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CLASSROOM PRACTICES FOR CONTEXT OF MATHEMATICS WORD PROBLEMS

ABSTRACT. How do teachers conceptualize and deal with context of mathematics word problems in their teaching? This question is discussed based on a study of 14 experienced teachers at the elementary, junior high and senior high school levels. Bruner's notions of paradigmatic and narrative modes of knowing formed the basis of analysis of data from sources that include interviews and classroom observations. The findings highlight the teachers' conceptions of problem context and teaching approaches for each of these modes of knowing. All of the teachers used the paradigmatic mode in their teaching but with different depth and most engaged in some form of the narrative mode to create a classroom environment that was motivational for students to learn word problems. The paper also highlights characteristics of these two modes as they relate directly to word problems and discusses implications for instruction, learning and teacher development.

KEY WORDS: instructional approaches, mathematics teachers' conceptions, narrative knowing, paradigmatic knowing, social context, word problems

Mathematical problems in school mathematics are typically presented in verbal, pictorial, or mathematical symbolic forms or some combination of the three. The verbal form could involve stating a symbolic form in words; instructions or goal for finding a solution; a mathematical context; or a 'real world' or social/cultural context, real or imagined. In this paper the focus is on the last form, that is, those with a 'real world' or social context, real or imagined, for example,

> The first time Tonya and Emma raced 20 km on bicycles, Emma was 2 km from the finish line when Tonya finished. The next day, Tonya agreed to start 2 km behind Emma, so that Tonya would ride 22 km and Emma, 20 km. If each of them rode at the same speed as she did during the first race, who won the second race?

Thus context and social context will be used interchangeably. Word problem is also being used in its broadest sense as opposed to the traditional view associated with it, that is, problems that are generally algorithmic in underlying problem-solving structure. In its broadest form, a word problem can be algorithmic or non-algorithmic, closed or open.

The focus of this paper is on such word problems in relation to making sense of how their contexts are used in instruction in the mathematics classroom. In particular, the paper reports on a study of how teachers conceptualized and dealt with social context in word problems in their teaching. It

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discusses the paradigmatic and narrative modes of knowing as conveyed by the teachers' conceptions of context and their instructional approaches. It thus provides a basis for understanding and enhancing classroom practices for teaching word problems.

PROBLEM CONTEXT IN LEARNING MATHEMATICS

Context, in the form of word problems, is viewed as an important aspect of learning mathematics. Reform recommendations in mathematics education, for example, the National Council of Teachers of Mathematics (NCTM, 1989, 1999), assign a significant role to problem contexts in developing meaning for mathematics. Word problems can be used as a basis for application and a basis of integrating the real world in mathematics education. They can provide practice with real life problem situations, motivate students to understand the importance of mathematics concepts, and help students to develop their creative, critical and problem solving abilities. Verschaffel (2002) described their goal as "to bring reality into the mathematics classroom, to create occasions for learning and practising the different aspects of applied problem solving, without the practical ... inconveniences of direct contact with the real world situation" (p. 64). Boaler (1994) added to this by offering three reasons for learning in context, i.e., to provide students with a familiar metaphor, to motivate and interest students, and to enhance the "transfer of mathematical learning through a demonstration of the links between school mathematics examples and real world problems" (p. 552). These goals, however, seem to be a challenge in terms of being realized in the classroom, to judge by studies of word problems.

There is a large body of literature on word problems that focuses mainly on the learner. Studies have analyzed students' thinking and actions as problem solvers in a variety of ways and highlighted the challenges word problems pose for, or elicit from, students. Context has played a key role, directly or indirectly, in accounting for this challenge. In some earlier studies, students' ways of dealing with context have been associated with level of success in solving word problems and ability as a problem solver in mathematics. For example, Suydam (1980) explained that good problem solvers tended to forget the details of a problem and recall its structural features, whereas poor problem solvers tended to recall the specific details. Silver and Smith (1980) pointed out that students' difficulty with thinking of a related problem as a problem-solving strategy can be linked to attending to non-mathematical features present in a problem. Students often make judgments of problem relatedness on the basis of factors other

than mathematical structure, in particular, the context or cover story and the question asked in the problem. They added that such behavior can be nurtured in the classroom in that students are sometimes encouraged to focus on nonstructural aspects of a problem through predetermined categorizing of problems as, for example, age, distance, and coin problems. Other studies have focused on students' errors in relation to interpreting word problems. For example, Cummins (1991) found that children's errors were caused by misinterpretation of certain verbal expressions commonly used in problem texts. She concluded that a critical determinant of solution success was the interpretation a child assigned to certain phrases used in the problems.

