

NUMBER SYSTEMS

Natural numbers are -1, 2, 3,.....denoted by N.

Whole numbers are – 0, 1, 2, 3,..... denoted by W.

Integers - -3, -2, -1, 0, 1, 2, 3, denoted by Z.

Rational numbers – All the numbers which can be written in the form p/q, $q \neq 0$ are called rational numbers where p and q are integers.

Irrational numbers – A numbers is called irrational, if it cannot be written in the form p/q where p and q are integers and $q \neq 0$

The decimal expansion of a rational number is either terminating or non terminating recurring. Thus we say that a number whose decimal expansion is either terminating r non terminating recurring is a rational number.

The decimal expansion of a irrational number is non terminating non recurring.

All the rational numbers and irrational numbers taken together.

Make a collection of real number.

A real no is either rational or irrational.

If r is rational and s is irrational then r + s, r-s are always irrational numbers but r/s may be rational or irrational.

Every irrational number can be represented on a number line using Pythagoras theorem.

Rationalization means to remove square root from the denominator.

Integers are a combination of whole numbers and negative natural numbers

POLYNOMIALS

<u>Constants</u>: A symbol having a fixed numerical value is called a constant.

Variables : A symbol which may be assigned different numerical values is known available.

<u>Algebraic Expressions</u>- A combination of constants and variables connected by operators + , - , x and / is known as algebraic expression.

<u>Polynomials</u>: An algebraic expression in which the variables involved have only nonnegative integral powers is called a polynomial.

<u>Coefficient</u>- A numerical or constant quantity placed before and multiplying the variable in an algebraic expression

Degree of a polynomial – Degree of a polynomial in one variable is the highest power of the variable in the expression.

Classification of Polynomials based on Degree:

	Degree	Polynomial
(a)	1	Linear
(b)	2	Quadratic
(c)	3	Cubic
(d)	4	Biquadratic

Classification of Polynomials based on Number of Terms:

No. of terms		Polynomial
(i)	1	Monomial
(ii)	2	Binomial –
(iii)	3	Trinomial –

<u>Constant polynomial</u>: A polynomial containing one term only, consisting a constanterm is called a constant polynomial the degree of non-zero constant polynomial is zero.

Zero polynomial : A polynomial consisting of one term, namely zero only is called a zero polynomial. The degree of zero polynomial is not defined.

Zeroes of a polynomial : Let p(x). be a polynomial. If $p(\alpha) = 0$, then we say that is zero of the polynomial of p(x)

Finding the zeroes of polynomial p(x) means solving the equation p(x) = 0.

<u>Remainder Theorem</u>: Let f(x) be a polynomial of degree ≥ 1 and let a be any real number. When f(x) is divided (x - a) by then the remainder is f(a)

Factor theorem : Let f(x) be a polynomial of degree n > 1 and let a be any real number.

(i) If f(a) = 0 then (x - a) is factor of f(x)

(ii) If (x-a) is a factor of f(x) then f(a) = 0

Factor: A polynomial p(x) is called factor of q(x) divides q(x) exactly.

Factorization : To express a given polynomial as the product of polynomials each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorization.

Modes of Factorization

Factorization by taking out the common factor Factorization of quadratic trinomials by middle term splitting method.

Identity: Identity is a equation (trigonometric, algebraic) which is true for every value of variable. Some algebraic identities useful in factorization :

(i) $(x + y)^2 = x^2 + 2xy + y^2$ (ii) $(x - y)^2 = x^2 - 2xy + y^2$ (iii) $x^2 - y^2 = (x - y) (x + y)$ (iv) $(x + a)(x + b) = x^2 + (a + b)x + ab$ (v) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$ (vi) $(x + y)^3 = x^3 + y^3 + 3xy (x + y)$ (vii) $(x - y)^3 = x^3 - y^3 - 3xy (x - y)$ (viii) $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$ $x^3 + y^3 + z^3 = 3xyz$ if x + y + z = 0

COORDINATE GEOMETRY

- The horizontal real line is called x-axis and the vertical line is called y-axis
- The point of intersection of these two axes is called the origin and it is denoted by O.
- The X-Co-ordinate is also called abscissa and the y-co-ordinate is also called ordinate.
- The axes divide the plane into four regions called quadrants.
- The coordinates of the origin are (0, 0).
- The positive numbers appear to the right of the origin on the x-axis and above the origin on the y-axis.
- The negative numbers appear to the left of the origin on the x-axis and below the origin on the y-axis.
- In an ordered pair the first number denoted the x-axis and the second number denoted the y-axis.
- The coordinates of a point on the x-axis are of the form (x, 0).
- The coordinates of a point on the y-axis are of the form (x, y).
- In first quadrant both are positive; second quadrant y is positive and x is negative; the third quadrant y is negative and x is negative; the fourth quadrant x is positive y is negative.

