

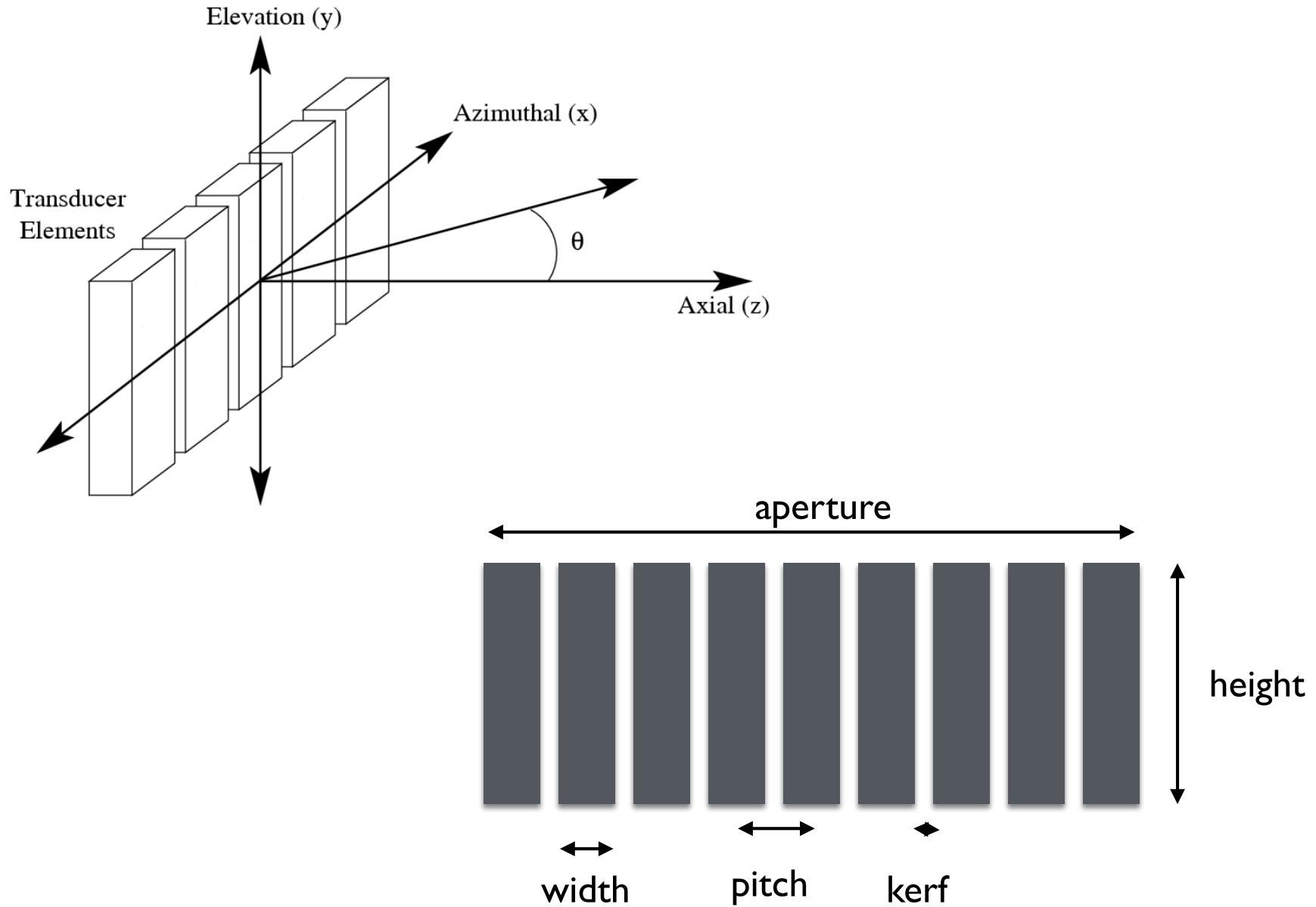
Class 7: Arrays

- Arrays
- Grating Lobes with Linear Arrays
- Phased Arrays and More Grating Lobes
- Pulse-Echo or Transmit-Receive and Dynamic Receive Focusing

Class 7

- **Arrays**
- Grating Lobes with Linear Arrays
- Phased Arrays and More Grating Lobes
- Pulse-Echo or Transmit-Receive and Dynamic Receive Focusing

Definitions



Most Common Transducers

Linear



<http://sinaiem.us/tutorials/pneumothorax>

High Frequency
~4cm field of view

Curvilinear

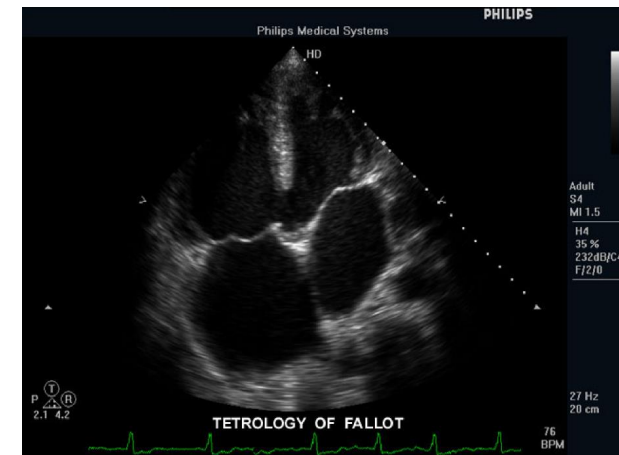
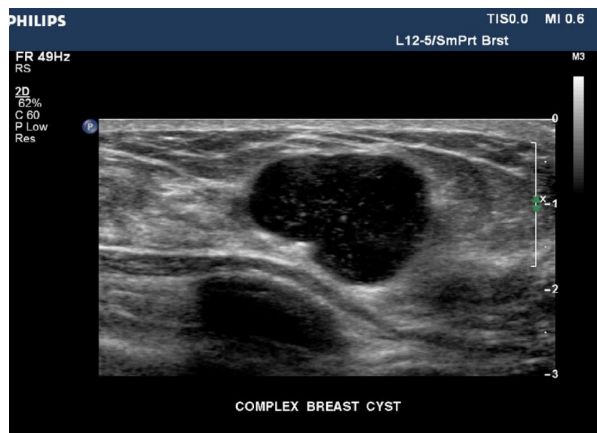


Low Frequency
-wide field of view

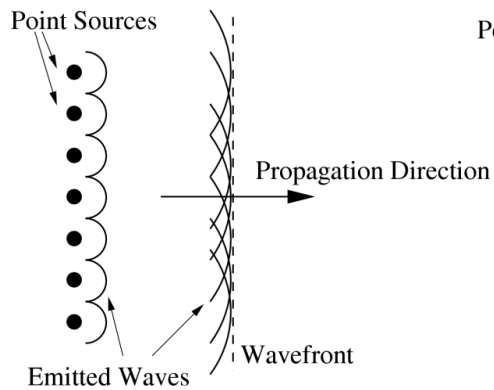
Phased



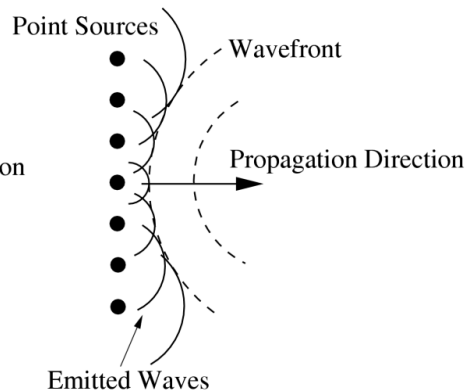
Low Frequency
-small acoustic window
-wide field of view at depth



