Comme Appelé du Néant— As If Summoned from the Void: The Life of Alexandre Grothendieck

Allyn Jackson

This is the second part of a two-part article about the life of Alexandre Grothendieck. The first part of the article appeared in the October 2004 issue of the *Notices*.

A Different Way of Thinking

> Dans le travail de découverte, cette attention intense, cette sollicitude ardente sont une force essentielle, tout comme la chaleur du soleil pour l'obscure gestation des semences enfouies dans la terre nourricière, et pour leur humble et miraculeuse éclosion à la lumière du jour.
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> In the work of discovery, this intense attention, this ardent solicitude, are an essential force, just like the warmth of the sun for the obscure gestation of seeds covered in nourishing soil, and for their humble and miraculous blossoming in the light of day.
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> —*Récoltes et Semailles*, page P49

Grothendieck had a mathematical style all his own. As Michael Artin of the Massachusetts Institute of Technology commented, in the late 1950s and 1960s "the world needed to get used to him, to his power of abstraction." Nowadays Grothendieck's point of view has been so thoroughly absorbed into algebraic geometry that it is standard fare for graduate students starting in the field, many of whom do not realize that things were once quite different. Nicholas Katz of Princeton University said that when as a young mathematician he first encountered Grothendieck's way of thinking, it seemed completely different and new. But it is hard to articulate what the difference was. As Katz put it, the change in point of view was so fundamental and profound and, once adopted, so completely natural "that it's sort of hard to imagine the time before you thought that way."

Although Grothendieck approached problems from a very general point of view, he did so not for generality's sake but because he was able to use generality in a very fruitful way. "It's a kind of approach that in less gifted hands just leads to what most people would say are sterile generalities," Katz commented. "He somehow knew what general things to think about." Grothendieck always sought the precise level of generality that would provide precisely the right leverage to gain insight into a problem. "He seemed to have the knack, time after time, of stripping away just enough so that it wasn't a special case, but it wasn't a vacuum either," commented John Tate of the University of Texas at Austin. "It's streamlined; there is no baggage. It's just right."

One striking characteristic of Grothendieck's mode of thinking is that it seemed to rely so little on examples. This can be seen in the legend of the so-called "Grothendieck prime". In a mathematical conversation, someone suggested to Grothendieck that they should consider a particular prime number. "You mean an actual number?" Grothendieck asked. The other person replied, yes, an actual prime number. Grothendieck suggested, "All right, take 57."

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But Grothendieck must have known that 57 is not prime, right? Absolutely not, said David Mumford of Brown University. "He doesn't think concretely." Consider by contrast the Indian mathematician Ramanujan, who was intimately familiar with properties of many numbers, some of them huge. That way of thinking represents a world antipodal to that of Grothendieck. "He really never worked on examples," Mumford observed. "I only understand things through examples and then gradually make them more abstract. I don't think it helped Grothendieck in the least to look at an example. He really got control of the situation by thinking of it in absolutely the most abstract possible way. It's just very strange. That's the way his mind worked." Norbert A'Campo of the University of Basel once asked Grothendieck about something related to the Platonic solids. Grothendieck advised caution. The Platonic solids are so beautiful and so exceptional, he said, that one cannot assume such exceptional beauty will hold in more general situations.

One thing Grothendieck said was that one should never try to prove anything that is not almost obvious. This does not mean that one should not be ambitious in choosing things to work on. Rather, "if you don't see that what you are working on is almost obvious, then you are not ready to work on that yet," explained Arthur Ogus of the University of California at Berkeley. "Prepare the way. And that was his approach to mathematics, that everything should be so natural that it just seems completely straightforward." Many mathematicians will choose a well-formulated problem and knock away at it, an approach that Grothendieck disliked. In a well-known passage of *Récoltes et Semailles*, he describes this approach as being comparable to cracking a nut with a hammer and chisel. What he prefers to do is to soften the shell slowly in water, or to leave it in the sun and the rain, and wait for the right moment when the nut opens naturally (pages 552–553). "So a lot of what Grothendieck did looks like the natural landscape of things, because it looks like it grew, as if on its own," Ogus noted.

Grothendieck had a flair for choosing striking, evocative names for new concepts; indeed, he saw the act of naming mathematical objects as an integral part of their discovery, as a way to grasp them even before they have been entirely understood (*R&S*, page P24). One such term is *étale*, which in French is used to describe the sea at slack tide, that is, when the tide is neither going in nor out. At slack tide, the surface of the sea looks like a sheet, which evokes the notion of a covering space. As Grothendieck explained in *Récoltes et Semailles*, he chose the word *topos*, which means "place" in Greek, to suggest the idea of "the 'object *par excellence*' to which topological intuition applies" (pages 40–41). Matching the concept, the word *topos* suggests the most fundamental, primordial notion of space. The term *motif* ("motive" in English) is intended to evoke both meanings of the word: a recurrent theme and something that causes action.

Grothendieck lecturing at the IHÉS.

Grothendieck's attention to choosing names meant that he loathed terminology that seemed unsuitable: In *Récoltes et Semailles*, he said he felt an "internal recoiling" upon hearing for the first time the term *perverse sheaf*. "What an idea to give such a name to a mathematical thing!" he wrote. "Or to any other thing or living being, except in sternness towards a person—for it is evident that of all the 'things' in the universe, we humans are the only ones to whom this term could ever apply" (page 293).

Although Grothendieck possessed great technical power, it was always secondary; it was a means for carrying out his larger vision. He is known for certain results and for developing certain tools, but it is his creation of a new viewpoint on mathematics that is his greatest legacy. In this regard, Grothendieck resembles Evariste Galois; indeed, in various places in *Récoltes et Semailles* Grothendieck wrote that he strongly identified with Galois. He also mentioned that as a young man he read a biography of Galois by Leopold Infeld [Infeld] (page P63).

Ultimately, the wellspring of Grothendieck's achievement in mathematics is something quite humble: his love for the mathematical objects he studied.

A Spirit in Stagnation

> [P]endant vingt-cinq ans, entre 1945 (quand j'avais dix-sept ans) et 1969 (quand j'allais sur les quarante-deux), j'ai investi pratiquement la totalité de

> mon énergie dans la recherche mathématique. Investissement démesuré, certes. Je l'ai payé par une longue stagnation spirituelle, par un "épaississement" progressif, que j'aurai plus d'une fois l'occasion d'évoquer dans les pages de *Récoltes et Semailles*.
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> [F]or twenty-five years, between 1945 (when I was seventeen years old) and 1969 (when I reached forty-two), I invested practically my entire energy into mathematical research. An excessive investment, certainly. I paid for it with a long spiritual stagnation, with a progressive "dulling", that I have more than once found occasion to evoke in the pages of *Récoltes et Semailles*.
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> —*Récoltes et Semailles*, page P17

During the 1960s, Barry Mazur of Harvard University visited the Institut des Hautes Études Scientifiques (IHÉS) with his wife. Although by that time Grothendieck had a family and a house of his own, he also kept an apartment in the same building where the Mazurs were living and frequently worked there late into the night. Because the apartment keys did not open the outside doors, which were locked at 11:00 p.m., one might have trouble getting into the building after an evening in Paris. But "I remember we never had any problems," Mazur recalled. "We would take the last train back, absolutely certain that there would be Grothendieck working, his desk by the window. We would throw some gravel at his window and he would open the outside door for us." Grothendieck's apartment was sparely furnished; Mazur remembered a wire sculpture in the outline of a goat and an urn filled with Spanish olives.