One area of ongoing concern related to students' treatment of context is the apparent tendency for students to solve word problems without associating them with their informal understanding of real life practice. Greer (1997) noted that there is widespread tendency of children to treat word problems with apparent disregard for the reality of the situations described by the text of the problem. He attributed this to the culture of the classroom. Some studies have discussed this behavior of students as lack of sense making (Silver et al., 1993) or suspension of sense making (Schoenfeld, 1991; Verschaffel et al., 2000). Students are known to give senseless answers even to unsolvable problems that lack essential information about the contextual situation. Schoenfeld (1991) suggests that as a result of classroom conditioning, students come to believe that executing mechanical calculations is more important than considering the real life meaningfulness of their actions in mathematics activities. As a result, there is suspension of sense making in problem solving.

These studies imply that students' failure to evaluate their problem solving moves in relation to common sense understanding of real life practices is not necessarily because of a cognitive deficit, but their understanding of how to play the traditional classroom game of dealing with problem context. This is supported by studies that have taken the nature of problem context and students' involvement with it into consideration. For example, some studies (e.g., DeFranco and Curcio, 1997; Verschaffel et al., 2000) have indicated that authentic problem contexts, where students participate directly in the problem situation, improved students' inclination to apply their real-world knowledge to the solution process. Other studies show positive results when context is linked to that of students. For example, Nesher and Hershkovitz (1997) found that children use real-world considerations when given non-routine problems taken from their lives while Davis-Dorsey (1991) found that elementary students benefited from personalizing word problems. Finally, students' natural orientation to draw on their real life experiences was reported by Barwell (2003) who showed

how two students used attention to narrative experience as one way to make sense of writing and solving word problems.

There is, therefore, evidence that suggests that the nature of problem context and how this context is attended to in instruction are important considerations for word problems to make more meaningful contributions to students' learning of mathematics. As Boaler (1994) argued, "mathematics questions should not train students to ignore real world variables but instead enable them to consider and examine the underlying structures and processes which connect classroom questions with real situation" (p. 562). Earlier, Freudenthal (1978) argued for the importance of classroom discussions of contextual problems that would allow students to learn that, in addition to the mathematical context, there exists something like a social context and that they (the students) are obliged to provide arguments if they insist on eliminating it. He queried, regarding the participants in his study: "Didn't the B-pupils stick to the social context because it had not been dealt with in the lesson? ... Shouldn't it have been our first task to do justice ... to the social context and its great wealth?" (p. 245). The teacher, then, is a critical factor in determining how or whether students are exposed to such experiences. But, how do teachers deal with the social context of word problems?

In recent years, mathematics teachers have become a focus of research and have been studied in terms of their knowledge, beliefs, classroom practices, and learning. A few studies have indicated that preservice teachers, in particular, have similar issues with context as do students. For example, Verschaffel et al. (1997) found that there was a strong tendency among preservice teachers to exclude real-world knowledge from their own spontaneous solutions of arithmetic word problems as well as from their appreciations of students' solutions. Contreras and Martínez-Cruz (2001) also found that preservice elementary teachers did not always base their responses on realistic considerations of the context situation. However, in general, a review of the literature suggests that there has been little attention given explicitly to the teaching of word problems and, in particular, no focus on how context is dealt with from the perspective of the teacher or the classroom. The intent of this paper, then, is to highlight teachers' thinking and practices in dealing with context.

THEORETICAL PERSPECTIVES

One theoretical perspective that has been offered as a basis for making sense of how students relate to problem context is based on Gilligan's (1982) two ways of knowing – separate and connected knowing. Buerk (1986) and

Brown (1984) suggested that if a student's way of knowing were oriented to one or the other, then the way he or she deals with problem context would be influenced by this orientation. This means that separate knowers will deal with problem context differently from connected knowers regardless of the mode of instruction. For example, as Buerk (1986) noted, separate knowers get right to a solution in a structured, algorithmic way, stripping away any context. On the other hand, connected knowers try to experience the problem, relate it to their personal world, clarify the language, create context and remove ambiguity. Similarly, Brown (1984) pointed out that connected knowers in mathematics would likely display a plea for more information by searching for a way of locating the episode within a broader context, a disinclination to set general principles to be used in future cases and a concern with connectedness among people.