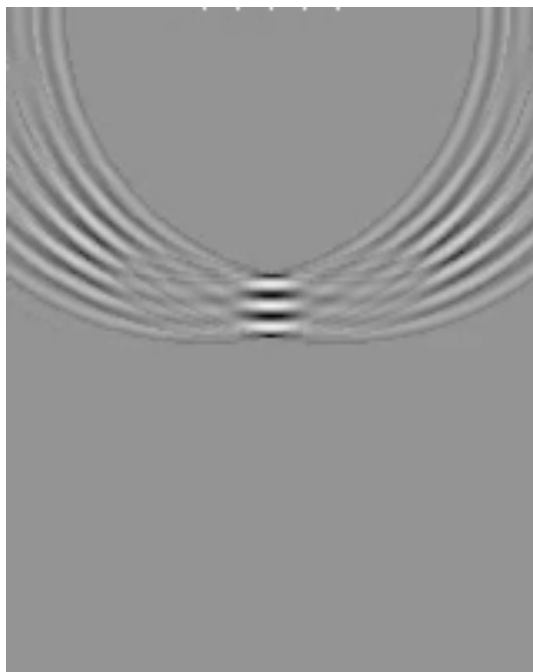
Array Beamforming



Plane Wave



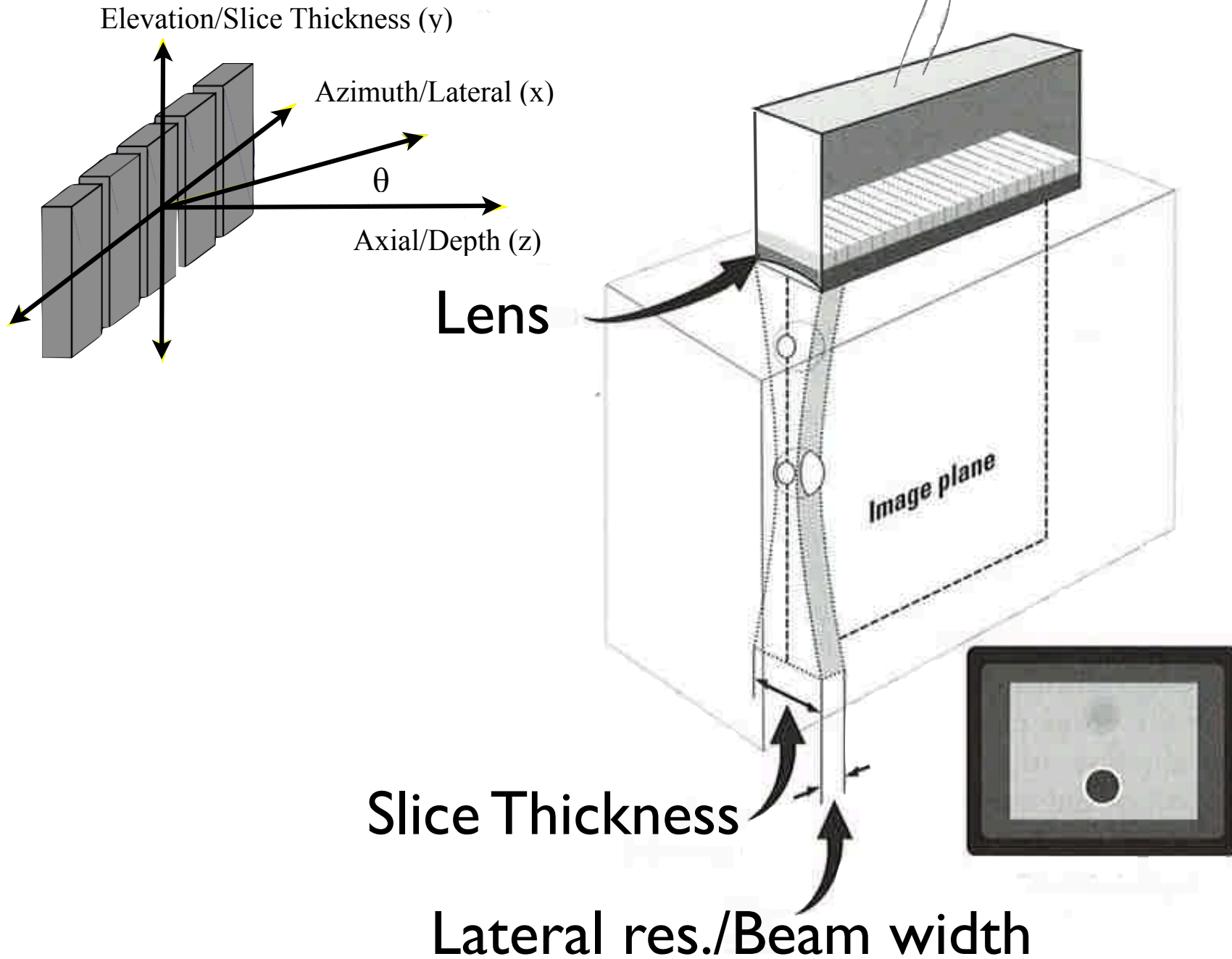
Converging or Focused
Wave



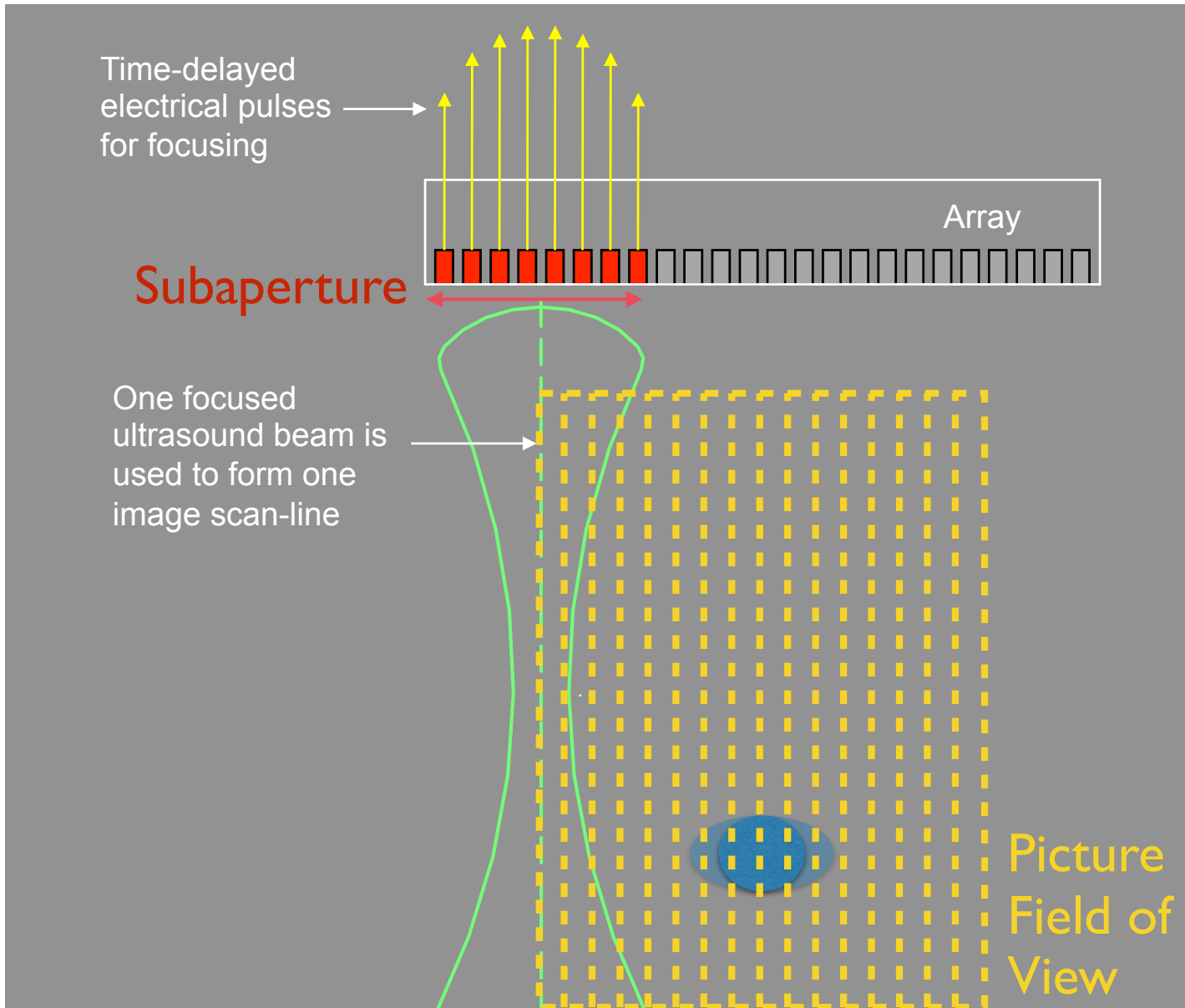
*Translate which
elements are used*

*Image one line
at a time*

Beam From Subaperture



Linear Array Imaging



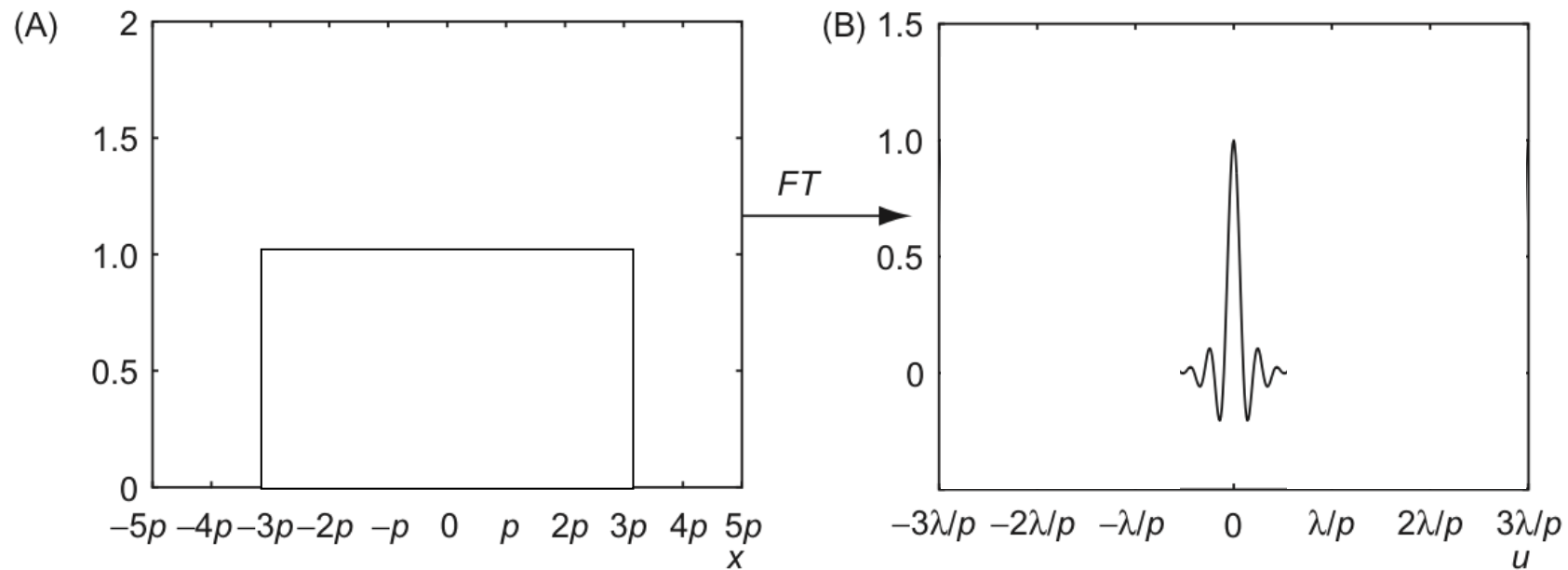
- FOV smaller than array
- Wider beam, any one object goes into multiple lines, lower resolution
- Choice of subaperture size presents a tradeoff between resolution and FOV

$$W = \frac{1.22\lambda F}{D}$$

Class 7

- Arrays
- **Grating Lobes with Linear Arrays**
- Phased Arrays and More Grating Lobes
- Pulse-Echo or Transmit-Receive and Dynamic Receive Focusing

Single Element Rectangular Aperture



$$rect\left(\frac{x}{L_x}\right) \xrightarrow{FT} \text{sinc}\left(\frac{L_x u}{\lambda}\right)$$

$$rect\left(\frac{x}{L_x}\right) \Leftrightarrow \text{sinc}(L_x f)$$

$$u = f \lambda$$

Ideal Array of point sources

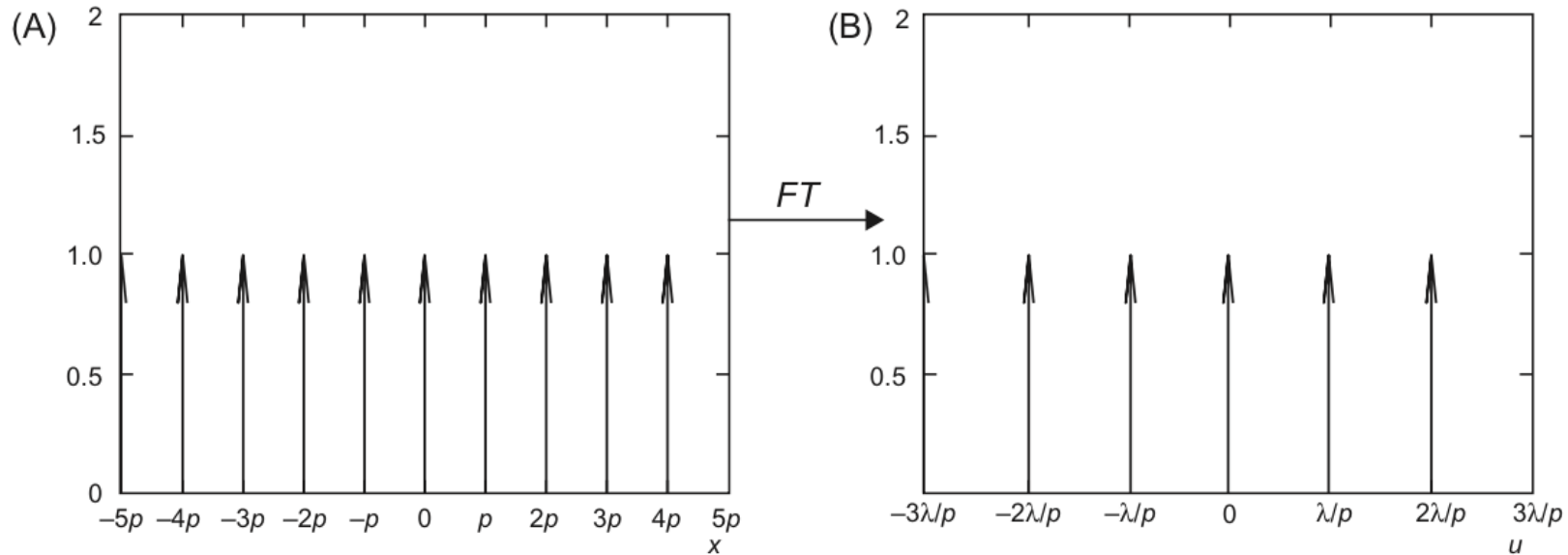


Figure 7.11

$$\overset{\text{FT}}{\text{comb}}\left(\frac{x}{p}\right) \Leftrightarrow \text{comb}\left(\frac{u}{\lambda/p}\right)$$

