefficient, x(t) is the displacement and $F_a(t)$ is the applied force:



The differential equation for the above Mass-Spring system can be derived as follows

$$M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t) = F_a(t)$$

Transfer Function:

Applying the Laplace transformation while assuming the initial conditions are zeros, we get

$$(Ms^2 + Bs + K) * X(s) = F_a(s)$$

Then the transfer function representation of the system is given by

$$TF = \frac{Output}{Input} = \frac{F_a(s)}{X(s)} = \frac{1}{(Ms^2 + Bs + K)}$$

Linear Time-Invariant Systems in MATLAB:

Control System Toolbox in MATLAB offers extensive tools to manipulate and analyze linear time-invariant (LTI) models. It supports both continuous- and discrete-time systems. Systems can be single-input/single-output (SISO) or multiple-input/multiple-output (MIMO). You can specify LTI models as:

Transfer functions (TF), for example,

$$P(s) = \frac{s+2}{s^2 + s + 10}$$

Note: All LTI models are represented as a ratio of polynomial functions

Examples of Creating LTI Models

Building LTI models with Control System Toolbox is straightforward. The following sections show simple examples. Note that all LTI models, i.e. TF, ZPK and SS are also MATLAB objects.

Example of Creating Transfer Function Models

You can create transfer function (TF) models by specifying numerator and denominator coefficients. For example,

```
>>num = [1 0];
>>den = [1 2 1];
>>sys = tf(num,den)
```

```
Transfer function:
    s
-------------
s^2 + 2 s + 1
```

A useful trick is to create the Laplace variable, s. That way, you can specify polynomials using s as the polynomial variable.

```
>>s=tf('s');
>>sys= s/(s^2 + 2*s + 1)
```

```
Transfer function:
      s
-------------
s^2 + 2 s + 1
```

This is identical to the previous transfer function.

Example of Creating Zero-Pole-Gain Models

To create zero-pole-gain (ZPK) models, you must specify each of the three components in vector format. For example,

```
>>sys = zpk([0],[-1 -1],[1])
```

```
Zero/pole/gain:

s
-------
(s+1)^2
```

produces the same transfer function built in the TF example, but the representation is now ZPK. This example shows a more complicated ZPK model.

```
>>sys=zpk([1 0], [-1 -3 -.28],[.776])
```

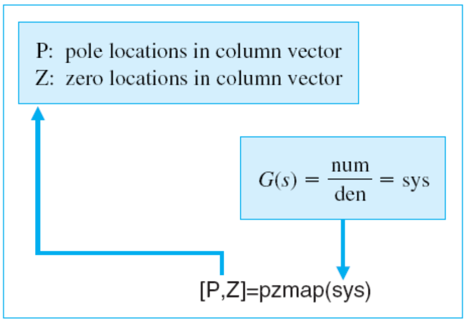
```
Zero/pole/gain:
   0.776 s (s-1)
--------------------
(s+1) (s+3) (s+0.28)
```

Plotting poles and zeros of a system:

pzmap

Compute pole-zero map of LTI models

```
pzmap(sys)
pzmap(sys1,sys2,...,sysN)
[p,z] = pzmap(sys)
```



P: pole locations in column vector
Z: zero locations in column vector

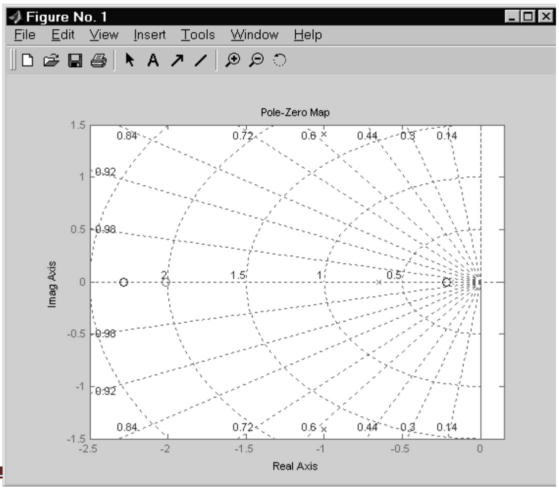
$G(s) = \frac{\text{num}}{\text{den}} = \text{sys}$

[P,Z]=pzmap(sys)

Description:

pzmap(sys) plots the pole-zero map of the continuous- or discrete-time LTI model sys. For SISO systems, pzmap plots the transfer function poles and zeros. The poles are plotted as x's and the zeros are plotted as o's.

pzmap(sys1,sys2,...,sysN) plots the pole-zero map of several LTI models on a single figure. The LTI models can have different numbers of inputs and outputs. When invoked with left-hand arguments,



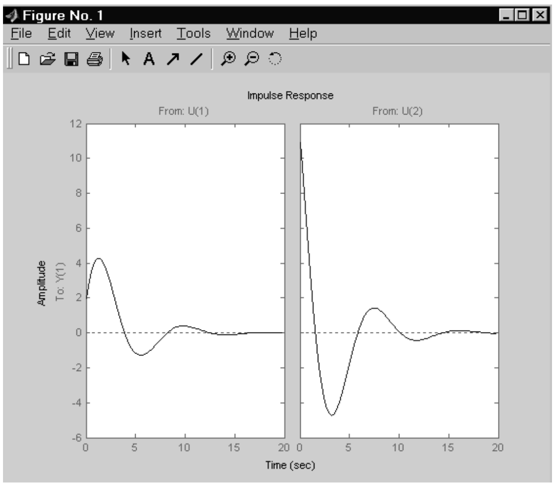
[p,z] = pzmap(sys) returns the system poles and zeros in the column vectors p and z. No plot is drawn on the screen. You can use the functions sgrid or zgrid to plot lines of constant damping ratio and natural frequency in the s- or z- plane.

Example
Plot the poles and zeros of the continuous-time system.

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

```
>>H = tf([2 5 1],[1 2 3]); sgrid
>>pzmap(H)
```

Simulation of Linear systems to different inputs



impulse, step and lsim

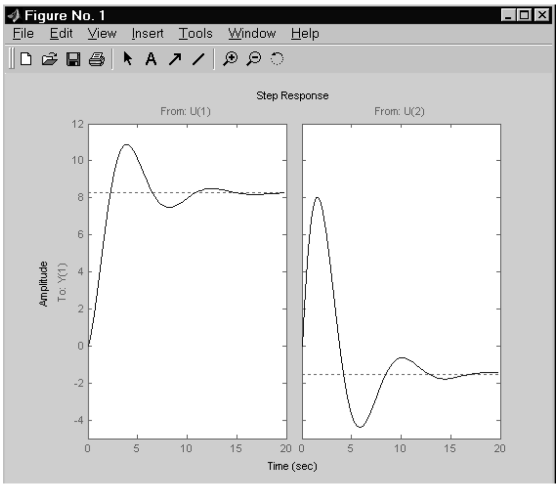
You can simulate the LTI systems to inputs like impulse, step and other standard inputs and see the plot of the response in the figure window. MATLAB command ‘impulse’ calculates the unit impulse response of the system, ‘step’ calculates the unit step response of the system and ‘lsim’ simulates the (time) response of continuous or discrete linear systems to arbitrary inputs. When invoked without left-hand arguments, all three commands plots the response on the screen. For example:

To obtain an impulse response

```
>> H = tf([2 5 1],[1 2 3]);
>>impulse(H)
```

To obtain a step response type

```
>>step(H)
```



Time-interval specification:

To contain the response of the system you can also specify the time interval to simulate the system to. For example,

```
>> t = 0:0.01:10;
>> impulse(H,t)
```

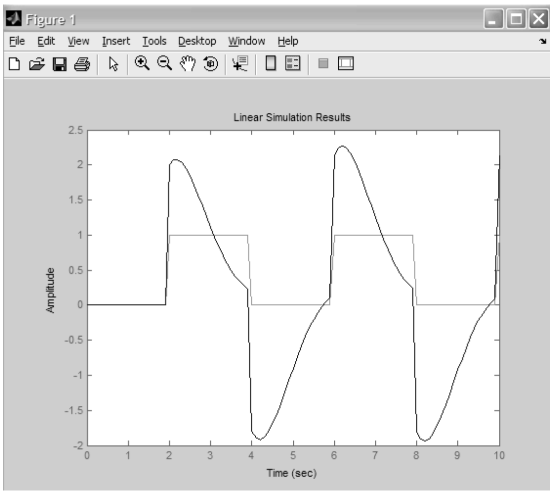
Or

```
>> step(H,t)
```

Simulation to Arbitrary Inputs:

To simulates the (time) response of continuous or discrete linear systems to arbitrary inputs use ‘lsim’. When invoked without left-hand arguments, ‘lsim’ plots the response on the screen.

lsim(sys,u,t) produces a plot of the time response of the LTI model sys to the input time history ‘t’,’u’. The vector ‘t’ specifies the time samples for the simulation and consists of regularly spaced time samples.



T = 0:dt:Tfinal

The matrix u must have as many rows as time samples (length(t)) and as many columns as system inputs. Each row u(I,☺ specifies the input value(s) at the time sample t(i).

Simulate and plot the response of the system

$$H(s) = \frac{2s^2 + 5s + 1}{s^2 + 2s + 3}$$

to a square wave with period of four seconds.

First generate the square wave with gensig. Sample every 0.1 second during 10 seconds:

```
>>[u,t] = gensig(‘square’,4,10,0.1);
```

Then simulate with lsim.

```
>> H = tf([2 5 1],[1 2 3])
```

```
Transfer function:
2 s^2 + 5 s + 1
-----------------
 s^2 + 2 s + 3
```

```
>> lsim(H,u,t)
```

Exercise 1:

Consider the transfer function

$$G(s) = \frac{6s^2 + 1}{s^3 + 3s^2 + 3s + 7}$$

Using MATLAB plot the pole zero map of the above system

Exercise 2:

a. Obtain the unit impulse response for the following system

$$\frac{B(s)}{A(s)} = \frac{1}{s^2 + 0.2s + 1}$$

b. Obtain the unit step response for the following system

$$\frac{B(s)}{A(s)} = \frac{s}{s^2 + 0.2s + 1}$$

c. Explain why the results in a. and b. are same?

Exercise 3:

A system has a transfer function

$$\frac{X(s)}{R(s)} = \frac{(15/z)(s + z)}{s^2 + 3s + 15}$$

Plot the response of the system when R(s) is a unit impulse and unit step for the parameter z=3, 6 and 12.

Exercise 4:

Consider the differential equation $\ddot{y} + 4\dot{y} + 4y = u$ where $y(0) = \dot{y}(0) = 0$ and $u(t)$ is a unit step. Determine the solution analytically and verify by co-plotting the analytical solution and the step response obtained with ‘step’ function.

CISE 302
Linear Control Systems

Lab Experiment 5: Block Diagram Reduction

Objective: The objective of this exercise will be to learn commands in MATLAB that would be used to reduce linear systems block diagram using series, parallel and feedback configuration.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

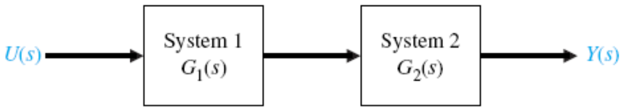
Category Soft-Experiment

Deliverables

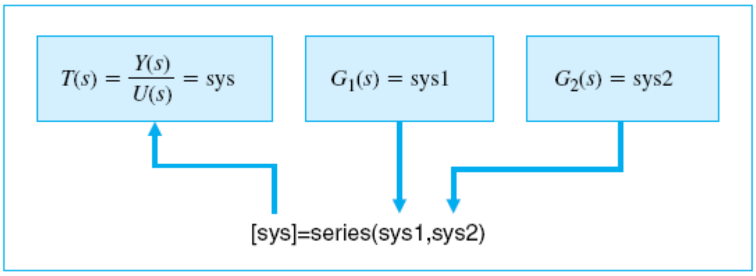
A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results for examples, exercises and Dorf (text book) related material of this lab should be reported properly.

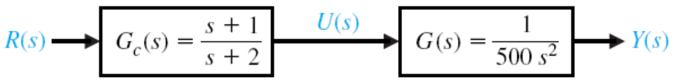
Series configuration: If the two blocks are connected as shown below then the blocks are said to be in series. It would like multiplying two transfer functions. The MATLAB command for the such configuration is "series".



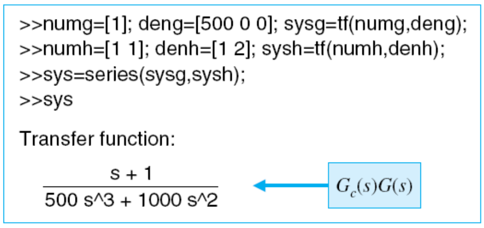
The series command is implemented as shown below:



Example 1: Given the transfer functions of individual blocks generate the system transfer function of the block combinations.

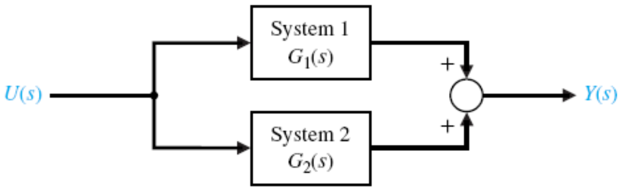


The result is as shown below:

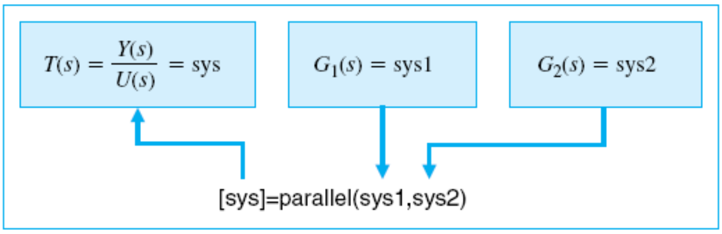


```
>>numg=[1]; deng=[500 0 0]; sysg=tf(numg,deng);
>>numh=[1 1]; denh=[1 2]; sysh=tf(numh,denh);
>>sys=series(sysg,sysh);
>>sys

Transfer function:
        s + 1
 ------------------
 500 s^3 + 1000 s^2
```

Parallel configuration: If the two blocks are connected as shown below then the blocks are said to be in parallel. It would like adding two transfer functions.



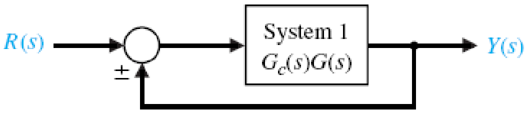
The MATLAB command for implementing a parallel configuration is "parallel" as shown below:



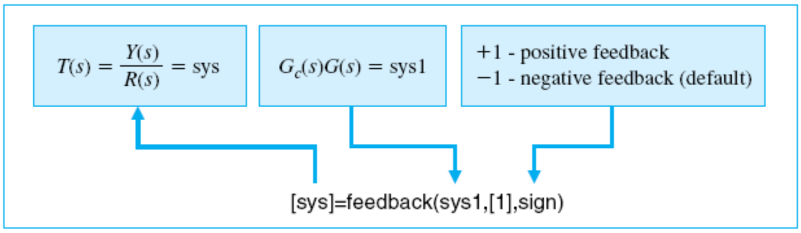
```
[sys]=parallel(sys1,sys2)
```

Example 2: For the previous systems defined, modify the MATLAB commands to obtain the overall transfer function when the two blocks are in parallel.

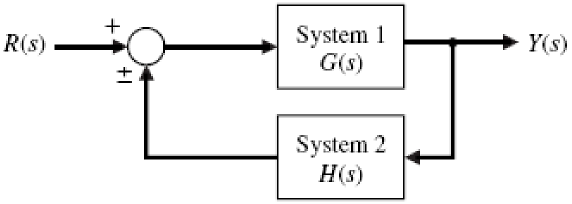
Feedback configuration: If the blocks are connected as shown below then the blocks are said to be in feedback. Notice that in the feedback there is no transfer function H(s) defined. When not specified, H(s) is unity. Such a system is said to be a unity feedback system.



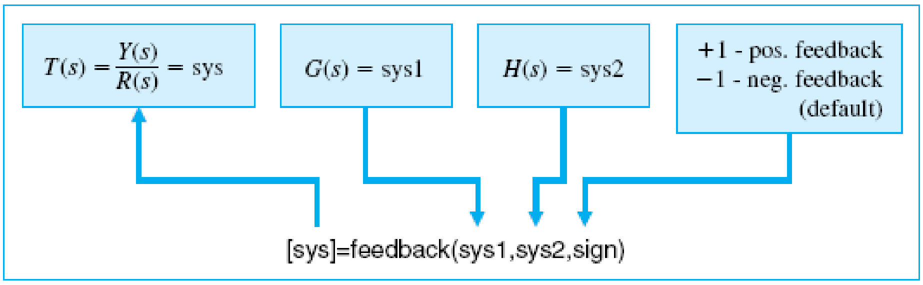
The MATLAB command for implementing a feedback system is "feedback" as shown below:


$T(s) = \frac{Y(s)}{R(s)} = \text{sys}$
$G_c(s)G(s) = \text{sys1}$
+1 - positive feedback
−1 - negative feedback (default)
[sys]=feedback(sys1,[1],sign)

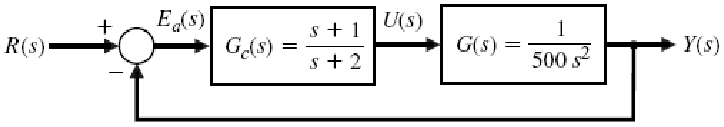
When H(s) is non-unity or specified, such a system is said to be a non-unity feedback system as shown below:



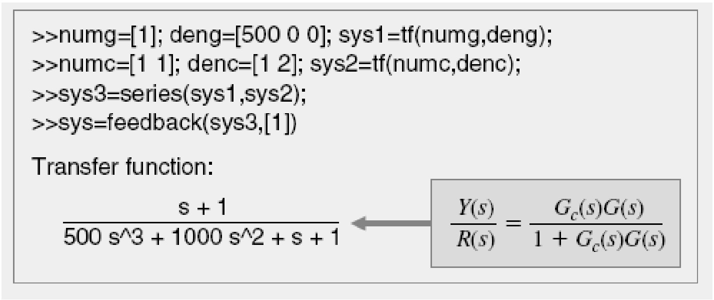
A non-unity feedback system is implemented in MATLAB using the same “feedback” command as shown:



Example 3: Given a unity feedback system as shown in the figure, obtain the overall transfer function using MATLAB:



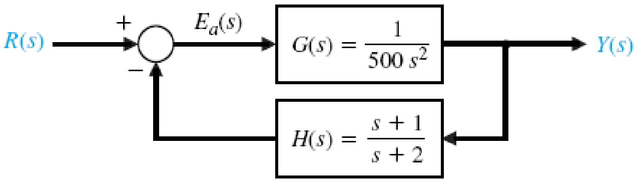
The result is as shown below:



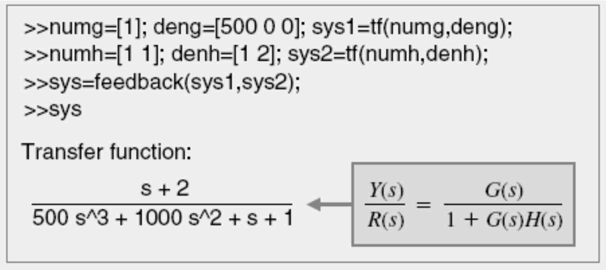
```
>>numg=[1]; deng=[500 0 0]; sys1=tf(numg,deng);
>>numc=[1 1]; denc=[1 2]; sys2=tf(numc,denc);
>>sys3=series(sys1,sys2);
>>sys=feedback(sys3,[1])

Transfer function:
            s + 1
  ---------------------------
  500 s^3 + 1000 s^2 + s + 1
```

$$\frac{Y(s)}{R(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}$$

Example 4: Given a non-unity feedback system as shown in the figure, obtain the overall transfer function using MATLAB:



The result is as shown below:

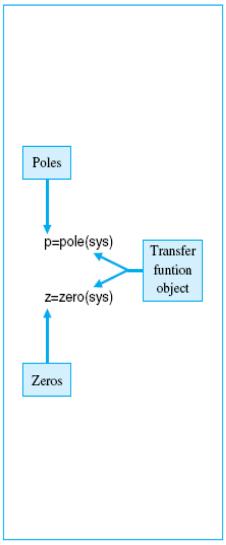


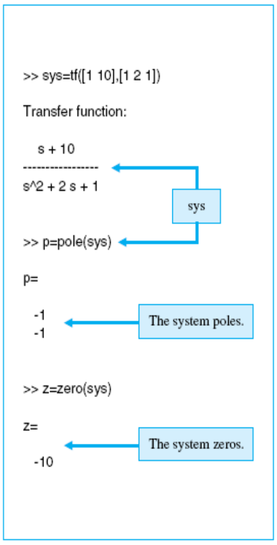
```
>>numg=[1]; deng=[500 0 0]; sys1=tf(numg,deng);
>>numh=[1 1]; denh=[1 2]; sys2=tf(numh,denh);
>>sys=feedback(sys1,sys2);
>>sys

Transfer function:
              s + 2
--------------------------------
500 s^3 + 1000 s^2 + s + 1
```

Poles and Zeros of System: To obtain the poles and zeros of the system use the MATLAB command “pole” and “zero” respectively as shown in example 5. You can also use MATLAB command “pzmap” to obtain the same.