This perspective becomes problematic if it suggests that students are clearly either separate or connected knowers or they have some natural disposition towards one or the other. While there are students who display connected behavior, treating them as one or the other could unduly discriminate against them and limit their learning. It also implies that nothing, or little, can be done to change their mathematical problem solving behavior, if necessary. In this paper, the focus is on an alternative theoretical perspective that embodies separate and connected knowing, but does not use them to categorize the learner. Thus the focus is not on the mode of knowing of the learner, but the mode of knowing that can be offered by a word problem.

The theoretical perspective being adopted here is based on the two of modes of knowing proposed by Bruner (1985, 1986) – paradigmatic and narrative knowing. Bruner (1986) describes paradigmatic and narrative knowing as two irreducible modes of cognitive functioning. "Each provides a way of ordering experience, of constructing reality, and the two (though amenable to complementary use) are irreducible to one another. . . . Each also provides ways of organizing representation in memory and of filtering the perceptual world. Efforts to reduce one mode to the other or to ignore one at the expense of the other inevitably fail to capture the rich ways in which people "know" and describe events around them" (p. 97).

The paradigmatic mode of knowing, which Bruner describes as logico-scientific, is centered on the narrow epistemological question of how to know truth. "It is based upon categorization or conceptualization and the operations by which categories are established, instantiated, idealized, and related one to the other to form a system" (Bruner, 1986, p. 98). It focuses on context-free and universal explications. It establishes truth by formal verification procedures and empirical proof. It deals in general causes and in their establishment. It leads to good theory, tight analysis, logical proof,

and empirical discovery guided by reasoned hypothesis. In contrast, the narrative mode of knowing, which Bruner describes as humanistic, is centered on the broader and more inclusive epistemological question of the meaning of experience. It operates by constructing two landscapes simultaneously. "One is the landscape of action, where the constituents are the arguments of action: agent, intention or goal, situation, instrument. Its other landscape is the landscape of consciousness: what those involved in the action know, think, or feel" (Bruner, 1986, p. 99). Narrative knowing thus focuses on context-sensitive and particular explications. It establishes truth-likeness by criticism and interpretation of text. It is concerned with the explication of human intentions in the context of action. It deals in human or human-like intentions and action. It leads to good stories, gripping drama, and believable historical accounts. These characteristics of the two modes of knowing are summarized in Table I.

Paradigmatic knowing, in relation to word problems, would require a focus on mathematical models or mathematical structures that are universal and context-free. Thus the approach to a solution of a word problem would involve strategies and ways of thinking that are independent of a particular social context. For example, one has to distinguish and separate a structure from its surrounding context. Narrative knowing, in relation to word problems, would require a focus on the social context of the problem. This could involve focusing on the cover story of the word problem in order to understand or relate to the storyline, plot, characters, objects, situations, actions, relationships, and/or intentions. These two modes of knowing, then, could provide a basis for making sense of how teachers deal with context in their teaching of word problems.

TABLE I
Paradigmatic and narrative modes of knowing

Paradigmatic [logico-scientific] Mode	Narrative [humanistic] mode
1. How to know truth	1. Meaning of experience
2. Context-free and universal explications	2. Context-sensitive and particular explications
3. General causes: Categorization and conceptualization	3. Human intentions and action
4. Formal verification procedures and empirical proof	4. Criticism and interpretation of text
5. Good theory, tight analysis, logical proof, empirical discovery guided by reasoned hypothesis	5. Good stories, gripping drama, believable historical accounts

RESEARCH METHOD

This study of teachers' treatment of context of word problems in their teaching is based on data from a larger project (the word-problem project) that investigated mathematics teachers' thinking in teaching word problems. The focus here is on fourteen experienced teachers, from different local schools, who were participants in the word-problem project. There were four elementary teachers (grades 1, 2, 5, and 6), two junior high teachers (grades 7 to 9); two junior-senior high teachers (grades 7, 9, 10, 11, 12), and six senior high teachers (grades 10, 11, 12; two with experience teaching junior college mathematics). The main criterion for selecting these teachers was willingness to participate. However, in addition to this, the two junior high, one junior-senior high and four senior high (without college experience) teachers were included because they were considered to be exemplary teachers in their school systems in relation to teaching a reform-oriented curriculum. This was validated during data collection in that their teaching engaged students in a significant way in problem solving, communication, reasoning and connections. There was also emphasis on teacher-led inquiry discussions and student-oriented approaches (e.g., group inquiry). For the other teachers in the sample, who taught the same curriculum, there was more emphasis on direct instruction but they also integrated characteristics noted for the exemplary teachers to varying degree. All of the participants were articulate and open about their thinking and experiences with word problems.

