# Coherency Based Dynamic Equivalencing of Electric Power System

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***Abstract*—Electric power networks, all over the world, have been constantly increasing in size and complexity due to several interconnections and high load demand. Power system studies of such large networks require excessive time and storage. In this paper, coherency based dynamic equivalencing technique for reducing the size of the power system network is proposed and implemented in PSS/E (Power System Simulator for Engineers) software to get the dynamic equivalent of a network having high voltage direct current transmission link. A comparison of various softwares available for static network reduction has been carried out. Transient stability simulation of the original and reduced network is performed and compared by creating a three-phase fault. The results are also implemented in Real-Time Digital Simulator (RTDS) to study the performance of equivalent network.**

***Index Terms*—coherency, dynamic equivalencing, high voltage direct current transmission, real-time digital simulator, transient stability simulation.**

## I. INTRODUCTION

The rapid expansion of the power system networks leads to many problems in their modeling and simulation. More often, the modeling of a large power system is not possible due to the limitations on the number of nodes in the various power system softwares like PSCAD (Power System Computer Aided Design) and RTDS (Real Time Digital Simulator). Also, when the behaviour of a part of a system is of interest, then the distant portion of a large interconnected network do not require a detailed modeling. Distant portions hardly affect the overall system dynamics. For a given power system, by reducing the system size, computer storage memory and simulation time can be significantly saved, especially for more frequent system programs (i.e., load-flow and short-circuit analysis).

Integration of the renewable energy sources (i.e., wind power, PV systems, and other distributed generations) requires transmission of power over long distances. For transmitting huge power over long distances, High Voltage Direct Current (HVDC) transmission lines are preferred. To study the power transfer capability and control characteristics of HVDC lines, a detailed modeling and simulation analysis is carried out using advanced power system softwares (e.g., PSS/E, PSCAD, RTDS, etc.). The research in this field is limited by the number of nodes the simulator can handle. All these facts contribute to the necessity of developing an equivalent that can replace the original network for all simulation purposes. The equivalent network is the reduced order model of the complete power system. It reduces the system complexity and retains all the important features of the entire system.

The research in power system equivalents started in the year 1960's. *Modal reduction* was the first network reduction program which was based on computation of eigen-values and eliminating the less important roots of the system equations [1]-[3]. But, this method finds difficulty in determining the modes to be eliminated. Also, computation of eigen-values is a tedious task. In 1970's, Podmore [4] came up with the idea of coherency based network reduction which became very popular and widely accepted, as it is able to give a physical picture of the reduce system. Later on, *Slow coherency algorithm* was developed, which was the combination of the above two methods [5]. All these techniques are having difficulty in their implementation to the non-linear time domain simulation as they required the modifications in the original dynamic simulation program in order to make use of the state matrix of the equivalent. Moreover, all these techniques are based on rigorous programming, so it is difficult to understand them.

In this paper, different approaches to obtain the static and dynamic equivalent of power system network are described. Static equivalents are only useful for the static programs, i.e., load flow, short circuit analysis, optimization, planning, and load forecasting. For the dynamic analysis of power system networks (e.g., transient stability studies and security assessment), the dynamic equivalents are used. This paper provides the static equivalent using E-TRAN. It bridges the gap between load flow programs and EMT (electromagnetic transients) programs which means, once we enter the network data in load flow program format, it automatically translates it into the PSCAD readable graphical format used for EMT studies. Equivalent obtained using E-TRAN can be directly run into PSCAD. Another software PSS/E is used to obtain to the dynamic equivalent. PSS/E provides non-linear time domain simulation of large power system network using graphical user interface. Also, it is possible to import the PSS/E data files directly into RTDS. A comparison of various softwares available for static network reduction has been carried out. Transient stability simulation of the original and reduced network having HVDC link is performed and compared by creating a three-phase fault. The results are also implemented in Real-Time Digital Simulator (RTDS) to study the performance of equivalent network.

