# CLASS12 DETERMINANTS



# Deepak Sir 9811291604

In this chapter, we shall study determinants up to order three only with real entries. Also, we will study various properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle, adjoint and inverse of a square matrix, consistency and inconsistency of system of linear equations and solution of linear equations in two or three variables using inverse of a matrix.

SHRI SAI MATERS TUITION CENTER 113-F ARJUN NAGAR,S J E NEW DELHI 110029 9811291604

# MATHEMATICS for CLASS 12 (CBSE NEW PATTEN)

By-DEEPAK SIR 9811 29 16 04

# **CHAPTER 4**

# DETERMINANTS

# **BV-DEEPAK SIR 9811291604**

To every square matrix A = [aij ] of order n, we can associate a number (real or complex) called determinant of the square matrix A, where aij= (i, j) th element of A.

This may be thought of as a function which associates each square matrix with a unique number (real or complex). If M is the set of square matrices, K is the set of numbers (real or complex) and *f*:  $M \rightarrow K$  is defined by f(A) = k, where  $A \in M$  and  $k \in K$ , then f(A) is called the determinant of A. It is also denoted by |A| or det A or A.

# Remarks

(i) For matrix A, | A | is read as determinant of A and not modulus of

(ii) Only square matrices have determinants.

Determinant of a matrix of order one

Let A = [a] be the matrix of order 1, then determinant of A is defined to be equal to a

Determinant of a matrix of order two

 $\begin{vmatrix} a11 & a12 \\ a21 & a22 \end{vmatrix} = a11a22 - a12a21$ 

Determinant of a matrix of order  $3X_3$ 

 $\begin{vmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{vmatrix}$ = a11(a22a33 - a32a23) - a12(a21a33 - a31a23) + a13(a21a32) - a22a31)

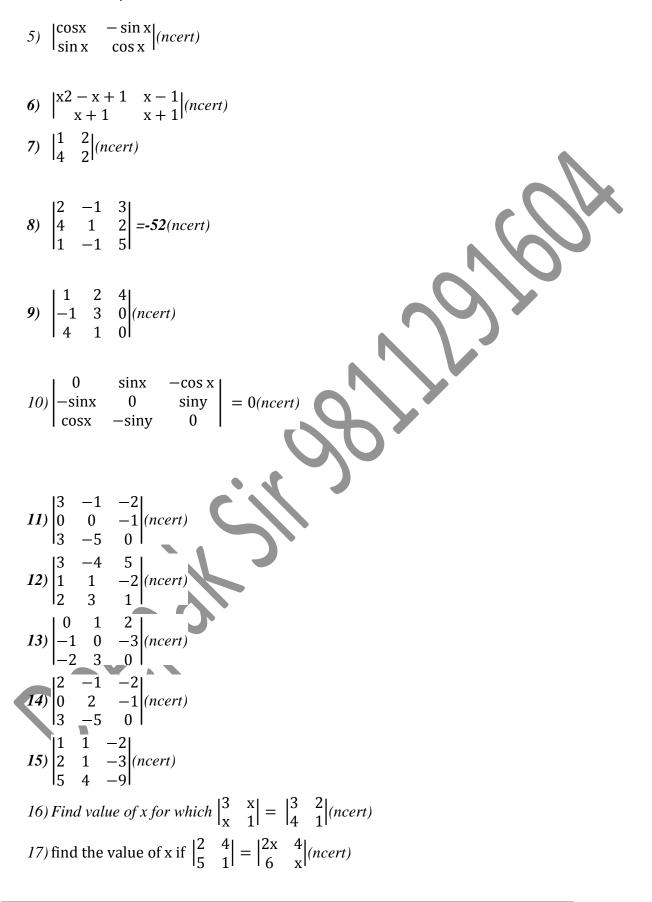
# Find the value of the following determinant.

# Tip

For easier calculations, we shall expand the determinant along that row or column which contains maximum number of zeros.