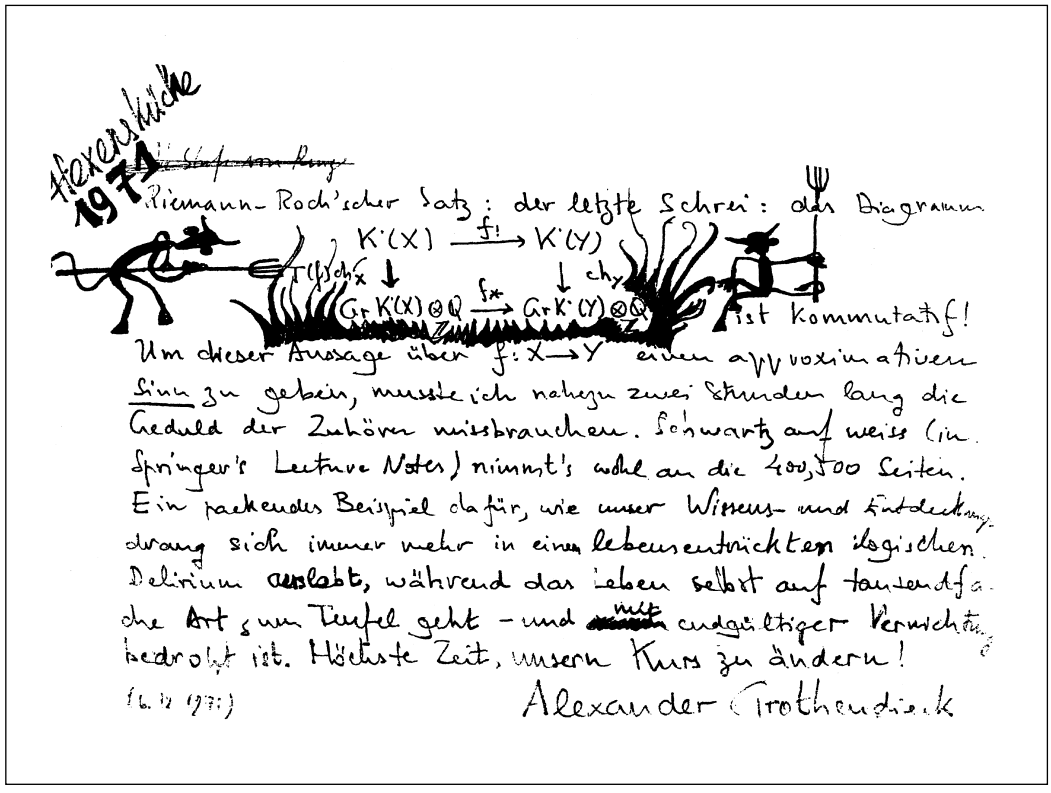
This somewhat lonely image of Grothendieck working away into the night in a spartan apartment captures one aspect of his life during the 1960s. At this time he did mathematics nonstop. He was talking to colleagues, advising students, lecturing, carrying on extensive correspondence with mathematicians outside of France, and writing the seemingly endless volumes of *EGA* and *SGA*. It is no exaggeration to say that he was single-handedly leading a large and thriving segment of worldwide research in algebraic geometry. He seemed to have few interests outside of mathematics; colleagues have said that he never read a newspaper. Even among mathematicians, who tend to be single-minded and highly devoted to their work, Grothendieck was an extreme case. "Grothendieck was working on the foundations of algebraic geometry seven days a week, twelve hours a day, for ten years," noted his IHÉS colleague David Ruelle. "He had achieved level -1 and was working on level 0 of something that must be 10 levels high.…At a certain age it becomes clear you will never be able to finish the building."

The extremity of Grothendieck's focus on mathematics is one reason for the "spiritual stagnation" he referred to in *Récoltes et Semailles*, which in turn is one of the reasons behind his departure, in 1970, from the world of mathematics in which he had been a leading figure. One step toward that departure was a crisis within the IHÉS, which led to his resignation. Starting in late 1969, Grothendieck became embroiled in a conflict with the founder and director of the IHÉS, Léon Motchane, over military funding for the institute. As historian of science David Aubin explained [Aubin], during the 1960s, the IHÉS finances were rather precarious, and in some years the institute received a small portion of its budget, never more than about 5 percent, from sources within the French military. All of the permanent IHÉS professors had misgivings about military funding, and in late 1969 they insisted that Motchane quit accepting such funding. Motchane agreed, but, as Aubin noted, he went back on his word just a few months later, when the IHÉS budget was stretched thin and he accepted a grant from the minister of the army. Outraged, Grothendieck tried in vain to persuade the other professors to resign along with him, but none did. Less than a year earlier, Pierre Deligne had joined the IHÉS faculty as a permanent professor, largely on the recommendation of Grothendieck, who now pressed his newly appointed colleague to join him in resigning. Deligne too refused. "Because I was very close to him mathematically, Grothendieck was surprised and deeply disappointed that this closeness of ideas did not extend outside of mathematics," Deligne recalled. Grothendieck's letter of resignation was dated May 25, 1970.

His rupture with the IHÉS was the most visible sign of a profound shift taking place in Grothendieck's life. Toward the end of the 1960s there were other signs as well. Some were small. Mazur recalled that when he was visiting the IHÉS in 1968, Grothendieck told him he had gone to the movies—for the first time in perhaps a decade. Other signs were larger. In 1966, when he was to receive the Fields Medal at the International Congress of Mathematicians (ICM) in Moscow, Grothendieck refused to attend as a protest against the Soviet government. In 1967 Grothendieck made a three-week trip to Vietnam, which clearly left an impression on him. His written account of the trip [Vietnam] described the many air raid alerts and a bombing that left two mathematics teachers dead, as well as the valiant efforts of the Vietnamese to cultivate a mathematical life in their country. A friendship with a Romanian physician named Mircea Dumitrescu led Grothendieck to make

in the late 1960s a fairly serious foray into learning some biology. He also discussed physics with Ruelle.

The events of the extraordinary year of 1968 must also have had an impact on Grothendieck. That year saw student protests and social upheavals all over the world, as well as the Soviet Union's brutal crushing of the "Prague Spring". In France the boiling point came in May 1968, when students objecting to university and government policies carried out massive protests that soon turned into riots. In Paris hundreds of thousands of students, teachers, and workers took to the streets to protest police brutality, and the French government, fearing revolution, stationed tanks around the perimeter of the city. Millions of workers went on strike, paralyzing the nation for about two weeks. Karin Tate, who was living in Paris with her husband at the time, John Tate, recalled the chaos that reigned. "Paving stones, batons, and any other missiles that were handy flew through the air," she said. "Soon the entire country was at a standstill. There was no gasoline (truckers were on strike), there were no trains (train workers were on strike), garbage was piling up in Paris (sanitation workers were on strike), there was very little food on the shelves." She and John fled to Bures-sur-Yvette, where her brother, Michael Artin, was visiting the IHÉS. Many Parisian mathematicians took the side of the students in the conflict. Karin Tate said the protests dominated conversations among the mathematicians she knew, though she did not remember discussing the topic with Grothendieck.



Grothendieck wrote this abstract into the colloquium book at the Universität Bielefeld when he spoke there in 1971. The abstract says: "Witch's Kitchen 1971. Riemann-Roch Theorem: The 'dernier cri': The diagram [displayed] is commutative! To give an approximate sense to the statement about $f : X \to Y$, I had to abuse the listeners' patience for almost two hours. Black on white (in Springer Lecture Notes) it probably takes about 400, 500 pages. A gripping example of how our thirst for knowledge and discovery indulges itself more and more in a logical delirium far removed from life, while life itself is going to Hell in a thousand ways—and is under the threat of final extermination. High time to change our course!"

Shortly after his resignation from the IHÉS, Grothendieck plunged into a world completely new to him, the world of protest politics. In a June 26, 1970, lecture at the Université de Paris in Orsay, he spoke not about mathematics but about the threat of nuclear proliferation to the survival of humankind and called upon scientists and mathematicians not to collaborate in any way with the military. Nicholas Katz, who had recently arrived for a visit at the IHÉS and was surprised to hear of Grothendieck's resignation, attended the lecture, which he said drew an audience of hundreds in a very crowded lecture hall. Katz remembered that in the lecture Grothendieck went so far as to say that doing mathematical research was actually "harmful" (*"nuisible"*), given the impending threats to the human race.

A written version of the lecture, "Responsabilité du savant dans le monde d'aujourd'hui: Le savant et l'appareil militaire" ("The responsibility of the scholar in today's world: The scholar and the military apparatus"), circulated as an unpublished manuscript. An appendix described the hostile reactions of the students who attended the lecture and who handed out flyers mocking Grothendieck. One of the flyers is reproduced in the appendix; a typical slogan: "Réussissez, ossifiez-vous, détruisez-vous vous-mêmes: devenez un petit schéma télécommandé par Grothendieck" ("Succeed, ossify, self-destruct: become a little scheme remote-controlled by Grothendieck"). He was clearly seen as a detested member of the establishment.

In another appendix in this manuscript, Grothendieck called for the founding of a group to fight

Tata Institute International Colloquium in 1968. Grothendieck (standing, left) and Armand Borel (seated, facing camera).

for the survival of the human race against environmental degradation and the dangers of military conflict. This group, called "Survival" ("Survivre et Vivre" in French) came into being in July 1970 when Grothendieck delivered his Orsay lecture a second time, at a summer school on algebraic geometry at the University of Montreal. The main activity of Survival was the publication of a newsletter by the same name, the first issue of which was written in English by Grothendieck and is dated August 1970. The newsletter describes an ambitious agenda of publication of books on science, organization of public courses on science aimed at nonexperts, and boycotts of scientific institutions that accept military funds.

