CLASS XII (2019-20)
PHYSICS (042)
SAMPLE PAPER-3

Time : 3 Hours **Maximum Marks : 70**

General Instructions :

(i) All questions are compulsory. There are 37 questions in all.

(ii) This question paper has four sections: Section A, Section B, Section C, Section D.

(iii) Section A contains twenty questions of one mark each, Section B contains seven questions of two marks each, Section C contains seven questions of three marks each and Section D contains three questions of five marks each.

(iv) There is no overall choice. However, internal choices has been provided in two question of one marks each, two question of two marks, one question of three marks and three questions of five marks weightage. You have to attempt only one of the choices in such questions.

(v) You may use the following values of physical constants wherever necessary.

$c = 3 \times 10^8$ m/s, $h = 6.63 \times 10^{-34}$Js, $e = 1.6 \times 10^{-19}$C, $\mu_0 = 4\pi \times 10^{-7}$TmA^{-1},

$\varepsilon_0 = 8.854 \times 10^{-12}$C^{2}N^{-1}m^{-2}, $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$Nm^{2}C^{-2}, $m_e = 9.1 \times 10^{-31}$kg,

Mass of neutron $= 1.675 \times 10^{-27}$kg,

Mass of proton $= 1.673 \times 10^{-27}$kg, Avogardro's number $= 6.023 \times 10^{23}$ per gram mole,

Boltzmann constant $= 1.38 \times 10^{-23}$JK^{-1}.

Section A

DIRECTION : (Q 1-Q 10) Select the most appropriate option from those given below each question

1. A polaroid is used to **(1)**

(a) reduce intensity of light
(b) produce polarised light
(c) increase intensity of light
(d) produce unpolarised light

Ans : (b) produce polarised light

We know that a polaroid is a device which is used to produce polarised light, when unpolarised light (i.e., ordinary light) is incident on it.

2. A strong argument for the particle nature of cathode rays is that they **(1)**

(a) cast shadow
(b) produce fluorescence
(c) travel through vacuum
(d) get deflected in magnetic field

Ans : (a) cast shadow

We know that cathode rays travel in straight lines towards anode and cast shadow of obstacle in their way. It indicates the particle nature of cathode rays.

3. A radioactive substance emits **(1)**

(a) α-rays (b) β-rays
(c) γ-rays (d) all of these

Ans : (d) all of these

We know that radioactivity is the spontaneous disintegration of the nucleus of a radioactive substance, from which α, β and γ-rays are emitted.

4. The torque acting on electric dipole of dipole moment $\vec{P}$ placed in electric field of intensity $\vec{E}$ is **(1)**

(a) $\vec{P} \times \vec{E}$ (b) $\vec{P} \cdot \vec{E}$
(c) pE (d) $\vec{P}/\vec{E}$

Ans : (a) $\vec{P} \times \vec{E}$

Torque = Either force $\times$ Perpendicular distance between the two forces

$$\begin{aligned}\tau &= qE \times 2a\sin\theta \\ &= (q \times 2a)E\sin\theta \text{(Since, } P = q\times 2l) \\ &= PE\sin\theta \\ &= \vec{P} \times \vec{E}\end{aligned}$$

5. A 10 μF capacitor is charged to a potential difference 50 V and it is then connected to another uncharged capacitor in parallel. If common potential difference becomes 20 V, then capacitance of the second capacitor is **(1)**

(a) 10 μF (b) 15 μF

(c) 20 μF (d) 30 μF

Ans : (b) 15 μF

Given, Capacitance of first capacitor,

$$C_1 = 10\,\mu\text{F}$$

potential difference of first capacitor,

$$V_1 = 50\text{ V}$$

Potential difference of second capacitor,

$$V_2 = 0 \text{ (because it is uncharged)}$$

and common potential,

$$V = 20\text{ V}$$

We Know that common potential difference,

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

or $$20 = \frac{(10 \times 50) + C_2 \times 0}{10 + C_2}$$

$$20 = \frac{500}{10 + C_2}$$

or $$200 + 20C_2 = 500$$

or $$20\,C_2 = 300$$

or $$C_2 = \frac{300}{20} = 15\,\mu\text{F}$$

Where, $C_2 =$ Capacitance of second capacitor.

6. If a bar magnet of length 10 cm and pole strength 40 A-m is placed at an angle of 30° in a uniform magnetic field of intensity 2×10^{-4} T, then torque acting on it is **(1)**

(a) 8×10^{-4} N-m (b) 6×10^{-4} N-m

(c) 4×10^{-4} N-m (d) 2×10^{-4} N-m

Ans : (c) 4×10^{-4} N-m

Given, Length of bar magnet,

$$2l = 10 \text{ cm} = 0.1 \text{ m}$$

Pole strength, $m = 40\,\text{A-m}$

Angle between bar magnet and magnetic field,

$$\theta = 30°$$

and intensity of magnetic field,

$$B = 2 \times 10^{-4}\text{ T}$$

We know that magnetic moment,

$$M = m \times 2l$$

$$= 40 \times 0.1$$

$$= 4 \text{ A-m}^2$$

Therefore, torque acting on the bar magnet in uniform magnetic field

$$\tau = MB\sin\theta$$

$$= 4 \times (2 \times 10^{-4}) \times \sin 30°$$

$$= (8 \times 10^{-4}) \times 0.5$$

$$= 4 \times 10^{-4}\text{ N-m.}$$

7. In a circuit with a coil of resistance 2Ω, the magnetic flux changes from 2 Wb to 10 Wb in 0.2 s. The charge that flows in the coil during this time is **(1)**

(a) 5 C (b) 4 C

(c) 1 C (d) 0.8 C

Ans : (b) 4 C

Given, Resistance of coil, $R = 2\Omega$,

Initial magnetic flux, $\phi_1 = 2\text{ Wb}$,

Final magnetic flux, $\phi_2 = 10\text{ Wb}$

and time-taken, $t = 0.2\text{ s}$

We know that change in magnetic flux,

$$\Delta\phi = \phi_2 - \phi_1$$

$$= 10 - 2 = 8\text{ Wb}$$

Therefore charge flowing through the coil

$$q = \frac{\Delta\phi}{R} = \frac{8}{2} = 4\text{ C}$$

8. Reactance of a capacitor of capacitance C for an alternating current of frequency $\frac{400}{\pi}$ Hz is 25 Ω. The value of C is **(1)**