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## II. STATIC EQUIVALENCING TECHNIQUES

Static equivalents can be divided into three groups: i) physical reduction; ii) topological reduction; and, iii) modal reduction. In physical reduction, the power system components like generators, transformers, transmission lines, and loads are modelled to the degree to which they are required for particular analysis. Elements electrically close to the disturbance are generally modelled more accurately than the elements which are far away. In topology reduction, certain nodes are removed or combined together to reduce the size of the network. Model reduction uses linearized model of the power system to be reduced and eliminates the unexcited modes [6]. The equivalent model obtained is in the form of reduced set of differential equations. The softwares that have been used, in this paper, are fundamentally based on the following static equivalencing techniques:

### *A. Kron's Reduction*

This is the most popular method for network reduction and also known as ward reduction. It uses the Gauss elimination method to reduce the size of the bus admittance matrix. This approach was originally presented by J. B. Ward in 1949 [7]. The power system under consideration is divided into two parts: the study system and the external system. The study system is left untouched while external system is the part of the system to be equivalenced. The buses which separate the study system to the external system are called boundary buses as shown in Fig. 1.

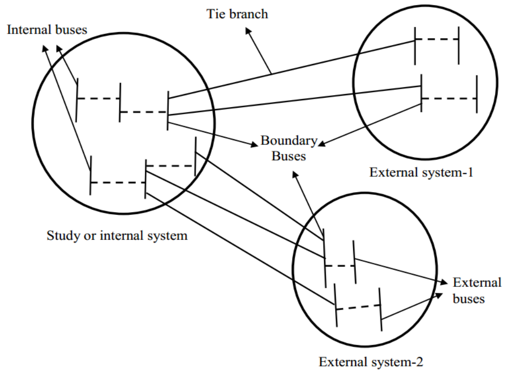

Fig. 1. Interconnected Power System Network

The objective of ward reduction is to eliminate the external buses with some modifications at the boundary buses. This treats the generation and load in the external network as constant current quantities. As the internal system is connected to the external system only through the boundary buses, thus the mutual admittance between any internal and external bus is zero. The following system admittance matrix is formulated in which buses are ordered in the sequence of internal, boundary, and external buses.

$$\begin{pmatrix} Y_{II} & Y_{IB} & 0 \\ Y_{BI} & Y_{BB} & Y_{BE} \\ 0 & Y_{EB} & Y_{EE} \end{pmatrix}\begin{pmatrix} V_I \\ V_B \\ V_E \end{pmatrix}=\begin{pmatrix} I_I \\ I_B \\ I_E \end{pmatrix} \tag{1}$$

where, the diagonal elements represent the self admittance matrices of the internal, boundary, and external system. The off-diagonal elements represent the mutual admittance matrix between the internal and boundary & between the boundary and external system. By applying the Gauss elimination to the system (1), the admittance matrix for the reduced system is obtained as:

$$\begin{pmatrix} Y_{II} & Y_{IB} \\ Y_{BI} & Y_{BB}' \end{pmatrix}\begin{pmatrix} V_I \\ V_B \end{pmatrix}=\begin{pmatrix} I_I \\ I_B' \end{pmatrix} \tag{2}$$

where,

$$Y_{BB}' = Y_{BB} - Y_{BE}Y_{EE}^{-1}Y_{EB} \tag{3}$$

$$I_B' = I_B - Y_{BE}Y_{EE}^{-1}I_E \tag{4}$$

$$Y_{Eq} = -Y_{BE}Y_{EE}^{-1}Y_{EB} \tag{5}$$

$$I_{Eq} = -Y_{BE}Y_{EE}^{-1}I_E \tag{6}$$

The equivalent admittance is physically realized as equivalent transmission lines between the boundary buses. The equivalent injection current is converted back to the power injection based on the voltage at the boundary buses. The final form of ward equivalent is shown in Fig. 2.

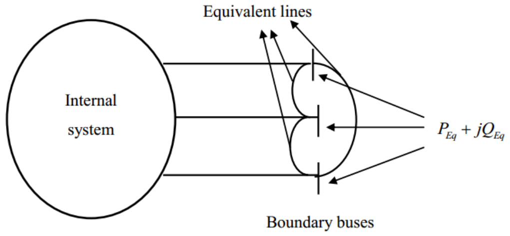

Fig. 2. Ward Equivalent

### *B. REI Reduction*

REI stands for *Radial, Equivalent, and Independent*. This method was developed by P. Dimo in 1975. The basic idea of this method is to aggregate the current and power injection of the group of eliminated buses on to a fictitious 'REI' node [8], [9]. Thus, the group of buses is replaced by a single node. The nodes that are being eliminated become passive and the fictitious node is connected through a virtual radial network, called the REI network, with a star point 'G' to these passive nodes as shown in Fig. 3. All the passive nodes and the star point are removed by Gaussian elimination.

