

CLASS12 DETERMINANTS



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In this chapter, we shall study determinants up to order three only with real entries. Also, we will study various properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle, adjoint and inverse of a square matrix, consistency and inconsistency of system of linear equations and solution of linear equations in two or three variables using inverse of a matrix.

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CHAPTER 4

DETERMINANTS

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To every square matrix $A = [a_{ij}]$ of order n , we can associate a number (real or complex) called determinant of the square matrix A , where $a_{ij} = (i, j)$ th element of A .

This may be thought of as a function which associates each square matrix with a unique number (real or complex). If M is the set of square matrices, K is the set of numbers (real or complex) and $f: M \rightarrow K$ is defined by $f(A) = k$, where $A \in M$ and $k \in K$, then $f(A)$ is called the determinant of A . It is also denoted by $|A|$ or $\det A$ or A .

Remarks

- (i) For matrix A , $|A|$ is read as determinant of A and not modulus of A .
- (ii) Only square matrices have determinants.

Determinant of a matrix of order one

Let $A = [a]$ be the matrix of order 1, then determinant of A is defined to be equal to a

Determinant of a matrix of order two

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

Determinant of a matrix of order 3×3

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22}a_{33} - a_{32}a_{23}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Find the value of the following determinant.

Tip

For easier calculations, we shall expand the determinant along that row or column which contains maximum number of zeros.

Evaluate

1) $\begin{vmatrix} 1 & -3 \\ 5 & 4 \end{vmatrix} (ncert)$

2) $\begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix} (ncert)$

3) $\begin{vmatrix} 11 & 75 \\ 17 & 2 \end{vmatrix} (ncert)$

4) $\begin{vmatrix} 1 & 8 \\ 6 & 3 \end{vmatrix} (ncert)$

$$5) \begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix} (ncert)$$

$$6) \begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} (ncert)$$

$$7) \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} (ncert)$$

$$8) \begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & 2 \\ 1 & -1 & 5 \end{vmatrix} = -52 (ncert)$$

$$9) \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix} (ncert)$$

$$10) \begin{vmatrix} 0 & \sin x & -\cos x \\ -\sin x & 0 & \sin y \\ \cos x & -\sin y & 0 \end{vmatrix} = 0 (ncert)$$

$$11) \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix} (ncert)$$

$$12) \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} (ncert)$$

$$13) \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix} (ncert)$$

$$14) \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix} (ncert)$$

$$15) \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix} (ncert)$$

$$16) \text{ Find value of } x \text{ for which } \begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} (ncert)$$

$$17) \text{ find the value of } x \text{ if } \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix} (ncert)$$

18) find the value of x if $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$ (ncert)

19) find the value of x if $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$ (ncert)

20) If $A = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$, then show that $|2A| = 4|A|$ (ncert)

21) if $A = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$, then show that $|3A| = 27|A|$ (ncert)

A square matrix is a singular matrix if its determinant is zero, otherwise it is non singular matrix

22) $\begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$, if it is a singular matrix then find x. (ncert) $x = -1$

23) $\begin{bmatrix} x+1 & -3 & 4 \\ -5 & x+2 & 2 \\ 4 & 1 & x-6 \end{bmatrix}$, if it is a singular matrix then find x (ncert) $x = 0, 3 \pm \sqrt{205}$

24) $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28$, find x. (ncert) $x = 2$

25) $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$ (ncert)

Properties of Determinants

Property 1 The value of the determinant remains unchanged if its rows and columns are interchanged.

if A is a square matrix, then $\det(A) = \det(A')$, where $A' =$ transpose of A.

Property 2 If any two rows (or columns) of a determinant are interchanged, then sign of determinant changes.

Property 3 If any two rows (or columns) of a determinant are identical (all corresponding elements are same), then value of determinant is zero.

Property 4 If each element of a row (or a column) of a determinant is multiplied by a constant k, then its value gets multiplied by k. If A is a square matrix of order n, then $|kA| = k^n |A|$

By this property, we can take out any common factor from any one row or any one column of a given determinant.

