Collaborative Filtering

Practical Machine Learning, CS 294-34

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Based on slides by Aleksandr Simma

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Outline

What is Collaborative Filtering?

Group of users

Group of items

What is Collaborative Filtering?

Group of users

Group of items

- Observe some user-item preferences
- Predict new preferences:

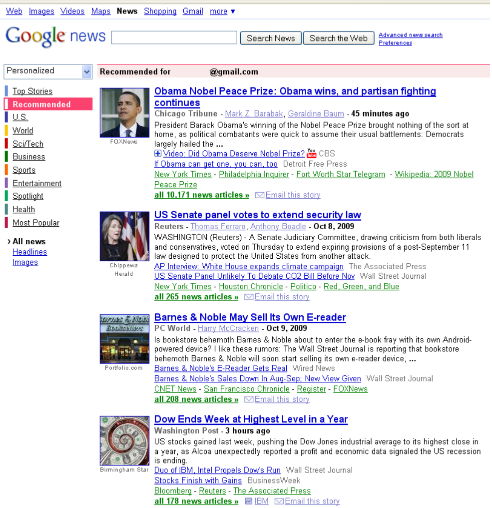
Does Bob like strawberries???

Collaborative Filtering in the Wild...

Amazon.com recommends products based on purchase history

Linder et al., 2003

Collaborative Filtering in the Wild...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Obama Nobel Peace Prize: Obama wins, and partisan fighting continues
Chicago Tribune - Mark Z. Barabak, Geraldine Baum - 45 minutes ago
President Barack Obama's winning of the Nobel Peace Prize brought nothing of the sort at home, as political combatants were quick to assume their usual battlements: Democrats largely hailed the ...
Video: Did Obama Deserve Nobel Prize? CBS
If Obama can get one, you can, too Detroit Free Press
New York Times - Philadelphia Inquirer - Fort Worth Star Telegram - Wikipedia: 2009 Nobel Peace Prize
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Chippewa Herald

US Senate panel votes to extend security law
Reuters - Thomas Ferraro, Anthony Boadle - Oct 8, 2009
WASHINGTON (Reuters) - A Senate Judiciary Committee, drawing criticism from both liberals and conservatives, voted on Thursday to extend expiring provisions of a post-September 11 law designed to protect the United States from another attack.
AP Interview: White House expands climate campaign The Associated Press
US Senate Panel Unlikely To Debate CO2 Bill Before Nov Wall Street Journal
New York Times - Houston Chronicle - Politico - Red, Green, and Blue
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Barnes & Noble May Sell Its Own E-reader
PC World - Harry McCracken - Oct 9, 2009
Is bookstore behemoth Barnes & Noble about to enter the e-book fray with its own Android-powered device? I like these rumors: The Wall Street Journal is reporting that bookstore behemoth Barnes & Noble will soon start selling its own e-reader device, ...
Barnes & Noble's E-Reader Gets Real Wired News
Barnes & Noble's Sales Down In Aug-Sep; New View Given Wall Street Journal
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Birmingham Star

Dow Ends Week at Highest Level in a Year
Washington Post - 3 hours ago
US stocks gained last week, pushing the Dow Jones industrial average to its highest close in a year, as Alcoa unexpectedly reported a profit and economic data signaled the US recession is ending.
Duo of IBM, Intel Propels Dow's Run Wall Street Journal
Stocks Finish with Gains BusinessWeek
Bloomberg - Reuters - The Associated Press
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- **Google News** recommends new articles based on click and search history
- Millions of users, millions of articles

Das et al., 2007

Collaborative Filtering in the Wild...

Netflix predicts other “Movies You’ll ♡” based on past numeric ratings (1-5 stars)

- Recommendations drive 60% of Netflix’s DVD rentals
- Mostly smaller, independent movies (Thompson 2008)

http://www.netflix.com

Collaborative Filtering in the Wild...

- **Netflix Prize:** Beat Netflix recommender system, using Netflix data → Win $1 million
- **Data:** 480,000 users 18,000 movies 100 million observed ratings = only 1.1% of ratings observed

"The Netflix Prize seeks to substantially improve the accuracy of predictions about how much someone is going to love a movie based on their movie preferences."

What is Collaborative Filtering?

Insight: Personal preferences are correlated

- If Jack loves A and B, and Jill loves A, B, and C, then Jack is more likely to love C

Collaborative Filtering Task

- Discover patterns in observed preference behavior (e.g. purchase history, item ratings, click counts) across community of users
- Predict new preferences based on those patterns

Does not rely on item or user attributes (e.g. demographic info, author, genre)

- Content-based filtering: complementary approach

What is Collaborative Filtering?

Given:

- Users $u \in \{1, \ldots, U\}$
- Items $i \in \{1, \ldots, M\}$
- Training set $\mathcal{T}$ with observed, real-valued preferences r_{ui} for some user-item pairs (u, i)
  - r_{ui} = e.g. purchase indicator, item rating, click count ...

Goal: Predict unobserved preferences

- Test set $\mathcal{Q}$ with pairs (u, i) not in $\mathcal{T}$

View as matrix completion problem

- Fill in unknown entries of sparse preference matrix

$$\mathbf{R} = \underbrace{\left.\begin{bmatrix} ? & ? & 1 & \ldots & 4 \\ 3 & ? & ? & \ldots & ? \\ ? & 5 & ? & \ldots & 5 \end{bmatrix}\right\} U \text{ users}}_{M \text{ items}}$$

What is Collaborative Filtering?