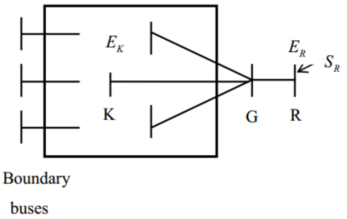

Fig. 3. REI Equivalent

## III. STATIC EQUIVALENCING USING E-TRAN

In this paper, static network equivalent of New-England 39-bus system is obtained using E-TRAN software. Various other softwares like PSS/E, DigSILENT PowerFactory and, PowerWorld simulator are also available nowadays to obtain the static equivalent network but, the advantage of using E-TRAN is that it provides the multi-port network equivalent whereas the DigSILENT is capable of providing only the single port equivalent network. In multi-port equivalent network, it does not only build the equivalent voltage source behind the impedance but also builds the interconnections between the boundary buses in the external network [10]. This equivalent includes both the diagonal and off-diagonal terms in the admittance matrix. However, the single-port network equivalent does not consider the off-diagonal terms of the reduced admittance matrix. The static network equivalent obtained through PSS/E is only capable of providing the interconnections between the boundary buses and does not provide the equivalent voltage source. Consequently, the equivalent network obtained using E-TRAN can be used for load flow analysis as well as for short circuit analysis.

## IV. DYNAMIC EQUIVALENCING

The dynamics of the power system is mainly affected by the generators. Thus, it is important to take into account the generator buses in the equivalencing process. Static equivalencing techniques replace the generation as negative loads and thus, treat it as a constant voltage source. This paper uses the coherency based method to build a dynamic equivalent. Coherency is the term used for a group of generators which swing together following a remote disturbance. Generator buses are combined together if they are coherent. The basic steps of coherency based dynamic equivalencing are:

### *A. Coherency determination*

Two generator buses are defined as coherent if the angular difference between them is constant within a certain tolerance over a certain time interval when the power system is perturbed [11]. Coherency is independent of the size of the disturbance. Also, a linearized system model can be used for identification of coherent generators and hence, a classical synchronous machine model is considered and the excitation and the turbine-governor systems are ignored.

In this paper, an inertia constant index along with the time-domain simulation of rotor angles is used for the identification of coherent generators. The inertia constant $\beta_{ij}$ is based on the normalized value of the inertia differences among the machines [12]. This is defined by the expression:

$$\beta_{ij} = \frac{|M_i - M_j|}{Max(M_i, M_j)} = \beta_{ji} \tag{7}$$

for $i = 1,2,3,\ldots.n$ and $j = 1,2,3,\ldots.(i\text{-}1)$

$M_i$ and $M_j$ are the inertia constants of machines *i* and *j*. For perfect coherency, $\beta_{ij}$ must be zero but, in practice $\beta_{ji} \le 0.2$ is considered to be satisfactory to conclude that the machines are coherent.

Generators in the external system which are found to be coherent based on the inertia constant index are further tested by applying a fault in the study system and observing the rotor angles of these generators. This paper uses the tolerance based coherency identification criterion as:

$$\max\left|\Delta\delta_i(t) - \Delta\delta_j(t)\right| \le \varepsilon \tag{8}$$

where,

$\tau$ = 1~3 seconds; is the recorded simulation time.

$\varepsilon = 5^\circ \sim 10^\circ$; is maximum rotor angle deviation.

$\delta_i$ and $\delta_j$ are the rotor angles for machines *i* and *j*.

In this paper, time domain simulation of rotor angles is carried out in PSS/E. Equation (8) states that within a simulation time $\tau$, after a large disturbance, the difference of two coherent generator's rotor angle deviation is not larger than a small constant $\varepsilon$ at every sampling point.