grating lobes

Subaperture Array of Point Sources

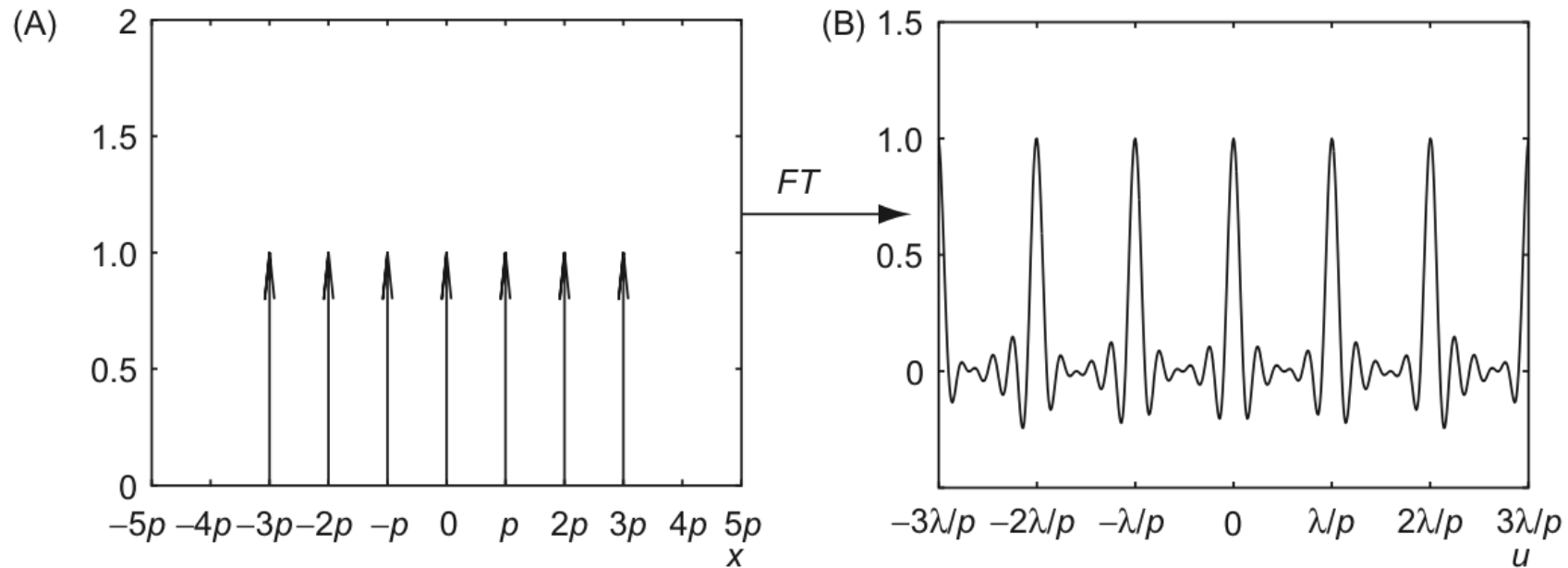


Figure 7.12

$$\text{FT}$$

$$\text{comb}\left(\frac{x}{p}\right)\text{rect}\left(\frac{x}{L_x}\right) \Leftrightarrow \text{comb}\left(\frac{u}{\lambda/p}\right) * \text{sinc}\left(\frac{L_x u}{\lambda}\right)$$

grating lobes

Subaperture Array of Finite Width Sources

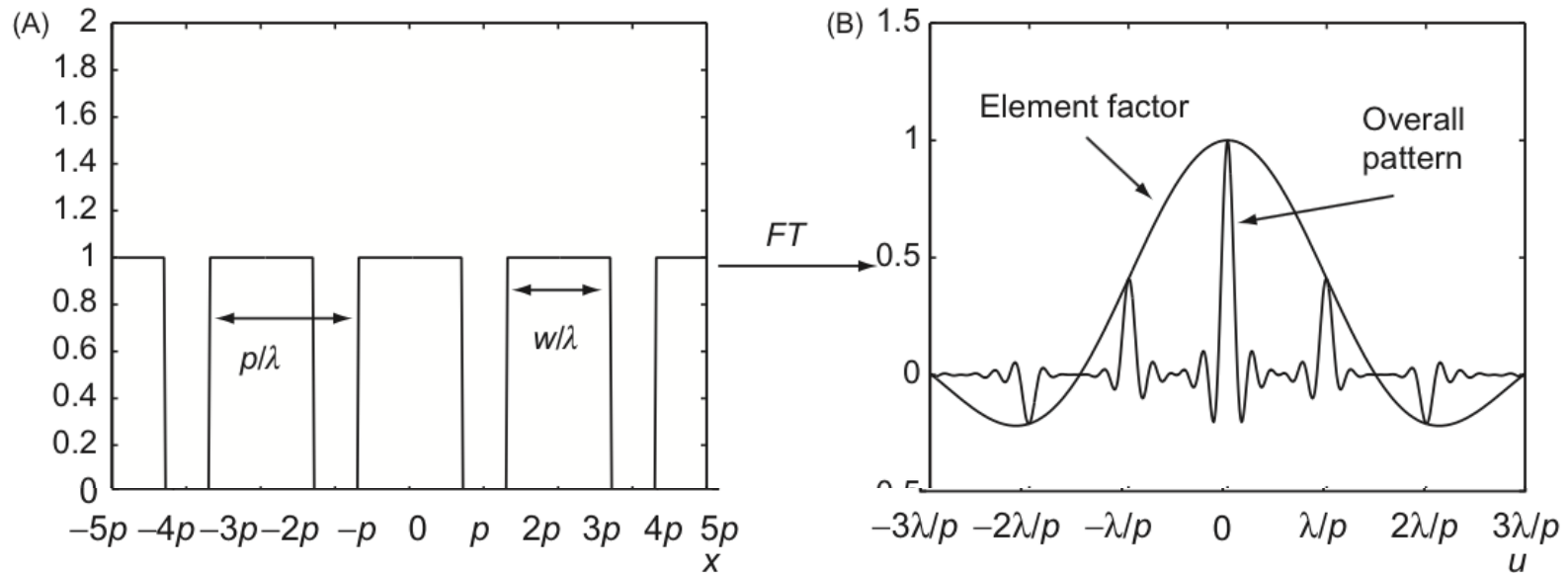


Figure 7.15

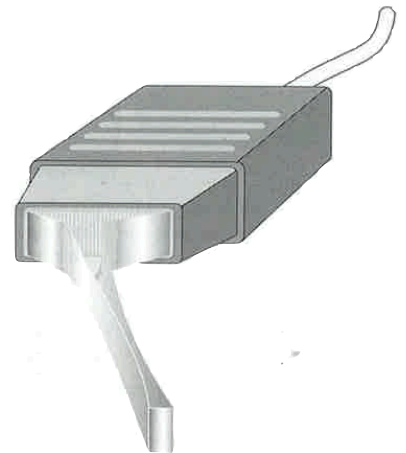
FT

$$\left(\text{comb}\left(\frac{x}{p}\right) * \text{rect}\left(\frac{x}{w}\right) \right) \text{rect}\left(\frac{x}{L_x}\right) \Leftrightarrow \left(\text{comb}\left(\frac{u}{\lambda/p}\right) \text{sinc}\left(\frac{u}{\lambda/w}\right) \right) * \text{sinc}\left(\frac{L_x u}{\lambda}\right)$$

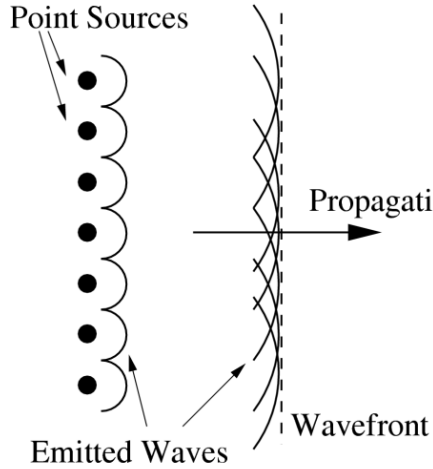
envelope on the grating lobes

Class 7

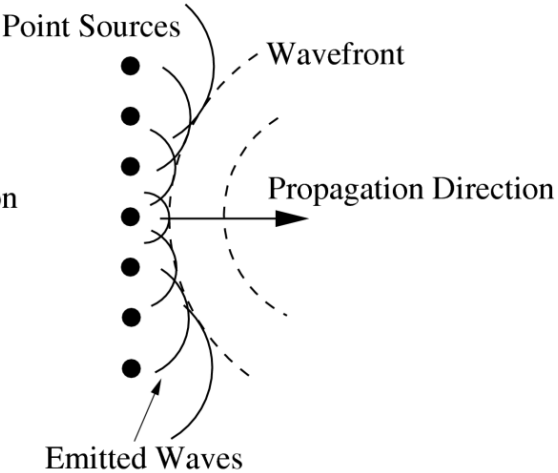
- Arrays
- Grating Lobes with Linear Arrays
- **Phased Arrays and More Grating Lobes**
- Pulse-Echo or Transmit-Receive and Dynamic Receive Focusing