Measuring success

- Interested in error on unseen test set Q, not on training set
- For each (u, i) let r_{ui} = true preference, $\hat{r}_{ui}$ = predicted preference
- Root Mean Square Error
  - $\text{RMSE} = \sqrt{\frac{1}{|Q|} \sum_{(u,i)\in Q} (r_{ui} - \hat{r}_{ui})^2}$
- Mean Absolute Error
  - $\text{MAE} = \frac{1}{|Q|} \sum_{(u,i)\in Q} |r_{ui} - \hat{r}_{ui}|$
- Ranking-based objectives
  - e.g. What fraction of true top-10 preferences are in predicted top 10?

Centering Your Data

- What?
  - Remove bias term from each rating before applying CF methods: $\tilde{r}_{ui} = r_{ui} - b_{ui}$
- Why?
  - Some users give systematically higher ratings
  - Some items receive systematically higher ratings
  - Many interesting patterns are in variation around these systematic biases
  - Some methods assume mean-centered data
    - Recall PCA required mean centering to measure variance around the mean

Centering Your Data

- What?
  - Remove bias term from each rating before applying CF methods: $\tilde{r}_{ui} = r_{ui} - b_{ui}$
- How?
  - Global mean rating
    - $b_{ui} = \mu := \frac{1}{|\mathcal{T}|} \sum_{(u,i)\in\mathcal{T}} r_{ui}$
  - Item's mean rating
    - $b_{ui} = b_i := \frac{1}{|R(i)|} \sum_{u\in R(i)} r_{ui}$
    - $R(i)$ is the set of users who rated item i
  - User's mean rating
    - $b_{ui} = b_u := \frac{1}{|R(u)|} \sum_{i\in R(u)} r_{ui}$
    - $R(u)$ is the set of items rated by user u
  - Item's mean rating + user's mean deviation from item mean
    - $b_{ui} = b_i + \frac{1}{|R(u)|} \sum_{i\in R(u)} (r_{ui} - b_i)$

Shrinkage

- What?
  - Interpolating between an estimate computed from data and a fixed, predetermined value
- Why?
  - Common task in CF: Compute estimate (e.g. a mean rating) for each user/item
  - Not all estimates are equally reliable
  - Some users have orders of magnitude more ratings than others
  - Estimates based on fewer datapoints tend to be noisier

$$\mathbf{R} = \begin{array}{ccccccccc} & A & B & C & D & E & F & \text{User mean} \\ \textit{Alice} & 2 & 5 & 5 & 4 & 3 & 5 & 4 \\ \textit{Bob} & 2 & ? & ? & ? & ? & ? & 2 \\ \textit{Craig} & 3 & 3 & 4 & 3 & ? & 4 & 3.4 \end{array}$$

- Hard to trust mean based on one rating

Shrinkage

- What?
  - Interpolating between an estimate computed from data and a fixed, predetermined value
- How?
  - e.g. Shrunk User Mean:

$$\tilde{b}_u = \frac{\alpha}{\alpha + |R(u)|} * \mu + \frac{|R(u)|}{\alpha + |R(u)|} * b_u$$

  - μ is the global mean, α controls degree of shrinkage
  - When user has many ratings, $\tilde{b}_u \approx$ user's mean rating
  - When user has few ratings, $\tilde{b}_u \approx$ global mean rating

$\mathbf{R} =$

	A	*B*	*C*	*D*	*E*	*F*	User mean	Shrunk mean
Alice	2	5	5	4	3	5	4	3.94
Bob	2	?	?	?	?	?	2	2.79
Craig	3	3	4	3	?	4	3.4	3.43

Global mean $\mu = 3.58$, $\alpha = 1$

Classification/Regression for CF

Interpretation: CF is a set of M classification/regression problems, one for each item

- Consider a fixed item i
- Treat each user as incomplete vector of user's ratings for all items except i: $\vec{r}_u = (3, ?, ?, 4, ?, 5, ?, 1, 3)$
- Class of each user w.r.t. item i is the user's rating for item i (e.g. $1, 2, 3, 4,$ or 5)
- Predicting rating $r_{ui} \equiv$ Classifying user vector $\vec{r}_u$

Classification/Regression for CF

Approach:

- Choose your favorite classifier/regression algorithm
- Train separate predictor for each item
- To predict r_{ui} for user u and item i, apply item i's predictor to vector of user u's incomplete ratings vector

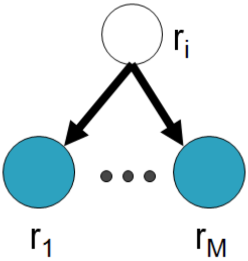
Pros:

- Reduces CF to a well-known, well-studied problem
- Many good prediction algorithms available

Cons:

- Predictor must handle missing data (unobserved ratings)
- Training M independent predictors can be expensive
- Approach may not take advantage of problem structure
  - Item-specific subproblems are often related

Naive Bayes Classifier



- Treat distinct rating values as classes
- Consider classification for item i
- Main assumption
  - For any items $j \neq k \neq i$, r_j and r_k are conditionally independent given r_i
  - When we know rating r_{ui} all of a user's other ratings are independent
- Parameters to estimate
  - Prior class probabilities: $P(r_i = v)$
  - Likelihood: $P(r_j = w | r_i = v)$