### *B. Reduction of generator buses*

To reduce the number of generator buses, the coherent generator buses are lumped together into a single equivalent bus. Load flow is run on the PSS/E raw data file and the results obtained after load flow are used to aggregate the parameters of the coherent generators. The load flow data file is then modified to incorporate the parameters of equivalent bus in the system. The procedure can be described in the following steps:

1) The voltage and phase angle on the equivalent bus is defined. Either an average voltage of the group or the voltage of the individual bus; called as reference bus, is selected. Similarly the phase angle is defined. Each terminal bus is connected through an ideal transformer with complex turns ratio to the equivalent bus, defined as

$$a_k = \frac{V_k}{V_t} \tag{9}$$

where, $V_k$ is the voltage on coherent bus k and $V_t$ is the voltage of the equivalent bus.

2) The angular frequencies of the coherent generators are almost identical and thus assumed to be $\omega$, the swing equation of equivalent generator can be written as:

$$\left(\sum_{i=1}^{n} M_i\right)\frac{d\omega}{dt} = \sum_{i=1}^{n} P_{m_i} - \sum_{i=1}^{n} P_{e_i} - \left(\sum_{i=1}^{n} D_i\right)\omega \tag{10}$$

where, *n* is the number of coherent generators. The mechanical and electrical power of the equivalent generator is the sum of the mechanical and electrical power of coherent generators. Also, the inertia and damping constant of the equivalent generator can be defined as the sum of the inertia and damping constant of the coherent generators. The transient reactances of all the coherent generators are combined in parallel to get the transient reactance of equivalent generator. If the generator buses in the coherent group have load, then the equivalent load consists of the sum of the PQ load of all coherent buses.

3) In the last step of generator aggregation, the control units associated with the generators are combined together to transfer to the equivalent generator. In this paper, only the exciter and governor unit is considered as a control unit and

the control unit of reference generator has been taken as equivalent control unit.

4) The reference generator is replaced by the equivalent generator and the remaining generator buses in coherent group are removed.

### *C. Elimination of load buses*

After reducing the generator buses, the network reduction is performed using the PSS/E network equivalence tool [13]. It eliminates all the load buses from the external system. All the boundary buses are retained in this process.

## V. RESULTS AND DISCUSSION

In this paper, New-England 39-bus system is considered for application of the above procedure. The system has 10 generators, out of which 6 belong to the external system. The system is divided into two parts as shown in Fig. 4. The area above dashed line is study system (area-1) and the area below it, is taken as external system (area-2). Assuming that the faults will occur only in the internal area, the external system is reduced as an equivalent network.

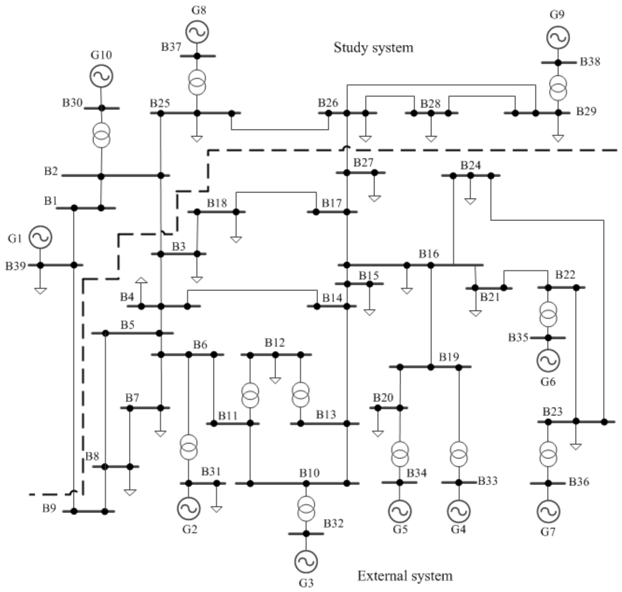

Fig. 4. New-England 39-bus system

The buses belonging to each area are:

Area-1: 1, 2, 25, 26, 28, 29, 30, 37, 38, 39
Area-2: 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 31, 32, 33, 34, 35, 36
Boundary buses: 3, 9, 27
Slack bus: 31

The external network of system is first reduced as a static equivalent by using E-TRAN and then the same system is reduced as a dynamic equivalent implementing coherency technique in PSS/E software. The comparison between the load flow results of original and reduced network obtained using E-TRAN are shown in Table I. For short-circuit analysis, a three-phase bus fault is applied to bus-29 using PSCAD software in both the systems and the line flows are compared in Table II. These results show that the boundary bus voltages and slack bus power of original system match exactly with the reduced system. Also, the line flows after the short circuit is exactly matching in both the systems and thus, the equivalent is the correct representation of the original system and therefore, can be used as load flow and short-circuit study.