Evaluate

1) 
$$\begin{vmatrix} 1 & -3 \\ 5 & 4 \end{vmatrix}$$
 (ncert)  
2)  $\begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix}$  (ncert)  
3)  $\begin{vmatrix} 11 & 75 \\ 17 & 2 \end{vmatrix}$  (ncert)  
4)  $\begin{vmatrix} 1 & 8 \\ 6 & 3 \end{vmatrix}$  (ncert)



18) find the value of x if 
$$\begin{vmatrix} 2 & 3 \ 1 = \begin{vmatrix} x & 3 \ 2x & 5 \end{vmatrix} (ncert)$$
  
19) find the value of x if  $\begin{vmatrix} 1 & 2 \ 1 = \begin{vmatrix} 6 & 2 \ 1 & 8 & 6 \end{vmatrix} (ncert)$   
20) If  $A = \begin{vmatrix} 1 & 2 \ 1 & 2 \ 2 \end{vmatrix}$ , then show that  $|2A| = 4|A|(ncert)$   
21) if  $A = \begin{vmatrix} 1 & 2 & 4 \ -1 & 3 & 0 \ 4 & 1 & 0 \end{vmatrix}$ , then show that  $|3A| = 27|A|(ncert)$   
A squre matrix is a singular matrix if its determinant is zero, other wise it is non singular matrix  
matrix  
22)  $\begin{vmatrix} 1 & -2 & 3 \ 1 & 2 & -3 \ 1 & 2 & -3 \ 1 & -3 & 4 \ -5 & x + 2 & 2 \ 4 & 1 & x - 6 \end{vmatrix}$ , if it is a singular matrix then find x . (ncert)  $x = 0.3 \pm \sqrt{205}$   
24)  $\begin{vmatrix} x^2 & x & 1 \ 3 & 1 & 4 \ 2 & -3 \ 1 & 4 \ -5 & -5 & -4 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -6 \ 1 & 1 \ 2 & -7 \ 2 &$ 

Shri sai maters tuition center , arjun nagar, sje, nd. Deepak sir 9811291604

# MATHEMATICS for CLASS 12 (CBSE NEW PATTEN)

### By-DEEPAK SIR 9811 29 16 04

**Property 6** If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e., the value of determinant remain same if we apply the operation  $R. \rightarrow R. + kR$ . or  $C \rightarrow C + kC$ 

*Property 5* If all the elements of any row or column of a determinant are zeros, then the value of a determinant is zero.

*Property 6* If the corresponding elements of any two rows or columns of a determinant are identical or proportional, then the value of the determinant is zero.

**Property 7** If in a determinant any two rows or columns are interchanged, then the value of the determinant obtained is negative of the value of the given determinant. If we make *n* such changes of rows (columns) in determinant A and obtain determinant A then  $A = (-1)^n A$ .

**Property 8** To the elements of any row or column of a determinant if we add or subtract the multiples of corresponding elements of any other row or column, then the value of determinant remains unchanged.

If more than one operation like  $R1 \rightarrow R1 + kR2$  is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operations.

$26) \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} (ncert)$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$28) \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0(ncert)$ $29) \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0(ncert)$
$29)\begin{vmatrix} a - b & b - c & c - a \\ b - c & c - a & a - b \\ c - a & a - b & b - c \end{vmatrix} = 0(ncert)$
$30)\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0(ncert)$
31) $\begin{vmatrix} \sin x & \cos x & \cos(x+w) \\ \sin y & \cos y & \cos(y+w) \\ \sin z & \cos z & \cos(z+w) \end{vmatrix} = 0(ncert)$
32) if a, b, c are in A. Pthen find value $\begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix}$ ( <i>ncert</i> )