The research process was framed in a qualitative, naturalistic research perspective (Creswell, 1998). The main sources of data for the larger word-problem project were open-ended interviews, classroom observations, role-play, and teaching artifacts. The interviews included examining the participants' thinking about, and experiences with, word problems in relation to their practice with particular emphasis on their classroom processes, planning and intentions. Interview questions were framed in both a cognitive context to allow the teachers to share *their* way of thinking and a phenomenological context to allow them to describe their teaching behaviors as lived experiences (i.e., stories of actual events). For example, some questions were of the form: How do you view word problems? What do you think is the role of word problems in mathematics/curriculum/your teaching? How do you think word problems should be taught? Other questions were in the form of open situations to address, for example, telling stories of memorable, liked and disliked classes involving word problems that they taught; giving a presentation on word problems at a teacher conference; and having a conversation with a preservice teacher about word problems. The interviews were audio taped and transcribed. Classroom

observations focused on the teachers' actual instructional behaviors during lessons involving/related to word problems. Special attention was given to what the teachers and students did during instruction and how their actions interacted during the lessons. These lessons were audio taped and detailed field notes were taken. Role-play scenarios allowed the teachers to act out, instead of talk about, a situation, for example, presenting a word problem to the class. Teaching artifacts consisted mainly of relevant teaching notes, lesson plans and textbooks.

Data analysis for the larger word-problem project involved the researcher and two research assistants working independently to review the data and identify attributes of the teachers' thinking and actions that were characteristic of their perspective of teaching word problems. The review consisted of open-ended coding focusing on significant statements and actions that reflected judgments, intentions, expectations, and values of the teachers regarding word problems that occurred on several occasions in different contexts. These attributes were grouped into themes and validated by comparison of findings by the three reviewers and triangulation of findings from interviews, classroom observations and role-play. Member checks were conducted with six of the participants (one elementary, one junior high and four senior high) who were available for this quality check.

It should be noted that data collection for the larger word-problem project was not explicitly focused on problem context. So there was not a specific set of predetermined interview questions about it. However, probing questions during the interviews and the comprehensive list and open-endedness of the predetermined interview questions allowed the teachers to share information about how they dealt with context. Classroom observation data were also detailed enough to capture this aspect. So it was not until data analysis of the larger word-problem project that the treatment of context of word problems emerged as a formal theme in understanding the teachers' teaching of word problems and became a focus of further analysis. After a review of the literature, Bruner's modes of knowing seemed to offer a meaningful way of conducting this further analysis. Thus, the data sorted into this theme were further coded based on situations that suggested paradigmatic knowing and those that suggested narrative knowing in order to obtain a landscape of the different ways, collectively, in which the teachers conceptualized and dealt with context in their teaching, i.e., what they did or allowed the students to do with the social context when working with word problems. This resulted in the teachers' paradigmatic-oriented and narrative-oriented perspectives of context. These are described as orientations to indicate that they are not necessarily identical matches with the paradigmatic and narrative perspectives but a fit to key characteristics. The findings are presented to highlight these orientations.

TEACHERS' PARADIGMATIC-ORIENTED PERSPECTIVE OF CONTEXT

The paradigmatic-orientated perspective of context was evident in both the teachers' conceptions and teaching approaches in dealing with context. Their conceptions of problem context included the following: a barrier to the mathematics or the solution of the problem; a nuisance factor that distracted students from the mathematics; added extraneous or redundant information; not important to solve the problem. As one teacher, SH5, explained, "The context from the students' point of view is important, from my point of view, it's not important at all . . . so long as they can derive the particular equation out of it. . . . The storyline does not matter."

The teachers' conceptions also indicated that in solving a word problem, the focus is on the non-humanistic aspects of the context, that is, ignoring or eliminating aspects of the context that do not point to an underlying mathematics concept or process and thus converting the problem to a decontextualized or impersonal situation. The following are examples of the teachers' thinking.

> You have to strip it bare of all the interesting stuff and then you get down to just the facts. . . . You can strip away the stuff we don't really need . . . to come up with some kind of model to solve the problem. . . . It's bare bones stuff, because this is what math does. [SH1]

> You have to teach kids . . . how to say, 'what is completely not needed in this sentence?' . . . 'Oh, so George's brother Bob who built the fence doesn't have anything to do with it.' . . . They need to get practice in getting rid of the extraneous stuff and getting down to, "Oh, there are five ducks, period". [ES1]

> You have got to try to extract from some underlying meaning and then connecting to what was learned in the past. . . . Whether the context relates to the real world or not, it probably doesn't matter for reasoning. . . . To me the math is the underlying essence of things. [SH6]

Thus, while the social context was viewed as problematic, the impersonalized problem situation was valued as a way for students to make sense of the mathematics.

