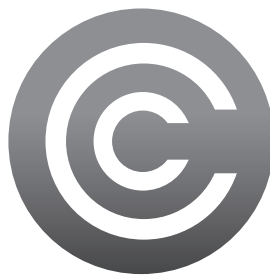


**Common Core Investigations  
Teacher's Guide**

**Grade Seven**



**PEARSON**

Boston, Massachusetts  
Upper Saddle River, New Jersey

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# CMP and the Common Core State Standards

**C**onnected Mathematics (CMP) is a field-tested and research-validated program that focuses on a few big ideas at each grade level. Students explore these ideas in depth, thereby developing deep understanding of key ideas that they carry from one grade to the next. The sequencing of topics within a grade and from grade to grade, the result of lengthy field-testing and validation, helps to ensure the development of students' deep mathematical understanding and strong problem-solving skills. By the end of grade 8, CMP students will have studied all of the content and skills in the Common Core State Standards for Mathematics\* (CCSSM) for middle grades (Grades 6-8). The focus on helping students develop deep mathematical understanding and strong problem solving skills aligns well to the intent of the Common Core State Standards for Mathematics, which articulates 3 to 5 areas of emphasis at each grade level from Kindergarten through Grade 8.

The sequence of content and skills in CMP varies in some instances from that in the CCSSM, so in collaboration with the CMP2 authors, Pearson has created a set of investigations for each grade level to further support and fully develop students' understanding of the content standards of the CCSSM. The authors are confident that the CMP2 curriculum supplemented with the additional investigations at each grade level will address all of the content and skills of the CCSSM, but even more, will contribute significantly to advancing students' mathematical proficiency as described in the Standards for Mathematical Practices of the CCSSM. Through the in-depth exploration of concepts, students become confident in solving a variety of problems with flexibility, skill, and insightfulness, and are able to communicate their reasoning and understanding in a variety of ways.

In this supplement, you will find support for all of the Common Core (CC) Investigations.

- The At-A-Glance page includes Teaching Notes and answers to all problems and exercises for the CC Investigation.
- The Additional Practice and Skill Practice pages can be reproduced for your students. These offer opportunities for students to reinforce the core concepts of the CC Investigation.
- Use the Check-Up to assess your students' understanding of the concepts presented in the investigation.
- The answers for all of the ancillary pages are found at the back of this book.
- The reduced student pages are provided for your convenience as you read through the teaching support and plan for implementing each investigation.

In the Pacing Guide (pp. xii–xiii), we propose placement for teaching each CC Investigations. CC Investigations 1–3 build on the concepts of linear relationships, so they should be taught after *Moving Straight Ahead*. CC Investigation 4 involves geometric concepts, so it should be used after the unit *Filling and Wrapping*. CC Investigation 5 on Sampling couples well with *Data Distributions* Investigation 1.

# Common Core State Standards Correlations

The following alignment of the Common Core State Standards for Mathematics to Pearson's *Connected Mathematics 2* (CMP2) ©2009 program includes the supplemental investigations that complete the CMP2 program.

COMMON CORE STATE STANDARDS GRADE 7	CMP2 UNITS	CONTENT
<b>Ratios and Proportional Relationships</b>		
<b>Analyze proportional relationships and use them to solve real-world and mathematical problems.</b>		
<b>7.RP.1</b> Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.	<i>CC Investigations</i>	<b>CC Inv. 1:</b> Graphing Proportions
<b>7.RP.2</b> Recognize and represent proportional relationships between quantities.	<i>Comparing and Scaling</i>	<b>Inv. 1:</b> Making Comparisons <b>Inv. 2:</b> Comparing Ratios, Percents, and Fractions <b>Inv. 3:</b> Comparing and Scaling Rates <b>Inv. 4:</b> Making Sense of Proportions
<b>7.RP.2.a</b> Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	<i>Comparing and Scaling</i>  <i>CC Investigations</i>	<b>Inv. 4:</b> Making Sense of Proportions  <b>CC Inv. 1:</b> Graphing Proportions
<b>7.RP.2.b</b> Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.	<i>Comparing and Scaling</i>          <i>Moving Straight Ahead</i>	<b>Inv. 3:</b> Comparing and Scaling Rates <b>Inv. 4:</b> Making Sense of Proportions  <b>Inv. 1:</b> Walking Rates <b>Inv. 2:</b> Exploring Linear Functions With Graphs and Tables <b>Inv. 3:</b> Solving Equations <b>Inv. 4:</b> Exploring Slope

COMMON CORE STATE STANDARDS GRADE 7	CMP2 UNITS	CONTENT
<p><b>7.RP.2.c</b> Represent proportional relationships by equations.</p>	<p><i>Variables and Patterns</i></p> <p><i>Comparing and Scaling</i></p> <p><i>Stretching and Shrinking</i></p> <p><i>Moving Straight Ahead</i></p>	<p><b>Inv. 1:</b> Variables, Tables, and Coordinate Graphs  <b>Inv. 2:</b> Analyzing Graphs and Tables  <b>Inv. 3:</b> Rules and Equations  <b>Inv. 4:</b> ACE 13</p> <p><b>Inv. 4:</b> Making Sense of Proportions</p> <p><b>Inv. 4:</b> Similarity and Ratios  <b>Inv. 5:</b> Using Similar Triangles and Rectangles</p> <p><b>Inv. 1:</b> Walking Rates  <b>Inv. 2:</b> Exploring Linear Functions With Graphs and Tables  <b>Inv. 3:</b> Solving Equations  <b>Inv. 4:</b> Exploring Slope</p>
<p><b>7.RP.2.d</b> Explain what a point <math>(x, y)</math> on the graph of a proportional relationship means in terms of the situation, with special attention to the points <math>(0, 0)</math> and <math>(1, r)</math> where <math>r</math> is the unit rate.</p>	<p><i>Variables and Patterns</i></p> <p><i>Comparing and Scaling</i></p> <p><i>Moving Straight Ahead</i></p> <p><i>CC Investigations</i></p>	<p><b>Inv. 2:</b> Analyzing Graphs and Tables</p> <p><b>Inv. 3:</b> ACE 9, 10</p> <p><b>Inv. 1:</b> Walking Rates  <b>Inv. 2:</b> Exploring Linear Functions With Graphs and Tables  <b>Inv. 3:</b> Solving Equations  <b>Inv. 4:</b> Exploring Slope</p> <p><b>CC Inv. 1:</b> Graphing Proportions</p>
<p><b>7.RP.3</b> Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.</p>	<p><i>Variables and Patterns</i></p> <p><i>Comparing and Scaling</i></p> <p><i>Stretching and Shrinking</i></p>	<p><b>Inv. 4:</b> ACE 12</p> <p><b>Inv. 1:</b> Making Comparisons  <b>Inv. 2:</b> Comparing Ratios, Percents, and Fractions  <b>Inv. 3:</b> Comparing and Scaling Rates  <b>Inv. 4:</b> Making Sense of Proportions</p> <p><b>Inv. 4:</b> Similarity and Ratios  <b>Inv. 5:</b> Using Similar Triangles and Rectangles</p>

COMMON CORE STATE STANDARDS GRADE 7	CMP2 UNITS	CONTENT
<b>The Number System</b>		
<b>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.</b>		
<b>7.NS.1</b> Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	<i>Accentuate the Negative</i>	<b>Inv. 2:</b> Adding and Subtracting Integers <b>Inv. 4:</b> Properties of Operations
<b>7.NS.1.a</b> Describe situations in which opposite quantities combine to make 0.	<i>Accentuate the Negative</i>	<b>Inv. 2:</b> Adding and Subtracting Integers
<b>7.NS.1.b</b> Understand $p + q$ as the number located a distance $ q $ from $p$ , in the positive or negative direction depending on whether $q$ is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.	<i>Accentuate the Negative</i>	<b>Inv. 1:</b> Extending the Number System <b>Inv. 2:</b> Adding and Subtracting Integers
<b>7.NS.1.c</b> Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$ . Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.	<i>Accentuate the Negative</i>	<b>Inv. 2:</b> Adding and Subtracting Integers
<b>7.NS.1.d</b> Apply properties of operations as strategies to add and subtract rational numbers.	<i>Accentuate the Negative</i>	<b>Inv. 2:</b> Adding and Subtracting Integers <b>Inv. 4:</b> Properties of Operations
<b>7.NS.2</b> Fluently divide multi-digit numbers using the standard algorithm.	<i>Accentuate the Negative</i>	<b>Inv. 3:</b> Multiplying and Dividing Integers <b>Inv. 4:</b> Properties of Operations
<b>7.NS.2.a</b> Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.	<i>Accentuate the Negative</i>	<b>Inv. 3:</b> Multiplying and Dividing Integers <b>Inv. 4:</b> Properties of Operations
<b>7.NS.2.b</b> Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$ and $q$ are integers, then $-(p/q) = (-p)/q = p/(-q)$ . Interpret quotients of rational numbers by describing real-world contexts.	<i>Accentuate the Negative</i>	<b>Inv. 3:</b> Multiplying and Dividing Integers
<b>7.NS.2.c</b> Apply properties of operations as strategies to multiply and divide rational numbers.	<i>Accentuate the Negative</i>	<b>Inv. 3:</b> Multiplying and Dividing Integers <b>Inv. 4:</b> Properties of Operations
<b>7.NS.2.d</b> Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats	<i>Comparing and Scaling</i>  <i>Accentuate the Negative</i>	<b>Inv. 3:</b> Comparing and Scaling Rates  <b>Inv. 3:</b> Multiplying and Dividing Integers
<b>7.NS.3</b> Solve real-world and mathematical problems involving the four operations with rational numbers. NOTE: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.	<i>Accentuate the Negative</i>	<b>Inv. 2:</b> Adding and Subtracting Integers <b>Inv. 3:</b> Multiplying and Dividing Integers <b>Inv. 4:</b> Properties of Operations

COMMON CORE STATE STANDARDS GRADE 7	CMP2 UNITS	CONTENT
<b>Expressions and Equations</b>		
<b>Use properties of operations to generate equivalent expressions.</b>		
<b>7.EE.1</b> Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	<i>Moving Straight Ahead</i>  <i>CC Investigations</i>	<b>Inv. 3:</b> Solving Equations <b>Inv. 4:</b> Exploring Slope  <b>CC Inv. 2:</b> Equivalent Expressions
<b>7.EE.2</b> Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.	<i>CC Investigations</i>	<b>CC Inv. 2:</b> Equivalent Expressions
<b>7.EE.3</b> Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.	<i>Variables and Patterns</i>  <i>Accentuate the Negative</i>  <i>Moving Straight Ahead</i>	<b>Inv. 2:</b> Analyzing Graphs and Tables <b>Inv. 3:</b> Rules and Equations <b>Inv. 4:</b> Calculator Tables and Graphs  <b>Inv. 1:</b> Extending the Number System <b>Inv. 2:</b> Adding and Subtracting Integers <b>Inv. 3:</b> Multiplying and Dividing Integers <b>Inv. 4:</b> Properties of Operations  <b>Inv. 1:</b> Walking Rates <b>Inv. 2:</b> Exploring Linear Functions With Graphs and Tables <b>Inv. 3:</b> Solving Equations <b>Inv. 4:</b> Exploring Slope
<b>7.EE.4</b> Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.	<i>Variables and Patterns</i>  <i>Moving Straight Ahead</i>	<b>Inv. 1:</b> Variables, Tables, and Coordinate Graphs <b>Inv. 2:</b> Analyzing Graphs and Tables <b>Inv. 3:</b> Rules and Equations  <b>Inv. 1:</b> Walking Rates <b>Inv. 2:</b> Exploring Linear Functions With Graphs and Tables <b>Inv. 3:</b> Solving Equations <b>Inv. 4:</b> Exploring Slope
<b>7.EE.4.a</b> Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$ , where $p$ , $q$ , and $r$ are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.	<i>Variables and Patterns</i>  <i>Moving Straight Ahead</i>	<b>Inv. 1:</b> Variables, Tables, and Coordinate Graphs <b>Inv. 2:</b> Analyzing Graphs and Tables <b>Inv. 3:</b> Rules and Equations  <b>Inv. 1:</b> Walking Rates <b>Inv. 2:</b> Exploring Linear Functions With Graphs and Tables <b>Inv. 3:</b> Solving Equations <b>Inv. 4:</b> Exploring Slope
<b>7.EE.4.b</b> Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$ , where $p$ , $q$ , and $r$ are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.	<i>Moving Straight Ahead</i>  <i>CC Investigations</i>	<b>Inv. 2:</b> ACE 44  <b>CC Inv. 3:</b> Inequalities





COMMON CORE STATE STANDARDS GRADE 7	CMP2 UNITS	CONTENT
<b>Statistics and Probability</b>		
<b>Use random sampling to draw inferences about a population.</b>		
<b>7.SP.1</b> Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	<i>CC Investigations</i>	<b>CC Inv. 5: Variability</b>
<b>7.SP.2</b> Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.	<i>CC Investigations</i>	<b>CC Inv. 5: Variability</b>
<b>Draw informal comparative inferences about two populations.</b>		
<b>7.SP.3</b> Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.	<i>Data Distributions</i>  <i>CC Investigations</i>	<b>Inv. 2: Making Sense of Measures of Center</b>  <b>CC Inv. 5: Variability</b>
<b>7.SP.4</b> Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.	<i>Data Distributions</i>	<b>Inv. 3: Comparing Distributions: Equal Numbers of Data Values</b> <b>Inv. 4: Comparing Distributions: Unequal Numbers of Data Values</b>
<b>Investigate chance processes and develop, use, and evaluate probability models.</b>		
<b>7.SP.5</b> Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	<i>What Do You Expect?</i>	<b>Inv. 1: ACE 14</b>
<b>7.SP.6</b> Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.	<i>What Do You Expect?</i>	<b>Inv. 1: Evaluating Games of Chance</b> <b>Inv. 2: Analyzing Situations Using an Area Model</b> <b>Inv. 3: Expected Value</b> <b>Inv. 4: Binomial Outcomes</b>
<b>7.SP.7</b> Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.	<i>What Do You Expect?</i>	<b>Inv. 1: Evaluating Games of Chance</b> <b>Inv. 2: Analyzing Situations Using an Area Model</b> <b>Inv. 3: Expected Value</b> <b>Inv. 4: Binomial Outcomes</b>
<b>7.SP.7.a</b> Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.	<i>What Do You Expect?</i>	<b>Inv. 1: Evaluating Games of Chance</b> <b>Inv. 2: Analyzing Situations Using an Area Model</b> <b>Inv. 3: Expected Value</b> <b>Inv. 4: Binomial Outcomes</b>
<b>7.SP.7.b</b> Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.	<i>What Do You Expect?</i>	<b>Inv. 1: Evaluating Games of Chance</b> <b>Inv. 2: Analyzing Situations Using an Area Model</b> <b>Inv. 3: Expected Value</b> <b>Inv. 4: Binomial Outcomes</b>

COMMON CORE STATE STANDARDS GRADE 7	CMP2 UNITS	CONTENT
<p><b>7.SP.8</b> Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.</p>	<p><i>What Do You Expect?</i></p>	<p><b>Inv. 1:</b> Evaluating Games of Chance  <b>Inv. 2:</b> Analyzing Situations Using an Area Model  <b>Inv. 3:</b> Expected Value  <b>Inv. 4:</b> Binomial Outcomes</p>
<p><b>7.SP.8.a</b> Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.</p>	<p><i>What Do You Expect?</i></p>	<p><b>Inv. 1:</b> Evaluating Games of Chance  <b>Inv. 2:</b> Analyzing Situations Using an Area Model  <b>Inv. 3:</b> Expected Value  <b>Inv. 4:</b> Binomial Outcomes</p>
<p><b>7.SP.8.b</b> Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.</p>	<p><i>What Do You Expect?</i></p>	<p><b>Inv. 1:</b> Evaluating Games of Chance  <b>Inv. 2:</b> Analyzing Situations Using an Area Model  <b>Inv. 3:</b> Expected Value  <b>Inv. 4:</b> Binomial Outcomes</p>
<p><b>7.SP.8.c</b> Design and use a simulation to generate frequencies for compound events.</p>	<p><i>What Do You Expect?</i></p>	<p><b>Inv. 1:</b> Evaluating Games of Chance  <b>Inv. 2:</b> Analyzing Situations Using an Area Model  <b>Inv. 3:</b> Expected Value  <b>Inv. 4:</b> Binomial Outcomes</p>

# Common Core State Standards Pacing Guide

This Pacing Guide offers suggestions as you look to implement the Grade 7 Common Core State Standards for Mathematics in the CMP2 classroom. The Chart shows placement recommendations for the Common Core Investigations provided in this supplement.

Investigations labeled as Review (R) offer timely practice of concepts from earlier grades, helping to activate students' prior knowledge as they are introduced to new concepts that build on these concepts. Investigations labeled as Extending (\*) offer students the opportunity to explore concepts in greater depth or to extend their study of concepts.

The suggested number of standard days for each unit is based on a 45-minute class period; a block period is assumed to be 90 minutes of instructional time. Common Core students entering Grade 7 were introduced to ratios and rates, expressions and equations, integers, and volumes and nets of solids in Grade 6. Because of these prior understandings, students will begin the units *Variables and Patterns*, *Stretching and Shrinking*, *Comparing and Scaling*, and *Filling and Wrapping* with tools needed to complete these units at an accelerated pace. This will leave time in the year to cover the CC Investigations needed for Grade 7.

Variables and Patterns		Standard 8 days • Block 4 days
Inv. 1 Variables, Tables, and Coordinate Graphs	Reviews 6.NS.6, 6.EE.9	R
Inv. 2 Analyzing Graphs and Tables	Reviews 6.NS.6, 6.EE.9	R
Inv. 3 Rules and Equations	7.EE.4, 7.EE.4.a	✓
Inv. 4 Calculator Tables and Graphs	Extends 7.EE.4, 7.EE.4.a	*
Stretching and Shrinking		Standard 22 days • Block 11 days
Inv. 1 Enlarging and Reducing Shapes	7.G.1, 7.G.4 (ACE 10, 13)	✓
Inv. 2 Similar Figures	7.G.1, 7.G.6	✓
Inv. 3 Similar Polygons	7.G.1, 7.G.5 (ACE 22–24), 7.G.6	✓
Inv. 4 Similarity and Ratios	7.RP.2.c, 7.RP.3, 7.G.1	✓
Inv. 5 Using Similar Triangles and Rectangles	7.RP.2.c, 7.RP.3, 7.G.1	✓
Comparing and Scaling		Standard 20 days • Block 10 days
Inv. 1 Making Comparisons	7.RP.2, 7.RP.3	✓
Inv. 2 Comparing Ratios, Percents, and Fractions	7.RP.2, 7.RP.3	✓
Inv. 3 Comparing and Scaling Rates	7.RP.2, 7.RP.2.b, 7.RP.2.c, 7.RP.2.d (ACE 9, 10), 7.RP.3	✓
Inv. 4 Making Sense of Proportions	7.RP.2, 7.RP.2.a, 7.RP.2.b, 7.RP.3, 7.G.1	✓

## KEY

✓ Core Content

R Review

\* Extending

<b>Accentuate the Negative</b>		<b>Standard 18 days • Block 9 days</b>
Inv. 1 Extending the Number System	7.NS.1.b, 7.EE.3	✓
Inv. 2 Adding and Subtracting Integers	7.NS.1, 7.NS.1.a, 7.NS.1.b, 7.NS.1.c, 7.NS.1.d, 7.NS.3, 7.EE.3	✓
Inv. 3 Multiplying and Dividing Integers	7.NS.2, 7.NS.2.a, 7.NS.2.b, 7.NS.2.c, 7.NS.2.d, 7.NS.3, 7.EE.3	✓
Inv. 4 Properties of Operations	7.NS.1, 7.NS.1.d, 7.NS.2, 7.NS.2.c, 7.NS.3, 7.EE.3	✓
<b>Moving Straight Ahead</b>		<b>Standard 30 days • Block 15 days</b>
Inv. 1 Walking Rates	7.EE.3, 7.EE.4, 7.EE.4.a	✓
Inv. 2 Exploring Linear Functions With Graphs and Tables	7.EE.3, 7.EE.4, 7.EE.4.a, 7.EE.4.b (ACE 44)	✓
Inv. 3 Solving Equations	7.EE.1, 7.EE.3, 7.EE.4, 7.EE.4.a	✓
Inv. 4 Exploring Slope	7.EE.1, 7.EE.3, 7.EE.4, 7.EE.4.a	✓
CC Inv. 1 Graphing Proportional Relationships	7.RP.1, 7.RP.2.a, 7.RP.2.d	✓
CC Inv. 2 Equivalent Expressions	7.EE.1, 7.EE.2	✓
CC Inv. 3 Inequalities	7.NS.3, 7.NS.4.b	✓
<b>Filling and Wrapping</b>		<b>Standard 24 days • Block 12 days</b>
Inv. 1 Building Boxes	7.G.6	✓
Inv. 2 Designing Rectangular Boxes	7.G.6	✓
Inv. 3 Prisms and Cylinders	7.G.4, 7.G.6	✓
Inv. 4 Cones, Spheres, and Pyramids	7.G.6	✓
Inv. 5 Scaling Boxes	7.G.6	✓
CC Inv. 4 Geometry Topics	7.G.2, 7.G.3, 7.G.4, 7.G.5	✓
<b>What Do You Expect?</b>		<b>Standard 22 ½ days • Block 11 days</b>
Inv. 1 Evaluating Games of Chance	7.SP.5 (ACE 14), 7.SP.6, 7.SP.7. 7.a, b, 7.SP.8, 7.SP.8.a, 7.SP.8.b, 7.SP.8.c	✓
Inv. 2 Analyzing Situations Using an Area Model	7.SP.6, 7.SP.7, 7.SP.7.a, 7.SP.7.b, 7.SP.8, 7.SP.8.a, 7.SP.8.b, 7.SP.8.c	✓
Inv. 3 Expected Value	7.SP.6, 7.SP.7, 7.SP.7.a, 7.SP.7.b, 7.SP.8, 7.SP.8.a, 7.SP.8.b, 7.SP.8.c	✓
Inv. 4 Binomial Outcomes	7.SP.6, 7.SP.7, 7.SP.7.a, 7.SP.7.b, 7.SP.8, 7.SP.8.a, 7.SP.8.b, 7.SP.8.c	✓
<b>Data Distribution</b>		<b>Standard 23 days • Block 12 days</b>
Inv. 1 Making Sense of Variability	Reviews 6.SP.1, 6.SP.2, 6.SP.3	R
CC Inv. 5 Sampling and Variability	7.SP.1, 7.SP.2, 7.SP.3	✓
Inv. 2 Making Sense of Measures of Center	7.SP.3, 7.SP.4	✓
Inv. 3 Comparing Distributions: Equal Numbers of Data Values	7.SP.4	✓
Inv. 4 Comparing Distributions: Unequal Numbers of Data Values	7.SP.4	✓



## CC Investigation 1: Graphing Proportions

PACING 2 days

### Mathematical Goals

© DOMAIN: Ratios and Proportional Relationships

- Compute unit rates associated with ratios of fractions, including quantities measured in like or different units.
- Decide whether two quantities are in a proportional relationship.
- Explain what any point  $(x, y)$ , including  $(0, 0)$  and  $(1, r)$  where  $r$  is a unit rate, on a graph of a proportional relationship means in terms of the situation.

### Teaching Notes

In this investigation, students will extend their understanding of ratios and develop understanding of proportionality to solve problems. In Problem 1.1, students will explore and graph  $y = kx$  relationships. Discuss with them the importance of the points  $(0, 0)$  and  $(1, r)$ , where  $r$  represents the unit rate, on the graph. Students will convert rates to unit rates in Problem 1.2. Continue your discussions of  $y = kx$  proportional relationships in Problem 1.3. Assist students as necessary in completing graphs of the data.