<b>33</b> ) $\begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6X & 9X & 12X \end{vmatrix}$ USE PROPERTIES TO SOLVE FOLLOWING
$34)\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a2b2c2(ncert)$
$35) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a) (ncert)$
<b>36</b> ) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c) (ncert)$
<b>37)</b> $\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx) (ncert)$
$38) \begin{vmatrix} x + y & y + z & z + x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$
$39) \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a) (ncert)$
$40) \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 \text{ then show that } 1+xyz=0(ncert)$
$\begin{aligned} 40 & \begin{vmatrix} x & x^{2} & 1 + x^{3} \\ y & y^{2} & 1 + y^{3} \\ z & z^{2} & 1 + z^{3} \end{vmatrix} = 0 \text{ then show that } 1 + xyz = 0(ncert) \\ 41 & \begin{vmatrix} a & a^{2} & bc \\ b & b^{2} & ca \\ c & c^{2} & ab \end{vmatrix} = \begin{vmatrix} 1 & a^{2} & c^{3} \\ 1 & b^{2} & b^{3} \\ 1 & c^{2} & c^{3} \end{vmatrix} (ncert) \end{aligned}$
42) $\begin{vmatrix} x & x^2 & y+z \\ y & y^2 & y+x \\ z & z^2 & x+y \end{vmatrix} = (y-z)(z-x)(x-y)(x+y+z)(ncert)$
$(1 - y) \begin{vmatrix} y & y & y \\ z & z^2 & y + z \\ y & y^2 & y + x \\ z & z^2 & x + y \end{vmatrix} = (y - z)(z - x)(x - y)(x + y + z)(ncert)$ $(1 - y)(x + y + z)(ncert)$ $(1 - y)(x + y + z)(ncert)$ $(2 - x)(x - y)(y - z)(z - x), \text{ where } p \text{ is any scler number.}$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
$45)\begin{vmatrix}b+c & q+r & y+z\\c+a & r+p & z+x\\a+b & p+q & x+y\end{vmatrix} = 2\begin{vmatrix}a & p & x\\b & q & y\\c & r & z\end{vmatrix}(ncert)$

$$46) \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1(ncert)$$

$$47) \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1+c \end{vmatrix} = abc(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}) = abc+bc+ac+ab(ncert)$$

$$48) \begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0(ncert (R2 \rightarrow R2 - (R1+2R3)))$$

$$49) \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc(ncert) R1 \rightarrow R1 - R2 - R3 , expend$$

$$50) \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0(ncert)$$

$$510 \begin{vmatrix} x+4 & 2x & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

$$52) \begin{vmatrix} y+k & y & y \\ y & y & y+k \end{vmatrix} = k^{2}(3y+k) (ncert)$$

$$731 \begin{vmatrix} 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = 2(x+y+z)^{2}(ncert)$$

$$R1 \rightarrow R1 + R2 + R3, take common (a+b+c) and then c2=c2-c1, c3=c3-c1$$

$$54) \begin{vmatrix} x & x^{2} \\ x & x^{2} & 1 \\ x & x^{2} & 1 \end{vmatrix} = (1-x^{3})^{2} (ncert)$$

$$55) \begin{vmatrix} 1+a^{2}-b^{2} & 2ab & -2b \\ 2b & 1-a^{2}+b^{2} & 2a \\ 2b & 1-a^{2}+b^{2} & 2a \\ 2b & 1-a^{2}-b^{2} \end{vmatrix} = (1+a^{2}+b^{2})^{3}(ncert)$$

$$(c1=c1-bc3, c2=c2+c3, sthen take common (1+a2+b2), r3=r3-br1+ar2)$$

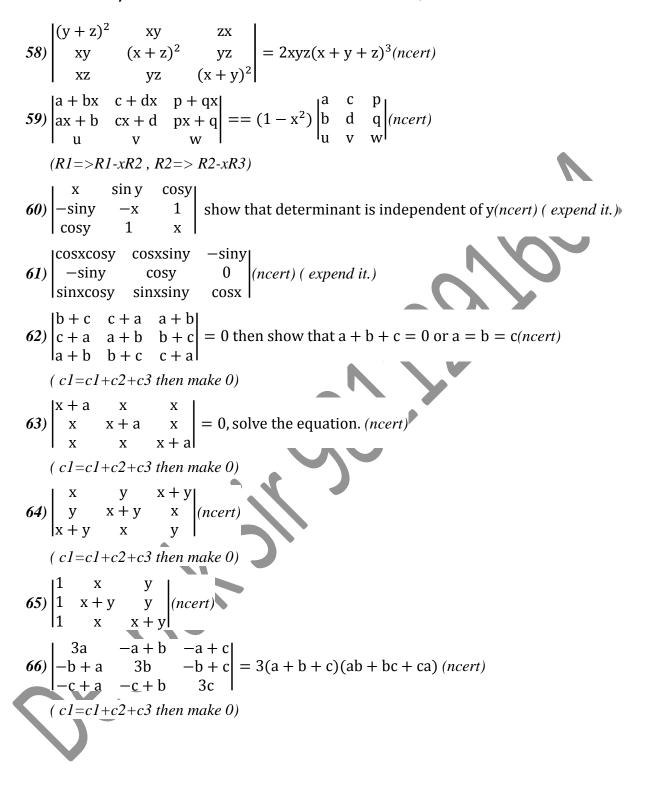
$$56) \begin{vmatrix} a^{2} + 1 & ab & ac \\ ab & b^{2} + 1 & bc \\ ca & cb & c^{2} + 1 \end{vmatrix} = 1+a^{2}+b^{2}+c^{2}(ncert)$$