Naive Bayes Classifier

Train classifier with all users who have rated item i

- Use counts to estimate prior and likelihood

$$P(r_i = v) = \frac{\sum_{u=1}^{U} \mathbf{1}(r_{ui} = v)}{\sum_{w=1}^{V} \sum_{i=1}^{U} \mathbf{1}(r_{ui} = w)}$$

$$P(r_j = w | r_i = v) = \frac{\sum_{u=1}^{U} \mathbf{1}\left(r_{ui} = v, r_{uj} = w\right)}{\sum_{z=1}^{V} \sum_{u=1}^{U} \mathbf{1}\left(r_{ui} = v, r_{uj} = z\right)}$$

- Complexity
  - $O(\sum_{u=1}^{U} |R(u)|^2)$ time and $O(M^2 V^2)$ space for all items

Predict rating for (u, i) using posterior

$$P(r_{ui} = v | r_{u1}, \ldots, r_{uM}) = \frac{P(r_{ui} = v) \prod_{j \neq i} P(r_{uj} | r_{ui} = v)}{\sum_{w=1}^{V} P(r_{ui} = w) \prod_{j \neq i} P(r_{uj} | r_{ui} = w)}$$

Naive Bayes Summary

Pros:

- Easy to implement
- Off-the-shelf implementations readily available

Cons:

- Large space requirements when storing parameters for all M predictors
- Makes strong independence assumptions
- Parameter estimates will be noisy for items with few ratings
  - E.g. $P(r_j = w | r_i = v) = 0$ if no user rated both i and j

Addressing cons:

- Tie together parameter learning in each item's predictor
- Shrinkage/smoothing is an example of this

K Nearest Neighbor Methods

Most widely used class of CF methods

- Flavors: Item-based and User-based
- Represent each item as incomplete vector of user ratings: $\vec{r}_{\cdot i} = (3, ?, ?, 4, ?, 5, ?, 1, 3)$
- To predict new rating r_{ui} for query user u and item i:
  1. Compute similarity between i and every other item
  2. Find K items rated by u most similar to i
  3. Predict weighted average of similar items' ratings
- Intuition: Users rate similar items similarly.

KNN: Computing Similarities

How to measure similarity between items?

- Cosine similarity

$$S(\vec{r}_{.i}, \vec{r}_{.j}) = \frac{\langle \vec{r}_{.i}, \vec{r}_{.j} \rangle}{\|\vec{r}_{.i}\| \, \|\vec{r}_{.j}\|}$$

- Pearson correlation coefficient

$$S(\vec{r}_{.i}, \vec{r}_{.j}) = \frac{\langle \vec{r}_{.i} - \mathrm{mean}(\vec{r}_{.i}), \vec{r}_{.j} - \mathrm{mean}(\vec{r}_{.j}) \rangle}{\|\vec{r}_{.i} - \mathrm{mean}(\vec{r}_{.i})\| \, \|\vec{r}_{.j} - \mathrm{mean}(\vec{r}_{.j})\|}$$

- Inverse Euclidean distance

$$S(\vec{r}_{.i}, \vec{r}_{.j}) = \frac{1}{\|\vec{r}_{.i} - \vec{r}_{.j}\|}$$

Problem: These measures assume complete vectors

Solution: Compute over subset of users rated by both items

Complexity: $O(\sum_{u=1}^{U} |R(u)|^2)$ time

KNN: Choosing K neighbors

How to choose K nearest neighbors?

- Select K items with largest similarity score to query item i

Problem: Not all items were rated by query user u

Solution: Choose K most similar items rated by u

Complexity: $O(min(KM, M \log M))$

Herlocker et al., 1999

KNN: Forming Weighted Predictions

Predicted rating for query user u and item i

- $N(i;u)$ is the *neighborhood* of item i for user u
  - i.e. the K most similar items rated by u
- $\hat{r}_{ui} = b_{ui} + \sum_{N(i;u)} w_{ij}(r_{uj} - b_{uj})$

How to choose weights for each neighbor?

- Equal weights: $w_{ij} = \frac{1}{|N(i;u)|}$
- Similarity weights: $w_{ij} = \frac{S(i,j)}{\sum_{j \in N(i;u)} S(i,j)}$ (Herlocker et al., 1999)
- Learn optimal weights for each user (Bell and Koren, 2007)
- Learn optimal global weights (Koren, 2008)

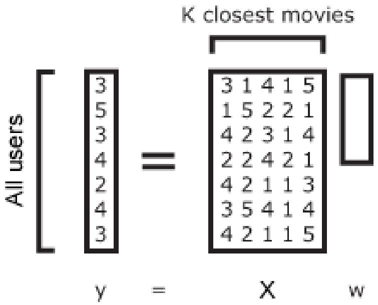
Complexity: $O(K)$

KNN: User Optimized Weights

Intuition: For a given query user u and item i, choose weights that best predict other known ratings of item i using only $N(i;u)$:

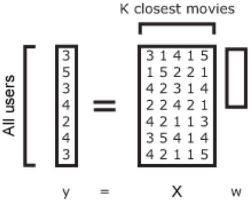
$$\min_{\mathbf{w}_{i\cdot}} \sum_{s \in R(i), s \neq u} \left(r_{si} - \sum_{j \in N(i;u)} w_{ij} r_{sj} \right)^2$$

With no missing ratings, this is a linear regression problem:



Bell and Koren, 2007

KNN: User Optimized Weights



Bell and Koren, 2007

- Optimal solution: $w = A^{-1} b$ for $A = X^T X, b = X^T y$
- Problem: X contains missing entries
  - Not all items in $N(i;u)$ were rated by all users
- Solution: Approximate A and b

$$
\begin{aligned}
\hat{A}_{jk} &= \frac{\sum_{s \in R(j) \cap R(k)} r_{sj} r_{sk}}{|R(j) \cap R(k)|} \\
\hat{b}_{k} &= \frac{\sum_{s \in R(i) \cap R(k)} r_{si} r_{sk}}{|R(i) \cap R(k)|} \\
\hat{w} &= \hat{A}^{-1} \hat{b}
\end{aligned}
$$