LINEAR EQUATIONS IN 2 VARIABLE

Linear Equation In Two Variables

An equation of the form ax + by + c = 0 where a, b, c are real numbers and x, y are variables, is called a linear equation in two variables.

Here 'a' is called coefficient of x, 'b' is called coefficients of y and c is called constant term.

Solutions Of Linear Equation

The value of the variable which when substituted for the variable in the equation satisfies the equation i.e. L.H.S. and R.H.S. of the equation becomes equal, is called the solution or root of the equation.

Rules For Solving An Equation

- (i) Same quantity can be added to both sides of an equation without changing the equality
- (ii) Same quantity can be subtracted from both sides of an equation without changing the equality
- (iii) Both sides of an equation may be multiplied by a same non-zero number without changing the equality
- (iv) Both sides of an equation may be divided by a same non-zero number without changing the equality.

• A Linear Equation in 2 Variables can have infinitely many solutions.

Graph Of Linear Equations

The graph of an equation in x and y is the set of all points whose coordinates satisfy the equation : In order to draw the graph of a linear equation ax + by + c = 0 may follow the following algorithm. **Step 1**: Obtain the linear equation ax + by + c = 0

- **Step 2 :** Express y in terms of x i.e. y = -((ax + b)/c))
- **Step 3**: Put any two or three values for x and calculate the corresponding values of y from the expression values of y from the expression obtained in step 2. Let we get points as (α_1, β_1) , (α_2, β_2) , (α_3, β_3)
- **Step 4** : Plot points (α_1, β_1) , (α_2, β_2) , (α_3, β_3) on graph paper.
- Step 5 : Join the points marked in step 4 to obtain. The line obtained is the graph of the equation ax + by + c = 0

- Note : (i) The reason that a degree one polynomial equation ax + by + c = 0 is called a linear equation is that its geometrical representation is a straight line
 - (ii) The graph of the equation of the form y = kx is a line which always passes through the origin

Equations Of Lines Parallel To The X— Axis And Y— Axis

Every point on the x – axis is of the form (x, 0). The equation of the x – axis is given by y=0. Similarly, observe that the equation of the y – axis is given by x = 0. Consider the equation x - 2 = 0. If this is treated as an equation in one variable x only, then it has the unique solution x = 2, which is a point on the number line. An equation in two variables, x - 2 = 0 is represented by the line AB in the graph in below given fig.



Note : (i) The graph of x = a is a straight line parallel to the y - axis (ii) The graph of y = a is straight line parallel to the x - axis

INTRODUCTION TO EUCLID'S GEOMETRY

The Greeks developed geometry is a systematic manner Euclid (300 B.C.) a greek mathematician, father of geometry introduced the method of proving mathematical results by using deductive logical reasoning and the previously proved result. The Geometry of plane figure is know as "Euclidean Geometry" Axioms :

The basic facts which are taken for granted without proof are called axioms some Euclid's axioms are (i) Things which are equal to the same thing are equal to one another.i.e.

a = b, $b = c \Rightarrow a = c$

(ii) If equals are added to equals, the wholes are equal i.e. $a = b \Rightarrow a + c = b + c$

(iii) If equals are subtracted from equals, the remainders are equal i.e.

 $a = b \implies a-c = b - c$

- (iv) Things which coincide with one another are equal to one another.
- (v) The whole is greater than the part.
- **Postulates :** Axioms are the general statements, postulates are the axioms relating to a particular field.

Educlid's five postulates are.

- (i) A straight line may be drawn any one point to any other point
- (ii) A terminated line can be produced indefinitely
- (iii) A circle can be drawn with any centre and any radius
- (iv) All right angles are equal to one another
- (v) If a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines, if produced indefinitely meet on that side on which the angles are less than two right angles.

Statements : A sentence which is either true or false but not both, is called a statement.

Theorems : A statement that requires a proof is called a theorem.

Corollary - Result deduced from a theorem is called its corollary

LINES AND ANGLES

- (1) Point We often represent a point by a fine dot made with a fine sharpened pencil on a piece of paper
- (2) Line A line is completely known if we are given any two distinct points. Line AB is represented by as $AB \cdot A$ line or a straight line extends indefinitely in both the directions.