TABLE I
COMPARISON BETWEEN ORIGINAL AND STATIC EQUIVALENT

| **Full system** | | | **Reduced system** | | |
|---|---|---|---|---|---|
| ***Boundary bus voltages*** | ***Magnitude (per unit)*** | ***Angle (deg)*** | ***Boundary bus voltages*** | ***Magnitude (per unit)*** | ***Angle (deg)*** |
| V3 | 1.029 | -8.60 | V3 | 1.029 | -8.60 |
| V9 | 1.027 | -10.32 | V9 | 1.027 | -10.32 |
| V27 | 1.037 | -7.50 | V27 | 1.037 | -7.50 |
| ***Slack bus Power*** | | | ***Slack bus Power*** | | |
| ***Active Power (MW)*** | ***Reactive Power (MVAr)*** | | ***Active Power (MW)*** | ***Reactive Power (MVAr)*** | |
| P31=521.2 | Q31=198.2 | | P31=521.16 | Q31=198.28 | |

TABLE II
RESULTS FOR SHORT CIRCUIT TEST

| **Full System** | | **Reduced System** | |
|---|---|---|---|
| ***Active Power Flow from bus-29 (MW)*** | | ***Active Power Flow from bus-29 (MW)*** | |
| From 29 to 26 | 191.976 | From 29 to 26 | 191.99 |
| From 29 to 28 | 348.94 | From 29 to 28 | 348.99 |
| From 29 to 38 | -825.46 | From 29 to 38 | -825.38 |
| ***Reactive Power Flow from bus-29 (MVAr)*** | | ***Reactive Power Flow from bus-29 (MVAr)*** | |
| From 29 to 26 | -67.21 | From 29 to 26 | -67.20 |
| From 29 to 28 | -38.85 | From 29 to 28 | -38.83 |
| From 29 to 38 | 79.06 | From 29 to 38 | 79.05 |

For dynamic equivalencing, generator G2 is not considered for coherency test because it is connected to the swing bus of the system. Rest of the generators present in the external system are found to be coherent using the inertia constant index and their rotor angle simulation and aggregated into a single machine. The network is reduced in PSS/E and is tested for transient analysis. Table III gives the percentage of reduction in number of components in equivalent network. Table IV compares the simulation time taken by both the systems for a fault at bus-29. It shows that there is a saving of about 59% in simulation time of reduced system. Table V compares the load flow and short-circuit results in PSS/E between both the systems which tells that the values for both the systems are exactly matching.

TABLE III
COMPARISON OF NUMBER OF COMPONENTS

| | **Original system** | **Reduced system** | **Percentage of reduction** |
|---|---|---|---|
| **Buses** | 39 | 15 | 61.53 |
| **Loads** | 19 | 10 | 47.36 |
| **Plants** | 10 | 6 | 40.00 |
| **Branches** | 34 | 14 | 58.82 |
| **2-winding transformer** | 12 | 9 | 25.00 |

A three-phase fault at *t*=5sec. is applied on bus-29 and cleared after 5 cycle duration, i.e., 0.0833 sec. The transient simulation results in PSS/E for original and reduced systems are compared in the Fig. 5 to Fig. 7. These results show that the transient response of both the systems is following the same nature with a little difference between their settling times. Hence, the equivalent is satisfying the dynamic equivalencing criteria.

TABLE IV
COMPARISON OF SIMULATION TIME

| | **Original system** | **Reduced system** | **Saving in time (%)** |
|---|---|---|---|
| **Simulation time** | 353 sec | 145 sec | 58.92 |