#### Vocabulary

- rate
- unit rate
- proportional relationship

#### Problem 1.1

Before students begin Problem 1.1, review with them equivalent fractions, and how to simplify fractions. Ask: *How are the fractions  $\frac{1}{2}$ ,  $\frac{2}{4}$ , and  $\frac{3}{6}$  related? How do you know?* (They have equal values;  $\frac{2}{4}$  and  $\frac{3}{6}$  are equivalent to  $\frac{1}{2}$ .)

During Problem 1.1, ask:

- *Which variable will you place on each axis of your graph?* (The independent variable should go on the horizontal axis.)
- *Does your choice of which variable to place on which axis affect the graph?* (The rate will be different depending on which variable is graphed horizontally and which is graphed vertically.)
- *How do you find the slope of a line?* (Divide the difference in the  $y$ -values of two points by the difference in their  $x$ -values.)

### Problem 1.2

Before Problem 1.2, review the terms *rate* and *unit rate* with students. Refer to the example rate of  $\frac{140 \text{ miles}}{3.5 \text{ hours}}$ . Ask:

- *What two units are being compared in 140 miles in 3.5 hours? (miles to hours)*
- *How could you use this rate to find the distance that could be traveled in 7 hours? (Set up a proportion, or multiply  $140 \times 2$ .)*
- *Would it be helpful to find out how many miles are traveled in 1 hour? Why? (Yes, you then could multiply the number of miles in 1 hour by the number of hours traveled to get the total distance.)*

Explain that a unit rate has a unit of 1 in the denominator. Ask: *How can you find the unit rate for the rate  $\frac{140 \text{ miles}}{3.5 \text{ hours}}$ ? (Divide both numerator and denominator by 3.5.)*

During Problem 1.2, ask: *How can converting rates to unit rates make comparing those rates easier? (With unit rates, all of the rates have a denominator of 1, so you need to compare only the numerators.)*

### Problem 1.3

During Problem 1.3, guide students to discover that graphs of proportional relationships will be in the form  $y = kx$ , and always will pass through the origin. Ask:

- *How far does Karl jog in 0 hours? (0 miles)*
- *How far does Rebecca jog in 0 hours? (0 miles)*
- *What do these answers tell you about where the graphs of the runners' data will cross the y-axis? (The graphs will cross the y-axis at the origin.)*

### Summarize

To summarize the lesson, ask:

- *What is a rate? (a ratio that compares quantities in different units)*
- *How can you find a unit rate when given a rate? (Divide the numerator by the denominator to find a rate with a denominator of 1 unit.)*
- *When might finding a unit rate help you solve a problem? (when comparing prices to find the best deal)*
- *How can you tell from looking at the graph of a relationship if that relationship is proportional? (The relationship is proportional if the graph is a straight line that passes through the origin.)*



## Assignment Guide for Investigation 1

Problem 1.1, Exercise 1

Problem 1.2, Exercises 2–9

Problem 1.3, Exercises 10–16

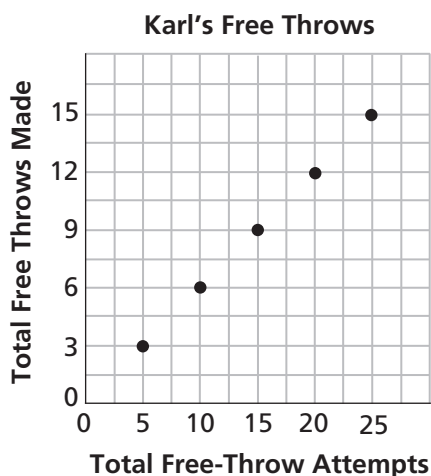
## Answers to Investigation 1

### Problem 1.1

A.

Game Number	1	2	3	4	5
Total Number of Free-throw Attempts	5	10	15	20	25
Total Number of Free Throws Made	3	6	9	12	15

B. 1. (10, 6), (15, 9), (20, 12), (25, 15)



2. The points form a straight line. In 20 games, Karl will attempt  $5(20) = 100$  free throws and make  $3(20) = 60$  of them.

3. 0; If Karl makes no free-throw attempts, then he will make no free throws.

4. 0.6; for every free-throw attempt, Karl will make 0.6 free throws. This means he makes free throws at a 60% success rate.

5. The slope is  $\frac{3}{5}$ . I can find the slope by

dividing the difference in any two points' y-values by the difference in their x-values. For the points (5, 3) and (15, 9),

$$\frac{9 - 3}{15 - 5} = \frac{6}{10} = \frac{3}{5}.$$

### Problem 1.2

A. 1. 640 mi to 19 gal, or  $\frac{640 \text{ mi}}{19 \text{ gal}}$

2.  $640 \div 19 \approx 33.7 \text{ mi/gal}$

B. 1. \$9.98 to 12 balls, or  $\frac{\$9.98}{12 \text{ balls}}$ ;

\$17.98 to 25 balls, or  $\frac{\$17.98}{25 \text{ balls}}$ ;

\$39.99 to 50 balls, or  $\frac{\$39.99}{50 \text{ balls}}$

2.  $9.98 \div 12 \approx \$0.83/\text{ball}$ ;  $17.98 \div 25 \approx \$0.72/\text{ball}$ ;  $39.99 \div 50 = \$0.80/\text{ball}$

C. 1.  $8\frac{1}{4} \text{ lb}$  to 6 balls, or  $\frac{33 \text{ lb}}{24 \text{ balls}}$

2.  $8\frac{1}{4} \div 6 \approx 1\frac{3}{8} \text{ lb/ball}$

D. 1. The unit rate tells you the quantity for an individual unit.

2. Find the unit rate by dividing the numerator of any rate by its denominator so that the new rate has a denominator of 1. For example, if it costs \$42 for 3 video

games, the rate of cost to games is  $\frac{\$42}{3 \text{ games}}$ .

Find  $42 \div 3$  to get the unit rate of \$14 per game.

**Problem 1.3**

A. 1. The distance always is 5 times the time.

$$2. \frac{3}{0.6} = 3 \div 0.6 = 5 \text{ mi/h}; \frac{5}{1} = 5 \div 1 = 5 \text{ mi/h};$$

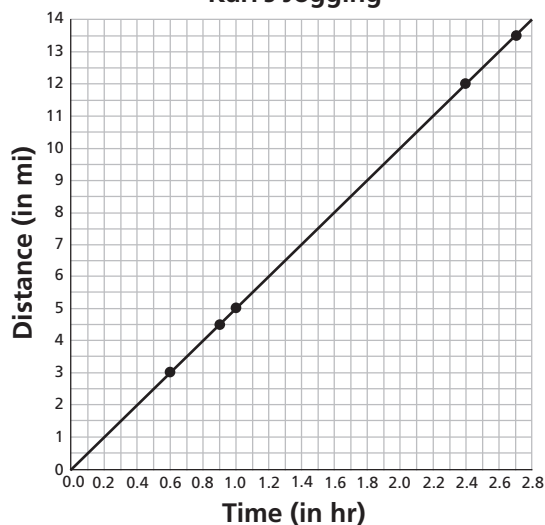
$$\frac{4.5}{0.9} = 4.5 \div 0.9 = 5 \text{ mi/h}; \frac{12}{2.4} = 12 \div 2.4$$

$$= 5 \text{ mi/h}; \frac{13.5}{2.7} = 13.5 \div 2.7 = 5 \text{ mi/h}$$

3. The ratios all are equivalent; Karl jogged at the same average speed for each run.

4. I can multiply the unit rate, 5 mi/h, by the number of hours to find the number of miles:  $5 \cdot 1.5 = 7.5$  mi.

B. 1. **Karl's Jogging**



The graph is a straight line; there is a proportional relationship between distance and time.

2. 0.3; It takes Karl 0.3 h to jog 1.5 mi.

3. a. (0, 0)

b. If Karl jogs for 0 hours, he will travel 0 miles.

C. 1. The unit rate is the rate with a denominator, or time, of 1. In the table, the corresponding distance for a time of 1 h is 5 mi, so  $r = 5$  mi/h. For each hour Karl jogs, he goes 5 miles.

2. Yes, the point (1, 5) lies on the line.

3. The point (1, 5) represents Karl's unit rate of speed.

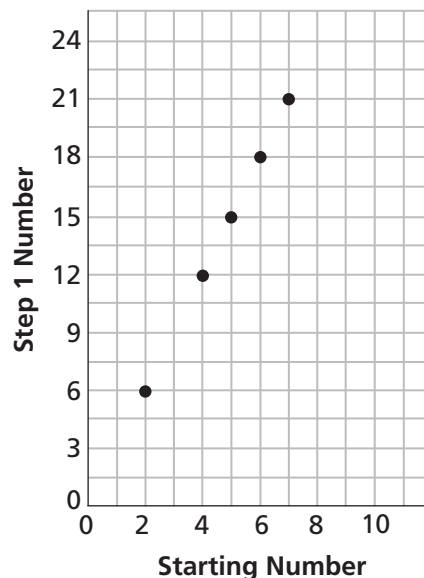
4. For  $x = 1, y = 6$ . The point (1, 6) represents Karl's faster unit rate of speed, 6 mi/h.

**Exercises**

1. a. 24

b. 6, 12, 15, 18, 21

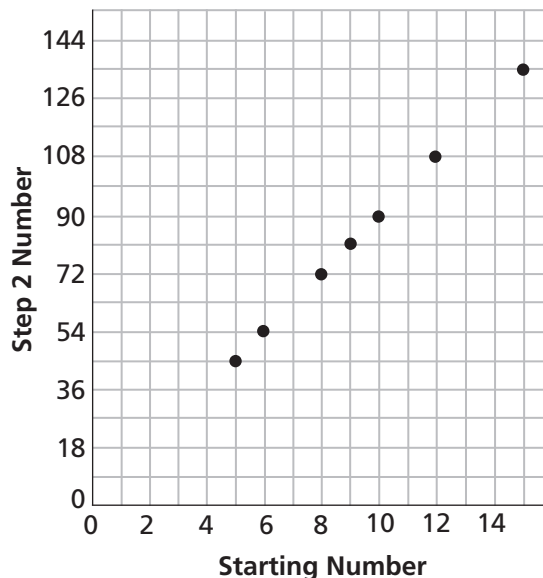
c. **Puzzle Numbers**



d. **Puzzle Numbers**

<b>Starting Number</b>	5	6	8	9	10	12	15
<b>Step 1 Number</b>	15	18	24	27	30	36	45
<b>Step 2 Number</b>	45	54	72	81	90	108	135

e. **Puzzle Numbers**



The points lie on a line.

f. The graph in part (e) would be more steep than the graph in (c).

2. 759 mi to 22 gal, or  $\frac{759 \text{ mi}}{22 \text{ gal}}$ ; 34.5 mi/gal

3. \$3.01 to 1.21 lb, or  $\frac{\$3.01}{1.21 \text{ lb}}$ ; about \$2.49/lb

4. \$25.92 to 12 key chains, or  $\frac{\$25.92}{12 \text{ key chains}}$ ; \$2.16/key chain

5. 72 calls to 6 h, or  $\frac{72 \text{ calls}}{6 \text{ h}}$ ; 12 calls/h

6. 2,220 Cal to 6 servings, or  $\frac{2,220 \text{ Cal}}{6 \text{ servings}}$ ; 370 Cal/serving

7. \$270 to 144 patches, or  $\frac{\$270}{144 \text{ patches}}$ ; about \$1.88/patch

8. a.  $\$2.19 \div 4 \approx \$0.55/\text{bottle}$ ;  $\$3.59 \div 8 \approx \$0.45/\text{bottle}$ ;  $\$6.99 \div 24 \approx \$0.29/\text{bottle}$

b. 24-pack

c.  $\$1.99 \div 8 \approx \$0.25/\text{bottle}$

d. three 8-packs

9. a. \$1.33

b. 0 cans

c. 16 cans

10. a. pages per day and days per page

b.  $168 \div 7 = 24$  pages/day, which means Amanda must read at a daily rate of 24 pages to finish on time;

$7 \div 168 = \frac{1}{24}$  day/page, which means that

Amanda must read at a rate of  $\frac{1}{24}$  day per page to finish on time.

c. 73 pages

d. Yes,  $144 \div 5 = 28.8$ ;  $28.8 > 24$ ; Amanda is reading at a greater rate than the required unit rate.

11. No, the ratios are not equivalent:  $\frac{1}{4} \neq \frac{3}{16}$ .

12. Yes, the ratios are equivalent:

$\frac{1}{2} = \frac{3}{6} = \frac{5}{10} = \frac{7}{14}$ .

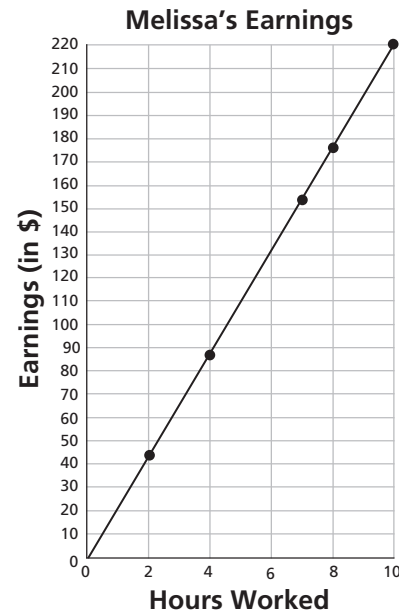
13. Yes, the graph is a straight line that passes through the origin.

14. No, the graph does not pass through the origin.

15. a.  $\frac{44}{2}$ ;  $\frac{88}{4}$ ;  $\frac{154}{7}$ ;  $\frac{176}{8}$ ;  $\frac{220}{10}$

b. Yes, all of the ratios are equivalent.

c.



d. If Melissa works 0 hours, she makes \$0.

e. Melissa earns \$22/h. The corresponding y-value for an x-value of 1 is 22.

f. (1, 22)

g.  $6 \text{ h} \times \$22/\text{h} = \$132$

16. a. There does not seem to be any pattern.

b.  $2 \div 0.3 = \text{about } 6.7 \text{ mi/h}$ ;  $4 \div 0.7 = \text{about } 5.7 \text{ mi/h}$ ;  $6 \div 1 = 6 \text{ mi/h}$ ;  $3 \div 0.5 = 6 \text{ mi/h}$ ;  $10 \div 2 = 5 \text{ mi/h}$

c. Most of the ratios are different; Rebecca did not jog at the same average speed each day.

d. Possible answer: No, the distance jogged and time do not form a proportional relationship.



**Additional Practice****Investigation 1****Graphing Proportions**

Michelle's grandfather pays her to do his yard work. The table shows how much she earned over several weeks.

Earnings				
Week	1	2	3	4
Time Spent Working (hours)	4	3	6	8
Earnings (\$)	28	21	42	56

1.
  - a. Describe any pattern you see in how the times and earnings are related.
  - b. Find the ratio of dollars per hour for each week.
  - c. What do you notice about the ratios? What does that tell you about Michelle's earnings?
  - d. The next month, Michelle spent a total of 16 hours doing yard work. Explain how to use the ratios to find how much Michelle earned for that work.
  
2.
  - a. Graph the data pairs from the table. Plot time along the  $x$ -axis and earnings along the  $y$ -axis. Connect the data points.
  - b. What shape does the graph take? What does this tell you about the relationship between time and earnings?
  - c. If you extend your graph, what value on the  $y$ -axis corresponds to an  $x$ -value of 1? What does that mean in terms of Michelle's earnings?
  - d. If you extend your graph, where does the graph cross the  $y$ -axis? Write the data pair represented by that point and explain what the point means.
  
3. Michelle's grandfather gives her a \$1 per hour raise. Graph Michelle's earnings at the new pay rate on the same graph. How are the graphs different? How are they alike?

**Skill: Find the Unit Rate****Investigation 1****Graphing Proportions**

For Exercises 1–10, write the fraction as a unit rate.

1.  $\frac{25}{5}$

2.  $\frac{8}{4}$

3.  $\frac{16}{28}$

4.  $\frac{30}{18}$

5.  $\frac{21}{14}$

6.  $\frac{6}{36}$

7.  $\frac{26}{39}$

8.  $\frac{42}{12}$

9.  $\frac{108}{45}$

10.  $\frac{35}{56}$

11. Marcus rode his bike 40 miles in  $2\frac{1}{2}$  hours. Write his average speed as a unit rate.

12. Angela paid \$5.56 for 4 pounds of beans. Write the cost of the beans as a unit rate.

**Skill: Proportional Rates**

For Exercises 13–20, tell whether all of the values are proportional.

13.  $\frac{4}{8}, \frac{6}{12}, \frac{3}{6}, \frac{5}{10}$

14.  $\frac{2}{6}, \frac{5}{15}, \frac{10}{35}, \frac{7}{21}$

15.  $\frac{28}{4}, \frac{64}{8}, \frac{77}{11}, \frac{49}{7}$

16.  $\frac{16}{4}, \frac{44}{11}, \frac{28}{7}, \frac{20}{5}$

17.  $\frac{5}{35}, \frac{7}{56}, \frac{9}{72}, \frac{11}{88}$

18.  $\frac{18}{3}, \frac{72}{12}, \frac{63}{9}, \frac{78}{13}$

19.  $\frac{7}{35}, \frac{3}{15}, \frac{14}{70}, \frac{8}{40}$

20.  $\frac{22}{11}, \frac{46}{23}, \frac{78}{39}, \frac{55}{22}$

21. Amir recorded the amounts of time it took him to complete his last three math worksheets. He finished 45 problems in 15 minutes, 60 problems in 20 minutes, and 75 problems in 25 minutes. Did he complete each set of problems at the same rate?

22. Store A sells 3-lb bags of almonds for \$14.37, store B sells 4-lb bags for \$19.16, and store C sells 2-lb bags for \$9.68. Do all of the stores charge the same amount for almonds?

**Check-Up****Investigation 1****Graphing Proportions**

1. Sahil found 4 different stores that sell seeds for the flowers he wants to plant in his garden. The number of seeds per packet and the packet price for each store are shown below.

<b>Andy's Seeds</b>	<b>Jenny's Seeds</b>	<b>Garden Place</b>	<b>Blooming Acres</b>
10 flower seeds	12 flower seeds	20 flower seeds	15 flower seeds
\$2.50	\$3.12	\$4.60	\$3.60

- a. Write the rate of seeds to dollars for each store. Then write a unit rate for each.
- b. Sahil needs 75 flower seeds for his garden. He wants to spend the least amount of money and have the fewest seeds left over. Where should Sahil buy his seeds? Explain your answer.
- c. Sahil's friend has \$10 and wants to buy as many seeds as he can. Where should Sahil's friend buy his seeds? Explain.
2. Four students are reading the same 200-page book. On Monday night, they record the numbers of pages they have read and the time it took them.

Name	Pages Read	Time (min)
Emilio	15	20
Luz	8	10
Erika	3	5
Jerome	7	10

Emilio says that he'll finish his book in the shortest amount of time, since he read the most pages. Is he correct? Explain why or why not.

**Check-Up** *(continued)***Investigation 1****Graphing Proportions**

3. The table shows the prices for different lengths of rescue rope.

Rescue Rope (8,000-lb Strength)				
Length (ft)	4	8	12	20
Price (\$)	6	12	18	30

- a. Graph the data pairs from the table and connect the points. Describe the shape the graph takes.
- b. Where does the graph cross the  $y$ -axis? Write the data pair represented by that point and explain what the point means.
- c. What price value corresponds on the graph to a length value of 1 ft? What does that data pair mean in terms of the price of the rescue rope?
4. The table shows the prices for different lengths of a stronger rescue rope.

Rescue Rope (13,000-lb Strength)				
Length (ft)	8	9	15	30
Price (\$)	20	22.5	37.5	75

- a. Write the unit rate for this rope in dollars per foot.
- b. Explain how the graph of the data pairs from this table would be different than the graph you made in Part 3. Explain how the graphs would be similar.



# CC Investigation 1: Graphing Proportions

© DOMAIN: Ratios and Proportional Relationships

## Problem 1.1

During the first basketball game of the season, Karl made 3 of his 5 free-throw attempts. Karl then made 3 of 5 free-throw attempts in each of the second game, the third game, the fourth game, and the fifth game.

A. Copy and complete the table.

Karl's Free Throws					
Game Number	1	2	3	4	5
Total Number of Free-throw Attempts	5	10	■	■	■
Total Number of Free Throws Made	3	■	■	■	■

- B. For each game, you can write the coordinates of the point (*free-throw attempts, free throws made*). The first point is (5, 3).
1. List the coordinates of the other four points. Graph all five points.
  2. What do you notice about these points? If the pattern continues for 20 games, how many free throws will Karl attempt? How many free throws will he make? Explain.
  3. What  $y$ -value on the graph corresponds to the  $x$ -value of 0? Explain in words what this point represents.
  4. What  $y$ -value on the graph corresponds to the  $x$ -value of 1? What does this point,  $(1, r)$  represent?
  5. Connect the points. What is the slope of the line? Explain how you found the slope.

Notes

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A **rate** is a ratio that compares quantities in different units such as miles to hours. An example of a rate is  $\frac{140 \text{ miles}}{3.5 \text{ hours}}$ .

A **unit rate** is a rate for which one of the numbers being compared is 1 unit. So,  $\frac{140 \text{ miles}}{3.5 \text{ hours}}$  is  $140 \text{ miles} \div 3.5 \text{ hours}$ , or 40 miles per 1 hour. The rate of miles to hours becomes miles per hour, which is written mi/h. The word “per” can be replaced with “for one.”

### Problem 1.2

**A.** On a recent road trip, the team van traveled 640 miles on 19 gallons of gasoline.

1. What is the rate of miles to gallons?
2. What is the rate of miles to one gallon?

**B.** The team wants to sell mini basketballs to raise money. The table shows different package sizes for purchasing basketballs.

Mini Basketballs	
Price	Quantity
\$9.98	12
\$17.98	25
\$39.99	50

1. What is the rate of dollars per balls for each package size?
2. What is the rate of dollars to one ball for each package size?

**C.** A 6-pack of basketballs weighs  $8\frac{1}{4}$  pounds.

1. What is the rate of pounds to balls?
2. What is the rate of pounds to one ball?

**D. 1.** What information does a unit rate provide?

2. Describe how you can find a unit rate. Use an example to illustrate your method.

Two quantities are in a **proportional relationship** if a change in one quantity corresponds to a change by the same factor in the other quantity. The ratios between the quantities do not change when the quantities themselves change.

## Notes

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**Problem 1.3**

Karl stays in shape between basketball seasons by jogging. The table shows his distances and times for one week.

Karl's Jogging					
Day	Mon.	Tues.	Wed.	Thurs.	Sat.
Distance (in miles)	3	5	4.5	12	13.5
Time (in hours)	0.6	1	0.9	2.4	2.7

- A.** Use the information in the table.
- Describe any pattern you see in how the distances and times are related.
  - Find the ratio of miles per hour for each day.
  - What do you notice about the ratios? What does that tell you about Karl's jogging?
  - Explain how to use the ratios to find how far Karl can jog in 1.5 hours.
- B.** Graph the data pairs from the table. Plot distance along the vertical axis and time along the horizontal axis.
- Connect the data points and describe the shape the graph takes. What does this tell you about the relationship between distance and time?
  - What value on the horizontal axis corresponds on the graph to a vertical value of 1.5? What does this mean in terms of Karl's jogging?
  - Where does the line of the graph cross the vertical axis? Write the data pair represented by that point.
    - Explain what that point means.
- C.** Karl's jogging speed in miles per hour is a unit rate,  $r$ .
- Explain how to use the table to find Karl's jogging speed,  $r$ .
  - Look at the point  $(1, r)$  on the graph. Does that point lie on the line you drew to represent Karl's jogging data?
  - Explain what point  $(1, r)$  represents.
  - Suppose Karl begins jogging faster, and jogs 3 miles in 0.5 hour one day, and 8 miles in  $1\frac{1}{3}$  hours another day. If these points were graphed, and a line drawn through them, what  $y$ -value would correspond to an  $x$ -value of 1? What would that point represent?

