Collapse and Revival in the Jaynes-Cummings-Paul Model

Departmental Honors Defense in Physics

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The Jaynes-Cummings-Paul Model

- First described in 1963 (Jaynes & Cummings).
  - Independently described in 1963 by Harry Paul.
- Second major paper in 1965 (Cummings).
- Experimentally confirmed in the 1980s.
- One method for bringing purely quantum effects into optics.

The Jaynes-Cummings-Paul Model

- **Defining characteristics of the model.**
  - Atom in a lossless cavity.
  - Single-mode electric field.
  - Two accessible atomic levels.
  - Atom oscillates between energy levels.

The Jaynes-Cummings-Paul Model

- **Interesting properties of the model.**
    - Non-zero transition probability in the absence of electric field.
    - Periodic collapse and revival of atomic oscillations.

Outline of the Project

- Construct the Jaynes-Cummings Hamiltonian.
    - Quantize the electric field.
    - Write down the atomic energy levels.
    - Work out the interaction term.
- Apply the Hamiltonian to a pair of demonstrations.
    - Definite photon states.
    - Coherent field states.

The Jaynes-Cummings Hamiltonian

$$\hat{H}_{\mathrm{JC}} = \hbar\omega\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\hbar\omega_0\hat{\sigma}_z + \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^{\dagger}\right)$$

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- Put forward by Jaynes and Cummings in a pair of papers.
- A series of approximations allow for the simple form of the Hamiltonian.
- 3 components:
    - energy of the field.
    - energy of the atomic transitions.
    - energy from interaction of the field with the atom.

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The Free Field Hamiltonian

$$\hat{H}_{\text{field}} = \hbar\omega\hat{a}^{\dagger}\hat{a}$$

Free Field Hamiltonian

- One-dimensional cavity, boundary at $z = 0$ and $z = L$.

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2} = 0$$

- Separate variables and solve.

$$\begin{aligned} E_x(z,t) &= Z(z) \cdot q(t) \\ &= E_0\, q(t) \sin(kz) \end{aligned}$$

Free Field Hamiltonian

- We can obtain the magnetic field from Ampere's Law.

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

$$B_y = -\frac{1}{c^2}\int \frac{\partial E_x}{\partial t}\,\mathrm{d}z$$

$$= \frac{\mu_0 \varepsilon_0}{k}\,E_0\,\dot{q}(t)\cos(kz)$$

Free Field Hamiltonian

- Introduce creation and annihilation operators.

$$\hat{a} = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} + i\hat{p})$$
$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar\omega}}(\omega\hat{q} - i\hat{p})$$

- Classical functions go to quantum operators.

$$q(t) \rightarrow \hat{q}$$
$$\dot{q}(t) \rightarrow \hat{p}$$

Free Field Hamiltonian

- We now have a quantum expression for the fields.

$$\hat{E}_x(z,t) = E_0 \left[\hat{a}(t) + \hat{a}^\dagger(t)\right] \sin(kz)$$
$$\hat{B}_y(z,t) = B_0 \left[\hat{a}(t) - \hat{a}^\dagger(t)\right] \cos(kz)$$

- The Hamiltonian of the field may then be calculated.

$$\hat{H} = \frac{1}{2}\int_{\text{cavity}} \left[\varepsilon_0 \hat{E}_x^2 + \frac{1}{\mu_0}\hat{B}_y^2\right] \mathrm{d}V$$
$$= \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$
$$\approx \hbar\omega\hat{a}^\dagger\hat{a}$$

The Atomic Hamiltonian

$$\hat{H}_{\text{atom}} = \frac{1}{2}\hbar\omega_0\hat{\sigma}_z$$

The Atomic Hamiltonian

- Limited to two accessible states implies a 2D basis.

$$|+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |-\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Hamiltonian is a sum over accessible energies.

$$\hat{H}_{\text{atom}} = E_+ |+\rangle\langle+| + E_- |-\rangle\langle-|$$

$$= \begin{pmatrix} E_+ & 0 \\ 0 & E_- \end{pmatrix}$$

The Atomic Hamiltonian

- Simplify the Hamiltonian.

$$\hat{H}_{\text{atom}} = \frac{1}{2}\begin{pmatrix} E_+ + E_- & 0 \\ 0 & E_+ + E_- \end{pmatrix} + \frac{1}{2}\begin{pmatrix} E_+ - E_- & 0 \\ 0 & E_- - E_+ \end{pmatrix}$$

$$\hat{H}_{\text{atom}} = \frac{1}{2}\left(E_+ + E_-\right)\hat{\mathbf{I}} + \frac{1}{2}\Delta E \hat{\sigma}_z$$

- The Hamiltonian is then expressed in terms of a Pauli matrix.

$$\Delta E = E_+ - E_- \equiv \hbar\omega_0$$

$$\hat{H}_{\text{atom}} \approx \frac{1}{2}\hbar\omega_0\hat{\sigma}_z$$

The Pauli Spin Matrices

- Pauli spin matrices.

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- Raising & lowering operators.

$$\hat{\sigma}_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{\sigma}_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

The Interaction Hamiltonian

$$\hat{H}_{\text{int}} = \hbar\lambda \left(\hat{\sigma}_+ \hat{a} + \hat{\sigma}_- \hat{a}^\dagger\right)$$

The Interaction Hamiltonian

- Begin with minimal coupling.

$$H = \frac{1}{2m}\left[\mathbf{p} - q\mathbf{A}(\mathbf{r},t)\right]^2 + q\Phi(\mathbf{r},t)$$

- Apply coupling to each particle (no scalar potential).

$$\begin{aligned} H_p &= \tfrac{1}{2m_p}\left[\mathbf{p}_p^2 - e\mathbf{p}_p \cdot \mathbf{A}(\mathbf{r}_p,t) - e\mathbf{A}(\mathbf{r}_p,t)\cdot\mathbf{p}_p + e^2\mathbf{A}^2(\mathbf{r}_p,t)\right] \\ H_e &= \tfrac{1}{2m_e}\left[\mathbf{p}_e^2 + e\mathbf{p}_e \cdot \mathbf{A}(\mathbf{r}_e,t) + e\mathbf{A}(\mathbf{r}_e,t)\cdot\mathbf{p}_e + e^2\mathbf{A}^2(\mathbf{r}_e,t)\right] \end{aligned}$$

$$H_{\text{int}} = H_p + H_e - \frac{e^2}{4\pi\varepsilon_0}\frac{\mathbf{r}_e - \mathbf{r}_p}{|\,\mathbf{r}_e - \mathbf{r}_p\,|^3}$$

The Interaction Hamiltonian

- Introduce center of mass coordinates.

