Real numbers:

Euclid's division lemma

Given positive integers a and b, there exist whole numbers q and r satisfying $a = bq + r$, $0 \le r < b$.

Euclid's division algorithm:

This is based on Euclid's division lemma.

According to this, the HCF of any two positive integers a and b, with $a > b$, is obtained as follows:

Step 1: Apply the division lemma to find q and r where $a = bq + r$, $0 \le r < b$.

Step 2: If $r = 0$, then HCF is b. If $r \neq 0$, apply Euclid's lemma to b and r.

Step 3: Continue the process till the remainder is zero. The divisor at this stage will be HCF (a, b).

Also, HCF (a, b) = HCF (b, r).

The fundamental theorem of arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur.

- Let $x = p/q$ be a rational number, such that prime factorisation of 'q' is of the form $2^n 5^m$, where m,n are non-negative integers. Then x has a decimal expansion which is terminating.

- Let $x = p/q$ be a rational number, such that prime factorization of 'q' is not of the form $2^n 5^m$, where m, n are non-negative integers. Then x has a decimal expansion which is non-terminating repeating.

Polynomial:

- A quadratic polynomial in x with real coefficients is of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.

- The zeroes of a polynomial p(x) are precisely the x-coordinates of the points, where the graph of $y = p(x)$ intersects the x -axis.

- A quadratic polynomial can have at most 2 zeroes and a cubic polynomial can have at most 3 zeroes.

- If α and β are the zeroes of the quadratic polynomial $ax^2 + bx + c$, then

Sum of zeroes (α +β) = $\frac{-b}{a}$

Product of zeroes (αβ) = $\frac{c}{a}$

• If α, β, γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d = 0$, then

Sum of zeroes taken one at time (α +β + γ) = $\frac{-b}{a}$

Sum of zeroes taken two at time (αβ+ βγ+ γα) = $\frac{c}{a}$

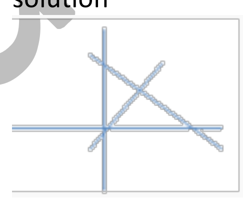
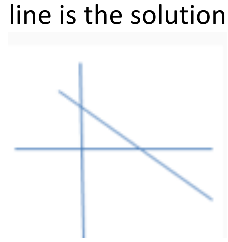
Product of zeroes (αβγ) = $\frac{-d}{a}$

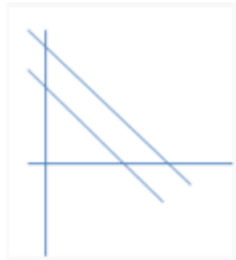
Pair of linear equations in two variables:

• If the lines intersect at a point, then that point gives the unique solution of the two equations. In this case, the pair of equations is **consistent**.

• If the lines coincide, then there are infinitely many solutions — each point on the line being a solution. In this case, the pair of equations is **dependent (consistent).**

• If the lines are parallel, then the pair of equations has no solution. In this case, the pair of equations is **inconsistent.**

Simultaneous pair of Linear equation	Condition	Graphical representation	Algebraic interpretation
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines; The intersecting point coordinate is the only solution 	One unique solution only
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines; The any coordinate on the line is the solution. 	Infinite solutions

$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel Lines 	No solution

Cross multiplication method:

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$
$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{a_1c_2 - a_2c_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Value of x can be obtained by using first and last expression.

Value of y can be obtained by using second and last expression.

Quadratic equation:

The roots of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ are given by $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ provided $D \geq 0$.

Discriminant of the quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by

$D = b^2 - 4\,ac$

• $b^2 - 4ac > 0$ then we will get **two real solutions** to the quadratic equation.

• $b^2 - 4ac = 0$ then we will get **two equal real solutions** to the quadratic equation.

• $b^2 - 4ac > 0$ then we will **no real solution** to the quadratic equation.

• A quadratic equation can also be solved by method of completing square.