- Estimates based on users who rated each pair of items

KNN: User Optimized Weights

Benefits

- Weights optimized for the task of rating prediction
    - Not just borrowed from the neighborhood selection phase
- Weights not constrained to sum to 1
    - Important if all nearest neighbors are dissimilar
- Weights derived simultaneously
    - Accounts for correlations among neighbors
- Outperforms KNN with similarity or equal weights
- Can compute entries of $\hat{A}$ and $\hat{b}$ offline in parallel

Drawbacks

- Must solve additional *KxK* system of linear equations per query

Bell and Koren, 2007

KNN: Globally Optimized Weights

Consider the following KNN prediction rule for query (u, i):

$$\hat{r}_{ui} = b_{ui} + |N(i;u)|^{-\frac{1}{2}} \sum_{j \in N(i;u)} w_{ij}(r_{uj} - b_{uj})$$

Could learn a single set of KNN weights w_{ij}, shared by all users, that minimize regularized MSE:

$$E = \frac{1}{|\mathcal{T}|} \sum_{(u,i) \in \mathcal{T}} \frac{1}{2}(\hat{r}_{ui} - r_{ui})^2 + \lambda \sum_{i=1}^{M} \sum_{j=1}^{M} \frac{1}{2} w_{ij}^2 = \frac{1}{|\mathcal{T}|} \sum_{(u,i) \in \mathcal{T}} E_{ui}$$

Optimize objective using **stochastic gradient descent**:

- For each example $(u, i) \in \mathcal{T}$, update w_{ij} $\forall j \in N(i; u)$

$$\begin{aligned} w_{ij}^{t+1} &= w_{ij}^t - \gamma \frac{\partial}{\partial w_{ij}} E_{ui} \\ &= w_{ij}^t - \gamma(|N(i;u)|^{-\frac{1}{2}}(\hat{r}_{ui} - r_{ui})(r_{uj} - b_{uj}) + \lambda w_{ij}^t) \end{aligned}$$

KNN: Globally Optimized Weights

Benefits

- Weights optimized for the task of rating prediction
  - Not just borrowed from the neighborhood selection phase
- Weights not constrained to sum to 1
  - Important if all nearest neighbors are dissimilar
- Weights derived simultaneously
  - Accounts for correlations among neighbors
- Outperforms KNN with similarity or equal weights

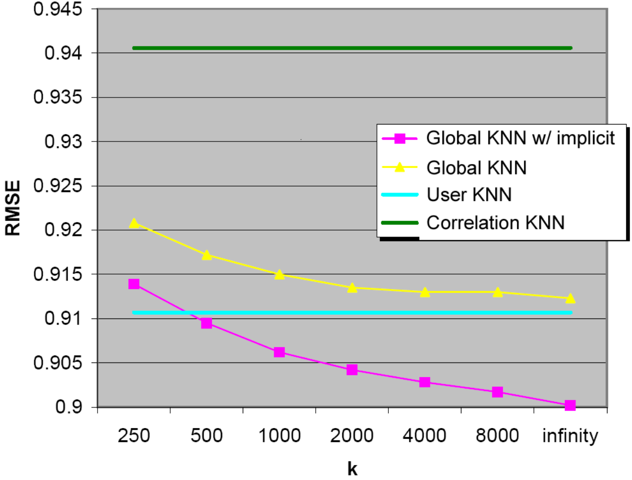
Drawbacks

- Must solve global optimization problem at training time
- Must store $O(M^2)$ weights in memory

Koren, 2008

KNN: Summary

Comparison of KNN weighting schemes on Netflix quiz data



Koren, 2008

KNN: Summary

Pros

- Intuitive interpretation
- When weights not learned…
  - Easy to implement
  - Zero training time
- Learning prediction weights can greatly improve accuracy for little overhead in space and time

Cons

- When weights not learned…
  - Need to store all item (or user) vectors in memory
  - May redundantly recompute similarity scores at test time
  - Similarity/equal weights not always suitable for prediction
- When weights learned…
  - Need to store $O(M^2)$ or $O(U^2)$ parameters
  - Must update stored parameters when new ratings occur

Low Dimensional Matrix Factorization

Matrix Completion

- Filling in the unknown ratings in a sparse $U \times M$ matrix R

$$\mathbf{R} = \begin{bmatrix} ? & ? & 1 & \dots & 4 \\ 3 & ? & ? & \dots & ? \\ ? & 5 & ? & \dots & 5 \end{bmatrix}$$

Low dimensional matrix factorization

- Model R as a product of two lower dimensional matrices

- A is $U \times K$ "user factor" matrix, $K \ll U, M$
- B is $M \times K$, "item factor" matrix
- Learning A and B allows us to reconstruct all of R

Low Dimensional Matrix Factorization

Interpretation: Rows of A and B are low dimensional feature vectors a_u and b_i for each user u and item i

Motivation: Dimensionality reduction

- Compact representation: only need to learn and store $UK + MK$ parameters
- Matrices can often be adequately represented by low rank factorizations

Low Dimensional Matrix Factorization

Very general framework that encapsulates many ML methods

- Singular value decomposition
- Clustering
  - A can represent cluster centers
  - B probabilities of belonging to each cluster
- Factor Analysis/Probabilistic PCA