$$\mathbf{R} = \frac{m_p \mathbf{r}_p + m_e \mathbf{r}_e}{M}, \quad \mathbf{r} = \mathbf{r}_e - \mathbf{r}_p$$

$$\mathbf{r}_p = \mathbf{R} - \frac{\mu}{m_p}\mathbf{r}, \quad \mathbf{r}_e = \mathbf{R} + \frac{\mu}{m_e}\mathbf{r}$$

- Also, center of mass momenta.

$$\mathbf{P} = \mathbf{p}_p + \mathbf{p}_e \qquad \mathbf{p} = \frac{\mu}{m_e}\mathbf{p}_e - \frac{\mu}{m_p}\mathbf{p}_p$$

The Dipole Approximation

$$\mathbf{r}_p = \mathbf{R} - \frac{\mu}{m_p}\mathbf{r}, \quad \mathbf{r}_e = \mathbf{R} + \frac{\mu}{m_e}\mathbf{r}$$

- We can now make the dipole approximation to simplify the Hamiltonian.

$$\mathbf{A}(\mathbf{r}_p, t) \sim \mathbf{A}(\mathbf{R} + \delta\mathbf{r}, t) \sim \mathbf{A}(\mathbf{R}, t)$$

- Then write out the full interaction Hamiltonian.

$$H_{\text{int}} = \left(\frac{\mathbf{P}^2}{2M} - \frac{e^2}{4\pi\varepsilon_0} \frac{\mathbf{r}}{|\,\mathbf{r}\,|^3} \right) + \frac{1}{2\mu} \left[\mathbf{p} + e\mathbf{A}(\mathbf{R}, t)\right]^2$$

The Dipole Approximation

- A second formulation takes the form of a dipole in a field.

$$H_{\text{int}} = \left(\frac{\mathbf{P}^2}{2M} - \frac{e^2}{4\pi\varepsilon_0} \frac{\mathbf{r}}{|\,\mathbf{r}\,|^3} \right) + \frac{\mathbf{p}^2}{2\mu} - \mathbf{d} \cdot \mathbf{E}(\mathbf{R}, t)$$

- We can compare the two Hamiltonians through Lagrangians.

$$H^{(0)} = \frac{1}{2\mu} \left[\mathbf{p} + e\mathbf{A}(\mathbf{R}, t)\right]^2$$

$$\mathcal{L}^{(0)} = \dot{\mathbf{r}} \cdot \mathbf{p} - H^{(0)}$$

$$\dot{\mathbf{r}} = \frac{\partial H^{(0)}}{\partial p} = \frac{1}{\mu} \left[\mathbf{p} + e\mathbf{A}(\mathbf{R}, t)\right]$$

The Dipole Approximation

$$\dot{\mathbf{r}} = \frac{1}{\mu}\left[\mathbf{p} + e\mathbf{A}(\mathbf{R}, t)\right] \Leftrightarrow \mathbf{p} = \mu\dot{\mathbf{r}} - e\mathbf{A}(\mathbf{R}, t)$$

- The Lagrangian for minimal coupling.

$$\mathcal{L}^{(0)} = \frac{\mu}{2}\dot{\mathbf{r}}^2 - e\dot{\mathbf{r}} \cdot \mathbf{A}(\mathbf{R}, t)$$

- Subtracting a complete time-derivative will not change variation, leading to the same equations of motion.

$$\mathcal{L}' = \mathcal{L}^{(0)} - \frac{d}{dt}\left(-e\mathbf{r} \cdot \mathbf{A}(\mathbf{R}, t)\right)$$

The Dipole Approximation

- The Lagrangian for minimal coupling.

$$\frac{d}{dt}\left[-e\mathbf{r}\cdot\mathbf{A}(\mathbf{R},t)\right] = -e\dot{\mathbf{r}}\cdot\mathbf{A}(\mathbf{R},t) - e\mathbf{r}\cdot\frac{d}{dt}\mathbf{A}(\mathbf{R},t)$$

$$= -e\dot{\mathbf{r}}\cdot\mathbf{A}(\mathbf{R},t) - e\mathbf{r}\cdot\frac{\partial}{\partial t}\mathbf{A}(\mathbf{R},t)$$

- We get exactly the form we were looking for - a dipole will give exactly the same dynamics.

$$\mathcal{L}' = \frac{\mu}{2}\dot{\mathbf{r}}^2 - e\mathbf{r}\cdot\mathbf{E}(\mathbf{R},t)$$

The Dipole Approximation

- Make the inverse Legendre transformation.

$$H' = \frac{\mathbf{p}^2}{2\mu} - e\mathbf{r} \cdot \mathbf{E}(\mathbf{R}, t)$$

- Now we may quantize the field & dipole.

$$\begin{aligned} \hat{H}_{\text{int}} &= -\hat{\mathbf{d}} \cdot \hat{\mathbf{E}} \\ &= -\hat{\mathbf{d}} \cdot \mathbf{E}_0 \left(\hat{a} + \hat{a}^\dagger\right) \sin(kz) \\ &= \hat{d} g \left(\hat{a} + \hat{a}^\dagger\right) \end{aligned}$$

The Dipole Operator

- Fix the dipole operator in the basis.

$$\langle + | \hat{d} | + \rangle = \langle - | \hat{d} | - \rangle = 0$$

$$\langle + | \hat{d} | - \rangle = \Big(\langle - | \hat{d} | + \rangle \Big)^* = d$$

- The dipole operator is responsible for "moving" the atom between energy levels.

The Pauli Spin Matrices

- Raising & lowering operators.

$$\hat{\sigma}_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{\sigma}_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$\hat{\sigma}_+ \, | + \rangle = 0, \quad \hat{\sigma}_- \, | - \rangle = 0$$

$$\hat{\sigma}_+ \, | - \rangle = | + \rangle \, , \quad \hat{\sigma}_- \, | + \rangle = | - \rangle$$

$$\hat{d} = d \, (\hat{\sigma}_+ + \hat{\sigma}_-)$$

The Rotating-Wave Approximation

- Multiply out the interaction Hamiltonian.

$$\hat{H}_{\text{int}} = \hbar\lambda \left(\hat{\sigma}_+ + \hat{\sigma}_-\right)\left(\hat{a} + \hat{a}^\dagger\right)$$
$$= \hbar\lambda \left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a} + \hat{\sigma}_+\hat{a}^\dagger + \hat{\sigma}_-\hat{a}^\dagger\right)$$

- The operators gain time-dependence in the interaction picture.

$$\hat{a}^\dagger(t) = \hat{a}^\dagger e^{i\omega t} \qquad \hat{a}(t) = \hat{a}e^{-i\omega t}$$
$$\hat{\sigma}_+(t) = \hat{\sigma}_+ e^{i\omega_0 t} \qquad \hat{\sigma}_-(t) = \hat{\sigma}_- e^{-i\omega_0 t}$$