TABLE V
COMPARISON OF LOAD FLOW RESULTS IN PSS/E

| **Full system** | | | **Reduced system** | | |
|---|---|---|---|---|---|
| ***Boundary bus voltages*** | ***Magnitude (per unit)*** | ***Angle (deg)*** | ***Boundary bus voltages*** | ***Magnitude (per unit)*** | ***Angle (deg)*** |
| V3 | 1.0302 | -8.61 | V3 | 1.0301 | -8.61 |
| V9 | 1.0282 | -10.33 | V9 | 1.0282 | -10.33 |
| V27 | 1.0377 | -7.51 | V27 | 1.0377 | -7.51 |
| ***Slack bus Power*** | | | ***Slack bus Power*** | | |
| ***Active Power (MW)*** | ***Reactive Power (MVAr)*** | | ***Active Power (MW)*** | ***Reactive Power (MVAr)*** | |
| P31=521.2 | Q31=198.3 | | P31=521.2 | Q31=198.7 | |
| ***Power flow from area-1 to area-2*** | | | ***Power flow from area-1 to area-2*** | | |
| P=647 MW | Q= 231 MVAr | | P=647 MW | Q=231 MVAr | |
| ***Short-circuit MVA*** | | | ***Short-circuit MVA*** | | |
| 46.02 pu | | | 46.18 pu | | |

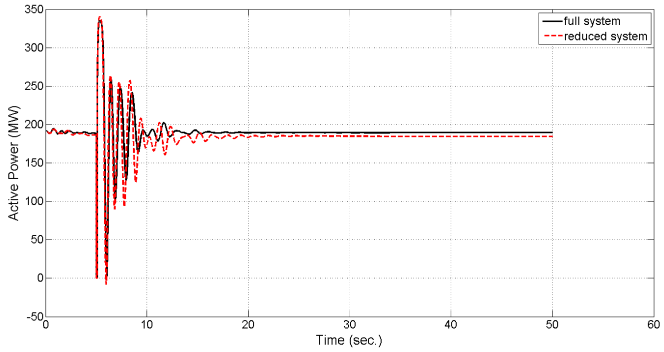

Fig. 5. Active power flow from bus-29 to bus-26

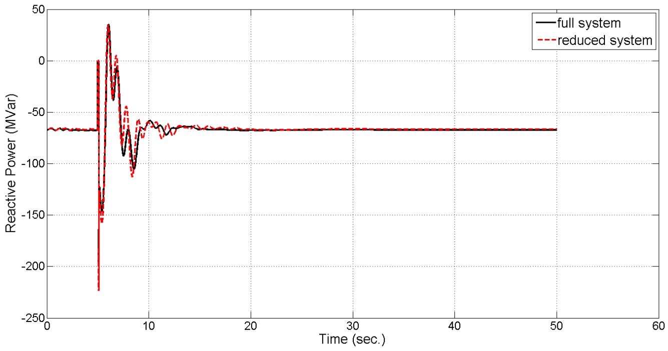

Fig. 6. Reactive power flow from bus-29 to bus-26

A HVDC line is connected between bus-1 and bus-2 of the study system and the same external network is considered for dynamic equivalencing. The load flow results before and after equivalencing are compared in Table VI and VII which are exactly matching. The systems have also been tested for transient simulation shown in Fig. 8 to Fig. 10. The response is matching with some tolerance.

To see the performance of dynamic equivalent in real-time digital simulator (RTDS), the original and reduced system are imported in RTDS and tested for a three-phase fault at bus-29 in run time environment of RTDS window. Results in RTDS have been in Table VIII and IX. These results show that the reduced network satisfy the dynamic equivalencing criteria.

TABLE VI
COMPARISON OF LOAD FLOW RESULTS WITH HVDC LINE

| **Full system** | | | **Reduced system** | | |
|---|---|---|---|---|---|
| ***Boundary bus voltages*** | ***Magnitude (per unit)*** | ***Angle (deg)*** | ***Boundary bus voltages*** | ***Magnitude (per unit)*** | ***Angle (deg)*** |
| V3 | 0.9832 | -8.30 | V3 | 0.9831 | -8.30 |
| V9 | 1.0156 | -19.27 | V9 | 1.0156 | -19.28 |
| V27 | 1.0087 | -7.35 | V27 | 1.0086 | -7.35 |
| ***Slack bus Power*** | | | ***Slack bus Power*** | | |
| ***Active Power (MW)*** | ***Reactive Power (MVAr)*** | | ***Active Power (MW)*** | ***Reactive Power (MVAr)*** | |
| P31=636.7 | Q31=288.9 | | P31=636.8 | Q31=289.3 | |
| ***Power flow from area-1 to area-2*** | | | ***Power flow from area-1 to area-2*** | | |
| P=539 MW | Q= 110 MVAr | | P=539 MW | Q=111 MVAr | |