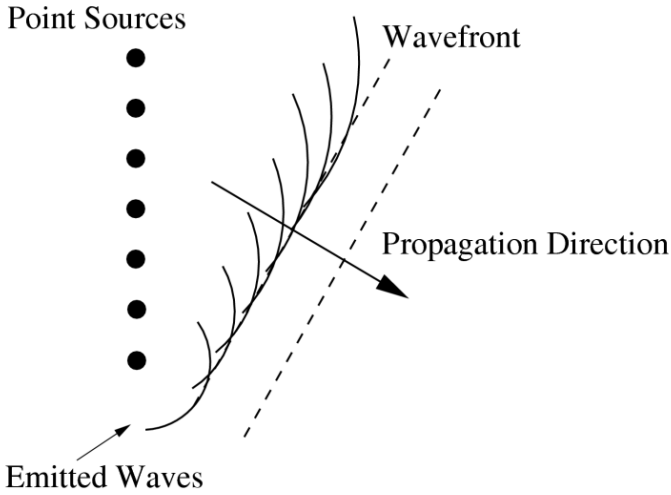
Steering



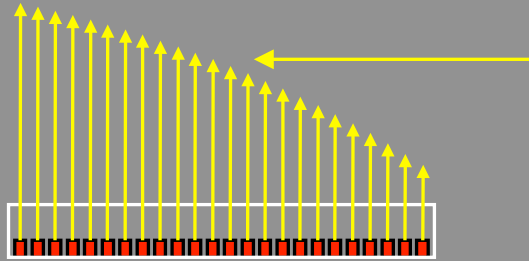
Plane Wave



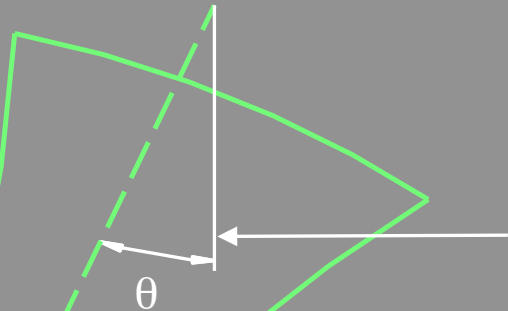
Converging or Focused Wave



Steered Wave

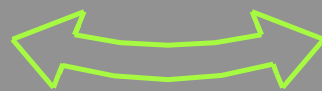


Utilizes the entire aperture for each scan line



Electronic sector scanning is achieved through varying the steering angle θ

Beam Steering



Scanning
Direction

Steered Subaperture Array of Finite Width Sources

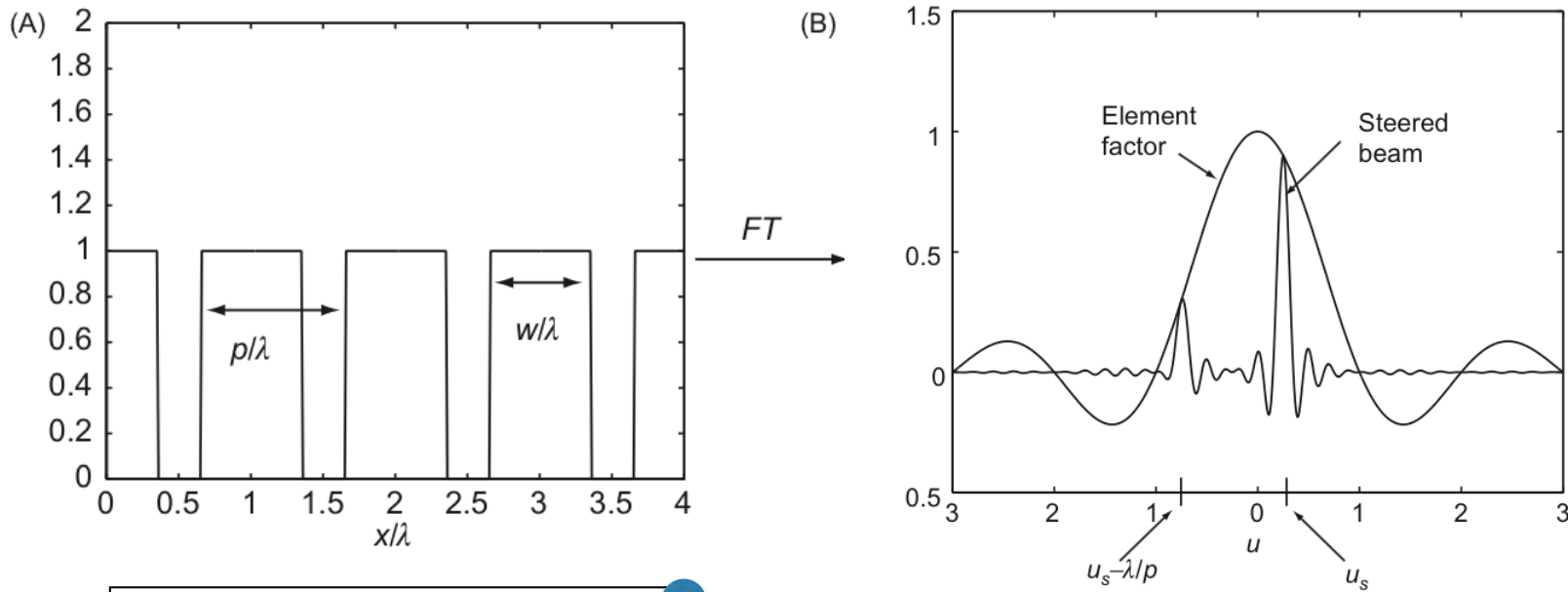
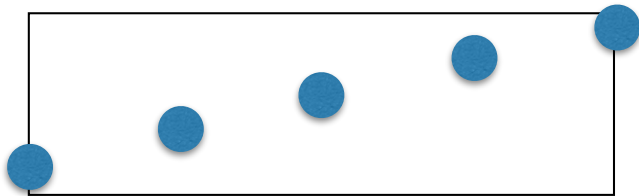


Figure 7.17

Array angular response when steered at θ_s .

Phase



$$\left(\text{comb}\left(\frac{x}{p}\right) * \text{rect}\left(\frac{x}{w}\right) \right) \text{rect}\left(\frac{x}{L_x}\right) \Leftrightarrow \left(\text{comb}\left(\frac{u}{\lambda/p}\right) \text{sinc}\left(\frac{u}{\lambda/w}\right) \right) * \text{sinc}\left(\frac{L_x u}{\lambda}\right)$$

linear phase

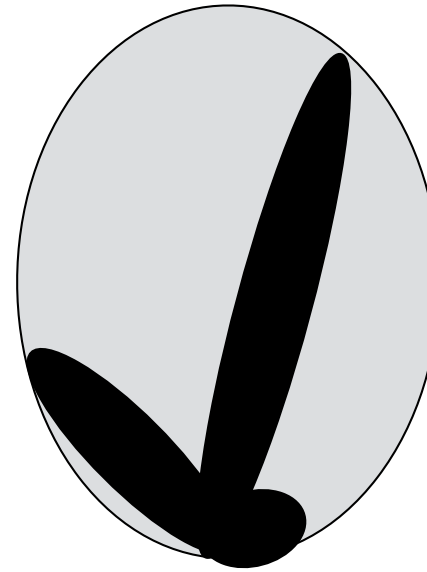
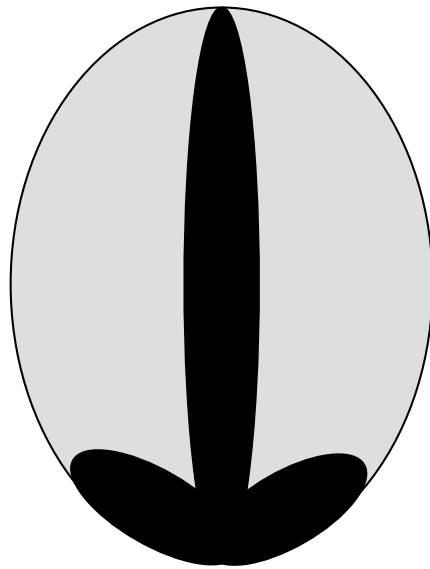
→

translation

grating lobes modulated by the envelope

Beam pattern follows the envelope

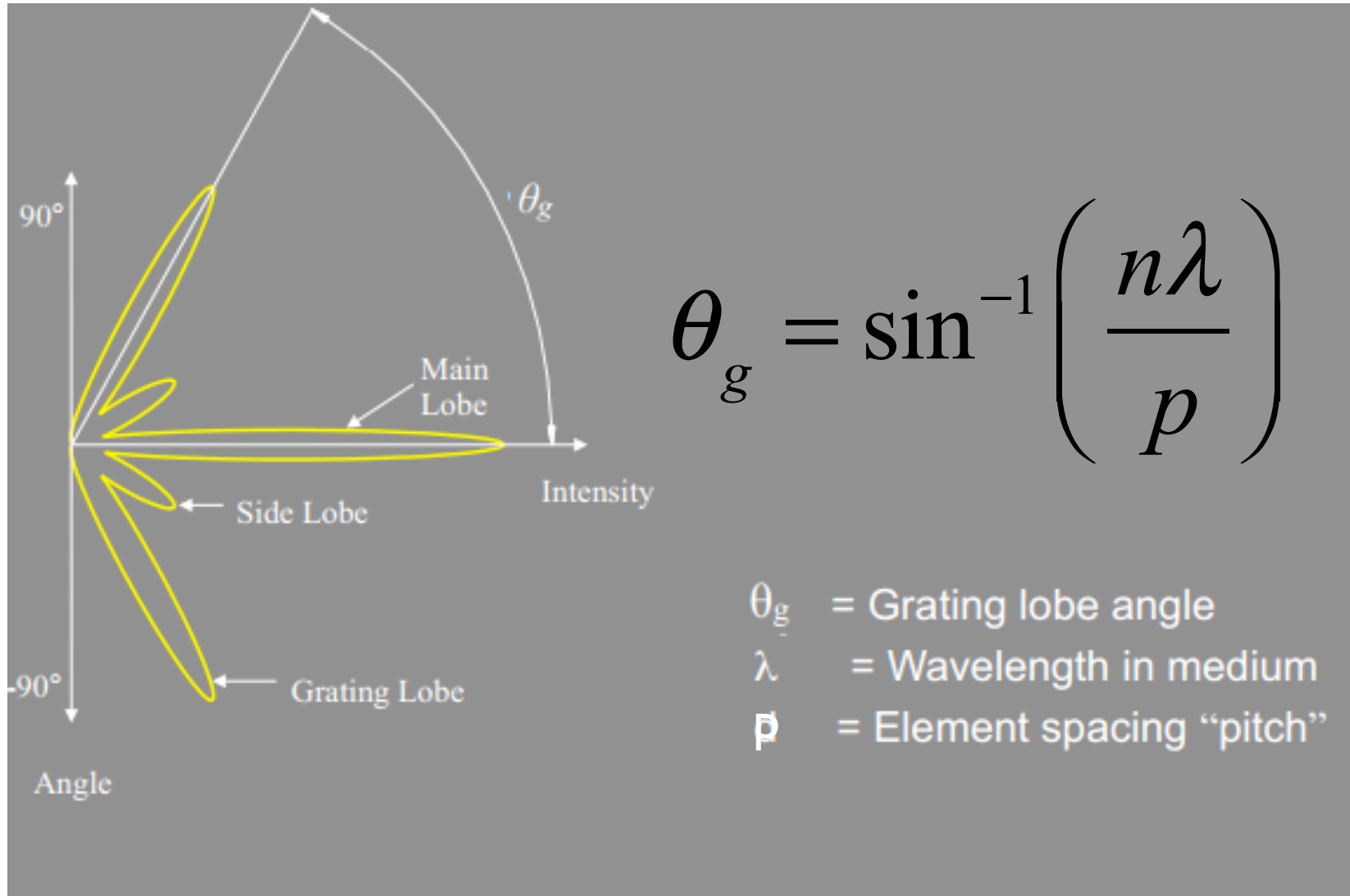
Grating lobes at 60°



Steered 15° to right
Main lobe intensity decreases
Grating lobe intensity increases