$(a + b)^2 = a^2 + b^2 + 2ab$

$(a - b)^2 = a^2 + b^2 - 2ab$

Arithmetic Progression:

• If a, b, c are in AP, then $2b = a + c$

• nth term of an arithmetic progression:

$\mathbf{T_n = a + (n - 1)d}$

• Number of terms of an arithmetic progression:

n = $\frac{(l-a)}{d}+1$

Where n = number of terms, a= the first term, l = last term, d= common difference

Additional notes on AP

To solve most of the problems related to AP, the terms can be conveniently taken as:

•3 terms: (a – d), a, (a +d)

•4 terms: (a – 3d), (a – d), (a + d), (a +3d)

•5 terms: (a – 2d), (a – d), a, (a + d), (a +2d)

• The nth term of an A.P is the difference of the sum to the first n terms and the sum to first (n-1) terms of it:

$T_n = S_n - S_n-1$

• If each term of an AP is increased, decreased, multiplied or divided by the same non-zero constant, the resulting sequence also will be in AP.

• In an AP, sum of terms equidistant from beginning and end will be constant.

• A.P which contain finite term is called finite A.P and which contains infinite terms is called infinite term.

Triangles:

Δ ABC ~ Δ PQR $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$	By A.A. test or S.A.S test or by S.S.S. test C.P.S.T.
$\frac{A_1}{A_2} = \left\|\frac{S_1}{S_2}\right\|^2$	Areas of ~ Δ's are proportional to squares of their corresponding sides.
$A_1 = A_2$	A median divides a Δ into 2 Δ's with equal area.
$\frac{A_1}{A_2} = \frac{base_1}{base_2} =$	An area of Δ's meeting at common vertex and base through the same straight line is proportional to their bases.

• If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, then the triangles on both sides of the perpendicular are similar to the whole triangle and also to each other.

Coordinate geometry:

Distance formula

• Distance =$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$. (The same formula is to be used to find the length of line segment, sides of a triangle, square, rectangle, parallelogram etc.)

Section formula

Point (x, y) = $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

- If point P(x, y) divides AB in k:1, then the coordinates of point P will be ($kx_2/k+1$, $ky_2+y_1/k+1$)
- Mid-point = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- Centroid of a triangle = $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
- To prove **co-linearity** of the given three points A,B, and C, You have to find length of AB, BC, AC then use the condition AB + BC = AC.

Area of triangle

- Area of triangle: $1/2[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$
- Area cannot be negative so, we shall ignore negative sign if it's occurring in the problem.
- If the area of the triangle is zero, then vertices of the triangle are collinear.
- To find the area of quadrilateral we shall divide it into two triangles by joining two opposite vertices, find their areas and add them.

Introduction to trigonometry:

Trigonometry Ratio Table

Angles (In Degrees)	0°	30°	45°	60°	90°	180°	270°	360°
Angles (In Radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	1
cot	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined
csc	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1

- Wherever 'Square' appears think of using the identities

(i) $\sin^2\theta + \cos^2\theta = 1$ (ii) $\sec^2\theta - \tan^2\theta = 1$ (iii) $\mathrm{Cosec}^2\theta - \cot^2\theta = 1$

- Try to convert all the values of the given problem in terms of Sin θ and Cos θ
- Cosec θ may be written as 1/Sin θ
- Sec θ may be written as 1/Cos θ
- Cot θ may be written as 1/Tan θ
- Tan θ may be written as Sin θ / Cos θ
- Wherever fractional parts appears then think of taking their 'LCM'
- Think of using $(a + b)^2$, $(a - b)^2$, $(a + b)^3$, $(a - b)^3$ formulae etc.

- Rationalise the denominator [If a + b, (or) a – b format is given in the denominator]
- You may separate the denominator

  For Ex: $\frac{\text{Sin }\theta + \text{Cos }\theta}{\text{Sin }\theta}$ as $\frac{\text{Sin }\theta}{\text{Sin }\theta} + \frac{\text{Cos }\theta}{\text{Sin }\theta}$ = 1 + Cot θ

- Sin (90 – θ) = Cos θ : Cos (90 – θ) = Sin θ
- Sec (90 – θ) = Cosec θ : Cosec (90 – θ) = Sec θ
- Tan (90 – θ) = Cot θ : Cot (90 – θ) = Tan θ

Circles:

•The tangent to a circle is perpendicular to the radius through the point of contact.

•The lengths of the two tangents from an external point to a circle are equal.

Area related to circle:

- Area of a Circle = πr^2
- Perimeter of a Circle = 2πr
- Area of sector = θ/360° (πr^2)
- Length of an arc = θ/360° (2πr)
- Area of ring = π ($R^2 - r^2$)
- Area of segment = Area of the corresponding sector – Area of the corresponding triangle

$= \frac{R^2}{2}\left(\frac{\theta\pi}{180^0} - sin\theta\right)$

Where, θ is the central angle in degrees.