The Interaction Picture

- States in the interaction picture evolve in time slightly differently that in the Schrödinger picture.

$$\frac{d}{dt}\,|\,\Psi_I(t)\,\rangle = \frac{i}{\hbar}\hat{H}_{0,S}\,|\,\Psi_I(t)\,\rangle + e^{i\hat{H}_{0,S}t/\hbar}\frac{d}{dt}\,|\,\Psi_S(t)\,\rangle$$

$$= \frac{i}{\hbar}\hat{H}_{0,S}\,|\,\Psi_I(t)\,\rangle + e^{i\hat{H}_{0,S}t/\hbar}\left(-\frac{i}{\hbar}\hat{H}_S\,|\,\Psi_S(t)\,\rangle\right)$$

$$= e^{i\hat{H}_{0,S}t/\hbar}\hat{V}_S e^{-i\hat{H}_{0,S}t/\hbar}\,|\,\Psi_I(t)\,\rangle$$

$$= \hat{V}_I(t)\,|\,\Psi_I(t)\,\rangle$$

The Interaction Picture

- The state vectors in the interaction picture evolve in time according to the interaction term only.

$$\frac{d}{dt}\,|\,\Psi_I(t)\,\rangle = \hat{V}_I(t)\,|\,\Psi_I(t)\,\rangle$$

- It can be easily shown through differentiation that operators in the interaction picture evolve in time according only to the free Hamiltonian.

$$\frac{d\hat{\mathcal{O}}_I}{dt} = \frac{i}{\hbar}[\,\hat{H}_{0,I}\,,\hat{\mathcal{O}}\,] + \left(\frac{\partial\hat{\mathcal{O}}_I}{\partial t}\right)$$

The Rotating-Wave Approximation

- The interaction Hamiltonian now carries oscillating phase terms.

$$\hat{H}_{\text{int}} = \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a} + \hat{\sigma}_+\hat{a}^\dagger + \hat{\sigma}_-\hat{a}^\dagger\right)$$

$$= \hbar\lambda\Big(\hat{\sigma}_+\hat{a}e^{i(\omega_0-\omega)t} + \hat{\sigma}_+\hat{a}^\dagger e^{i(\omega_0+\omega)t} + \hat{\sigma}_-\hat{a}e^{-i(\omega_0+\omega)t} + \hat{\sigma}_-\hat{a}^\dagger e^{-i(\omega_0-\omega)t}\Big)$$

- Setting the detuning $\Delta = \omega - \omega_0$ to 0 removes time-dependence.

$$\hat{H}_{\text{int}} = \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger\right)$$

The Jaynes-Cummings Hamiltonian

- We now have the full Jaynes-Cummings Hamiltonian.

$$\hat{H}_{\mathrm{JC}} = \hbar\omega\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^{\dagger}\right)$$

Demonstration 1: Definite Photon States

$$|\,\Psi(t)\,\rangle = C_+(t)\,|+\rangle\,|\,n\rangle + C_-(t)\,|-\rangle\,|\,n+1\,\rangle$$

The Jaynes-Cummings Hamiltonian

- We now have the full Hamiltonian.

$$\hat{H}_{\mathrm{JC}} = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega\hat{\sigma}_z + \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger\right)$$

- 2 commuting terms.

$$\hat{H}_0 = \hbar\omega\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega\hat{\sigma}_z, \quad \hat{H}' = \hbar\lambda\left(\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{a}^\dagger\right)$$

- Schrödinger equation in the interaction picture.

$$i\hbar\frac{d}{dt}\,|\,\Psi(t)\,\rangle = \hat{H}'\,|\,\Psi(t)\,\rangle$$

Definite Photon States

- The atomic states may be in a linear combination of the two energy levels.

$$|\,\Psi(t)\,\rangle = C_+\,|+\rangle\,|\,n\,\rangle + C_-\,|-\rangle\,|\,n+1\,\rangle$$

- Only 2 modes of transition.

Stimulated emission $\quad |+\rangle\,|\,n\,\rangle \rightarrow |-\rangle\,|\,n+1\,\rangle$

Stimulated absorption $\quad |-\rangle\,|\,n\,\rangle \rightarrow |+\rangle\,|\,n-1\,\rangle$

Definite Photon States

- Solving the Schrödinger equation and equating coefficients yields two coupled differential equations.

$$\dot{C}_+(t) = -i\lambda\sqrt{n+1}\,C_-(t)$$

$$\dot{C}_-(t) = -i\lambda\sqrt{n+1}C_+(t)$$

- These are easily solved with initial conditions. For instance, choose $|\,\Psi(0)\,\rangle = |\,+\,\rangle \Rightarrow C_+ = 1,\; C_- = 0\,.$

$$C_+(t) = \cos\left(\sqrt{n+1}\,\lambda\,t\right)$$

$$C_-(t) = -i\sin\left(\sqrt{n+1}\,\lambda\,t\right)$$

Definite Photon States

- The wave function of the total system oscillates in time.

$$|\,\Psi(t)\,\rangle = \cos(\lambda\sqrt{n+1}\,t)\,|\,+\,\rangle\,|\,n\,\rangle - i\sin(\lambda\sqrt{n+1}\,t)\,|\,-\,\rangle\,|\,n+1\,\rangle$$

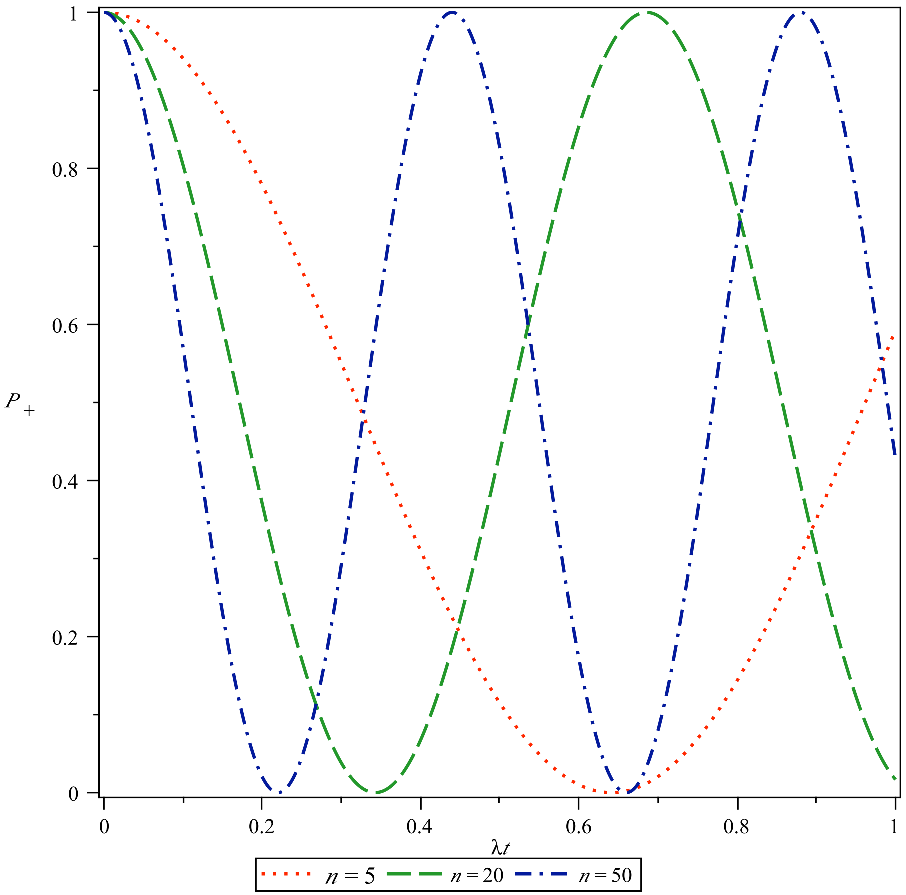
- The probability amplitudes are given through inner products and also oscillate in time.