For this paradigmatic-oriented perspective, while the teachers' conceptions about context were similar, there were significant differences in how they treated the context during solution of a problem in their teaching. Some used approaches that led to a fragmented view of the problem context. Others used approaches with holistic views of the problem context. A comparison of the teachers is provided later in the paper. Following are three instructional approaches that emerged from their teaching.

Approach P1

This approach involved showing students how, and guiding them, to fragment and translate the context into mathematical representations. For example, for the topic systems of equations, teacher SH5 read to the class the word problem he was illustrating. He then identified key words and phrases, wrote them on the chalkboard, and wrote the corresponding symbolic representations while explaining what he was doing. For the next problem, he first asked students to read the problem, then asked leading questions to get them to reproduce the process. As students practiced, he reminded them, "Break it down first and translate the words into the mathematical sentence". He would also illustrate and direct students to delete sentences when the problem context was long and to then treat the remaining text as the complete problem. Other teachers used variations of this approach to lead students to identify the "known" and "unknown" in the context to get to a solution. For some topics, these teachers first focused on translation of verbal sentences into mathematical representations before dealing with actual problems that incorporated such sentences. In the case of grade 1, and often grade 2, the teacher would read the problem and stress the key words. They would explain the keywords and sometimes get students to model them with manipulatives, for example, "take away".

Approach P2

This approach involved helping students to see how the mathematics structure could be independent of the context. Three ways in which this was done are: (1) The teacher illustrated the solutions of two problems that were identical except for differences in the context (e.g., different names, objects, or places). He then drew students' attention to how the answer was the same and the names, objects or places did not matter. (2) Instead of using a second problem, the teacher orally proposed changes and posed questions about the relevance of the context. For example, "Suppose I change this from Jean's to Jake's salary, would it make a difference?" "Does it make a difference if it was a green car or a red car? Does it matter if it's a car or not?" (3) The teacher gave students a set of three to five problems of similar mathematics structure but different context to solve. The students were required to compare the problems to determine any patterns they noticed that will be useful in solving future problems.

Approach P3

This approach involved helping students to think for themselves in interpreting the context mathematically. After reading the problem, students

were asked to explain (orally or in writing first, then orally) their meanings or understandings of the problem, which then formed the basis of unpacking the context in relation to arriving at a mathematical solution. The teachers used questioning, prompts and discussions to facilitate this unpacking. For example: (i) The teacher questioned students about the assumptions they made about the problem context. They talked about which should be eliminated and why and whether there were others that could be introduced. (ii) If students offered contextual information as part of their meanings, they were asked if or how they thought it was helpful in solving the problem. If no one offered contextual information, the teacher sometimes asked questions like: Does it matter who's driving the car? Do we care about how Mary feels that she lost the race? (iii) Students were prompted to consider an alternative meaning if the teacher perceived a flaw in theirs in relation to arriving at a solution. (iv) Students were encouraged to compare and challenge each other's meanings and justify their own in terms of how it related to what the problem required them to find.

The preceding treatment of context is paradigmatic because it focuses on suppressing the humanistic aspects of the problem context and emphasizing abstract relationships embedded in it. The next section considers situations in which the humanistic aspects are recognized.

TEACHERS' NARRATIVE-ORIENTED PERSPECTIVE OF CONTEXT

The narrative-orientated perspective of context was evident in both the teachers' conceptions and teaching approaches in dealing with context. Their conceptions of problem context included that it is important in playing an affective role as a way to connect students to mathematics and vice versa. It has little to do with mathematics, but without it the mathematics could be unbearable for many students. As this teacher, SH6, explained, "I pay a lot of attention to the psychological side so students don't get discouraged. So it's not just the math, it's the people part. . . .What I'm dealing with is the students' attitude. . . . If they feel there is a point to something, then it will make sense to them. If they feel there's no point to it, then it would just be meaningless computation." Consistent with this, some teachers' conceptions also included that context should be meaningful, interesting and relevant for students and that students' personal experiences, real or imagined, are important as a basis of problem context. These teachers talked about the importance of context as a basis for making connections, particularly to personal experiences; making the problem real; and attaching meaning, for example:

You make the problem real to them first, then get into, "now what are we concerned about here?" . . . But you have to get them interested in it first. It has to make sense for them. [SH1]

Kids need to connect the problems to the world otherwise it's just hanging in an empty space. . . . If they can do an association of a word problem with their own physical experiences, with their past experiences, or with something that takes the problem from the context of being out there to something that's in here, in their own minds, then that's more comfort, they can create a comfort zone. [SH2]