That first issue carried a list of the names, professions, and addresses of the group's members, who numbered twenty-five at the time. On the list were several mathematicians, Grothendieck's mother-in-law, and his son Serge. The directors of the group were Grothendieck and three other mathematicians: Claude Chevalley, Denis Guedj, and Pierre Samuel (*R&S*, page 758). Survival was one of many leftist groups that emerged in the wake of the tumultuous 1960s; a similar organization in the United States was the Mathematics Action Group. Too small and diffuse to accumulate much influence, Survival was more active in Paris than in the United States and Canada, due mostly to Grothendieck's presence. When he moved out of Paris in 1973, the group petered out.

At the ICM in Nice in the summer of 1970, Grothendieck tried to recruit members for Survival. He wrote, "I expected massive enrollments—there were (if I remember correctly) two or three" (*R&S*, page 758). Nevertheless, his proselytizing drew a good deal of attention. "First of all, he was one of the world stars in mathematics at that time," said Pierre Cartier of the IHÉS, who attended the congress. "Also, you have to remember the political climate at the time." Many mathematicians opposed the Vietnam War and sympathized with Survival's antimilitary stance. During the congress, Cartier said, Grothendieck sneaked a table in between two publishers' booths in the exhibit area and, assisted by his son Serge, began to hand out the Survival newsletter. This caused a heated row between him and his old colleague and friend, Jean Dieudonné, who had become the first dean of the science faculty at the Université de Nice when it was founded in 1964 and who was responsible for the ICM being held there. Cartier said that he and others tried unsuccessfully to persuade Dieudonné to permit this "unofficial booth". Eventually Grothendieck took the table out to the street in front of the hall in which the congress was being held. But another problem loomed: in delicate negotiations with the mayor of Nice, the congress organizers had promised there would be no street demonstrations. Police officers began to question Grothendieck, and finally the chief of police showed up. Grothendieck was asked to move his table just a few yards back so that it was off the sidewalk. "But he refused," Cartier recalled. "He wanted to be put in jail. He really wanted to be put in jail!" Finally, Cartier said, he and some others moved the table back sufficiently to satisfy the police.

Although Grothendieck's plunge into politics was sudden, he was by no means alone. His good friend Cartier has a long history of political activism. For example, he was among the mathematicians who used the holding of the ICM in Warsaw in 1983 to negotiate the release of one hundred fifty political prisoners in Poland. Cartier traces his activism to the example set by his teacher and mentor, Laurent Schwartz, who was one of the most politically vocal and active academics in France. Schwartz was the thesis adviser of Grothendieck. Another mathematician Grothendieck knew well, Pierre Samuel, is one of the founders of the French Green Party. Outside of France, many mathematicians were politically active. Among the best-known examples in North America are Chandler Davis and Stephen Smale, who were deeply involved in protests against the Vietnam War.

But despite his strong convictions, Grothendieck was never effective in the real world of politics. "He was always an anarchist at heart," Cartier observed. "On many issues, my basic positions are not very far from his positions. But he was so naive that it was totally impossible to do anything with him politically." He was also rather ignorant. Cartier recalled that, after an inconclusive presidential election in France in 1965, the newspapers carried

headlines saying that de Gaulle had not been elected. Grothendieck asked if this meant that France would no longer have a president. Cartier had to explain to him what a runoff election is. "Grothendieck was politically illiterate," Cartier said. But he did want to help people: it was not unusual for Grothendieck to give shelter for a few weeks to homeless people or others in need. "He was very generous, he has always been very generous," Cartier said. "He remembered his youth, his difficult youth, when his mother had nothing, and he was always ready to help—but in a nonpolitical way."

The Wild '70s

> [In 1970 J]'ai alors quitté un milieu pour entrer dans un autre—le milieu des gens "des premiers rangs" pour le "marais"; soudain, la plupart de mes nouveaux amis étaient de ceux justement qu'un an avant encore j'aurais tacitement situés dans cette contrée sans nom et sans contours. Le soi-disant marais soudain s'animait et prenait vie par les visages d'amis liés à moi par une aventure commune—une autre aventure!
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> [In 1970] I left one milieu to enter another—the milieu of people "of the first rank" for the "swamp"; suddenly, the majority of my new friends were those who just a year before I had tacitly situated in this region without name and without shape. The so-called swamp suddenly moved around and took on life through the faces of friends tied to me by a common adventure—another adventure!
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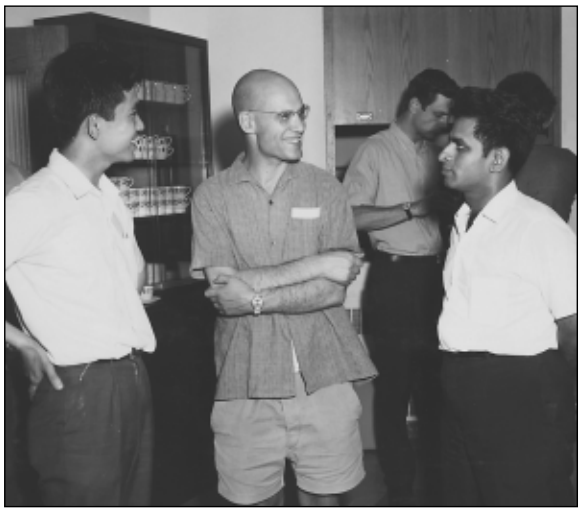
> —*Récoltes et Semailles*, page 38

"Légion d'Honneur! Légion d'Honneur!" Grothendieck was shouting from the back of the auditorium, waving a paper facsimile of the Légion d'Honneur cross, a distinction conferred by the French government. The scene was the opening day of a summer school on modular functions, held in Antwerp in the summer of 1972 and supported by the North Atlantic Treaty Organization (NATO). Grothendieck's longtime friend Jean-Pierre Serre of the Collège de France, who had recently received the Légion d'Honneur, was presenting the opening speech. Grothendieck approached Serre and asked, "Do you mind if I go to the blackboard and say something?" Serre replied, "Yes, I mind" and left the room. Grothendieck then mounted the podium and began speaking against NATO support for the conference. Other mathematicians sympathized with this view: One example was Roger Godement,



Grothendieck, center, University of Montreal, around 1970.

who in April 1971 wrote an open letter explaining his reasons for refusing to attend the conference.

Unbeknownst to Grothendieck, Cartier and some other mathematicians who were uncomfortable about the NATO support had conducted extensive negotiations to have a NATO representative come to the conference for a public debate. Cartier and others eased Grothendieck off the podium, but the damage had been done: Cartier soon received an angry phone call from the NATO representative, who had heard about the outburst and refused to come, believing that conditions for an orderly debate had been ruined. "To me, it was sad, because from what I remember, I think that the audience was mostly on Grothendieck's political side," Cartier noted. "Even people who were close to his political views or his social views were antagonized by his behavior.…He behaved like a wild teenager."

By the time of the Antwerp meeting, Grothendieck had cut many of the ties that had bound him to an orderly life focused on mathematics. For one thing, he no longer had a permanent position. After he left the IHÉS in 1970, Serre arranged for him to have a visiting position at the Collège de France for two years. This elite institution operates differently from other universities in France (or anywhere else for that matter). Each professor at the Collège must submit for approval by the assembly of all the professors a program of the lectures he or she plans to deliver during the year. Serre recalled that Grothendieck offered two possible programs: one on mathematics and one on the political themes that occupied the Survival group. The committee approved the mathematical program and rejected the other one. So Grothendieck presented mathematical lectures prefaced by long discourses about politics. After two years he applied for a permanent position at the Collège de France, a position that had become vacant with the retirement of Szolem Mandelbrojt. The curriculum vitae

A. Grothendieck with children Serge (left) and Johanna in 1960.

Grothendieck submitted plainly showed that he intended to give up mathematics to focus on tasks he believed to be far more urgent: "the imperatives of survival and the promotion of a stable and humane order on our planet." How could the Collège appoint to a position in mathematics someone who had declared that he would no longer do any mathematics? "He was rightly refused," Serre said.

It was also during the period just after he left the IHÉS that Grothendieck's family life crumbled and he and his wife separated. In the two years after he left the IHÉS, Grothendieck spent a fair amount of time lecturing in mathematics departments in North America. He spread the gospel of Survival by insisting he would give a mathematics lecture only if arrangements were made for him also to give a political lecture. On one such trip in May 1972, he visited Rutgers University and met Justine Bumby (Skalba), then a graduate student of Daniel Gorenstein. Captivated by Grothendieck's charismatic personality, Bumby left behind her life as a graduate student to follow him, first on the remainder of his trip in the United States, and then on to France, where she lived with him for two years. "He's the most intelligent person I've ever met," she said. "I was very much in awe of him."