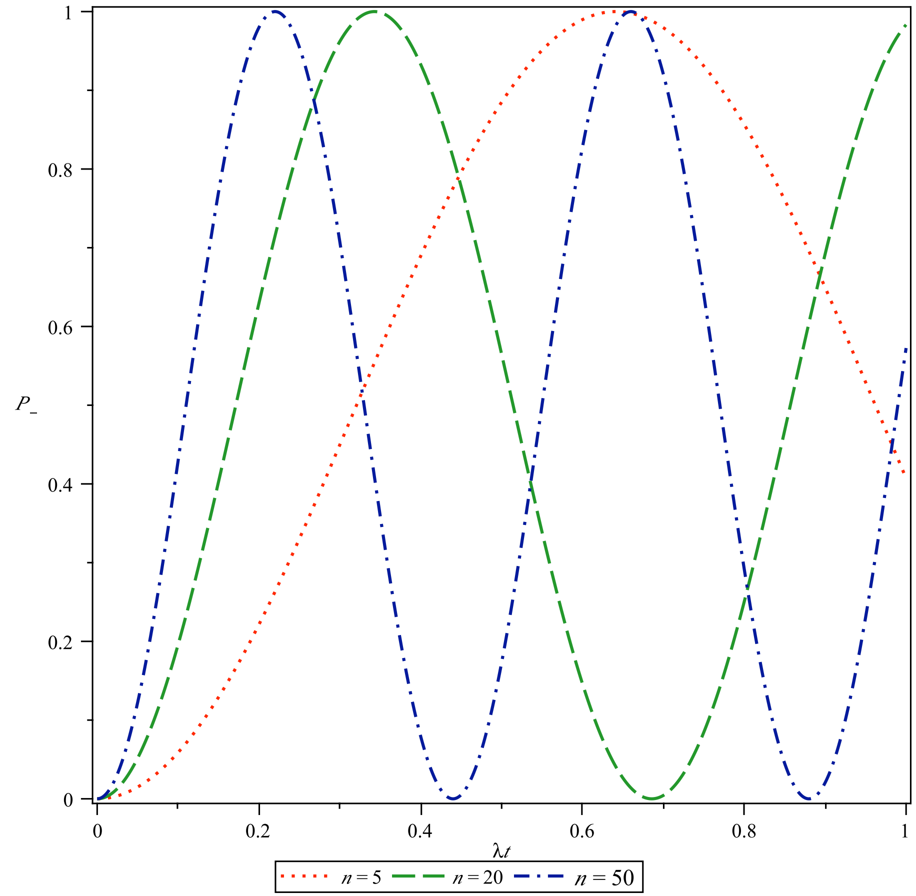
$$P_+(t) = |\,C_+\,|^2 = \cos^2(\lambda\sqrt{n+1}\,t)$$

$$P_-(t) = |\,C_-\,|^2 = \sin^2(\lambda\sqrt{n+1}\,t)$$

$$P_+(t) + P_-(t) = 1$$

Definite Photon States





Oscillation of the probability amplitudes in time.

Definite Photon States

- We are interested in measuring the atomic population inversion W(t) (the expectation value of the inversion operator).

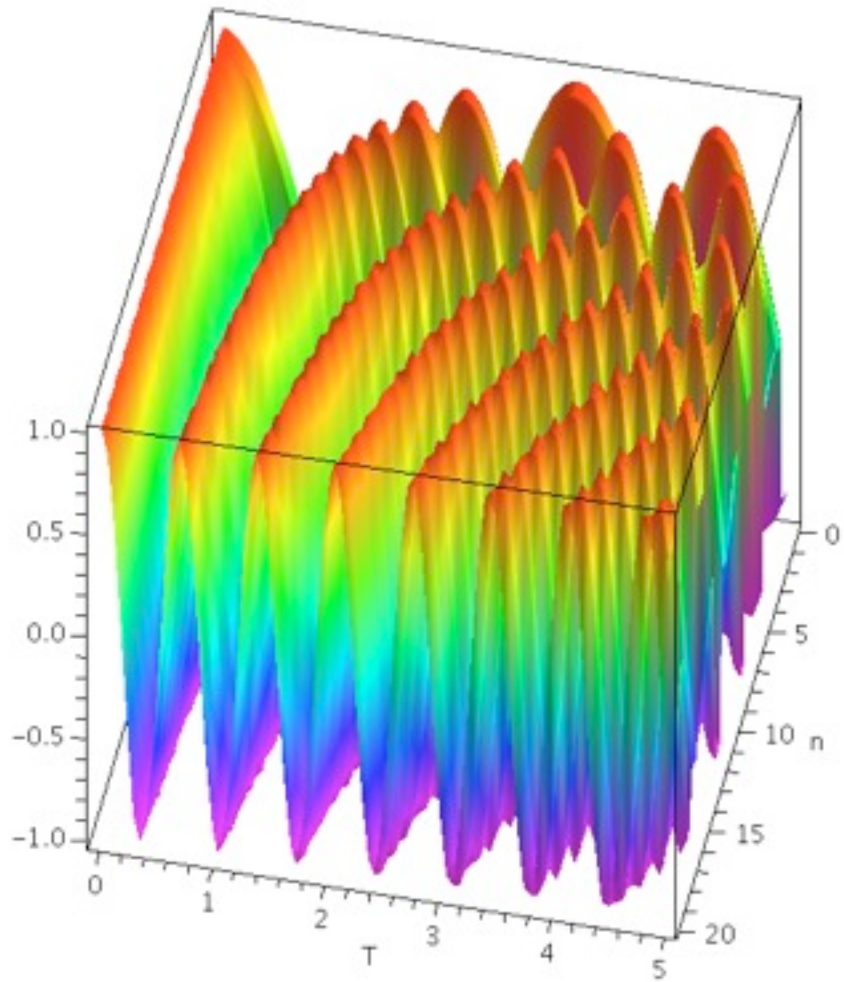
$$W(t) = \langle\, \Psi(t) \,|\, \hat{\sigma}_z \,|\, \Psi(t) \,\rangle$$

$$= |\, C_+ \,|^2 - |\, C_- \,|^2$$

$$= \cos^2(\lambda\sqrt{n+1}\, t) - \sin^2(\lambda\sqrt{n+1}\, t)$$

$$W(t) = \cos\left(2\lambda\sqrt{n+1}\, t\right)$$

Definite Photon States



Atomic inversion for several periods and a range of electric field strengths.