**Notes**


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## Exercises

1. **a.** In the first step of a number puzzle, you multiply your starting number by 3. If you start with 8, what is the new number after Step 1?
- b.** What are the new numbers after Step 1 for the following starting numbers: 2, 4, 5, 6, and 7?
- c.** Graph the (*starting number, Step 1 number*) points using the numbers from part (b).
- d.** Step 2 of the puzzle is to take the Step 1 number and multiply that number by 3. Copy and complete the table below.

**Puzzle Numbers**

<b>Starting Number</b>	5	6	8	9	10	12	15
<b>Step 1 Number</b>	■	■	24	■	■	■	■
<b>Step 2 Number</b>	■	■	72	■	■	■	■

- e.** Graph the (*starting number, Step 2 number*) points. What do you notice?
- f.** If the graphs in parts (c) and (e) are drawn using the same scales, would the graph in part (e) be *more steep* or *less steep* than the graph in (c)?

**For Exercises 2–7, write the comparison as a rate. Then find the unit rate.**

2. 759 miles per 22 gallons
3. \$3.01 for 1.21 pounds of nectarines
4. \$25.92 for 12 key chains
5. 72 phone calls in 6 hours
6. 2,220 Calories in 6 servings
7. \$270 for 144 American Flag patches
8. **a.** Find the unit price for each size of packaging.
- b.** Which size offers the best unit price?
- c.** Find the new unit price for the 8-pack if it goes on sale for \$1.99.
- d.** What is the least expensive way to buy 24 bottles of water during the sale period?

Bottled Water	
Price	Quantity
\$2.19	4
\$3.59	8
\$6.99	24

## Notes

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9. Canned soup is marked at 3 for \$3.99.
- What is the price per can?
  - How many cans of soup can you buy for \$1?
  - How many cans of soup can you buy with \$22?
10. For a report, Amanda must read 168 pages in 7 days.
- What are the two unit rates that she might compute?
  - Compute each unit rate and tell what it means.
  - Amanda plans to read the same number of pages each day. How many pages should Amanda have read by the end of the third day?
  - If she has read 144 pages by day 5, can she expect to finish in time? Explain.

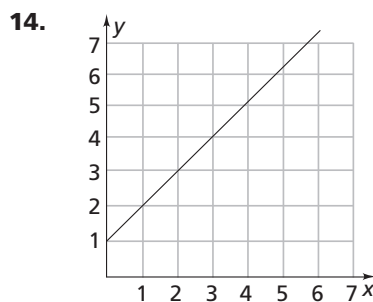
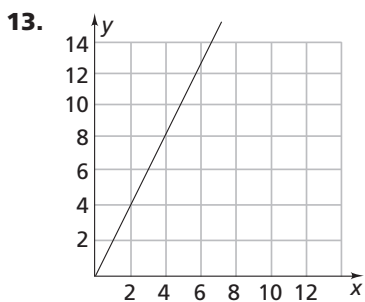
**For Exercises 11–14, tell whether the table or graph represents a proportional relationship. Explain how you know.**

11. 

$x$	1	2	3	4
$y$	4	8	16	32

12. 

$x$	1	3	5	7
$y$	2	6	10	14



**Notes** \_\_\_\_\_

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15. The table shows the numbers of hours Melissa works and the amounts that she earns.

Melissa's Earnings					
Hours Worked	2	4	7	8	10
Earnings (in \$)	44	88	154	176	220

- Write a ratio for each data pair in the table.
  - Is the relationship between time and earnings proportional? Explain why or why not.
  - Graph the data pairs from the table. Plot time along the horizontal axis and earnings along the vertical axis.
  - What does the point  $(0, 0)$  on the graph represent?
  - Use the graph to determine how much Melissa earns per hour,  $r$ . Explain how you used the graph to find your answer.
  - Give the ordered pair that represents Melissa's rate per hour.
  - How much does Melissa earn if she works 6 hours?
16. The table shows Rebecca's jogging times and distances for one week.

Rebecca's Jogging					
Day	Mon.	Tues.	Wed.	Thurs.	Sat.
Distance (in miles)	2	4	6	3	10
Time (in hours)	0.3	0.7	1	0.5	2

- Describe any pattern you see in how the distances and times are related.
- Find the ratio of miles per hour for each day.
- What do you notice about the ratios? What does that tell you about Rebecca's jogging?
- Explain whether you can use the ratios to find how far Rebecca can jog in 1.5 hours.

## Notes

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**CC Investigation 2: Equivalent Expressions**

PACING 1 day

**Mathematical Goals**© **DOMAIN:** Expressions and Equations

- Apply the properties of operations to add, subtract, factor, and expand algebraic expressions.
- Understand that writing an equivalent expression in a problem context can shed light on how quantities in the problem are related.

**Teaching Notes**

In order for students to understand and appreciate the purpose of finding equivalent expressions, they first must understand that algebraic expressions can be written to represent problem situations. Give students practice in writing simple algebraic expressions by having them represent these situations:

- 3 more than a number
- 8 degrees less than yesterday's temperature
- some boxes of pencils with 25 pencils in each box
- splitting the cost of dinner equally among 4 friends

Have students explain what each value in their expressions represents.

Before beginning the problems, review the associative, commutative, and distributive properties, using different types of rational numbers.

**Associative Property**

$$4 + (3 + 8) = (4 + 3) + 8$$

$$(3 \times 7.5) \times 2 = 3 \times (7.5 \times 2)$$

**Commutative Property**

$$\frac{1}{4} + \frac{3}{8} = \frac{3}{8} + \frac{1}{4}$$

$$4 \times \frac{5}{6} = \frac{5}{6} \times 4$$

**Distributive Property**

$$6 \times (8.5 + 9) = (6 \times 8.5) + (6 \times 9)$$

**Vocabulary**

- algebraic expression
- associative property
- commutative property
- distributive property

### **Problem 2.1**

During Problem 2.1 A, ask: *Why would you want to simplify the expression  $25j + 11.5(2 + j)$ ? (to make it easier to find the value of the expression when given a value for  $j$ )*

During Problem 2.1 A, guide students through the steps of the simplification. Ask:

- *How is the second expression different than the first? (The coefficient 11.5 has been multiplied separately by each addend inside the parentheses.)*
- *What property does that demonstrate? (distributive property)*
- *How is the third expression different than the second? (The order of the addends is changed.)*
- *What property does that demonstrate? (commutative property)*
- *How is the fourth expression different than the third? (The coefficients of  $j$  have been placed together inside parentheses.)*
- *What property does that demonstrate? (distributive property)*

Before Problem 2.1 A, Part 4, explain that to evaluate an expression, students should substitute the given value for  $j$  and then simplify.

After Problem 2.1, ask: *Was it easier to evaluate the expression in Part 4 or in Part 5? Why? (Part 5 was easier to evaluate because there were fewer terms.)*

### **Problem 2.2**

Before 2.2 A, review with students how to express a percent as an equivalent decimal. Ask: *What decimal is equivalent to 20%? (0.2)*

During Problem 2.2 A, ask: *What equivalent expression can you write for  $p - 0.2p$ , using the distributive property? ( $p(1 - 0.2)$ )*

During Problem 2.2 A, Part 4, ask:

- *What expression can you write to represent the sale price of an item that is 25% off? ( $p - 0.25p$ )*
- *What equivalent expression can you write using the distributive property? ( $p(1 - 0.25)$ )*
- *What expression can you write to represent the price of an item that is on sale for 75% of its original price? ( $0.75p$ )*

During Problem 2.2 C, ask: *How can you tell from looking at the expression which term represents the discount and which represents the tax? (The term being subtracted represents a discount off the original price, while the term being added represents the tax added to the price.)*

### **Summarize**

To summarize the lesson, ask:

- *When might you use a property of operations to write an equivalent expression? (to simplify an expression so that you can evaluate it using mental math)*
- *What property is used to rewrite the expression  $3.6a + 4.4a$  as  $(3.6 + 4.4)a$ ? (distributive property)*
- *What is another way to express 0.7 times a number? (the number minus 0.3 times the number)*



## Assignment Guide for Investigation 2

Problem 2.1, Exercises 1–17

Problem 2.2, Exercises 18–23

### Answers to Investigation 2

#### Problem 2.1

A. 1. the cost of the pairs of jeans Chris buys

2.  $11.5(2 + j)$

3.

Step	Reason
$25j + 11.5(2 + j)$	original expression
$25j + 11.5(2) + 11.5(j)$	distributive property
$25j + 11.5(j) + 11.5(2)$	commutative property of addition
$(25 + 11.5)j + 11.5(2)$	distributive property
$(36.5)j + 11.5(2)$	addition
$36.5j + 23$	multiplication

4. For 2 pairs of jeans:  $25j + 11.5(2 + j) = 25(2) + 11.5(2 + 2) = 50 + 11.5(4) = 50 + 46 = \$96$ ; for 4 pairs of jeans:  $25j + 11.5(2 + j) = 25(4) + 11.5(2 + 4) = 100 + 11.5(6) = 100 + 69 = \$169$ .

5. For 2 pairs of jeans:  $36.5j + 23 = 36.5(2) + 23 = 73 + 23 = \$96$ ; for 4 pairs of jeans:  $36.5j + 23 = 36.5(4) + 23 = 146 + 23 = \$169$ ; the expressions have the same values for a given value of  $j$ , so the expressions are equivalent.

B. 1. 78 represents the money Chris has;  $20t$  represents the cost of  $t$  dress shirts;

$\frac{1}{2}(12t)$  is the cost of  $t$  ties at  $\frac{1}{2}$  off.

2.

Step	Reason
$78 - 20t - \frac{1}{2}(12t)$	original expression
$78 - 20t - 6t$	multiplication
$78 - (20 + 6)t$	distributive property
$78 - 26t$	addition

3.  $78 - 20t - \frac{1}{2}(12t) = 78 - 60 - 18 = 0$ ;

$78 - 26t = 78 - 78 = 0$ ; yes, the expressions have the same value for  $t = 3$  because they are equivalent expressions.

#### Problem 2.2

A. 1.  $p$  represents the full price of the item;  $0.2p$  represents the 20% discount.

2. The sale price is the full price,  $p$ , minus the 20% discount, or  $p - 0.2p$ .

3. Yes, according to the distributive property,  $p - 0.2p = (1 - 0.2)p = 0.8p$ .

4. The sales are equivalent. A sale of 25% off gives a sale price of  $100\% - 25\% = 75\%$ .

B. 1.  $c$  represents the price of an item;  $0.06c$  represents the 6% sales tax on the item.

2. The price, including tax, is the item's price,  $c$ , plus the tax,  $0.06c$ , or  $c + 0.06c$ .

3. Factor  $c$  out of both terms, and then add the remaining terms:  
 $c + 0.06c = (1 + 0.06)c = 1.06c$ .

4. The expressions are equivalent:  
 $c + 0.06c = 1.06c$ .

5.  $c + 0.06c = 20 + (0.06)20 = 20 + 1.2 = \$21.20$ ;  
 $1.06c = 1.06(20) = \$21.20$ ; The expressions  
 have the same value because they are  
 equivalent.
6. The cost of the jacket with tax is  
 $1.06c = 1.06(225) = \$238.50$ ;  $\$235 < \$238.50$ ,  
 so it would cost Chris less to buy the jacket  
 online.
- C. 1. 15%; the term  $-0.15d$  represents a savings  
 of 15% of the original price,  $d$ .
2. 7%; the term  $+0.07d$  represents a tax of  
 7% on the original price,  $d$ .
3.  $d - 0.15d + 0.07d = (1 - 0.15 + 0.07)d = 0.92d$

### Exercises

1.

Step	Reason
$4(p + 7) - 2p$	original expression
$4p + 28 - 2p$	distributive property
$4p - 2p + 28$	commutative property
$(4 - 2)p + 28$	distributive property
$2p + 28$	subtraction

2.  $10t$ ; distributive property
3.  $\frac{5}{3}x$ ; distributive property
4.  $15m - 55$ ; distributive property
5.  $4p - 12$ ; distributive property, commutative  
 property
6.  $1.2n + 8.4$ ; distributive property, commutative  
 property
7.  $-2g + 4h - 22$ ; distributive property,  
 commutative property, associative property
8.  $4(x - 3)$ ;  $4(-3 + x)$
9. a.  $2m + 20$ , where  $m$  represents the amount of  
 money Marty has.
- b.  $m + 2m + 20 = (1 + 2)m + 20 = 3m + 20$ ;  
 distributive property
- c.  $\frac{1}{2}(3m + 20) = \frac{3}{2}m + 10$ ; distributive  
 property
- d. Aimee:  $2m + 20 = 2(30) + 20 = \$80$ ;  
 Jack:  $\frac{3}{2}m + 10 = \frac{3}{2}(30) + 10 =$   
 $45 + 10 = \$55$
10. No,  $x + 8 + 3x + 12y = 4x + 8 + 12y$ ;  
 $4(x + 8) + 12y = 4x + 32 + 12y$ ;  
 $4x + 8 + 12y \neq 4x + 32 + 12y$ .
11. first week: 8; second week:  $h$ ; third week:  $2h$ ;  
 total:  $8 + h + 2h = 8 + 3h$
12. D
13. B
14. F
15. E
16. A
17. C
18. a.  $w + 0.07w$   
 b.  $1.07w$
19.  $d + 0.04d$ ;  $1.04d$
20.  $x - 0.25x$ ;  $0.75x$
21.  $c + 0.75c$ ;  $1.75c$
22. Yes,  $0.83p = (1 - 0.17)p = p - 0.17p$ .
23. Yes,  $y - \frac{1}{5}y = \left(1 - \frac{1}{5}\right)y = \frac{4}{5}y$ .

**Additional Practice****Investigation 2****Equivalent Expressions**

Marcus is shopping at Sound World. Prices are listed in the table.

Sound World	
Item	Price
DVD	\$24
CD	\$11
Video Game	\$37.50
Computer Game	\$22.50

- Marcus buys some DVDs. He buys 3 more CDs than he buys DVDs. He writes the expression  $24d + 11(3 + d)$  to represent the total cost.
  - Describe what each part of Marcus' expression represents.
  - Copy and complete the table to show the steps to simplifying the expression.

Step	Reason
$24d + 11(3 + d)$	original expression
$24d + 11(3) + 11(d)$	
$24d + 11(d) + 11(3)$	
$(24 + 11)d + 11(3)$	
$(35)d + 11(3)$	
$35d + 33$	

- Evaluate the original expression,  $24d + 11(3 + d)$ , to find the total cost if Marcus buys 3 DVDs or 5 DVDs.
  - Evaluate the simplified expression,  $35d + 33$ , for the same values of  $d$ . Compare the answers to what you found. What does this tell you about the expressions?
- Marcus can save 15% on his purchase by signing up for a Sound World credit card. Write two equivalent expressions to show the discounted price of an item with an original price of  $p$ .
    - Marcus writes the expression  $p + 0.06p$  to represent the price of an item including sales tax. Tell what tax rate the store charges, and write an equivalent expression to represent the total price.

**Skill: Evaluate Expressions****Investigation 2****Equivalent Expressions**

For Exercises 1–6, evaluate the expression for  $p = 4$ .

1.  $3p$

2.  $8p + 9$

3.  $16 \div p$

4.  $p(4 + p)$

5.  $6 - \frac{p}{2}$

6.  $\frac{p + 24}{7}$

For Exercises 7–12, evaluate the expression for  $g = 0.7$ .

7.  $1 - g$

8.  $4(g + 7)$

9.  $13g$

10.  $g + 0.8 + g$

11.  $45g + g$

12.  $12g + 12$

**Skill: Simplify Expressions**

Simplify the expression.

13.  $6y - (4 + y)$

14.  $2.5(4g - 2) + 1.8g$

15.  $\frac{2}{5}r + \frac{4}{5}r$

16.  $-(t + 4) - 7(t + 8)$

17.  $8(14 - 6f)$

18.  $2d + 13d - 6d$

19.  $9(a - 6) + 6(9 - a)$

20.  $\frac{1}{2}(u - 6) + \frac{u}{2}$

21.  $9q + \frac{3}{2}(6q + 4)$

22.  $4(4.1 - 2v) + 3v$

**Check-Up****Investigation 2****Equivalent Expressions**

1. At the start of the season, a baseball team buys some balls for each player on the team. It also buys 4 fewer bats than balls for each player. The expression  $12b + 12(b - 4)$  represents the total number of bats and balls the team buys for 12 players.
- Describe what each part of the expression represents.
  - Complete the table to show the steps to simplifying the expression.

Step	Reason
$12b + 12(b - 4)$	original expression
$12[b + (b - 4)]$	
$12[(b + b) - 4]$	
$12(2b - 4)$	
$12(2b) - 12(4)$	
$24b - 48$	

- Evaluate the original expression,  $12b + 12(b - 4)$ , to find the total number of bats and balls if the team buys 6 balls for each player. Show your work.
  - Evaluate the simplified expression,  $24b - 48$ , for the same value of  $b$ . Show your work. Compare the answer to the answer in Part c. What does this tell you about the expressions?
2. SportsTown is having a 30%-off sale on baseball equipment. Andy finds the sale price of a baseball glove that normally costs \$49 by evaluating the expression  $49 - 0.3(49)$ . Juan finds the sale price by multiplying  $0.7(49)$ . Who found the correct sale price? Explain how you know.

**Check-Up** *(continued)***Investigation 2****Equivalent Expressions**

3. The table shows the prices of two stocks.

Stock Prices	
Stock	Price Per Share
Fine Motors, Inc.	\$68
AutoMart Corp.	\$70

- a. Fine Motors' stock gains 5% per share. Write two equivalent expressions to represent the stock's new price. Evaluate each expression to find the new price. Show your work.
- b. Over one month, Fine Motors' stock gains 12% per share over the price shown in the table. What percent gain would AutoMart's stock need to have over the same period for the two stocks to have the same price? Explain how you know.
4. Lisa buys a computer on sale for 25% off. The store adds 6% sales tax to the price of the computer after the discount.
- a. The expression  $0.75c + 0.06(c - 0.25c)$  represents the final price, including tax, of a computer with a regular price of  $c$  dollars. Simplify the expression. List the properties of operations you use.
- b. How much does Lisa pay for a computer with a regular price of \$700? Show your work.

# CC Investigation 2: Equivalent Expressions

© DOMAIN: Expressions and Equations

You can use the properties of operations, such as the associative, commutative, and distributive properties, to simplify algebraic expressions.

## Problem 2.1

**A.** Chris buys some pairs of jeans. He buys 2 more pairs of shorts than he buys jeans. He writes the expression  $25j + 11.5(2 + j)$  to represent the total cost.

1. What does the term  $25j$  in the expression mean?
2. What part of the expression represents the cost of the shorts Chris buys?
3. Copy and complete the table to show the steps to simplifying the expression.

Item	Price
Pair of Shorts	\$11.50
Pair of Jeans	\$25.00
Dress Shirt	\$20.00
Tie	\$12.00

Step	Reason
$25j + 11.5(2 + j)$	original expression
$25j + 11.5(2) + 11.5(j)$	
$25j + 11.5(j) + 11.5(2)$	
$(25 + 11.5)j + 11.5(2)$	
$(36.5)j + 11.5(2)$	
$36.5j + 23$	

4. Evaluate the original expression,  $25j + 11.5(2 + j)$ , to find the total cost if Chris buys 2 pairs of jeans or 4 pairs of jeans.
  5. Evaluate the simplified expression,  $36.5j + 23$ , for the same values of  $j$ . What does this tell you about the expressions?
- B.** Chris has \$78 and a coupon that lets him buy one tie for  $\frac{1}{2}$  off for each dress shirt he buys. He writes the expression  $78 - 20t - \frac{1}{2}(12t)$  to represent how much money he will have left if he buys  $t$  shirts and ties.
1. Explain what each term in the expression represents.
  2. Simplify the expression. Give a reason for each step.
  3. Evaluate each expression for  $t = 3$ . Does each expression have the same value? Explain why or why not.

Equivalent Expressions 7

STUDENT PAGE

Notes

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**Problem 2.2**

- A.** The store where Chris is shopping is having a 20%-off sale. Chris writes the expression  $p - 0.2p$  to represent the sale price of any item.
1. What do the terms  $p$  and  $0.2p$  in the expression represent?
  2. Explain how the expression  $p - 0.2p$  represents the sale price of an item.
  3. Does the expression  $0.8p$  also represent the sale price of an item? Explain why or why not.
  4. Two other stores also are having sales. One store offers 25% off, and the second store offers its items at 75% of their original prices. Which sale is better? Explain your choice.
- B.** A 6% sales tax is added to all purchases at the store. Chris writes the expression  $c + 0.06c$  to represent the cost of an item, including tax.
1. What do the terms  $c$  and  $0.06c$  in the expression represent?
  2. Explain how the expression  $c + 0.06c$  represents the cost of an item, including tax.
  3. Describe how to use the distributive property to simplify the expression  $c + 0.06c$ . Write the simplified expression.
  4. Explain why both expressions represent the same situation.
  5. Evaluate each expression for  $c = \$20$ . Do the expressions have the same value? Explain why or why not.
  6. Chris wants to buy a jacket that costs \$225 before the 6% tax. He can buy the same jacket online for \$235 with no sales tax. Where should Chris buy the jacket? Explain your choice.
- C.** Chris buys a pair of shoes with an original price of  $d$  dollars. He writes the expression  $d - 0.15d + 0.07d$  to represent the final cost of the shoes.
1. The shoes are on sale. What percent off is the sale? Explain how you know.
  2. Chris includes sales tax in the final cost of the shoes. What sales tax rate did he use? Explain how you know.
  3. Simplify the expression. Show your work.

**Notes**

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**Exercises**

1. Copy and complete the table by providing the property or reason for each step.

Step	Reason
$4(p + 7) - 2p$	original expression
$4p + 28 - 2p$	
$4p - 2p + 28$	
$(4 - 2)p + 28$	
$2p + 28$	

For Exercises 2–7, simplify the expression. List the properties you used.

2.  $3t - 7t + 14t$
3.  $\frac{1}{3}x + \frac{4}{3}x$
4.  $5(3m - 11)$
5.  $5p - (12 + p)$
6.  $1.2(3n + 7) - 2.4n$
7.  $-2(g + 5) + 4(h - 3)$
8. Write an expression that can be simplified to  $4x - 12$  using the distributive property. Write another equivalent expression using the commutative property.
9. Marty has some money. Aimee has \$20 more than twice as much money as Marty has. Jack has one half of the sum of the amounts that Marty and Aimee have.
- Write an expression for the amount of money Aimee has.
  - Write and simplify an expression for the sum of the amounts of money that Aimee and Marty have. Which property or properties did you use to simplify the expression?
  - Write and simplify an expression for the amount of money Jack has. Which property or properties did you use to simplify the expression?
  - If Marty has \$30, how much money do Aimee and Jack each have?
10. James says the expressions  $x + 8 + 3x + 12y$  and  $4(x + 8) + 12y$  are equivalent. Is James correct? Explain your answer.
11. You worked a total of 8 hours the first week of this month. The second week you worked less, but you can't remember how many hours less. You know you worked twice as many hours in the third week as in the second week. Write and simplify an expression for the number of hours you worked the first three weeks of this month. Show your steps.