- Distance moved by a wheel in one revolution = Circumference of the wheel.
- Angle described by minute hand in 60 minutes = 360^0
- Angle described by minute hand in 1minute = $\frac{360^0}{60^0} = 6^0$
- Number of revolutions = $\frac{\text{Total distance moved}}{\text{Circumference of the wheel}}$

Surface areas and volume:

Cylinder

Volume of a cylinder = $\pi r^2 h$

Curved surface area = 2πrh

Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r (h + r)$

Volume of hollow cylinder = $\pi R^2 h - \pi r^2 h = \pi(R^2 - r^2)h$

TSA of hollow cylinder = Outer CSA + Inner CSA + 2 Area of ring

= $2\pi Rh + 2\pi rh + 2[\pi R^2 - \pi r^2]$

Cone

Volume of a Cone = $\frac{1}{3}\pi r^2 h$

CSA of a Cone = πrℓ (Here 'ℓ' refers to 'Slant Height') [where $\ell = \sqrt{(h^2 + r^2)}$]

TSA of a Cone = $\pi r\ell + \pi r^2 = \pi r (\ell + r)$

Frustum

Volume of a frustum $= \frac{1}{3}\pi h\,[R^2 + r^2 + Rr]$

CSA of a frustum = $\pi\ell[R + r]$ (Here 'ℓ' refers to 'Slant height') [where $\ell = \sqrt{(h^2 + (R-r)^2)}$]

TSA of a frustum = $\pi\ell(R + r) + \pi r^2 + \pi R^2 =$

Sphere

Surface area of a Sphere = $4\pi r^2$ (In case of Sphere, CSA = TSA i.e. they are same)

Volume of hemisphere = $\frac{2}{3}\pi r^3$ [Take half the volume of a sphere]

CSA of hemisphere = $2\pi r^2$ [Take half the SA of a sphere]

TSA of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Volume of a sphere $= \frac{4}{3}\pi r^3$

Volume of spherical shell = Outer volume – Inner volume = $\frac{4}{3}\pi\,(R^3 - r^3)$

While solving the problems based on combination of solids it would be better if you take common.

- T.S.A of combined solid = C.S.A of solid 1 + C.S.A of solid 2 + C.S.A of solid 3
- If a solid is melted and, recast into number of other small solids, then

Volume of the larger solid = No of small solids x Volume of the smaller solid

For Ex: A cylinder is melted and cast into smaller spheres. Find the number of spheres

Volume of Cylinder = No of sphere × Volume of sphere

- If an 'ice cream cone with hemispherical top' is given then you have to take

(a) Total Volume = Volume of Cone + Volume of Hemisphere

(b) Surface area = CSA of Cone + CSA of hemisphere

Statistics:

Mean

The mean for grouped data can be found by:

(i) The direct method: $\bar{X} = \frac{\sum fixi}{\sum fi}$

(ii) The assumed mean method: $\bar{X} = a + \frac{\sum f_i d_i}{\sum fi}$, where $d_i = x_i - a$

(iii) The step deviation method: $\bar{X} = a + \frac{\sum f_i u_i}{\sum fi} \times h$, where $u_i = \frac{x_i - a}{h}$

Mode

The mode for the grouped data can be found by using the formula:

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

l = lower limit of the modal class.

f_1 = frequency of the modal class.

f_0 =frequency of the preceding class of the modal class.

f_2 = frequency of the succeeding class of the modal class.

h = size of the class interval.

Median

The median for the grouped data can be found by using the formula:

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

l = lower limit of the median class.
n = number of observations.
cf = cumulative frequency of class interval preceding the median class.
f = frequency of median class.
h = class size.

• **Empirical Formula: Mode = 3 median –2 mean.**

Probability:

Probability of an event: P (event) = $\frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$

In a deck of playing cards, there are four types of cards:
♠ (Spades in Black colour) having A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, K, and Q total 13 cards
♣ (Clubs in Black colour) having A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, K, and Q total 13 cards
♥ (Hearts in Red colour) having A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, K, and Q total 13 cards
♦ (Diamond in Red colour) having A, 2, 3, 4, 5, 6, 7,8, 9, 10, J, K, and Q total 13 cards
52 cards

• Jack, King and Queen are known as 'Face Cards', as these cards are having some pictures on it.
• Always remember **Ace is not a face card** as it doesn't carry any face on it.