Definite Photon States

- An interesting property of the model is a non-zero transition probability in the absence of electric field.

$$W(t)|_{n=0} = \cos(2\lambda t)$$

$$|\,(\langle\, 0\,|\,\langle +|)\;|\,\Psi(t)\,\rangle\,|^2 = P_+^{(0)}(t) = \cos^2(\lambda t)$$

$$|\,(\langle\, 1\,|\,\langle -|)\;|\,\Psi(t)\,\rangle\,|^2 = P_-^{(0)}(t) = \sin^2(\lambda t)$$

Demonstration 2: Coherent Photon States

$$| \Psi(t) \rangle = \sum_{n=0}^{\infty} C_n \left(C_+ \, | + \rangle + C_- \, | - \rangle \right) | \, n \rangle$$

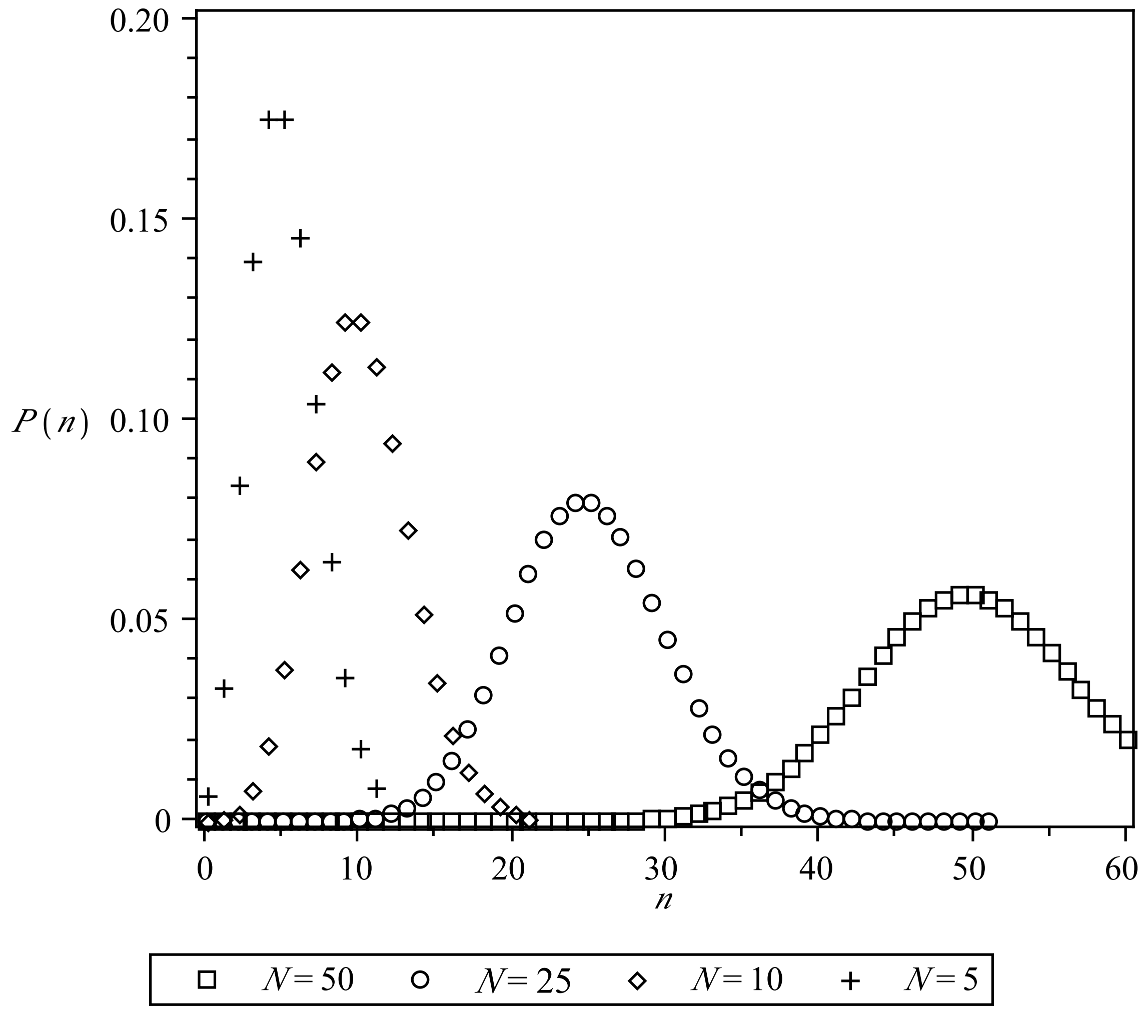
Coherent States

- "Near classical" photon states.
- Superposition of photon number states.
- $|\,\alpha\,|^2 = N$ is mean photon number.

$$|\,\psi_{\text{field}}\,\rangle = e^{-N/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\,|\,n\,\rangle$$

$$P(n) = |\,\langle\, n\,|\,\psi_{\text{field}}\,\rangle\,|^2 = e^{-N}\frac{N^n}{n!}$$

Coherent States



Coherent States

- The general state of the coherent system is a direct product of the atom and field states.

$$|\,\psi_{\text{field}}\,\rangle = \sum_{n=0}^{\infty} C_n\,|\,n\,\rangle \qquad\qquad |\,\psi_{\text{atom}}\,\rangle = C_+\,|\,+\,\rangle + C_-\,|\,-\,\rangle$$

$$|\,\Psi(t)\,\rangle = |\,\psi_{\text{atom}}\,\rangle \otimes |\,\psi_{\text{field}}\,\rangle = \sum_{n=0}^{\infty} C_n\,[C_+\,|\,+\,\rangle + C_-\,|\,-\,\rangle]\,|\,n\,\rangle$$

- We will choose a similar initial condition as before.

$$|\,\Psi(0)\,\rangle = \sum_{n=0}^{\infty} C_n\,|\,+\,\rangle\,|\,n\,\rangle$$

Coherent States

- Solve the Schrödinger equation again with the initial condition to obtain the general wave function.

$$|\,\Psi(t)\,\rangle = \sum_{n=0}^{\infty} C_n \left\{\cos(\lambda\sqrt{n+1}\,t)\,|\,+\,\rangle\,|\,n\,\rangle \right.$$

$$\left. -i\sin(\lambda\sqrt{n+1}\,t)\,|\,-\,\rangle\,|\,n+1\,\rangle\right\}$$

Coherent States

- This leads to transition probabilities that oscillate, but also consist of superpositions of photon states.