(a) 25 μF (b) 50 μF

(c) 75 μF (d) 100 μF

Ans : (b) 50 μF

Given,Capacitance of capacitor $= C$

Frequency of current, $f = \frac{400}{\pi}\text{Hz}$

Reactance of capacitor, $X_C = 25\,\Omega$

We know that reactance of the capacitor,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$C = \frac{1}{2\pi f X_C}$$

$$= \frac{1}{2\pi \times \frac{400}{\pi} \times 25}$$

$$= 50 \times 10^{-6}\,F$$

$$F = 50\,\mu\text{F}$$

9. In an oscillating LC-circuit, effective inductance is 200 μH. If frequency of oscillation is 1200 kHz, then capacitance of capacitor in the circuit is **(1)**

(a) 11 pF (b) 22 pF
(c) 44 pF (d) 88 pF

Ans : (d) 88 pF

Given, Inductance, $L = 200\,\mu\text{H} = 200 \times 10^{-6}\,\text{H}$

and frequency of oscillation,

$$f = 1200\,\text{kHz} = 1200 \times 10^{3}\,\text{Hz}$$

We know that frequency of LC-oscillation,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

or

$$C = \frac{1}{4\pi^2 f^2 L}$$

$$= \frac{1}{4\pi^2(1200 \times 10^3)^2 \times (200 \times 10^{-6})}$$

$$= 88 \times 10^{-12}\,\text{F} = 88\,\text{pF}$$

10. The magnifying power of a magnifying glass of power 12 dioptre is **(1)**

(a) 4 (b) 1200
(c) 3 (d) 25

Ans : (a) 4

Given,Magnifying power of glass= 12

Power of the lens, $P = \frac{1}{f}$

$$f = \frac{1}{P} = \frac{1}{12}\,\text{m}$$

$$= \frac{100}{12}\,\text{cm}$$

Magnifying lower of simple microscope is given by,

$$m = 1 + \frac{D}{f}$$

$$= 1 + \frac{25 \times 12}{100}$$

$$= 1 + 3 = 4$$

DIRECTION : (Q11-Q15) Fill in the blanks with appropriate answer.

11. .... spectrum is also called molecular spectrum. **(1)**

Ans : Band

We know that band spectrum is obtained from molecules in the gaseous state. Therefore band spectrum is also called molecular spectrum.

12. According to classical theory, the path of an electron in Rutherford's atom is .... **(1)**

Ans : spiral

We know according to classical theory that the path of an electron in Rutherford's atom is spiral towards the nucleus due to the continuous loss of energy of electron.

13. If the current gain in common-emitter is 100, then the emitter current in a transistor for a base current of 5 mA, is .... mA. **(1)**

Ans : 0.505 mA

Given,
Current gain in common-emitter,

$$\beta = 100$$

Base current, $I_B = 5\,\mu\text{A} = 0.005\,\text{mA}$
We know that collector current,

$$I_C = \beta \times I_B$$

$$= 100 \times 0.005 = 0.5\,\text{mA}$$

Therefore,

Emitter current, $I_E = I_C + I_B$

$$= 0.5 + 0.005 = 0.505\,\text{mA}$$

14. In a circuit, the internal resistance of cell is equal to the external resistance. If e.m.f. of the cell is 4 V, then the potential difference across the terminals of the cell is .... V. **(1)**

Ans : 2 V

Given, Internal resistance of cell, $r = R$

and e.m.f. of cell, $E = 4\,\text{V}$
We know that current in the circuit,

$$I = \frac{E}{R + r} = \frac{E}{R + R} = \frac{E}{2R}$$

Therefore potential difference across the terminals of the cell,

$$V = I \times R$$

$$= \frac{E}{2R} \times R = \frac{E}{2}$$

$$= \frac{4}{2} = 2\,\text{V}$$

where, R = external resistance in the circuit

or

A galvanometer having a resistance of 8 Ω is shunted by a wire of resistance 2 Ω. If the total current is 1 A, then current passing through the shunt will be .... A. **(1)**

Ans : 0.8 A

Given, Resistance of galvanometer,

$$G = 8\,\Omega$$

Shunt resistance, $S = 2\,\Omega$

And, total current, $I = 1\text{ A}$

We know that current passing through the galvanometer,

$$I_g = \left(\frac{S}{S+G}\right)\times I$$

$$= \left(\frac{2}{2+8}\right)\times 1 = 0.2\text{ A}$$

Therefore, current passing through the shunt,

$$= I - I_g = 1 - 0.2 = 0.8\text{ A}$$

15. The pressure exerted by an electromagnetic wave of intensity I on a non-reflecting surface is (where $c =$ Velocity of light) .... **(1)**

Ans : $\frac{I}{c}$

Given, Intensity of electromagnetic wave $= I$

and, Velocity of light $= c$

We know that intensity of electromagnetic wave,

$$I = \frac{E}{At} = \frac{F\times d}{At} = \frac{Fc}{A}$$

or $\frac{F}{A} = \frac{I}{c}$

or $P = \frac{I}{c}$

where $P =$ Pressure exerted by the electromagnetic wave equal to $\frac{F}{A}$.

DIRECTION : (Q16-Q20) Answer the following:

16. The force $\vec{F}$ experienced by a particle of charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by $\vec{F} = q(\vec{v}\times\vec{B})$. Which pair of vectors is always at right angles to each other?

Ans :

Pair of vectors always at right angle are $\vec{F}$ and $\vec{v}, \vec{F}$ and $\vec{B}$. **(1)**

17. What is the relationship between amplitudes of electric and magnetic fields in free space?

Ans :

$$\frac{\text{Amplitude of electric field }(E_0)}{\text{Amplitude of magnetic field}(B_0)} = c \text{ (speed of light)}$$ **(1)**

18. Find the ratio of De-Broglie wavelengths associated with electrons accelerated through 25V and 36 V.

Ans :

Since, $\lambda \propto \frac{1}{\sqrt{V}}$. Therefore,

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{V_2}{V_1}}$$

$$= \sqrt{\frac{36}{25}} = \frac{6}{5}$$ **(1)**

19. Why is it said that nuclear forces are saturated forces?

Ans :

A nucleon in a nucleus experiences force due to nearest neighbours only and not due to all nucleons, hence nuclear force is said to be saturated. **(1)**

20. The radius of inner most orbit of hydrogen atom is 5.1×10^{-11} m. What is the radius of orbit in second excited state?

Ans :

Radius of orbit is given by

$$r_n \propto n^2$$

where, $r_n =$ radius of nth orbit

$n =$ principal quantum number

For ground state, $n = 1$

For second excited state,

$$n = 3$$

$$\frac{r_3}{r_1} = \left(\frac{3}{1}\right)^2 = 9$$

$$r_3 = 9r_1$$

$$= 9\times 5.1\times 10^{-11}$$

$$= 4.59\times 10^{-10}\text{ m}$$ **(1)**

or

Compare the radii of two nuclei having mass numbers 3 and 81, respectively.