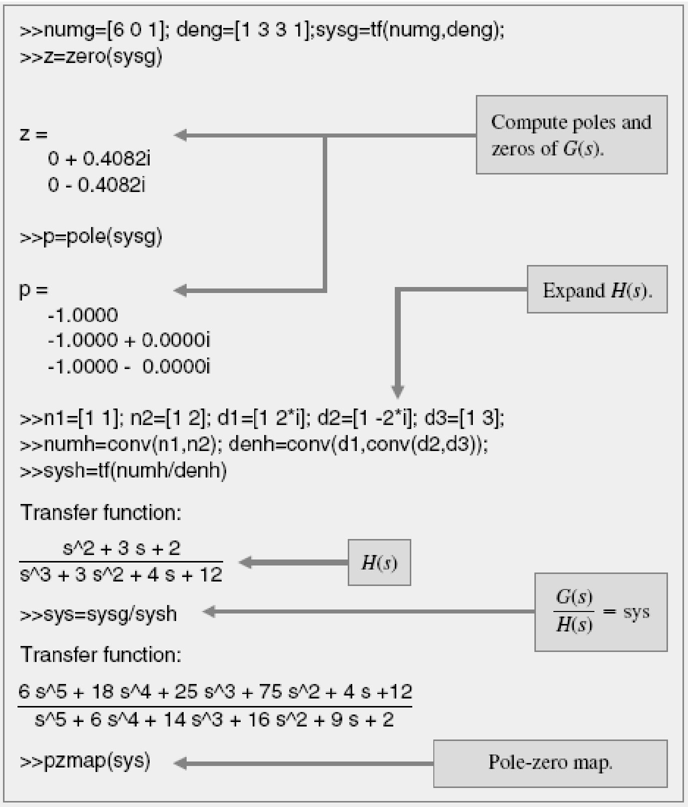
Example 5: Given a system transfer function plot the location of the system zeros and poles using the MATLAB pole-zero map command.

For example:





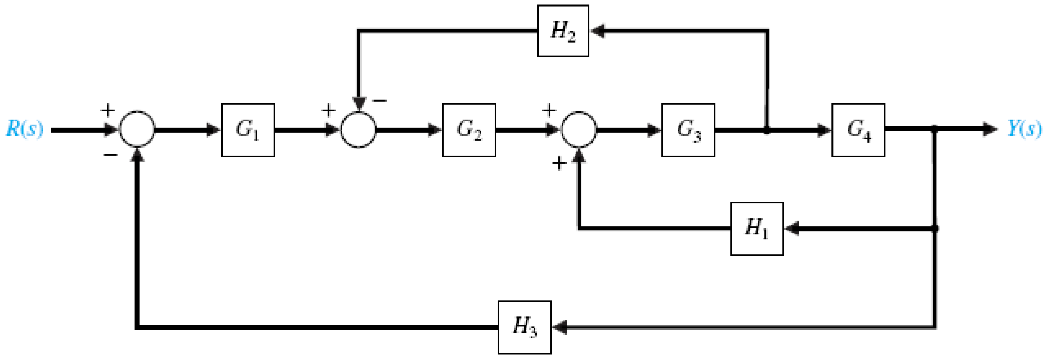
```
>> sys=tf([1 10],[1 2 1])

Transfer function:

  s + 10
---------------
s^2 + 2 s + 1

>> p=pole(sys)

p=

  -1
  -1

>> z=zero(sys)

z=

  -10
```



```
>>numg=[6 0 1]; deng=[1 3 3 1];sysg=tf(numg,deng);
>>z=zero(sysg)

z =
    0 + 0.4082i
    0 - 0.4082i

>>p=pole(sysg)

p =
    -1.0000
    -1.0000 + 0.0000i
    -1.0000 -  0.0000i

>>n1=[1 1]; n2=[1 2]; d1=[1 2*i]; d2=[1 -2*i]; d3=[1 3];
>>numh=conv(n1,n2); denh=conv(d1,conv(d2,d3));
>>sysh=tf(numh/denh)

Transfer function:
      s^2 + 3 s + 2
--------------------------
s^3 + 3 s^2 + 4 s + 12

>>sys=sysg/sysh

Transfer function:
6 s^5 + 18 s^4 + 25 s^3 + 75 s^2 + 4 s +12
--------------------------------------------
   s^5 + 6 s^4 + 14 s^3 + 16 s^2 + 9 s + 2

>>pzmap(sys)
```

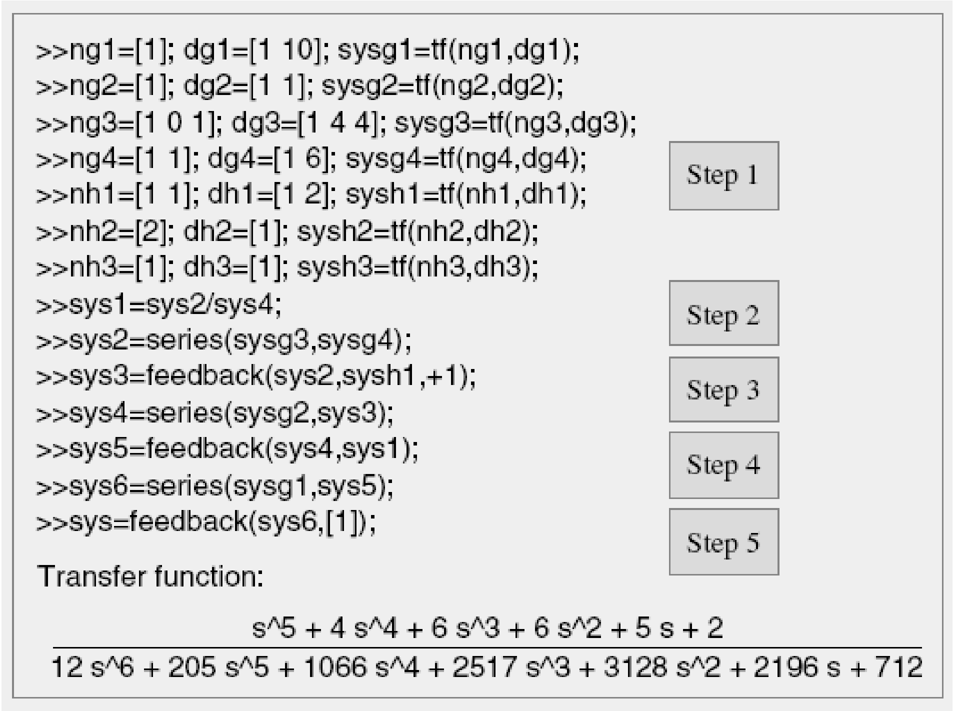
Compute poles and zeros of $G(s)$.
Expand $H(s)$.
$H(s)$
$\frac{G(s)}{H(s)} = \text{sys}$
Pole-zero map.

Exercise 1: For the following multi-loop feedback system, get closed loop transfer function and the corresponding pole-zero map of the system.



Given $G_1 = \frac{1}{(s+10)}$; $G_2 = \frac{1}{(s+1)}$; $G_3 = \frac{s^2+1}{(s^2+4s+4)}$; $G_4 = \frac{s+1}{(s+6)}$; $H_1 = \frac{s+1}{(s+2)}$; $H_2 = 2$; $H_3 = 1$ (Reference: Page 113, Chapter 2, Text: Dorf.)

MATLAB solution:

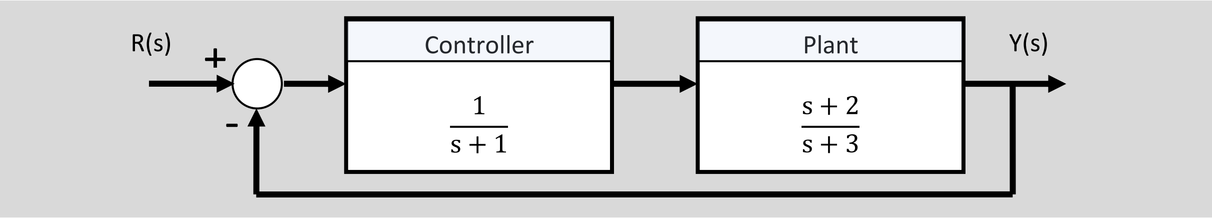


```
>>ng1=[1]; dg1=[1 10]; sysg1=tf(ng1,dg1);
>>ng2=[1]; dg2=[1 1]; sysg2=tf(ng2,dg2);
>>ng3=[1 0 1]; dg3=[1 4 4]; sysg3=tf(ng3,dg3);
>>ng4=[1 1]; dg4=[1 6]; sysg4=tf(ng4,dg4);
>>nh1=[1 1]; dh1=[1 2]; sysh1=tf(nh1,dh1);
>>nh2=[2]; dh2=[1]; sysh2=tf(nh2,dh2);
>>nh3=[1]; dh3=[1]; sysh3=tf(nh3,dh3);
>>sys1=sys2/sys4;
>>sys2=series(sysg3,sysg4);
>>sys3=feedback(sys2,sysh1,+1);
>>sys4=series(sysg2,sys3);
>>sys5=feedback(sys4,sys1);
>>sys6=series(sysg1,sys5);
>>sys=feedback(sys6,[1]);

Transfer function:

              s^5 + 4 s^4 + 6 s^3 + 6 s^2 + 5 s + 2
 ----------------------------------------------------------------
 12 s^6 + 205 s^5 + 1066 s^4 + 2517 s^3 + 3128 s^2 + 2196 s + 712
```

Instruction: Please refer to Section 2.6 and Section 2.2 in Text by Dorf.

Exercise 2: Consider the feedback system depicted in the figure below

a. Compute the closed-loop transfer function using the 'series' and 'feedback' functions
b. Obtain the closed-loop system unit step response with the 'step' function and verify that final value of the output is 2/5.



Reference: Please see Section 2.5 of Text by Dorf for Exercise 3.

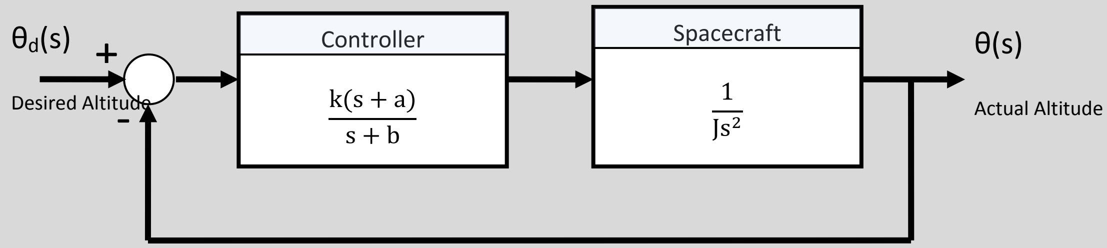
Exercise 3: A satellite single-axis altitude control system can be represented by the block diagram in the figure given. The variables 'k', 'a' and 'b' are controller parameters, and 'J' is the spacecraft moment of inertia. Suppose the nominal moment of inertia is 'J' = 10.8E8, and the controller parameters are k=10.8E8, a=1, and b=8.

a. Develop an m-file script to compute the closed-loop transfer function

$$T(s) = \theta(s)/\theta_d(s).$$

b. Compute and plot the step response to a 10^o step input.

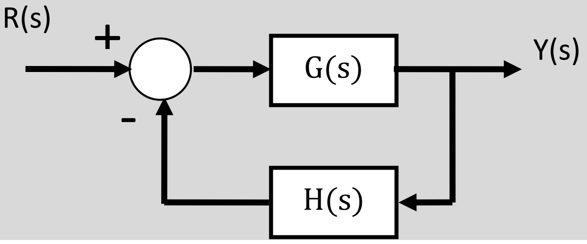
c. The exact moment of inertia is generally unknown and may change slowly with time. Compare the step response performance of the spacecraft when J is reduced by 20% and 50%. Discuss your results.



Reference: Please see Section 2.9 of Text by Dorf for Exercise 4.

Exercise 4: Consider the feedback control system given in figure, where

$$G(s) = \frac{s+1}{s+2} \text{ and } H(s) = \frac{1}{s+1}.$$



a. Using an m-file script, determine the close-loop transfer function.

b. Obtain the pole-zero map using the 'pzmap' function. Where are the closed-loop system poles and zeros?

c. Are there any pole-zero cancellations? If so, use the 'minreal' function to cancel common poles and zeros in the closed-loop transfer function.

d. Why is it important to cancel common poles and zeros in the transfer function?

Exercise 5: Do problem CP2.6 from your text

CISE 302
Linear Control Systems

Lab Experiment 6: Performance of First order and second order systems

Objective: The objective of this exercise will be to study the performance characteristics of first and second order systems using MATLAB.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

Category Soft-Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- MATLAB scripts and their results for Exercise 1 & 2 should be reported properly.

Overview First Order Systems:
An electrical RC-circuit is the simplest example of a first order system. It comprises of a resistor and capacitor connected in series to a voltage supply as shown below on Figure 1.

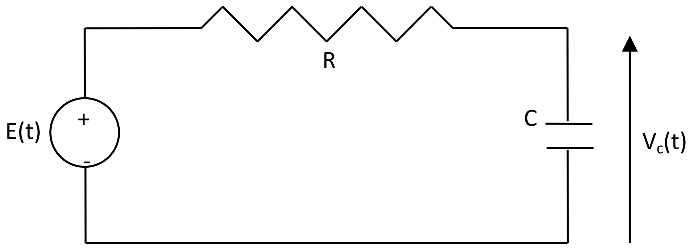


Figure 1: RC Circuit

If the capacitor is initially uncharged at zero voltage when the circuit is switched on, it starts to charge due to the current ‘i' through the resistor until the voltage across it reaches the supply voltage. As soon as this happens, the current stops flowing or decays to zero, and the circuit becomes like an open circuit. However, if the supply voltage is removed, and the circuit is closed, the capacitor will discharge the energy it stored again through the resistor. The time it takes the capacitor to charge depends on the time constant of the system, which is defined as the time taken by the voltage across the capacitor to rise to approximately 63% of the supply voltage. For a given RC-circuit, this time constant is $\tau = RC$. Hence its magnitude depends on the values of the circuit components.

The RC circuit will always behave in this way, no matter what the values of the components. That is, the voltage across the capacitor will never increase indefinitely. In this respect we will say that the system is passive and because of this property it is stable.

For the RC-circuit as shown in Fig. 1, the equation governing its behavior is given by

$$\frac{dv_c(t)}{dt} + \frac{1}{RC} v_c(t) = \frac{1}{RC} E \quad \text{where } v_c(0) = v_0 \qquad (1)$$

where $v_c(t)$ is the voltage across the capacitor, R is the resistance and C is the capacitance. The constant $\tau = RC$ is the time constant of the system and is defined as the time required by the system output i.e. $v_c(t)$ to rise to 63% of its final value (which is E). Hence the above equation (1) can be expressed in terms of the time constant as:

$$\tau \frac{dv_c(t)}{dt} + v_c(t) = E \quad \text{where } v_c(0) = v_0 \qquad (1)$$

Obtaining the transfer function of the above differential equation, we get

$$\frac{V_c(s)}{E(s)} = \frac{1}{\tau s + 1} \qquad (2)$$

where τ is time constant of the system and the system is known as the first order system. The performance measures of a first order system are its time constant and its steady state.

Exercise 1:

a) Given the values of R and C, obtain the unit step response of the first order system.
   a. R=2KΩ and C=0.01F
   b. R=2.5KΩ and C=0.003F
b) Verify in each case that the calculated time constant ($\tau = RC$) and the one measured from the figure as 63% of the final value are same.
c) Obtain the steady state value of the system.

Overview Second Order Systems:

Consider the following Mass-Spring system shown in the Figure 2. Where K is the spring constant, B is the friction coefficient, x(t) is the displacement and F(t) is the applied force:

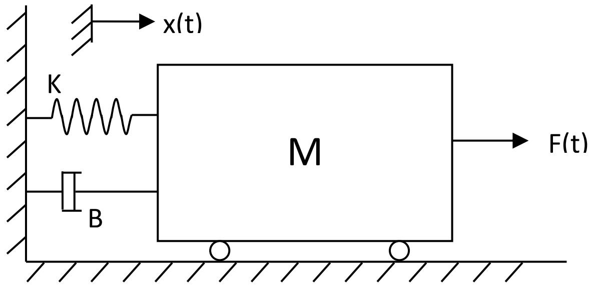


Figure 2. Mass-Spring system

The differential equation for the above Mass-Spring system can be derived as follows

$$M\frac{d^2x(t)}{dt^2} + B\frac{\partial x(t)}{\partial t} + Kx(t) = F(t)$$

Applying the Laplace transformation we get

$$(Ms^2 + Bs + K) * X(s) = F(s)$$

provided that, all the initial conditions are zeros. Then the transfer function representation of the system is given by

$$TF = \frac{Output}{Input} = \frac{F(s)}{X(s)} = \frac{1}{(Ms^2 + Bs + K)}$$

The above system is known as a second order system.