(3) Line segment – A part (or portion) of a line with two end points is called a line segment.



(4) Ray - A part of line with one end points is called a ray



(5) Collinear points – If three or more points lie on the same line, they are called collinear points otherwise they are called non-collinear points.

Types of Angles –

- (1) Acute angle An acute angle measure between 0° and 90°
- (2) **Right angle** A right angle is exactly equal to 90°
- (3) **Obtuse angle** An angle greater than 90° but less than 180°
- (4) Straight angle A straight angle is equal to 180°
- (5) **Reflex angle** An angle which is greater than 180° but less than 360° is called a reflex angle
- (6) **Complementary angles** Two angles whose sum is 90° are called complementary angles
- (7) **Supplementary** Two angles whose sum is 180° are called supplementary angles
- (8) Adjacent angles Two angles are adjacent, if they have a common vertex, a common arm and their non common arms are on different sides of common arm.
- (9) Linear pair Two angles form a linear pair, if their non-common arms form a line
- (10) **Vertically opposite angles** Vertically opposite angles are formed when two lines intersect each other at a point.

Lines And A Transversal



When a transversal intersects 2 lines, following angles are formed (refer the figure above)-

(a) Corresponding angles :	
(i) ∠1 and ∠5	(ii) ∠2 and ∠6
(iii) ∠4 and ∠8	(iv) ∠3 and ∠7
(b) Alternate interior angles :	
(i) ∠4 and ∠6	(ii) ∠3 and ∠5
(c) Alternate exterior angles :	
(i) ∠1 and ∠7	(ii) ∠2 and ∠8
(d) Interior angles on the same side	e of the transversal :
(i) ∠4 and ∠5	(ii) ∠3 and ∠6

If a transversal intersects two parallel lines, then

- (i) each pair of corresponding angles is equal.
- (ii) each pair of alternate interior angles is equal.
- (iii) each pair of interior angle on the same side of the transversal is supplementary.

If a transversal interacts two lines such that, either

- (i) any one pair of corresponding angles is equal, or
- (ii) any one pair of of alternate interior angles is equal or
- (iii) any one pair of interior angles on the same side of the transversal is supplementary then the lines are parallel.

Lines which are parallel to a given line are parallel to each other.

The sum of the three angles of a triangle is 180°.

If a side of a triangle is produced, the exterior angle so formed is equal to the sum of two interior opposite angles.

- Triangle A closed figure formed by three intersecting lines is called a triangle. A triangle has three sides, three angles and three vertices.
- Congruent figures Congruent means equal in all respects or figures whose shapes and sizes are both the same for example, two circles of the same radii are congruent. Also two squares of the same sides are congruent.
- Congruent Triangles –two triangles are congruent if and only if one of them can be made to superpose on the other, so as to cover it exactly.
- If two triangles ABC and PQR are congruent under the correspondence A ↔ P, B ↔ Q and C ↔ R then symbolically, it is expressed as ΔABC ≅ ΔPQR



- In congruent triangles corresponding parts are equal and we write 'CPCT' for corresponding parts of congruent triangles.
- SAS congruency rule –Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle. For example ΔABC and ΔPQR as shown in the figure satisfy SAS congruent criterion.



• ASA Congruence Rule –Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle. For examples Δ ABC and ΔDEF shown below satisfy ASA congruence critertion.



• AAS Congruence Rule – Two triangle are congruent if any two pairs of angles and one pair of corresponding sides are equal for example ΔABC and ΔDEF shown below satisfy AAS congruence criterion.



- AAS criterion for congruence of triangles is a particular case of ASA criterion.
- Isosceles Triangle A triangle in which two sides are equal is called an isosceles triangle. For example ABC shown below is an isosceles triangle with AB = AC.



- Angle opposite to equal sides of a triangle are equal.
- Sides opposite to equal angles of a triangle are equal.
- Each angle of an equilateral triangle is 60°.
- SSS Congruence Rule If three sides of one triangle are equal to the three sides of another triangle then the two triangles are congruent for example ΔABC and ΔDEF as shown in the figure satisfy SSS congruence criterion.



 RHS Congruence Rule – If in two right triangle the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle then the two triangle are congruent. For example ΔABC and ΔPQR shown below satisfy RHS congruence criterion.



RHS stands for right angle –Hypotenuse side.