Ans :

Radius of the nucleus,

$$R = R_0 A^{1/3}$$

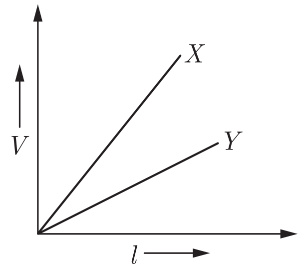
where, $A =$ mass number

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3}$$

$$\frac{R_1}{R_2} = \left(\frac{3}{81}\right)^{1/3} = \frac{1}{3}$$

Section B

21. The variation of potential deference V with length in case of two potentiometers X and Y is as shown in the given diagram. Which one of these two, you prefer for comparing e.m.f's of two cells and why?



Ans :

Here, slope of $V-l$ graph $=\frac{V}{l}=k=$ Potential gradient

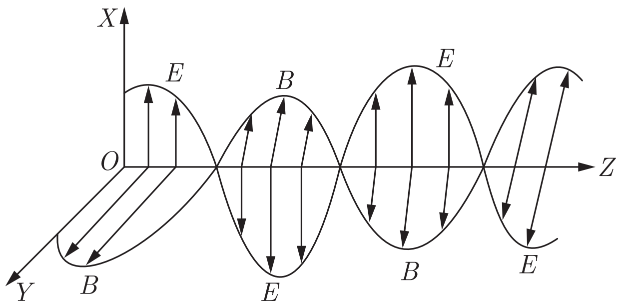
Slope of X > Slope of Y.

$\therefore$ Y has a smaller potential gradient than X. A potentiometer with smaller potential gradient is preferred because a larger length of the wire will be required to balance the e.m.f so that measurement will be accurate. **(2)**

22. Draw a sketch of a plane electromagnetic wave propagating along the Z-direction. Depict clearly the directions of electric and magnetic fields varying sinusoidally with Z.

Ans :

The propagation of electromagnetic waves along Z-axis is shown below.



(2)

23. What is (1) momentum and (2) energy of photon of wavelength 0.01Å?

Ans :

(1) Momentum, $p=\frac{h}{\lambda}=\frac{h}{0.01\times10^{-10}}$

$=10^{12}\times6.6\times10^{-34}$

$=6.6\times10^{-22}$ kg/s

(2) Energy, $E=\frac{hc}{\lambda}$

$$=\frac{6.6\times10^{-34}\times3\times10^{8}}{0.01\times10^{-10}}\cong2\times10^{-13}\text{ J}$$

(2)

24. When a capacitor is connected in series LR circuit, the alternating current flowing in the circuit increases. Explain why?

Ans :

Impedance of series LR circuit

$=Z=\sqrt{R^2+X_L^2}$

When capacitor is also connected in circuit, impedance,

$$Z'=\sqrt{R^2+(X_L-X_C)^2}$$

Clearly $Z'<Z$, so value of current $\left[I=\frac{V}{Z}\right]$ increases.

(2)

25. A short bar magnet placed with its axis at 30° to a uniform magnetic field of $0.2\,T$ experiences a torque of 0.060 Nm.

1. Calculate magnetic moment of the magnet
2. Find out what orientation of the magnet corresponds to its stable equilibrium in the magnetic field.

Ans :

Given : $\theta=30°$

$B=0.2\,T$

$\tau=0.060\,Nm$

$\tau=MB\sin\theta$

$$M=\frac{\tau}{B\sin\theta}=\frac{0.060}{0.2\times\sin30°}$$

$=0.6\,J/T$

For stable equilibrium, $\tau=0$

$\therefore$ $\sin\theta=0$

$\theta=0°$ **(2)**

26. Deduce ohm's law using the concept of drift velocity

Ans :

As we know,

$$v_d=-\frac{eE\tau}{m}$$

and $E=-\frac{V}{l}$

$$v_d=\frac{eV\tau}{ml}$$

and $I=nAev_d$

$\therefore$ $I=neA\left[\frac{eV}{ml}\right]$

$$\tau=\left[\frac{ne^2A\tau}{ml}\right]$$

$$I=\frac{1}{R}V$$

or $V=IR$

where, $R=\frac{m\ell}{nAe^2\tau}$ is a constant for a particular conductor at a particular temperature and is called the resistance of the conductor. **(2)**

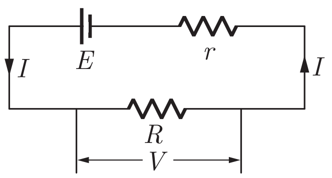
or

Derive an expression for the internal resistance of a cell in terms of e.m.f and terminal potential difference of a cell.

Ans :

Let R be the external resistance and r be the internal resistance of the cell of e.m.f E.

$\therefore$ In closed circuit, total resistance of the circuit $= R \times r$.



$\therefore$ Current, $I = \dfrac{E}{R+r}$

Potential difference, $V = E - Ir$

($Ir =$ Potential drop across internal resistance)

$$Ir = E - V$$

$$r = \frac{E-V}{I}$$

By ohm's law, $V = IR$

$\therefore$ $I = \dfrac{V}{R}$

$\therefore$ $r = \left[\dfrac{E-V}{V}\right]R$

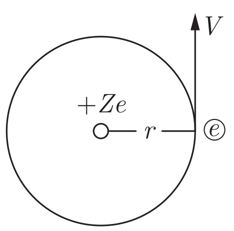
$$r = \left[\frac{E}{V} - 1\right]R \qquad \textbf{(2)}$$

27. Prove that the radius of the nth Bohr orbit of an atom is directly proportional to n^2, where n is principal quantum number.

Ans :

Radius of nth Bohr orbit:

To keep electron in orbit, centripetal force equal to electrostatic force



Therefore, $\dfrac{mv^2}{r} = \dfrac{k'Ze^2}{r^2}$

where, $k' = \dfrac{1}{4\pi\varepsilon_0}$

$$r = k'Ze^2/mv^2 \quad ...(1)$$

where, m is the mass of the electron and v its speed in an orbit of radius r. Bohr's quantisation condition for angular momentum is,