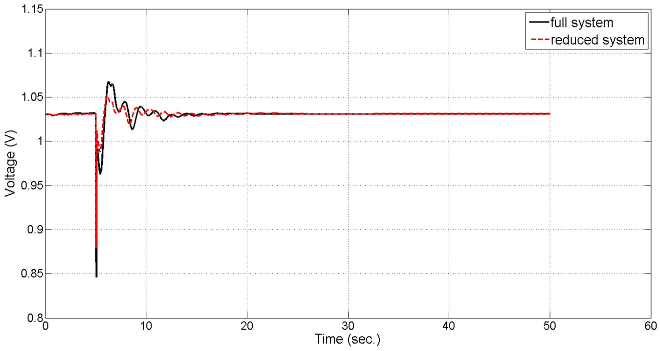

Fig. 7. Voltage profile at boundary bus-3

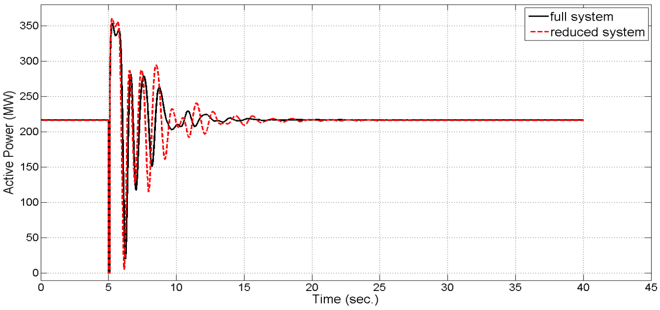

Fig. 8. Active power flow from bus-29 to bus-26 with HVDC line

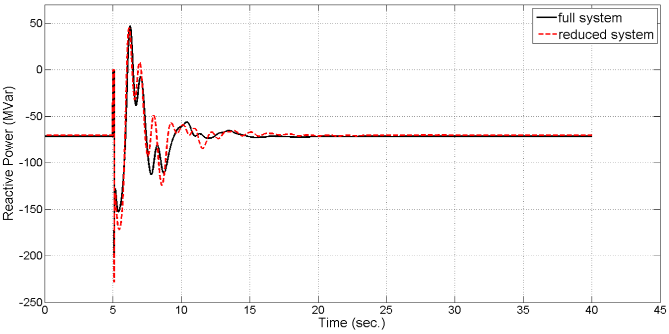

Fig. 9. Reactive power flow from bus-29 to bus-26 with HVDC line

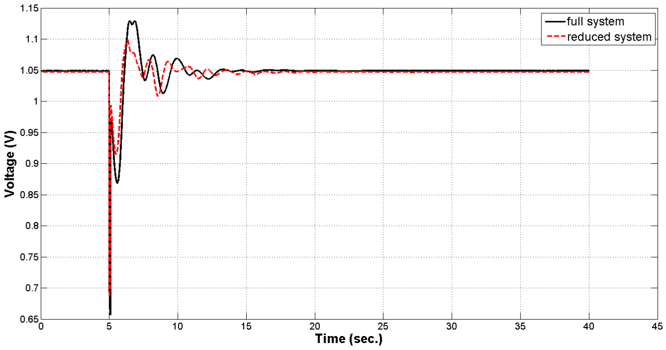

Fig. 10. Voltage profile at boundary bus-27

TABLE VII
COMPARISON OF POWER FLOW IN HVDC LINE

| | **Power flow in HVDC link** | | | |
|---|---|---|---|---|
| | ***Active Power MW*** | ***Reactive Power MVAr*** | ***Convertor transformer*** | |
| | | | ***Ratio*** | ***Angle*** |
| **Full System** | 904.8 | 509.3 | 0.962 | 14.2 |
| **Reduced System** | 904.8 | 509.2 | 0.962 | 14.2 |