- A point equidistant from two given points lies on the perpendicular bisector of the line segment joining the two points and its converse.
- A point equidistant from two intersecting lines lies on the bisectors of the angles formed by the two lines.
- In a triangle, angle opposite to the longer side is larger (greater)
- In a triangle, side opposite to the larger (greater) angle is longer.
- Sum of any two sides of a triangle is greater than the third side.

A quadrilateral is a closed figure obtained by joining four point (with no three points collinear) in an order. Every quadrilateral has : (i) Four vertices, (ii) Four sides, (iii) Four angles and (iv) Two diagonals.

SUM OF THE ANGLES OF A QUADRILATERAL

Statement : The sum of the angles of a quadrilateral is 360°.

Types Of Quadrilaterals

- 1. Trapezium : It is quadrilateral in which one pair of opposite sides are parallel.
- 2. Parallelogram : It is a quadrilateral in which both the pairs of opposite sides are parallel.
- 3. Rectangle : It is a quadrilateral whose each angle is 90°. ABCD is a rectangle.

(i) $\angle A + \angle B = 90^{\circ} + 90^{\circ} = 180^{\circ} \Leftrightarrow AD ||BC$ (ii) $\angle B + \angle C = 900 + 900 = 180^{\circ} \Leftrightarrow AB ||DC$



Rectangle ABCD is a parallelogram also.

- 4. Rhombus : It is a quadrilateral whose all the sides are equal.
- 5. Square : It is a quadrilateral whose all the sides are equal and each angle is 90°.
- 6. Kite : It is a quadrilateral in which two pairs of adjacent sides are equal.

Note :

- Square, rectangle and rhombus are all parallelograms.
- Kite and trapezium are not parallelograms.
- A square is a rectangle.
- A square is a rhombus.
- A parallelogram is a trapezium.

PARALLELOGRAM :

A parallelogram is a quadrilateral in which opposite sides are parallel. It is denoted by



PROPERTIES OF PARALLELOGRAM :

- 1. A diagonal of a parallelogram divides it into two congruent triangles.
- 2. The opposite sides of a parallelogram are equal.

Theorem : If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.

3. The opposite angles of a parallelogram are equal.

Theorem : If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.

4. The diagonals of a parallelogram bisect each other.

Theorem : If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.

MID POINT THEOREM

(BASIC PROPORTIONALITY THEOREM)

Statement 1 :

The line segment joining the mid-points of any two sides of a triangle is parallel to the third side



Statement 2 :

The line drawn thorugh the mid –point of one side of a triangle, parallel side bisects the third side.

- Area of a figure is a unit associated with the part of the plane enclosed by that figure.
- Two congruent figures have equal area but the figures having equal areas need not be congruent.
- If a planar region formed by a figure X is made up of of two non –overlapping planar regions formed by figures A and B, then ar (X) = ar (A) +ar (B), where ar (P) denotes the area of figure P.
- Two figures are said to be on the same base and between the same parallels, if they have a common base and the vertices opposite to the common base of each figure lie on a line parallel to the base.
- Parallelograms on the same base and between the same parallels are equal in area.
- Area of a parallelogram is the product of its base and the corresponding altitude.
- Parallelograms on the same base and having equal areas lie between the same parallels.
- If a parallelogram and a triangle are on the same base and between the same parallels, then area of the triangle is half the area of the parallelogram.
- Triangles on the same base and between he same parallels ae equal in area.
- Area of triangle is half the product of its base and the corresponding altitude.
- Triangles on the same base and having equal areas lie between the same parallels.
- A median of a triangle divides it into two triangles of equal areas.

The collection of all the points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

A circle is a closed curve all of whose points lie in the same plane and are at the same distance from the centre. The fixed point is called the centre of the circle and the fixed distance is called the raidus of the circle. **Note** : The line segment joining the centre and any point on the circle is also called a radius of the circle.

INTERIOR AND EXTERIOR OF A CIRCLE

A circle divides the plane on which it leis into three parts. They are (i) inside the circle, which is laos called the interior of the circle. (ii) the circle and (iii) outside the circle, which is also called the exterior of the circle

CHORD

A chord of a circle is a line joining two points of the circumference. A chord passes through the centre is called diameter.

Note : Diameter is the longest chord and all diameters have the same length, which is equal to two times the radius.

A piece of a circle between two points is called an arc. In a circle equal chords have equal ares.

When P and Q are ends of a diameter, then both arcs are equal and each is called a semicircle.

The length of the complete circle is called its circumference.

The region between a chord and either of its arcs is called segment of the circle.