$$mvr = \frac{nh}{2\pi}$$

$$r = \frac{nh}{2\pi mv} \qquad ...(2)$$

From Eqs. (1) and (2), we get

$$\frac{k'Ze^2}{mv^2} = \frac{nh}{2\pi mv}$$

$$v = \frac{2\pi k'Ze^2}{nh}$$

Putting this value of v in Eq. (2), we get

$$r = \frac{nh}{2\pi m}\frac{nh}{2\pi k'Ze^2}$$

$$= \frac{n^2h^2}{4\pi^2 mk'Ze^2}$$

$$r \propto n^2 \qquad \textbf{(2)}$$

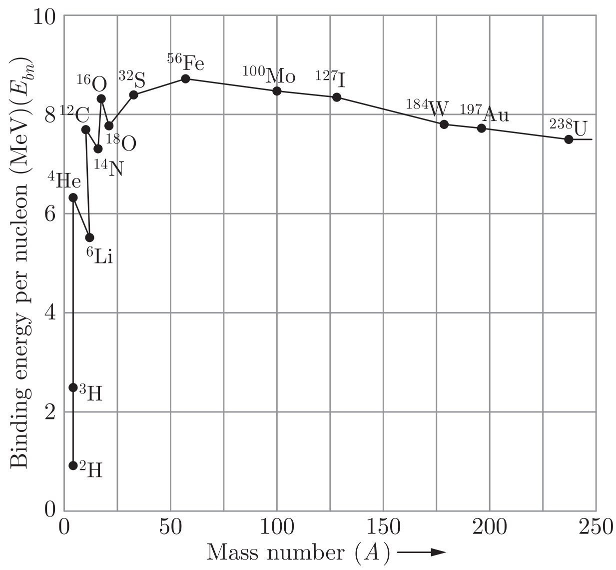
or

Draw a graph showing the variation of binding energy per nucleon with mass number. Hence, from the graph, explain why elements having mass number A between 30 and 170 have almost same binding energy.

Ans :

Binding Energy Curve:

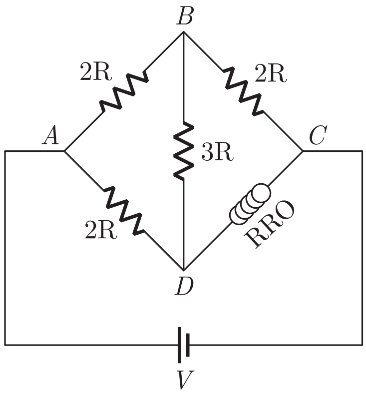
It is a plot of the binding energy per nucleon E_{bn} versus the mass number A for a large number of nuclei.



Elements having $30 < A < 170$ have almost constant E_{bn} as the nuclear force are short range forces. In large nuclei, a nucleon will inside the nucleus experiences force only due to its immediate neighbours. Its binding energy will thus depend only on the member of immediate neighbours. Thus, addition of extra nucleons as A increases for larger nuclei, does not change the binding energy per nucleon (E_{bn}) of the corresponding nuclei. Thus, E_{bn} remain almost constant. **(2)**

Section C

28. 1. Use Kirchhoff's rules to obtain the balance condition in a Wheatstone bridge.
2. Calculate the value of R in the balance condition of the Wheatstone bridge, if the carbon resistor connected across the arm CD has the colour sequence red, red and orange, as is shown in the figure.



Ans :

1. In balance condition of Wheatstone bridge,

$$I_g = 0$$

Using Kirchhoff's loop law for loop ABDA,

$$PI_1 - R(I - I_1) = 0$$

$$PI_1 = R(I - I_1) \quad ...(i)$$

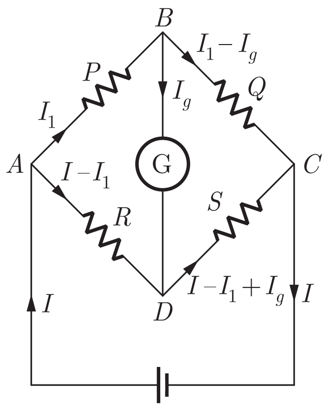
For loop BCDB,

$$Q(I_1 - I_g) = S(I - I_1 - I_g)$$

$$I_g = 0$$

$$QI_1 = S(I - I_1) \quad ...(ii)$$

Solving (i) & (ii), we get



$$\frac{P}{Q} = \frac{R}{S}$$

or $\quad \dfrac{P}{R} = \dfrac{Q}{S}$

2. Let a carbon resistor S is given to the bridge arm CD Then,

$$\frac{2R}{R} = \frac{2R}{S}$$

$$\therefore \quad \frac{R}{S} = 1$$

$$R = S = 22 \times 10^3\,\Omega$$ **(3)**

29. A long straight wire AB carries a current of 4 A. A proton P travels at 4×10^6 m/s parallel to the wire 0.2 m from it and in a direction opposite to the current. Calculate the force which the magnetic field of current exerts on the proton. Also specify the direction of the force.

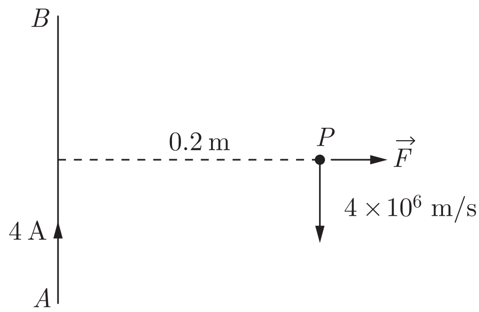
Ans :

Here, $\quad I = 4A$,

$v = 4 \times 10^6$ m/s

$r = 0.2$ m

$\mu_0 = 4\pi \times 10^{-7}$ Tm/A



Magnetic field due to a straight wire carrying current,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 4}{2\pi \times 0.2}$$

$$= 4 \times 10^{-6}\,T$$

The field acts at right angles to the direction of moving proton.

$\therefore$ Force exerted by the magnetic field on the moving proton.

$$F = qvB\sin\theta [\text{Here } \theta = 90°]$$

$$= 1.6 \times 10^{-19} \times 4 \times 10^6 \times 4 \times 10^{-6} \times 1$$

$$= 2.56 \times 10^{-20} N. \text{ [Away from the wire]}$$

(3)

30. For a single slit of width "a", the first minimum of the interference pattern of a monochromatic light of wavelength λ occurs at an angle of λ/a. At the same angle of λ/a, we get a maximum for two narrow slits separated by a distance "a" Explain.

Ans :

The path difference between two secondary wavelets,

$$n\lambda = a\sin\theta$$

Since, θ is very small $\sin\theta = \theta$. So, for the first order diffraction $n = 1$, the angle is λ/a.

Now we know that must be very small $\theta = 0$ (nearly) because of which the diffraction pattern is minimum.

Now for interference case, for two interfering waves of intensity I_1 and I_2 we must have two slits separated by a distance. We have the resultant intensity,

$$I = I_1 + I_2 + 2\sqrt{I_1 + I_2\cos\theta}$$

Since, $\theta = 0$ (nearly) corresponding to angle λ/a so, $\cos\theta = 1$ (nearly)

So, $I = I_1 + I_2 + 2\sqrt{I_1 + I_2\cos\theta}$

$$I = I_1 + I_2 + 2\sqrt{I_1 + I_2\cos(0)}$$

$$I = I_1 + I_2 + 2\sqrt{I_1 + I_2}$$

Thus, resultant intensity is sum of the two intensities, so there is a maxima corresponding to the angle λ/a.