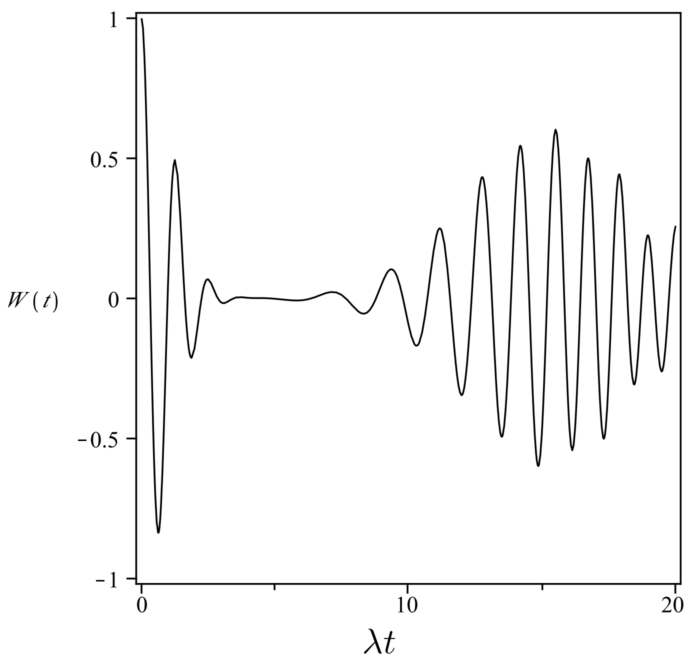
$$P_+(t) = |\langle + | \Psi(t) \rangle|^2 = \sum_{n=0}^{\infty} e^{-N} \frac{N^n}{n!} \cos^2(\lambda\sqrt{n+1}\,t)$$

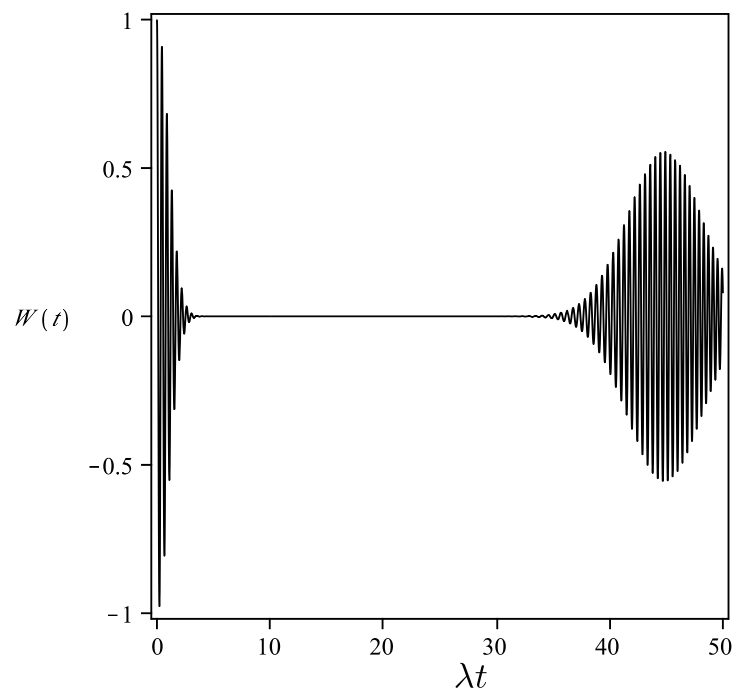
$$P_-(t) = |\langle - | \Psi(t) \rangle|^2 = \sum_{n=0}^{\infty} e^{-N} \frac{N^n}{n!} \sin^2(\lambda\sqrt{n+1}\,t)$$

Coherent States

- Collapse and revival are strongly displayed in the atomic inversion of coherent states.

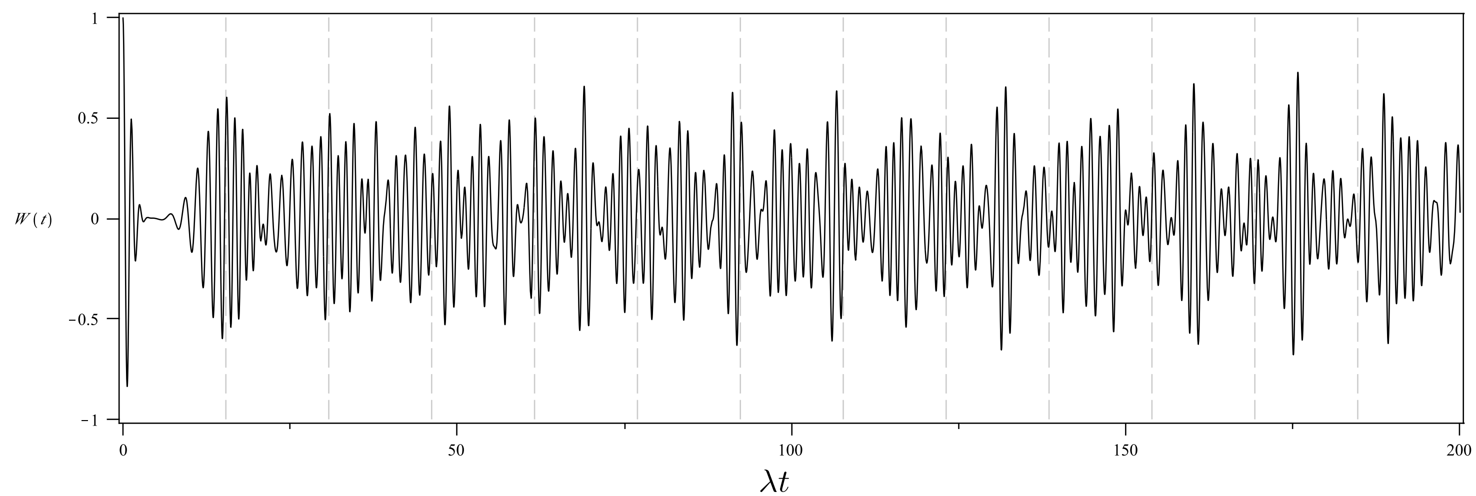
$$W(t) = P_+(t) - P_-(t) = e^{-N} \sum_{n=0}^{\infty} \frac{N^n}{n!} \cos(2\lambda\sqrt{n+1}\,t)$$