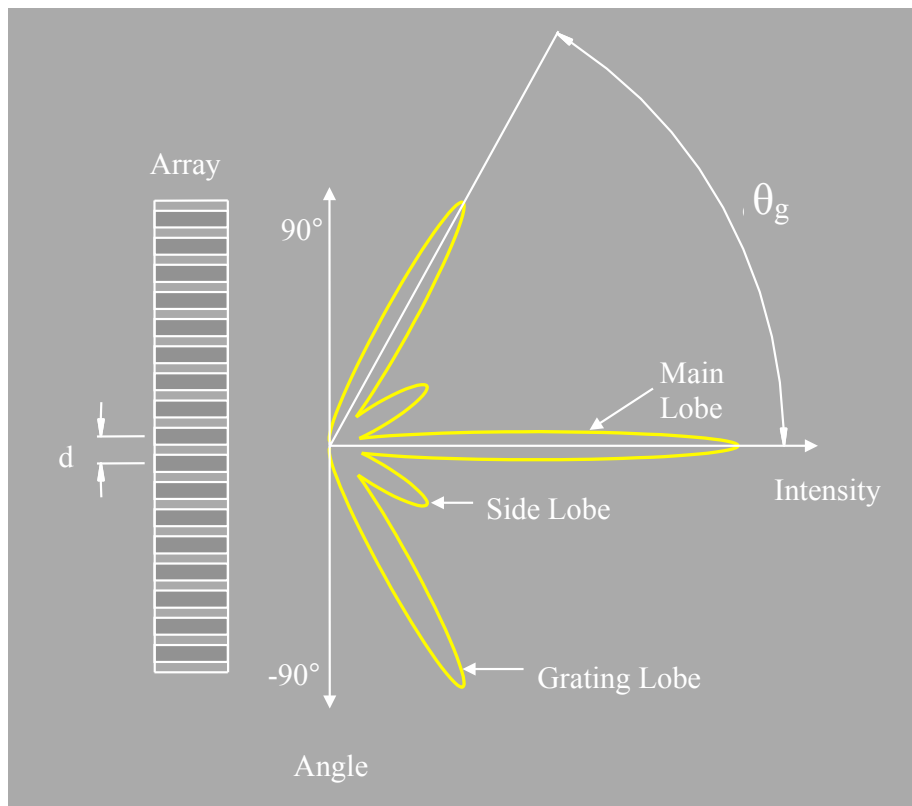
Grating Lobes

Energy goes into the near field from grating lobes



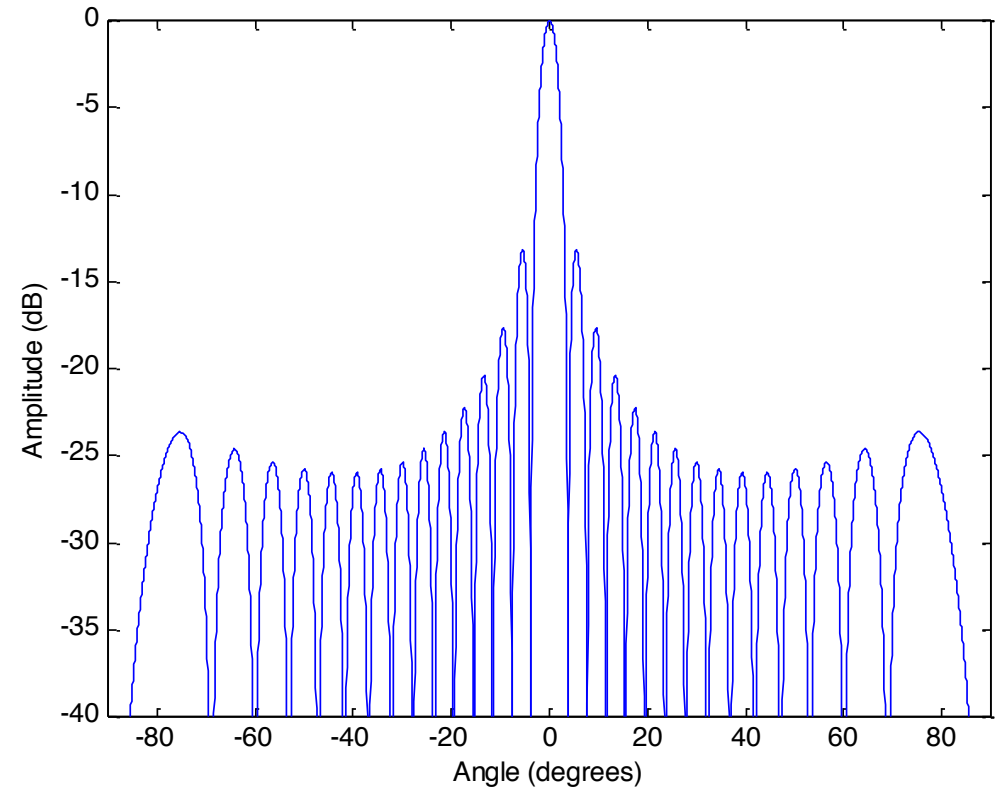
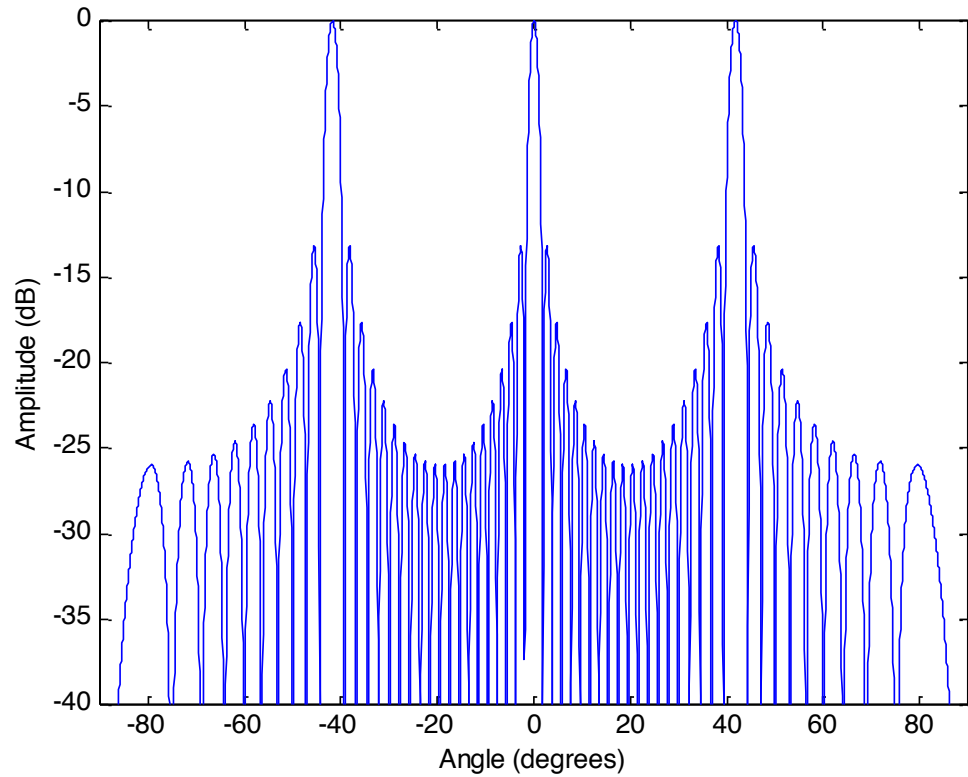
Correcting for Grating Lobes

- If the element spacing is less than λ , the grating lobe is greater than 90 degrees



$$\sin(\theta_g) = \left(\frac{\lambda}{p} \right)$$

What is the pitch?



$$\sin(\theta_g) = \left(\frac{\lambda}{p} \right)$$

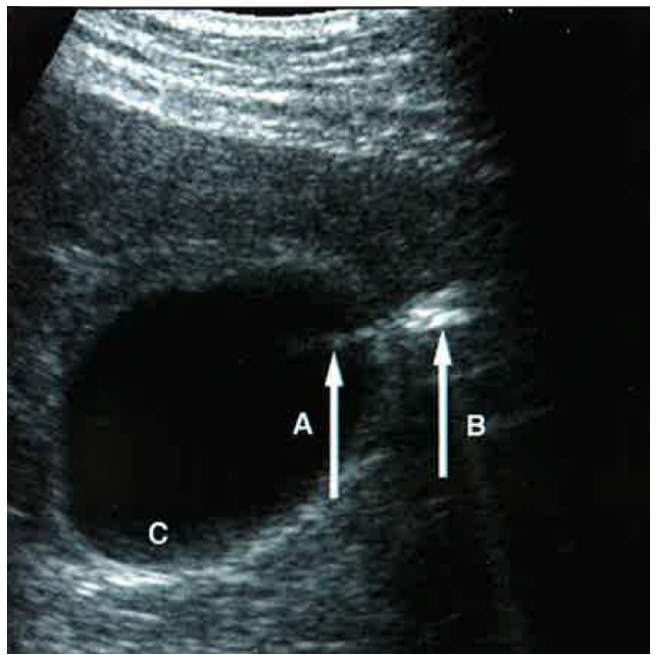
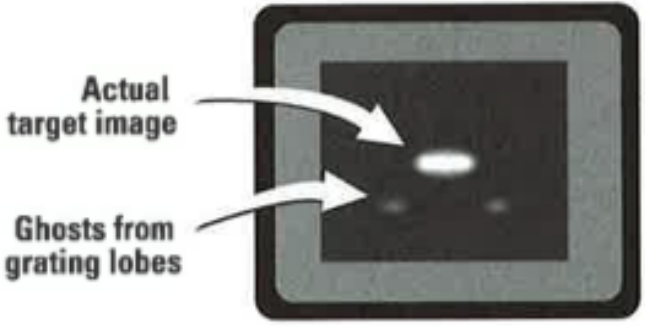
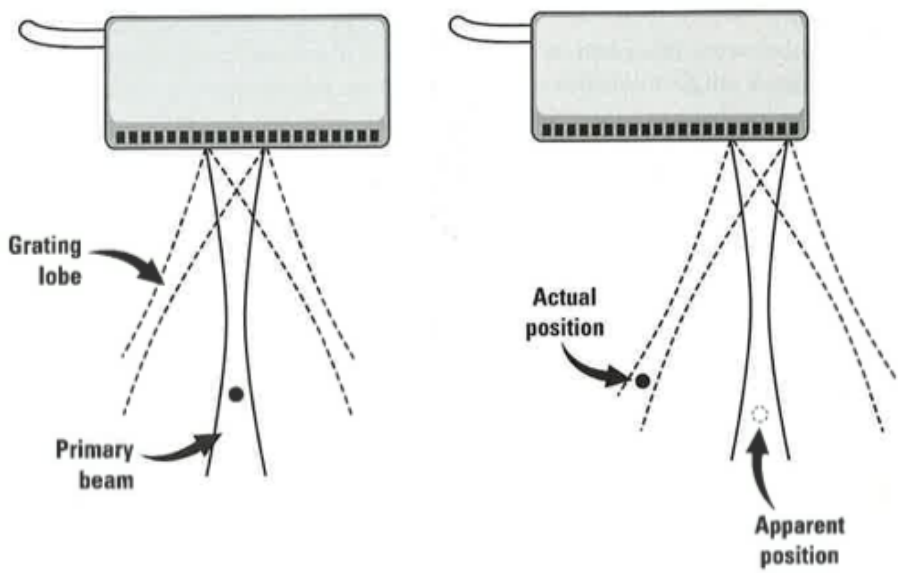
$$\sin(40^\circ) = \left(\frac{\lambda}{p} \right)$$

$$d = 1.55\lambda$$

$$\sin(90^\circ) = \left(\frac{\lambda}{d} \right)$$

$$d = \lambda$$

Grating Lobe Artifacts

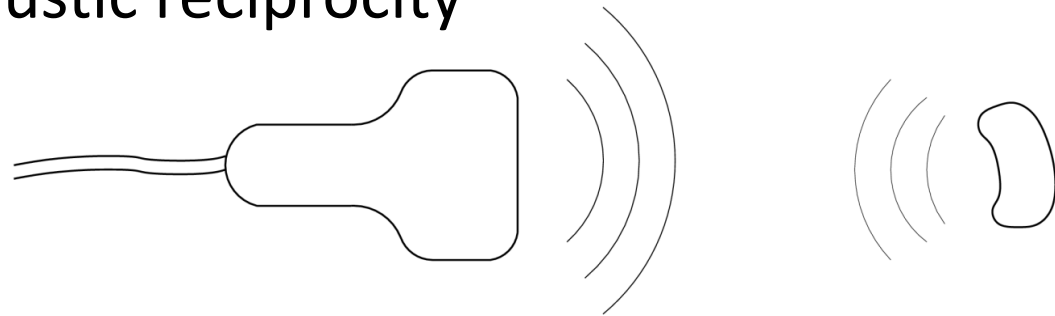


Class 7

- Arrays
- Grating Lobes with Linear Arrays
- Phased Arrays and More Grating Lobes
- **Pulse-Echo or Transmit-Receive and Dynamic Receive Focusing**

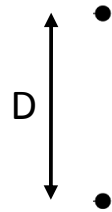
Pulse - Echo

- Because US is pulse-echo, we are concerned with both the **transmitted pressure field (Tx)** and the **receiver sensitivity (Rx)**
 - Rx sensitivity is the locations in the field where the receiver is sensitive to incoming waves
- Rx can be determined using the principle of acoustic reciprocity



Acoustic Reciprocity

- The receiver sensitivity is equal to the transmit diffraction pattern of the receiver acting as a source