The generalized notation for a second order system described above can be written as

$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

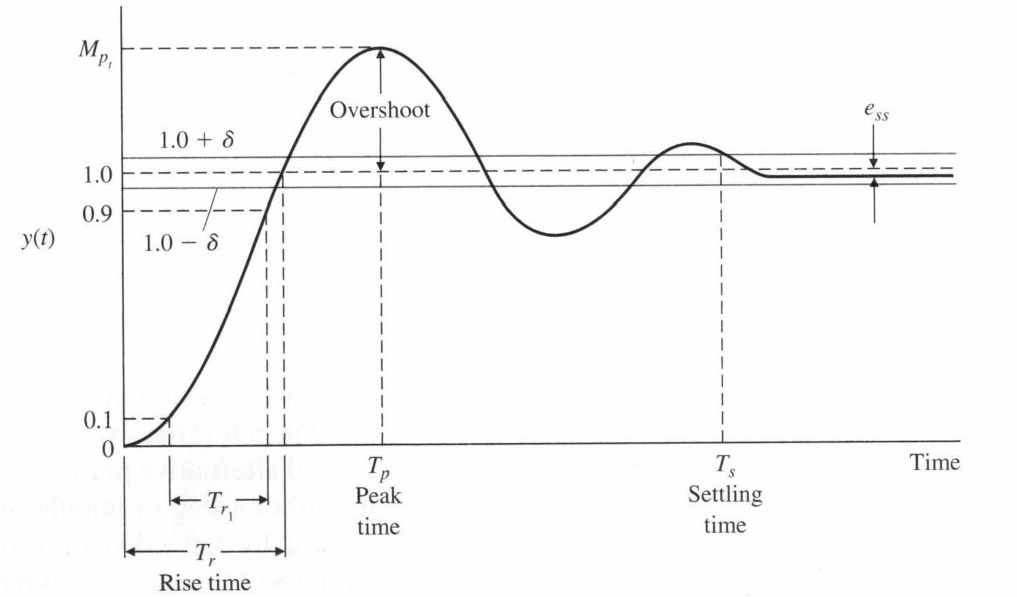
With the step input applied to the system, we obtain

$$Y(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

for which the transient output, as obtained from the Laplace transform table (Table 2.3, Textbook), is

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \cos^{-1}(\zeta))$$

where 0 < ζ < 1. The transient response of the system changes for different values of damping ratio, ζ. Standard performance measures for a second order feedback system are defined in terms of step response of a system. Where, the response of the second order system is shown below.



The performance measures could be described as follows:

Rise Time: The time for a system to respond to a step input and attains a response equal to a percentage of the magnitude of the input. The 0-100% rise time, T_r, measures the time to 100% of the magnitude of the input. Alternatively, T_{r1}, measures the time from 10% to 90% of the response to the step input.

Peak Time: The time for a system to respond to a step input and rise to peak response.

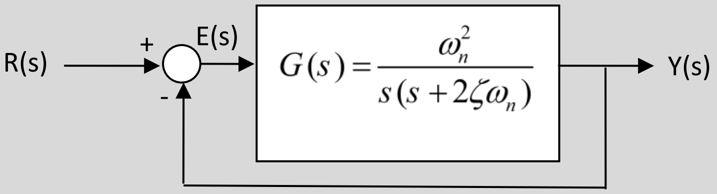
Overshoot: The amount by which the system output response proceeds beyond the desired response. It is calculated as

$$\text{P.O.}=\frac{M_{p_t}-f\upsilon}{f\upsilon}\times 100\%$$

where M_{Pt} is the peak value of the time response, and fv is the final value of the response.
Settling Time: The time required for the system's output to settle within a certain percentage of the input amplitude (which is usually taken as 2%). Then, settling time, T_s, is calculated as

$$T_s = \frac{4}{\zeta\omega_n}$$

Exercise 2: Effect of damping ratio ζ on performance measures. For a single-loop second order feedback system given below



Find the step response of the system for values of ωn = 1 and ζ = 0.1, 0.4, 0.7, 1.0 and 2.0. Plot all the results in the same figure window and fill the following table.

ζ	Rise time	Peak Time	% Overshoot	Settling time	Steady state value
0.1					
0.4					
0.7					
1.0					
2.0					

CISE 302
Linear Control Systems

Lab Experiment 7: DC Motor Characteristics

Objective: The objective of the experiment is to show how a permanent magnet D.C. motor may be controlled by varying the magnitude and direction of its armature current and recognize the torque/speed characteristic of the D.C. Motor

List of Equipment/Software

Following equipment/software is required:

- MATLAB
- LabVIEW
- DC Servo System (feedback equipment)

a.	OU150A	Op Amp Unit
b.	AU150B	Attenuator Unit
c.	PA150C	Pre-Amplifier Unit
d.	SA150D	Servo Amplifier
e.	PS150E	Power Supply
f.	DCM150F	DC Motor
g.	IP150H	Input Potentiometer
h.	OP150K	Output Potentiometer
i.	GT150X	Reduction Gear Tacho
j.	DC Voltmeter	

Category Software-Hardware Experiment

Note: This lab exercise may take two weeks.

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- Clearly show the model development and the Simulink model.
- Show the parameter identification graphs and calculations properly.
- Connection diagram of the hardware experimental part.
- Report the results in the table and graphical way with summarized learning outcomes.

Introduction:

This experiment will illustrate the characteristics of the D.C. motor used in the Modular Servo and show how it can be controlled by the Servo Amplifier.

The motor is a permanent magnet type and has a single armature winding. Current flow through the armature is controlled by power amplifiers as in figure so that rotation in both directions is possible by using one, or both of the inputs. In most of the later assignments the necessary input signals are provided by a specialized Pre-Amplifier Unit PA150C, which connected to Inputs 1 and 2 on SA150D

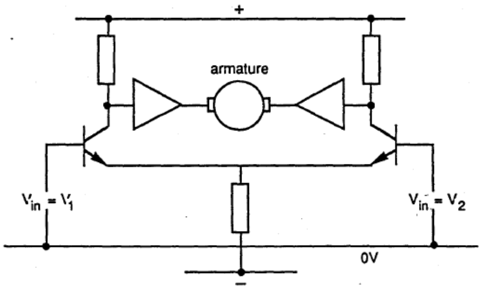


Figure: Armature Control

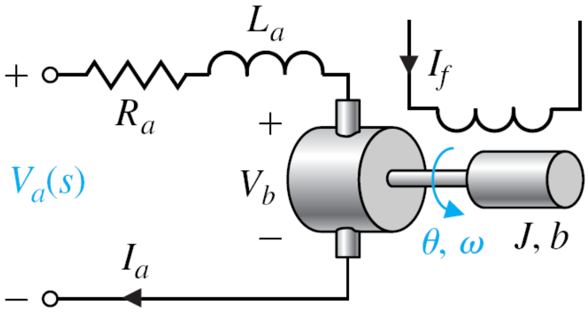


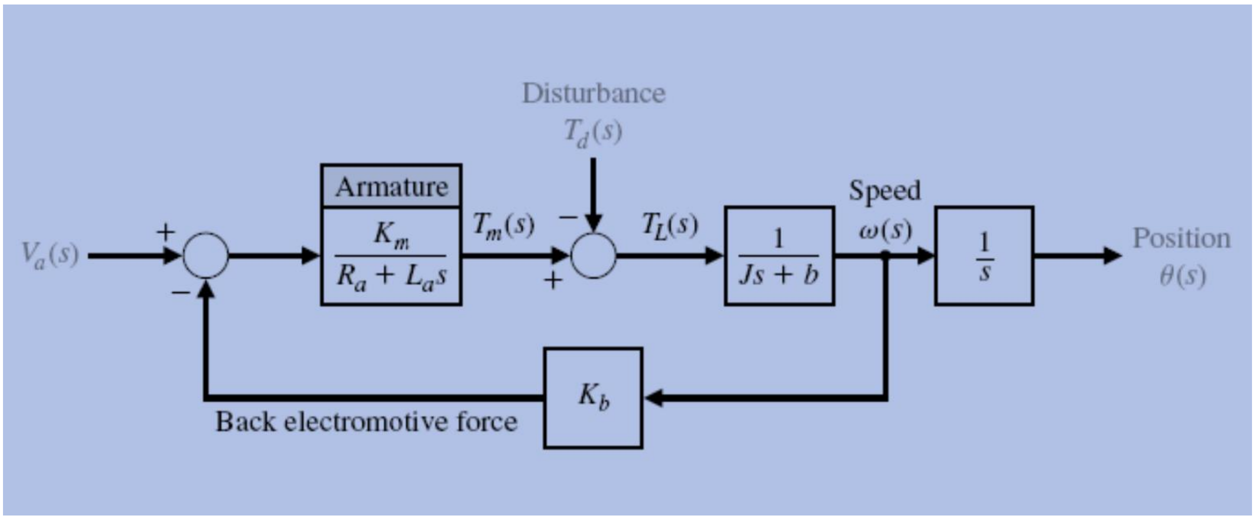
Figure: DC motor armature-controlled rotational actuator

As the motor accelerates the armature generates an increasing 'back-emf' V_a tending to oppose the driving voltage V_{in}. The armature current is thus roughly proportional to (V_{in} - V_a). If the speed drops (due to loading) V_a reduces, the current increases and thus so does the motor torque. This tends to oppose the speed drop. This mode of control is called 'armature-control' and gives a speed proportional to V_{in} as in figure.

Model of the armature-controlled DC motor:

The model of the armature-controlled DC motor has been developed in many text books in particular (Dorf and Bishop, 2008).
Assignment: Read (Dorf and Bishop, 2008) page 62-65

The final block diagram is as follows:



Model Simulation using Simulink:

Prerequisite to this section is a mathematical understanding of the elctro-mechanical model of a DC motor. Student should be able to understand how electrical terms (voltage, current, emf) interact with mechanical terms (speed, position) via electro-magnetic circuit (inductance). The students should be able to understand and derive the mathematical model of a DC motor.

The motor torque, **T**, is related to the armature current, **i**, by a constant factor **Kt**. The back emf, **e**, is related to the rotational velocity by the following equations:

$$T = K_t i$$

$$e = K_e \frac{d\theta}{dt}$$

In SI units (which we will use), **Kt** (armature constant) is equal to **Ke** (motor constant).

1. Building the Model

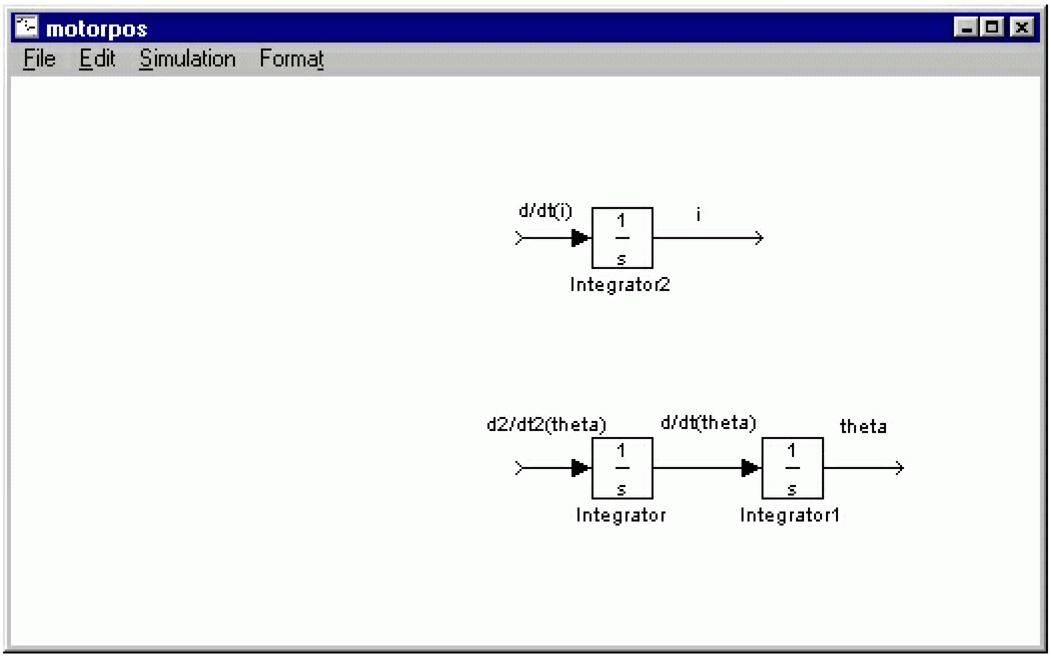
This system will be modeled by summing the torques acting on the rotor inertia and integrating the acceleration to give the velocity, and integrating velocity to get position. Also, Kirchoff's laws will be applied to the armature circuit.

Open Simulink and open a new model window. First, we will model the integrals of the rotational acceleration and of the rate of change of armature current.

$$\iint \frac{d^2\theta}{dt^2} = \int \frac{d\theta}{dt} = \theta$$

$$\int \frac{di}{dt} = i$$

- Insert an Integrator block (from the Linear block library) and draw lines to and from its input and output terminals.
- Label the input line "d2/dt2(theta)" and the output line "d/dt(theta)" as shown below. To add such a label, double click in the empty space just above the line.
- Insert another Integrator block attached to the output of the previous one and draw a line from its output terminal.
- Label the output line "theta".
- Insert a third Integrator block above the first one and draw lines to and from its input and output terminals.
- Label the input line "d/dt(i)" and the output line "i".



Next, we will start to model both Newton's law and Kirchoff's law. These laws applied to the motor system give the following equations:

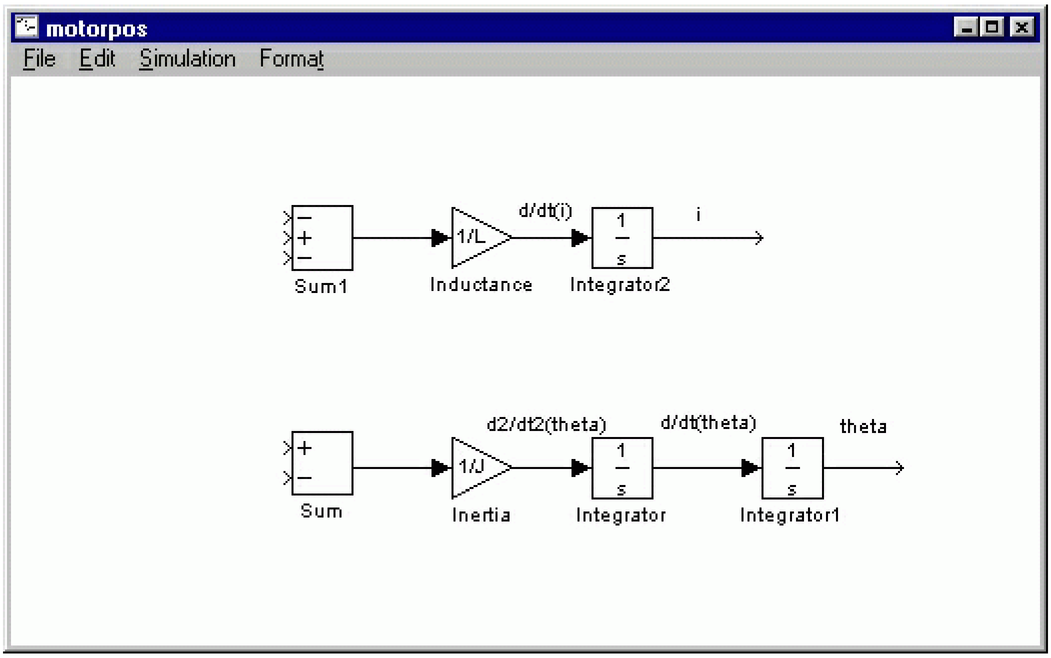
$$J\,\frac{d^2\theta}{dt^2} = T - b\,\frac{d\theta}{dt} \;=>\; \frac{d^2\theta}{dt^2} = \frac{1}{J}\left(K_t i - b\,\frac{d\theta}{dt}\right)$$

$$L\,\frac{di}{dt} = -Ri + V - e \;=>\; \frac{di}{dt} = \frac{1}{L}\left(-Ri + V - K_e\,\frac{d\theta}{dt}\right)$$

The angular acceleration is equal to 1/J multiplied by the sum of two terms (one pos., one neg.). Similarly, the derivative of current is equal to 1/L multiplied by the sum of three terms (one pos., two neg.).

- Insert two Gain blocks, (from the Linear block library) one attached to each of the leftmost integrators.
- Edit the gain block corresponding to angular acceleration by double-clicking it and changing its value to "1/J".
- Change the label of this Gain block to "inertia" by clicking on the word "Gain" underneath the block.

- Similarly, edit the other Gain's value to "1/L" and it's label to Inductance.
- Insert two Sum blocks (from the Linear block library), one attached by a line to each of the Gain blocks.
- Edit the signs of the Sum block corresponding to rotation to "+-" since one term is positive and one is negative.
- Edit the signs of the other Sum block to "-+-" to represent the signs of the terms in Kirchoff's equation.

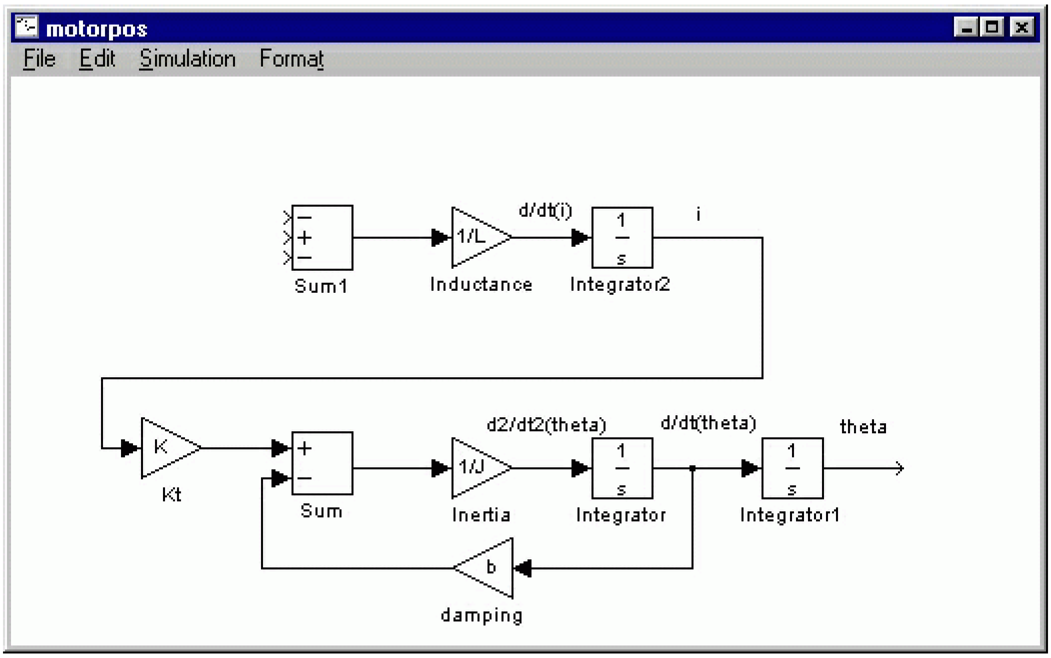


Now, we will add in the torques which are represented in Newton's equation. First, we will add in the damping torque.