Singular Value Decomposition

Squared error objective for MF

$$\operatorname*{argmin}_{A,B} \|R - AB^T\|_2^2 = \operatorname*{argmin}_{A,B} \sum_{u=1}^{U} \sum_{i=1}^{M} (r_{ui} - \langle a_u, b_i \rangle)^2$$

- Reasonable objective since RMSE is our error metric

When all of R is observed, this problem is solved by singular value decomposition (SVD)

- **SVD:** $R = H\Sigma V^T$
  - H is $U \times U$ with $H^T H = I_{U\times U}$
  - V is $M \times M$ with $V^T V = I_{M\times M}$
  - Σ is $U \times M$ and diagonal
- **Solution:** Take first K pairs of singular vectors
  - Let $A = H_{U\times K}\Sigma_{K\times K}$ and $B = V_{M\times K}$

SVD with Missing Values

Weighted SE objective

$$\operatorname*{argmin}_{A,B} \sum_{u=1}^{U} \sum_{i=1}^{M} W_{ui}(r_{ui} - \langle a_u, b_i \rangle)^2$$

Binary weights

- $W_{ui} = 1$ if r_{ui} observed, $W_{ui} = 0$ otherwise
- Only penalize errors on known ratings

How to optimize?

- Straightforward singular value decomposition no longer applies
- Local minima exist $\Rightarrow$ algorithm initialization is important

SVD with Missing Values

Insight: Chicken and egg problem

- If we knew the missing values in R, could apply SVD
- If we could apply SVD, we could find the missing values in R
- Idea: Fill in unknown entries with best guess; apply SVD; repeat

Expectation-Maximization (EM) algorithm

- Alternate until convergence:
  1. E step: $X = W * R + (1 - W) * \hat{R}$
     (* represents entrywise product)
  2. M step: $[H, \Sigma, V] = SVD(X)$, $\hat{R} = H_{U\times K}\Sigma_{K\times K}V^T_{M\times K}$

Complexity: $O(UM)$ space and $O(UMK)$ time per EM iteration

- What if UM or UMK is very large?
  - $UM = 8.5$ billion for Netflix Prize dataset
- Complete ratings matrix may not even fit into memory!

Srebro and Jaakkola, 2003

SVD with Missing Values

Regularized weighted SE objective

$$\operatorname*{argmin}_{A,B} \sum_{u=1}^{U} \sum_{i=1}^{M} W_{ui}(r_{ui} - \langle a_u, b_i \rangle)^2 + \lambda(\sum_{u=1}^{U} ||a_u||^2 + \sum_{i=1}^{M} ||b_i||^2)$$

Equivalent form

$$\operatorname*{argmin}_{A,B} \sum_{(u,i)\in\mathcal{T}} (r_{ui} - \langle a_u, b_i \rangle)^2 + \lambda(\sum_{u=1}^{U} ||a_u||^2 + \sum_{i=1}^{M} ||b_i||^2)$$

Motivation

- Counters *overfitting* by implicitly restricting optimization space
  - Shrinks entries of A and B toward 0
- Can improve *generalization error*, performance on unseen test data

SVD with Missing Values

Insight: If we knew B, could solve for each row of A via ridge regression and vice-versa

- Alternate between optimizing A and optimizing B with the other matrix held fixed

Alternating least squares (ALS) algorithm

- Alternate until convergence:
  1. For each user u, update
     $a_u \leftarrow (\sum_{i \in R(u)} b_i b_i^T + \lambda I)^{-1} \sum_{i \in R(u)} r_{ui} b_i$
  2. For each item i, update
     $b_i \leftarrow (\sum_{u \in R(i)} a_u a_u^T + \lambda I)^{-1} \sum_{u \in R(i)} r_{ui} a_u$

Complexity: $O(UK + MK)$ space, $O(UK^3 + MK^3)$ time per iteration

- Note: updates for vectors a_u can all be performed in parallel (same for b_i)
- No need to store completed ratings matrix

Zhou et al., 2008

SVD with Missing Values

Insight: Use standard gradient descent

- $\nabla_{a_u} E = \lambda a_u + \sum_{i \in R(u)} b_i(\langle a_u, b_i \rangle - r_{ui})$
- $\nabla_{b_i} E = \lambda b_i + \sum_{u \in R(i)} a_u(\langle a_u, b_i \rangle - r_{ui})$

Gradient descent algorithm

- Repeat until convergence:
  1. For each user u, update
     $a_u \leftarrow a_u - \gamma(\lambda a_u + \sum_{i \in R(u)} b_i(\langle a_u, b_i \rangle - r_{ui}))$
  2. For each item i, update
     $b_i \leftarrow b_i - \gamma(\lambda b_i + \sum_{u \in R(i)} a_u(\langle a_u, b_i \rangle - r_{ui}))$
- Can update all a_u in parallel (same for b_i)

Complexity: $O(UK + MK)$ space, $O(NK)$ time per iteration

- No need to store completed ratings matrix
- No K^3 overhead from solving linear regressions

SVD with Missing Values

Insight: Update parameter after each observed rating

- $\nabla_{a_u} E_{ui} = \lambda a_u + b_i(\langle a_u, b_i \rangle - r_{ui})$
- $\nabla_{b_i} E_{ui} = \lambda b_i + a_u(\langle a_u, b_i \rangle - r_{ui})$

Stochastic gradient descent algorithm

- Repeat until convergence:
  1. For each $(u, i) \in \mathcal{T}$
     1. Calculate error: $e_{ui} \leftarrow (\langle a_u, b_i \rangle - r_{ui})$
     2. Update $a_u \leftarrow a_u - \gamma(\lambda a_u + b_i e_{ui})$
     3. Update $b_i \leftarrow b_i - \gamma(\lambda b_i + a_u e_{ui})$