Their life together was in some ways emblematic of the counterculture years of the 1970s. Once, at a peaceful demonstration in Avignon, the police intervened, harassing and pushing away the demonstrators. Grothendieck got angry when they started pestering him, Bumby recalled. "He was a good boxer, so he was very fast," she said. "We see the policemen approaching us, and we are all scared, and then the next thing we know, the two policemen are on the ground." Grothendieck had singlehandedly decked two police officers. After some other officers had subdued him, Bumby and Grothendieck were bundled into a wagon and taken to the police station. When his identification papers revealed that he was a professor at the Collège de France, the two were taken in to see the chief of police, who spoke to them in English, as Bumby spoke no French. After a short conversation, in which the police chief expressed his desire to avoid trouble between police and professors, the two were released and no charges were brought.

Shortly after Bumby came to France with Grothendieck, he started a commune in a large house he had rented just south of Paris in Chatenay-Malabry, and they lived there together. She said he sold organically grown vegetables and sea salt out of the basement of the house. The commune was a bustling place: Bumby said that Grothendieck held meetings, which might attract up to a hundred people, about the issues raised in the Survival group, and these attracted considerable media attention. However, the commune dissolved fairly rapidly as a result of complicated personal relationships among the members. It was around this time that Grothendieck's position ended at the Collège de France, and in the fall of 1972 he took a temporary position teaching for one year at the Université de Paris in Orsay. After that, Grothendieck obtained a position called *professeur à titre personnel*, which is attached to a single individual and can be taken to any university in France. Grothendieck took his to the Université de Montpellier, where he was to remain until his retirement in 1988.

In early 1973 he and Bumby moved to Olmet-le-sec, a rural village in the south of France. This area was at the time a magnet for hippies and others in the counterculture movement who wanted to return to a simpler lifestyle close to the land. Here Grothendieck again attempted to start up a commune, but personality conflicts led to its collapse. At various times three of Grothendieck's children came to live in the Paris commune and in the one in Olmet. After the latter commune dissolved, he moved with Bumby and his children to Villecun, a short distance away. Bumby noted that Grothendieck had a hard time adjusting to the ways of the people attracted to the counterculture movement. "His students in mathematics had been very serious, and they were very disciplined, very hardworking people," she said. "In the counterculture he was meeting people who would loaf around all day listening to music." Having been an undisputed leader in mathematics, Grothendieck now found himself in a very different milieu, in which his views were not always taken seriously. "He was used to people agreeing with his opinions when he was doing algebraic geometry," Bumby remarked. "When he switched to politics all the people who would have agreed with him before suddenly disagreed with him.... It was something he wasn't used to."

Although most of the time Grothendieck was very warm and affectionate, Bumby said, he

sometimes had violent outbursts followed by periods of silent withdrawal. There were also disturbing episodes in which he would launch into a monologue in German, even though she understood no German. "He would just go on as if I wasn't there," she said. "It was kind of scary." He was frugal, sometimes compulsively so: one time, to avoid throwing away three quarts of leftover coffee, he drank it—with the predictable result that he got quite sick afterward. Bumby said she believes that his speaking German and his extreme frugality may have been connected psychologically to the hardships he endured as a child, especially the time when he lived with his mother in the internment camps.

Grothendieck may have been experiencing some kind of psychological breakdown, and Bumby today wonders whether she should have sought treatment for him. Whether he would have submitted to such treatment is unclear. They parted ways not long after their son, John, was born in the fall of 1973. After spending some time in Paris, Bumby moved back to the United States. She married a mathematician who was a widower, Richard Bumby of Rutgers University, and they raised John and Richard's two daughters. John exhibited a good deal of mathematical talent and was a mathematics major at Harvard University. He recently finished his Ph.D. in statistics at Rutgers. Grothendieck has had no contact with this son.

During the early 1970s, Grothendieck's interests were very far from those of the mathematical world he had left behind. But that world intruded in a major way in the summer of 1973, when, at a conference in honor of W. V. D. Hodge in Cambridge, England, Pierre Deligne presented a series of lectures about his proof of the last and most stubborn of the Weil conjectures. Grothendieck's former student Luc Illusie was at the conference and wrote to him with the news. Wanting to know more, Grothendieck, accompanied by Bumby, visited the IHÉS in July 1973.

In 1959 Bernard Dwork proved by p-adic methods the first Weil conjecture (which says that the zeta function of a variety over a finite field is a rational function). Grothendieck's 1964 l-adic proof of this conjecture was more general and introduced his "formalism of the six operations." In the 1960s Grothendieck also proved the second Weil conjecture (which says that the zeta function of a variety satisfies a functional equation). Finding a way to prove the last Weil conjecture (sometimes called the "congruence Riemann Hypothesis") was a major inspiration for much of his work. He formulated what he called the "standard conjectures," which, if they could be proved, would imply all of the Weil conjectures. The standard conjectures were also formulated independently around the same time by Enrico Bombieri. To this day, the standard conjectures remain inaccessible. Deligne found a clever way to circumvent them when he proved the last Weil conjecture. One of the key ideas he used came from a paper by R. A. Rankin [Rankin], which is about the classical theory of modular forms and of which Grothendieck was unaware. As John Tate put it, "For the proof of the last Weil conjecture, you needed another ingredient that was more classical. That was Grothendieck's blind spot."

When Bumby and Grothendieck turned up at the IHÉS that summer, among the visitors was William Messing of the University of Minnesota. Messing first met Grothendieck in 1966, when as a graduate student at Princeton he attended a series of lectures Grothendieck gave at Haverford College. These lectures made a deep impression on Messing, and Grothendieck became his informal thesis adviser. In 1970 Messing joined the Survival group at the Montreal meeting at which it was founded. The following year, while Grothendieck was visiting Kingston University in Ontario, he and Messing made a car trip to visit Alex Jameson, an Indian activist living on a reservation near Buffalo, New York. Grothendieck was pursuing a quixotic hope of helping the Indians resolve a dispute over a land treaty.

In the summer of 1973 Messing was living in a small studio in the Ormaille, the housing complex for IHÉS visitors. Excitement was bubbling among the mathematicians over Deligne's breakthrough. "Grothendieck was with Justine," Messing recalled. "They came for dinner, and Katz and I spent the evening explaining to Grothendieck the main new and different things in Deligne's proof of the last of the Weil conjectures. He was pretty excited." At the same time, Grothendieck expressed disappointment that the proof bypassed the question of whether or not the standard conjectures were true. "I think he certainly would have been very happy to have proven [all the Weil conjectures] himself," Katz remarked. "But in his mind, the Weil conjectures were important because they were the tip of the iceberg reflecting some fundamental structures in mathematics that he wanted to discover and develop." A proof of the standard conjectures would reveal that structure in a much deeper way.

Later during that visit Grothendieck also met with Deligne to discuss the proof. Deligne recalled that Grothendieck was not as interested in the proof as he would have been had it used the theory of motives. "If I had done it using motives, he would have been very interested, because it would have meant the theory of motives had been developed," Deligne remarked. "Since the proof used a trick, he did not care." In trying to develop the theory of motives, Grothendieck had run into a major technical difficulty. "The most serious problem was that, for his idea of motives to work, one had to be able to construct enough algebraic cycles," Deligne explained. "I think he tried very hard and

he failed. And since then nobody has been able to succeed." According to Deligne, this technical obstacle to developing the theory of motives was probably far more frustrating to Grothendieck than his inability to prove the last Weil conjecture.

A Distant Voice

> [J]'ai quitté "le grand monde" mathématique en 1970.…Après quelques années de militantisme anti-militariste et écologique, style "révolution culturelle", dont tu as sans doute eu quelque écho ici et là, je disparais pratiquement de la circulation, perdu dans une université de province Dieu sait où. La rumeur dit que je passe mon temps à garder des moutons et à forer des puits. La vérité est qu'à part beaucoup d'autres occupations, j'allais bravement, comme tout le monde, faire mes cours à la Fac (c'était là mon peu original gagne-pain, et ça l'est encore aujourd'hui).
>
> I left "the great world" of mathematics in 1970.…After several years of anti-military and ecological militancy, "cultural revolution"-style, of which you have no doubt heard an echo here and there, I just about disappeared from circulation, lost in a university in some province, God knows where. Rumor has it that I pass my time tending sheep and drilling wells. The truth is that apart from many other occupations, I bravely went, like everyone else, to teach my courses in the Department (this was the way I originally earned my bread, and it's the same today).
>
> —*Récoltes et Semailles*, page L3

When Grothendieck came to the Université de Montpellier in 1973, Yves Ladegaillerie, then twenty-five years old, was a *maître des conferences* there, having finished his doctorate at the Institut Henri Poincaré in Paris three years earlier. Grothendieck proposed that Ladegaillerie do a *thèse d'état* with him in topology and spent a great deal of time initiating the younger mathematician into his vision and methods. In a brief memoir about Grothendieck, Ladegaillerie wrote: "I had had as professors in Paris some of the great mathematicians of the day, from Schwartz to Cartan, but Grothendieck was completely different, an extra-terrestrial. Rather than translating things into another language, he thought and spoke directly in the language of modern structural mathematics, to whose creation he had contributed greatly" [Ladegaillerie]. Once, in order to verify a certain algebraic computation involving braids, Ladegaillerie made a little model using some string and a small plank with holes. This made Grothendieck laugh out of sheer delight: "At that moment, he was like a child before a wizard who performed a trick, and he told me: 'I would never have thought of doing that'."