$$(r1/a, r2/b, r3/c, then ac1, bc2, cc3 then c1=c1+c2+c3)$$

57) If a,b, c are positive and unequal, show that value of determinant is negative

a b c b c a (ncert) lc a bl

6 | Page



7 | Page

#### Area of a Triangle

In earlier classes, we have studied that the area of a triangle whose vertices are (x1, y1), (x2, y2) and (x3, y3), is given by the expression 1/2 [x1(y2-y3) + x2 (y3-y1) + x3 (y1-y2)].

- a. Since area is a positive quantity, we always take the absolute value of the determinant in (1).
- b. If area is given, use both positive and negative values of the determinant for calculation.
- c. The area of the triangle formed by three collinear points is zero.

*d.* area of triangle = 
$$\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

- 67) Find area of the triangle with vertices at the point given in each of the following : (i) (1, 0), (6, 0), (4, 3) (ii) (2, 7), (1, 1), (10, 8) (iii) (-2, -3), (3, 2), (-1, -8)
- 68) Show that points

5 6

3

2 1

4

A (a, b + c), B (b, c + a), C (c, a + b) are collinear.

- 69) Find values of k if area of triangle is 4 sq. units and vertices are (k, 0), (4, 0), (0, 2) (ii) (-2, 0), (0, 4), (0, k)
- 70) Find equation of line joining (1, 2) and (3, 6) using determinants.
- 71) Find equation of line joining (3, 1) and (9, 3) using determinants.
- 72) If area of triangle is 35 sq units with vertices (2, -6), (5, 4) and (k, 4). Then find k.
- 73) Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find k if D(k, 0) is a point such that area of triangle ABD is 3sq units.
- 0) is a point such that area of triangle ABD is 3sq units. 74)Find the vale of k so that point (k, k-2k), (-k+1, 2k), (-4-k, 6-2k) may be collinear. (-1,1/2).

# Minors and Cofactors

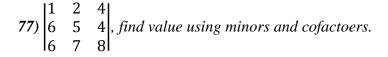
Minor of an element a.. of a determinant is the determinant obtained by deleting its ith row and Jth column in which element aij. lies. Minor of an element aij. is denoted by Mij.

*Minor of an element of a determinant of order* n(n > 2) *is a determinant of order* n - 1*.* 

#### Find minors of the following

Expanding the determinant  $\Delta$ , along R1, we have =  $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ 

Hence  $\Delta = sum$  of the product of elements of any row (or column) with their corresponding cofactors.