Complexity: $O(UK + MK)$ space, $O(NK)$ time per pass through training set

- No need to store completed ratings matrix
- No K^3 overhead from solving linear regressions

Takacs et al., 2008, Funk, 2006

Constrained MF as Clustering

Insight: Soft clustering of items is MF

- Row b_i represents item i's fractional belonging to each cluster
- Columns of A are cluster centers
- Yields greater interpretability

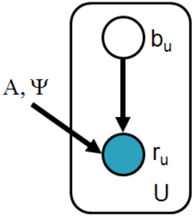
Constrained weighted SE objective

$$\operatorname*{argmin}_{A,B} \sum_{u=1}^{U} \sum_{i=1}^{M} W_{ui}(r_{ui} - \langle a_u, b_i \rangle)^2 \text{ s.t. } \forall i \;\; b_i \geq 0, \sum_{k=1}^{K} b_{ik} = 1$$

- Wu and Li (2008) penalize constraints in the objective and optimize via stochastic gradient descent

Takeaway: Can add your favorite constraints and optimize with standard techniques

Factor Analysis



Motivation

- Explain data variability in terms of latent *factors*
- Provide model for how data is generated

The Model

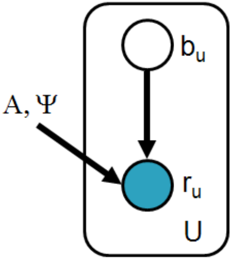
- For each user, r_u = partially observed ratings vector in $\mathbb{R}^M$
- For each user, b_u = latent factor vector in $\mathbb{R}^K$
- A is an $M \times K$ matrix of parameters (*factor loading matrix*)
- Ψ is an $M \times M$ covariance matrix
  - Probabilistic PCA: Special case when $\Psi = \sigma^2 I$
- To generate ratings for user u:
  1. Draw $b_u \sim \mathcal{N}(0, I_K)$
  2. Draw $r_u \sim \mathcal{N}(Ab_u, \Psi)$

Canny, 2002

Factor Analysis

Parameter Learning

- Only need to learn A and Ψ
- b_u are variables to be integrated out
- Typically use EM algorithm (Canny, 2002)
  - Can be very slow for large datasets
- Alternative: Stochastic gradient descent on negative log likelihood (Lawrence and Urtasun, 2009)



Low Dimensional MF: Summary

Pros

- Data reduction: only need to store $UK + MK$ parameters at test time
  - $MK + M^2$ needed for Factor Analysis
- Gradient descent and ALS procedures are easy to implement and scale well to large datasets
- Empirically yields high accuracy in CF tasks
- Matrix factors could be used as inputs into other learning algorithms (e.g. classifiers)

Cons

- Missing data MF objectives plagued by many local minima
- Initialization is important
- EM approaches tend to be slow for large datasets

Incorporating Implicit Feedback

Implicit feedback

- In addition to explicitly observed ratings, may have access to binary information reflecting implicit user preferences
  - Is a movie in a user's queue at Netflix?
  - Was this item purchased (but never rated)?
- Test set can be a source of implicit feedback
  - For each (u, i) in the test set, we know u rated i; we just don't know the rating.
  - Data is not "missing at random"
  - The fact that a user rated an item provides information about the rating.
    - E.g. People who rated Lord of The Rings I and II tend to rate LOTR III more highly.
- Can extend several of our algorithms to incorporate implicit feedback as additional binary preferences

Incorporating Implicit Feedback

KNN: Globally Optimized Weights

- Let $T(i;u)$ be the set of K items most similar to i for which u has positive implicit feedback
  - E.g. Positive implicit feedback: Every item purchased by u or every movie in the queue of u
- Augment the KNN prediction rule with implicit feedback weights c_{ij}:

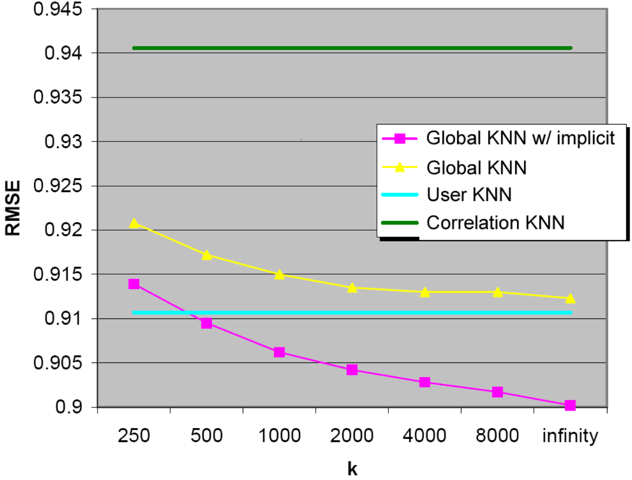
$$\hat{r}_{ui} = b_{ui} + |N(i;u)|^{-\frac{1}{2}} \sum_{j \in N(i;u)} w_{ij}(r_{uj} - b_{uj}) + |T(i;u)|^{-\frac{1}{2}} \sum_{j \in T(i;u)} c_{ij}$$

- Each c_{ij} is an offset of the baseline KNN prediction
- c_{ij} is large when implicit feedback about j is informative about i
- Optimize w_{ij} and c_{ij} jointly using stochastic gradient descent