Grothendieck lived an ascetic, unconventional life in an old house without electricity in Villecun, about thirty-five miles outside of Montpellier. Ladegaillerie remembered seeing Justine Bumby and her baby there, though she soon was gone. Many friends, acquaintances, and students went to visit Grothendieck, including people from the ecology movement. In 1974 the leader of a group of Buddhist missionaries from Japan came to visit Grothendieck, and after that many other adherents of Buddhism passed through his home (*R&S*, page 759). Once, after being host to a Buddhist monk whose travel documents were not in order, Grothendieck became the first person in France ever to be charged under an obscure 1949 law against "gratuitously lodging and feeding a stranger in an irregular situation" (*R&S*, page 53). As someone who had been stateless all his life, Grothendieck was outraged at the charge and tried to launch a campaign against it. He even traveled to Paris to speak about it at a Bourbaki seminar. His campaign made headlines in French national newspapers. Ultimately he paid a fine and received a suspended sentence.

It was around this time that Grothendieck learned to drive. He had an ancient Citroën of a model called 2CV and known informally as a *deux chevaux*. One of his students, Jean Malgoire, now a *maître des conferences* at Montpellier, recalled a terrifying journey through a torrential rainstorm with Grothendieck at the wheel. In addition to being a poor driver, Grothendieck was far more occupied with the discourse he was presenting to his passengers than with the condition of the road. "I was sure we would never get there alive!" Malgoire said. "I understood then that Alexandre had a very special relationship with reality.… Rather than adapting to what was real, he believed that reality would adapt itself to him." One time, while driving a moped, Grothendieck collided head-on with an automobile. According to Ladegaillerie, he had turned his eyes from the road to get an apricot out of a bag that was behind him. Although he had a leg fracture serious enough to require surgery, he requested acupuncture as the only anesthetic. He agreed to take antibiotics only when the surgeon told him that the alternative was to amputate the broken leg.

At the Université de Montpellier, Grothendieck had a regular faculty position and taught at all levels. Although the students were not as strong as the ones he had had in Paris, he nevertheless poured a great deal of energy, enthusiasm, and patience into

his teaching. He had an unconventional teaching style. For an examination involving polyhedra, he had students submit paper-and-glue models, much to the dismay of those who had to shepherd the exam papers through the grading process. One person who took undergraduate courses from him at Montpellier is Susan Holmes, now a statistician at Stanford University. "I found him very inspiring, as he was both unconventional and kind to the students, who really didn't understand at all that he was a great mathematician," she recalled. He showed up in the worn-out attire of a hippie and distributed his homegrown organic apples in class. "He definitely did not explain in a linear fashion suited to undergraduates, but his teaching was very inspiring, and one got the impression of some wonderful mysterious 'big picture'," Holmes said.

Grothendieck was never one for reading as a way to learn about and understand mathematics. Talking to others had always been his primary way of finding out what was going on in the field. His departure from the intense, stimulating atmosphere of the IHÉS, where oral exchanges were his primary mode of communication about mathematics, was an enormous change for him. Compared with the pace he kept during the 1960s, Grothendieck's later mathematical work was sporadic. Although he had several Ph.D. students at Montpellier, he did not establish anything like the thriving school he had headed at the IHÉS. Some of Grothendieck's former students and colleagues from his Paris days traveled to Montpellier to visit him. The most frequent of these visitors was Deligne, who during the 1970s was the main person keeping Grothendieck aware of new developments.

At Montpellier, Grothendieck did not have a seminar that met consistently. He formed a small working group with Ladegaillerie, Malgoire, and some of his other students, but according to Ladegaillerie it never really got off the ground. During 1980–81, he ran a seminar, whose sole attendee was Malgoire, on relations between Galois groups and fundamental groups. This is the subject of his 1,300-page manuscript *La Longue Marche à Travers la Théorie de Galois* (*The Long March through Galois Theory*), completed in 1981. Grothendieck did not publish *La Longue Marche*, but through Malgoire's efforts part of it was published in 1995 by the Université de Montpellier [Marche]. There was also a small working seminar in which Ladegaillerie gave some lectures on William Thurston's work on Teichmüller spaces, which stimulated Grothendieck's interest in this subject.

By the 1980s Grothendieck felt he had done all he could in trying to motivate the less-than-enthusiastic students at Montpellier and decided to apply for a position as a researcher in the Centre National de la Recherche Scientifique (CNRS). The CNRS, an agency of the French government, employs mathematicians and scientists to do research. Based at universities or research institutions, CNRS positions usually entail no teaching. In the 1950s, before he went to the IHÉS, Grothendieck had held a CNRS position. In the 1970s he applied to reenter the CNRS but was turned down. At that time, Michel Raynaud of the Université de Paris-Orsay was on the committee of mathematicians that reviewed CNRS applications. Raynaud said the CNRS administration had been hesitant to take Grothendieck on, arguing that it was unclear whether he would continue doing mathematics. The committee could not contradict this argument, and the application was turned down.

When Grothendieck reapplied to the CNRS in 1984, his application was once again controversial. Jean-Pierre Bourguignon, now director of the IHÉS, chaired the committee in charge of reviewing applications in mathematics, among which was Grothendieck's. According to Bourguignon, in the handwritten letter required for the application, Grothendieck listed several tasks he would not perform, such as supervising research students. Because CNRS contracts obligate researchers to perform some of these tasks, this letter was viewed by the CNRS administration as proof of Grothendieck's ineligibility. Bourguignon said he tried to get Grothendieck to amend his application so that it did not state explicitly all the tasks he refused to carry out, but Grothendieck would not budge. After considerable effort on the part of several people, Grothendieck was eventually put on a special kind of position, called a *position asterisquée*, that was acceptable to him and to the CNRS. The CNRS did not actually hire him but was in charge only of paying his salary, and he retained his university affiliation. So for his last few years at Montpellier before his retirement in 1988, Grothendieck did not teach and spent less and less time at the university.

The mathematical part of Grothendieck's 1984 application to the CNRS was the now-famous manuscript *Esquisse d'un Programme*. In it he outlines, in a somewhat mysterious but nevertheless penetrating and visionary fashion, a new area that he called "anabelian algebraic geometry". He also muses on the inadequacy of general topology and presents ideas for a renewal in the form of what he called "tame topology". The *Esquisse* also contains his ideas about *dessins d'enfants*, which he originally developed in order to have a simple way of explaining to students some notions in algebraic geometry and which have since spawned a good deal of research. Grothendieck sent the *Esquisse* to mathematicians who he thought might take an interest in it, and the manuscript circulated unpublished for several years.

Leila Schneps of the Université de Paris VI read the *Esquisse* in 1991. Before that she had identified

Grothendieck with the foundational works of *EGA* and *SGA*, and she found that the *Esquisse* was completely different. "It was a wild expression of mathematical imagination," she recalled. "I loved it. I was bowled over, and I wanted to start working on it right away." She became an enthusiastic evangelist for the research program described in the *Esquisse*, and she and others have made a good deal of progress on it. She said, "Some of it doesn't even seem to make sense at first, but then you work for two years, and you go back and look at it, and you say, 'He *knew* this'." She edited a book on *dessins d'enfants*, which appeared in 1994 [Schneps1], and in 1995 she and Pierre Lochak, also of the Université de Paris VI, organized a conference around the *Esquisse*. The *Esquisse* appeared for the first time in print in the proceedings of that conference [Schneps2].