TABLE IX
COMPARISON OF TRANSIENT SIMULATION RESULTS IN RTDS

| **Full System** | | **Reduced System** | |
|---|---|---|---|
| ***Steady state value of active Power flow from bus-29 (MW)*** | | ***Steady state value of active Power flow from bus-29 (MW)*** | |
| From 29 to 26 | 194.9 | From 29 to 26 | 201.9 |
| From 29 to 28 | 350.6 | From 29 to 28 | 357.6 |
| From 29 to 38 | -828.8 | From 29 to 38 | -841.0 |
| ***Steady state value of reactive Power flow from bus-29 (MVAr)*** | | ***Steady state value of reactive power Flow from bus-29 (MVAr)*** | |
| From 29 to 26 | -65.54 | From 29 to 26 | -64.95 |
| From 29 to 28 | -37.14 | From 29 to 28 | 36.40 |
| From 29 to 38 | 76.58 | From 29 to 38 | 76.19 |

## VI. CONCLUSIONS

This paper provides a comparative study of the static and dynamic equivalents using the power system softwares. Coherency based technique is adopted for dynamic equivalencing and the results obtained has been varified on the test case network. Static equivalent can not be used for transient simulation studies and hence, the correct dynamic equivalent is obtained using PSS/E. The main benefit of this paper is to provide the dynamic equivalent using the graphical user interface of modern power system softwares and hence, it avoids the use of rigorous prgramming in MATLAB. The results have also been tested on real-time digital simulator. The transient simulation results in PSS/E shows that the line flows and voltage profile match exactly in original and reduced network. The maximum steady state error obtained between the transient response of two syatems is 2.5%.

Network reduced with HVDC line, is also tested for a three-phase fault and the transient simulation of both the systems is compared. The maximum steady-state error beween the two networks found in this case is 3.17%. The simulaion time for both the system is also compared and it is found that reduced system provides 59% saving in time. Also the number of components in equivalent network is reuced and thus, it will have less number of state variables and differential equation which helps in saving the computer storage requirement and computaion time.

## REFERENCES

[1] Savo D. Dukic, Andrija T. Saric, “Dynamic model reduction: An overview of available techniques with application to power systems,” *Serbian Journal of Electrical Engineering*, vol. 9, no. 2, pp.131-169, Jun 2012.

[2] Sebastiao E.M. de Oliveira, J.F. de Queiroz, “Modal dynamic equivalent for electric power systems, Part I: Theory”, *IEEE Trans. on Power Syst.*, vol.3, no.4, Nov 1988.

[3] J.M. Undrill, A.E.Turner, “Construction of power system electromechanical equivalents by modal analysis” *IEEE Winter Power meeting*, New York, 1971.

[4] R. Podmore, “A comprehensive program for computing coherency based dynamic equivalents,” *Power Industry Computer Applications Conference*, pp. 298-306, 15-18 May 1979.

[5] J.H. Chow, R. Galarza, P. Acaari, W. W. Price, “Inertial and slow coherency aggregation algorithms for power system dynamic model reduction,” *IEEE Trans. on Power Syst.*, vol. 10, no. 2, pp. 680-685, May 1995.

[6] J. Machowski, J. Bialek and J. Bumby, *Power System Dynamics: Stability and control*, Wiley, 2008, Ch. 14.

[7] J. B. Ward, “Equivalent circuits for Power flow Studies,” *AIEE Trans. on Power Apparatus and Systems*, vol. 68, pp. 373-382, 1949.

[8] P. Dimo, “Nodal analysis of power systems,” England, Abacus Press, 1975.

[9] W.F. Tinney, W.L. Powell, “The REI approach to power network equivalents,” *PICA 77th conference*, Toronto, Canada, pp. 314-320, May 1997.

[10] G. Irwin, D. Woodford, “E-TRAN: Translation of load flow/stability data into electromagnetic transients programs,” *International Conference on Power System Transients*, New Orleans, USA, 2003.

[11] J.P. Yang, G.H. Cheng, Z. Xu, “Dynamic reduction of large power system in PSS/E,” *IEEE/PES Transmission and Distribution Conference & Exhibition: Asia and Pacific*, Dalian, China, pp. 1-4, 2005.

[12] Shaikh Rashedur Rahman, Md. Yeakub. Hussain, Md. Sekendar Ali, “A new approach to coherency identification in large multi-machine power system,” *International Conference on Electrical and Computer Engineering, Dhaka*, Bangladesh, pp. 587-590, Dec 2012.

[13] Siemens PTI, Program Operation Manual, PSS/E 33, Mar 2013.

[14] New England 39-bus system generator and exciter data available at http://sys.elec.kitami-it.ac.jp/ueda/demo/WebPF/39-New-England