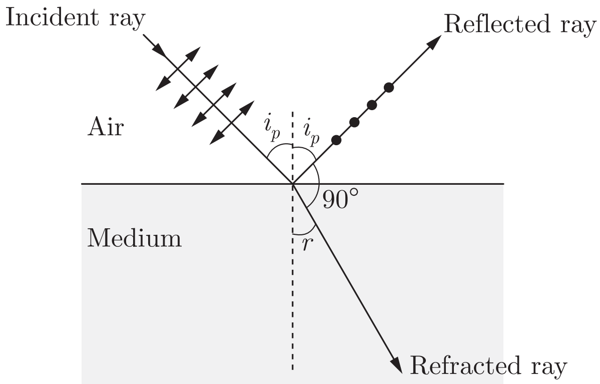
This is why, at the same angle of λ/a, we get a maximum for two narrow slits separated by a distance "a". **(3)**

31. (i) Show, with a suitable diagram, how unpolarised light can be polarised by reflection.

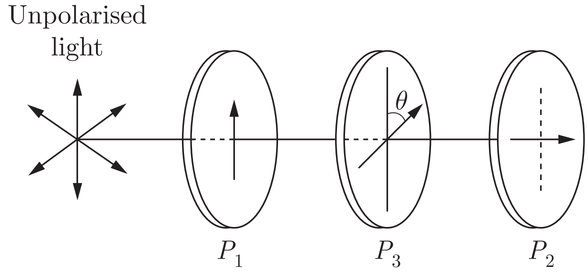
(ii) Two polaroids P_1 and P_2 are placed with their pass axes perpendicular to each other. An unpolarised light of intensity I_0 is incident on P_1. A third polaroid P_3 is kept in between P_1 and P_2 such that its pass axis makes an angle of 60° with that of P_1. Determine the intensity of light transmitted through P_1, P_2 and P_3.

Ans :

(i) When unpolarised light is incident on the interface of two transparent media the reflected light is polarised. It the unpolarised light is incident at the angles 0° or 90° the reflected ray remains unpolarised. When the reflected wave is perpendicular to the refracted wave, the reflected wave is totally polarised. The angle of incidence in this case is called polarising angle or Brewster's angle (i_p).



(ii) According to questions P_1, P_2 and P_3 are placed as shown in the diagram



$$I'' = I'\cos^2\theta = \frac{I_0}{2}\cos^2(60^\circ)$$

$$I'' = \frac{I_0}{2}\left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$$

Therefore, a light of intensity $\frac{I_0}{8}$ will pass through P_3 and the angle between P_3 and P_2 will be 30° because of the condition given in the questions. Intensity of light after talling on P_2,

$$I' = I''\cos^2(\theta) = \frac{I_0}{8}\cos^2(30^\circ)$$

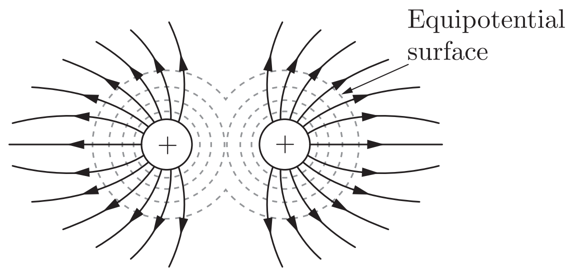
$$= \frac{I_0}{32}$$ **(3)**

32. (i) Depict the equipotential surfaces for a system of two identical positive point charges placed a distance d apart.

(ii) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points with positions r_1 and r_2 respectively in presence of external electric field E.

Ans :

(i) The figure is shown below,



(ii) By definition, electric potential energy of any charge q placed in the region of electric field is equal to the work done in bringing charge q from infinity to that point and given by

$$U = qV$$

Now, considering the electric potentials at positions r_1 and r_2 as V_1 and V_2 respectively. Therefore, total potential energy of the system of two charges q_1 and q_2 placed at points with position vectors r_1 and r_2 in the region of E is given by $U=$ Work done in bringing charge q, from infinite to that position in E is equal to work done for charge q_2 from infinite to that position in $E+$ work done to that of charge q_2 at these positions in presence of q_1.

$$\text{i.e.,} U = q_1 V_1 + q_2 V_2$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{|r_2 - r_1|} \quad \textbf{(3)}$$

or

A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor but has the thickness $2d/3$, where d is the separation between the plates. Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor.

Ans :

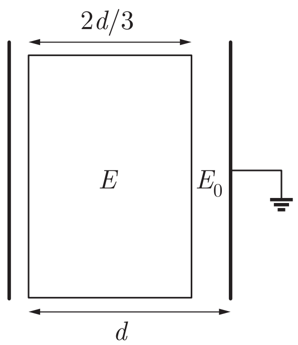
Let the potential difference across the plates of a parallel plate capacitor be V and d is the distance between them,

$$A = \text{Area of the plates}$$

Then electric field E_0 between them is given by,

$$E_0 = \frac{V}{d} = \frac{Q}{A\varepsilon_0}$$

When a slab of thickness $t = \frac{2}{3}d$ and dielectric constant K is introduced between the plates.



Then, $$V = E_0\left[d - \frac{2d}{3}\right] + E \times \frac{2d}{3}$$

$$= E_0\frac{d}{3} + \frac{E_0}{K}\frac{2d}{3} = E_0\frac{d}{3}\left[1 + \frac{2}{K}\right]$$

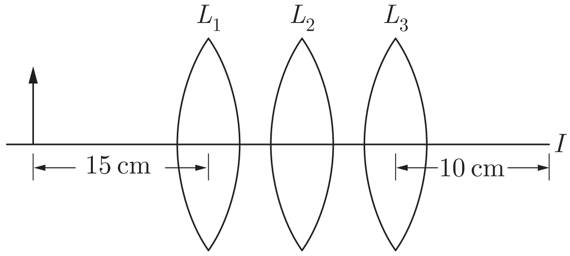
or $$V = \frac{Q}{A\varepsilon_0}\frac{d}{3}\left[1 + \frac{2}{K}\right] \left[\because E_0 = \frac{Q}{A\varepsilon_0}\right]$$

Therefore capacitance,

$$C = \frac{Q}{V} = \frac{3A\varepsilon_0}{d\left[1 + \frac{2}{K}\right]}$$

This is the required expression. **(3)**

33. You are given three lenses L_1, L_2 and L_3 each of focal length 10 cm. An object is kept at 15 cm in front of L_1, as shown. The final real image is formed at the focus I of L_3. Find the separation between L_1, L_2 and L_3.