Aside from the *Esquisse* and *La Longue Marche*, Grothendieck wrote at least one other mathematical work during the 1980s. *À la Poursuite des Champs* (*Pursuing Stacks*), which runs 1,500 pages, began as a letter to Daniel Quillen of the University of Oxford. Completed in 1983, it sketches Grothendieck's vision of a synthesis of homotopical algebra, homological algebra, and topos theory. *À la Poursuite des Champs* circulated widely among mathematicians but was never published. Although its topic is mathematics, the style of *À la Poursuite des Champs* is completely different from the style of his earlier mathematical writings. It was written as a sort of "log book" on a mathematical voyage of discovery, which includes all the false starts, wrong turns, and sudden inspirations that characterize mathematical discovery but that are typically omitted from written mathematical works. When nonmathematical matters drew his attention, they become part of the log book too: for example, *À la Poursuite des Champs* contains a digression about the birth of one of his grandchildren. During the 1990s he wrote a 2,000-page mathematical work on the foundations of homotopy theory called *Les Dérivateurs*, which he gave to Malgoire in 1995 and which is now being made available on the Web [Deriv].

While he was at Montpellier, Grothendieck's uncompromising, "anti-establishment" bent seems to have become more pronounced. After Ladegaillerie's thesis was finished, Grothendieck wrote to Springer-Verlag to suggest that it be published in the Lecture Notes series. He was outraged when he received the reply that the series no longer published theses. The thesis was submitted for publication anyway, with the predictable result that it was rejected. According to Ladegaillerie, Grothendieck wrote letters about this to colleagues, in an effort to build a campaign against Springer. Ladegaillerie decided to publish his thesis in the form of several papers rather than as a whole, and the main part appeared in *Topology*. Grothendieck reproached him for having cut the work into publishable pieces. As Ladegaillerie put it, Grothendieck tried to enlist him in his "fight against the establishment," but Ladegaillerie resisted, believing that such a fight was unreasonable and unjustified.

"Despite such disagreements, we have stayed friends, with highs and lows," Ladegaillerie said. Of his work with Grothendieck, Ladegaillerie said, "It was fascinating to work with a genius. I don't like this word, but for Grothendieck there is no other word possible.…It was fascinating, but it was also frightening, because the man was not ordinary." Memories of working on mathematics with Grothendieck long into the night, by the light of a kerosene lamp, are "the greatest memories of my life as a mathematician."

Reaping and Sowing

> Il y a beaucoup de choses dans *Récoltes et Semailles*, et les uns et les autres y verront sans doute beaucoup de choses différentes: un *voyage* à la découverte d'un passé; une *méditation* sur l'existence; un *tableau de moeurs* d'un milieu et d'une époque (ou le tableau du glissement insidieux et implacable d'une époque à une autre…); une *enquête* (quasiment policière par moments, et en d'autres frisant le roman de cape et d'épée dans les bas-fonds de la mégapolis mathématique…); une vaste *divagation mathématique* (qui sèmera plus d'un…); un traité pratique de psychanalyse appliquée (ou, au choix, un livre de *"psychanalyse-fiction"*); une panégyrique de la *connaissance de soi*; "Mes *confessions*"; un *journal* intime; une psychologie de la *découverte et de la création*; un *réquisitoire* (impitoyable, comme il se doit…), voire un *règlement de comptes* dans "le beau monde mathématique" (et sans faire de cadeaux).
>
> There are many things in *Récoltes et Semailles*, and different people will no doubt see in it many different things: a *voyage* to the discovery of a past; a *meditation* on existence; a *portrait of the morals* of a milieu and of an era (or the portrait of an insidious and relentless sliding of one era into another…); an *inquest* (almost detective-style at times, and at others bordering on cloak-and-dagger fiction set in the underbelly of the mathematical megapolis); a vast *mathematical ramble* (which will leave more than one reader in the dust…); a practical treatise on applied psychology (or, if you like, a book of

"psychoanalytic-fiction"); a panegyric on *self-knowledge*; "My *confessions*"; a private *diary*; a psychology of *discovery and creation*; an indictment (pitiless, as is fitting), even a *settling of scores* in "the world of elite mathematics" (and without any gifts).

—*Récoltes et Semailles*, page L2

Between June 1983 and February 1986, Grothendieck wrote *Récoltes et Semailles: Réflexions et témoignage sur un passé de mathématicien* (*Reapings and Sowings: Reflections and testimony about the past of a mathematician*). It is a work that defies categorization. The title suggests a memoir, but *Récoltes et Semailles* is something more and less than a memoir. It is more, in that it contains not only memories of events in his life but also analyses, often quite deep and minute, of the moral and psychological significance of those events and his attempts to reconcile their meaning with his view of himself and the world. These analyses lead him into philosophical musings about the role of discovery and creativity in mathematics and in life more generally. At the same time, *Récoltes et Semailles* is something less than a memoir, in that it does not attempt a systematic and comprehensive account of events in Grothendieck's life. He is not writing for future biographers or historians, but primarily for himself. *Récoltes et Semailles* is a probing examination of matters closest to his heart. He brings to this work the searching curiosity, the same drive to get to the very bottom of things, that he brought to his mathematics. The result is a dense, multilayered work that reveals a great and sometimes terrifying mind carrying out the difficult work of trying to understand itself and the world.

Needless to say, *Récoltes et Semailles* is not an easy read, and Grothendieck makes a lot of demands on his readers. Much of it has a quotidian feel, and in some parts he is obviously setting down his thoughts as they evolve from one day to the next. As a result, within the space of a page there can be sudden and sometimes disconcerting changes in mood and topic. The organization is complex. The main text is divided into numbered sections, each with its own carefully chosen and evocative title. Within each section there are cross-references to other sections, as well as numerous footnotes, some quite long and substantial, and sometimes even footnotes to the footnotes. The wide-ranging vocabulary presents special challenges for those whose native language is not French, as does his penchant for using colloquialisms, some of them rather vulgar. Through it all Grothendieck writes with great care, insight, and clarity, in a pungent and arresting style. He often succeeds at describing things that at first glance would seem quite ineffable.

One of the reasons for the complexity of the structure of *Récoltes et Semailles*, and for its spontaneity, is that Grothendieck wrote it without a definite plan in mind. He started writing it as an introduction to *À la Poursuite des Champs*, which was to mark his return to making a serious investment of time and energy in doing and publishing mathematics. The introduction was intended to explain the new spirit of his research, which would not focus on the precise and exhaustive foundation-building of his earlier work, but would take readers on a "voyage of discovery" of new mathematical worlds. Grothendieck envisioned *Récoltes et Semailles* as the first volume of a series called *Réflexions*, which would contain his thoughts and reflections on things mathematical and otherwise. The second volume was to have been *À la Poursuite des Champs*, and *La Longue Marche à Travers la Théorie de Galois* and *Esquisse d'un Programme* were also to have been included.

In the first part of *Récoltes et Semailles*, which he called "Fatuité et Renouvellement" ("Complacency and Renewal"), Grothendieck does a lot of soul-searching about the mathematical community in which he worked. The welcoming atmosphere he encountered upon joining that community as a newcomer in 1948 began to disappear, he says, as mathematicians came to use their reputations to set themselves in a superior position. Mathematics became a way to gain power, and the elite mathematicians of the day became smug, feared figures who used that power to discourage and disdain when it served their interests. He ruefully recounts some instances in which he himself displayed attitudes of conceit and haughtiness and realizes that these attitudes had coalesced into a "sportive" or competitive approach to mathematics that had begun to hamper his ability to open himself to the beauty of mathematical things.

It was after writing "Fatuité et Renouvellement" that he was suddenly struck by "the insidious reality of a *Burial* of my oeuvre and at the same time of my person, which suddenly imposed itself on me, with an irresistible force and with this very name, 'The Burial', on [April 19, 1984]." (*R&S*, page L8). On that date he began writing what eventually became a three-part series called "L'Enterrement" ("The Burial"), comprising more than one thousand pages. In it he strongly attacks some of his former students and colleagues, whom he believes tried to "bury" his work and his style of mathematics by pilfering his ideas and not according proper credit to him. He also champions the work of Zoghman Mebkhout, who during the 1970s developed some of Grothendieck's ideas and whose work Grothendieck believes was unfairly marginalized and ignored. "L'Enterrement" presents six mathematical areas, or "construction sites" (*"chantiers"*), that he says were abandoned when

he left the IHÉS in 1970 and that he believes his students should have developed. Throughout "L'Enterrement" he closely analyzes his relationship with Pierre Deligne, the most brilliant of all of his students and the one with whom he had the closest mathematical affinity.

"L'Enterrement (II) ou La Clef du Yin et du Yang" ("The Burial (II) or the Key to Yin and Yang") is rather different from the other two parts of "L'Enterrement" in being less directly concerned with the investigation of the "burial". This second part, which Grothendieck notes is the most personal and deepest part of *Récoltes et Semailles*, constitutes a wide-ranging meditation on diverse themes such as creativity, intuition, violence, conflict, and the self. He uses the "yin-yang" dialectic to analyze different styles of doing mathematics, concluding that his own style is fundamentally "yin", or feminine. This style is captured in one especially evocative section called "La mer qui monte…" ("The rising sea…"). He likens his approach to mathematics to a sea: "The sea advances imperceptibly and without sound, nothing seems to happen and nothing is disturbed, the water is so far off one hardly hears it. But it ends up surrounding the stubborn substance, which little by little becomes a peninsula, then an island, then an islet, which itself is submerged, as if dissolved by the ocean stretching away as far as the eye can see" (*R&S*, page 553).

