Name:_________________________________

Student ID:_____________________________

Classical Mechanics Comprehensive Exam

Fall 2018

You may use any intermediate results in the textbook. No electronic devices (calculator, computer, cell phone etc) are allowed.

For Administrative use only:

#1:_____________________

#2:_____________________

#3:_____________________

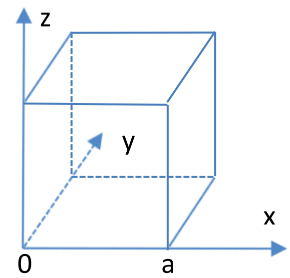
Total:___________________

1. (16pts) **Motion of a charged particle--** The equation of motion for a particle in electric field **E** and magnetic field **B** is $\frac{d}{dt}m\mathbf{v} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, where m is the mass and q is the charge. Define scalar potential ϕ and vector potential **A** as $\mathbf{E} = -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A}$, and $\mathbf{B} = \nabla \times \mathbf{A}$. Using Cartesian position coordinates **x** as generalized coordinates, derive velocity-dependent potential energy, Lagrangian, canonical momenta, and Hamiltonian. Verify that the canonical equations of Hamilton recover the equation of motion. Is the Hamiltonian the total energy of the particle?

Answer:

2. (16pts) **Inertia tensor for a solid cube**— A uniform solid cube of mass M and side a has a corner sitting at the origin of the Cartesian coordinates and three sides along the three positive coordinates direction (see figure below). (A) Calculate the inertia tensor $\overleftrightarrow{\mathrm{I}}$. (B) Calculate the angular momentum **L** of an angular velocity **ω** for **ω**=ω(1,0,0) and for **ω**=ω(1,1,1).

Answer:



3. (18pts) **Relativistic collision—** A particle with mass m_a and velocity $\mathbf{v}_a$ collides elastically with another particle at rest with mass m_b. The collision is head-on, i.e., two particles move along the line of the incident velocity after the collision. (A) What is the final velocity of the target particle $\mathbf{v}_b$? (B) What is $\mathbf{v}_b$ if m_a=m_b? (C) What is $\mathbf{v}_b$ if v_a<<c (speed of light)?

Answer:

Physics Qual - Statistical Mechanics

(Fall 2018)

I. Use the equipartition theorem of classical statistical mechanics to obtain an expression for the average energy of a diatomic molecule. What is the specific heat per molecule? Discuss the reason behind the observation that the specific heat per molecule of H_2 below $100^0 K$ (and above its liquefaction temperature) is only $\frac{3}{2}KT$. Give three distinct examples of specific phenomena for which experimental observations are at variance with the results of classical statistical mechanics. Discuss the reasons underlying the failure of the classical approach in each case.

II. Derive the Planck radiation law for the energy density per unit area in a two dimensional space. Using this result, derive the Stephan-Boltzmann law for the total energy density per unit area of a two dimensional space. How does the temperature dependence of the two dimensional Stephan-Boltzmann law compare with the three dimensional case? It might be useful to know that

$$\int_0^\infty dx \, \frac{x^2}{e^x - 1} = 2\zeta(3) = 2.4041 \quad .$$

III. Near room temperature, as the temperature increases, the electrical resistance of a conductor increases, whereas the resistance of a semiconductor decreases. Explain these two phenomena. Why is it appropriate to use the Fermi distribution in metals at room temperature, and why is the low temperature approximation for the latter justified in this context.

Quantum Mechanics Ph.D. Qualifying Exam (Spring 2018)

Show all your work. Make sure to explain all your answers, especially if the answer does not require a calculation.

Some useful integrals

$$\int_{-\infty}^{\infty} e^{iax} e^{-bx^2/2} dx = \sqrt{\frac{2\pi}{b}} e^{-\frac{a^2}{2b}}\,, \qquad \int_{-\infty}^{\infty} e^{iax} x e^{-bx^2/2} dx = \frac{i(2\pi)^{1/2} a}{b^{3/2}} e^{-\frac{a^2}{4b}}$$

1. (30 points) Consider a system of two spins, 1 and 2. The system can be described by in the basis $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, where the first entry denotes state of spin 1 and the second entry denotes state of spin 2. The interaction between the particles is described by the Hamiltonean

$$H = E_0 + \frac{A}{4}\hat{\boldsymbol{\sigma}}_1 \cdot \hat{\boldsymbol{\sigma}}_2\,, \tag{1}$$

where $\hat{\boldsymbol{\sigma}}_1$ and $\hat{\boldsymbol{\sigma}}_2$ are vectors of Pauli matrices. Assume that at time $t = 0$ the system is in state $|\psi(0)\rangle = \alpha|\uparrow\uparrow\rangle + \beta|\uparrow\downarrow\rangle$ with the normalization $\alpha^2 + \beta^2 = 1$.