Ans :

For lens L_1, $u = -15$ cm

$$v = ?$$

$$f = +10 \text{ cm}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{10} = \frac{1}{v} + \frac{1}{15}$$

Distance of image from lens L_1,

$$v = 30 \text{ cm}$$

For lens L_3, $\frac{1}{f''} = \frac{1}{v''} - \frac{1}{u''}$

Distance of image from lens L_3,

$$v'' = 10 \text{ cm}$$

$$\frac{1}{10} = \frac{1}{10} + \frac{1}{u''}$$

$$u'' = \infty$$

The refracted rays from lens L_1 becomes parallel to principal axis. It is possible only when image formed by L_1 lies at first focus of L_2 i.e., at a distance of 10 cm from L_2.

$\therefore$ Separation between L_1 and L_2

$$= 30 + 10 = 40 \text{ cm}$$

The distance between L_2 and L_3 may take any value. **(3)**

34. Obtain the relation between the decay constant and half life of a radioactive sample. The half life of a certain radioactive material against $a-$ decay is 100 days. After how much time, will the undecayed fraction of the material be 6.25%?

Ans :

Relation between decay constant and half life of a radioactive substance:

The number of atoms at any instant in a radioactive sample is given by,

$$N = N_0 e^{-\lambda t}$$

N = total number of atoms at any instant

N_0 = number of atoms in radioactive substance at $t = 0$

When, $t = T$ (where, T is the half life of the sample)

$$N = \frac{N_0}{2}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda T}$$

$$e^{\lambda T} = 2$$

Taking log on both the sides, we get

$$\lambda T = \log_e 2 = 2.303 \log_{10} 2$$

$$T = \frac{2.303 \log_{10} 2}{\lambda} = \frac{0.693}{\lambda}$$

Let t be the required time after which the undecayed fraction of the material will be 6.25 %.

$$\frac{6.25}{100} = \frac{1}{16}$$

$$N = \frac{N_0}{16}$$

But, $N = N_0\left(\frac{1}{2}\right)^n$

where, $N = \frac{t}{T}$

$$\frac{N_0}{16} = N_0\left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{2}\right)^4 = \left(\frac{1}{2}\right)^n$$

Hence $n = 4$

Time, $t = n \times T = 4 \times 100$

$= 400$ days **(3)**

Section D

35. (i) A point charge causes an electric flux of $-1 \times 10^3 \text{N-m}^2\text{C}^{-1}$ to pass through a spherical Gaussian surface of 10 cm radius centered on the charge.

(a) How much flux will pass through the surface, if the radius of the Gaussian surface is doubled?

(b) Find the value of the point charge.

(ii) Two point charges $q_A = 3\mu C$ and $q_B = -3\mu C$ are located 20 cm apart in vacuum. What is the electric field and its direction at the mid-point O of the line AB joining the two charges?

Ans :

(i) (a) Same since the charge enclosed in both cases is same, hence amount of flux does not change. **(1)**

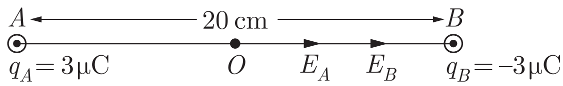
(b) As, we know $\phi_E = \frac{q}{\varepsilon_0}$ (Gauss's law)

$$q = \phi_E \varepsilon_0$$

$$= -1 \times 10^3 \times 8.85 \times 10^{-12}$$

$$\approx -8.8 \text{ nC}$$ **(1)**

(ii) Given, $AB = 20$ cm



$$AO = OB = 10 \text{ cm} = 0.1 \text{ m}$$

$$q_A = 3\,\mu C = 3 \times 10^{-6} \text{ C}$$

$$q_B = -3\mu C = -3 \times 10^{-6} \text{ C}$$

The electric field at a point due to a charge q is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$ **(1)**

where, r is the distance between charge and the point. Electric field due to q_A at O is E_A.

$$E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_A}{(AO)^2}$$

$$E_A = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2}$$

$$= \frac{27 \times 10^3}{0.1 \times 0.1}$$

$$= 2.7 \times 10^{-6} \text{ N/C}$$

The direction of E_A is A to O, i.e., towards O or towards OB as the electric field is always directed away from positive charge.

Electric field due to q_B at O is E_B.

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_B}{(OB)^2}$$

$$E_B = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2}$$

$$= \frac{27 \times 10^3}{0.1 \times 0.1}$$

$= 2.7 \times 10^6$ N/C **(1)**

The direction of E_B is O to B, i.e. towards B or towards OB as the electric field is always directed towards the negative charge.

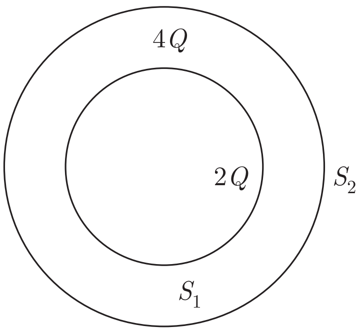
Now, we see that both E_A and E_B are in same direction. So, the resultant electric field at O is E. Hence,

$$E = E_A + E_B$$
$$= 2.7 \times 10^6 + 2.7 \times 10^6$$
$$= 5.4 \times 10^6 \text{ N/C}$$

The direction of E (resultant electric field) will be from O to B or towards B. **(1)**

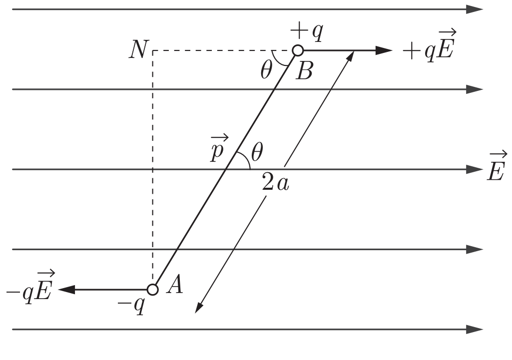
or

1. Deduce the expression for the torque acting on a dipole of dipole moment $\vec{p}$ in the presence of a uniform electric field $\vec{E}$.
2. Consider two hollow concentric spheres, S_1 and S_2, enclosing charges 2Q and 4Q respectively as shown in the figure.
   (a) Find out the ratio of the electric flux through them.
   (b) How will the electric flux through the sphere S_1 change if a medium of dielectric constant 'ε_r' is introduced in the space inside S_1 in place of air? Deduce the necessary expression.