- Insert a gain block below the inertia block, select it by single-clicking on it, and select Flip from the Format menu (or type Ctrl-F) to flip it left-to-right.
- Set the gain value to "b" and rename this block to "damping".
- Tap a line (hold Ctrl while drawing) off the first rotational integrator's output (d/dt(theta)) and connect it to the input of the damping gain block.
- Draw a line from the damping gain output to the negative input of the rotational Sum block.

Next, we will add in the torque from the armature.

- Insert a gain block attached to the positive input of the rotational Sum block with a line.
- Edit it's value to "K" to represent the motor constant and Label it "Kt".
- Continue drawing the line leading from the current integrator and connect it to the Kt gain block.

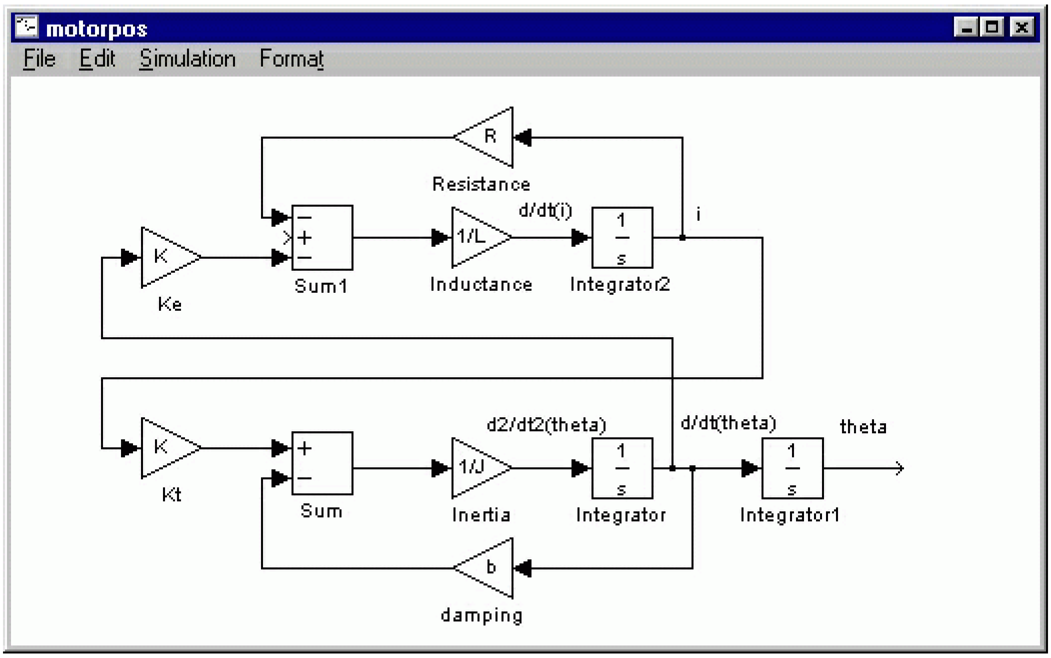


Now, we will add in the voltage terms which are represented in Kirchoff's equation. First, we will add in the voltage drop across the coil resistance.

- Insert a gain block above the inductance block, and flip it left-to-right.
- Set the gain value to "R" and rename this block to "Resistance".
- Tap a line (hold Ctrl while drawing) off the current integrator's output and connect it to the input of the resistance gain block.
- Draw a line from the resistance gain output to the upper negative input of the current equation Sum block.

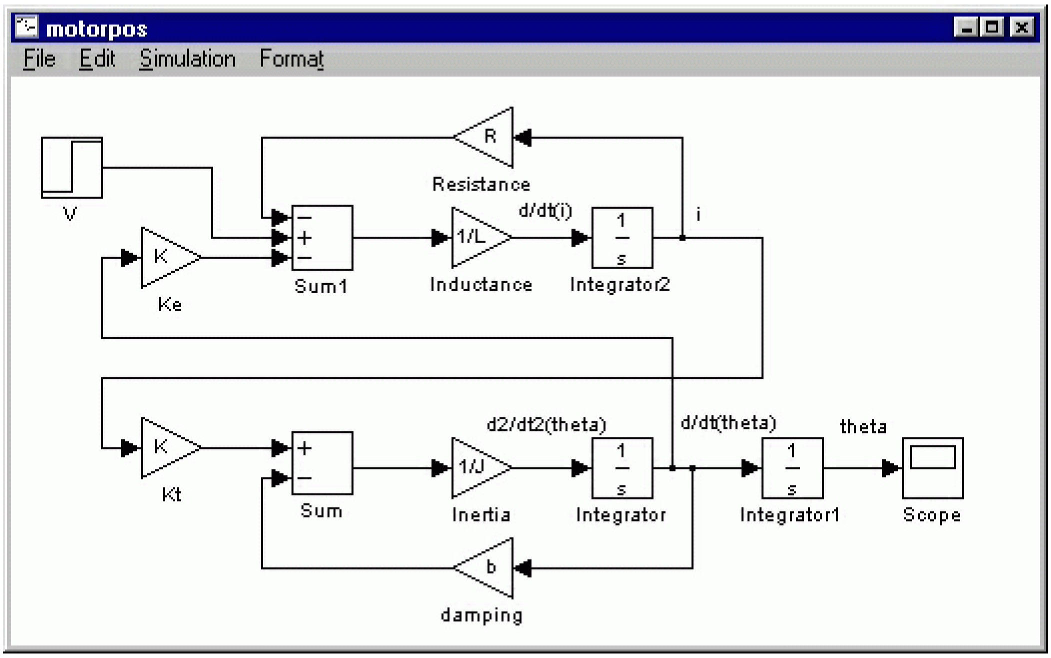
Next, we will add in the back emf from the motor.

- Insert a gain block attached to the other negative input of the current Sum block with a line.
- Edit it's value to "K" to represent the motor constant and Label it "Ke".
- Tap a line off the first rotational integrator's output (d/dt(theta)) and connect it to the Ke gain block.



The third voltage term in the Kirchoff equation is the control input, V. We will apply a step input.

- Insert a Step block (from the Sources block library) and connect it with a line to the positive input of the current Sum block.
- To view the output speed, insert a Scope (from the Sinks block library) connected to the output of the second rotational integrator (theta).
- To provide a appropriate unit step input at t=0, double-click the Step block and set the Step Time to "0".



2. DC motor nominal values

- moment of inertia of the rotor (J) = 3.2284E-6 kg.m^2/s^2
- damping ratio of the mechanical system (b) = 3.5077E-6 Nms
- electromotive force constant (K=Ke=Kt) = 0.0274 Nm/Amp
- electric resistance (R) = 4 ohm
- electric inductance (L) = 2.75E-6 H
- input (V): Source Voltage
- output (theta): position of shaft

Assumption: The rotor and shaft are assumed to be rigid

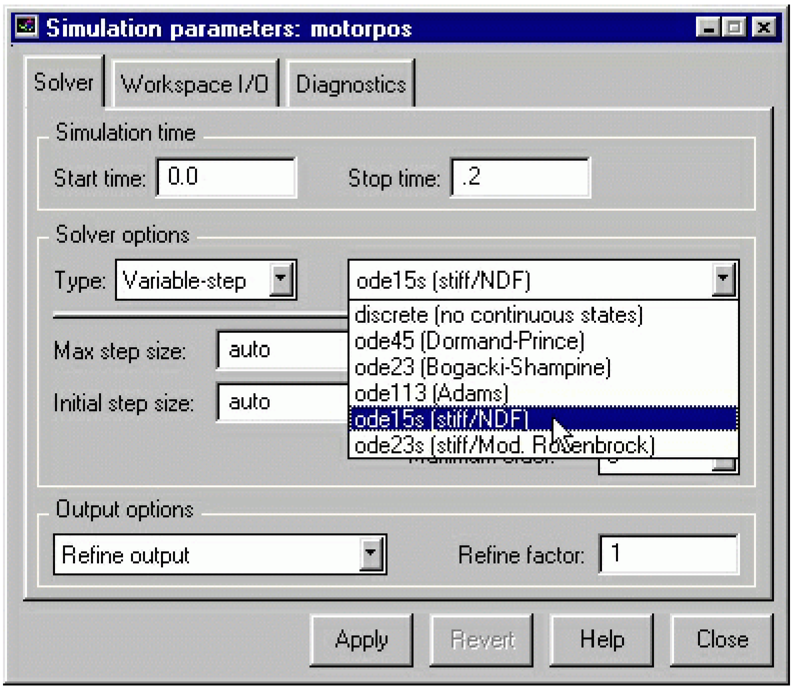
The physical parameters must now be set. Run the following commands at the MATLAB prompt:

```
J=3.2284E-6;
b=3.5077E-6;
K=0.0274;
R=4;
L=2.75E-6;
```

Run the simulation (Ctrl-t or Start on the Simulation menu).

3. Simulation:

To simulate this system, first, an appropriate simulation time must be set. Select Parameters from the Simulation menu and enter "0.2" in the Stop Time field. 0.2 seconds is long enough to view the open-loop response. Also in the Parameters dialog box, it is helpful to change the Solver Options method. Click on the field which currently contains "ode45 (Dormand-Prince)". Select the option "ode15s (stiff/NDF)". Since the time scales in this example are very small, this stiff system integration method is much more efficient than the default integration method.



Simulation parameters: motorpos

Simulation time

Start time: 0.0

Stop time: .2

Solver options

Type: Variable-step

ode15s (stiff/NDF)

Max step size: auto

Initial step size: auto

Output options

Refine output

Refine factor: 1

Step input:

- Use step input from 0 volts to 2 volts and observe the response.
- Save the response to workspace variable to further compare with the experimental DC motor (DCM 150F).
- Now Step the input voltage from 2 volts to 4 volts. Save the response to further compare with experimental motor.
- This is the simulation section for the Exercise 2 – Step input.

Sine wave input:

- Remove the Step Input and connect a Function Generator from the Simulink-Library to the input of the motor model in Simulink.
- Select Sinusoidal function in the function generator.
- Fix the amplitude of the sine wave to 2.
- Take several responses by varying the frequency of the sinusoidal wave keeping the amplitude fixed.
- Save the input and output of the DC motor model to further compare with experimental motor response.
- This is the simulation section for the Exercise 2 – Sine input

Parameter Identification:

Purpose: Modeling in Simulink requires system parameters of DC motor. If the parameters of the DC motor system are unknown, students should use this section to determine the system parameters of the DC motor system. The motor is attached to a tachometer, flywheel, and a load (magnetic load disk). Data should be acquired for the unloaded spin up of the motor, as well as the response of the system upon the application of a load. The data from the experimental setup can be collected using USB DAQ from National Instruments and LABVIEW software.

Derivations

To measure the constants km and R, we can use equations following equations,

$$e_m = k_m \omega$$

$$i_a = \frac{e_a - e_m}{R}$$

Divide by e_a,

$$\frac{i_a}{e_a} = \frac{1}{R} - \frac{k_m}{R e_a}\omega$$

$$\omega \frac{k_m}{R e_a} + \frac{i_a}{e_a} = \frac{1}{R}$$

$$\frac{\omega k_m + R i_a}{R e_a} + \frac{i_a}{e_a} = \frac{1}{R}$$

$$\frac{R e_a}{\omega k_m + R i_a} = R$$

$$e_a = \omega k_m + R i_a$$

$$\frac{e_a}{i_a} = \frac{\omega}{i_a} k_m + R$$

Hint: The last equation resembles with the well known $y = mx + c$ equation of a line.

Since we have measured values for e_a, i_a, and ω, we can plot $\frac{e_a}{i_a}$ versus $\frac{\omega}{i_a}$ to determine the slope and intercept, which reveals the constant values of k_m and R.

To derive B, t_f, and k_w we use the equation

$$\tau_m = k_i\, i_a = J\dot{\omega} + B\omega + \tau_f$$

For steady state $\dot{\omega} = 0$, so we have

$$\tau_m = B\omega + \tau_f$$

If we plot the graph of τ_m and ω, and plot a best fit line, we can get the corresponding values of B and τ_f.

Once we know τ_m, we can find k_i using

$$\tau_m = k_i\, i_a$$

Note that k_ω is the slope of the e_t vs ω graph.

Summarize the system parameters for the DC motor in corresponding graphs and tables. Use these parameters in the Simulink Model simulation and validate the Simulation results with the experimental set up.

Viscous Friction vs. Coulomb Friction

We can estimate the speed at which the viscous friction is greater than the coulomb friction, that is, when $B\omega \geq T_f$. The speed can be calculated by dividing T_f by B .

The approximate tachometer voltage corresponding to this speed can be determined by using the best fit equation for the relationship between e_t and ω .

After determining the best fit equation for e_t and ω , we can plug in the value of ω calculated above to get an expected tachometer voltage e_t .

Note that the voltage estimated may be greater than the tachometer voltage determined by the experimental setup in lab. This case, we can assume that the Coulomb friction in the motor system dominates.

Calculating J from the Motor Spinup Data

Applying a step input of to the motor and acquiring data samples, we can obtain a curve for the tachometer voltage vs. time. Given the form of the first order response of the system to an input, we can further fit the data with an appropriate exponential to determine the time constant.

$$Transfer\ Function(s) = \frac{E_t(s)}{E_a(s)} = \frac{1}{s}.\frac{K}{s+a} \Leftrightarrow e_t(t) = K.\frac{1}{a}(1-e^{-at})$$

Where,

$$a = \frac{1}{\tau} = \frac{BR + k_m k_i}{R\,J}\text{, and } K = \frac{k_i\, k_\omega}{R\,J}$$

Using the above equations and the best fit constants for the motor spin up obtained above, we can calculate the value of J .

Figures to be plotted:

1. e_t (volts) versus $\omega(rad/s)$: To determine k_ω .
2. $\frac{e_a}{i_a}$ (volts/ampere = ohms) versus $\frac{\omega}{i_a}$ (rad/(sec*Amp)): To determine constants k_m and R.
3. T_m (N*m) versus ω (rad/s): To determine B and T_f .

Table of values: k_ω B R k_m T_f k_T

CISE 302
Linear Control Systems

Lab Experiment 8: Validation of DC Motor Characteristics

Objective: The objective of the experiment is to validate the learning outcomes of the last experiment (Exp. 6) for the characteristic of the D.C. Motor.

List of Equipment/Software

Following equipment/software is required:

- LabVIEW
- DC Servo System (feedback equipment)

a.	OU150A	Op Amp Unit
b.	AU150B	Attenuator Unit
c.	PA150C	Pre-Amplifier Unit
d.	SA150D	Servo Amplifier
e.	PS150E	Power Supply
f.	DCM150F	DC Motor
g.	IP150H	Input Potentiometer
h.	OP150K	Output Potentiometer
i.	GT150X	Reduction Gear Tacho
j.	DC Voltmeter	

Category Software-Hardware Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- Show the parameter identification graphs and calculations properly.
- Connection diagram of the hardware experimental part.
- Report the results in the table and graphical way with summarized learning outcomes.

Model validation:

Preliminary Procedure:

- Attach the AU150B, SA150D and PS150E, to the baseplate by means of the magnetic bases.
- Fit the eddy-current brake disc to the motor shaft.
- Now attach the DCM150F to the baseplate by means of the magnetic fixings and fix the plug into the SA150D.

- Attach the GT150X to the baseplate by means of the magnetic fixings and position it, so that it is connected to the motor shaft by means of the flexible coupling.
- Attach the LU150L to the baseplate by means of the magnetic fixings and position it so that when the cursor is on position 10 the eddy-current disc lies midway in the gap with its edge flush with the back of the magnet.
- Fix the plug from the SA150D into the PS150E
- Connect the Power Supply to the mains supply line, DO NOT switch on yet.

Procedure:

- Connect the equipment as shown in the figure.
- The system provides a tacho-generator coupled to the motor. For use in later assignments, it will be necessary to calibrate this generator by finding the factor Kg, which are the volts generated per thousand rev/min of motor shaft.
- Use the switch on the top of the GT150X to display the tacho volts or speed as required.

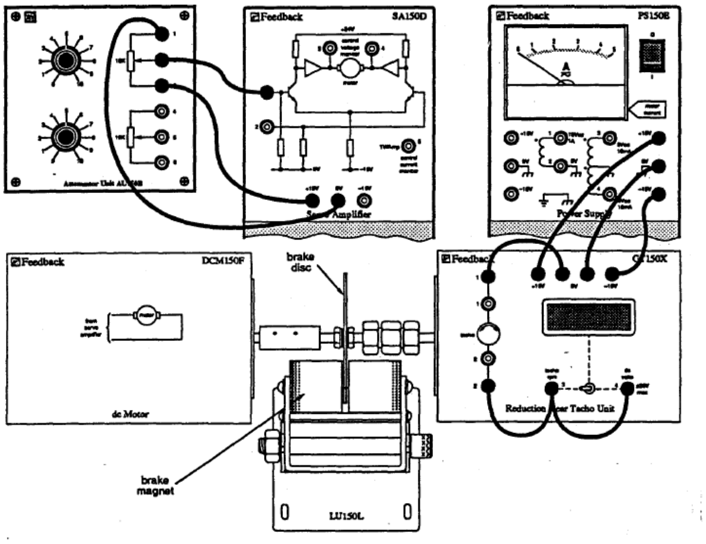


Figure: Connections for DC Motor

Exercise 1:

1. Set the magnetic brake to the unloaded position and turn the potentiometer till there is a reading of 1V on the voltmeter.
2. Repeat this reading with a 2V output. Then repeat for 3V, 4V and 5V.

   Now record the speed. Tabulate your results in a copy of the table given below.

Tacho-generator Volts	Speed r/min

3. Plot a graph of your results, as in figure below, of Speed against Tacho-generator volts.

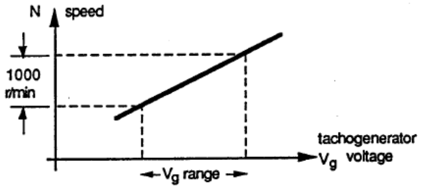


Figure: Speed vs Tacho-generator volts

The calibration factor Kg = Vg/N-r/min. It should be about 2.OV to 3.OV per 1000 r/min.

Exercise 2: Compare model and real system

- Step the input voltage from 0V to 2 V and compare the output with MATLAB response.
- While the system is stable and the input is at 2 V, Step the input voltage from 2V to 4V. Record the input and output and compare the output with the same experiment in MATLAB.
- Connect a frequency generator to the input voltage and fix the input to 2*sin(wt) where $w = 2\pi f$. Record the output and compare it to MATLAB response.