**Notes**

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For Exercises 12–17, match the expression with an equivalent expression from the box at the right.

12.  $x - 0.7x$

13.  $1.5x$

14.  $2x - 2$

15.  $x - 0.5x$

16.  $x + 0.3x$

17.  $2(x + 1)$

A.  $1.3x$

B.  $x + 0.5x$

C.  $2x + 2$

D.  $0.3x$

E.  $0.5x$

F.  $2(x - 1)$

18. A 7% sales tax is added to all purchases at an electronics store. Carmella wants to buy a digital music player that costs  $w$  dollars before tax.

- Write an expression using addition that shows the total cost of the music player, including tax.
- Write an equivalent expression using only multiplication, that also shows the total cost of the music player, including tax.

For Exercises 19–21, write two equivalent expressions you could use to solve the problem.

- A new employee earns  $d$  dollars per year. She gets a raise of 4% after her first year. What is her new salary?
- You are buying some CDs. You have a coupon for 25% off of your purchase. How much do you spend if your purchase before the discount is taken is  $x$  dollars?
- The number of cells,  $c$ , in a Petri dish increases by 75% in the first hour. How many cells are in the dish after the first hour?
- The population of a city decreased this year by 17% from the population,  $p$ , last year. A resident of the city says that you can find the city's population this year by finding  $0.83p$ . Is the resident correct? Explain your answer.
- Can the expression  $y - \frac{1}{5}y$  be used to represent the same situation as represented by the expression  $\frac{4}{5}y$ ? Explain why or why not.

## Notes

## CC Investigation 3: Inequalities

PACING 2 days

### Mathematical Goals

© DOMAIN: Expressions and Equations

- Solve word problems leading to one- and two-step inequalities.
- Graph the solutions to one- and two-step inequalities and interpret the solution set in the context of the problem.

### Teaching Notes

This investigation focuses on representing situations and solving problems using inequalities. A connection between solving equations and solving inequalities focuses on similar procedures. When students solve one- and two-step inequalities in Problem 3.2, emphasize the similarities with solving linear equations, but be careful to point out that there are some differences as well. Students will explore these differences in Problem 3.3.

#### Vocabulary

- inequality

#### Materials

- grid paper

#### Problem 3.1

*Before* students begin Problem 3.1, review the inequality symbols and make sure students have an understanding of inequalities. As students answer the Getting Ready questions, ask them to explain their choices of inequality symbols. Discuss other phrases that could be represented as inequalities.

*During* Problem 3.1 A, ask: *Why are you asked to write an inequality rather than an equation?* (The problem states that Caitlyn wants a television with “at least” a 26-in. screen. So, Caitlyn will need to spend an amount equal or greater than the cost of the 26-in. television, \$330.)

*During* Problem 3.1 B, emphasize the meaning of open and closed circles in the graph of an inequality. Ask: *What does “at least” tell you about the graph?* (There should be a closed circle at 330 since \$330 is included and the arrow should point to the right.)

#### Problem 3.2

*Before* Problem 3.2, during Getting Ready, ask students to explain the steps involved in solving the inequality  $3x - 15 \geq 12$ , and then compare these steps to solving the equation  $3x - 15 = 12$ .

*During* Problem 3.2 A, Part 1, ask: *Why is 129 being subtracted from the variable  $a$ ?* (Caitlyn is going to spend \$129 for softball equipment, so it must be subtracted from the amount in the community center’s savings account.)

*During* Problem 3.2 A, Part 3, ask: *Why is the phrase “possible value” used?* (There are an infinite number of solutions to the inequality.)

During Problem 3.2 B, Part 1, ask:

- *What does each term in  $15h + 40 \leq 140$  represent?* ( $15h$  represents the hourly rental rate for  $h$  hours, 40 represents the set-up fee, and 140 represents the amount that the group has to spend.)
- *What do you notice about this inequality compared to the one you solved in Problem 3.2 A?* (The variable  $h$  is multiplied by a number, 15.)
- *What does this mean when solving the inequality?* (Two operations are needed; subtract 40 from both sides of the inequality, then divide both sides of the inequality by 15.)

During Problem 3.2 B, Part 2, ask: *How will you decide what inequality to write? Explain.* (The dining room costs \$12 per hour, so the hourly rental cost will be represented by  $12h$ . The set-up fee is \$80, and the maximum amount to spend is \$140. The total cost of the rental is  $12h + 80$ , which must be less than or equal to 140.)

### **Problem 3.3**

Before Problem 3.3, during Getting Ready, ask: *How are the solutions to the two inequalities,  $12x < 60$  and  $-12x < 60$ , different?* (The direction of the inequality sign is different, and the sign of the number, 5, has changed.)

After Problem 3.4 A, ask: *What could you have done with the inequality  $175 - 35g \geq 0$  before solving it to avoid having to divide by a negative number?* (Add  $35g$  to both sides of the inequality to get the new inequality  $175 \geq 35g$ .)

During Problem 3.3 A, Part 3, ask: *What does 0 represent?* (It represents the least number of games the center could buy.)

During Problem 3.3 B, Part 1, ask:

- *What does each term in the inequality represent?* (175 represents the money that can be spent on games;  $8m$  represents the monthly fee for  $m$  months; 99 represents the equipment cost.)
- *How else could you write an inequality to represent the same situation?* ( $175 - 99 \geq 8m$ )

### **Summarize**

To summarize the lesson, ask:

- *When graphing the solution of an inequality, when do you use an open circle and when do you use a closed circle?* (An open circle is used with the symbols  $<$  and  $>$  to represent that the number is not part of the solution. A closed circle is used with the symbols  $\leq$  and  $\geq$  to represent that the number is part of the solution.)
- *How is solving one- and two-step inequalities similar to solving one- and two-step linear equations?* (You can use inverse operations to solve both inequalities and linear equations.)
- *What must you do if you multiply or divide both sides of an inequality by a negative number?* (You must reverse the direction of the inequality symbol.)

## Assignment Guide for Investigation 3

Problem 3.1, Exercises 1–4, 23

Problem 3.2, Exercises 5–14, 17, 20–22, 24–28

Problem 3.3, Exercises 15–16, 18–19

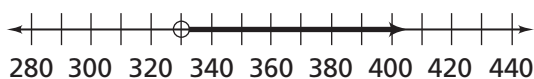
## Answers to Investigation 3

### Problem 3.1

A.  $m \geq \$330$

- B. 1. Caitlyn drew a solid circle because the least that she can spend to get the television she wants is \$330, and using a closed circle includes \$330 in the solution.

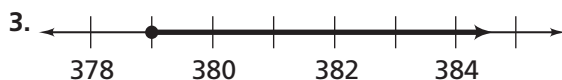
2. Draw an open circle at 330;



### Problem 3.2

A. 1.  $a - 129$  represents the amount of money Caitlyn has to spend on the video game system. The total amount in the community center's bank account is  $a$  and 129 is the amount she must keep in the bank to spend on softball equipment.

2.  $a - 129 \geq 250$ ;  $a - 129 + 129 \geq 250 + 129$ ;  $a \geq 379$ ; This solution means that the community center must have \$379 or more in its bank account in order to purchase the video game system and softball equipment.



B. 1.  $15h + 40 \leq 140$ ;  $15h + 40 - 40 \leq 140 - 40$ ;  $15h \leq 100$ ;  $15h \div 15 \leq 100 \div 15$ ;  $h \leq \text{about } 6.67$

2.  $12h + 80 \leq 140$ ;  $12h + 80 - 80 \leq 140 - 80$ ;  $12h \leq 60$ ;  $12h \div 12 \leq 60 \div 12$ ;  $h \leq 5$

3. The group should rent the main hall since it can rent it for more time than it can rent the dining room.

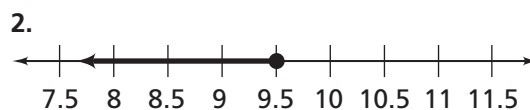
### Problem 3.3

A. 1.  $175 - 35g \geq 0$ ;  $175 - 35g - 175 \geq 0 - 175$ ;  $-35g \geq -175$ ;  $-35g \div 35 \geq -175 \div 35$ ;  $-g \geq 5$ ;  $g \leq 5$ ; The community center can buy 5 or fewer games.



3. The variable  $g$  represents the number of games the community center can buy. The center cannot buy a negative number of games. The least it can buy is 0 games.

B. 1.  $175 - 8m \geq 99$ ;  $175 - 8m - 175 \geq 99 - 175$ ;  $-8m \geq -76$ ;  $-8m \div 8 \geq -76 \div 8$ ;  $-m \geq 9.5$ ;  $m \leq 9.5$ ; The community center can rent games for up to 9 months.

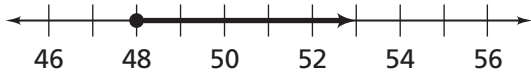


3.  $175 - 19m - 175 \geq 19 - 175$ ;  $-19m \geq -156$ ;  $-19m \div 19 \geq -156 \div 19$ ;  $-m \geq \text{about } 8.2$ ;  $m \leq \text{about } 8.2$ ; The community center can rent games for up to 8 months.

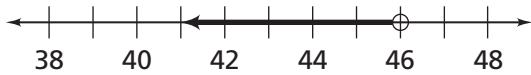
4. The center should rent from NetGames since they can rent longer for the same amount of money. Also, if they continue the rentals past that period, it will be cheaper than Anytime Games.

**Exercises**

1.  $h \geq 48$  in.;



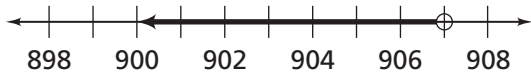
2.  $h < 46$  in.;



3. Yes; someone who is 36 inches tall could ride the Spiral ( $36 > 35$ ), but not ride the Leapin' Lizard ( $36 < 38$ ).

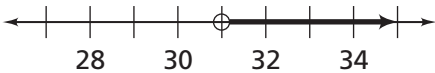
4. No; For the Tilt-a-Whirl, a rider must be at least 48 inches tall, and for the Ladybug, a rider must be less than 46 inches tall. There is no height such that meets both requirements.

5.  $p < 907$  points;



6. B

7.  $d > 31$  days;



8. C

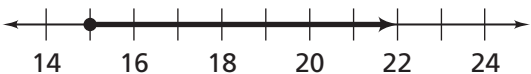
9.  $12.5b + 37.5 \geq 75$ ;  $b \geq 3$  times

10.  $4f \leq 39$ ;  $f \leq 9.75$  figurines

11.  $x < 2$ ;



12.  $y \geq 15$ ;



13.  $z > 21$ ;



14.  $a \leq 6$ ;



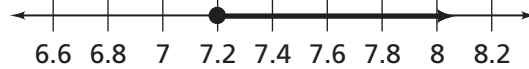
15.  $b > -5$ ;



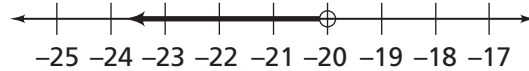
16.  $c \leq 11$ ;



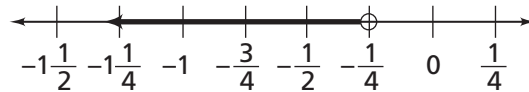
17.  $p \geq 7.2$



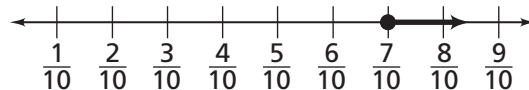
18.  $y < -20$



19.  $x < -\frac{1}{4}$



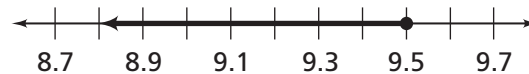
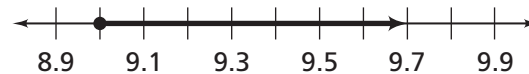
20.  $t \geq \frac{7}{10}$



21. C

22.  $p + 6 \leq 2(6 + 10 + 15)$ ;  $p \leq 56$  sq ft, so Janine needs to buy enough paint to cover up to 56 square feet.

23. a.  $l \geq 9$  and  $l \leq 9.5$ ;



b. Yes;



24.  $2.4s \leq 140$ ;  $s \leq 58.3$  min/mi

25.  $b \leq 5$  min/mi; Pauline needs to bike 5 minutes or less per mile.



27. B

28. Martin can rent the bicycle for 4 hours or less. The closed circle indicates that 4 hours is included in the solution.

**Additional Practice****Investigation 3****Inequalities**

1. The community theater company pays \$500 per night to rent a theater for its performances. It charges \$6 for a regular-price ticket to an evening performance.
  - a. Write and solve an inequality to show how many tickets the theater company must sell to at least pay for the theater rental for the night. Graph the solution. Explain what your solution means.
  - b. At one performance, the theater company sold 8 T-shirts and 3 DVDs. Write and solve an inequality to show how many tickets they need to sell to pay for the theater rental that night. Graph the solution. Explain what your solution means.
2. Chen's cell phone plan costs \$39 per month, and gives him 450 free minutes each month. Each minute over 450 costs Chen \$0.25. His cell phone company also offers a plan with unlimited minutes for \$99 per month.
  - a. Write an inequality that shows how many minutes Chen can talk each month under his current plan for \$99 or less.
  - b. Chen uses his cell phone about 700 minutes per month. Should he switch to the unlimited plan? Explain.
3. Adita and her family are planning a trip to Europe. They really like the hot weather, and don't want to travel where the average temperature for the month is less than 80°F. Adita knows the formula to convert temperatures from degrees Fahrenheit to degrees Celsius is  $F = 1.8C + 32$ .
  - a. Write an inequality to show Adita's preferred temperature in °F.
  - b. Use the formula  $F = 1.8C + 32$  to write an inequality to show the family's preferred temperature in °C.
  - c. Adita found a city in Spain that has an average temperature of 27°C. She found a city in France where the average is 26°C. Do these cities meet Adita's preferences for temperature?

**Show Souvenirs**

T-shirt	\$10
DVD	\$15

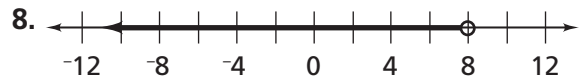
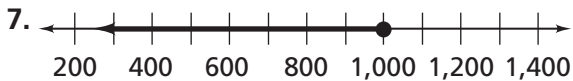
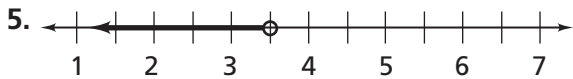
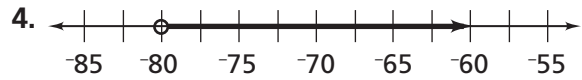
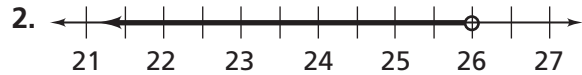
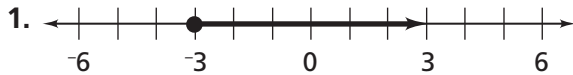
**Support YOUR  
Community Theater!**

## Skill: Read Graphs of Inequalities

### Investigation 3

#### Inequalities

Write an inequality shown by the graph.



## Skill: Solve Inequalities

Solve the inequality.

9.  $t + 5 < 7$

10.  $\frac{g}{2} \geq 6$

11.  $6t \leq 42$

12.  $\frac{1}{4} - p > \frac{3}{4}$

13.  $\frac{3}{8} + u \leq 4$

14.  $w \div 4 < 9$

15.  $62 + x \geq -8$

16.  $3v \leq 120$

17.  $14 + \frac{q}{2} > 3$

18.  $4 - 7r < 53$

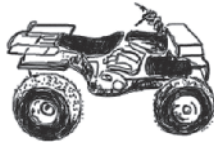


**Check-Up****Investigation 3****Inequalities**

1. Carmen and Tracie are spending the day at the beach. They are deciding whether to rent two beach ATVs or one dune buggy for the afternoon. The rental shop will rent equipment by the hour or by the half hour.

**Magic Mike's Beach Rentals**

**Beach ATV**  
(1 rider)



**\$30 per hour**

**Dune Buggy**  
(2 riders)

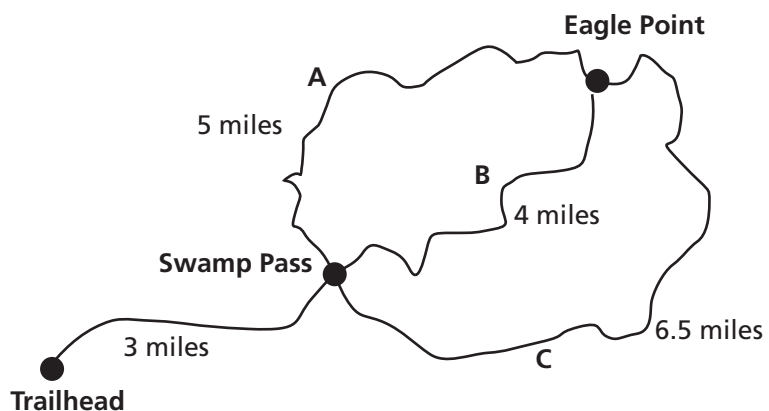


**\$50 per hour**

- a. Carmen and Tracie have agreed they want to spend no more than \$150 on rentals for the afternoon. Carmen says that with their budget of \$150, they can rent the ATVs for 5 hours or the dune buggy for 3 hours. Is she correct? Explain your reasoning.
- b. Tracie found a newspaper coupon for \$30 off any dune buggy rental of 2 hours or more. Write and solve an inequality to find how long they can rent the dune buggy and spend at most \$150, using the coupon. Graph the solution.
- c. Carmen suggests they rent the dune buggy for only 2.5 hours so they can save some money for dinner at The Pelican Grill. At The Pelican Grill, dinner will cost \$35 for the two of them. If the girls rent the dune buggy for 2.5 hours, and use the coupon, will they have enough money for dinner?

**Check-Up** *(continued)***Investigation 3****Inequalities**

2. You are going on a hike to Eagle Point. It will take you 45 minutes to hike from the trailhead to Swamp Pass. The amount of time it takes to get from there to Eagle Point along any trail is  $\frac{d}{r}$  where  $d$  is the distance, and  $r$  is the rate, in miles per minute, that you can hike the trail.



- Your goal is to make the entire hike from the trailhead to Eagle Point in under 3 hours. Write an inequality to show this time.
- You want the total distance you hike to Eagle Point to be under 9 miles. Write an inequality using  $d$  to represent the distance of a trail from Swamp Pass to Eagle Point. Graph the solution.
- Can you hike from the trailhead to Eagle Point, taking Trail A, in 3 hours or less? Write and solve an inequality to show the rate you would need to hike along Trail A. Graph the solution, and explain what the solution means.
- How fast would you need to hike along Trail B to make it from the trailhead to Eagle Point in the same time? Write and solve an inequality to answer the question.

# CC Investigation 3: Inequalities

An **inequality** is a mathematical sentence that compares two quantities that are not equal. Use the following symbols to represent inequalities:

- $<$  means “is less than.”
- $>$  means “is greater than.”
- $\neq$  means “is not equal to.”
- $\leq$  means “is less than or equal to.”
- $\geq$  means “is greater than or equal to.”

### Getting Ready for Problem 3.1

Like equations, you can write inequalities to represent a situation.

- How could you represent, *Lisa will spend less than \$25?*
- How could you represent, *Rodney ran at least 30 miles last week?*
- How could you use a number line to show *greater than 2?*

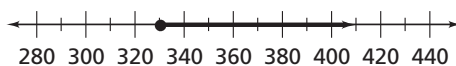
### Problem 3.1

Caitlyn volunteers with some friends at a community center.

- A.** Caitlyn is shopping online to find a new television for the center. Caitlyn wants a television with at least a 26-in. screen. Write an inequality to show how much money,  $m$ , the center will need to spend.

Television Prices	
Screen Size	Price
22 in.	\$300
26 in.	\$330
32 in.	\$370
40 in.	\$420

- B.** Caitlyn drew the graph below to represent the solution to her inequality.



1. Why do you think she drew a solid circle at 330?
  2. Caitlyn wants to have money left over for accessories. How can Caitlyn change her graph to show that they need to have more than \$330? Draw a graph to show how much they need to have.
- C.** The center has a stand for the television that will support up to 30 lb of weight. Draw a graph to show how much the television she buys can weigh.

### Notes

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**Getting Ready for Problem 3.2**

Solve an inequality just as you would an equation.

$$3x - 15 \geq 12 \quad \text{Write the inequality.}$$

$$3x - 15 + 15 \geq 12 + 15 \quad \text{Add 15 to each side of the inequality.}$$

$$3x \geq 27 \quad \text{Simplify.}$$

$$\frac{3x}{3} \geq \frac{27}{3} \quad \text{Divide each side of the inequality by 3.}$$

$$x \geq 9 \quad \text{Simplify.}$$

- How is solving the inequality above similar to solving the equation  $3x - 15 = 12$ ?

**Problem 3.2**

- A.** Caitlyn plans to use money from the community center's savings account to buy a video game system. She also needs to leave at least \$129 in the savings account to buy some softball equipment.

Video Game System Sale This Month Only  
\$250.00  
Selected Extra Games on Sale  
\$35.00 each

The inequality  $a - 129 \geq 250$  represents this situation, where  $a$  represents the amount of money the center has in its savings account.

1. What does the algebraic expression  $a - 129$  represent?
2. Solve the inequality. What does the solution mean?
3. Use a number line to show all possible values for  $a$ .

- B.** The center rents rooms for an hourly rate, plus a set-up fee.

Room Rentals		
Room	Rental Rate per Hour	Set-up Fee
Main Hall	\$15	\$40
Dining Room	\$12	\$80

1. A group has \$140 to spend. The inequality  $15h + 40 \leq 140$  represents the cost to rent the main hall, where  $h$  represents the number of hours the group can rent the room. Solve the inequality.
2. The same group also is considering renting the dining room. Write and solve an inequality to represent this situation.
3. Which room should the group rent? Explain.

**Notes**

**Getting Ready for Problem 3.3**

When the solution of an inequality requires dividing or multiplying by a negative number, you need to change the direction of the inequality sign. Compare these solutions.

$$\begin{array}{rcl} 12x < 60 & & -12x < 60 \\ \frac{12x}{12} < \frac{60}{12} & & \frac{-12x}{-12} > \frac{60}{-12} \\ x < 5 & & x > -5 \end{array}$$

- Choose a number included in the solution to  $12x < 60$ . Is this also a solution to  $-12x < 60$ ? Explain.

**Problem 3.3**

- A.** The community center has \$175 to spend on video games for its new game system. Games are on sale for \$35 each.
1. The inequality  $175 - 35g \geq 0$  represents the number of games the center could buy. Solve the inequality and explain the solution.
  2. Graph the solution on a number line.
  3. Explain why a value of  $g$  that is less than 0 does not make sense for this situation.
- B.** The center is considering signing up for an online game-rental service rather than buying the games. The table shows the equipment cost and monthly fees for two services.

Game Rental Services		
Service	Equipment Cost	Monthly Fee
NetGames	\$99	\$8
Anytime Games	\$19	\$19

1. The inequality  $175 - 8m \geq 99$  represents the number of months the center could rent games from NetGames with its \$175. Solve the inequality and explain the solution.
2. Graph the solution on a number line.
3. Write and solve an inequality to represent the number of months the center could rent games from Anytime Games.
4. Which service should the community center use? Explain your choice.