Ans :

1. Dipole in a Uniform External Field,



Consider an electric dipole consisting of charges $-q$ and $+q$ and of length $2a$ placed in a uniform electric field $\vec{E}$ making an angle θ with electric field.

Force on charge $-q$ at $A = q\vec{E}$ (opposite to $\vec{E}$)

Force on charge $+q$ at $B = q\vec{E}$ (along $\vec{E}$).

When electric dipole is placed under the action of two equal and unlike parallel forces, it gives rise a torque on the dipole.

τ =Force×Perpendicular distance between the two forces

$$\tau = qE(AN) = qE(2a\sin\theta)$$
$$\tau = q(2a)E\sin\theta$$
$$\tau = pE\sin\theta$$
$$\therefore \quad \vec{\tau} = \vec{p} \times \vec{E}$$ **(2)**

2. (a) Charge enclosed by sphere,

$$S_1 = 2Q$$

By Gauss law, electric flux through sphere S_1 is,

$$\phi_1 = 2Q/\varepsilon_0$$

Charge enclosed by sphere,

$$S_2 = 2Q + 4Q = 6Q$$
$$\therefore \quad \phi_1 = 6Q/\varepsilon_0$$

The ratio of the electric flux is,

$$\phi_1\phi_2 = 2Q/\varepsilon_0/6Q/\varepsilon_0$$
$$= 2/6$$
$$= 1/3$$

(b) For sphere S_1, the electric flux is,

$$\phi' = 2Q/\varepsilon_r$$
$$\therefore \quad \phi'/\phi_1 = \varepsilon_0/\varepsilon_r$$
$$\phi' = \phi_1.\varepsilon_0/\varepsilon_r$$
$$\because \quad \varepsilon_r > \varepsilon_0$$
$$\therefore \quad \phi' < \phi_1$$ **(3)**

Therefore, the electric flux through the sphere S_1 decreases with the introduction of the dielectric inside it.

36. (i) Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it.

(ii) The current flowing through an inductor of self inductance L is continuously increasing. Plot a graph showing the variation of

(a) Magnetic flux versus the current
(b) Induced emf versus dI/dt
(c) Magnetic potential energy stored versus the current.

Ans :

(i) In the experiment, when N-pole of a magnet is moved towards the coil, the upper face of the coil acquires north polarity. So work has to be done against the force of repulsion in bringing the magnet closer to the coil.

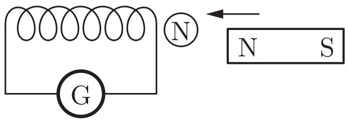


Fig. (a)

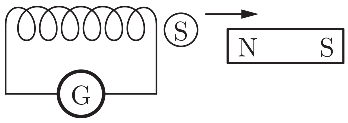


Fig. (b)

When the N-pole is moved away, south polarity is developed on the upper face of the coil. Therefore, work has to be done against the force of attraction in taking the magnet away from the coil

$\therefore$ Mechanical work done is converted into electrical energy of the coil.

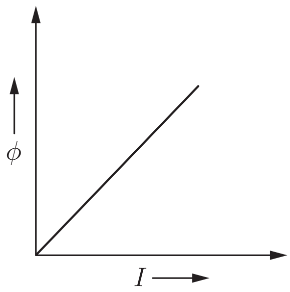
When the magnet does not move work done is zero. Therefore, induced current is also not produced **(2)**

(ii) (a) where

$l =$ Strength of current through the coil at any time

$\phi =$ Amount of magnetic flux linked with all turns of the coil at that time

and, $L =$ Constant of proportionality called coefficient of self induction



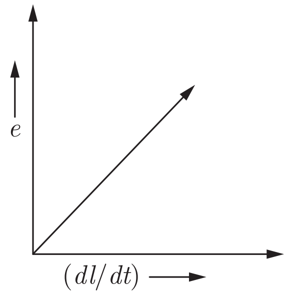
(1)

(b) Induced emf,

$$e = \frac{-d\phi}{dt}$$

$$= \frac{-d}{dt}(LI)$$

i.e., $e = -L\frac{dI}{dt}$



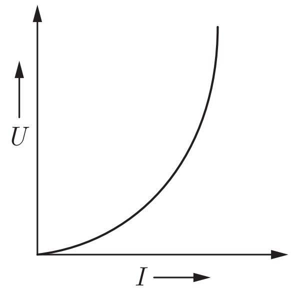
[The graph is drawn considering only magnitude of e. **(1)**

(c) Since magnetic potential energy is given by,

$$U = \frac{1}{2}LI^2$$

$$U \propto I^2$$

So, graph will be parabolic upwards.



(1)

or

A series L-C-R circuit is connected to an AC source having voltage $V = V_m \sin \omega t$. Derive the expression for the instantenous current I and its phase relationship to the applied voltage.

Obtain the condition for resonance to occur.

Define 'power factor'. State the conditions under which it is maximum and minimum

Ans :

Phase difference between voltage and current,

$$\tan\phi = \frac{X_L - X_C}{R} \quad \text{....(i)}$$

and, $$I_0 = \frac{V_0}{2} = \frac{V_0}{\sqrt{(X_L - X_C^2 +)R^2}}$$

$\therefore$ Expression of AC,

$$I = I_0 \sin(\omega t = \phi)$$

Condition for Resonance

Inductive reactance must be equal to capacitive reactance,

i.e., $X_L = X_C$

As, $X_L = X_C$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

where, $\omega_0 =$ resonant angular frequency.
Impedance becomes minimum and equal to ohmic resistance

i.e., $Z = Z_{minimum} = R$

AC becomes maximum,

$$\therefore \quad I_{max} = \frac{V_{max}}{Z_{min}} = \frac{V_{max}}{R}$$

Voltage and current arrives in same phase.

Power factor

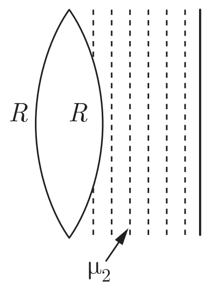
In an AC circuit, the ratio of true power consumption and virtual power consumption is termed as power factor.

i.e., $\cos\phi = \frac{P_{av}}{V_{rms}I_{rms}} = \frac{\text{True power}}{\text{Apparent power}}$

Also, $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C^2)}}$

The power factor is maximum i.e., $\cos\phi = +1$, in L-C-R series AC circuit when circuit is in resonance. The power factor is minimum when phase angle between V and I is 90° i.e., either pure inductive circuit or pure capacitive AC circuit. **(5)**

37. A biconvex lens with its two faces of equal radius of curvature R is made of a transparent medium of refractive index μ_1. It is kept in contact with a medium of refractive index μ_2 as shown in the figure.