Nonlinear characteristics

The difference between the MATLAB model and the real system can be explained by the presence of a nonlinear dynamic that was ignored during modeling. Indeed, Due to brush friction, a certain minimum input signal is needed to start the motor rotating. Above figure shows how the speed varies with load torque. The first experiment will be to obtain the characteristics of the motor.

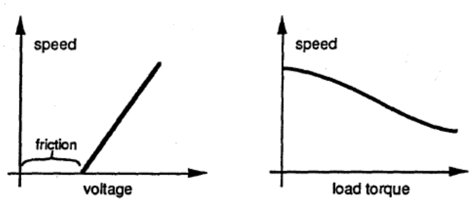


Figure: Armature control characteristics

Exercise 3: determination of the nonlinear DC motor characteristics

- Reduce the input voltage till the motor is just turning then measure with your voltmeter the voltages between OV and potentiometer slider and the tacho-generator output. Then tabulate as in fig 3.3.6. Increase the input voltage in one-volt steps, take readings of the input voltage and tachogenerator voltage up to approximately 2000 r/min which is the maximum speed of the motor.
- Plot the input voltages against speed, your results should be similar to the figure which shows plot of speed vs voltage.

V_{in} Volts	V_g Volts	Speed rpm

- Calculate the slope (input volts per thousand r/min).

Exercise 4: Effect of the load

- To measure the torque/speed characteristics, fix the brake so that it passes over the disc smoothly while the motor is running.
- Set the brake at position 0 and increase the input voltage until the motor rotates at close to its maximum speed.
- Then set the brake at position 10 and if necessary reduce the input voltage so that the ammeter on the PS150E is just below 2 amps; note the value of the input voltage.

Take tacho-generator readings over the range of the brake down to zero position, tabulating your results.

Brake Position	V_g Volts	Speed Rpm

- Now reset the brake back to maximum position and reduce the signal input voltage so that the motor is slowly rotating. Note the actual value of the input voltage.
- Take readings over the brake range tabulating the further results.

Brake Position	V_g Volts	Speed rpm

- Plot the two sets of results, as in figure of Speed against Torque (brake position) for the two input voltage values.
- Below figure shows the approximate brake position/g.cm characteristics of the motor at 1000 r/min. For other speeds, the torque will be proportional to the speed.

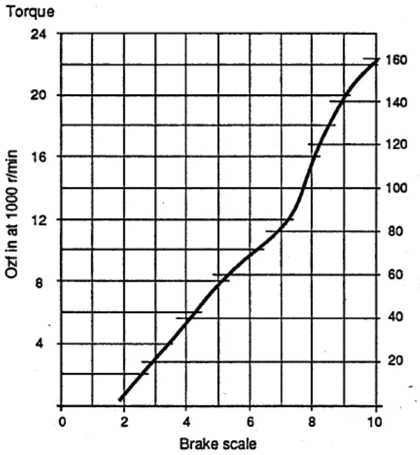


Figure: Approximate brake characteristics at 1000 r/min

With armature control the negative feedback of the back emf will oppose the input signal and so tend to maintain a steady motor current; this results in a more constant speed over the torque range. As a result the torque/speed curve becomes more similar to that produced by a shunt wound motor. The armature-controlled shunt-wound motor is extensively used in control systems.

CISE 302
Linear Control Systems

Lab Experiment 9: Effect of Feedback on disturbance & Control System Design

Objective: The objective of this exercise will be to study the effect of feedback on the response of the system to step input and step disturbance taking the practical example of English Channel boring machine and design a control system taking in account performance measurement.

List of Equipment/Software

Following equipment/software is required:

- MATLAB

Category Soft - Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- The Simulink model.
- MATLAB scripts and results for Exercise 1 & 2.

Overview:

The construction of the tunnel under the English Channel from France to the Great Britain began in December 1987. The first connection of the boring tunnels from each country was achieved in November 1990. The tunnel is 23.5 miles long and bored 200 feet below sea level. Costing $14 billion, it was completed in 1992 making it possible for a train to travel from London to Paris in three hours.

The machine operated from both ends of the channel, bored towards the middle. To link up accurately in the middle of the channel, a laser guidance system kept the machines precisely aligned. A model of the boring machine control is shown in the figure, where Y(s) is the actual angle of direction of travel of the boring machine and R(s) is the desired angle. The effect of load on the machine is represented by the disturbance, $T_d(s)$.

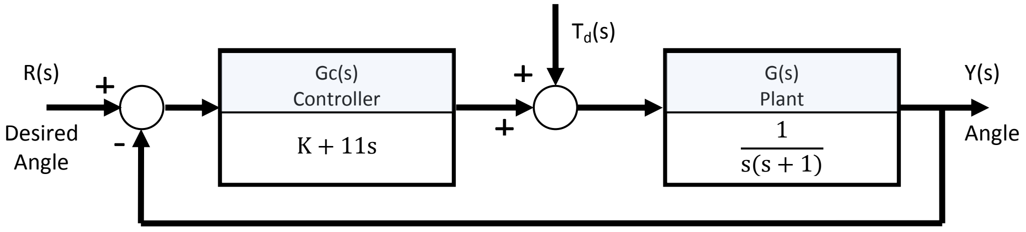


Figure: A block diagram model of a boring machine control system

Exercise 1:

a) Get the transfer function from R(s) to Y(s)
b) Get the transfer function from D(s) to Y(s)
c) Generate the system response; for K= 10, 20, 50, 100; due to a unit step input - r(t)
d) Generate the system response; for K= 10, 20, 50, 100; due to a unit step disturbance - d(t)
e) For each case find the percentage overshoot(%O.S.), rise time, settling time, steady state of y(t)
f) Compare the results of the two cases
g) Investigate the effect of changing the controller gain on the influence of the disturbance on the system output

M-files for two cases of K=20 and K=100 are shown below

Due to unit step – r(s)

```
% Response to a Unit Step Input R(s)=1/s
for K=20 and K=100
%
numg=[1];deng=[1                       1
0];sysg=tf(numg,deng);
K1=100;K2=20;
num1=[11 K1];num2=[11 K2];den=[0 1];
sys1=tf(num1,den);sys2=tf(num2,den);
%
sysa=series(sys1,sysg);sysb=series(sys2,sys
d);
sysc=feedback(sysa,[1]);sysd=feedback(sys
b,[1]);
%
t=[0:0.01:2.0];
[y1,t]=step(sysc,t);[y2,t]=step(sysd,t);
subplot(211);plot(t,y1);title('Step   Response
for K=100');
xlabel('Time    (seconds)');ylabel('y(t)');grid
on;
subplot(212);plot(t,y2);title('Step   Response
for K=20');
xlabel('Time    (seconds)');ylabel('y(t)');grid
on;
```

Due to unit disturbance – T_d(s)

```
% Response to a Disturbance Input D(s)=1/s
for K=20 and K=100
%
numg=[1];deng=[1                       1
0];sysg=tf(numg,deng);
K1=100;K2=20;
num1=[11 K1];num2=[11 K2];den=[0 1];
sys1=tf(num1,den);sys2=tf(num2,den);
%
sysa=feedback(sysg,sys1);sysa=minreal(sys
a);
sysb=feedback(sysg,sys2);sysb=minreal(sys
b);
%
t=[0:0.01:2.5];
[y1,t]=step(sysa,t);[y2,t]=step(sysb,t);
subplot(211);plot(t,y1);title('Disturbance
Response for K=100');
xlabel('Time    (seconds)');ylabel('y(t)');grid
on;
subplot(212);plot(t,y2);title('Disturbance
Response for K=20');
label('Time     (seconds)');ylabel('y(t)');grid
on;
```

Exercise 2: Design of a Second order feedback system based on performances.

For the motor system given below, we need to design feedback such that the overshoot is limited and there is less oscillatory nature in the response based on the specifications provided in the table. Assume no disturbance (D(s)=0).

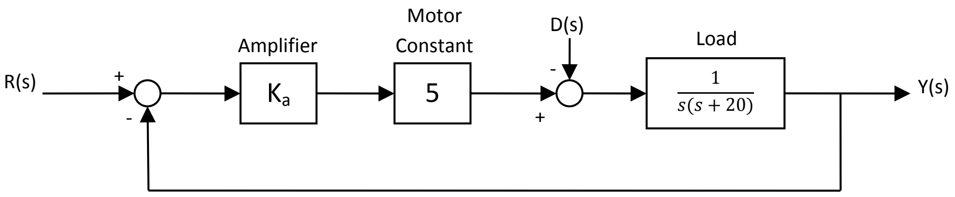


Table: Specifications for the Transient Response

Performance Measure	Desired Value

Percent overshoot	Less than 8%
Settling time	Less than 400ms

Use MATLAB, to find the system performance for different values of K_a and find which value of the gain K_a satisfies the design condition specified. Use the following table.

K_a	20	30	50	60	80
Percent Overshoot					
Settling time					

CISE 302
Linear Control Systems

Lab Experiment 10: Effect of Feedback on disturbance & Control System Design of Tank Level System

Objective: The objective of this exercise will be to study the effect of feedback on the response of the system to step input and step disturbance on the Two Tank System.

List of Equipment/Software

Following equipment/software is required:

- MATLAB
- LabVIEW
- NI USB 6009 Data Acquisition Card
- Two Tank System (CE 105)

Category Software Hardware Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- LabVIEW programming files (Block diagram and Front Panel)
- Graphical representation of data collected for several cases of disturbance (leakage via valve at bottom). (Instructors should provide the data collection VI file)
- Controller performance and parameters for each case of disturbance.

Overview:

A model of the tank system and the controller is shown in the figure, where Y(s) is the actual level of the tank and R(s) is the desired level. The effect of disturbance to the tank system is represented by the disturbance, $T_d(s)$. The $T_d(s)$ is the leakage that can be generated from the hand valve at the bottom of the tank system.

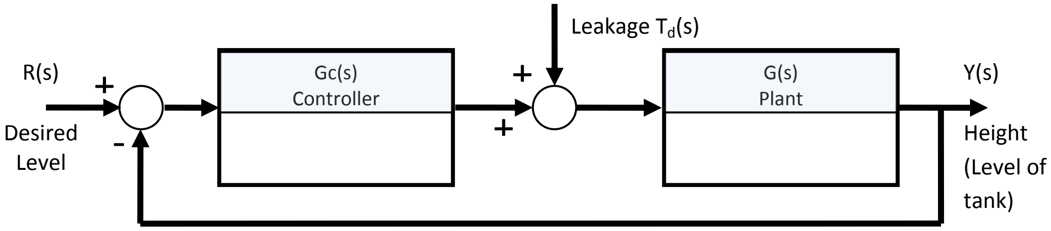


Figure: A block diagram model of a two tank level control system

NOTE: Instructors should manage to complete the hardware/software setup for the students to take reading and implement the controller.

Exercise:

a) Launch the data collection LabVIEW file and make the proper connections to the Two Tank System. This experiment uses only one tank of the system.

b) The disturbance to the tank is the hand-valve in the bottom of the first tank.

c) Collect the data for the FIVE cases of leakage valve (valve at bottom).

d) From the theory and understanding from last experiment (Exp 7), identify and prepare the transfer function and respective controllers for the specifications discussed during last experiment.

e) Generate the system response; for K= 10, 20, 50, 100; due to the five cases of leakage disturbance - d(t)

f) For each case find the percentage overshoot(%O.S.), rise time, settling time, steady state of y(t)

g) Compare the results of all the cases

h) Investigate the effect of changing the controller gain on the influence of the disturbance on the system output

CISE 302
Linear Control Systems

Lab Experiment 11: Introduction to PID controller

Objective: Study the three term (PID) controller and its effects on the feedback loop response. Investigate the characteristics of the each of proportional (P), the integral (I), and the derivative (D) controls, and how to use them to obtain a desired response.

List of Equipment/Software

Following equipment/software is required:

- MATLAB
- LabVIEW

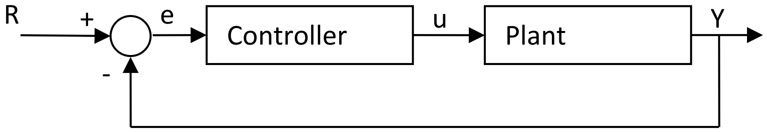
Category Soft - Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- LabVIEW programming files (Block diagram and Front Panel)
- Controller design and parameters for each of the given exercises.

Introduction: Consider the following unity feedback system:



Plant: A system to be controlled.

Controller: Provides excitation for the plant; Designed to control the overall system behavior.

The three-term controller: The transfer function of the PID controller looks like the following:

$$K_P + \frac{K_I}{s} + K_D s = \frac{K_D s^2 + K_P s + K_I}{s}$$

KP = Proportional gain

KI = Integral gain

KD = Derivative gain

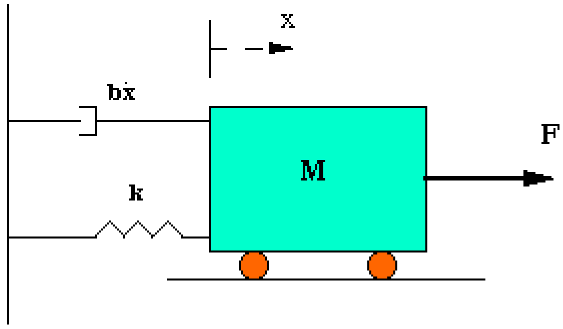
First, let's take a look at how the PID controller works in a closed-loop system using the schematic shown above. The variable (e) represents the tracking error, the difference between the desired input value (R) and the actual output (Y). This error signal (e) will be sent to the PID controller, and the controller computes both the derivative and the integral of this error signal. The signal (u) just past the controller is now equal to the proportional gain (KP) times the magnitude of the error plus the integral gain (KI) times the integral of the error plus the derivative gain (KD) times the derivative of the error.

$$u = K_P e(t) + K_I \int e(t)dt + K_D \frac{de(t)}{dt}$$

This signal (u) will be sent to the plant, and the new output (Y) will be obtained. This new output (Y) will be sent back to the sensor again to find the new error signal (e). The controller takes this new error signal and computes its derivatives and its internal again. The process goes on and on.

Example Problem:

Suppose we have a simple mass, spring, and damper problem.



The modeling equation of this system is

$$M\ddot{x} + b\dot{x} + kx = F$$

Taking the Laplace transform of the modeling equation (1), we get

$$Ms^2X(s) + bsX(s) + kX(s) = F(s)$$

The transfer function between the displacement X(s) and the input F(s) then becomes

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + bs + k}$$

Let

- M = 1kg
- b = 10 N.s/m
- k = 20 N/m
- F(s) = 1

Plug these values into the above transfer function

$$\frac{X(s)}{F(s)} = \frac{1}{s^2 + 10s + 20}$$

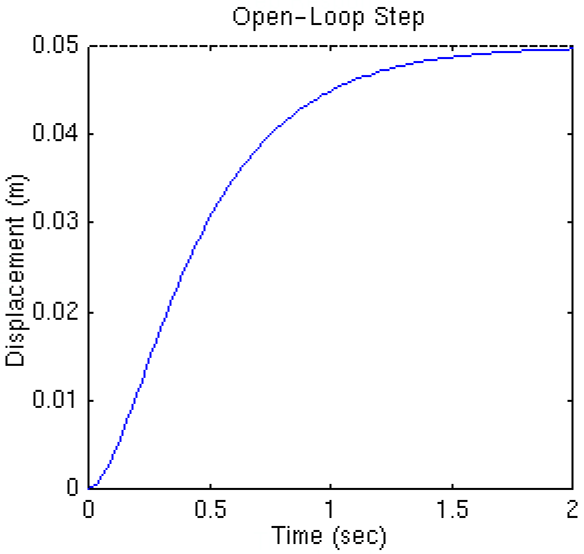
The goal of this problem is to show you how each of K_p, K_i and K_d contributes to obtain

- Fast rise time
- Minimum overshoot
- No steady-state error

Open-loop step response: Let's first view the open-loop step response.

```
num=1;
den=[1 10 20];
plant=tf(num,den);
step(plant)
```

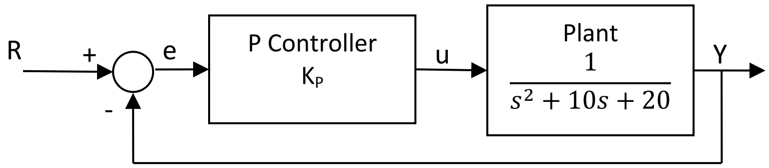
MATLAB command window should give you the plot shown below.



The DC gain of the plant transfer function is 1/20, so 0.05 is the final value of the output to a unit step input. This corresponds to the steady-state error of 0.95, quite large indeed. Furthermore, the rise time is about one second, and the settling time is about 1.5 seconds. Let's design a controller that will reduce the rise time, reduce the settling time, and eliminates the steady-state error.

Proportional control:

The closed-loop transfer function of the above system with a proportional controller is:

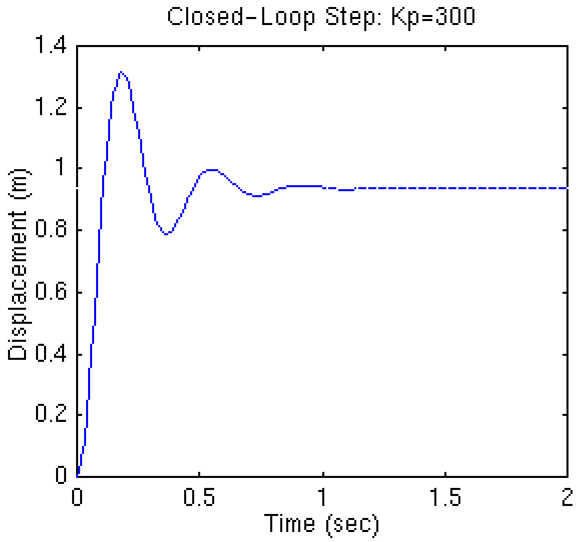


$$\frac{X(s)}{F(s)} = \frac{K_P}{s^2 + 10s + (20 + K_P)}$$

Let the proportional gain (K_P) equal 300:

```
Kp=300;
contr=Kp;
sys_cl=feedback(contr*plant,1);
t=0:0.01:2;
step(sys_cl,t)
```

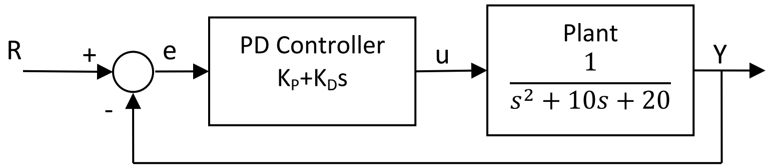
MATLAB command window should give you the following plot.



Note: The MATLAB function called feedback was used to obtain a closed-loop transfer function directly from the open-loop transfer function (instead of computing closed-loop transfer function by hand). The above plot shows that the proportional controller reduced both the rise time and the steady-state error, increased the overshoot, and decreased the settling time by small amount.