**Notes**


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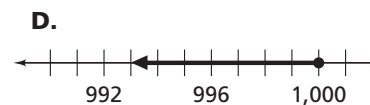
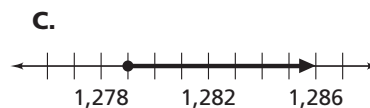
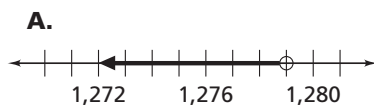
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## Exercises

For Exercises 1–4, use the information below.

Amusement Park Rides Height Requirements			
Ride	Height Requirement	Ride	Height Requirement
Jungle Jam	minimum of 40 in.	The Spiral	more than 35 in.
Tilt-a-Whirl	at least 48 in.	Ladybug	under 46 in.
Stargazer	more than 44 in.	Leapin' Lizard	at least 38 in.
Bunny Hop	60 in. maximum	Racetrack	over 42 in.

- Write and graph an inequality that represents the heights of people who can ride the Tilt-a-Whirl.
- Write and graph an inequality that represents the heights of people who can ride the Ladybug.
- Is it possible that someone is able to ride the Spiral but not the Leapin' Lizard? If so, give that person's height.
- Would someone be able to ride both the Tilt-a-Whirl and the Ladybug? Explain your answer.
- Playing a video game, Emily has gained some points, lost 107 points, and finishes at less than 800 points. The inequality  $p - 107 < 800$  represents this situation. Solve and graph the inequality.
- Multiple Choice** On level 2 of a video game, the maximum number of points is 1,000. Emily has lost 279 points and is on level 2. The inequality  $p - 279 \leq 1,000$  represents this situation. Which is the graph of its solution?



## Notes

For Exercises 7–9, use the information below.

Florida Hiking Trails		
Trail	Location	Length
Citrus Hiking Trail	Withlacoochee State Forest	43.3 miles
Myakka Hiking Trail	Myakka River State Park	49 miles
Big Oak Trail	Suwannee River State Park	12.5 miles
Florida Trail	Ocala National Forest	71 miles

7. Camille has a goal of hiking more than 350 miles this year. She already hiked the Florida Trail and now plans to hike 9 miles each day for  $d$  days. The inequality  $9d + 71 > 350$  represents this situation. Solve and graph the inequality.
8. **Multiple Choice** Camille’s brother Roberto hiked the Florida Trail with her and the Myakka Hiking Trail alone. He wants to hike no more than 400 miles this year and now plans to hike  $d$  day trips of 10 miles each. Which inequality could represent this situation?
- A.  $10d + 120 > 400$                       C.  $10d + 120 \leq 400$   
B.  $10d + 120 \geq 400$                       D.  $10d + 120 < 400$
9. Miquel does all of his hiking on the Big Oak Trail. He already has hiked it 3 times this year and has a goal of hiking at least 75 total miles this year. Write and solve an inequality representing the number of times  $b$  he still needs to hike the Big Oak Trail to reach his goal.
10. Jenna has \$39 to spend on materials to make pottery figures. It costs her \$4 to make one figure. Write and solve an inequality to represent this situation.

**Notes** \_\_\_\_\_  
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For Exercises 11–20, solve and graph each inequality.

11.  $x + 7 < 9$

12.  $y - 12 \geq 3$

13.  $7z - 49 > 98$

14.  $3a - 5 \leq 13$

15.  $-2b < 10$

16.  $-5c \geq -55$

17.  $p + 6.8 \geq 14$

18.  $36 < -1.8y$

19.  $\frac{1}{2} - 2x > 1$

20.  $4\frac{1}{4} \leq 5t + \frac{3}{4}$

21. **Multiple Choice** Which is the solution to the inequality  $3x - 12 > 9$ ?

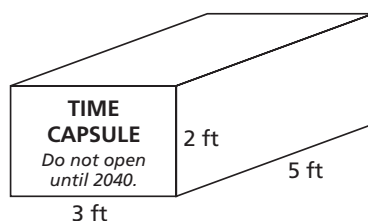
A.  $x > 21$

B.  $x < 7$

C.  $x > 7$

D.  $x > 3$

22. Janine is in charge of painting her school's time capsule. Her school's time capsule has a surface area that is less than or equal to the surface area of the time capsule shown below.



Janine already painted 6 square feet and needs to buy more paint to finish. Write and solve an inequality to show how much more area Janine needs to paint.

23. A machinist making steel rods for an airplane engine knows that each rod must be at least 9 mm long but no longer than 9.5 mm.

a. Write two inequalities that together represent the possible lengths. Graph the inequalities.

b. Can both solutions be shown on one graph? If so, draw the graph.

Notes \_\_\_\_\_

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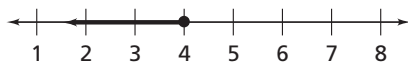
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For Exercises 24–26, use the information below.

Triathlon Distances	
Race Portion	Distance
Swim	2.4 mi
Bike	112 mi
Run	26.2 mi

- 24.** Pauline wants to finish the swim portion of the triathlon in 140 minutes or less. Write and solve an inequality to show the pace, in minutes per mile, that Pauline must swim.
- 25.** Athletes are given 630 minutes to complete both the swim and bike portions of the race. Pauline finishes the swim portion in 70 minutes. She averages a rate of  $b$  minutes per mile on the bike. The inequality  $112b + 70 \leq 630$  represents this situation. Solve the inequality and explain what the solution represents.
- 26.** Pauline completes the swim and bike portions in 538 minutes. Her goal is to complete the entire triathlon in 800 minutes or less. The inequality  $800 \geq 532 + 26.2r$  represents this situation, where  $r$  is the number of minutes it takes Pauline to run each mile. Graph the solution to the inequality.
- 27. Multiple Choice** An electrician has a roll with 45 ft of wire. She uses  $23\frac{1}{2}$  ft of the wire on one project, and will cut  $p$  3-ft pieces from the rest of the roll. Which inequality represents this situation?
- A.**  $3p + 23\frac{1}{2} \geq 45$                       **C.**  $3p - 23\frac{1}{2} \leq 45$
- B.**  $3p + 23\frac{1}{2} \leq 45$                       **D.**  $3p - 23\frac{1}{2} \geq 45$
- 28.** A bicycle shop rents bicycles for \$3.50 per hour, and helmets for \$6 per day. Martin has \$20 to spend to rent a helmet and a bicycle for  $h$  hours. The graph shows the solution to the inequality  $3.5h + 6 \leq 20$  representing this situation.



Explain what the graph shows about how long Martin can rent the bicycle.

## Notes

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**CC Investigation 4: Geometry Topics**

PACING 3 days

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**Mathematical Goals**

- Draw possible triangles when given three measures of their angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- Describe two-dimensional cross sections of three-dimensional figures.
- Use the formulas for the area and circumference of a circle to solve problems.
- Give an informal derivation of the relationship between a circle's area and its circumference.
- Use facts about complementary, vertical, and adjacent angles to write and solve simple equations for an unknown angle in a figure.

**Teaching Notes**

In this investigation, students will explore the geometry topics of cross sections, circles, drawing triangles, and special angle relationships.

You may want to have solids available to demonstrate cross sections. Students could create a table display of cross sections after you have cut these solids in different ways. Cutting solid shapes and then coloring the cuts is one way to help students understand what happens when solids are cut to make cross sections.

Review the parts of a circle using a diagram. Also, review the formulas for the circumference and area of a circle. When students are given the diameter of a circle, check that they are finding the radius before determining the area of the circle.

Drawing triangles with given conditions will require students to recall that the sum of the measures of the angles of any triangle is  $180^\circ$ . Also review with students the definition of similar figures and apply the definition to triangles. Students will use protractors and rulers to construct the triangles.

Students will use special relationships of complementary, vertical, and adjacent angles to find the missing measures of angles in a figure. The purpose of this investigation is to find the measures without actually measuring the angles, though giving students access to protractors will allow them to check their answers.

**Vocabulary**

- cross section
- diameter
- radius
- circumference
- complementary angles
- adjacent angles
- vertical angles

**Materials**

- geometric solids
- protractor
- ruler

**Problem 4.1**

Before Problem 4.1, distribute cylinder models to students, or display one for the class to examine. Ask:

- *What three-dimensional solid is this? (cylinder)*
- *What two-dimensional shape do you see when you look at one base? (circle)*
- *What two-dimensional shape do you see when you look at the side of the cylinder from any direction? (rectangle)*
- *How is cutting a block of cheese like making a cross section? (The resulting cut of a block of cheese reveals a cross section of the solid figure.)*

During Problem 4.1, ask: *What shapes of cheese should Marcus serve on rectangular or circular crackers? (rectangles or circles)*

**Problem 4.2**

Before Problem 4.2, ask:

- *Why is the numerical value of  $\pi$  written followed by three dots? (to show that it is not an exact value and the decimal digits go on forever with no repeating pattern)*
- *What is another approximation of pi that you know?  $\left(\frac{22}{7}\right)$*
- *If you want the answer for the circumference of a circle to be exact, what should you do? (Leave  $\pi$  in the answer.)*
- *Is the circumference a measure of length or of area? (a measure of length)*
- *Can two circles have the same radius but not have the same circumference? (No, every circle with the same radius has the same circumference.)*

After Problem 4.2, ask:

- *What type of units do you use to express area? (square units)*
- *Why can't you express the exact area of a circle using just numbers? (because the value used for  $\pi$  is an approximation)*
- *Use the ratio you wrote for Part B to write the ratio of the circumference of the dessert plate to its area.  $\left(\frac{2}{r} = \frac{2}{6} = \frac{1}{3}\right)$*

**Problem 4.3**

Before Problem 4.3, ask:

- *What do you know about the sum of the measures of the angles of any triangle? (It is always  $180^\circ$ .)*
- *How many different triangles with a given set of angle measures can be drawn? (an infinite number)*
- *What do you call two triangles which have the same angle measures, but different side lengths? (similar)*
- *What do you know about the side lengths of similar triangles? (Their side lengths are proportional.)*

During Problem 4.3 B, ask: *What do you know about the side lengths and angle measures of an isosceles triangle?* (Two of the sides are the same length, and two of the angles have the same measure.)

During Problem 4.3 C, ask: *From just looking at the sets, which measurements won't allow you to draw a triangle? If so, which one or ones?* (The side lengths of 2 in., 4 in., and 6 in. cannot make a triangle since the sum of the length of the two shorter sides is not greater than the length of the longest side. The angle measures of  $20^\circ$ ,  $40^\circ$ , and  $60^\circ$  can not make a triangle since their sum is not  $180^\circ$ .)

#### **Problem 4.4**

Review the definitions of the types of angles. Remind students that *supplementary angles* are two angles which have a sum of measures of  $180^\circ$ .

During Problem 4.4, ask:

- *How do you know the measures of the outside corners of the piece of poster board?* (The poster board is a rectangle, so the measure of each angle is  $90^\circ$ .)
- *How do you know that the measures of  $\angle j$  and  $\angle h$  are equal?* (We are told that the triangle with angles  $d$ ,  $h$ , and  $j$  is an isosceles triangle.)
- *What is the sum of the measures of angles that together form a straight line?* ( $180^\circ$ )

#### **Summarize**

To summarize the lesson, ask:

- *What two-dimensional shapes can be made by taking a cross section parallel to any face of a right rectangular prism?* (rectangles)
- *What are the formulas that relate a circle's area and circumference to its radius?* ( $A = \pi r^2$ ;  $C = 2\pi r$ )
- *What symbol does the expression for the area or circumference of a circle need to include for the measure to be exact?* ( $\pi$ )
- *How many different triangles could be drawn which have angle measures of  $35^\circ$ ,  $55^\circ$ , and  $90^\circ$ ?* (an infinite number)
- *How many different triangles could be drawn which have side lengths of 6 cm, 8 cm, and 10 cm?* (1)
- *How many different triangles could be drawn which have angle measures of  $70^\circ$ ,  $70^\circ$ , and  $50^\circ$ ?* (0)
- *What is the sum of the measures of complementary angles?* ( $90^\circ$ )
- *What is the measure of the angle that is vertical to an angle measuring  $45^\circ$ ?* ( $45^\circ$ )

## Assignment Guide for Investigation 4

Problem 4.1, Exercises 1–8

Problem 4.2, Exercises 9–15

Problem 4.3, Exercises 16–28

Problem 4.4, Exercises 29–36

## Answers to Investigation 4

### Problem 4.1

- A. 1. Slice the rectangular block to make rectangular slices; slice the cylindrical block to make rectangular slices.



2. The sizes and shapes of the slices of the rectangular block would not change. The rectangular slices from the cylindrical block would change size and shape, getting larger toward the center of the block, and smaller toward the edges.
- B. 1. Slice the rectangular block to make square slices; slice the cylindrical block to make circular slices.



2. No; the sizes and shapes of the slices from the rectangular block would not change.
- C. Marcus should cut the rectangular block into rectangular slices for the rectangular crackers and the cylindrical block into circular slices for the circular crackers so that the shapes of the slices match the shapes of the crackers. He also could slice the rectangular block into square slices to fit the circular crackers.

### Problem 4.2

- A. 1. Dinner plate:  $A = \pi r^2 = \pi(5)^2 = 25\pi$ ;  
dessert plate:  $A = \pi r^2 = \pi(3)^2 = 9\pi$ ;  
 $25\pi - 9\pi = 16\pi \text{ in.}^2$

2. 3 to 5, or  $\frac{3}{5}$

3. 9 to 25, or  $\frac{9}{25}$

- B.  $r^2$  to  $R^2$ , or  $\frac{r^2}{R^2}$

- C. 1.  $C = 2\pi r = 2\pi(5) = 10\pi \text{ in.}$

2.  $\frac{C}{A} = \frac{10\pi}{25\pi} = \frac{2}{5}$

3.  $\frac{C}{A} = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$

4. No, since  $\pi$  is a factor in both the numerator and the denominator of the ratio, it can be divided out of the ratio.

### Problem 4.3

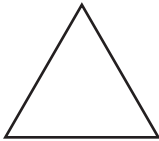
- A. 1. Check students' work. Side lengths will vary, but angle measures should be  $80^\circ$ ,  $80^\circ$ , and  $20^\circ$ .
2. No, the side lengths can vary and still keep the same angle measures.
- B. 1. Check students' work. Angle measures will vary, but the short side should be 6 in. long.
2. No, the lengths of the other two sides of the triangle can be any lengths longer than 3 in.
3. Check students' work. Yes, there is only one possible triangle with a side length of 6 in. and angle measures of  $75^\circ$ ,  $75^\circ$ , and  $30^\circ$ .
- C. 1. Check students' work. Only the third triangle is possible to draw.
2. Third triangle; that is the only triangle that can be drawn with the given measures. Its third angle measures  $80^\circ$ .

**Problem 4.4**

- A. angles  $a$  and  $d$ ; angles  $f$  and  $c$ ; angles  $e$  and  $b$
- B. Sample: angles  $a$  and  $b$ ; angles  $c$  and  $d$ ; angles  $e$  and  $f$
- C. Sample: angles  $g$  and  $h$
- D. 1. The corner of the poster board is the corner of a rectangle, so its measure is  $90^\circ$ . Angles  $j$  and  $k$  together make up the right angle.
2.  $j + h + d = 180$ ;  $j = h = 38$ ;  $38 + 38 + d = 180$ ;  $d = 104^\circ$ ; The sum of the angles of a triangle is  $180^\circ$ , so  $j + h + d = 180$ . Angle  $j$ 's measure is given as  $38^\circ$ , which also is the measure of angle  $h$ .
3.  $c + k + 90 = 180$ ;  $c + k = 90$ ;  $c + 52 = 90$ ;  $c = 38^\circ$ ; The sum of the angles of a triangle is  $180^\circ$ , so  $c + k + 90 = 180$ . Angle  $j$  and  $k$  are complementary, so the measure of angle  $k$  is  $52^\circ$ .
4.  $b + c + d = 180$ ;  $c = 38^\circ$ ,  $d = 104^\circ$ , so  $b + 38 + 104 = 180$ ;  $b = 38^\circ$ ; The sum of the angle measures is  $180^\circ$ , and I knew the measures of angle  $c$  and  $d$ .
5.  $a = d = 104^\circ$ ;  $e = b = 38^\circ$ ;  $f = c = 38^\circ$

**Exercises**

1. equilateral triangle



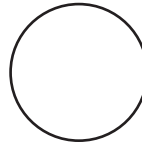
2. rectangle



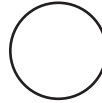
3. square



4. circle



5. circle



6. isosceles triangle



7. square



8. rectangle



9. a.
- $16\pi$
- ft

b.  $28\pi$  mmc.  $7.4\pi$  cmd.  $65\pi$  m

10. a.
- $100\pi$
- cm
- <sup>2</sup>

b.  $169\pi$  in.<sup>2</sup>c.  $70.56\pi$  mm<sup>2</sup>d.  $0.16\pi$  cm<sup>2</sup>

- 11.
- $81\pi$
- cm
- <sup>2</sup>

- 12.
- $24\pi$
- ft

13. a. \$84,403.20

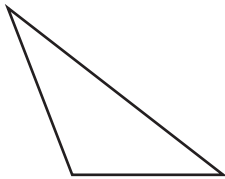
b. 125.6 ft

14. a. 90 revolutions

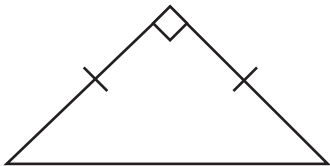
b. The area of the front wheel is 9 times greater than the area of the back wheel.

- 15.
- $(4 - \pi)$
- in.
- <sup>2</sup>

- 16. more than one triangle
- 17. no triangles
- 18. exactly one triangle
- 19. no triangles
- 20. more than one triangle
- 21. more than one triangle
- 22. more than one triangle
- 23. exactly one triangle
- 24. Sample:



- 25. Sample:



- 26. Check students' work.
- 27. Check students' work.
- 28. Design B, because that triangle is equilateral and a circle would fit best in it.

- 29. Sample: angles 1 and 2; angles 4 and  $45^\circ$
- 30. Sample: angles 1 and 4; angles 2 and 3
- 31. angles 1 and 4; angles 2 and  $45^\circ$
- 32.  $m\angle 1 = 45^\circ$ ; angles 1 and 2 are vertical, and angles 2 and  $45^\circ$  are complementary;  $m\angle 2 = 45^\circ$ ; angles 2 and  $45^\circ$  are complementary;  $m\angle 3 = 90^\circ$ ; angle 3 is vertical to a right angle;  $m\angle 4 = 45^\circ$ ; angle 4 is vertical to an angle with a measure of  $45^\circ$ .
- 33. No, the angles are complementary, so the sum of their measures is  $90^\circ$ , not  $180^\circ$ .
- 34.  $m\angle 1 = 102^\circ$ ;  $m\angle 2 = 38^\circ$ ;  $m\angle 4 = 50^\circ$ ;  $m\angle 5 = 52^\circ$
- 35. a.  $x + (x + 27) = 135$ ;  $2x + 27 = 135$ ;  $2x = 108$ ;  $x = 54^\circ$   
b.  $m\angle 1 = 45^\circ$ ;  $m\angle 2 = 45^\circ$ ;  $x = 54^\circ$ ;  $x + 27 = 81^\circ$
- 36. a.  $(x + 13) + (2x + 23) = 90$ ;  $3x + 36 = 90$ ;  $3x = 54$ ;  $x = 18$   
b.  $31^\circ, 59^\circ$

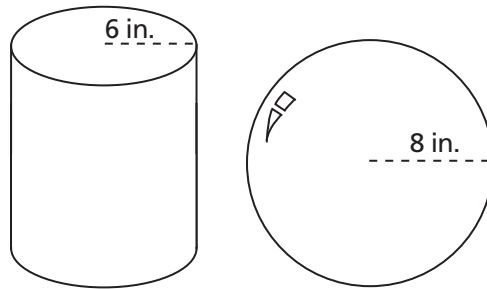


## Additional Practice

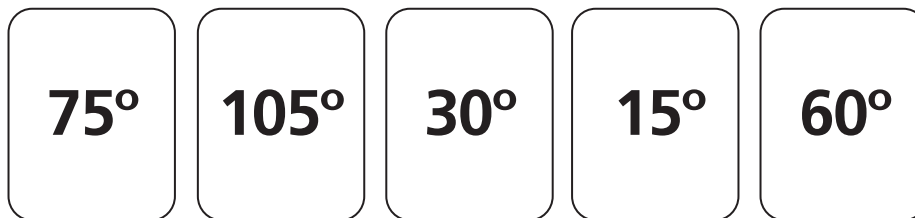
### Investigation 4

#### Geometry Topics

1. Lisa has the two solids shown below.



- Draw a cross section of the cylinder that is also a cross section of the sphere. Explain how the cross sections were taken.
  - Draw a cross section of the cylinder that is not a cross section of the sphere. Explain how you know.
  - Draw a cross section of the sphere that is not a cross section of the cylinder. Explain how you know.
2. Keith is playing a game called *Triangle Maker* and has these cards in his hand.



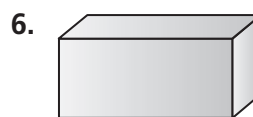
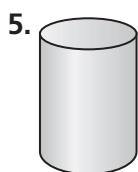
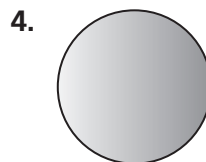
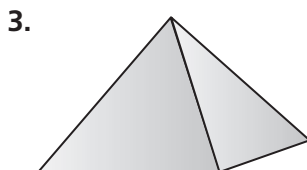
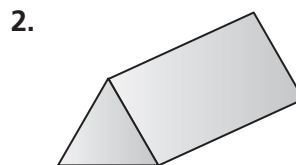
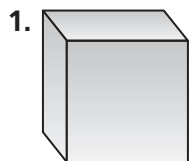
- Which three cards can Keith play as three angles to make a triangle?
- How many different triangles can be drawn that have the angle set that Keith played?
- After playing the triangle, Keith draws a card. What card does he need to make a triangle with his remaining two cards?

## Skill: Name Cross Sections

### Investigation 4

#### Geometry Topics

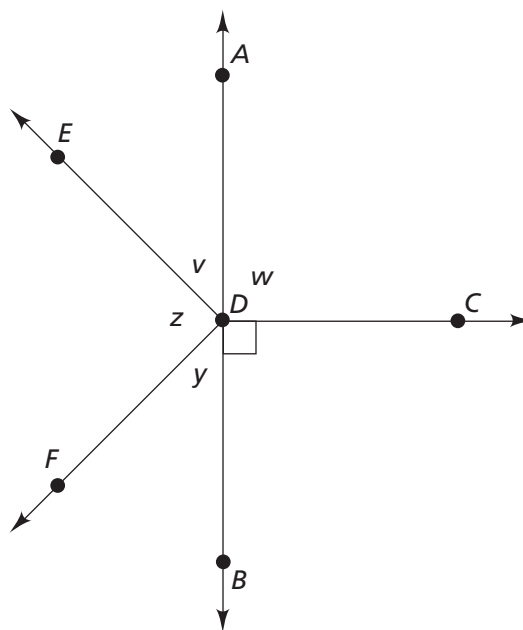
Name the cross section shapes that would result from a horizontal and a vertical cut of the figure.



## Skill: Angle Measures

Use the diagram at the right. Points *A* and *B* are on the same line.

7. Write another name for  $\angle ADE$ .
8. Which angle has a measure of  $90^\circ$ ?
9. What is the measure of  $\angle ADB$ ?
10. Write another name for angle *y*.
11. Name two angles that together make up  $\angle ADF$ .
12. Name two angles that together make up  $\angle ADB$ .
13. What do angles *w* and *y* have in common?
14. What do angles *v* and *z* have in common?
15. Name the angle that is made up of angles *z* and *v*.
16. Name the angle that is made up of  $\angle BDF$  and  $\angle FDE$ .

