

# Coefficient of Linear Expansion

## 1) Introduction

Most materials expand when heated through a temperature change that does not produce a change in phase. The added heat increases the average amplitude of vibration of the atoms in the material which increases the average separation between the atoms.

Suppose an object of length  $L$  undergoes a temperature change of magnitude  $\Delta T$ . We find experimentally that if  $\Delta T$  is reasonably small, the change in length  $\Delta L$ , is generally proportional to  $L$  and  $\Delta T$ . Therefore we can write:

$$\Delta L = \alpha L \Delta T$$

Where  $\alpha$  is the coefficient of linear expansion and has different values for different materials. For materials that are not isotropic, such as an asymmetric crystal,  $\alpha$  can have a different value depending on the axis along which the expansion is measured.  $\alpha$  can also vary with temperature so that the degree of expansion depends not only on the magnitude of the temperature change, but also on the absolute temperature as well. However, typically these variations are negligible compared to the accuracy with which engineering measurements need to be made. We can often safely take the coefficient of linear expansion as a constant for a given material. Shown below are some values of  $\alpha$  for some common solids.

Substance	$\alpha$ $10^{-6}/^{\circ}\text{C}$
Aluminum	23
Brass	19
Copper	17
Lead	29
Steel	11
Glass	9

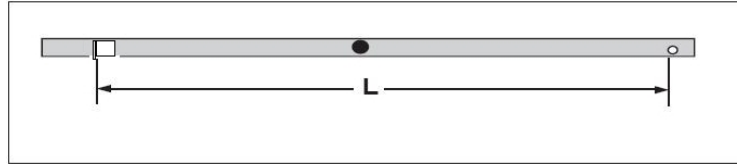
The order of magnitude of the expansion is about 1mm per meter length per 100  $^{\circ}\text{C}$ .

## 2) Equipment

- Steam Generator
- Copper, aluminum and steel bars
- Expansion base
- Ruler
- Thermometer
- Multi-meter
- Pump
- Ice water

### 3) Procedure

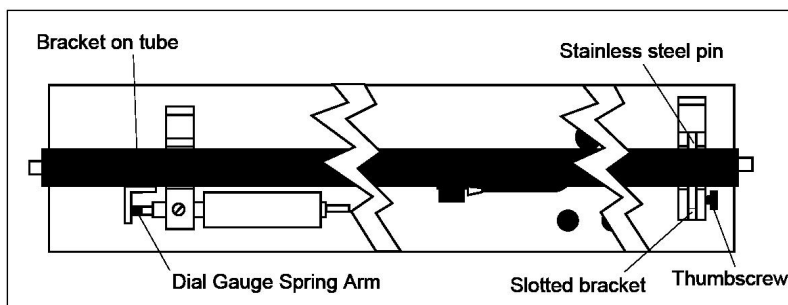
- 1) Measure the length  $L$  of the copper tube at room temperature. Measure from the inner edge of the stainless steel pin on one end, to the inner edge of the angle bracket at the other end (See Figure)



Measuring the tube length

Length of copper tube ( $L$ ) : \_\_\_\_\_

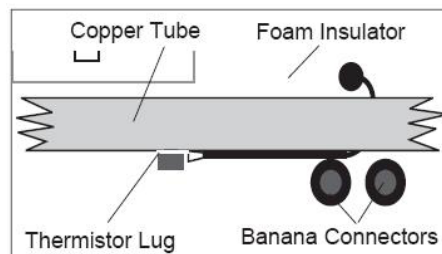
- 2) Mount the copper tube in the expansion base as shown in the figure. The stainless steel pin on the tube fits into the slot on the slotted mounting block and the bracket on the tube presses against the spring arm of the dial gauge.



Top view of equipment setup

Note: Slide or push the tube to one side of the slide support. Drive the thumbscrew against the pin until the tube can no longer be moved. Use this as your reference point.

- 3) Use one of the provided thumbscrews to attach the thermistor lug to the threaded hole in the middle of the copper tube. The lug should be aligned with the axis of the tube, as shown in the above figure, so there is maximum contact between the lug and the tube.
- 4) Place the foam insulator over the thermistor lug as shown in the figure



Thermistor attachment

- 5) Plug the leads of your ohmmeter into the banana plug connectors labeled THERMISTOR in the center of the expansion base.
- 6) Measure  $R_{\text{room}}$ , the resistance of the thermistor at room temperature.