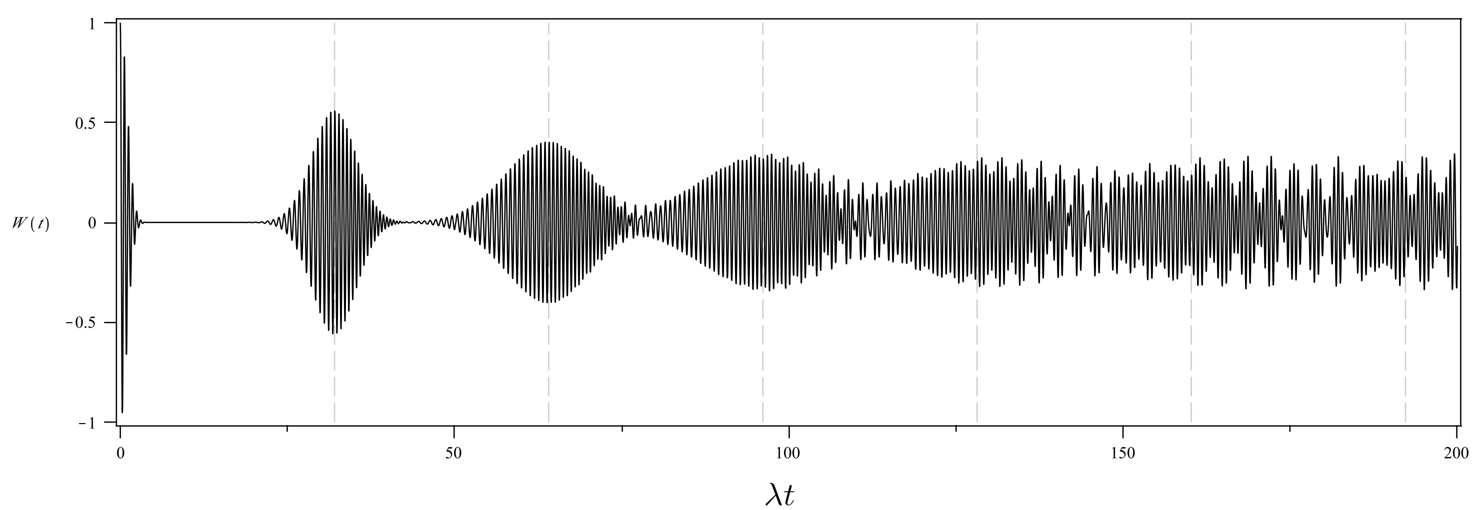




Coherent States

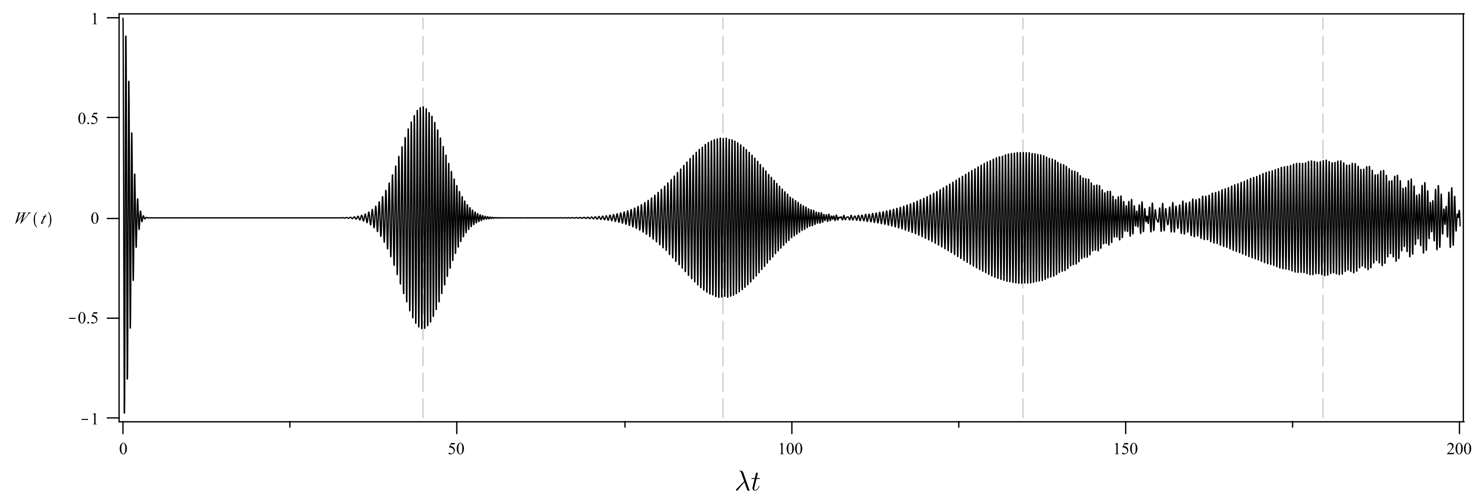
- Collapse and revival are approx. periodic over longer time-scales.