The minor are corresponds to the minor sector and the major are corresponds to the major sector. When two arcs are equal, that is, each is a semicircle, then both segments and both sectors become the same and each is known as a semicircular region.



Theorem 1: Equal chords of a circle subtend equal angles at the centre. **Theorem 1**: If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal,

PERPENDICULAR FROM THE CENTRE TO A CHORD

Theorem 3 : The perpendicular from the centre of a circle to a chord bisects the chord.
Theorem 4: The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
Theorem 5: There is one and only one circle passing through three given non –chollinear points.
Remark : If ABC is a triangle, then by above given Theorem there is a unique circle passing through the three vertices A, B and C of the triangle. This circle is called the circumcircle of the AABC. Its centre and radius are called respectively the circumcentre and the circumradius of the triangle.

EQUAL CHORDS AND THEIR DIST ANCES FROM THE CENTRE

Theorem 6 : Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres). **Theorem 7**: Chords equidistant from the centre of a circle are equal in length.

ANGLE SUBTENDED BY AN ARC OF A CIRCLE

Result : Congruent arcs (or equal arcs) of a circle subtend equal angles att the centre.

Theorem 8: The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Note: Theorem gives the relationship between the angles subtended by an are at the centre and at a point on the circle.

ANGLE FORMED IN THE SEGMENT

Theorem 9: Angles in the same segment of a circle are equal. **Note:** Angle in a semicircle is a right angle.

Theorem 10: If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle (i.e. they are concyclic).

CYCLIC QUADRILATERAL:

A quadrilateral ABCD is called cyclic if all the four vertices of it lie on a circle.



Theorem 11: The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Theorem 12: If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

HERON'S FORMULA



* Triangle with sides a, b and c

(i) Semi perimeter of triangle $s = \frac{a+b+b}{2}$

(ii) Area = $\sqrt{s(s-a)(s-b)(s-c)}$ square units.



SURFACE AREAS AND VOLUMES

SURFACE AREA OF A CUBOID:

Let us consider a cuboid of length = 1 units Breadth = b units and height = h units Then we have: (i) Total surface area of the cuboid =2(1*b+b*h+h*1)sq. units (ii) Lateral surface area of the cuboid =[2(1+b)*h]sq. units (iii)Area of four walls of a room = [2(1+b)*h]sq. units. =(Perimeter of the base * height)sq. units (iv) Surface area of four walls and ceiling of a room = lateral surface area of the room + surface area of ceiling =2(1+b)*h+1*b(v) Diagonal of the cuboid = $v1^2+b^2+h^2$

SURFACE AREA OF A CUBE: Consider a cube of edge a unit.

- (i) The Total surface area of the cube = $6a^2$ sq. units
- (ii) Lateral surface area of the cube = $4a^2$ sq. units.
- (iii) The diagonal of the cube = $\sqrt{3}$ a units.

SURFACE AREA OF THE RIGHT CIRCULAR CYLINDER

Cylinder: Solids like circular pillars, circular pipes, circular pencils, road rollers and gas cylinders etc. are said to be in cylindrical shapes.

Curved surface area of the cylinder

= Area of the rectangular sheet

= length * breadth

= Perimeter of the base of the cylinder * height

= 2 πr*h

Therefore, curved surface area of a cylinder = $2\pi rh$

Total surface area of the cylinder = $2\pi rh + 2\pi r^2$

So total area of the cylinder = $2\pi r(r+h)$

If a cylinder is a hollow cylinder whose inner radius is r1 and outer radius r2 and height h then

Total surface area of the cylinder

 $= 2\pi r_1 h + 2\pi r_2 h + 2\pi (r_2^2 - r_1^2)$ = $2\pi (r_1 + r_2)h + 2\pi (r_2 + r_1)(r_2 - r_1)$ = $2\pi (r_1 + r_2)[h + r_2 - r_1]$

SURFACE AREA OF A RIGHT CIRCULAR CONE:

curved surface area of a cone = $I/2 * 1 * 2 \pi = \pi I$ where r is base radius and I its slant height Total surface area of the right circular cone. = curved surface area + Area of the base = $\pi rI + \pi r2 = \pi r(I+r)$

Note: $l^2 = r^2 + h^2$ By applying Pythagoras Theorem, here h is the height of the cone. Thus $l = \sqrt{r^2} + h^2$ and $r = \sqrt{l^2} - h^2$ $h = \sqrt{l^2} + r^2$



SURFACE AREA OA A SPHERE

Sphere: A sphere is a three dimensional figure (solid figure) which is made up of all points in the space which lie at a constant distance called the radius, from a fixed point called the centre of the sphere.