Koren, 2008

Incorporating Implicit Feedback

Comparison of KNN weighting schemes on Netflix test data



Koren, 2008

Incorporating Implicit Feedback

NSVD

- Represent each user as a "bag of movies"
- Instead of learning a_u for each user explicitly, learn second set of item vectors, $\tilde{b}_i$
  - Let $a_u = |T(u)|^{-\frac{1}{2}} \sum_{i \in T(u)} \tilde{b}_i$ where $T(u)$ is the set of all items for which u has positive implicit feedback
- New MF objective:

$$\operatorname*{argmin}_{\tilde{B},B} \sum_{(u,i) \in \mathcal{T}} (r_{ui} - \langle |T(u)|^{-\frac{1}{2}} \sum_{j \in T(u)} \tilde{b}_j, b_i \rangle)^2$$

- Train via stochastic gradient descent with regularization
- Additional properties
  - $2MK$ parameters instead of $MK + UK$, useful when $M < U$
  - Handles new users without retraining
  - Empirically underperforms SVD techniques but captures different patterns in the data

Paterek, 2007

Incorporating Implicit Feedback

SVD++

- Integrate the missing-data SVD and NSVD objectives

$$\underset{A,\tilde{B},B}{\operatorname{argmin}} \sum_{(u,i)\in\mathcal{T}} \left(r_{ui} - \langle a_u + |T(u)|^{-\frac{1}{2}} \sum_{j\in T(u)} \tilde{b}_j, b_i\rangle\right)^2$$

- Learning both explicit user vectors, a_u, and implicit vectors, $|\mathrm{T(u)}|^{-\frac{1}{2}} \sum_{j\in T(u)} \tilde{b}_j$
- Train via stochastic gradient descent with regularization

Performance on Netflix Prize quiz set

Model	50 factors	100 factors	200 factors
SVD	0.9046	0.9025	0.9009
SVD++	0.8952	0.8924	0.8911

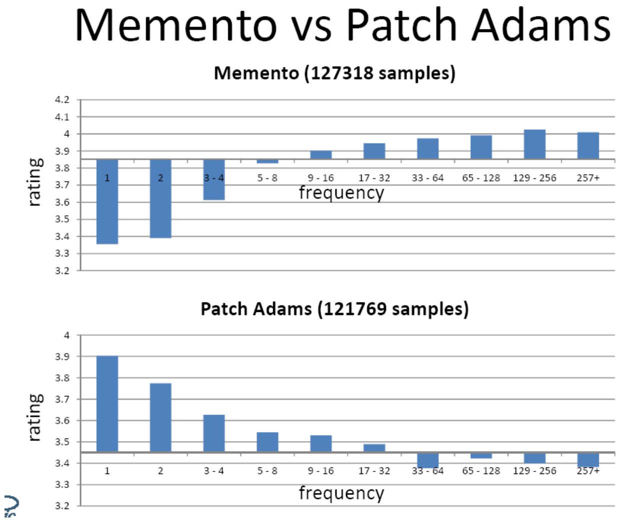
Koren, 2008

Adding Time Dependence

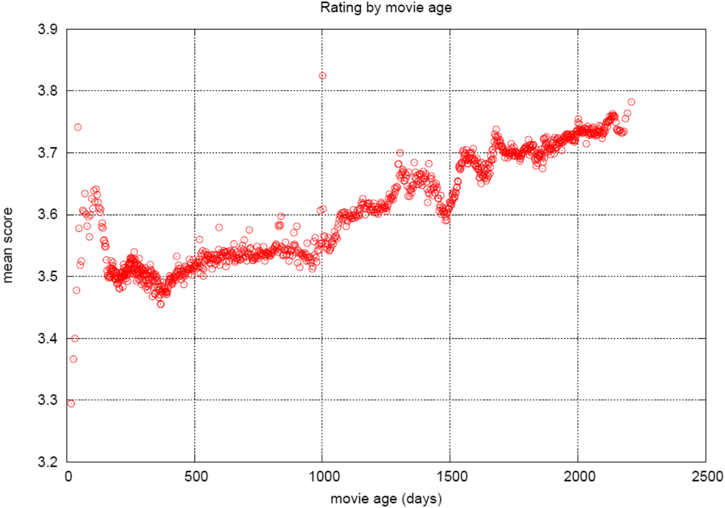
Claim: Preferences are time-dependent

- Items grow and fade in popularity
- User tastes evolve over time
- Decade, season, and day of the week all influence expressed preferences
- Even number of items rated in a day can be predictive of ratings (Pragmatic Theory Netflix Grand Prize Talk 2009)

Average movie rating versus number of movies rated that day in Netflix dataset (Piotte and Chabbert 2009)



Average movie rating versus number of days since first rating in Netflix dataset



Koren, 2009

Adding Time Dependence

Claim: Preferences are time-dependent

Claim: Rating timestamps routinely collected by companies

- Dates provided for each rating in Netflix Prize dataset

$\Rightarrow$ Valuable to introduce time dependence into CF algorithms

Adding Time Dependence

TimeSVD++

- Parameterize explicit user factor vectors by time

$$a_u(t) = a_u + \alpha_u \mathrm{dev}(t) + \aleph_{ut}$$

- a_u is a static baseline vector
- $\alpha_u \mathrm{dev}(t)$ is a static vector multiplied by the deviation from the user's average rating time
  - Captures linear changes in time
- $\aleph_{ut}$ is a vector learned for a specific point in time

Koren, 2009

Adding Time Dependence

TimeSVD++

- New objective

$$\underset{A(t),\tilde{B},B}{\text{argmin}} \sum_{(u,i)\in\mathcal{T}} (r_{ui} - \langle a_u(t) + |T(u)|^{-\frac{1}{2}} \sum_{j\in T(u)} \tilde{b}_j, b_i\rangle)^2$$

- Optimize via regularized stochastic gradient descent

Results on Netflix Quiz Set

Model	f=10	f=20	f=50	f=100	f=200
SVD	.9140	.9074	.9046	.9025	.9009
SVD++	.9131	.9032	.8952	.8924	.8911
timeSVD++	.8971	.8891	.8824	.8805	.8799

- f in this chart above is K in our model
- Note: $f = 200$ requires fitting billions of parameters with only 100 million ratings!