$R_{\text{room}}$ : \_\_\_\_\_

- 7) Use tubing to attach the steam generator to the end of the copper tube. Attach it to the end farthest from the dial gauge.
- 8) Use an object to raise the end of the expansion base at which steam enters the tube (a few centimeters is sufficient) This will allow any water that condenses in the tube to drain out. Place a container under the other end of the tube to catch the draining water.
- 9) Turn the outer casing of the dial gauge to align the zero point on the scale with the long indicator needle. As the tube expands, the indicator needle will move in a counterclockwise direction.
- 10) Turn on the steam generator. As steam begins to flow, watch the dial gauge and the ohmmeter. When the thermistor resistance stabilizes, record the resistance  $R_{\text{hot}}$

$R_{\text{hot}}$ : \_\_\_\_\_

Is the rate of change of the dial gauge and the temperature similar, or does one change faster than the other?

Also record the expansion of the tube length  $\Delta L$  as indicated by the displacement of the indicator on the dial gauge. (Each increment on the dial gauge is equivalent to 0.01 mm of tube expansion).

$\Delta L$ : \_\_\_\_\_

Turn off the steam generator to allow the copper tubes to cool.

Record the resistance and  $\Delta L$  as the tube cools.

$\Delta L$	Resistance	Temperature

Using the conversion table convert your thermistor resistance measurements into temperature measurements. You will need to interpolate to find the temperature measurement.

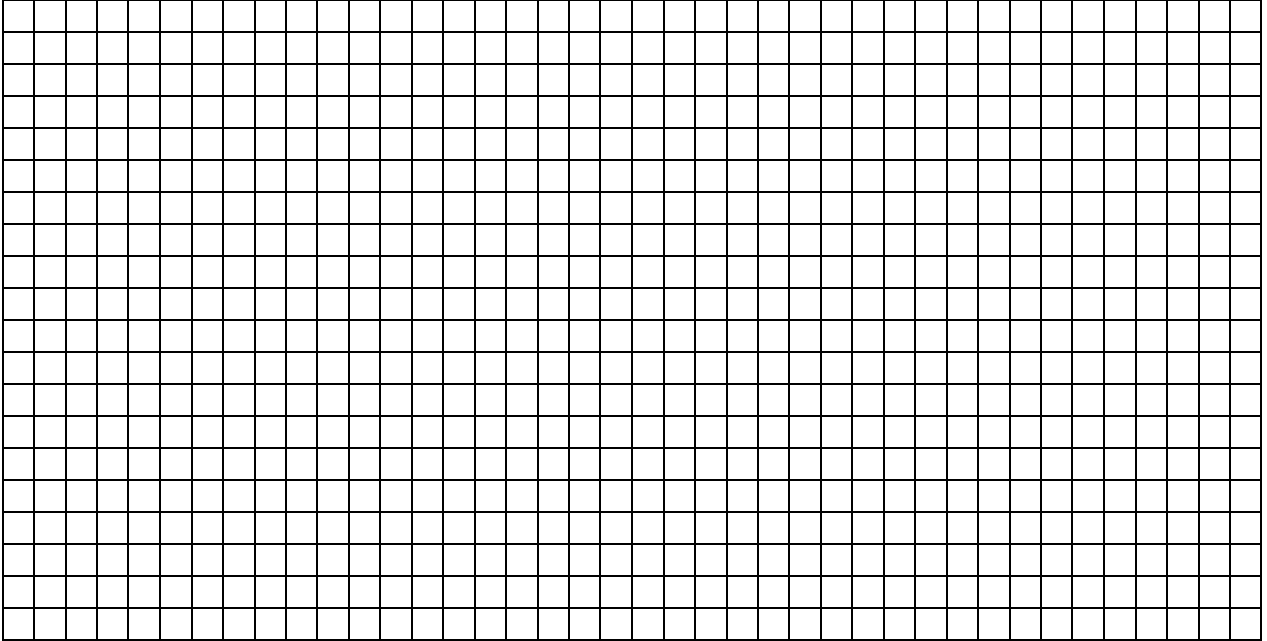
Calculate the coefficient of linear expansion for copper based on your first temperature measurement as the tube is heated. (Make sure you correctly calculate the uncertainty)

$\alpha$ : \_\_\_\_\_

Based on the two points you measure, can you determine if the expansion is linear ?