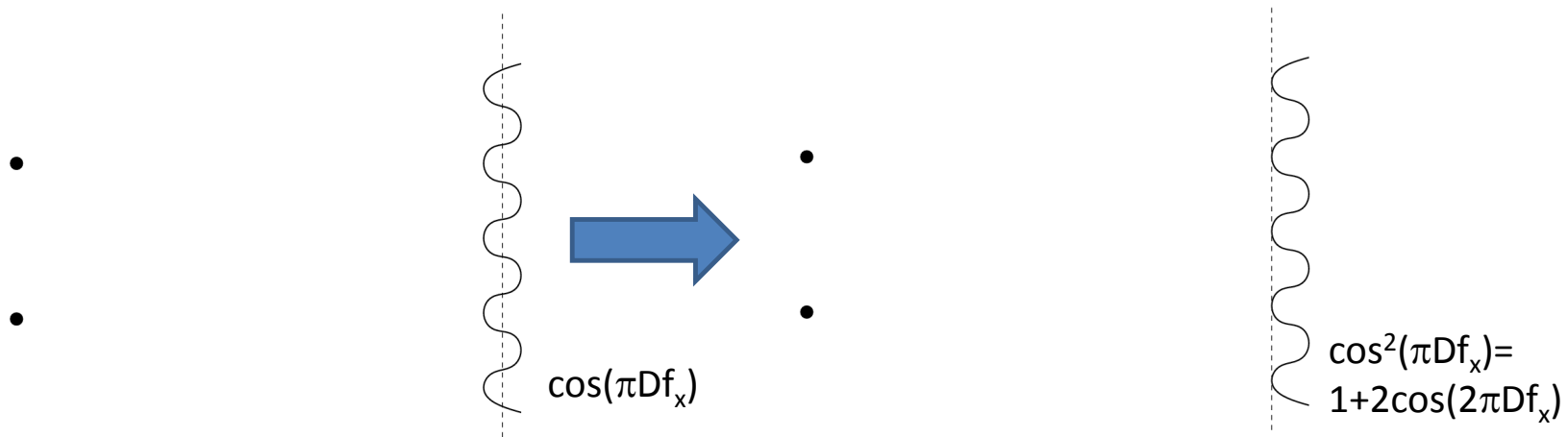


$$R_x = \cos(\pi D f_x)$$

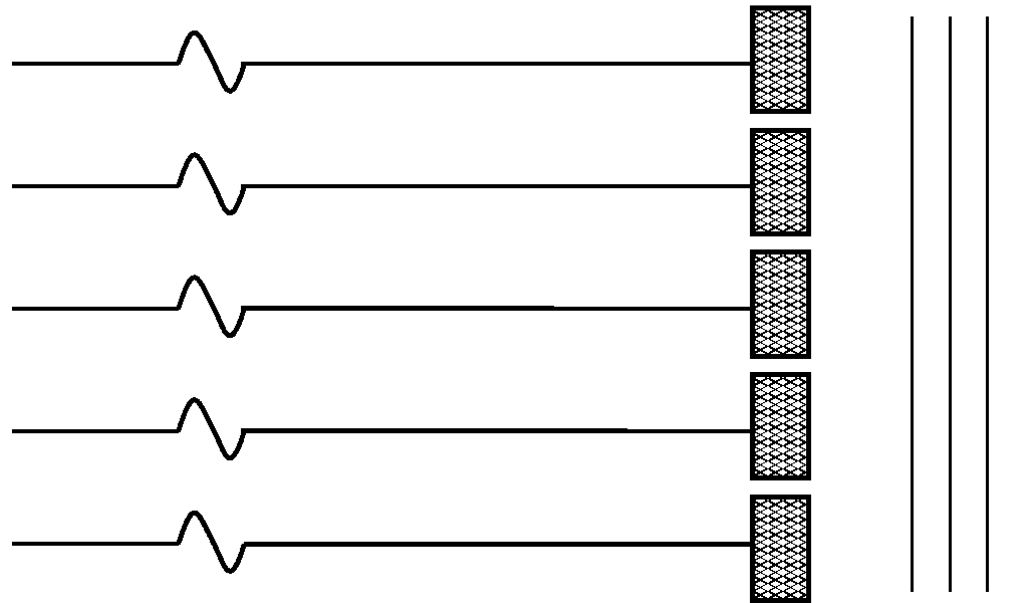
Pulse Echo Sensitivity

- The pulse-echo beam pattern (i.e. the sensitivity of the ultrasound system) is equal to the product of the transmit diffraction pattern and the receive sensitivity pattern:

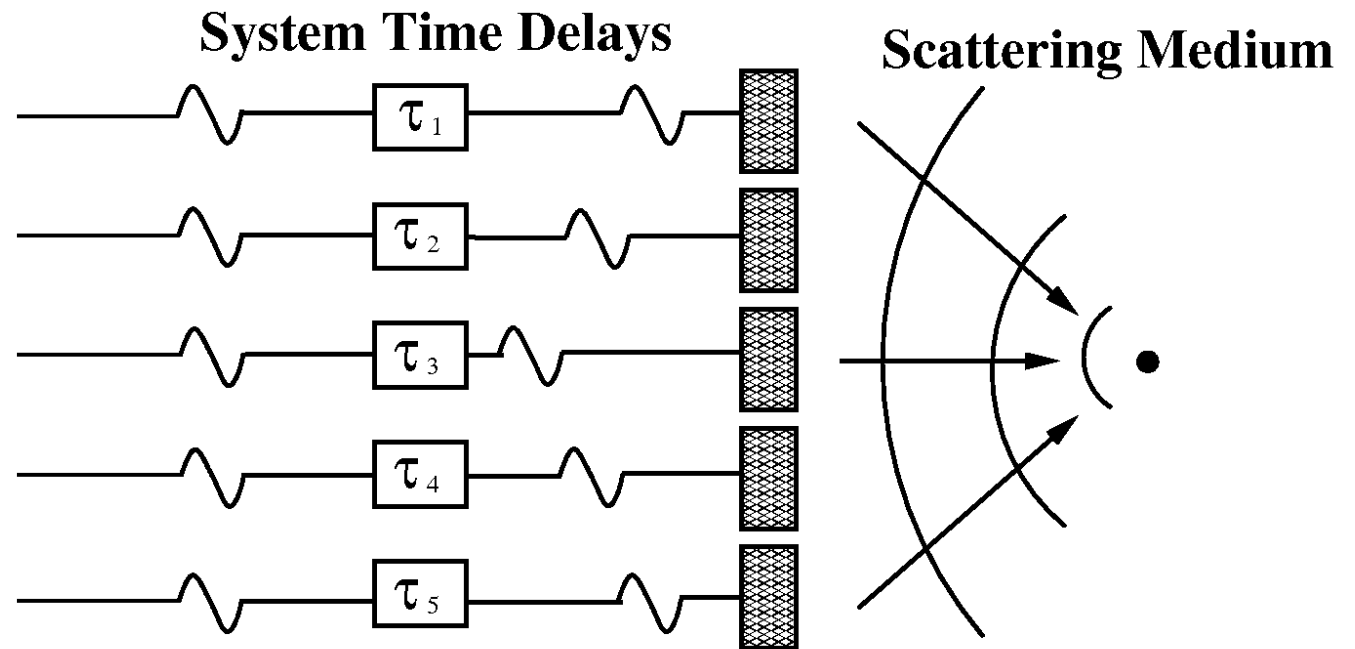
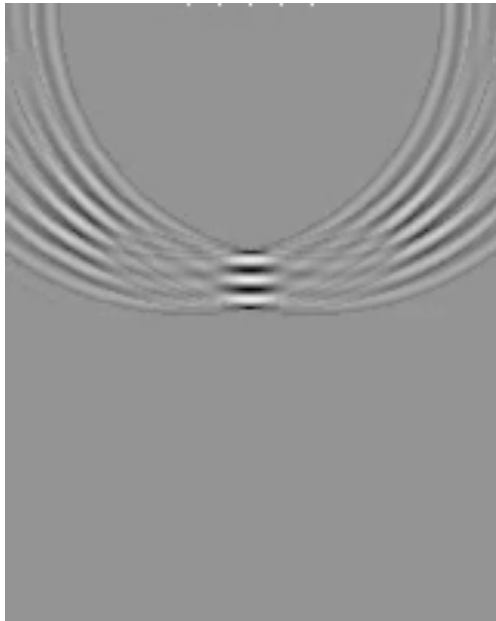
$$B(x, y) = Tx(x, y)Rx^*(x, y)$$