Grothendieck in a photograph from the 1950s.

In "L'Enterrement" he pursues some of the themes established in "Fatuité et Renouvellement" concerning the competitive, snobbish attitudes of the upper crust of the mathematical world. For example, he notes that much of his work in mathematics was marked by an "attitude of service": service to the mathematical community of writing clear and complete expositions that make fundamental and foundational ideas widely accessible. Although he candidly admits that his own conceit sometimes led him into elitist attitudes, he says that he never lost this spontaneous sense of service, "service to all those who leaped with me into a common adventure" (*R&S*, page 630, (*)). He believes that the mathematical community lost this sense of service as personal aggrandizement and the development of an exclusionary elite became the order of the day. He also decries the devaluation of vision and intuition in favor of technical mastery.

Apart from "Fatuité et Renouvellement" and the three parts that make up "L'Enterrement", *Récoltes et Semailles* has two introductory volumes, as well as an appendix to "La Clef du Yin et du Yang". About two hundred copies were sent out to his mathematical colleagues. Despite Grothendieck's intention to publish it, the original French-language version of *Récoltes et Semailles* has never appeared in print, as the strong attacks it contains could be deemed libelous. Nevertheless, it has circulated widely. Copies can be found on bookshelves in mathematicians' offices all over the world, especially in France, and in some libraries in universities and mathematics institutes. Historian of science Alain Herreman of the Université de Rennes has undertaken an effort to post on the Web html files containing the entire French original, and partial translations into English, Russian, and Spanish have appeared there too [*R&S*]. A Japanese translation of a large portion of *Récoltes et Semailles* was prepared by Yūichi Tsuji, who knew Grothendieck through the Survival group, and was published in the 1990s by Gendaisūgakusha, a mathematics publisher. According to Michel Waldschmidt of the Université de Paris VI, who was president of the Société Mathématique de France (SMF) during 2001–04, the society considered, during his presidency, the question of whether to publish *Récoltes et Semailles*. The question raised strong opinions both for and against, Waldschmidt said, and ultimately the SMF decided against publication.

Many mathematicians, especially some of Grothendieck's former students, were shocked and hurt by the accusations in *Récoltes et Semailles*. One of them, Luc Illusie of the Université de Paris-Orsay, recalled that he talked to another former student, Jean-Louis Verdier, about whether they should try to discuss the accusations with Grothendieck. According to Illusie, Verdier, who died in 1989, felt that Grothendieck's state of mind was such that there was no sound basis for discussion. But, Illusie said, "I thought, 'It is not possible that Grothendieck has become like that. I will try to reason and to discuss with him. Maybe I will agree with him on some points that he is right and on others he is not right.' Eventually, we settled the material points, but nothing really emerged, and he remained convinced that everyone was against him."

In *Récoltes et Semailles* Grothendieck says that, after he left the mathematical world in 1970, his style of doing mathematics was held in contempt and that many of the paths he had broken went undeveloped. It is true that after that time, research in algebraic geometry began to shift, mixing the highly general approach that characterized his work with investigation of specific problems. Deligne's proof of the Weil conjecture, which was

very much in the spirit of Grothendieck but which also incorporated many new ideas, was one of the great advances of the 1970s. Along with developments in the theory of D-modules and Deligne's mixed Hodge theory, greater attention began to be paid to more specific problems, such as the classification theory of varieties and questions about low-dimensional varieties. Also, after the Antwerp meeting of 1972, collaborations grew between algebraic geometry and representation theory, leading to advances in the theory of automorphic forms and the Langlands program. As Illusie put it, all these developments show that there has been "quite a natural balance between general theory and the study of specific examples at great length, to enrich the theory itself."

Récoltes et Semailles also contains the accusation that Grothendieck's work was not always properly credited. Indeed, his work was so well known and fundamental that credit was not always specifically accorded to him. "It is true that everybody knew he had invented motives, for instance, or l-adic cohomology, and so there was no need to quote his name every time one used them," remarked Jean-Pierre Serre. "His name was rarely mentioned because of that. On the other hand, it was well known that it was due to him. Nobody was saying that it was due to someone else." Serre noted that Grothendieck's complaining about lack of credit is in sharp contrast to his behavior during the 1960s, when he shared his ideas with great generosity and in some cases attached other people's names to ideas he himself had come up with. "It was sad to read *Récoltes et Semailles* because of that," Serre said.

Even granting that there was a shift away from Grothendieck's style of mathematics and that credit was not always specifically accorded to him, it is a long leap from there to the deliberate "burial" that he asserts took place. "In retrospect, very few mathematical ideas have been as widely used as Grothendieck's," said Illusie. "Everybody who is doing algebraic or arithmetic geometry now uses Grothendieck's language, ideas, theorems, and so on. So when you think one second, it is completely ridiculous that he suggested that he could have been buried." There is no question that mathematics suffered a great loss when Grothendieck halted his research career in 1970. But mathematics did not stop; others continued to work, following their own ideas and interests. In February 1986, after receiving a copy of *Récoltes et Semailles*, Serre wrote to Grothendieck: "You are surprised and indignant that your former students did not continue the work that you had undertaken and largely completed. But you do not ask the most obvious question, the one every reader expects you to answer: and you, why did you abandon the work in question?" [Corr].

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- Table des matières (pages T1–T10)
- Introduction (March 1984: sections 1–5, pages i–xi)
- Introduction (May–June 1985: sections 6–10, pages xi–xxii)

Première Partie: Fatuité et Renouvellement
(June 1983, February 1984: pages 1–171)

Deuxième Partie: L'Enterrement I, ou la Robe de l'Empereur de Chine
(April–June 1984: pages 173–420)

Troisième Partie: L'Enterrement II, ou la Clef du Yin et du Yang
(September 1984–January 1985: pages 421–774)

Quatrième Partie: L'Enterrement III, ou les Quatre Opérations
(February 1985–June 1985: pages 775–1252)

Les Portes sur l'Univers (Appendice à la Clef du Yin et du Yang)
(March–April 1986: pages PU1–PU127)

Although the accusations of a "burial" have generated a good deal of notoriety, there is much more to *Récoltes et Semailles*. Those who have read beyond those parts have been deeply touched by the work's beauty and insights. Grothendieck's critique of how the highly competitive atmosphere of the mathematical world stifles creativity and renewal of the field resonated with many. In *Récoltes et Semailles* Grothendieck puts the highest value on the innocent, childlike curiosity that gives birth to the creative impulse, and he mourns the way it is trampled on by competitiveness and the desire for power and prestige.

"I am one of quite probably a minority who think that *Récoltes et Semailles* is a miraculous document," said William Messing. "That is not to say that there are not parts that are excessive and have aspects of what might be referred to as paranoia. But it's very striking that the person who created *EGA* and *SGA* would write in such a style. The systematic and soul-searching aspect is of a piece with his approach to mathematics. Those who have really read it—as opposed to looking at five pages of negative comments—tend to think of it as an extraordinary document."

Lightness Descending

> [A]ujourd'hui je ne suis plus, comme naguère, le prisonnier de tâches

> interminables, qui si souvent m'avaient interdit de m'élancer dans l'inconnu, mathématique ou non. Le temps des *tâches* pour moi est révolu. Si l'âge m'a apporté quelque chose, c'est d'être plus léger.
>
> Today I am no longer, as I once was, the prisoner of interminable tasks, which so often prevented me from leaping into the unknown, mathematical or otherwise. The time of *tasks* for me is over. If age has brought me anything, it is lightness.
>
> —*Esquisse d'un Programme*

"[T]he ethics of the scientific profession (especially among mathematicians) have degraded to such a degree that pure and simple theft between colleagues (especially at the expense of those who have no position of power to defend themselves) has almost become the general rule and is in any case tolerated by all, even in the most flagrant and iniquitous cases." So wrote Grothendieck in an April 19, 1988, letter to the Royal Swedish Academy of Sciences in which he declined the 1988 Crafoord Prize. He also sent to the academy the introductory volumes of *Récoltes et Semailles*. The academy had awarded the prize of around US$200,000 to him and Pierre Deligne. Grothendieck's letter became widely known when it was published in *Le Monde* on May 4, 1988 [LeMonde]. To play into the game of accepting prizes and honors, Grothendieck wrote, would be to validate "a spirit and an evolution in the scientific world that I see as profoundly unhealthy, and condemned to disappear soon, so suicidal is it, spiritually as well as intellectually and materially." Evidently these sentiments resonated with many readers of *Le Monde*. One of the newspaper's editors told Jean-Pierre Bourguignon that the paper had received more reactions to Grothendieck's letter than to any other preceding it and that most of the letters registered approval that finally a scientist had recognized how corrupt the scientific milieu had become. News of the letter appeared in other magazines and newspapers, and it was avidly discussed within the mathematical community. An English translation was published in the *Mathematical Intelligencer* [Intell], and a short item appeared in the *Notices* [Notices].