Koren, 2009

Adding Time Dependence

KNN: Globally optimized time-decaying weights

- New prediction rule

$$\hat{r}_{ui} = b_{ui} + |N(i;u)|^{-\frac{1}{2}} \sum_{(j,t)\in N(i;u)} e^{-\beta_u |t-t_j|} w_{ij}(r_{uj} - b_{uj})$$
$$+ |T(i;u)|^{-\frac{1}{2}} \sum_{(j,t)\in T(i;u)} e^{-\beta_u |t-t_j|} c_{ij}$$

- **Intuition:** Allow the strength of item relationships to decay with time elapsed between ratings
- Optimize regularized weighted SE objective via stochastic gradient descent
- Netflix test set RMSE drops from .9002 (without time) to .8885

Koren, 2009

Combining Methods

Why combine?

- Diminishing returns from optimizing a single algorithm
- Different models capture different aspects of the data
- Statistical motivation
  - If X_1, X_2 uncorrelated with equal mean, $Var(\frac{X_1}{2} + \frac{X_2}{2}) = \frac{1}{4}(Var(X_1) + Var(X_2))$
  - Moral: Errors of different algorithms can cancel out

Combining Methods

Training on Errors

- Many CF algorithms handle arbitrarily real-valued preferences
- Treat the prediction errors of one algorithm as input "preferences" of second algorithm
- Second algorithm can learn to predict and hence offset the errors of the first
- Often yields improved accuracy

Data normalization	No interpolation ($k = 0$)	Correlation-based interpolation $k = 20$	$k = 35$	$k = 50$	Jointly derived interpolation $k = 20$	$k = 35$	$k = 50$
none (raw scores)	NA	0.9947	1.002	1.0085	0.9536	0.9596	0.9644
double centering	0.9841	0.9431	0.9470	0.9502	0.9216	0.9198	0.9197
global effects	0.9657	0.9364	0.9390	0.9413	0.9194	0.9179	0.9174
factorization	0.9167	0.9156	0.9142	0.9142	0.9071	0.9071	0.9071

Bell and Koren, 2007

Combining Methods

Stacked Ridge Regression

- Linearly combine algorithm predictions to best predict unseen ratings
- Withhold a subset of your training set ratings from algorithms during training
- Let columns of $\mathbf{P}$ = predictions of each algorithm on hold-out set
- Let $\mathbf{y}$ = true hold-out set ratings
- Solve for optimal regularized blending coefficients, β
  $\min_\beta \left\|\mathbf{y} - \mathbf{P}\beta\right\|^2 + \lambda \left\|\beta\right\|^2$
- Solution: $\beta = (\mathbf{P}^\top\mathbf{P} + \lambda\mathbf{I})^{-1}\mathbf{P}^\top\mathbf{y}$
- Blended predictions often more accurate than any single predictor on true test set

Breiman, 1996

Combining Methods

Integrating Models

- Largest boosts in accuracy come from integrating disparate approaches into a single unified model
- Integrated KNN-SVD++ predictor

$$\hat{r}_{ui} = \langle a_u + |T(u)|^{-\frac{1}{2}} \sum_{j \in T(u)} \tilde{b}_j, b_i \rangle + |T(i;u)|^{-\frac{1}{2}} \sum_{j \in T(i;u)} c_{ij}$$
$$+ b_{ui} + |N(i;u)|^{-\frac{1}{2}} \sum_{j \in N(i;u)} w_{ij}(r_{uj} - b_{uj})$$

- Optimize regularized weighted SE objective via stochastic gradient descent
- Results on Netflix Quiz Set

	50 factors	100 factors	200 factors
RMSE	0.8877	0.8870	0.8868
time/iteration	17min	20min	25min

Koren, 2008

Challenges for CF

Relevant objectives

- How will output of CF algorithms will be used in a real system?
- Predicting actual rating may be useless!
- May care more about ranking of items

Missing at random assumption

- Many CF methods incorrectly assume that the items rated are chosen randomly, independently of preferences
- How can our models capture information in choices of ratings?
  - Marlin et al, 2007, Salakhutdinov and Mnih, 2007

Challenges for CF

Preference versus intention

- Distinguish what people like from what people are interested in seeing/purchasing
- Worthless to recommend an item a user already has/was going to buy anyway

Scaling to truly large datasets

- Latest algorithms scale to 100 million rating Netflix dataset. Can they scale to 10 billion ratings? Millions of users and items?
- Simple and parallelizable algorithms are preferred

Challenges for CF

Multiple individuals using the same account

- Benefit in modeling their individual preferences?

Handling users and items with few ratings

- Use user and item meta-data: Content-based filtering
  - User demographics, movie genre, etc.
- Kernel methods seem promising
  - Basilico and Hofmann, 2004, Yu et al., 2009
- Subject of Netflix Prize 2
  http://www.netflixprize.com/community/viewtopic.php?id=1520
  - Answer is worth $500,000

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