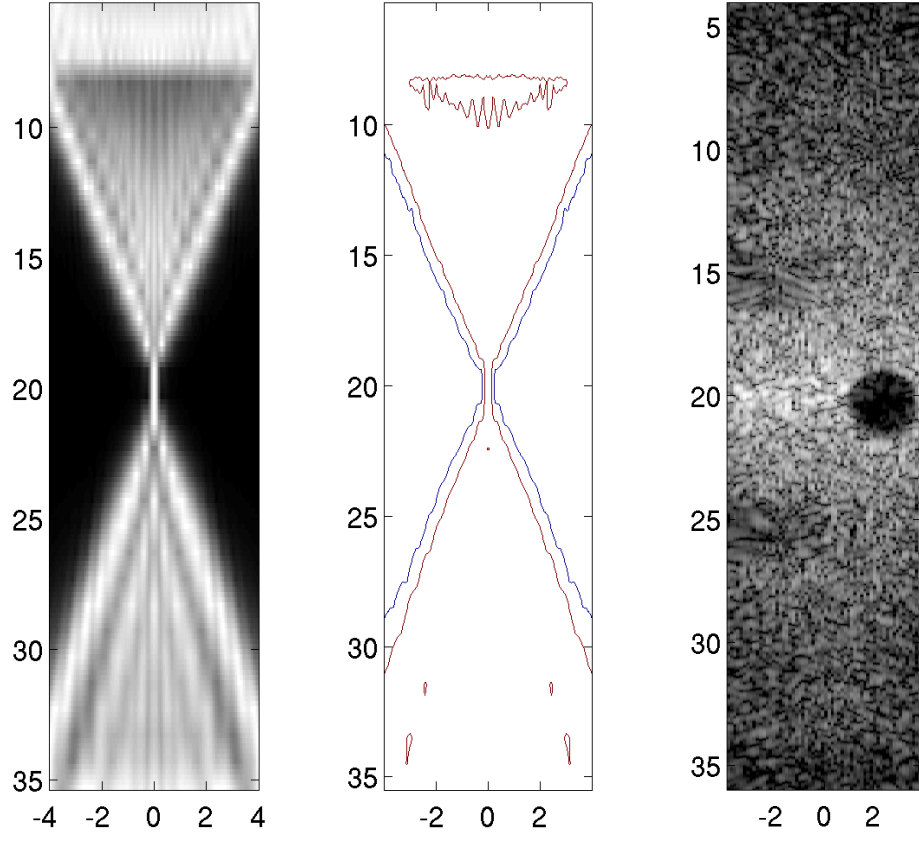
Transmit Beamforming: Plane Wave



Transmit Beamforming: Focused Wave

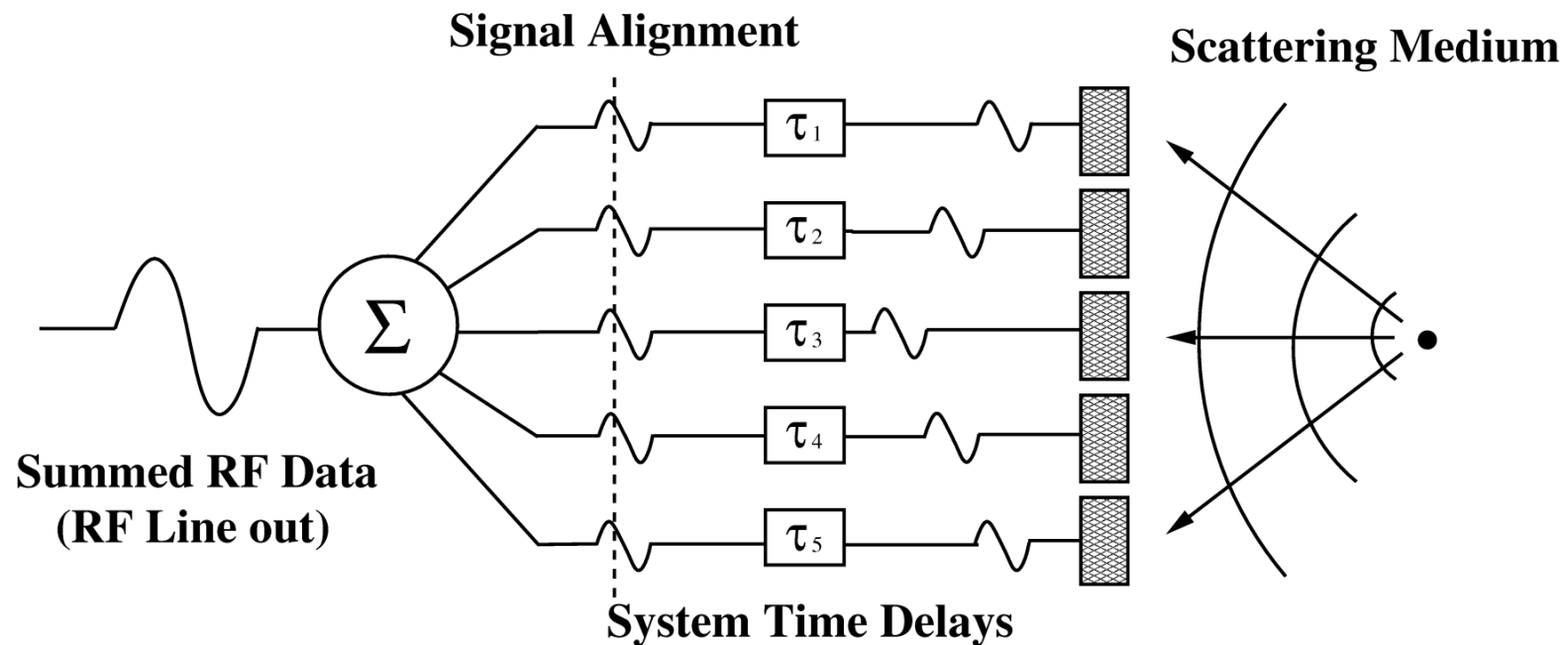


Transmit Beam



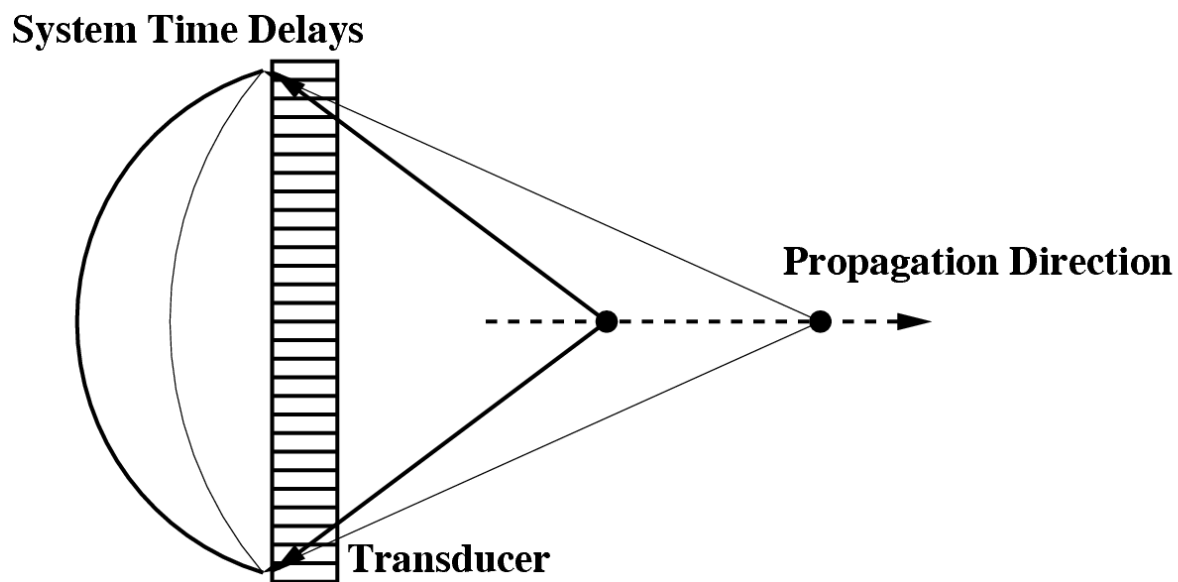
Receive Beamforming

- Transmit beamforming in reverse:



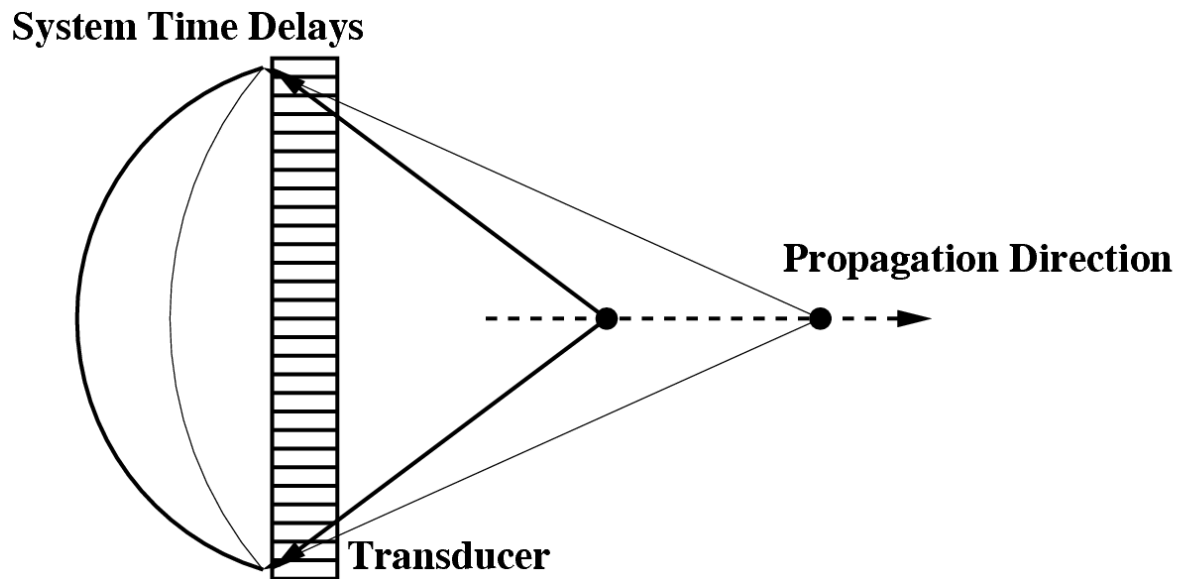
Use same equation as before to compute time delays for receiving!

Transmit Focusing at Different Depths

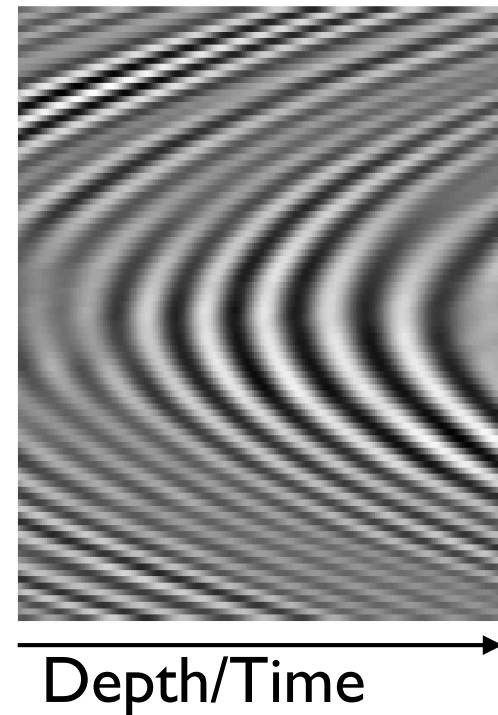


Dynamic Receive Focusing

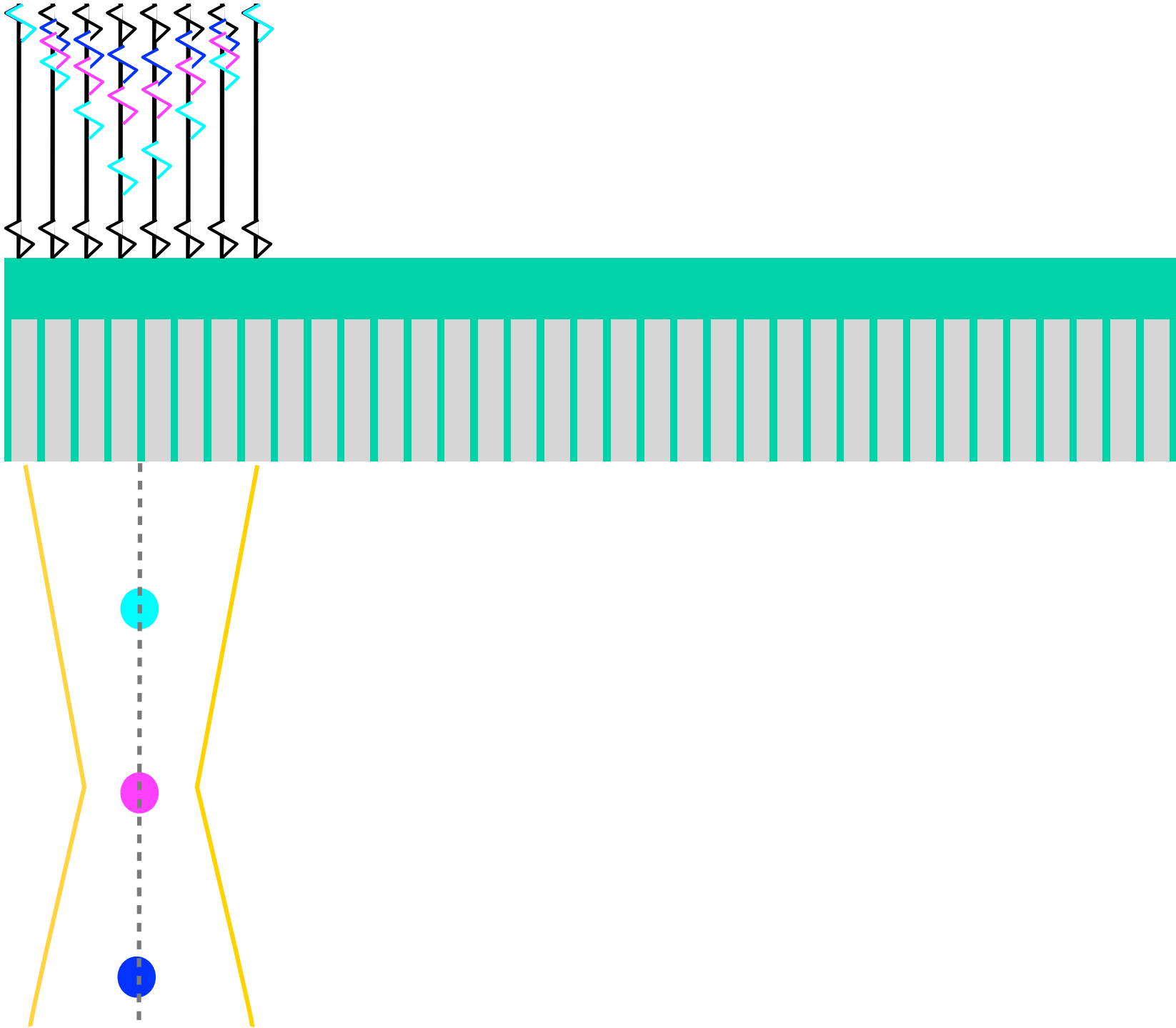
- Continuously adjust the delays



Raw (undelayed)