Proportional-Derivative control:

The closed-loop transfer function of the given system with a PD controller is:

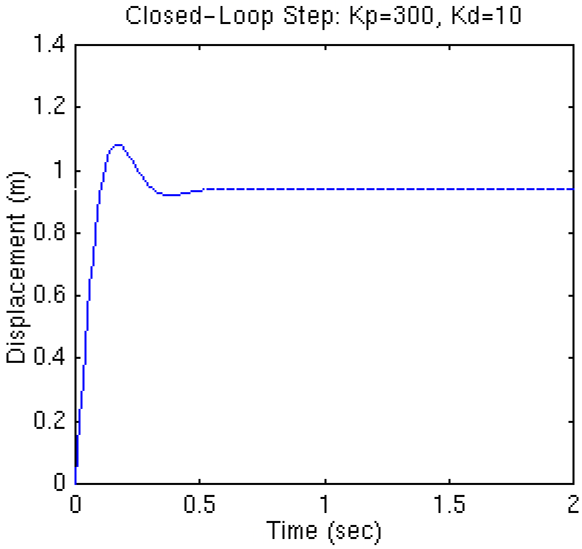


$$\frac{X(s)}{F(s)} = \frac{K_D s + K_P}{s^2 + (10 + K_D)s + (20 + K_P)}$$

Let K_P equal 300 as before and let K_D equal 10.

```
Kp=300;
Kd=10;
contr=tf([Kd Kp],1);
sys_cl=feedback(contr*plant,1);
t=0:0.01:2;
step(sys_cl,t)
```

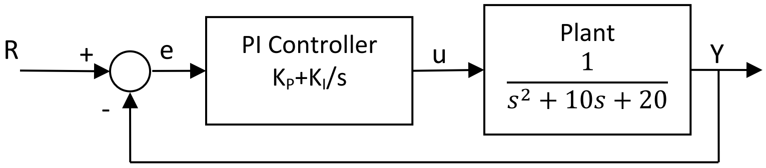
MATLAB command window should give you the following plot.



This plot shows that the derivative controller reduced both the overshoot and the settling time, and had a small effect on the rise time and the steady-state error.

Proportional-Integral control:

Before going into a PID control, let's take a look at a PI control. For the given system, the closed-loop transfer function with a PI control is:

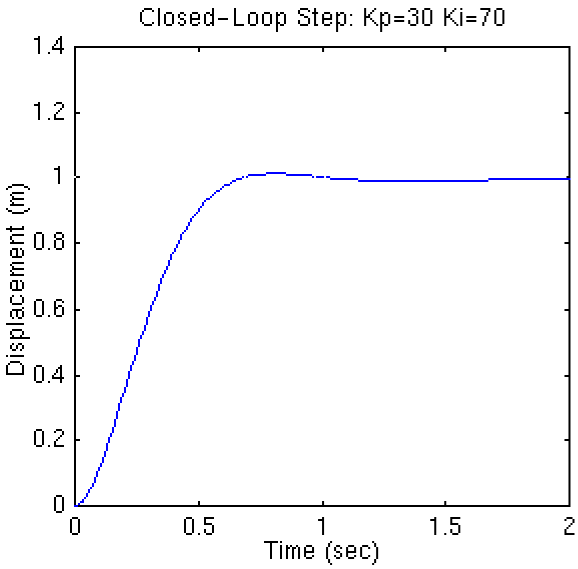


$$\frac{X(s)}{F(s)} = \frac{K_P s + K_I}{s^3 + 10s^2 + (20 + K_P)s + K_I}$$

Let's reduce the K_P to 30, and let K_I equal 70.

```
Kp=30;
Ki=70;
contr=tf([Kp Ki],[1 0]);
sys_cl=feedback(contr*plant,1);
t=0:0.01:2;
step(sys_cl,t)
```

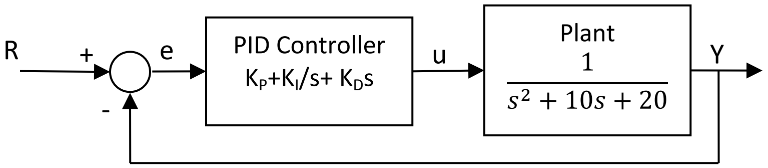
MATLAB command window gives the following plot.



We have reduced the proportional gain (Kp) because the integral controller also reduces the rise time and increases the overshoot as the proportional controller does (double effect). The above response shows that the integral controller eliminated the steady-state error.

Proportional-Integral-Derivative control:

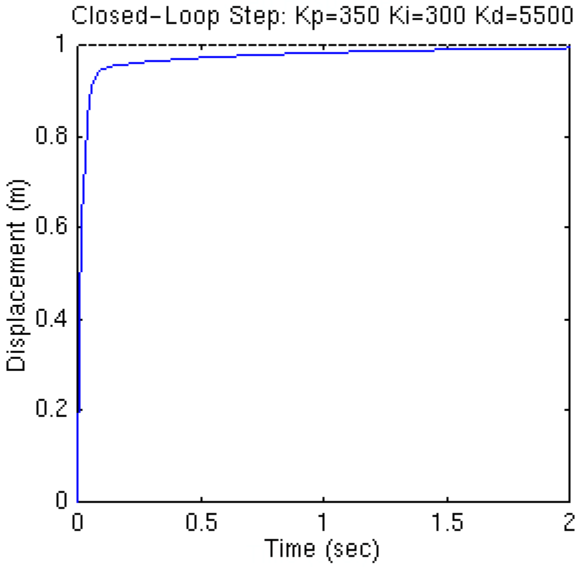
Now, let's take a look at a PID controller. The closed-loop transfer function of the given system with a PID controller is:



$$\frac{X(s)}{F(s)} = \frac{K_D s^2 + K_P s + K_I}{s^3 + (10 + K_D)s^2 + (20 + K_P)s + K_I}$$

After several trial and error runs, the gains Kp=350, Ki=300, and Kd=50 provided the desired response. To confirm, enter the following commands to an m-file and run it in the command window. You should get the following step response.

```
Kp=350;
Ki=300;
Kd=50;
contr=tf([Kd Kp Ki],[1 0]);
sys_cl=feedback(contr*plant,1);
t=0:0.01:2;
step(sys_cl,t)
```



Now, we have obtained a closed-loop system with no overshoot, fast rise time, and no steady-state error.

The characteristics of P, I, and D controllers:

The proportional controller (K_P) will have the effect of reducing the rise time and will reduce, but never eliminate, the steady state error. An integral controller (K_I) will have the effect of eliminating the steady state error, but it may make the transient response worse. A derivative control (K_D) will have the effect of increasing the stability of the system, reducing the overshoot and improving the transient response.

Effect of each controller K_P, K_I and K_D on the closed-loop system are summarized below

CL Response	Rise Time	Overshoot	Settling Time	S-S Error
K_P	Decrease	Increase	Small Change	Decrease
K_I	Decrease	Increase	Increases	Eliminate
K_D	Small Change	Decreases	Decreases	Small Change

Note that these corrections may not be accurate, because K_P, K_I, and K_D are dependent of each other. In fact, changing one of these variables can change the effect of the other two. For this reason the table should only be used as a reference when you are determining the values for K_P, K_I, and K_D.

Exersice:

Consider a process given below to be controlled by a PID controller,

$$G_p(s) = \frac{400}{s(s+48.5)}$$

a) Obtain the unit step response of Gp(s).
b) Try PI controllers with (Kp=2, 10, 100), and Ki=Kp/10. Investigate the unit step response in each case, compare the results and comment.
c) Let Kp=100, Ki=10, and add a derivative term with (Kd=0.1, 0.9, 2). Investigate the unit step response in each case, compare the results and comment.

Based on your results in parts b) and c) above what do you conclude as a suitable PID controller for this process and give your justification.

CISE 302
Linear Control Systems

Lab Experiment 12: Open Loop and Closed Loop position control of DC Motor

Objective: To familiarize the servo motor system and experience the open and closed loop control of servo system to be used in an automatic position control system.

List of Equipment/Software

Following equipment/software is required:

- LabVIEW
- DC Servo System (feedback equipment)

a.	OU150A	Op Amp Unit
b.	AU150B	Attenuator Unit
c.	PA150C	Pre-Amplifier Unit
d.	SA150D	Servo Amplifier
e.	PS150E	Power Supply
f.	DCM150F	DC Motor
g.	IP150H	Input Potentiometer
h.	OP150K	Output Potentiometer
i.	GT150X	Reduction Gear Tacho
j.	DC Voltmeter	

Category Software-Hardware Experiment

Deliverables

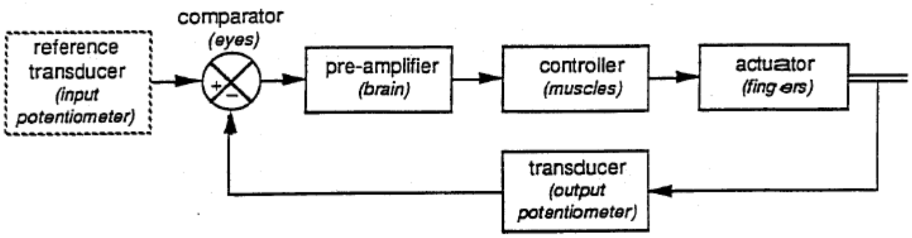
A complete lab report including the following:

- Summarized learning outcomes.
- Connection diagram of the hardware experimental part.
- Report the procedure and troubleshooting during the experiment.
- Show results for the open loop and closed loop position control via graphs and tables.
- Report the LabVIEW program components i.e., Front Panel and Block Diagram.

Procedure (Open loop position control):

1. Attach the AU150B, SA150D and DCM150F to the baseplate by means of the magnetic fixings.
2. Fix the plugs from the servo amplifier into the power supply.
3. Fix the plug from the motor unit into the servo amplifier.
4. Attach the GT150X to the baseplate by means of the magnetic fixing and position it so that it is connected to the motor shaft by means of the flexible coupling.
5. Set up for open loop as shown in Fig. 1.
6. Use a push-on coupling to link a low-speed shaft of the GT150X to the output potentiometer shaft.
7. Starting with AU150B the potentiometer knob at the fully counter-clockwise position gradually turn it till the motor just rotates and record:
8. Scale position at which the motor just rotates: ....(1)
9. Direction in which the output rotary potentiometer moves: .... (2)
10. Return the output rotary potentiometer cursor to zero by turning the GT150X high-speed shaft.
11. Decide on a position in the direction (2), you wish the potentiometer shaft to turn to and then turn the AU150B potentiometer knob to position (1). As the cursor nears the required angle, reduce this input signal so that the cursor comes to rest nearly at the required point.

The open loop system will have you as a feedback. Such a system could be shown as in the figure.



For this system connect the setup as shown below

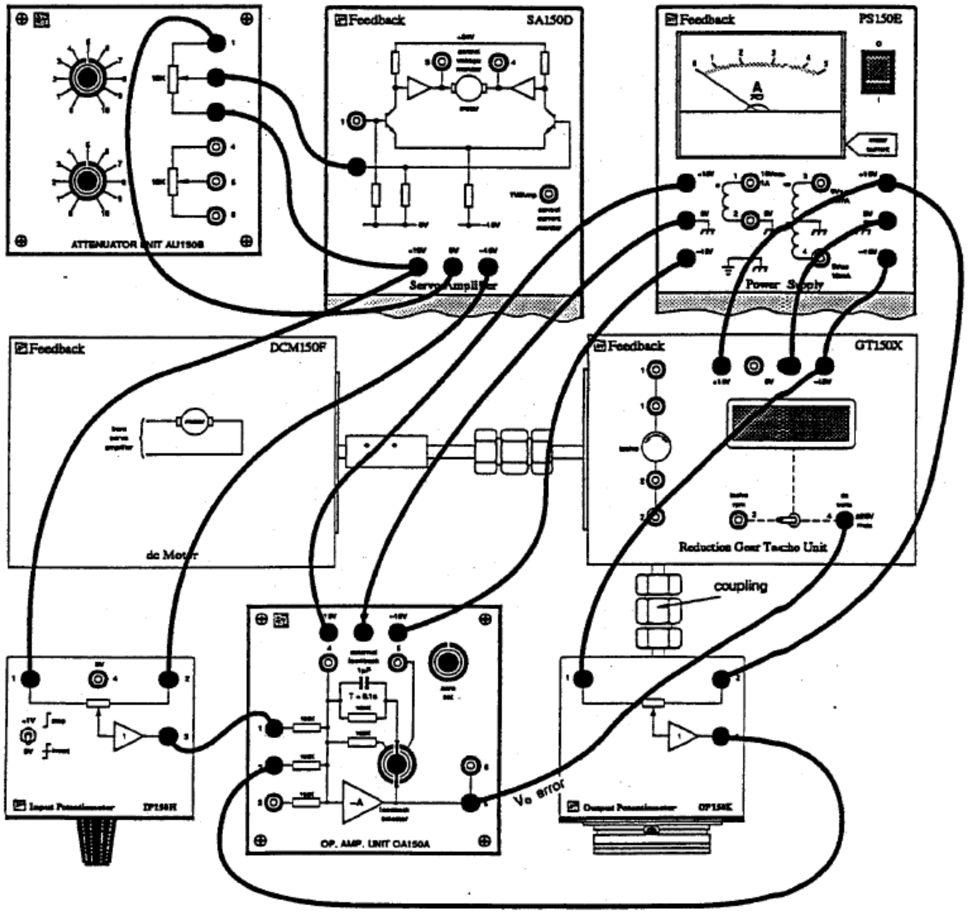


Figure 1: Open loop System (with you as feedback)

Procedure (Closed loop position control):

1. Set the apparatus as shown in Fig.2 for closed loop control.
2. This shall utilize the error signal output VO of the operational amplifier to drive the output potentiometer via the pre-amplifier and motor.
3. The upper potentiometer on the AU150B can now be used as a gain control and should initially be set to zero before switching on the power.
4. Adjust the PA150C `zero set' so that the motor does not rotate.
5. Now set the IP150H to some arbitrary angle and increase the gain control setting.
6. The output potentiometer cursor should rotate to an angle nearly equal to that of the input potentiometer cursor.

Trouble-shooting:

- **Output potentiometer (or the motor) is oscillating:**

  Make sure that the upper potentiometer on the AU150B is all down to zero (i.e. the gain is equal to zero) and then rotate the zero set knob on pre-amplifier PA150C so that the motor stop rotating or oscillating.

  After increasing the gain (i.e. make the upper potentiometer on the AU150B other than zero) the motor behaves in the same way then change the order of connection from pre-amplifier to servo-amplifier (i.e. if the output “3” and “4” of pre-amplifier PA150C is connected to input “1” and “2” of the servo-amplifier SA150D respectively then change it such that the output “3” and “4” of pre-amplifier PA150C is connected to input “2” and “1” of the servo-amplifier SA150D respectively or vice-versa)

- **Output potentiometer is not following input potentiometer:** in such cases there is misalignment. Please hold the outer disc of input potentiometer IP150H firmly and rotate its knob making sure that the disc does not rotate. Doing this make sure that the input potentiometer “0” angle matches with the output potentiometer “0” angle.