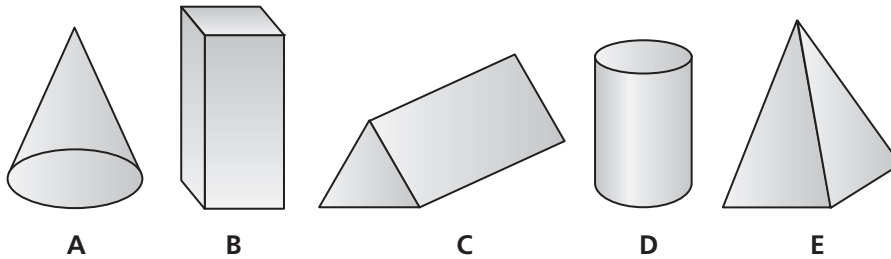


# Check-Up

## Investigation 4

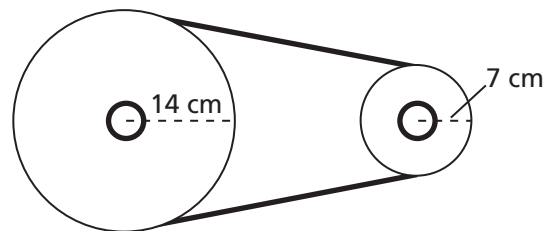
### Geometry Topics

1. Use the solid figures shown below.



- Which figures can have a circle as a cross section?
- Which figures can have a triangle as a cross section?
- Which figures can have a rectangle as a cross section?

2. An engine has two wheels that are joined by a belt. The belt makes the wheels turn at the same rate.



- Write a ratio of the area of the larger wheel to the area of the smaller wheel. Is the symbol  $\pi$  needed for the ratio? Explain why or why not.
- What part of a full turn does the large wheel complete for each full turn of the small wheel? Show your work.

### Check-Up *(continued)*

**Investigation 4**  
**Geometry Topics**

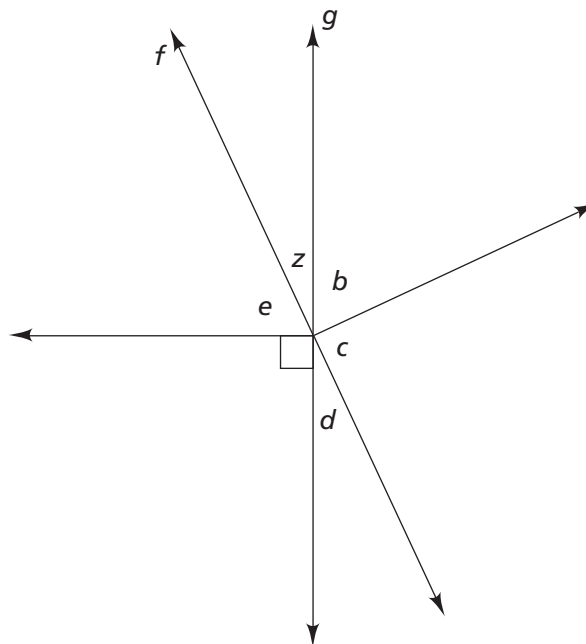
3. Ricardo is making a design using triangle patterns. He made the table below showing some of the angle measures he wants to use for the patterns.

Ricardo's Triangle Patterns				
Triangle Pattern	Type of Triangle	Angle 1	Angle 2	Angle 3
1		72°		55°
2	equilateral		60°	
3	isosceles		65°	65°
4	isosceles	102°		

- a. Complete the table.
- b. Explain whether you could find the missing angle measures for triangle 4 if it were not an isosceles triangle.

4. Line  $f$  intersects line  $g$ .

- a. Name a pair of vertical angles.
- b. Name two pairs of adjacent angles.
- c. Name a pair of complementary angles.
- d. The measure of  $\angle e$  is  $65^\circ$ , and  $\angle c$  is a right angle. Find the measures of all of the other angles. Explain how you found each measure.



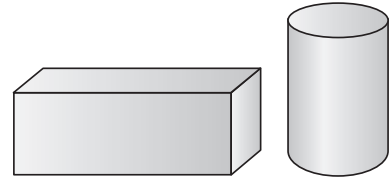
# CC Investigation 4: Geometry Topics

© DOMAIN: Geometry

When a three-dimensional figure is intersected by a plane, the resulting face is called a **cross section** of the figure. The two-dimensional shape of the new face depends on the slice that is taken of the three-dimensional figure.

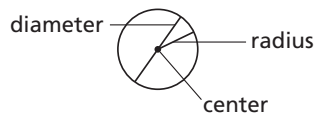
## Problem 4.1

Marcus is serving cheese and crackers at a party. He has one rectangular block of cheese and one cylindrical block of cheese. He wants to slice the cheese into different shapes.



- A. Marcus has a box of rectangular crackers.
  - 1. How should Marcus slice each block of cheese? Draw a picture of one slice from each block.
  - 2. Would the size or shape of the slices of cheese change as he slices through each block? If so, explain how they would change.
- B. Marcus has another box of crackers that are shaped like circles.
  - 1. How should Marcus slice each block of cheese? Draw a picture of one slice from each block.
  - 2. Would the size or shape of the slices of cheese change as he slices through each block? If so, explain how they would change.
- C. Which block of cheese should Marcus use for each box of crackers? Explain your choice.

A circle is a shape with all points the same distance from the center. A **diameter** is a line segment with endpoints on the circle and midpoint at the center. A **radius** is a line segment with endpoints at the center of the circle and on the circle.



The **circumference** is the distance around the circle. The ratio  $\frac{\text{circumference}}{\text{diameter}}$  for every circle is the number 3.14159... The exact value of this ratio is represented with the Greek letter  $\pi$ , pronounced “pi.”

Geometry Topics 19

STUDENT PAGE

Notes

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**Getting Ready for Problem 4.2**

Recall that the formula for the circumference of a circle is

$$\text{Circumference} = 2 \times \pi \times \text{radius, or } C = 2\pi r.$$

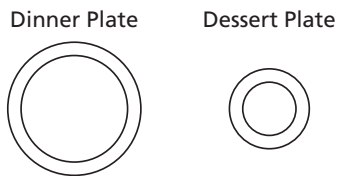
- What is the exact circumference of a helicopter landing pad with radius 15 meters? (Hint: Use  $\pi$  in your answer.)
- If you laid out the circumference of the landing pad in a straight line, exactly how many diameters could you fit on it?

**Problem 4.2**

**A.** The formula for the area of a circle is

$$\text{Area} = \pi \times \text{radius} \times \text{radius, or } A = \pi r^2.$$

- 1.** What is the exact difference in the areas of the two plates?



10-inch diameter    6-inch diameter

- 2.** What is the ratio of the radius of the dessert plate to the radius of the dinner plate?
- 3.** What is the ratio of the area of the dessert plate to the area of the dinner plate?
- B.** What ratio compares the area of a circular plate with radius  $r$  to the area of a circular plate with radius  $R$ ?
- C.** Consider the circumference and the area of the dinner plate.
- 1.** What is the exact circumference of the dinner plate?
- 2.** What is the ratio of the circumference of the dinner plate to its area?
- 3.** Use  $r$  to write the ratio of the circumference of any circle to its area.
- 4.** Will you need to estimate  $\pi$  to find this ratio for any circle? Explain why or why not.

**Notes**


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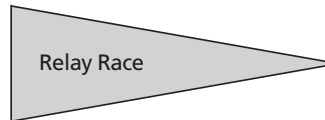
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**Problem 4.3**

Rebecca, Marcus, and Elian are making triangular flags to advertise events at Field Day.

- A.** Rebecca wants to make a flag that is the shape of an isosceles triangle. She thinks it will look best with angle measures of  $80^\circ$ ,  $80^\circ$ , and  $20^\circ$ .

1. Draw a triangle in the shape of Rebecca's flag on another piece of paper. What are the lengths of the sides of the triangle you drew?



2. Is the triangle you drew the only size flag Rebecca could make? If not, explain what other sizes she could make.

- B.** Marcus also wants to make a flag with the shape of an isosceles triangle, but he wants the short side of the flag to be 6 in. long to hang on a pole.

1. Draw a triangle in the shape of Marcus's flag on another piece of paper. What are the measures of the angles of the triangle you drew?



2. Is the triangle you drew the only possible isosceles flag Marcus could make? If not, explain how any other flags would be different.

3. Marcus decides he wants the angle opposite the 6-in. side to measure  $30^\circ$ . Draw a triangle in the shape of Marcus's flag on another piece of paper. Is the triangle you drew the only possible isosceles flag Marcus could make?

- C.** Elian likes even numbers and wants to use them as measurements for his flag. He is trying to decide which of these three sets of measurements he should use:

side lengths of 2 in., 4 in., and 6 in.

angle measures of  $20^\circ$ ,  $40^\circ$ , and  $60^\circ$

side length of 2 in., and angle measures of  $40^\circ$  and  $60^\circ$

1. On a separate piece of paper, try to draw each triangle that Elian is considering.
2. Which triangle should Elian choose for his flag? Explain how you decided.

**Notes**


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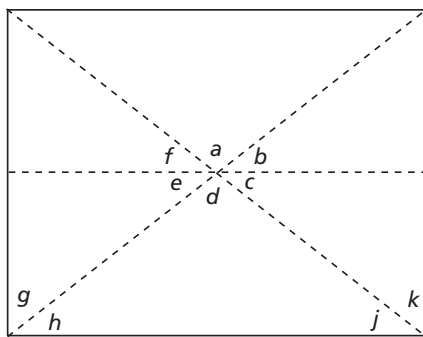


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Two angles are **complementary** if the sum of their measures is  $90^\circ$ . Angles are **adjacent** if they share a common side and a common vertex, but don't overlap. **Vertical** angles are pairs of non-adjacent angles that are formed by intersecting lines. Vertical angles are congruent.

### Problem 4.4

The diagram shows how Elian will use 3 straight cuts of a piece of rectangular poster board to make 2 isosceles triangles and 4 smaller right triangles for new flags.



- Name 3 pairs of vertical angles in the diagram.
- Name 3 pairs of adjacent angles in the diagram.
- Name a pair of complementary angles.
- The measure of  $\angle j$  is  $38^\circ$ . Elian knows that the sum of the measures of the angles of a triangle is  $180^\circ$ .
  - Elian writes the equation  $j + k = 90$ . Explain why this equation is true.
  - The measures of  $\angle j$  and  $\angle h$  are equal. Write and solve an equation to find the measure of  $\angle d$ . Explain your work.
  - The third angle of the triangle containing angles  $\angle c$  and  $\angle k$  is a right angle. Write and solve an equation to find the measure of  $\angle c$ . Explain your work.
  - Angles  $b$ ,  $c$ , and  $d$ , form a straight line. Write and solve an equation to find the measure of  $\angle b$ . Explain your work.
  - Use the properties of vertical angles to find the measures of angles  $a$ ,  $e$ , and  $f$ .

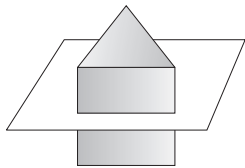
## Notes



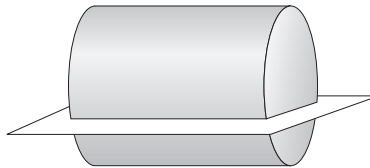
## Exercises

For Exercises 1–8, name and sketch the shape of each cross section.

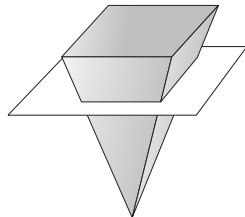
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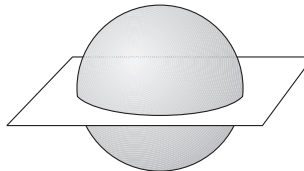
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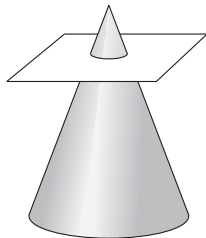
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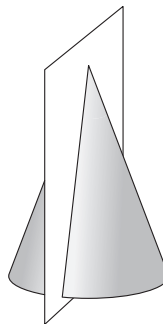
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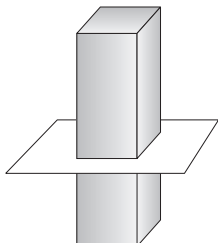
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6.



7.

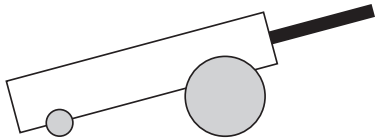
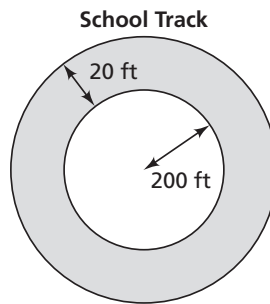


8.

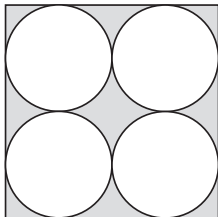


**Notes** \_\_\_\_\_  
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9. Find the exact circumference of each circle with the given radius or diameter.
- a. radius of 8 ft                                      b. diameter of 28 mm  
c. diameter of 7.4 cm                                d. radius of 32.5 m
10. Find the exact area of each circle with the given radius or diameter.
- a. diameter of 20 cm                                b. radius of 13 in.  
c. diameter of 16.8 mm                            d. radius of 0.4 cm
11. What is the exact area of a circle with a circumference of  $18\pi$  cm?
12. What is the exact circumference of a circle with an area of  $144\pi$  square feet?
13. a. The school board has voted to have the school track repaved. What is the total expense if the charge is \$3.20 per square foot? Use 3.14 for  $\pi$ .
- b. Approximately how much farther is it to run a lap along the outer edge than around the inner edge of the track? Use 3.14 for  $\pi$ .
14. The front wheels of a cart have a diameter that is 3 times the diameter of the back wheels.



- a. The front wheel makes 30 revolutions. How many revolutions does the back wheel make?
- b. How many times greater is the area of the front wheel than the area of the back wheel?
15. Raffle uses a circular cookie cutter with a diameter of 1 inch to cut four cookie shapes out of a 2-inch square sheet of dough. Exactly how many square inches of dough are left over?

**Notes**


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**For Exercises 16–23, tell whether *exactly one triangle, more than one triangle, or no triangles* are possible using the given measurements.**

16. angle measures:  $50^\circ$ ,  $30^\circ$ , and  $100^\circ$
17. angle measures:  $39^\circ$ ,  $21^\circ$ , and  $110^\circ$
18. side lengths: 3 ft, 5 ft, and 4 ft
19. side lengths: 7 cm, 1 cm, and 9 cm
20. angle measures:  $45^\circ$  and  $60^\circ$
21. side lengths: 6 in. and 6 in.
22. angle measures:  $57^\circ$  and  $43^\circ$ ; any side length: 4.3 in.
23. angle measures:  $40^\circ$  and  $40^\circ$ ; side length between those angles: 10 cm

**For Exercises 24–25, sketch the given type of triangle.**

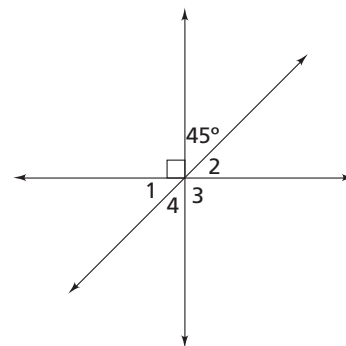
24. obtuse scalene
25. isosceles right

**For Exercises 26–27, draw the triangle with the given dimensions using a protractor, ruler, or compass.**

26. The side lengths are 2.5 in., 4 in., and 5 in.
27. Two angles measure  $50^\circ$  and  $40^\circ$ , and the side length between them is 3 in.
28. You are designing a logo that will include a triangle around a circular graphic of the earth. Which set of dimensions would you choose for the triangle? Explain your choice.
  - Design A side lengths 6 cm, 7 cm, and 10 cm
  - Design B side lengths 8 cm, 8 cm, 8 cm
  - Design C side lengths 2 cm, 7 cm, 7 cm

**For Exercises 29–32, use the diagram at the right.**

29. Identify two pairs of vertical angles.
30. Identify two pairs of adjacent angles.
31. Identify two pairs of complementary angles.
32. What are the measures of angles 1, 2, 3, and 4? Explain how you found each angle measure.



## Notes

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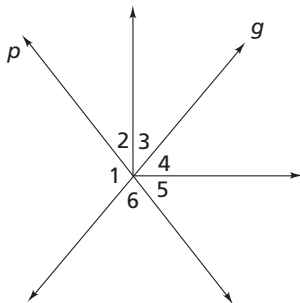


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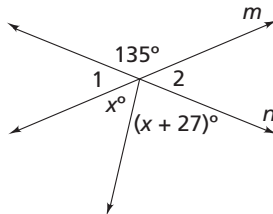
- 33.** An angle measuring  $x^\circ$  and an angle measuring  $63^\circ$  are complementary. A student wrote and solved the equation below to find the measure of angle  $x$ . Is the student correct? Explain why or why not.

$$\begin{aligned}x + 63 &= 180 \\x + 63 - 63 &= 180 - 63 \\x &= 117\end{aligned}$$

- 34.** Line  $p$  intersects line  $g$ . The measure of angle 3 is  $40^\circ$ , the measure of angle 6 is  $78^\circ$ , and angles 3 and 4 are complementary. Find the measures of the other angles.



- 35.** Line  $m$  intersects line  $n$ .



- a.** Write and solve an equation to find the value of  $x$ .  
**b.** What is the measure of each of the unknown angles?
- 36.** An angle measures  $(x + 13)^\circ$ . Another angle complementary to it measures  $(2x + 23)^\circ$ .
- a.** Write and solve an equation to find the value of  $x$ .  
**b.** What is the measure of each angle?

## Notes

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## CC Investigation 5: Variability

PACING 3 days

### Mathematical Goals

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- Understand that information can be gained about a population by examining statistics of a representative sample of the population, where random sampling tends to produce representative samples.
- Draw inferences about a population based on data from a random sample.
- Generate or simulate multiple samples of the same size to gauge the variation in estimates or predictions.
- Informally assess the degree of visual overlap of two data distributions with similar variabilities and express the difference between centers of the distributions as a multiple of a measure of variability.

### Teaching Notes

In this investigation, students will build on their previous work with single data distributions to compare two data distributions and answer questions about differences the distributions suggest about the populations. Students will begin informal work with random sampling to generate data sets and explore the importance of representative samples for drawing inferences.

#### Vocabulary

- representative sample
- interquartile range
- mean absolute deviation

#### Problem 5.1

Before Problem 5.1, present the following scenario to students:

*Plains Middle School is considering the following locations for a Grade 7 field trip: science museum, state park, or ballet company. The principal wants to survey a sample of students to find which location Grade 7 students would prefer. Ask:*

- *Should the principal select the members of the science club for the sample?* (No; not everyone in the population has an equal chance of being selected, and their opinions might be biased in favor of the science museum.)
- *Should the principal survey every tenth student to walk into the middle school on a given morning?* (No, because that sample would include students outside the population of seventh graders.)
- *Should the principal ask a group of randomly-selected Grade 7 girls?* (No, not everyone in the population has an equal chance of being selected.)
- *How can the principal get a random sample of Grade 7 students?* (Answers will vary, leading to a discussion of the principles of how to generate randomness in samples.)

*During Problem 5.1 B, ask: How do you find the mean of a set of data?* (Divide the sum of the data values by the number of values in the set.)

During Problem 5.1 C, ask:

- How do you find the probability that any one filling will be randomly chosen? (Divide 1 by the total number of choices, 6:  $1 \div 6 = \frac{1}{6}$ )
- What are the possible outcomes of tossing a number cube? (1, 2, 3, 4, 5, and 6)
- Why is an experiment with a number cube a good test of your conclusion in Part 1? (A number cube also has 6 equally-likely outcomes.)

### **Problem 5.2**

Before Problem 5.2, review with students how to read a box plot. Ask:

- What was the least amount Tim earned? (\$35)
- What was the greatest amount Tim earned? (\$115)
- How do you find the interquartile range for data shown in a box plot? (Subtract the lower quartile, indicated by the left side of the box, from the upper quartile, indicated by the right side of the box.)

During Problem 5.2 B, ask: *What do the range and interquartile range tell you about how data varies?* (The range sizes tell you how spread out the data are.)

After Problem 5.2 C, ask: *What does a symmetrical box plot tell you about how data vary?* (Data in the first and fourth quarters are distributed over the same range and data in the second and third quarters are distributed over the same range.)

During Problem 5.2 F, ask: *What does a cluster of data tell about a data set?* (More data is distributed over one small interval than over other intervals.)

### **Problem 5.3**

During Problem 5.3 A, ask: *How are the distributions the same and how are they different?* (The interquartile ranges are the same and the medians are different.)

Before Problem 5.3 B, review with students how to read a dot plot. Ask:

- What do the dots above the number 40 in the Bountiful Bistro plot represent? (Five diners spent 40 minutes at dinner at Bountiful Bistro.)
- How can you tell on each plot how long most of the diners spent at dinner? (Find the number with the most dots above it.)
- How can you find the median value? (Count the total number of dots, then count dots beginning at the lowest value until you reach the middle value.)

During Problem 5.3 B Part 3, ask: *How does the size of the interquartile range translate visually onto a dot plot?* (The greater the interquartile range, the more spread out the data will appear.)

## Summarize

To summarize the lesson, ask:

- *What is the best way to produce a sample that will support valid inferences about the population?* (a random sample in which every member of the population has an equal chance of being included)
- *How does the sample size affect how accurate predictions about the population sampled will be?* (The larger the sample size, the more reliable the results.)
- *What can you compare about two data sets just by looking at their distributions on a box or dot plot?* (medians and relative sizes of mean absolute deviations)

Students in the CMP2 program will further study standards 7.SP.1, 7.SP.2, and 7.SP.3 in the Grade 8 Unit *Samples and Populations*.

## Assignment Guide for Investigation 5

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Problem 5.1, Exercises 1–6, 12–14

Problem 5.2, Exercises 7–11, 15–18

Problem 5.3, Exercises 19–29

## Answers to Investigation 5

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### Problem 5.1

- A.**
1. No; by selecting only women in business attire, members of the population who are male or not wearing business attire have no chance of being selected.
  2. No; selecting diners only at lunch excludes breakfast or dinner diners who also are part of the population.
  3. Survey every tenth diner all day. Each member of the population would have an equal chance of being selected.
- B.**
1. Casual Café: 38.4; Bountiful Bistro: 21.75; The mean for Casual Café is much greater, so that restaurant attracts older diners on average.
  2. Casual Café: Because younger, college-age, diners are attracted to the restaurant, advertise in local college newspapers or on-line and by email; Bountiful Bistro: Since the restaurant attracts older diners, advertise in the newspaper or local magazines.
- C.**
1. 1 or 2 turkey sandwiches; In 6 orders, I would expect  $\frac{1}{6}$ , or 1, turkey sandwich, and in 12 orders I would expect  $\frac{2}{12}$ , or 2, turkey sandwiches.
  2. Check students' work.
  3.  $50 \times \frac{1}{6} = 8\frac{1}{3}$ ; about 8 turkey sandwiches; Check students' work.
  4. Check students' work.
  5. Check students' answers; Student outcomes generally should be closer to theoretical probabilities with more trials.