I think the student ought to be able to connect it. If this is not connected to the computations that are underlying it, then there is no real learning that will take place. So I think if they do not see that connection they are basically sunk. [SH6]

For this narrative-oriented perspective, the teachers' conceptions about context differed in terms of its nature and role in learning word problems, which corresponded to significant differences in how they treated the context during solution of a problem in their teaching. A comparison of the teachers is provided later in the paper. The narrative perspective was reflected in their teaching in terms of when and how they allowed students to resonate in or interpret the context based on the students' particular circumstances or experiences. Resonance is being used to describe situations in which students see themselves, or real world experiences, in the context, whether explicit or not, and use that in dealing with the problem. Following are four of the instructional approaches of the teachers.

Approach N1

This approach allowed students to resonate in the social context as an aside, that is, detached from the problem-solving process. The social context of the problem provided a basis for sharing real-life stories triggered by it during teacher-led discussion of the word problem, but with no link to the problem solution. For example, in a grade 6 class, the teacher presented a problem involving skating on the overhead and asked students to read it. One student then asked the teacher if she skated and proceeded to talk about her (the student's) skating experience on the weekend. Other students joined in with their experiences that led to a brief discussion of the hockey game the night before. This went on for about eight minutes before the teacher asked what the problem was asking them to do and ended the discussion.

Approach N2

This approach allowed students to resonate in the context but was unique in treating non-mathematical solutions as acceptable. Any interpretation of the context by the students that made sense was allowed regardless of

whether it led to a mathematical solution or a non-mathematical solution. For example, solutions grade 2 students gave when discussing the problem: "Crysta, Sally, and Linda were trying to line up at the water fountain during recess. How many ways can they line up if Linda always had to be first?" included: Who ever wanted to have a drink first should have been there faster. It could be that they are friends and would allow Linda to be first. The teacher treated these as acceptable solutions.

Approach N3

This approach allowed students to resonate in the social context to discuss specific aspects of it they were curious about or to critique it, for example, whether it was meaningful, relevant, or helpful to them. The teacher sometimes initiated this discussion, but more often the students did so. For example, one teacher asked the class (grade 10) to read the following problem from the textbook: "It took a patrol boat 5 h to travel 60 km up a river against the current, and 3 h for the return trip with the current. Find the speed of the boat in still water and the speed of the current?" She then jokingly asked, "What's the point of this?" "Why would anybody want to solve this type of problem"? The students laughed and talked about how it was a stupid problem in relation to the real world. The teacher responded, "Yes, that is kind of stupid, but let's have a little fun and try and see what we can do with it." An example from another teacher involved the following problem from the grade 11 textbook: "Peter and Sharif traveled from Acton to Beamsville on foot. Peter walked half the distance and ran half the distance. Sharif walked half the time and ran half the time. If they walked at the same speed and ran at the same speed, who arrived in less time, or was it a tie?" After reading the problem, the students initiated a brief discussion that included the cultural background of Sharif, the location of the towns, and whether it made sense to travel by foot. Depending on the problem and criticism, the teachers who used this approach sometimes asked students to revise or rewrite the problem to reflect what they considered to be interesting or meaningful.

Approach N4

This approach is complementary to approach P3 of the paradigmatic approaches and allowed students to resonate in the context to determine their meanings or understandings of the problem. This occurred in two stages of the problem-solving process. (i) Entry into the problem: Students were allowed to use their real-life experiences, or draw on that of the teacher when theirs were lacking, to interpret the context. This generally involved

clarifying meanings of, for example, an object, person/occupation, or situation in the context. (ii) Exit out of the problem: Students were allowed to use their real-life experiences to explain whether the solution made sense. This sometimes led to a critique of the problem and/or posing new problems.

The next section of the paper compares the teachers in relation to these approaches for both the narrative and paradigmatic modes. It also compares the characteristics of these two modes for word problems based on these approaches and discusses implications for instruction.

DISCUSSION AND CONCLUSIONS

The paradigmatic and narrative modes of knowing both played important roles in most of the teachers' ways of dealing with the context of word problems in their teaching. The paradigmatic mode was more dominant and was combined with the narrative mode in different ways among the teachers. Table II provides a summary of these combinations in terms of the teaching approaches previously discussed. The teachers are grouped based on the approaches that were most common in their classrooms and reflected their perspective of how to deal with context. For the grade 7–12 teachers, group A are the exemplary teachers and group B are the others. The table is a simplified representation of the teachers' practices in that the distribution of the approaches is not that clear cut. For example, there were instances when aspects of approach N3 emerged in N1 or P2; aspects of N1 emerged in N3; and aspects of P2 emerged in P1 and vice versa. Each teacher also had his or her personal instructional style for the approaches. Thus the groupings are not homogeneous in terms of how each approach was lived in the classrooms.