Using the values of  $\Delta L$  and  $\Delta T$  that you made while the rod was cooling, plot  $\Delta L$  vs  $\Delta T$ . What do you expect this plot to look like?

If the graph is linear, what should the value of the slope represent?



Why isn't the graph linear? Remember your answer to the question about the rate of change of temperature vs rate of change of the dial gauge.

To solve this problem, connect a water pump to the apparatus and place it in ice water. This will allow the temperature to stabilize at a cold temperature to allow a third measurement to be made.

$R_{ice}$ : \_\_\_\_\_

$\Delta L$ : \_\_\_\_\_

Plot  $\Delta L$  vs  $\Delta T$  for your 3 measurements. (Room, Steam, Ice) and determine  $\alpha$  using the graph (with the correct uncertainty)

Which measurement of  $\alpha$  gives you the smaller uncertainty (Room, Steam) or (Room, Steam, Ice)? Why

11) Repeat the above experiment for steel and aluminum. You do not have to repeat the section where the tube is cooled from steam temperature down to room temperature as we now know this does not work.







Question: How many standard deviations are you from the accepted values for the coefficients of linear expansion for copper, steel and aluminum?

Question: Based on your graphs, can you conclude that the coefficients of linear expansion are independent of the temperature and only depend on the temperature change? Based on your single measurements when the tubes are heated, can you conclude that  $\alpha$  is independent of the temperature and only depends on the temperature change?

Question: Based on what you know about linear expansion. Derive a relationship for how the area of an object,  $A$ , will change as a function of temperature change. i.e.  $\Delta A = \beta A \Delta T$ . Derive how  $\beta$  is related to  $\alpha$ . When you do the derivation, assume that you have a rectangular plate of length  $a$  and width  $b$ . Neglect small quantities of size  $\Delta a \Delta b$ .

Question: Based on your result for area, what do you expect for the relationship between volume change as a function of temperature change?

## THERMISTOR CONVERSION TABLE: Temperature versus Resistance

Res. (Ω)	Temp. (°C)	Res. (Ω)	Temp. (°C)	Res. (Ω)	Temp. (°C)	Res. (Ω)	Temp. (°C)
351,020	0	95,447	26	30,976	52	11,625	78
332,640	1	91,126	27	29,756	53	11,223	79
315,320	2	87,022	28	28,590	54	10,837	80
298,990	3	83,124	29	27,475	55	10,467	81
283,600	4	79,422	30	26,409	56	10,110	82
269,080	5	75,903	31	25,390	57	9,767.2	83
255,380	6	72,560	32	24,415	58	9,437.7	84
242,460	7	69,380	33	23,483	59	9,120.8	85
230,260	8	66,356	34	22,590	60	8,816.0	86
218,730	9	63,480	35	21,736	61	8,522.7	87
207,850	10	60,743	36	20,919	62	8,240.6	88
197,560	11	58,138	37	20,136	63	7,969.1	89
187,840	12	55,658	38	19,386	64	7,707.7	90
178,650	13	53,297	39	18,668	65	7,456.2	91
169,950	14	51,048	40	17,980	66	7,214.0	92
161,730	15	48,905	41	17,321	67	6,980.6	93
153,950	16	46,863	42	16,689	68	6,755.9	94
146,580	17	44,917	43	16,083	69	6,539.4	95
139,610	18	43,062	44	15,502	70	6,330.8	96
133,000	19	41,292	45	14,945	71	6,129.8	97
126,740	20	39,605	46	14,410	72	5,936.1	98
120,810	21	37,995	47	13,897	73	5,749.3	99
115,190	22	36,458	48	13,405	74	5,569.3	100
109,850	23	34,991	49	12,932	75		
104,800	24	33,591	50	12,479	76		
100,000	25	32,253	51	12,043	77		