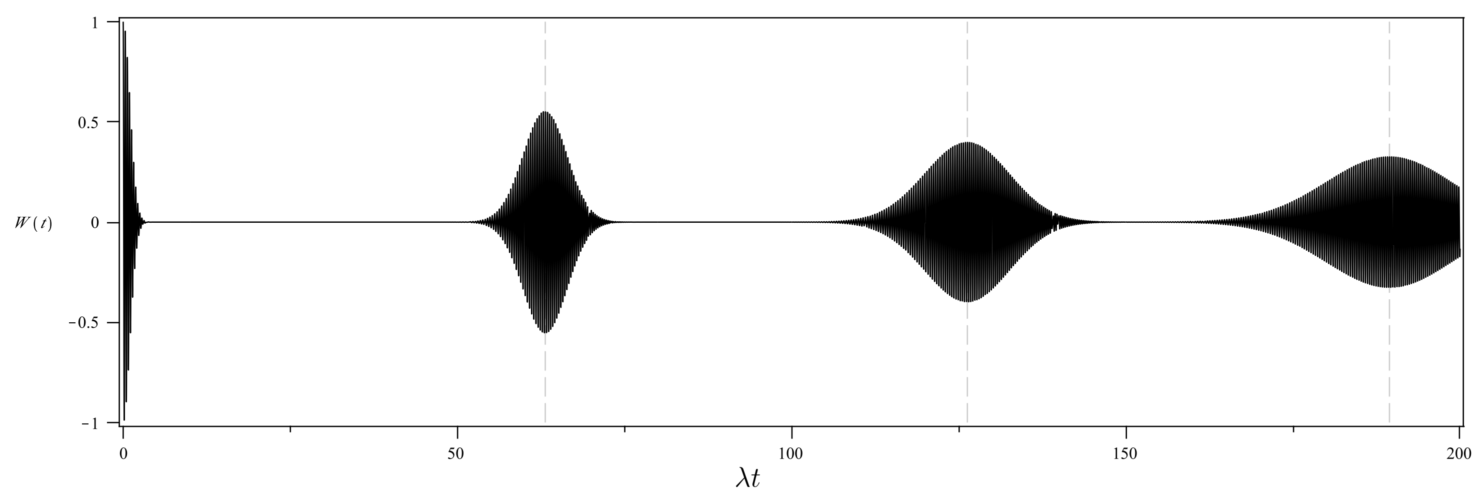




Coherent States

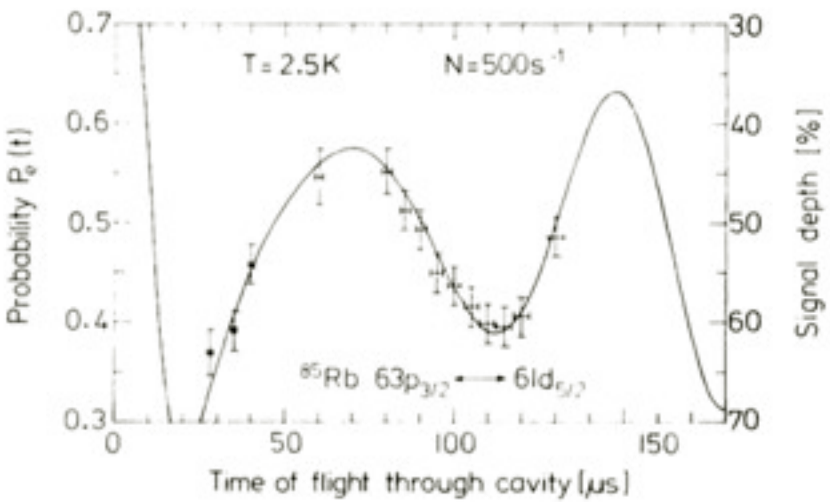
- More well-defined envelopes for large mean photon number.

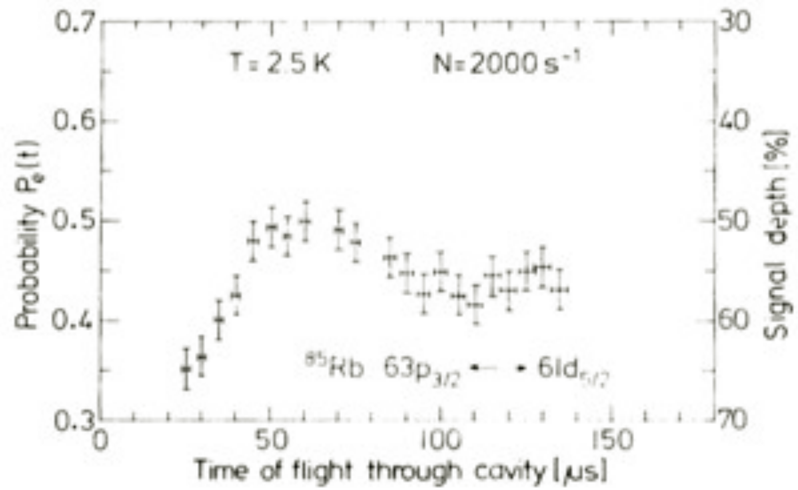




Experimental Confirmation

- Experimentally confirmed in the 1980s.





- Rubidium maser, 2.5 Kelvin cavity, Q factor of 6×10^7 .
- Large principal quantum number allows for only 2-level transitions.

Graphs & information from PRL vol. 58, no. 4, 26 January 1987, pp. 353--356.

Experimental Confirmation

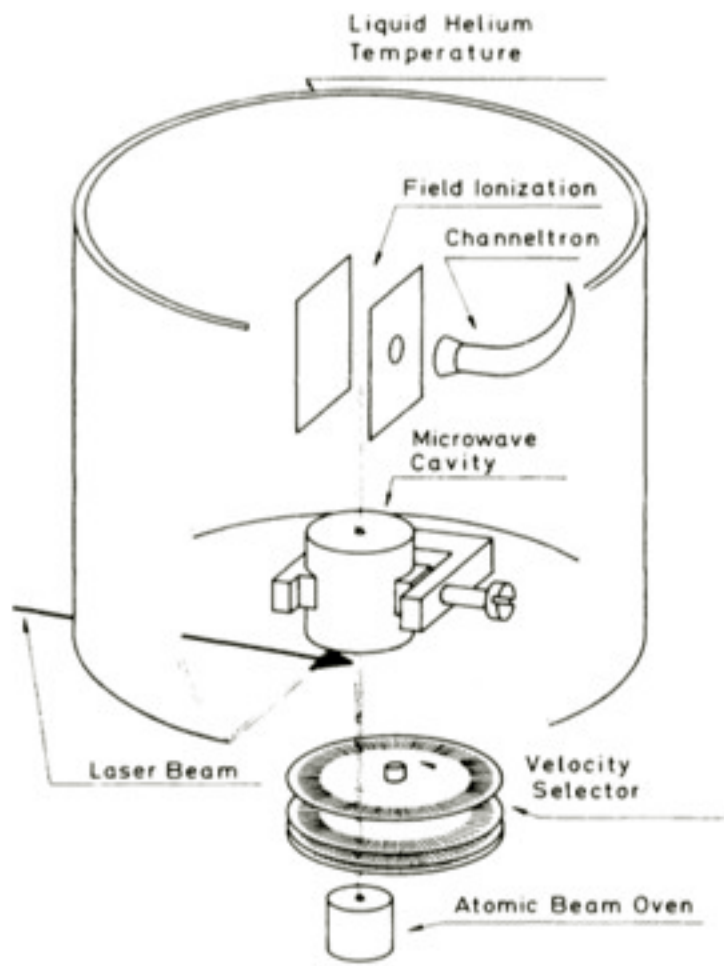


FIG. 1. Scheme of the experimental setup.

Graphs & information from PRL vol. 58, no. 4, 26 January 1987, pp. 353--356.

Conclusions & Extensions of the Jaynes-Cummings-Paul Model

Conclusions

- Collapse and revival are uniquely quantum mechanical in nature.

- Spontaneous emission is uniquely quantum mechanical.

- Simplified model allows for basic understanding about photon/atom interactions.

- The assumptions are very general and easily expounded upon.

Extensions of the Model

- Collapse and revival with nonzero detuning Δ.
- Cavity damping viz. photon loss (non-infinite Q factor).
- Multi-photon transitions.
- Time-dependent coupling constant $\lambda(t)$.

Thank You!

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