Property 5 If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

Property 6 If, to each element of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e., the value of determinant remain same if we apply the operation $R_i \rightarrow R_i + kR_j$ or $C_i \rightarrow C_i + kC_j$

Property 5 If all the elements of any row or column of a determinant are zeros, then the value of a determinant is zero.

Property 6 If the corresponding elements of any two rows or columns of a determinant are identical or proportional, then the value of the determinant is zero.

Property 7 If in a determinant any two rows or columns are interchanged, then the value of the determinant obtained is negative of the value of the given determinant. If we make n such changes of rows (columns) in determinant A and obtain determinant A then $A = (-1)^n A$.

Property 8 To the elements of any row or column of a determinant if we add or subtract the multiples of corresponding elements of any other row or column, then the value of determinant remains unchanged.

If more than one operation like $R_1 \rightarrow R_1 + kR_2$ is done in one step, care should be taken to see that a row that is affected in one operation should not be used in another operation. A similar remark applies to column operations.

$$26) \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix} (ncert)$$

$$27) \begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix} (ncert)$$

$$28) \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0 (ncert)$$

$$29) \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0 (ncert)$$

$$30) \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0 (ncert)$$

$$31) \begin{vmatrix} \sin x & \cos x & \cos(x+w) \\ \sin y & \cos y & \cos(y+w) \\ \sin z & \cos z & \cos(z+w) \end{vmatrix} = 0 (ncert)$$

$$32) \text{ if } a, b, c \text{ are in A.P then find value } \begin{vmatrix} 2y+4 & 5y+7 & 8y+a \\ 3y+5 & 6y+8 & 9y+b \\ 4y+6 & 7y+9 & 10y+c \end{vmatrix} (ncert)$$

$$33) \begin{vmatrix} 2 & 3 & 4 \\ 5 & 6 & 8 \\ 6X & 9X & 12X \end{vmatrix}$$

USE PROPERTIES TO SOLVE FOLLOWING

$$34) \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2 \text{ (ncert)}$$

$$35) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a) \text{ (ncert)}$$

$$36) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c) \text{ (ncert)}$$

$$37) \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx) \text{ (ncert)}$$

$$38) \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$39) \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a) \text{ (ncert)}$$

$$40) \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0 \text{ then show that } 1+xyz=0 \text{ (ncert)}$$

$$41) \begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & c^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \text{ (ncert)}$$

$$42) \begin{vmatrix} x & x^2 & y+z \\ y & y^2 & y+x \\ z & z^2 & x+y \end{vmatrix} = (y-z)(z-x)(x-y)(x+y+z) \text{ (ncert)}$$

$$43) \begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix} = (1+pxyz)(x-y)(y-z)(z-x), \text{ where } p \text{ is any scler number.}$$

$$44) \begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3 \text{ (ncert)}$$

$$45) \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \text{ (ncert)}$$

$$46) \begin{vmatrix} 1 & 1+p & 1+p+q \\ 2 & 3+2p & 4+3p+2q \\ 3 & 6+3p & 10+6p+3q \end{vmatrix} = 1(ncert)$$

$$47) \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}) = abc + bc + ac + ab(ncert)$$

$$48) \begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix} = 0(ncert) (R2 \rightarrow R2 - (R1 + 2R3))$$

$$49) \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = 4abc(ncert) R1 \rightarrow R1 - R2 - R3, expend$$

$$50) \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0(ncert)$$

$$51) \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} (ncert)$$

$$52) \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k) (ncert)$$

$$53) \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = 2(x+y+z)^2(ncert)$$

$R1 \rightarrow R1 + R2 + R3$, take common $(a+b+c)$ and then $c2 = c2 - c1$, $c3 = c3 - c1$

$$54) \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2 (ncert)$$

$$55) \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3(ncert)$$

$(c1 = c1 - bc3, c2 = c2 + ac3, \text{ then take common } (1+a^2+b^2), r3 = r3 - br1 + ar2)$

$$56) \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2(ncert)$$

$(r1/a, r2/b, r3/c, \text{ then } ac1, bc2, cc3 \text{ then } c1 = c1 + c2 + c3)$