In comparing the teachers, it makes sense that all of the teachers used the paradigmatic mode in dealing with context given that this it is characterized by "logico-scientific" thinking. The issue is the degree of depth to which it was used. The approach P1 reflected the least depth in that the

TABLE II
Teaching approaches matched to teachers

Number of teachers	Grades taught	Teaching approaches
2	1–2	P1, N1, N2
2	5–6	P1, N1
7	7–12 A	P2, P3, N3, N4
3	7–12 B	P1, P2

mode as described by Bruner was used in an instrumental way, i.e., the focus was on de-contextualization in a mechanical way by fragmenting the context into a collection of words, phrases and sentences to be translated to mathematical representations. This limited the "logico-scientific" thinking to an instrumental way of reducing a problem to a mathematical situation. Students were provided with a way of coding a problem context, e.g., to translate to symbols, to identify clue words, which reduced the paradigmatic mode to a coding scheme. Approach P3 reflected the most depth in that de-contextualization was treated as a way of reasoning behind/through the context to the mathematical representation. It required students, for example, to interpret the context and evaluate it for relevance, consider assumptions, and make connections to mathematical models. Table II thus shows that the elementary teachers displayed the least depth and the group A secondary teachers, the most depth in using the paradigmatic mode.

For the narrative mode, given its humanistic features, it makes sense that one of the goals of all of the teachers who used it was to create a classroom environment that was motivational for students to learn word problems. The issue here is the different levels to which it was used in relation to the problem solving process. In all of the teachers' classrooms, for the most part, the students initiated the narrative mode. However, the teacher controlled the extent to which it played out. The group B secondary teachers did not allow it to play out by reminding students that the context did not matter. The other teachers allowed it to play out, even if it had no relationship to solving the problem, and context was treated as a way to support, reflect and expand students' interests and as students' stories. Thus, in approach N1, students resonated in the context seemingly for fun and to establish rapport in the class, as the teachers involved explained. Approach N2 allowed students to treat the problem in a humanistic way and considered non-mathematical solutions to be acceptable. The teachers who used this approach felt it was acceptable for the younger students to give such solutions because their way of thinking was not developed enough to exclude such solutions. Approach N3, and in particular approach N4, had aspects that were important for students to enter and exit the problem in a way that was meaningful to them. For example, in approach N4, students could interpret the problem based on their real life experiences and could consider contextual information as being important to make sense of the problem situation, but with the understanding that this may not be relevant to obtain a mathematical solution. Approach N4 thus reflected the most depth in terms of treating context as a key component of understanding, experiencing and solving the problem. In Table II, then, while the group B secondary teachers did not use the narrative mode, the elementary teachers

used it with little or no depth and the group A secondary teachers used it with the most depth in terms of engaging students in the context in a mathematically relevant way.

In comparing the teachers in terms of both the paradigmatic and narrative modes, the following conclusions were drawn: The early grades elementary teachers related their teaching approaches to dealing with problem context to the developmental level of the students. The later grades elementary teachers related their approaches to the nature of mathematics and the developmental level of the students. The group A secondary teachers related their approaches to the nature of mathematics and the nature of the learner, e.g., they are inseparable from their experiences. The group B secondary teachers related their approaches mainly to the nature of mathematics. The assumption here is that the way the teachers dealt with context was influenced by how they viewed it mathematically and how they viewed the students as learners.

Table III summarizes key characteristics of what the paradigmatic- and narrative-oriented modes in dealing with word problems could look like based on the teachers' instructional approaches and corresponding items of Bruner's modes in Table I (thus the reference to Tables I and III in what follows). The table is not intended to capture the complexity of teaching or learning word problems, but to put the spotlight on characteristics directly related to context.