Note: A sphere is like the surface of a ball. The word solid sphere is used for the solid whose surface is a sphere.

Surface area of a sphere: The surface area of a sphere of radius $r = 4 \times area$ of a circle of radius $r = 4 \times \pi r^2$ = $4 \pi r^2$

Surface area of a hemisphere = $2\pi r^2$ Total surface area of a hemisphere = $2\pi r^2 + \pi r^2$ = $3\pi r^2$

Total surface area of a hollow hemisphere with inner and outer radius r₁ and r₂ respectively

$$= 2\pi r_1^2 + 2\pi r_2^2 + \pi (r_2^2 - r_1^2)$$

= $2\pi (r_1^2 + r_2^2) + \pi (r_2^2 - r_1^2)$

Volume of a cuboid: Volume of a cuboid = Area of the base * height V = I*b*hSo, volume of a cuboid = base area * height = length * breadth * height

Volume of a cube: Volume of a cube = edge * edge * edge $-a^3$ where a = edge of the cube

VOLUME OF A CYLINDER

Volume of a cylinder = $\pi r^2 h$ volume of the hollow cylinder $\pi r_2^2 h - \pi r_1^2 h$ = $\pi (r_2^2 - r_1^2) h$

VOLUME OF A RIGHT CIRCULAR CONE

Volume of a cone = $1/3 \pi r^2 h$, where r is the base radius and h is the height of the cone.

VOLUME OF A SPHERE

Volume of a sphere the sphere = $4/3 \, \pi r^3$, where r is the radius of the sphere. Volume of a hemisphere = $2/3 \, \pi r^3$ **APPLICATION:** Volume of the material of a hollow sphere with inner and outer radii r₁ and r₂ respectively

$$=\frac{4}{3}\pi r_2^3 - \frac{4}{3}\pi r_1^3 = \frac{4}{3}\pi (r_2^3 - r_1^3)$$

Volume of the material of a hemisphere with inner and outer radius r_1 and r_2 respectively = $\frac{2}{2}\pi(r_2^3 - r_1^3)$

STATISTICS

Primary data: Data which collected for the first time by the statistical investigator or with the help of his workers is called primary data.

Secondary data: These are the data already collected by a person or a society and these may be in published or unpublished form. These data should be carefully used

PRESENTATION OF DATA

Raw data : When the data is compiled in the same form and order in which it is collected, it is known as Raw data The difference of the highest and the lowest values in the data is called the range of the data

Frequency : It is a number, which tells that how many times does a particular data appear in a given set of data

Frequency Distribution :

A tabular arrangement of data showing their corresponding frequencies is called a frequency distribution. The table showing data with their corresponding frequencies is called a frequency table

Class Mark : The mid-point of a class is called the class mark of the class

Thus,

 $Class - mark = \frac{Upper \ class \ limit + Lower \ class \ limit}{2}$

MEASURES OF CENTRAL TENDENCY (AVERAGES)

Mean : The mean (or average) of a number of observations is the sum of the values of all the observations divided by the total number of observations.

It is denoted by the symbol x as 'x bar'

In general, for n observal as $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i}$

For an grouped frequency distribution,

 $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$ where f_i is the frequency corresponding to the observation x_i

Note :
$$\sum_{i=1}^{n} f_i x_i = f_1 x_1 + f_2 x_2 + f_3 x_3 + \dots + f_n x_n$$

<u>Median</u>

When the data is arranged in ascending (or descending) order the median of ungrouped data is calculated as follows:

When the number of observation(n) is odd, the median is the value of the ((n+1)/2)th observation.

MODE : The mode is that value of the observation which occurs most frequently, i.e., an observation with the maximum frequency is called the mode.

PROBABILITY

PROBABILITY - AN EXPERIMENTAL (EMPIRICAL) APPROACH.

A trial is an action which results in one or several outcomes. An event for an experiment is the collection of some outcomes of the experiment.

Let n be the total number of trials. The empirical probability P(E) of an event E happening is given by $P(E) = \frac{Number of trials in which the event happened}{The total number of trials}$

Number of trials in which the event happened

Note : The empirical probability depends on the number of trials.

The probability of an event lies between 0 and 1, i.e., It can be any fraction from 0 to 1. The sum of the probabilities of all the possible outcomes of a trial is 1. Probability of the occurrence of an event + Probability of the non-occurrence of that event = 1