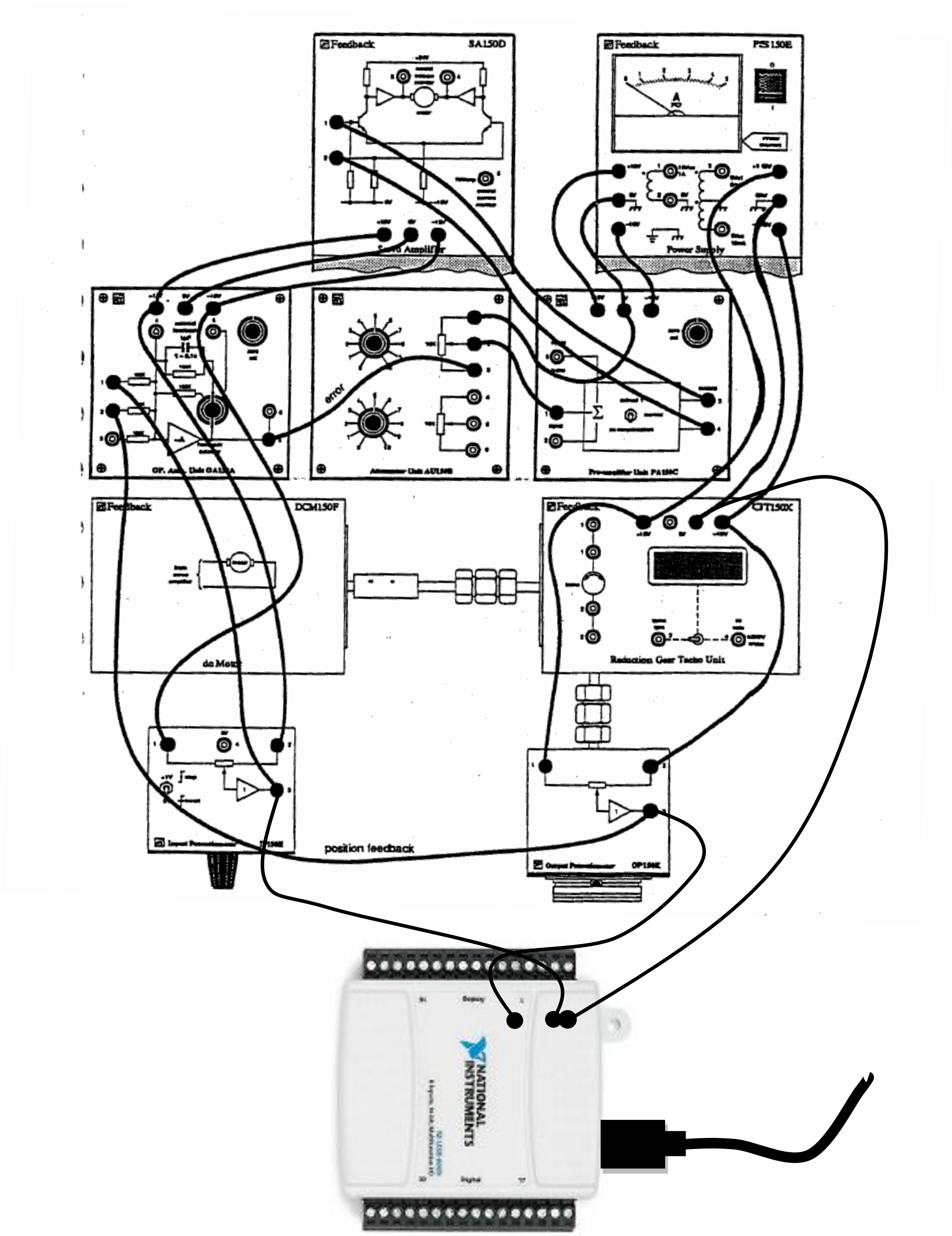
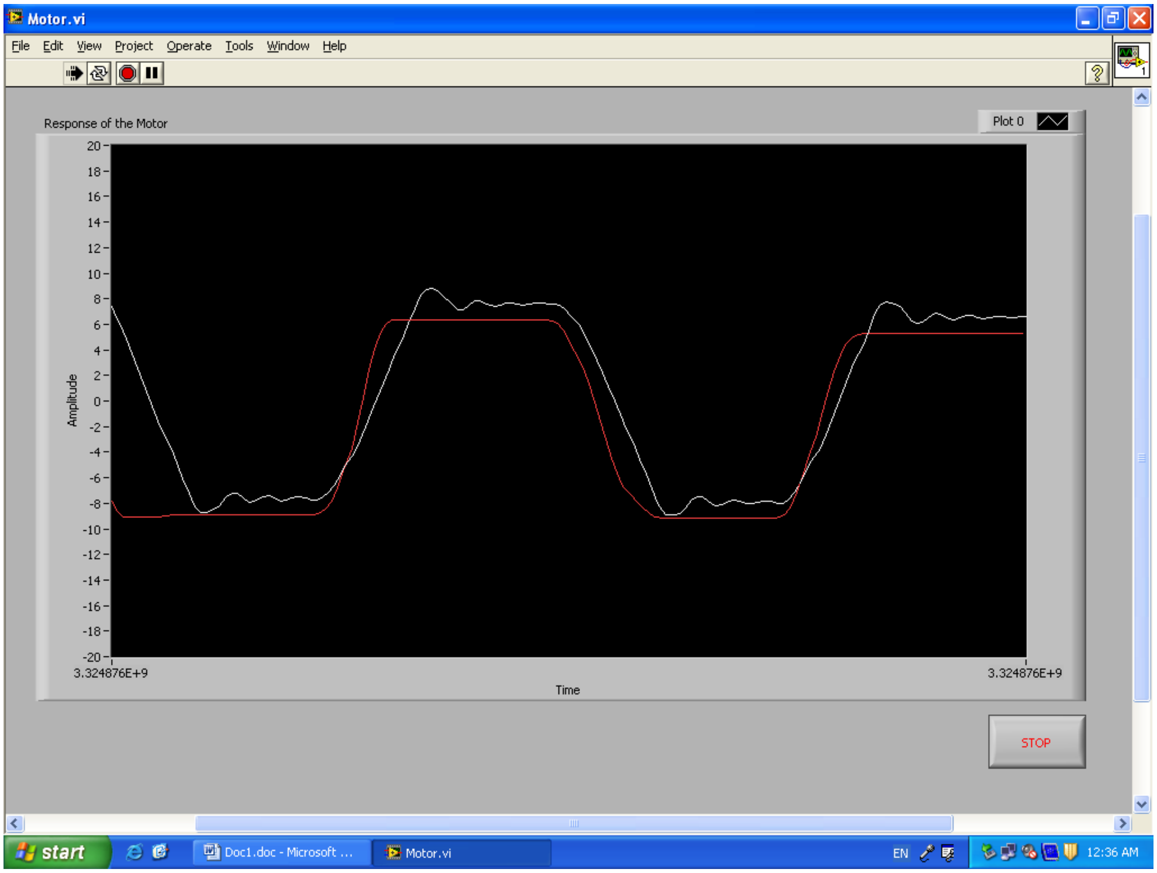
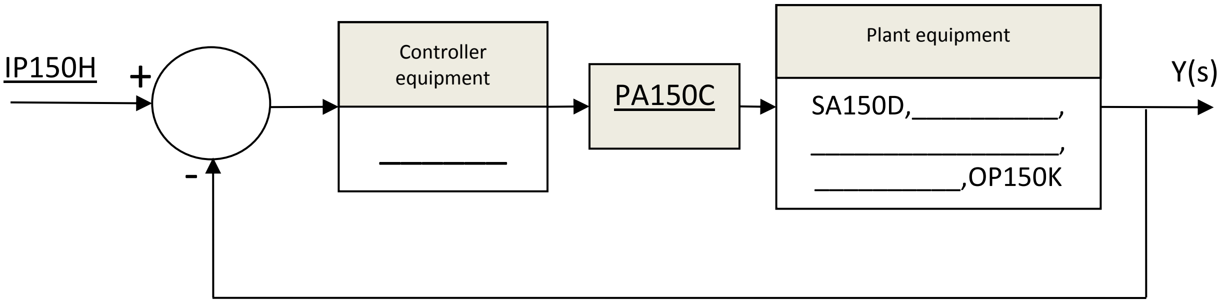


Figure 2: Closed Loop Position control of DC Motor.

Double-click the icon which says "motor" (Motor.vi) on the desktop to start the program to capture the signal. And click the "run" button which is the first button on the toolbox below the file menu. The program should be as shown below.



Identify from the setup the equipment which should be place in the following block diagram



CISE 302
Linear Control Systems

Lab Experiment 13: Simple Speed Control of DC Motor

Objective: Observe how the Simple Speed control system is constructed and appreciate the importance of Tacho-generator in closed-loop speed control system.

List of Equipment/Software

Following equipment/software is required:

- LabVIEW
- DC Servo System (feedback equipment)

a.	OU150A	Op Amp Unit
b.	AU150B	Attenuator Unit
c.	PA150C	Pre-Amplifier Unit
d.	SA150D	Servo Amplifier
e.	PS150E	Power Supply
f.	DCM150F	DC Motor
g.	IP150H	Input Potentiometer
h.	OP150K	Output Potentiometer
i.	GT150X	Reduction Gear Tacho
j.	DC Voltmeter	

Category Software-Hardware Experiment

Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- Connection diagram of the hardware experimental part.
- Report the procedure and troubleshooting during the experiment.
- Show results for the open loop and closed loop speed control via graphs and tables.

Introduction:

In the last experiments we saw how simple position control could be constructed. In today's assignment we shall see how simple speed control of motor could be done. In the experiment involving the DC Motor Characteristics we saw how the signal inputs into SA150D could vary the speed of the motor. This means that without any load you can set the motor to run at specified speed determining the value of the input signal. What kind of speed control was it?

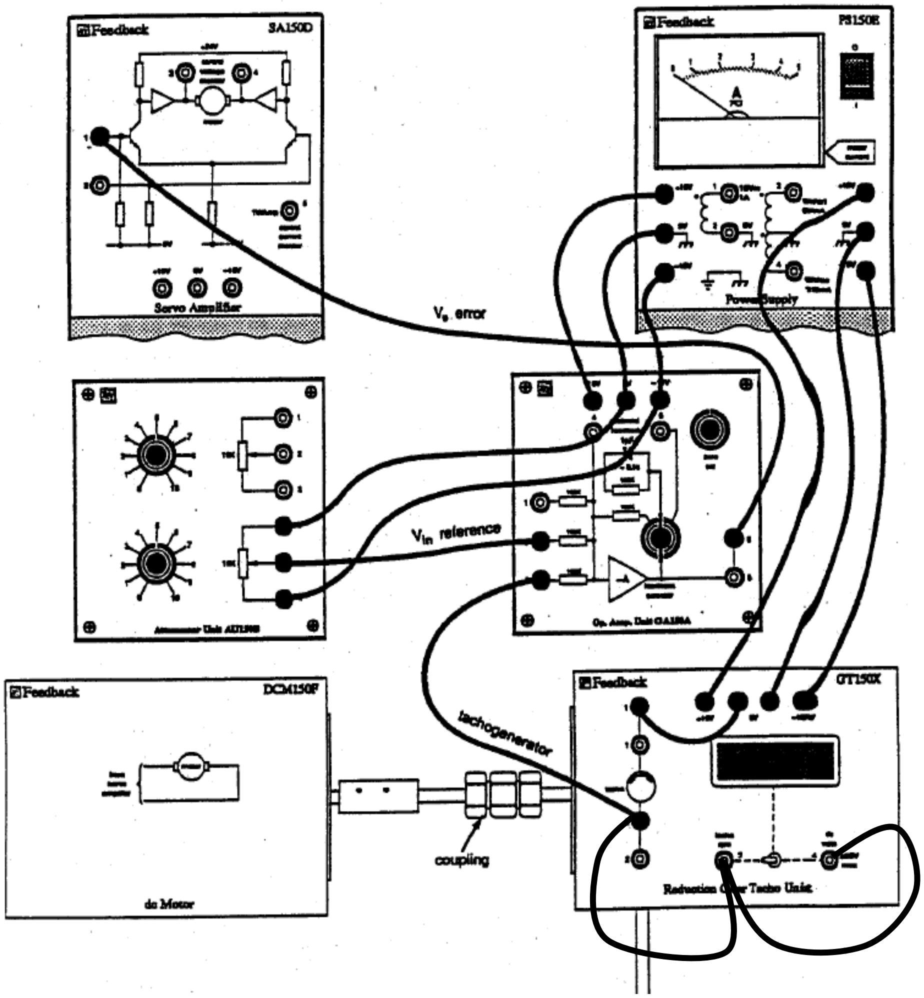
Now if we look at the torque/speed characteristics in the experiment, we can say that if load is placed on the motor the speed of the motor will change to some extent. With open-loop system the results show that there can be a reasonable speed control when operating without or with a fixed load but the system would be very unsuitable where the load was varying.

With closed load, we will show improvement in speed control with respect to varying load. That is, the actual speed will be compared to the required speed. This produces an error signal to actuate the servo amplifier output so that the motor maintains a more constant speed.

Exercise 1: Simple feedback speed-control without load.

In this exercise we will simply feedback a signal proportional to the speed, using the Tacho-generator. We then compare it with a reference signal of opposite polarity, so that the sum will produce an input signal into the servo amplifier of the required value. As comparator, we will use an operational amplifier.

On the OA150A set the 'feedback selector' to 100KΩ resistor



Before connecting the Tacho-generator to an input of the OA150A, increase the 'reference' voltage so that the motor revolves and on your voltmeter determine which the Tacho’s positive output is. The correct side can then be connected to OA150A input and the other side to 0V.

Reset the reference voltage to zero and then gradually increase it so that you can take readings over the motor speed range of upto approximately 2000 r/min for the reference, tacho-generator and error voltages.

Record your results in the following table

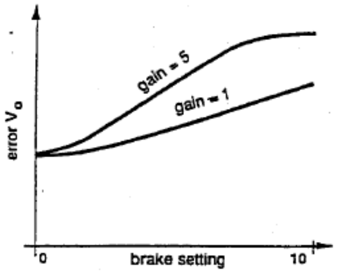
Speed	Reference Voltage	Tacho-generator voltage	Error Voltage
200			
400			
600			
800			
1000			
1200			
1400			
1600			
1800			
2000			

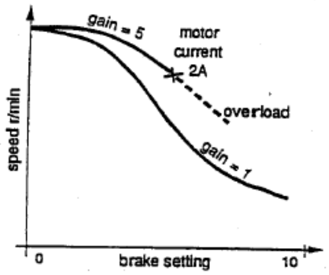
Plot the error voltage against speed.

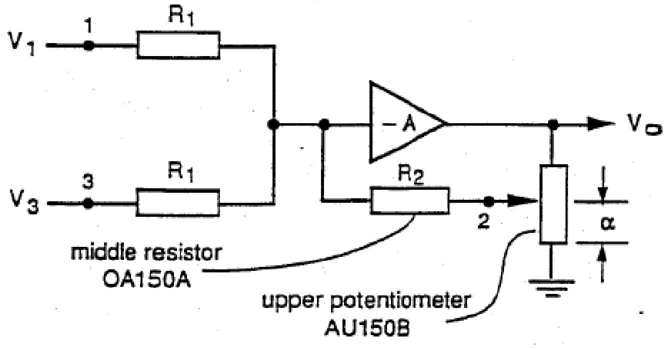
Exercise 2: Effect of Load on Speed

To find the effect of the load on speed we can user the magnetic brake as a load. The change in speed for a change in load will give us the regulation. Ensure that the eddy current brake disc is fitted to the motor. Also ensure that the load unit can be fully engaged without fouling either the motor mount or the eddy current disc.

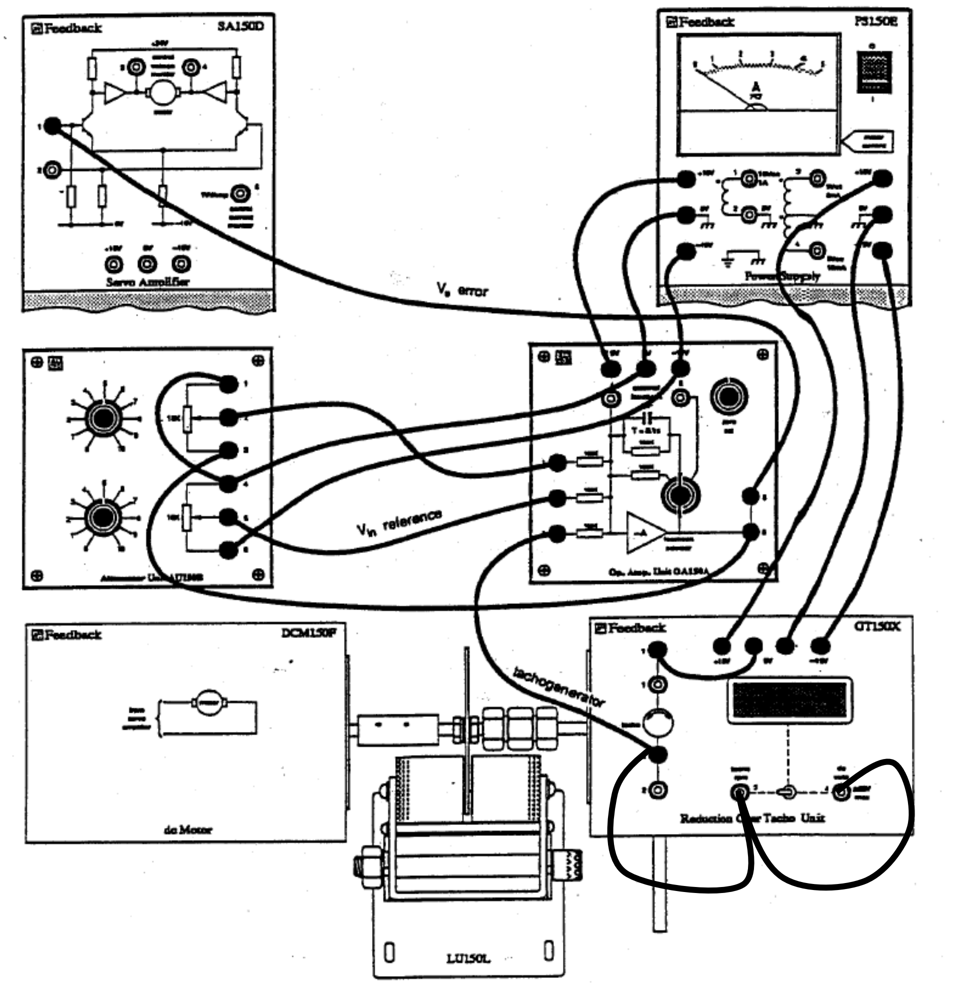
The exercise is concerned to show how an increase in the forward path gain will cause a given fall in speed to cause a larger increase in the value of the error V_0, so that for any change in load the speed drop or 'droop' will decrease with increase gain as shown in the figure.







For a gain control we can use the circuit given above, which has a gain of -1/α.



On the OA150A set the 'feedback selector' switch to 'external feedback'. On the LU150L swing the magnets clear. Initially set the gain to unity, that is to position 10 of the upper potentiometer and adjust the reference volts till the motor runs at 1000 r/min. Then take readings of the reference voltage, V_{in}, Error voltage, V_e and the Tacho-generator voltage, using the voltmeter, over the range of brake positions 0 – 10 and then tabulate your results in the following table. **Be careful that you do not exceed the 2A limiting current.** Repeat the readings for a gain of 5, which is to set the gain potentiometer to position 1. Re-adjust the reference potentiometer to give no-load motor speed of 1000 r/min.

For gain of 1

Brake Position	Reference (V_{in}) volts	Tacho-generator volts	Error (V_e) volts	Speed r/min
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

For gain of 5

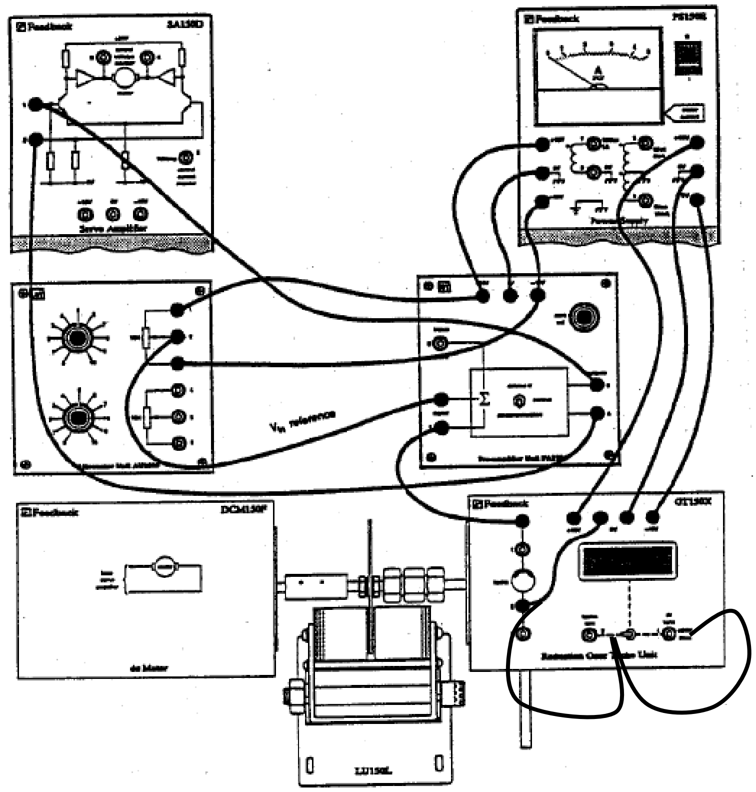
Brake Position	Reference (V_{in}) volts	Tacho-generator volts	Error (V_e) volts	Speed r/min
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Plot your results in the form of graphs of error voltage against brake setting and speed for gain values of 1 and 5.

Exercise 3: Reversible speed control

In the last part of the experiment we will assemble a simple reversible speed control system. From your reading you have seen that a high gain decreases the minimum reference signal needed for the motor to respond so this exercise we will use high gain.

The inputs into the SA150D can drive the motor in opposite directions but both inputs require positive voltages. As the output of the OA 150A varies from positive to negative it is necessary to use the PA150C pre-amplifier unit that is so designed that a negative input gives a positive voltage on output and a negative input gives a positive voltage on the other output with a gain of about 25.



Replace the OA150A with PA150C. Setup as shown in the above figure, adjusting the reference to zero output before coupling to the pre-amplifier. Set the pre-amplifier to '**ac compensation**', this will reduce the effect of ripple on the tacho-generator signal, which causes instability.

Set the potentiometer on AU150B to 5.

With no load on the motor, now find that you can invert the sign of the reference signal so that you can reverse the direction of the motor rotation, by slowly turning the reference potentiometer knob to either side of the center position 5. Record the reference voltage that just causes the motor to rotate.