### Problem 5.2

- A.** Tim: range =  $115 - 35 = \$80$ ,  
interquartile range =  $105 - 80 = \$25$ ;  
Dan: range =  $115 - 45 = \$70$ ,  
interquartile range =  $100 - 60 = \$40$
- B.** The ranges show that Tim's earnings, at their extremes, vary more widely than Dan's. But the interquartile ranges show that Tim's earnings are more clustered around the median value, which is higher than Dan's.

- C. Yes; the plot for Dan's earnings is symmetric.
  - D. Dan's earnings are more widely distributed, while Tim's earnings are more clustered.
  - E. The plot for Tim's earnings shows clustering about the median value, especially from 80 to 90.
  - F. Tim more consistently earns about the same amount in tips, while Dan's earnings are much less consistent, though they do not vary as greatly in the extremes as Tim's earnings.
  - G. Tim; the lower quartile of his tips is equal to the median of Dan's tips, so the overall earnings should be greater.
- 6. about 4 minutes; Since the songs are randomly selected, each sample should yield close to the same results. A song length of about 4 minutes is between the means of the first two samples.
  - 7. a. Casual Café: \$16; Bountiful Bistro: \$12.50  
b. Casual Café: \$22.50; Bountiful Bistro: \$20  
c. Casual Café: \$7.50; Bountiful Bistro: \$12.50
  - 8. Customers at Casual Café generally spent more than customers at Bountiful Bistro.
  - 9. At Casual Café, the difference between the least and greatest amounts is greater than at Bountiful Bistro, but the data are more clustered around the median.

### Problem 5.3

- A. 1. The values for Bountiful Bistro are generally less than the values for Casual Café, so the cost of dinner at Bountiful Bistro generally is less than the cost of dinner at Casual Café.
- 2. a. Bountiful Bistro: median = \$15, interquartile range =  $20 - 10 = \$10$ ;  
Casual Café: median = \$25, interquartile range =  $30 - 20 = \$10$   
b.  $25 - 15 = \$10$
- 3. Yes, the median price at Casual Café is greater than the median price at Bountiful Bistro.
- B. 1. Diners at Bountiful Bistro generally spend more time at dinner than diners at Casual Café.
- 2. Bountiful Bistro: median = 50 min; Casual Café: median = 20 min;  $50 - 20 = 30$  min
- 3. Bountiful Bistro; the data are more spread out for Bountiful Bistro, with a range of 70 min, than the data for Casual Café, with a range of 40 min.
- 10. The data for Casual Café are symmetric, so the amounts spent are evenly distributed. The data for Bountiful Bistro are not symmetric, so those amounts are less evenly distributed.
- 11. The data for Casual Café show clustering about the median, meaning more customers spend close to the median than customers do at Bountiful Bistro. The data for Bountiful Bistro show a cluster between \$5.00 and \$7.50; 25% of the purchases at this restaurant are in this range.
- 12. 17 cats
- 13. 32 dogs
- 14. 20 cats; 20 is closer to the mean for cats than the mean for dogs.
- 15. Casual Café is open 6 days each week and Bountiful Bistro is open 7 days each week. In the graph for Casual Café, there are 0 customers for Monday.
- 16. B
- 17. Both restaurants show peaks of numbers of customers on Saturdays.
- 18. The data for Casual Café are not symmetrically distributed, while the data for Bountiful Bistro are symmetric.
- 19. Sets A and C
- 20. Sets A and B
- 21. The review was effective because the students taking the review class scored higher than those who did not take the class. The box plots are the same size and shape, with the Review Class shifted right and showing higher scores.

### Exercises

- 1. No; your friends are not representative of the population.
- 2. Yes; every 20th student is representative of the population.
- 3. Yes; students at different grades is representative of the population.
- 4. No; the soccer team is not representative of the population.
- 5. Sample: Randomly select names by choosing every fifth name from a list of names of students in your class.

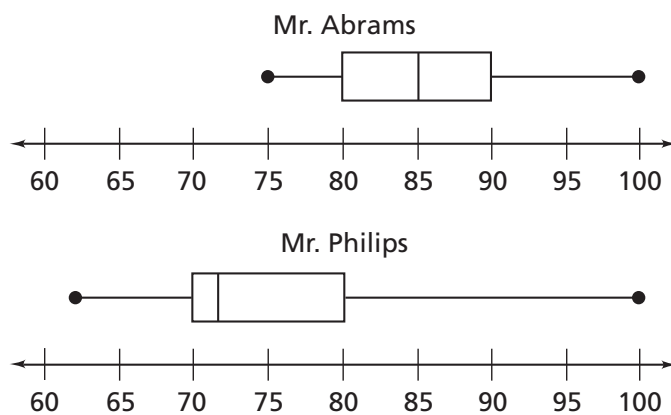


22. 10%
23. The difference in medians is almost twice the mean absolute deviation, which shows there is substantial difference between the data sets.
24. Player A: mean = 26 points; Player B: mean = 18 points; difference = 8 points
25. Player A: MAD = 2.4; Player B: MAD = 2.2
26. about three-and-a-half times as great
27. No, the data values show no overlap.
28. Yes, the mean absolute deviation would be more than twice the difference in means, so there should be a lot of overlap.
29. No; the difference in means is so much greater than the mean absolute deviations that there shouldn't be much overlap.



**Additional Practice****Investigation 5****Variability**

1. Rachel and Dwayne are running for Grade 7 President. Rachel is active in the drama club, and Dwayne plays on the school basketball team. Tell whether each sample would support a valid prediction about the outcome. Explain your reasoning.
  - a. a random sampling of members of the drama club
  - b. every member of the drama club and each player on the basketball team
  - c. a random sampling of Grade 7 girls
  - d. five randomly-selected students from each Grade 7 homeroom
  - e. every fifth student to walk into the middle school on a given morning
  
2. The box plots show the grades on history tests in two classes.



- a. What comparisons can you draw from the plots about the grades received by the two classes?
- b. What is the range and interquartile range of the data displayed in each box plot?
- c. What is the difference in the median values for the data sets?
- d. Use the ranges, interquartile ranges, and medians to compare how the grades between the classes vary.

## Skill: Representative Samples

### Investigation 5

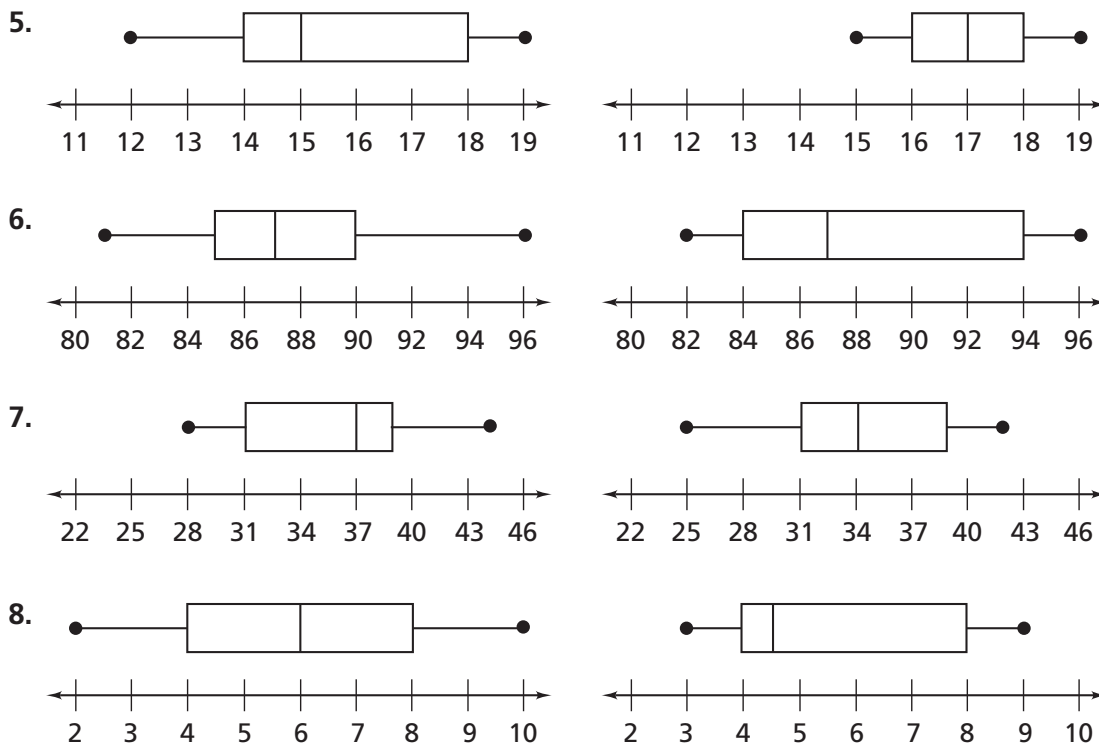
#### Variability

**Tell whether the sampling method will result in a representative sample. Explain your reasoning.**

1. Nicole is planning the games for a carnival for the first, second, and third graders at Bay Elementary School. To find out which games students would like to have, she asks 25 first graders during their lunch.
2. Mr. Williams is deciding what books to offer for the next book club for his seventh- and eighth-grade English classes. He randomly asks ten students from each of his classes to make their choices from three different books.
3. A city is surveying its residents to find out if an open space should be developed into a park or an office building. The city sends surveys to 100 randomly-selected residents of the city.
4. Alyssa is doing research for a report about the after-school activities of students at her school. She interviews every fifth student entering the gym after school.

## Skill: Reading Box Plots

**Compare the medians and interquartile ranges of each pair of plots, and explain what conclusions you can make about the data sets based on those measures.**



**Check-Up****Investigation 5****Variability**

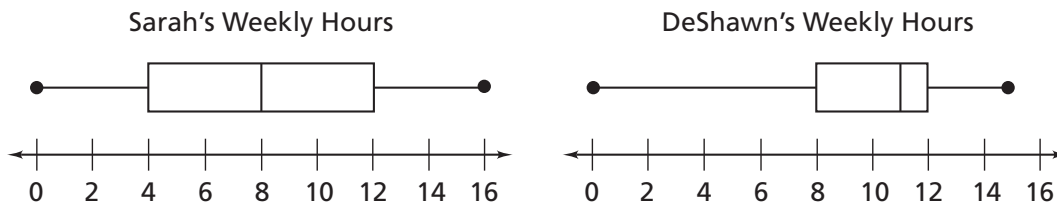
1. Brahim and Miguel are conducting a survey. They ask the question, “Which sport is your favorite to watch: soccer, basketball, or volleyball?”
  - a. Brahim wants to ask 25 of his classmates as they leave a basketball game. Will his results be reliable? Explain.
  - b. Miguel wants to ask 1 randomly-selected student from each of 5 gym classes. Will his results be reliable? Explain.
  - c. Describe a sampling method Brahim and Miguel could use to have the best chance of producing a representative sample for their survey.
2. Riders on a subway system can get on one of four different subway lines. Rita works for the subway system and wants an accurate prediction of how many of the 10,000 riders who use the station each day take each line.
  - a. Rita watches where 10 riders go, and sees that 2 take the green line. Based on Rita’s sample, how many riders would you predict take the green line each day? Explain your reasoning.
  - b. Rita watches the next 10 riders, and sees that 6 of them take the green line. Based on this sample, how many riders would you predict take the green line each day? Explain your reasoning.
  - c. Rita takes a third sample, of 200 riders, and sees that 70 of them take the green line. Based on this sample, how many riders would you predict take the green line each day? Explain your reasoning.
  - d. Were your estimates different? If so, explain why they were different. Tell which estimate you think is most accurate and explain your reasoning.

### Check-Up *(continued)*

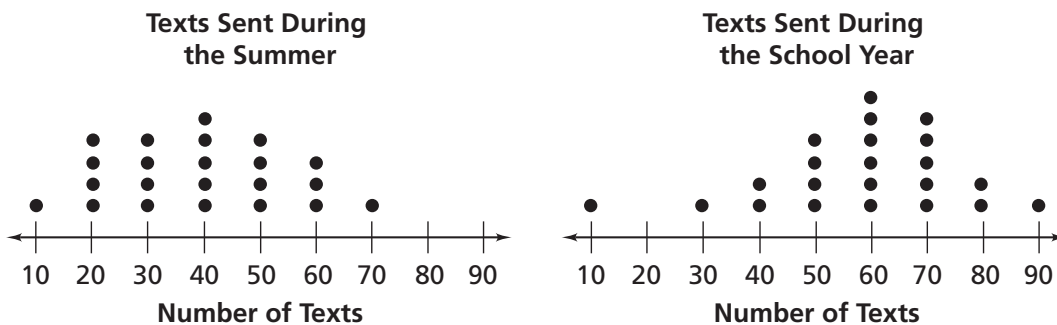
**Investigation 5**

**Variability**

3. Sarah and DeShawn work part-time at the bowling alley. The box plots show the number of hours they have worked each week this year.



- Find the ranges and interquartile ranges of the hours Sarah and DeShawn have worked, and use them to compare how their hours varied.
  - Compare how the amounts of hours Sarah and DeShawn worked are distributed.
  - Does either box plot show clustering or symmetry of data? If so, what does that show about the numbers of hours worked?
4. The dot plots show the numbers of texts, to the nearest 10, some students sent one week in the summer and one week during the school year.



- What comparisons can you draw from looking at the plots about the numbers of texts sent during each time of year?
- What is the difference in the median value for each set of data?
- For which set of data would you expect a greater interquartile range? Explain your answer.

# CC Investigation 5: Variability

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You can collect data from a random sample of a given population and use that data to make inferences about the population as a whole. Inferences will be valid only if the sample is representative of the population.

A sample is **representative** if every member of the population has an equal chance of being included in the sample. Random sampling is the best way to produce a representative sample that will support valid inferences.

## Problem 5.1

- A.** The owners of the Casual Café and the Bountiful Bistro want to know more about the types of customers that dine at their restaurants. They each conduct a survey to find their customers' ages and the price they would expect to pay for an entrée.
1. Suppose the owners took their samples by surveying the first fifteen women dressed in business attire. Do you think this sample is representative of the population? Explain.
  2. Suppose the owners took their samples by surveying every fifth customer at lunch. Do you think this sample is representative of the population? Explain.
  3. Describe a survey method that would give the restaurant owners a representative sample of the population. Explain how you decided on your method.
- B.** The table shows age data the owners gathered from a representative sample at each restaurant.

Casual Café	34	41	45	67	23	19	45	34	32	35	34	56	63	23	25
Bountiful Bistro	29	17	23	18	14	28	21	24							

1. What is the mean age for each restaurant's customers? What do the mean ages tell you about the customers that each restaurant attracts?
2. The owners are deciding how to advertise their restaurants. They want to advertise to the group of customers that they expect will want to dine with them. Give some recommendations to each owner about how and where they should advertise.

Variability 27

STUDENT PAGE

Notes

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Variability (27)46

**C.** Diners at Casual Café can make their own sandwiches starting with 1 of 6 fillings.

- 1.** If the sandwiches are randomly chosen, how many turkey sandwiches do you expect there will be in the next 10 sandwich orders? Explain how you found your answer.
- 2.** Do an experiment to test your conclusion. Toss a number cube 10 times and record the outcomes in a table. Did the number of times you tossed a 3 match your prediction for the number of turkey sandwiches ordered? Explain why or why not.
- 3.** How many turkey sandwiches would you expect out of the next 50 random sandwich orders? Do another experiment to test this conclusion. Toss a number cube 50 times and record the outcomes in a table.
- 4.** Repeat the experiment for another 50 tosses. Record the outcomes in a separate table.
- 5.** Were the experiments' outcomes closer to your predictions for 10 orders or for 50 orders? Explain why that might be so.

Make-Your-Own  
Sandwiches

1. Roast beef
2. Ham
3. Turkey
4. Grouper
5. Veggie
6. Hummus

## Notes

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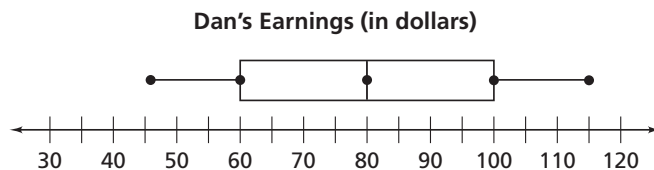
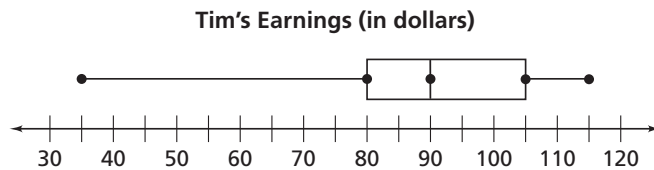
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You can use measures of variability, measures of center, and shape to compare the data displayed in two related graphs.

### Problem 5.2

Tim is on the wait staff at the Casual Café, and Dan is on the wait staff at Bountiful Bistro. The box plots below display the amounts they earned in tips on weekends during the past six months.



- A.** What is the range and interquartile range of the data displayed in each box plot?
- B.** Use the ranges and interquartile ranges of Tim's and Dan's tips. Compare how their tips vary.
- C.** Are either of the box plots symmetric?
- D.** Compare how the amounts of Tim's and Dan's tips are distributed.
- E.** Which, if any, of the box plots shows clusters of data?
- F.** Use the evidence of clusters or no clusters to compare Tim's and Dan's tips.
- G.** Overall, who do you think earns more tip money? Explain.

Notes \_\_\_\_\_

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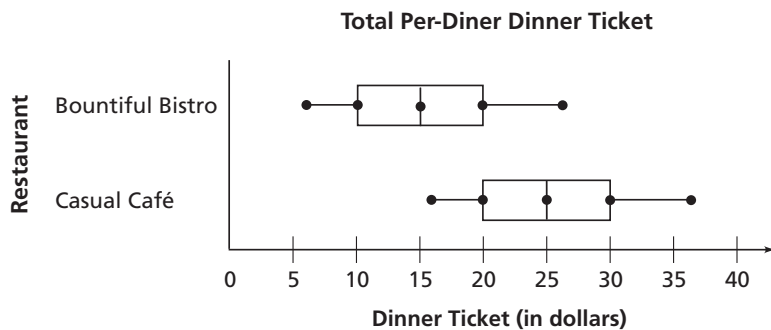
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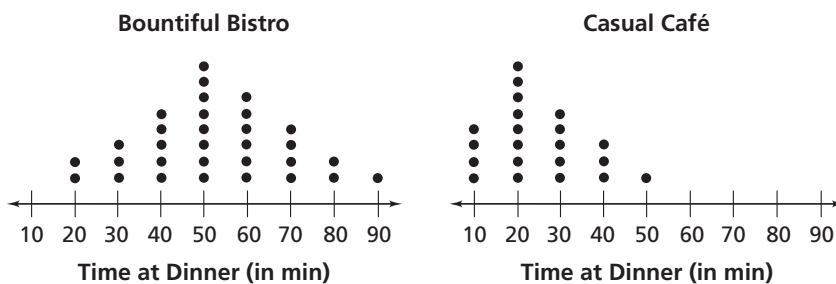
You can use measures of variability, such as interquartile range and mean absolute deviation, to make sense of data sets, both numerically and visually.

### Problem 5.3

A. The box plot compares the dinner ticket amounts for the two restaurants.



- Compare the distributions of the data shown in the box plot. What conclusions can you draw about the cost of dinner?
    - Find the median value and the interquartile range for each restaurant.
    - What is the difference in the medians?
  - Do the results you found support the conclusions you made about the data? Explain why or why not.
- B. The dot plots show the lengths of time, to the nearest 10 minutes, some diners spent at dinner at each restaurant.



- What comparisons can you draw from looking at the plots about the time diners spend having dinner at the restaurants?
- What is the difference in the median value for each set of data?
- For which set of data would you expect a greater interquartile range? Explain your answer.

## Notes

## Exercises

For Exercises 1–4, tell whether the sample is representative of the population.

1. You want to know what type of music students at your school like best. You ask a group of your friends which music they like best.
2. You want to know which type of food students at your school like best. You ask every 20th student in your school yearbook.
3. You want to know how many hours students at your school spend on the computer each day. You ask students from different grades as they leave school.
4. You want to know how many hours students at your school exercise each week. You ask the members of the soccer team how often they exercise each week.
5. Suppose you are taking a poll of students in your grade to see whom they are going to select in the election for president of your class. Describe one way you could find a sample that is representative of the population.
6. A student is trying to determine the average length of a song in her large music library. She randomly selects 20 songs and finds that the mean length is 4 minutes 9 seconds. Then, she randomly selects another 20 songs and finds that the mean length is 3 minutes 52 seconds. What would you expect the mean length of a third set of 20 songs would be? Why?

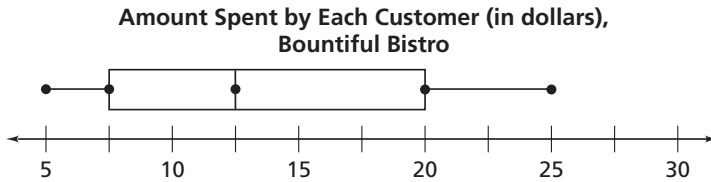
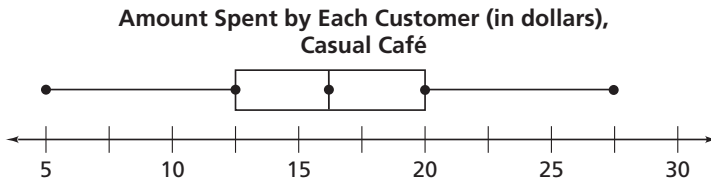
## Notes

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For Exercises 7–11, use the data displayed in the box plots below.



7. Find the following for each set of data.
  - a. median
  - b. range
  - c. interquartile range
8. Use the medians of the data to compare the amounts spent by customers at each restaurant.
9. Use the ranges and interquartile ranges of the data to compare how the amounts spent by customers at each restaurant vary.
10. Use the symmetry or lack of symmetry in each box plot. Compare how the amounts spent by customers at each restaurant are distributed.
11. Use the evidence of clusters or no clusters to compare the amounts spent by customers at each restaurant.

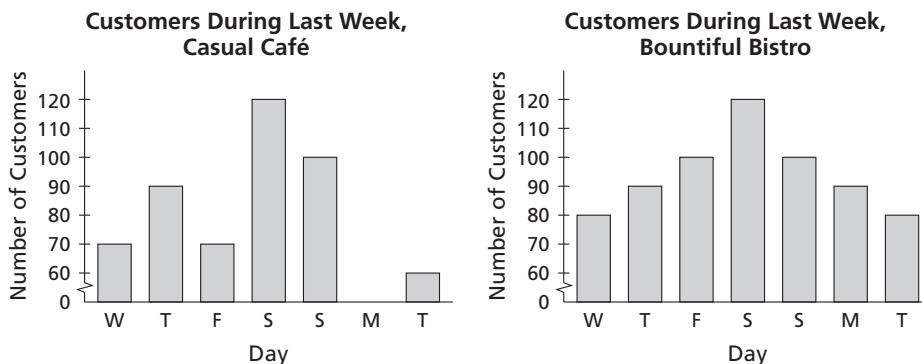
For Exercises 12–14, use the information about the numbers of cats and dogs that were adopted at a local shelter each month last year.

<b>Cats</b>	12	13	16	18	21	15	14	13	15	22	19	26
<b>Dogs</b>	25	30	38	29	27	40	33	26	32	34	41	29

12. What is the mean number of cats adopted each month?
13. What is the mean number of dogs adopted each month?
14. A worker knows that either 20 cats or 20 dogs were adopted one month recently. Based on your answers to Exercises 12 and 13, do you think it was 20 cats or 20 dogs? Explain.