57) If a, b, c are positive and unequal, show that value of determinant is negative

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} (ncert)$$

$$58) \begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3 (ncert)$$

$$59) \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = (1-x^2) \begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix} (ncert)$$

($R1 \Rightarrow R1 - xR2$, $R2 \Rightarrow R2 - xR3$)

$$60) \begin{vmatrix} x & \sin y & \cos y \\ -\sin y & -x & 1 \\ \cos y & 1 & x \end{vmatrix} \text{ show that determinant is independent of } y (ncert) \text{ (expend it.)}$$

$$61) \begin{vmatrix} \cos x \cos y & \cos x \sin y & -\sin y \\ -\sin y & \cos y & 0 \\ \sin x \cos y & \sin x \sin y & \cos x \end{vmatrix} (ncert) \text{ (expend it.)}$$

$$62) \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0 \text{ then show that } a+b+c=0 \text{ or } a=b=c (ncert)$$

($c1=c1+c2+c3$ then make 0)

$$63) \begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0, \text{ solve the equation. (ncert)}$$

($c1=c1+c2+c3$ then make 0)

$$64) \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix} (ncert)$$

($c1=c1+c2+c3$ then make 0)

$$65) \begin{vmatrix} 1 & x & y \\ 1 & x+y & y \\ 1 & x & x+y \end{vmatrix} (ncert)$$

$$66) \begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca) (ncert)$$

($c1=c1+c2+c3$ then make 0)

Area of a Triangle

In earlier classes, we have studied that the area of a triangle whose vertices are (x_1, y_1) , (x_2, y_2) and (x_3, y_3) , is given by the expression $1/2 [x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)]$.

- Since area is a positive quantity, we always take the absolute value of the determinant in (1).
- If area is given, use both positive and negative values of the determinant for calculation.
- The area of the triangle formed by three collinear points is zero.

d. area of triangle = $\frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

67) Find area of the triangle with vertices at the point given in each of the following :

- (i) (1, 0), (6, 0), (4, 3) (ii) (2, 7), (1, 1), (10, 8) (iii) (-2, -3), (3, 2), (-1, -8)

68) Show that points

A (a, b + c), B (b, c + a), C (c, a + b) are collinear.

69) Find values of k if area of triangle is 4 sq. units and vertices are

- (k, 0), (4, 0), (0, 2) (ii) (-2, 0), (0, 4), (0, k)

70) Find equation of line joining (1, 2) and (3, 6) using determinants.

71) Find equation of line joining (3, 1) and (9, 3) using determinants.

72) If area of triangle is 35 sq units with vertices (2, -6), (5, 4) and (k, 4). Then find k.

73) Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find k if D(k, 0) is a point such that area of triangle ABD is 3sq units.

74) Find the value of k so that point (k, k-2k), (-k+1, 2k), (-4-k, 6-2k) may be collinear. $(-1, 1/2)$.

Minors and Cofactors

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its i th row and j th column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij} .

Minor of an element of a determinant of order n ($n > 2$) is a determinant of order $n - 1$.

Find minors of the following

75) $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$

76) $\begin{vmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \\ 4 & 3 & 2 \end{vmatrix}$

77) $\begin{vmatrix} 1 & 2 & 4 \\ 6 & 5 & 4 \\ 6 & 7 & 8 \end{vmatrix}$, find value using minors and cofactors.

Expanding the determinant Δ , along R_1 , we have = $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

Hence Δ = sum of the product of elements of any row (or column) with their corresponding cofactors.

Adjoint of a matrix

The adjoint of a square matrix $A = [a_{ij}] n \times n$ is defined as the transpose of the matrix $[A_{ij}] n \times n$, where A_{ij} is the cofactor of the element a_{ij} . Adjoint of the matrix A is denoted by $\text{adj}A$.