The same year in which he turned down the Crafoord Prize, Grothendieck retired from the Université de Montpellier at the age of sixty. Also that year, six mathematicians decided to assemble a collection of articles as a "Festschrift" on the occasion of Grothendieck's sixtieth birthday [Festschrift] (there was also a special issue of the journal *K-Theory* dedicated to Grothendieck). The Festschrift seems to have been an attempt to make amends with Grothendieck and to show that he had not been "buried", as he asserted in *Récoltes et Semailles*. Some of the people contributing papers were among those he had most heavily criticized. When the Festschrift appeared in 1990, Illusie, who was one of the editors, sent a copy to Grothendieck, whose reaction was extremely bitter. In a letter to Illusie, he objected strongly to the brief foreword of the volume and also to the fact that he had not been told that the volume would appear. He said his work had been used like "confetti," like bright, worthless bits one throws into the air to give the pretense of happiness and celebration while ignoring the malaise underneath. Grothendieck submitted this letter for publication in the *Bulletin de la Société Mathématique de France*. When the SMF told him that the *Bulletin* carries only mathematics articles but that the letter could appear instead in the *SMF Gazette*, Grothendieck refused. The letter was never published.

After he retired, Grothendieck spent little time at the Université de Montpellier, though he continued to live in the area, in a village called Les Aumettes. At this time, Ladegaillerie said, Grothendieck seemed to be going through a deep spiritual crisis and wrote strange letters "that made us fear the worst about his condition." During 1987–88, Grothendieck wrote *La Clef des Songes ou Dialogue avec le Bon Dieu* (*The Key to Dreams or Dialogue with the Good Lord*), which expresses his conviction that God exists and that He speaks to people through their dreams. It also contains a good deal of material about Grothendieck's early life. *La Clef des Songes* runs about three hundred pages and is accompanied by another five hundred pages of notes. According to a lecture given in the summer of 2004 by Winfried Scharlau of the Universität Münster, Grothendieck subsumed *La Clef des Songes* under a collection of works that he called *Méditations* and that included the material making up *Réflexions*, as well as a poetical work called "Eloge de l'Inceste" ("The Eulogy to Incest"). Neither that work nor *La Clef des Songes* was ever widely distributed.

Many of Grothendieck's friends and colleagues became aware of his increasing preoccupation with spiritual matters when they received "La Lettre de la Bonne Nouvelle" ("The Letter of Good News"), which is dated January 26, 1990, and which he sent to about two hundred fifty people. The letter states: "You are part of a group of two to three thousand people, personally known to me, whom God destines for a great mission: That of announcing and preparing the *'New Age'* (or *Age of Liberation…*), which will commence on the *'Day of Truth'*, 14 October 1996." He says that God manifested Himself to him for the first time in 1986 and

communicated to him through dreams. He also describes encounters with a deity named Flora, who imparts revelations but also cruelly tests his faith. Although the content of the letter is baffling, the way it is written is perfectly lucid. Three months later Grothendieck sent a "correction", stating that he was no longer certain of the truth of the revelations described in "La Lettre de la Bonne Nouvelle". He writes: "That I was the victim of a mystification by one of more 'spirits' (among which my limited capacity could not distinguish), invested with prodigious powers over my body and in my psyche, I no longer have the least doubt." Together, the two letters impart an impression of deep disturbance and suffering.

In July 1990 Grothendieck asked Malgoire to take possession of all of his mathematical papers, including books, preprints, correspondence, and manuscripts in various states of preparation. Grothendieck wanted to "lighten" himself of many things, as Malgoire put it. He burned a huge amount of material, most of it nonmathematical, including letters that his parents had exchanged in the 1930s. He showed Malgoire a 200-liter oil drum filled with cinders and estimated he had destroyed a total of 25,000 pages. Grothendieck also left many papers and other items, including his mother's death mask, with a friend named Yolande Levine, to whom he had been very close for the preceding decade. He then disappeared into the Pyrenées to live in complete isolation. A small number of people knew where he was, and he instructed them not to forward any mail that arrived for him at the university. Malgoire said that even today, close to fifteen years after Grothendieck went into seclusion, the university still gets a great deal of correspondence addressed to him. In 1995 Grothendieck formally conferred the legal rights to his mathematical papers to Malgoire.

Grothendieck has had very little contact with mathematicians in the past fifteen years. Among the few who have seen him are Leila Schneps and Pierre Lochak, who met him in the mid-1990s. They told him about the progress made on the program he had outlined in the *Esquisse d'un Programme*, and he was surprised to learn that people were still interested in his work. He had developed a strong interest in physics but expressed frustration with what he felt was a lack of rigor in that field. Lochak and Schneps exchanged some letters with him and also sent him some books on physics that he had asked for. In one letter he asked a disarmingly simple question: What is a meter? His letters began to swing between warm friendliness and cold suspicion, and eventually he severed all contact with them. Although the friendship with Grothendieck could not be sustained, Lochak and Schneps retain a fervent admiration and a deep attachment to the man and his work. Together they painstakingly typed into TEX a large chunk of the handwritten *La Longue Marche à Travers la Théorie de Galois*. They have also started a website, the Grothendieck Circle, which contains a wealth of material about Grothendieck, his life, and his work [Circle].

The Dancing Star

> Ich sage euch: man muß noch Chaos in sich haben, um einen tanzenden Stern gebären zu können. Ich sage euch: ihr habt noch Chaos in euch.
>
> I tell you: one must have chaos inside, to give birth to a dancing star. I tell you: you have yet chaos in you.
>
> —Friedrich Nietzsche, *Also sprach Zarathustra*

The work of Alexandre Grothendieck has had a profound impact on modern mathematics and, more broadly, ranks among the most important advances in human knowledge during the twentieth century. The stature of Grothendieck can be compared to that of, for example, Albert Einstein. Each of them opened revolutionary new perspectives that transformed the terrain of exploration, and each sought fundamental, unifying connections among phenomena. Grothendieck's propensity for investigating how mathematical objects behave relative to one another echoes the relativistic viewpoint proposed by Einstein. Grothendieck's work also has parallels with another great twentieth-century advance, that of quantum mechanics, which turned conventional notions upside down by replacing point particles by "probability clouds". "[T]hese 'probability clouds', replacing the reassuring material particles of before, remind me strangely of the elusive 'open neighborhoods' that populate the toposes, like evanescent phantoms, to surround the imaginary 'points'," he wrote (*R&S*, page P60).

Yet, as extraordinary as Grothendieck's achievements are, he traced his creative capacity to something rather humble: the naive, avid curiosity of a child. "Discovery is the privilege of the child," he wrote in *Récoltes et Semailles* (page 1), "the child who has no fear of being once again wrong, of looking like an idiot, of not being serious, of not doing things like everyone else." For the work of discovery and creation, Grothendieck saw intellectual endowment and technical power as secondary to the child's simple thirst to know and understand. This child is inside each of us, though it may be marginalized, neglected, or drowned out. "Each of us can rediscover what discovery and creation are, and no one can invent them" (*R&S*, page 2).

One aspect of this childlike curiosity is a scrupulous fidelity to truth. Grothendieck taught his students an important discipline when writing about mathematics: never say anything false. Statements that were almost or essentially true were not permitted. It was acceptable to be vague, but when one gives precise details, one must say only things that are true. Indeed, Grothendieck's life has been a constant search for truth. From his mathematical work through *Récoltes et Semailles* and even "La Lettre de la Bonne Nouvelle", Grothendieck wrote with the unblinking honesty of a child. He spoke the truth—*his* truth, as he perceived it. Even when he made factual mistakes or was misled by incorrect assumptions, he presented candidly what was in his mind. He has never tried to hide who he is and what he thinks.

Grothendieck's search for truth took him to the very roots of mathematical ideas and to the far reaches of human psychological perception. He has had a long journey. "In his solitary retirement in the Pyrenées, Alexandre Grothendieck has the right to rest after all he has been through," wrote Yves Ladegaillerie [Ladegaillerie]. "He deserves our admiration and our respect but, above all, in thinking of what we owe him, we must leave him in peace."

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