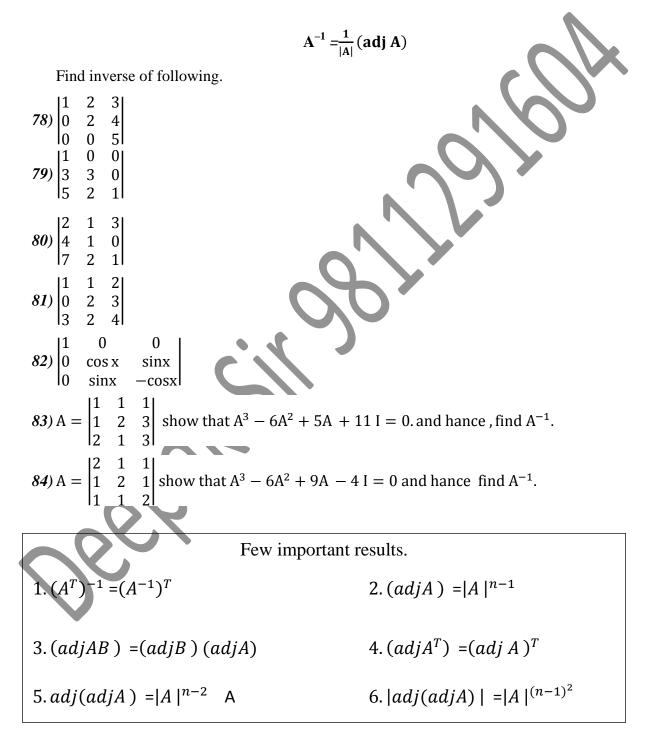


Adjoint of a matrix

The adjoint of a square matrix  $A = [aij] n \times n$  is defined as the transpose of the matrix [Aij]  $n \times n$ , where Aij is the cofactor of the element aij. Adjoint of the matrix A is denoted by adjA.

## Inverse of a Matrix

To find inverse of a matrix A, i.e.,  $A^{-1}$  we shall first define adjoint of a matrix.



## **Applications of Determinants and Matrices**

In this section, we shall discuss application of determinants and matrices for solving the system of linear equations in two or three variables and for checking the consistency of the system of linear equations.

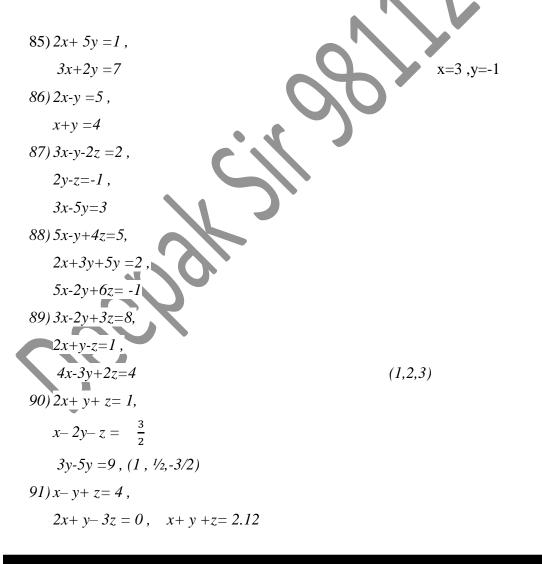
- A. **Consistent system :** A system of equations is said to be consistent if its solution (one or more) exists.
- B. **Inconsistent system:**A system of equations is said to be inconsistent if its solution does not exist.

CASE I when is A is non singular.  $X = A^{-1}B$ 

CASE II A is a singular matrix, then |A| = 0.

- In this case, we calculate (adjA) B.
- (i) If (adjA) B ≠O, (O being zero matrix), then solution does not exist and the system of equations is called inconsistent.
- (ii) If (adjA) B = O, then system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution

#### Examin consistency and Solve the system of equation



## MATHEMATICS for CLASS 12 (CBSE NEW PATTEN)

92) 2x + 3y + 3 z = 5,
x-2y+z=-4,
3x - y - 2z = 3.
93) x - y + 2z = 7,
3x+4y-5z=-5,
2x - y + 3z = 12
94) if $A = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$ find A-1. Using A-1 solve the following equations
2x - 3y + 5z = 11, $3x + 2y - 4z = -5$ , $x + y - 2z = -3$
95) Use product $A = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$ , $B = \begin{vmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{vmatrix}$ to solve the system of equations
x-y+2z=1, 2y-3z=1, 3x-2y+4z=2.
96) Use product $A = \begin{vmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{vmatrix}$ , $B = \begin{vmatrix} -1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$ to solve the system of equations.(mtc)
X+y+2z=1, $3x + 2y+z=7$ , $2x+y+3z=2$

- 97) The sum of three numbers is 6. If we multiply third number by 3 and addsecond number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.(1,2,3)
- 98) The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

$$99) A^{-1} = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix}, B^{-1} = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$100) A^{-1} = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}, find the inverse and verify that A^{-1} A = I_3$$

Shri sai maters tuition center , arjun nagar, sje, nd. Deepak sir 9811291604