1. Find the equivalent focal length of the combination.
2. Obtain the condition when this combination acts as a diverging lens.
3. Draw the ray diagram for the case $\mu_1 > (\mu_2 + 1)/2$, when the object is kept far away from the lens. Point out the nature of the image formed by the system.

Ans :

From the lens maker formula, we have

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Let f_1 and f_2 be the focal lengths of the two mediums. Then,

$$\frac{1}{f_1} = (\mu_1 - 1)\left[\frac{1}{R} - \left(-\frac{1}{R}\right)\right]$$

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{2}{R}\right)$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left[\left(-\frac{1}{R}\right) - \frac{1}{\infty}\right]$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(-\frac{1}{R}\right) \quad \textbf{(2)}$$

1. If f_{eq} is the equivalent focal length of the combination, then

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_{eq}} = \frac{2(\mu_1 - 1)}{R} - \frac{(\mu_2 - 1)}{R}$$

$$\frac{1}{f_{eq}} = \frac{2\mu_1 - \mu_2 - 1}{R}$$

$$f_{eq} = \frac{R}{2\mu_1 - \mu_2 - 1} \quad \textbf{(1)}$$

2. For the combination to behave as a diverging lens, $f_{eq} < 0$

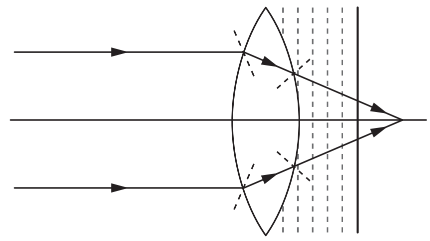
$$\frac{R}{2\mu_1 - \mu_2 - 1} < 0$$

$$2\mu_1 - \mu_2 - 1 < 0$$

$$\mu_1 < \frac{(\mu_2 + 1)}{2} \quad \textbf{(1)}$$

Which is the required condition

3. For $\mu_1 > (\mu_2 + 1)/2$ the combination will behave as the converging lens. So, an object placed far away from the lens will from image at the focus of the lens.



The image so formed will be real and diminished in nature. **(1)**

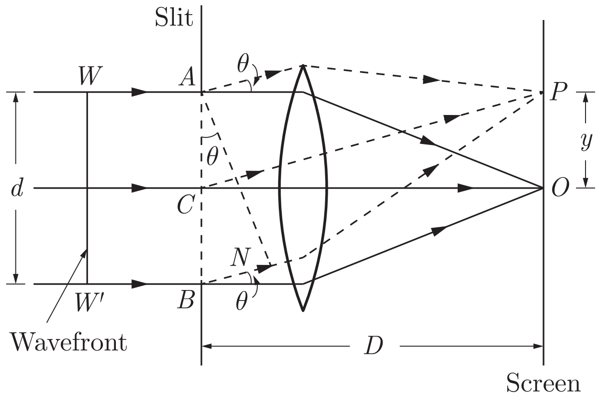
or

1. Using Huygen's construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.
2. Show that the angular width of the first diffraction fringe is half that of the central fringe.
3. Explain why the maxima at $\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{a}$

become weaker and weaker with increasing n.

Ans :

1. A Parallel beam of light with a plane wavefront WW' is made to fall on a single slit AB. As width of the slit $AB = d$ is of the order of wavelength of light, therefore diffraction occurs on passing through the slit.



Diffraction of light at single slit

The wavelets from the single wavefront reach the centre O on the screen in same phase and hence interfere constructively to give central maximum (bright fringe).
The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright band of decreasing intensity on both sides.
Consider a point P on the screen at which wavelets travelling in a direction, making angle θ with CO, are brought to focus by the lens. The wavelets from points A and B will have a path difference equal to BN.
From the right angled ΔANB, we have

$$BN = AB\sin\theta$$

or $$BN = d\sin\theta$$

To establish the condition for secondary minima, the slit is divide into 2, 4, 6, ... equal parts such that corresponding wavelets from successive regions interfere with path difference of $\lambda/2$.
or for nth secondary minimum, path difference

$$= d\sin\theta_n = n\lambda$$

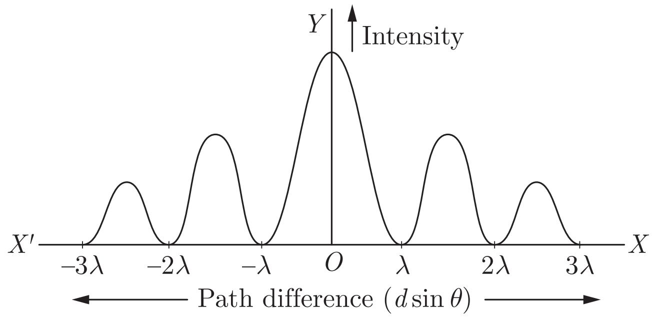
or $$\sin\theta_n = \frac{n\lambda}{d}(n = 1,2,3,...)$$

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7,..... equal parts such that corresponding wavelets from alternate regions interfere with path difference of $\lambda/2$.
or for nth secondary maximum, the slit can be divided into $(2n+1)$ equal parts.
Hence, for nth secondary maximum,

$$d\sin\theta_n = (2n+1)\frac{\lambda}{2}(n = 1,2,3,.....)$$

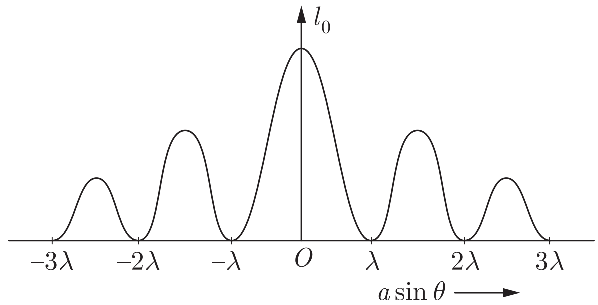
or $$\sin\theta_n = (2n+1)\frac{\lambda}{2d}$$

Hence, the diffraction pattern can be graphically shown as below. The point O corresponds to the position of point with path difference, $d\sin\theta = \lambda, 2\lambda,...$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima.



(2)

2. For central bright fringe, $\theta = 0^\circ$.



For first dark fringe,

$$a\sin\theta = \pm\lambda$$

or $$\sin\theta = \pm\frac{\lambda}{a}$$

If θ is small, then $\sin\theta \approx \theta$.

So, $$\theta = \pm\frac{\lambda}{a}$$

So, the half angular width of central maximum is,

$$\theta \approx \sin\theta = \frac{\lambda}{a}$$ **(2)**

3. On increasing the value of n, the part of slit contributing to the maximum decreases. Hence, the maxima becomes weaker. **(1)**