Minimum signal needed for motor response	
Forward	Reverse

Set the speed of rotation in one direction to 1000 r/min and then take readings over the brake position 0-10, and record them in the following table. To measure the error voltages place the voltmeter across both the PA150C outputs.

Then reverse direction and repeat the readings.

Practical Aspects:

So important has the tacho-generator been considered in the speed control, that it has very often been made an integral part of the motor.

Examples of speed control can be seen in every branch of industry and transport. They have become particularly important in continuous processes such as in the control of sheet-metal thickness in hot rolling mills, in generators and most industrial motors. In guidance systems, automatic pilots, lifts and overhead hoists both reverse speed and positional control may be used.

Forward				
Brake Position	Tacho-generator volts	Reference Voltage (V_{in})	Error Voltage (V_e)	Speed r/min
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Reverse				
Brake Position	Tacho-generator volts	Reference Voltage (V_{in})	Error Voltage (V_e)	Speed r/min
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

CISE 302
Linear Control Systems

Lab Experiment 14: PID Controller Design for Two Tank System

Objective: To familiarize the Two Tank System and experience the PID controller design to control the level of the tank system.

List of Equipment/Software

Following equipment/software is required:

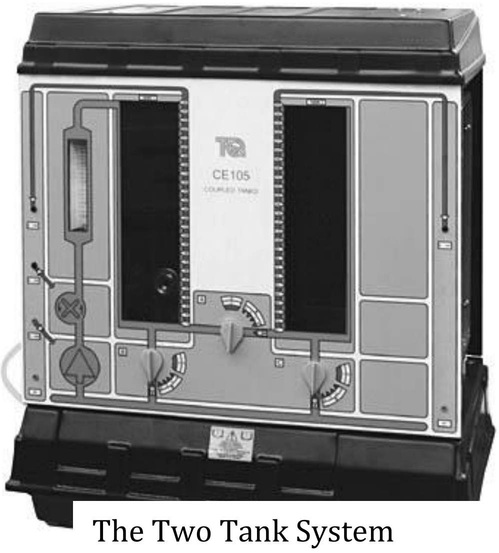
- LabVIEW
- NI USB 6009 Data Acquisition Card
- Two Tank System

Category Software-Hardware Experiment

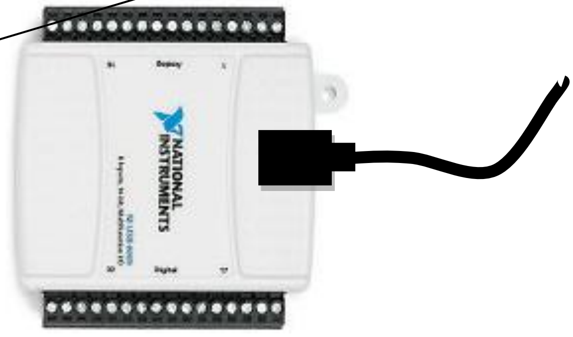
Deliverables

A complete lab report including the following:

- Summarized learning outcomes.
- Show the PID block diagram and the controller parameters with the process graphs.
- Report the LabVIEW program components i.e., Front Panel and Block Diagram.



The Two Tank System



The USB DAQ

Interfacing the Two tank

In this exercise, two-tank system is introduced. The two tank system is as shown in the figure. It has a water pump which takes in 0..10 voltages and pumps in water with speeds depending at the voltage supplied. Two outputs which are the speed of flow and the level of water in the tank are shown visually. There are speed and level sensors that provide voltages between 0 and 10 voltages to indicate speed and voltage. The yellow filled area shows the flow of water from pump to tank to reservoir through valves. The flow from tank to reservoir can be controlled using the value. At “0” indicator the valve is “fully closed” and at “5” it is “fully opened”.

This exercise will interface the tanks output to analog inputs to measure the tank level and speed of flow and use the analog voltage output of the USB 6009 to voltage input of the tank to run the pump motor. Use your knowledge of previous experiments to send a constant voltage out of the USB card and receive the two analog signals.

To conduct this experiment, we will have to first connect the 2-Tank system to LabVIEW through the NI DAQ card. The steps are as follows:

1. Connect the sensor of the tank system (top-most pin) to any Analog Input (AI) pin of the DAQ card.
2. Next connect the motor (2nd last/above ground) to an Analog Output (AO) pin.
3. Connect the ground of the tank (bottom most) pin to a ground of the DAQ.

Now the hardware wire connections are complete and we can start building the VI:

1. Use two sets of **“DAQ Assistant”** to capture the analog signal of level at the channel. And use one **“DAQ Assistant”** to send a signal from USB 6009 to the tank.
2. Use a **Knob** to select the voltage being sent from the USB 6009 to Tank. “**Knob**” can be found at **“Controls” >> “Num Ctrls” >> “Knob”**
3. Use the tank level indicator from **“Controls” >> “Numerical Ind” >> “Tank”** to display the output of tank level.
4. Use the “**Flat sequence**” in the block diagram from “**Functions**”>> **“Structures”** >> **“Flat Sequence”** to send the analog out signal first from computer to two tank and then read the two analog inputs signals from two tank to computer.
5. Add frames on the flat sequence by right clicking on the border of the flat sequence and selecting “Add frame after” from the menu.

Note: Connect all the wiring and use a **while** loop and **stop** button to run the VI.

Part – I: Design of Proportional Control in the PID Controller

Proportional Controller (part of the PID controller) is a common feedback loop component in industrial control systems. The controller takes a measured value from a process or other apparatus and compares it with a reference setpoint value. The difference (or "error" signal) is then used to adjust some input to the process in order to bring the process' measured value back to its desired setpoint. Basically, when the controller reads a sensor, it subtracts this measurement from the "setpoint" to determine the "error". It then uses the error to calculate a correction to the process's input variable (the "action") so that this correction will remove the error from the process's output measurement.

It is used mainly to handle the immediate error, which is multiplied by a constant *P* (for "proportional"), and added to the controlled quantity. *P* is only valid in the band over which a controller's output is proportional to the error of the system. This is known as the Propotional Band, often abbreviated as P_b. A controller setting of 100% proportional band means that a 100% change of the error signal (setpoint – process variable) will result in 100% change of the output, which is a gain of 1.0. A 20% proportional band indicates that 20% change in error gives a 100% output change, which is a gain of 5.

$$P_b = 100/\text{gain} \quad \text{OR} \quad K_p = \frac{1}{P_b}$$

With proportional band, the controller output is proportional to the error or a change in measurement (depending on the controller). So,

(controller output) = (error)*100/(proportional band)

This theory will be implemented on the 2-Tank system in this experiment. The controller will be designed in a VI while the hardware connections remain the same- as shown below:

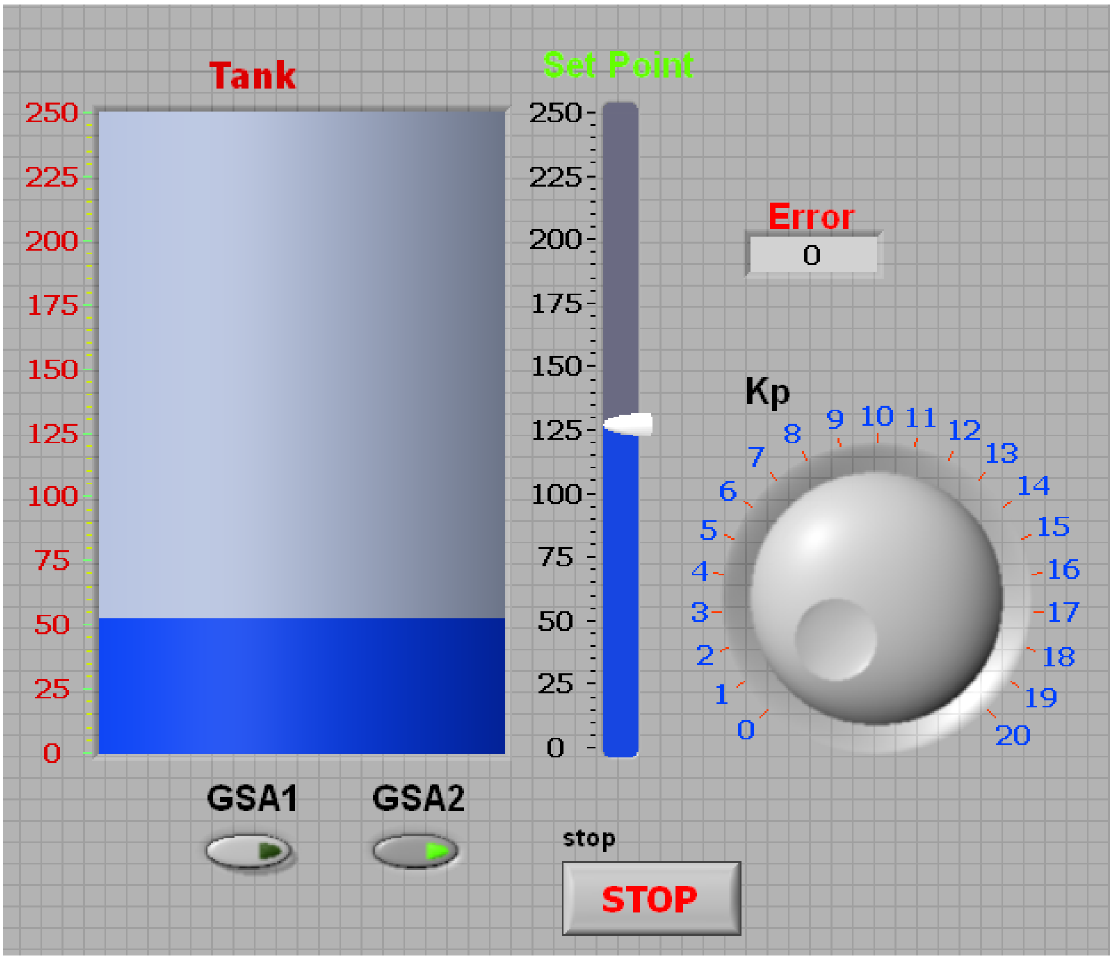
1. Connect the sensor of the tank system (top-most pin) to any Analog Input (AI) pin of the DAQ card.
2. Connect the +5 In and GND In pins of the Amplifier to the 5V and ground terminals of the Power supply.
3. Connect the negative input of the amplifier you are using, A- In or B- In, to ground and connect the positive inputs, A+ In or B+ In, to an Analog Output (AO) pin of the DAQ.
4. Next connect the motor (2nd last/above ground) to the Amplifier Output i.e. A Out or B Out.
5. Connect the ground of the Amplifier to the ground of the DAQ.
6. Connect the ground of the tank (bottom most) pin to a ground of the DAQ.

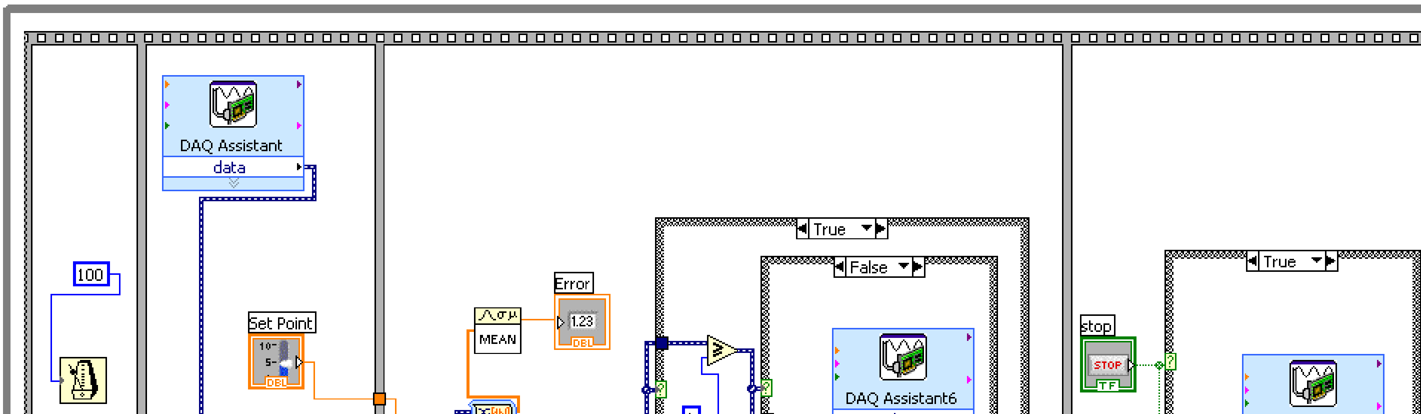
Note: Make sure that ALL devices are connected to a common ground.

The VI will be build as follows:

1. Click “New VI” button to create a new blank LabVIEW program.
2. In the Block diagram, go to **“Functions” >> Programming>> “Structures” >> “Flat Sequence”** and place it in the window. Next add 4 frames to it by right clicking on the border of the frame and selecting Add Frame After (or Before).
3. Keep the 1st two frames and the last frame the same as they were for On-Off Control viz.
   - The 1st frame to set the sampling time to 100ms- using **Wait Until Next ms Multiple** and **Numeric Constant**.
   - The 2nd frame to receive the sensor signal from the Tank, scale it properly and display it on the front panel in a graph as well as tank format- using **DAQ Assistant, Waveform Chart, Tank** and other numeric icons.
   - The last frame to manually terminate the execution of the program through a stop button on the front panel and make sure the motor is turned off at the end- using **DAQ Assistant**, **Case Structure** and **Numeric Constant.** (The entire Flat Sequence must be included in the while loop and the Stop button terminal must be connected to the stop button of the while loop)
4. Calculate the Error by subtracting (**“Functions” >> Programming>> “Numeric” >> “Subtract”**) the sensor value or level from the desired set point. The set point can be given in the form of a Numeric Constant in the Block diagram or through Vertical pointer slides, Numeric controls, etc. on the Front panel. This can be done in the 2nd or 3rd frame.

5. On the front panel, add a Control Knob from the Numeric palette. This will be used to control the Proportional gain $\boldsymbol{K_p}$. In the 3rd frame of the Block diagram sequence, multiply the error with the gain- connect the error and gain terminal to a multiplication block.
6. In the same frame check the above product (input to controller) and if it is greater than 1 send one to the Tank system- using DAQ. If it is lesser than 0 send the tank 0. If it is between 0 and 1, send the control input as it is. The comparison can be done using “Greater Or Equal?” and “Lesser Or Equal?” functions along with a Case Structure having another Case Structure inside (as in the On-Off Control). Here the control input is connected to the Case Selector.
7. After all the wiring is complete switch to Front Panel and press the RUN button to execute the VI.





Part – II: Design of Integral Part in the PID Controller

The next step in PID control is the inclusion of the **Integral** component – It is needed to learn from the past. The error is integrated (added up) over a period of time, and then multiplied by a constant K_i (making an average), and added to the controlled quantity. A simple proportional system either oscillates, moving back and forth around the setpoint because there's nothing to remove the error when it overshoots, or oscillates and/or stabilizes at a too low or too high value. By adding a proportion of the average error to the process input, the average difference between the process output and the setpoint is continually reduced. Therefore, eventually, a well-tuned PID loop's process output will settle down at the setpoint. As an example, a system that has a tendency for a lower value (heater in a cold environment), a simple proportional system would oscillate and/or stabilize at a too low value because when zero error is reached P is also zero thereby halting the system until it again is too low. Larger K_i implies steady state errors are eliminated quicker. The tradeoff is larger overshoot: any negative error integrated during transient response must be integrated away by positive error before we reach steady state. The integral component is always used with the proportional one and is so referred to as **PI controller**.

This theory will be implemented on the 2-Tank system in this experiment. The controller will be designed in a VI while the hardware connections remain the same- as shown below:

1. Connect the sensor of the tank system (top-most pin) to any Analog Input (AI) pin of the DAQ card.
2. Connect the +5 In and GND In pins of the Amplifier to the 5V and ground terminals of the Power supply.
3. Connect the negative input of the amplifier you are using, A- In or B- In, to ground and connect the positive inputs, A+ In or B+ In, to an Analog Output (AO) pin of the DAQ.
4. Next connect the motor (2nd last/above ground) to the Amplifier Output i.e. A Out or B Out.
5. Connect the ground of the Amplifier to the ground of the DAQ.
6. Connect the ground of the tank (bottom most) pin to a ground of the DAQ.

Note: Make sure that ALL devices are connected to a common ground.

The VI will be build as follows:

1. Click “New VI” button to create a new blank LabVIEW program.
2. In the Block diagram, go to **“Functions” >> Programming>> “Structures” >> “Flat Sequence”** and place it in the window. Next add 4 frames to it by right clicking on the border of the frame and selecting Add Frame After (or Before).
3. Keep the 1st two frames and the last frame the same as they were for On-Off Control viz.
1. The 1st frame to set the sampling time to 100ms- using **Wait Until Next ms Multiple** and **Numeric Constant**.
2. The 2nd frame to receive the sensor signal from the Tank, scale it properly and display it on the front panel in a graph as well as tank format- using **DAQ Assistant, Waveform Chart, Tank** and other numeric icons.

3. The last frame to manually terminate the execution of the program through a stop button on the front panel and make sure the motor is turned off at the end- using **DAQ Assistant**, **Case Structure** and **Numeric Constant.**
4. (The entire Flat Sequence must be included in the while loop and the Stop button terminal must be connected to the stop button of the while loop)
5. Calculate the Error by subtracting (**“Functions” >> Programming>> “Numeric” >> “Subtract”**) the sensor value or level from the desired set point. The set point can be given in the form of a Numeric Constant in the Block diagram or through Vertical pointer slides, Numeric controls, etc. on the Front panel. This can be done in the 2nd or 3rd frame.
6. On the front panel, add 2 Control Knobs from the Numeric palette. This will be used to control the Proportional gain $\boldsymbol{K_p}$ and the Integral Gain $\boldsymbol{K_I}$.
7. In the 3rd frame of the Block diagram sequence, check if the error is less than zero. If it is, then send the tank 0.If not go to the next step. The comparison can be made using the Case Structure and the “Lesser Or Equal?” function.
8. Multiply the error with the gain by connecting the error and gain terminal to a multiplication block. Also, integrate the error by sending it to the Integral block (**“Functions” >> Mathematics>> Integ & diff >>Time Domain Math-** select **Integral** in this block) and then multiply the integrated error with the Integral gain as was done with the Proportional gain. Next, add the 2 products together (use compound arithmetic or 2 add functions). Send the sum to the Tank through the DAQ Assistant.
9. In the same frame check the above product (input to controller) and if it is greater than 1 send one to the Tank system- using DAQ. If it is lesser than 0 send the tank 0. If it is between 0 and 1, send the control input as it is. The comparison can be done using “Greater Or Equal?” and “Lesser Or Equal?” functions along with a Case Structure having another Case Structure inside (as in the On-Off Control). Here the control input is connected to the Case Selector.
10. In the 2-Tank system open the exit valve a little to see the proper effect of the PI Control. The valve can be opened to the number 2 position or another one depending on the speed of the motor.
11. After all the wiring is complete switch to Front Panel and press the RUN button to execute the VI.