Inverse of a Matrix

To find inverse of a matrix A , i.e., A^{-1} we shall first define adjoint of a matrix.

$$A^{-1} = \frac{1}{|A|} (\text{adj} A)$$

Find inverse of following.

$$78) \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$$

$$79) \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & 1 \end{vmatrix}$$

$$80) \begin{vmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \\ 7 & 2 & 1 \end{vmatrix}$$

$$81) \begin{vmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 3 & 2 & 4 \end{vmatrix}$$

$$82) \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos x & \sin x \\ 0 & \sin x & -\cos x \end{vmatrix}$$

$$83) A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 1 & 3 \end{vmatrix} \text{ show that } A^3 - 6A^2 + 5A + 11I = 0. \text{ and hance , find } A^{-1}.$$

$$84) A = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{vmatrix} \text{ show that } A^3 - 6A^2 + 9A - 4I = 0 \text{ and hance find } A^{-1}.$$

Few important results.

$$1. (A^T)^{-1} = (A^{-1})^T$$

$$2. (\text{adj} A) = |A|^{n-1}$$

$$3. (\text{adj} AB) = (\text{adj} B) (\text{adj} A)$$

$$4. (\text{adj} A^T) = (\text{adj} A)^T$$

$$5. \text{adj}(\text{adj} A) = |A|^{n-2} A$$

$$6. |\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$$

Applications of Determinants and Matrices

In this section, we shall discuss application of determinants and matrices for solving the system of linear equations in two or three variables and for checking the consistency of the system of linear equations.

A. **Consistent system** : A system of equations is said to be consistent if its solution (one or more) exists.

B. **Inconsistent system**: A system of equations is said to be inconsistent if its solution does not exist.

CASE I when A is non singular. $X = A^{-1}B$

CASE II A is a singular matrix, then $|A| = 0$.

In this case, we calculate $(\text{adj}A) B$.

(i) If $(\text{adj}A) B \neq O$, (O being zero matrix), then solution does not exist and the system of equations is called inconsistent.

(ii) If $(\text{adj}A) B = O$, then system may be either consistent or inconsistent according as the system have either infinitely many solutions or no solution

Examin consistency and Solve the system of equation

85) $2x + 5y = 1$,

$3x + 2y = 7$

$x=3, y=-1$

86) $2x - y = 5$,

$x + y = 4$

87) $3x - y - 2z = 2$,

$2y - z = -1$,

$3x - 5y = 3$

88) $5x - y + 4z = 5$,

$2x + 3y + 5z = 2$,

$5x - 2y + 6z = -1$

89) $3x - 2y + 3z = 8$,

$2x + y - z = 1$,

$4x - 3y + 2z = 4$

$(1, 2, 3)$

90) $2x + y + z = 1$,

$x - 2y - z = \frac{3}{2}$

$3y - 5z = 9, (1, \frac{1}{2}, -\frac{3}{2})$

91) $x - y + z = 4$,

$2x + y - 3z = 0, \quad x + y + z = 2.12$

92) $2x + 3y + 3z = 5$,

$x - 2y + z = -4$,

$3x - y - 2z = 3$.

93) $x - y + 2z = 7$,

$3x + 4y - 5z = -5$,

$2x - y + 3z = 12$

94) if $A = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$ find A^{-1} . Using A^{-1} solve the following equations

$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

95) Use product $A = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$, $B = \begin{vmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{vmatrix}$ to solve the system of equations

$x - y + 2z = 1$, $2y - 3z = 1$, $3x - 2y + 4z = 2$.

96) Use product $A = \begin{vmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{vmatrix}$, $B = \begin{vmatrix} -1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{vmatrix}$ to solve the system of equations. (mtc)

$X + y + 2z = 1$, $3x + 2y + z = 7$, $2x + y + 3z = 2$

97) The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method. (1, 2, 3)

98) The cost of 4 kg onion, 3 kg wheat and 2 kg rice is Rs 60. The cost of 2 kg onion, 4 kg wheat and 6 kg rice is Rs 90. The cost of 6 kg onion 2 kg wheat and 3 kg rice is Rs 70. Find cost of each item per kg by matrix method.

99) $A^{-1} = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix}$, $B^{-1} = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$

100) $A = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix}$, find the inverse and verify that $A^{-1} A = I_3$