(a) Write the Hamiltonean $\hat{H}$ in a matrix representation in the above basis.

(b) Determine the eigenbasis of $\hat{H}$.

(c) Express the state $|\psi(0)\rangle$ in terms of the eigenbasis of $\hat{H}$.

(d) Calculate $|\psi(t)\rangle$ at a later time.

2. (25 points) Consider a one-dimensional simple harmonic oscillator with an additional anharmonic term

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \lambda x^4\,. \tag{2}$$

(a) What are the energy eigenvalues of H when $\lambda = 0$?

(b) What is the shift in the lowest energy eigenvalue found in part (a) to first order in λ?

(c) Find the energy shift to order λ^2.

(d) Using a Gaussian trial function, make a variational estimate of the energy of the ground state. You only need to obtain an explicit equation whose solution would give this estimate. Solve this equation to first order in λ and compare your result with that in part (b).

3. (35 points) Consider a spin-1/2 particle with spin $\boldsymbol{s}$ and magnetic moment $\boldsymbol{\mu}\gamma\boldsymbol{s}$ placed in a time-independent magnetic field pointing in the z-direction: $\boldsymbol{B} = B\hat{\boldsymbol{z}}$.

   (a) If at time $t = 0$ the particle's spin in the x direction is measured and found to be $+\hbar/2$, what is the quantum state of the system, expressed in the basis of eigenstates of s_z?

   (b) If the state resulting from the measurement in part (a) evolves with time, what is the resulting state at time $|\psi(t)\rangle$?

   (c) Find the average values of particle spin in the y direction at the time t.

   (d) If the spin of the state $|\psi(t)\rangle$ is measured in the $\hat{z}\cos(\theta) + \hat{x}\sin\theta$ direction and the result is $-\hbar/2$ obtained, what will be the resulting quantum state?

   (e) What is the probability that the value $-\hbar/2$ in the measurement of part (d) is found?

4. (12 points) A particle of mass m is contained within an impenetrable one-dimensional well extending from $x = -\frac{L}{2}$ to $x = \frac{L}{2}$. The particle is in its ground state.

   (a) Find the eigenfunctions of the ground state and the first excited state.

   (b) The walls of the well are instantaneously moved outward to form a new well extending from $-L < x < +L$. Calculate the probability that the particle will be found in the ground state (of the new configuration) after this sudden expansion.

   (c) Calculate the probability that the particle will be found in the first excited state after the expansion.

EM Fall 2018 Qualifying Exam

1. A parallel plate capacitor is made of two large conducting squares of area A separated by a distance h. A dielectric of dielectric constant ϵ fills the volume between the two plates. The capacitor is charged to potential V and then disconnected from the source. Find

(a) the electric field E between the plates

(b) the electric displacement D between the plates

(c) the capacitance.

(d) The dielectric is now removed from between the plates. Find the work required to remove the dielectric.

2. A charge Q is uniformly distributed along the z-axis from $z = -a$ to $z = a$. Find the potential at an arbitrary point (r, θ, ϕ) in a series expansion. You may assume $r > a$.

3. An antenna of length L has its ends at the origin and at $(0, 0, L)$. It carries a current $I = f(x)\cos(\omega t)$ where

$$f(x) = \begin{cases} x^2 & 0 \leq x \leq L/2 \\ (L-x)^2 & L/2 < x \leq L \end{cases} \tag{1}$$

Find the radiation pattern to lowest order in the frequency.

4. A solenoid of radius a has n turns per unit length. It carries a time varying current $I = I_0 \sin(\omega t)$. It is surrounded by a concentric loop of wire of radius $b > a$ and resistance per unit length ρ. Find the induced current in the second loop.