In the paradigmatic-oriented mode, the emphasis is on treating word problems as objects, with a focus on how to know "truth" (item 1 Table I), that is, mathematical solution(s) to the problem (item 1 Table III). Like the paradigmatic mode (item 2 Table I), the word problem requires

TABLE III
Paradigmatic- and narrative-oriented modes of word problems

Paradigmatic-oriented mode of a word problem	Narrative-oriented mode of a word problem
1. Solution(s) of problem	1. Meaning(s) of problem
2. Context-free and universal solution	2. Context-sensitive and particular explications of problem
3. Identifying, grouping, viewing problems by mathematical structure	3. Resonance in problem context
4. Verify mathematical model/procedure with different contexts, check solution with problem situation or real world constraints	4. Criticism and interpretation of problem text
5. Mathematical model/procedure/structure	5. Stories of experience

context-free and universal explications (item 2 Table III). This requires techniques/strategies (item 3 Table I) to distinguish and separate a mathematical structure from its surrounding context, for example, identifying, grouping, and viewing problems by mathematical structure (item 3 Table III). Verification or proof (item 4 Table I) in word problems could involve applying a particular structure or model to different context or checking a solution in terms of the situational context of the problem or the constraints of the real-world story situation (item 4 Table III). The process could lead to theory/discovery (item 5 Table I) in the form of generalizations based on the structure of the problem mathematical situations, construction of mathematical models, or how to choose mathematical procedures to obtain solutions to the problem (item 5 Table III).

In the narrative-oriented mode, the emphasis is on treating word problems as experience, with a focus on meaning of the experience (item 1, Table I), that is, meaning of the problem (item 1 Table III). Consistent with the narrative mode (item 2 Table I), there is context-sensitive and particular explications of the problem (item 2 Table III). This requires dealing in human intentions and actions (item 3 Table I) by resonating in the problem context (item 3 Table III). Also consistent with the narrative mode (item 4 Table I) is the use of criticism and interpretation of text of the problem (item 4 Table III). The process could lead to good stories and believable accounts (item 5 Table I), in particular, stories of experience (item 5 Table III).

The findings of the study suggest that both modes, together, could be an enriching experience for students. As Bruner (1986, p. 97) explained, the two modes are "amenable to complementary use" and "to ignore one at the expense of the other inevitably fails to capture the rich ways in which people 'know' and describe events around them". Incorporating both modes in instruction, with depth [as in approaches P3 and N4], seems to be more consistent with current perspectives of learning, e.g., constructivist or interpretive paradigms. If students are to construct their own meaning, it makes sense to attend to the narrative mode in instruction. Word problems have a "logico-scientific" and a "humanistic" component because of the mathematics structure and social context, respectively. Given that the humanistic component is likely to be more readily accessible to students because of the similarity (real or imagined) to their experiences, it would seem natural for their initial entry point into the problem to be through narrative knowing unless they have learnt how to suppress it. Not knowing how to suppress it could be a possible reason for the fear many students develop for contextual problems. On the other hand, suppressing it in an instrumental way could be a possible reason for the lack of sense-making or meaningful connection of the solution to reality as discussed earlier in this paper. However, what has

been viewed as students' unrealistic responses to word problems could be a result of their unique interpretations of the problem context when viewed from a narrative perspective.

The findings further suggest that in order for both of these modes to be integrated in instruction for word problems, it is important for the teacher to do the following: (i) Allow students to have a voice regarding what they value about the context and how to communicate it. (ii) Allow alternative meanings or interpretations of the problem that could be context-free or context-bound, but require justification from students of how they make sense mathematically and are relevant for students to arrive at a mathematical solution. (iii) Allow students to use both formal and informal knowledge to make sense of the problem and solution and communicate these understandings in class discussions. In general, then, students' knowledge of context should not be suppressed but treated as valuable to enter and exit the solution process. The goal should be to help students to construct for themselves a deep understanding of when, what, how, and why context could aid or hinder mathematical thinking in arriving at mathematical solutions to word problems.

Instruction that deals with context in the foregoing ways has the potential of achieving the goals of context discussed in the literature (e.g., Boaler, 1994; Verschaffel, 2002). It could encourage negotiation and interpretation, engage students' common sense and attend to reality, encourage a better connection between students' mathematical understanding and the real world, motivate students and encourage confidence, engage students' interests, counter the fear or perceived threat of word problems, and make mathematics more meaningful. This was reflected in claims made by the group A secondary teachers. However, further research focused on both teachers and learners will be necessary to understand the actual effect on learning and transfer within and outside mathematics. Further study is also needed to consider alternative types of problems in mathematics classes, for example, word problems in which a social context is not obvious or non-verbal problems in which students may express concerns about a lack of context.

Finally, this study has implications for teacher development. For example, the teachers who integrated the narrative mode in some form in their teaching learnt through experience how to do so. However, many of the teachers were significantly limited by their experiences in terms of how both modes unfolded in their teaching. This suggests that there is a need for teachers to have experiences to develop more depth in making sense of dealing with context in their teaching. This study provides examples of experiences that could be considered in this regard.

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