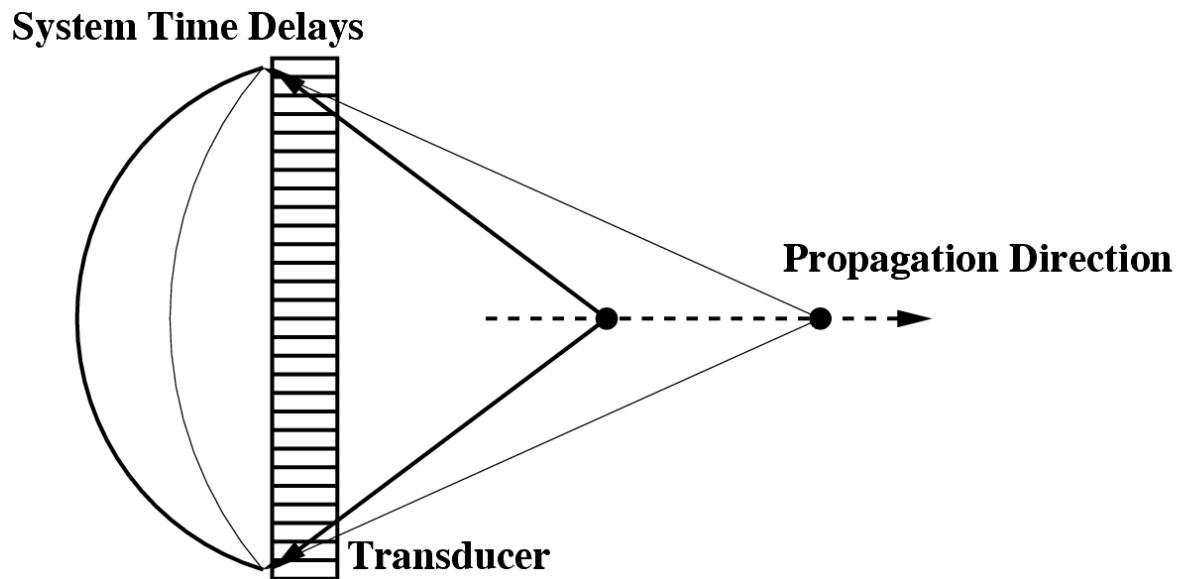


Dynamic Focusing on Receive

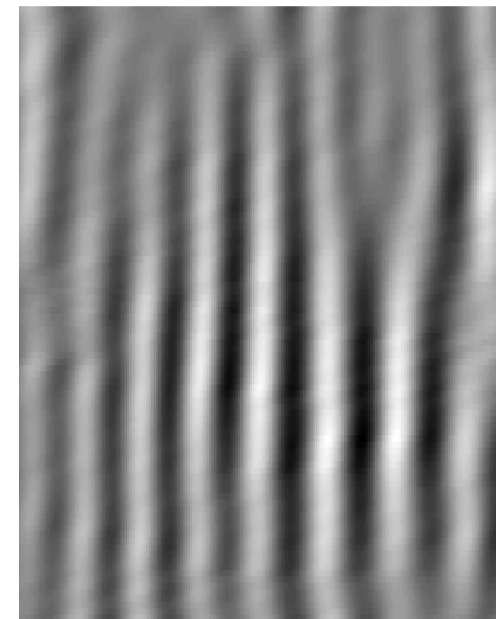


Dynamic Receive Focusing

- Continuously adjust the delays

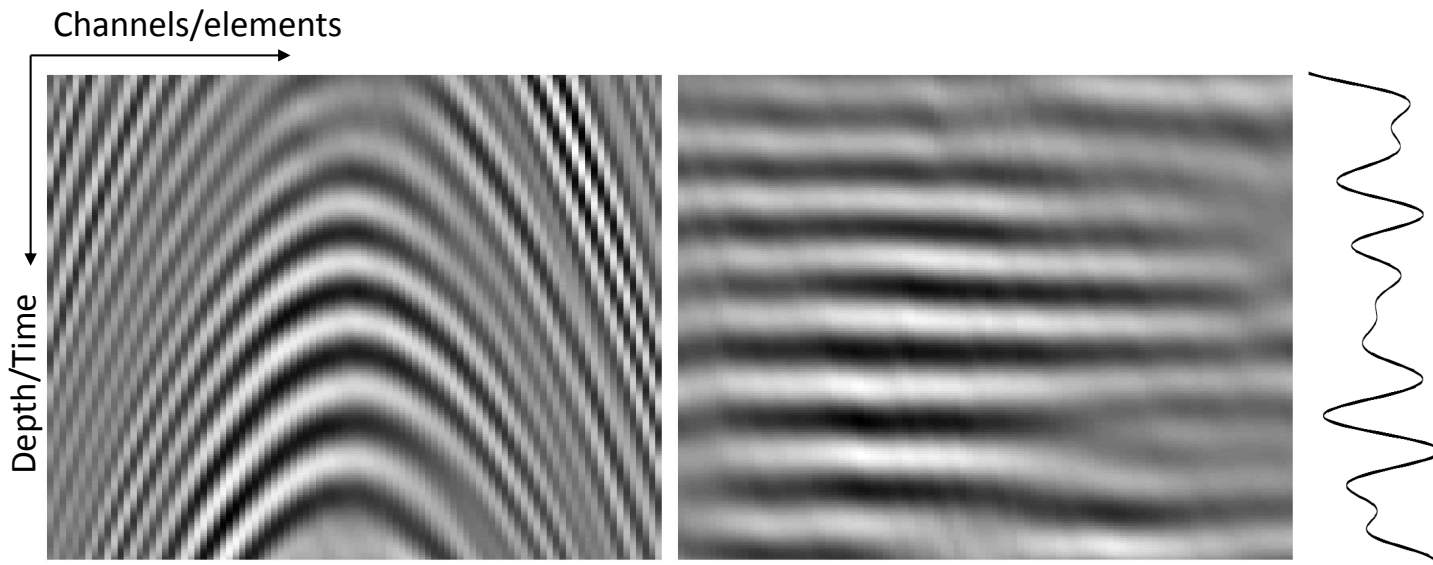
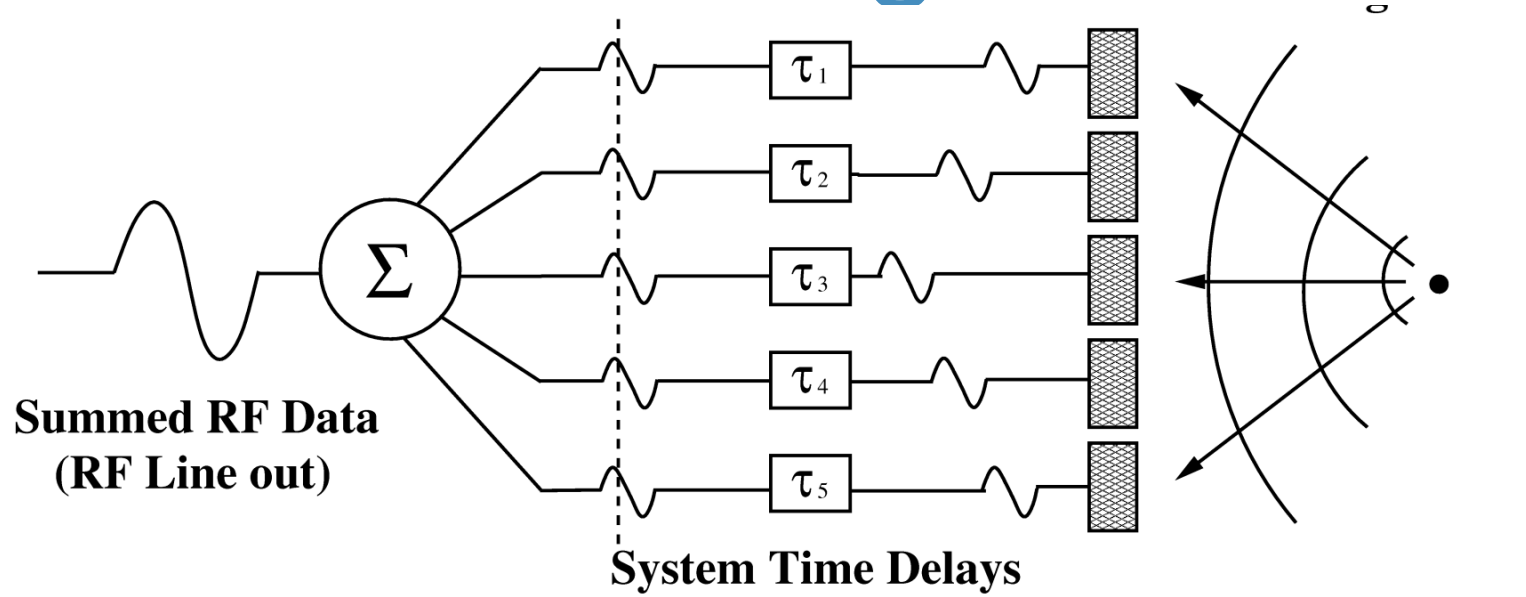


delayed



Depth/Time

Receive Beamforming

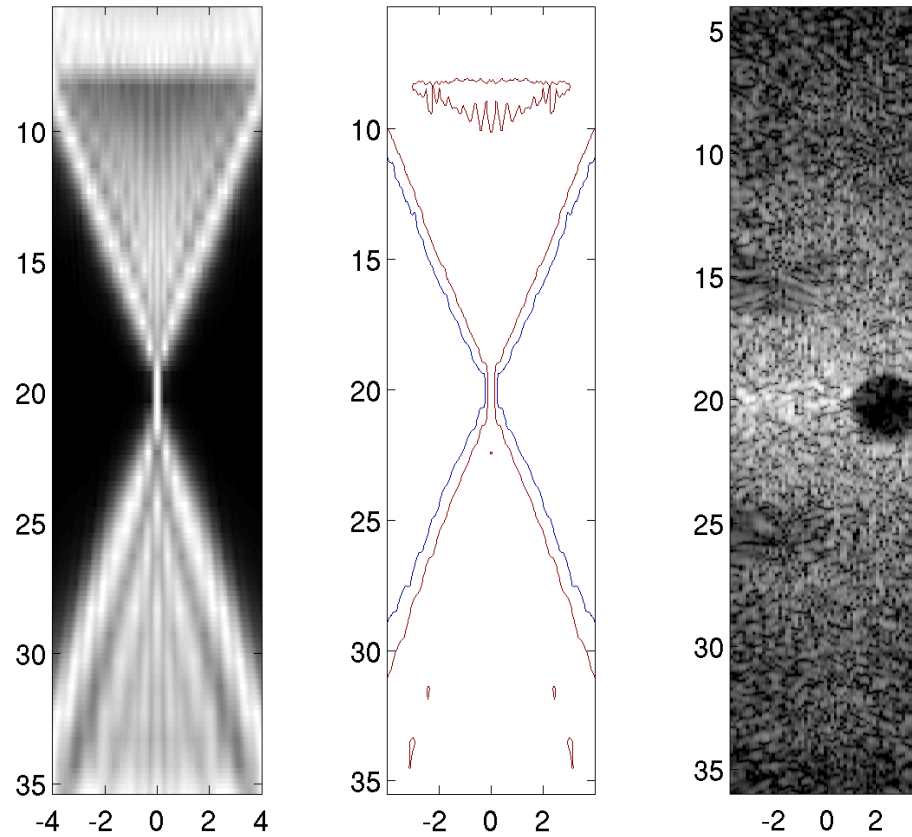


Unfocused signal received from the near field

Focused signal using time-delays: Note how wavefronts appear as "plane waves"

RF Sum

Transmit Beamforming Only



Dynamic Receive Beamforming