## Notes

For Exercises 15–18, use the data displayed in the bar graphs below.



15. Compare the numbers of days the restaurants are open. Explain how the graphs show this.
16. **Multiple Choice** Which statement is true about the mean numbers of customers during days that each restaurant is open?
  - A. The mean number of daily customers at Bountiful Bistro is about 20 more than at the Casual Café.
  - B. The mean number of daily customers at Bountiful Bistro is about 10 more than at the Casual Café.
  - C. The mean number of daily customers at the restaurants is about the same.
  - D. The mean number of daily customers at Bountiful Bistro is about 10 less than at the Casual Café.
17. Use any peaks in the data to compare the numbers of customers at the restaurants.
18. Use any symmetry or lack of symmetry to compare the distribution of data for the restaurants.

**For Exercises 19–20, determine which two sets of data will overlap more—Sets A and B or Sets A and C.**

19. Set A has a mean of 12 and a mean absolute deviation of 5.1.  
Set B has a mean of 23 and a mean absolute deviation of 4.9.  
Set C has a mean of 10 and a mean absolute deviation of 4.8.
20. Set A has a mean of 104 and a mean absolute deviation of 19.6.  
Set B has a mean of 84 and a mean absolute deviation of 25.  
Set C has a mean of 180 and a mean absolute deviation of 20.

**Notes**

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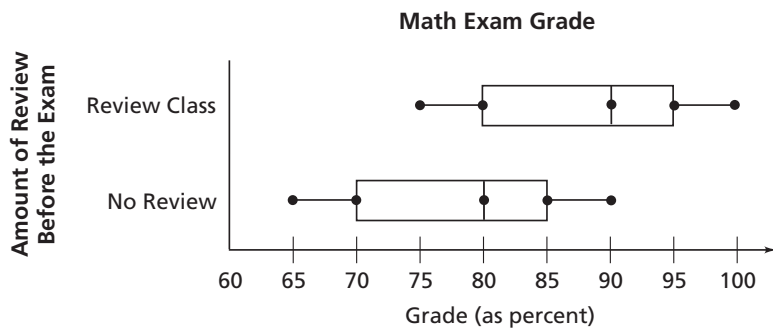


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For Exercises 21–23, use the information given about class test scores shown in this box plot.



21. What conclusions can you draw from looking at the plot about how effective the math exam review class was?
22. What is the difference in the medians between the sets of data?
23. The mean absolute deviation for both groups of students is 6.2. Compare that value to the difference in medians. What does that tell you about the data?

For Exercises 24–28, use the information given about the points that two basketball players scored in each of the games they played in this year.

Player A	30	26	21	28	24	28	25	26	30	22
Player B	16	18	15	18	22	14	16	23	18	20

24. Find the mean number of points scored for each player. Find the difference in the means.
25. Find and compare the mean absolute deviation for each player.
26. How many times greater is the difference in the means than the mean absolute deviation for each player?
27. Would you expect there to be a lot of overlap in dot plots of the data? Why or why not?
28. Suppose the mean number of points scored for Player B were 25 points, and the variability stays the same. Would you expect there to be a lot of overlap in a dot plot of the data? Why or why not?
29. Your mean quiz score is 15 points higher than your friend's mean quiz score, which is 3 times the mean absolute deviation of both of your scores. Do you think there will be a lot of overlap if you make a double histogram of the data? Explain.

## Notes

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# CC Investigation 1 Answers to Additional Practice, Skill Practice, and Check-Up

## Investigation 1 Additional Practice

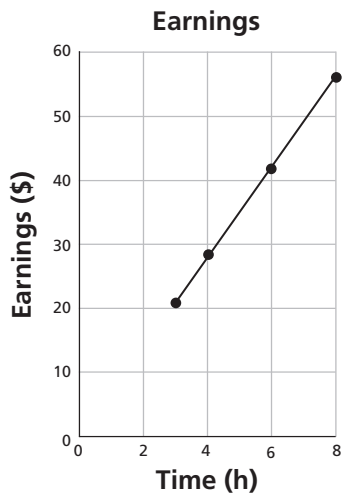
1. a. The earnings always are 7 times the time spent working.

b.  $\frac{\$28}{4 \text{ h}} = \frac{\$7}{1 \text{ h}}, \frac{\$21}{3 \text{ h}} = \frac{\$7}{1 \text{ h}}, \frac{\$42}{6 \text{ h}} = \frac{\$7}{1 \text{ h}},$   
 $\frac{\$56}{8 \text{ h}} = \frac{\$7}{1 \text{ h}}$

c. The ratios all are equivalent, so Michelle earned money at the same rate each hour.

d. Multiply the unit rate of \$7/h by the time, 16 h:  $16 \times 7 = \$112$ .

2. a.

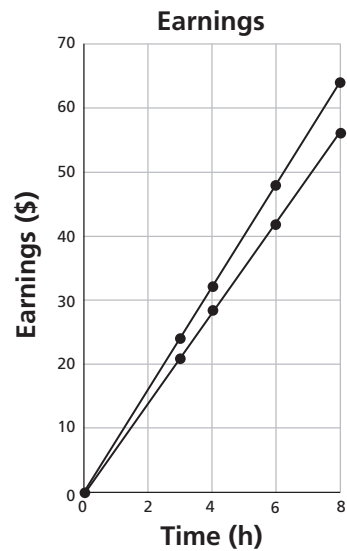


b. The graph is a straight line, so the rate of pay is constant.

c. \$7; Michelle's rate of pay is \$7/h.

d. at the origin; (0, 0); If Michelle works 0 hours, she gets paid \$0.

3.



Both graphs are straight lines that pass through the origin, but their slopes are different.

## Skill: Find the Unit Rate

1. 5
2. 2
3. about 0.57
4. about 1.67
5. 1.5
6. about 0.17
7. about 0.67
8. 3.5
9. 2.4
10. 0.625
11. 16 mi/h
12. \$1.39/lb

# CC Investigation 1 Answers to

## Additional Practice, Skill Practice, and Check-Up *(continued)*

### Skill: Proportional Rates

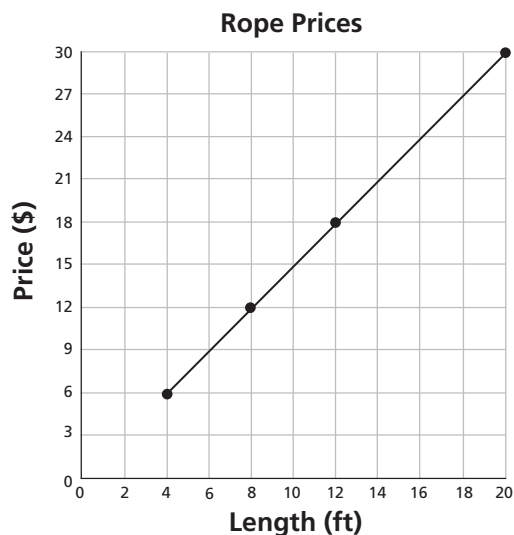
13. The values are proportional.
14. The values are not proportional.
15. The values are not proportional.
16. The values are proportional.
17. The values are not proportional.
18. The values are not proportional.
19. The values are proportional.
20. The values are not proportional.
21. Yes; the rates are equal.
22. No; the rates are not equal.

### Investigation 1 Check-Up

1. a. Andy's Seeds:  $\frac{10 \text{ seeds}}{\$2.50} = \frac{4 \text{ seeds}}{\$1}$ ;  
 Jenny's Seeds:  $\frac{12 \text{ seeds}}{\$3.12} \approx \frac{3.85 \text{ seeds}}{\$1}$ ;  
 Garden Place:  $\frac{20 \text{ seeds}}{\$4.60} \approx \frac{4.35 \text{ seeds}}{\$1}$ ;  
 Blooming Acres:  $\frac{15 \text{ seeds}}{\$3.60} \approx \frac{4.17 \text{ seeds}}{\$1}$
- b. Blooming Acres; To get 75 seeds at Garden Place, Sahil would spend  $4 \times \$4.60 = \$18.40$ , and have 5 seeds left over. To get 75 seeds at Blooming Acres, Sahil would spend  $5 \times \$3.60 = \$18.00$ , and have no seeds left over. Andy's is \$20 with 5 left over, and Jenny's is \$21.84 with 9 left over.
- c. Garden Place;  
 $4 \times 10 = 40$  seeds from Andy's Seeds,  
 $3 \times 12 = 36$  seeds from Jenny's Seeds,  
 $2 \times 20 = 40$  seeds from Garden Place, or  
 $2 \times 15 = 30$  seeds from Blooming Acres.  
 If he buys 40 seeds from Andy's, he will have no money left over. If he buys 40 seeds from Garden Place he still will have \$0.80 left over.

2. No, Emilio does not read at the fastest rate. Emilio reads at a rate of  $15 \div 20 = 0.75$  pages/min, but Luz reads at  $8 \div 10 = 0.8$  pages/min.

3. a.



The graph is a straight line.

- b. at the origin; (0, 0); a rope that is 0 ft long costs \$0.
  - c. \$1.50; the unit price of the rope is \$1.50/ft.
4. a. \$2.50/ft
  - b. Both would be straight lines that pass through the origin. The second line would be steeper.



# CC Investigation 2 Answers to Additional Practice, Skill Practice, and Check-Up

## Investigation 2 Additional Practice

1. a.  $d$  represents the number of DVDs  
 Marcus buys;  $24d$  represents the cost of  
 the DVDs,  $3 + d$  represents the number  
 of CDs he buys;  $11(3 + d)$  represents the  
 cost of the CDs.

b.

Step	Reason
$24d + 11(3 + d)$	original expression
$24d + 11(3) + 11(d)$	distributive property
$24d + 11(d) + 11(3)$	commutative property
$(24 + 11)d + 11(3)$	distributive property
$(35)d + 11(3)$	addition
$35d + 33$	multiplication

- c. 3 DVDs:  $24(3) + 11(3 + 3) = 72 + 11(6)$   
 $= 72 + 66 = \$138$ ; 5 DVDs:  $24(5) +$   
 $11(3 + 5) = 120 + 11(8) = 120 + 88$   
 $= \$208$
- d. 3 DVDs:  $35(3) + 33 = 105 + 33 = \$138$ ;  
 5 DVDs:  $35(5) + 33 = 175 + 33 = \$208$ ;  
 The values are the same, so the  
 expressions are equivalent.
2. a.  $0.85p$ ;  $p - 0.15p$   
 b. 6%;  $1.06p$

## Skill: Evaluate Expressions

- 12
- 41
- 4
- 32
- 4
- 4
- 0.3
- 30.8
- 9.1
- 2.2
- 32.2
- 20.4

## Skill: Simplify Expressions

- $5y - 4$
- $11.8g - 5$
- $\frac{6}{5}r$
- $-8t - 60$
- $-48f + 112$
- $9d$
- $3a$
- $u - 3$
- $18q + 6$
- $16.4 - 5v$

# CC Investigation 2 Answers to Additional Practice, Skill Practice, and Check-Up *(continued)*

## Investigation 2 Check-Up

1. a.  $b$  represents the number of balls bought for each player;  $12b$  represents the total number of balls;  $b - 4$  represents the number of bats bought for each player;  $12(b - 4)$  represents the total number of bats.

b.

Step	Reason
$12b + 12(b - 4)$	original expression
$12[b + (b - 4)]$	distributive property
$12[(b + b) - 4]$	associative property
$12(2b - 4)$	combine terms
$12(2b) - 12(4)$	distributive property
$24b - 48$	multiplication

- c.  $12(6) + 12(6 - 4) = 72 + 12(2) = 72 + 24 = 96$  bats and balls  
 d.  $24(6) - 48 = 144 - 48 = 96$  bats and balls;  
 The answers are the same, so the expressions are equivalent.

2. both;  $49 - 0.3(49) = (1 - 0.3)(49) = 0.7(49)$

3. a.  $1.05p = 1.05(68) = \$71.40$ ;  $p + 0.05p = 68 + (0.05)68 = 68 + 3.4 = \$71.40$

b. 8.8%; Fine Motors' price:  
 $1.12p = 1.12(68) = \$76.16$ ; Solve the equation  $76.16 = 70 + g(70)$ ;  $6.16 = 70g$ ;  
 $g = 0.088 = 8.8\%$ .

4. a.

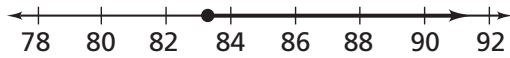
Step	Reason
$0.75c + 0.06(c - 0.25c)$	original expression
$0.75c + 0.06(c) - 0.06(0.25c)$	distributive property
$0.75c + 0.06c - 0.015c$	multiplication
$c(0.75 + 0.06 - 0.015)$	distributive property
$0.795c$	addition

b.  $\$556.50$ ;  $0.795(\$700) = \$556.50$

# CC Investigation 3 Answers to Additional Practice, Skill Practice, and Check-Up

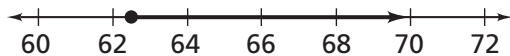
## Investigation 3 Additional Practice

1. a. 84 tickets;  $6t \geq 500$ ;  $6t \div 6 \geq 500 \div 6$ ;  
 $t \geq 83.33$ ;



The company must sell 84 or more tickets to pay for the theater rental.

- b. 63 tickets;  $6t + 8(10) + 3(15) \geq 500$ ;  
 $6t + 80 + 45 \geq 500$ ;  $6t + 125 \geq 500$ ;  
 $6t \geq 375$ ;  $t \geq 62.5$ ; the company must sell  
63 or more tickets.



2. a.  $39 + 0.25(m - 450) \leq 99$   
b. Yes; cost of current plan:  
 $39 + 0.25(700 - 450) = 39 + 0.25(250) =$   
 $39 + 62.5 = 101.50$ ;  $\$101.50 > \$99$ , so  
700 minutes cost more under the current  
plan than the unlimited plan.
3. a.  $t \geq 80$   
b.  $F = 1.8C + 32$ ;  $1.8C = F - 32$ ;  
 $C = \frac{5}{9}(F - 32)$ ;  $C = \frac{5}{9}(80 - 32)$   
 $= \frac{5}{9}(48) \approx 27$ ;  $t \geq 27$
- c. The temperature  $27^\circ\text{C}$  is a solution to  
the inequality  $t \geq 27$ , but the  
temperature  $26^\circ\text{C}$  is not a solution.

## Skill: Read Graphs of Inequalities

1.  $x \geq -3$
2.  $x < 26$
3.  $x \leq 160$
4.  $x > -80$
5.  $x < 3.5$
6.  $x \geq 2\frac{2}{3}$
7.  $x \leq 1,000$
8.  $x < 8$

## Skill: Solve Inequalities

9.  $t < 2$
10.  $g \geq 12$
11.  $t \leq 7$
12.  $p < -\frac{1}{2}$
13.  $u \leq 3\frac{5}{8}$
14.  $w < 36$
15.  $x \geq -70$
16.  $v \leq 40$
17.  $q > -22$
18.  $r > -7$

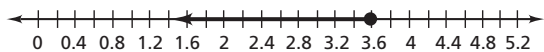
## CC Investigation 3 Answers to

### Additional Practice, Skill Practice, and Check-Up *(continued)*

#### Investigation 3 Check-Up

1. a. Yes;  $5(\$30) = \$150$  and  $3(\$50) = \$150$ .

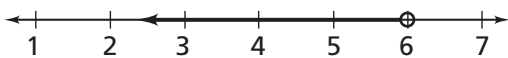
b. 3.5 h;  $50h - 30 \leq 150$ ;  $50h \leq 180$ ;  $h \leq 3.6$



c. Yes;  $2.5(50) - 30 = 125 - 30 = \$95$  rental cost;  $150 - 95 = \$55$  left for dinner.

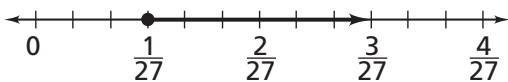
2. a.  $t < 180$

b.  $3 + d < 9$ ;  $d < 6$



c. Yes,  $45 + \frac{5}{r} \leq 180$ ;  $\frac{5}{r} \leq 135$ ;

$$r \geq \frac{1}{27} \text{ mi/min}$$



The rate needs to be  $\frac{1}{27}$  mi/min or greater to make the hike in 3 hours or less.

d.  $\frac{1}{45}$  mi/min;  $45 + \frac{3}{r} \leq 180$ ;  $\frac{3}{r} \leq 135$ ;

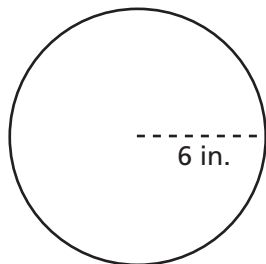
$r \geq \frac{1}{45}$  mi/min; The rate needs to be

$\frac{1}{45}$  mi/min or greater to make the hike in 3 hours or less.

# CC Investigation 4 Answers to Additional Practice, Skill Practice, and Check-Up

## Investigation 4 Additional Practice

1. a.



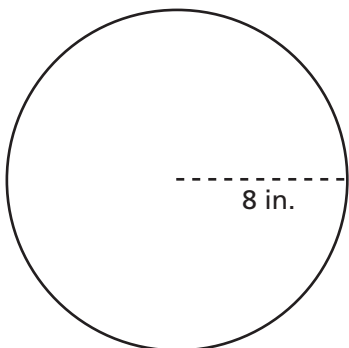
This is a horizontal cross section of the cylinder, and a cross section of the sphere at a level with a radius of 6 in.

b.



The cross section is a rectangle, and a sphere can have only circles as cross sections.

c.



The radius of the circular cross section is greater than the radius of the cylinder.

2. a.  $105^\circ$ ,  $15^\circ$ , and  $60^\circ$   
 b. an infinite number  
 c.  $75^\circ$ ;  $180^\circ - 75^\circ - 30^\circ = 75^\circ$

## Skill: Name Cross Sections

- horizontal: square; vertical: square
- horizontal: rectangle; vertical: triangle
- horizontal: square; vertical: triangle
- horizontal: circle; vertical: circle
- horizontal: circle; vertical: rectangle
- horizontal: rectangle; vertical: square

## Skill: Angle Measures

- $\angle v$
- $\angle CDB$  or  $w$
- $180^\circ$
- $\angle FDB$
- $\angle ADE$  and  $\angle EDF$ ; or  $v$  and  $z$
- $\angle ADC$  and  $\angle CDB$
- vertex  $D$
- ray  $DE$  and vertex  $D$
- $\angle ADF$
- $\angle BDE$

## CC Investigation 4 Answers to

### Additional Practice, Skill Practice, and Check-Up *(continued)*

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#### Investigation 4 Check-Up

1.
  - a. figures *A* and *D*
  - b. figures *A*, *C*, and *E*
  - c. figures *B*, *C*, *D*, and *E*
2.
  - a.  $\frac{\pi R^2}{\pi r^2} = \frac{\pi(14)^2}{\pi(7)^2} = \frac{\pi 196}{\pi 49} = \frac{196}{49} = \frac{4}{1}$ ;  
No;  $\pi$  is a common factor in both the numerator and denominator, so it can be divided out from the ratio.
  - b.  $\frac{1}{2}$  turn; Find the ratio of circumference of the small wheel to the circumference of the larger wheel:  
$$\frac{2\pi r}{2\pi R} = \frac{2\pi(7)}{2\pi(14)} = \frac{7}{14} = \frac{1}{2}$$
3.
  - a. Triangle 1: scalene;  $53^\circ$ ;  
Triangle 2:  $60^\circ$ ;  $60^\circ$ ;  
Triangle 3:  $50^\circ$ ;  
Triangle 4:  $39^\circ$ ;  $39^\circ$
  - b. The sum of the measures of the two angles is  $78^\circ$ , so many different triangles are possible.
4.
  - a. angles *z* and *d*
  - b. Possible answer: angles *e* and *z*, angles *z* and *b*
  - c. angles *e* and *z*; or angles *e* and *d*
  - d.  $m\angle z = 25^\circ$ ;  $m\angle b = 65^\circ$ ;  $m\angle d = 25^\circ$ ;  
Angles *e* and *z* are complementary, so  $m\angle z = 90 - m\angle e = 90 - 65 = 25^\circ$ ;  
Angles *z* and *d* are vertical angles, so  $m\angle d = m\angle z = 25^\circ$ ; Angles *b*, *c*, and *z* form a straight line, so  $m\angle b + m\angle c + m\angle z = 180$ ;  $m\angle b + 90 + 25 = 180$ ;  
 $m\angle b = 65^\circ$ . Angle *C* is a right angle, so the measure of angle *C* is  $90^\circ$ .

# CC Investigation 5 Answers to Additional Practice, Skill Practice, and Check-Up

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## Investigation 5 Additional Practice

1.
  - a. No; because Rachel is active in the drama club, a survey of its members might be biased.
  - b. No; the drama club members might be biased in favor of Rachel, and the basketball players might be biased in favor of Dwayne. Also, students who are not in these groups are not represented.
  - c. No; girls are not representative of the entire population.
  - d. Yes, each member of the population, Grade 7 students, has an equal chance of being selected for the sample.
  - e. No; the sample would include some people outside the population of Grade 7 students, such as Grade 6 or Grade 8 students.
2.
  - a. The students in Mr. Abrams' class overall received higher grades.
  - b. Mr. Abrams: range = 25, IQR = 10; Mr. Philips: range = 38, IQR = 10
  - c.  $85 - 72 = 13$
  - d. The interquartile ranges for the data sets are equal, so the middle part of each group of data is equally clustered about the median. The range is much greater for Mr. Philips' students, so their grades varied more widely. The median in Mr. Abrams' class is significantly higher than the median in Mr. Phillips' class. Overall, Mr. Abrams' students scored higher.

## Skill: Representative Samples

1. No; Grade 2 and Grade 3 students do not have any chance of being selected for the sample.
2. Yes; each member of the population, Mr. Williams' students, has an equal chance of being selected for the sample.
3. Yes; each member of the population, the city residents, has an equal chance of being selected for the sample.
4. No; the sample will include only students who use the gym after school.
5. Medians: 15 and 17; IQRs: 4 and 2; the values of the second set overall are greater and more clustered about the median.
6. Medians: 87 and 87; IQRs: 5 and 10; the middle values of the sets are the same, and the first set of values are more clustered about the median.
7. Medians: 37 and 34; IQRs: 8 and 8; the middle value of the first set is greater and both sets are clustered to the same degree about the median.
8. Medians: 6 and 4.5; IQRs: 4 and 4; the middle value of the first set is greater and both sets are clustered to the same degree about the median.

## CC Investigation 5 Answers to

### Additional Practice, Skill Practice, and Check-Up *(continued)*

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#### Investigation 5 Check-Up

1.
  - a. No, spectators or players from a basketball game might be biased in favor of watching basketball.
  - b. No, the sample size of only 5 students is too small to be reliable.
  - c. Possible answer: Ask a random sampling of classmates, such as every fifth student entering the school one morning.
2.
  - a. about 2,000 riders;  $\frac{2}{10} = 0.2$ ;  
 $0.2 \times 10,000 = 2,000$
  - b. about 6,000 riders;  $\frac{6}{10} = 0.6$ ;  
 $0.6 \times 10,000 = 6,000$
  - c. about 3,500 riders;  $\frac{70}{200} = 0.35$ ;  
 $0.35 \times 10,000 = 3,500$
  - d. Yes, the estimates were different based on the sample results. The estimate of about 3,500 riders probably is most accurate because it is based on the largest sample size.
3.
  - a. Sarah: range = 16; IQR = 8;  
DeShawn: range = 15; IQR = 4;  
The range in hours worked is about the same, but the numbers of hours DeShawn worked are much more clustered around his median time.
  - b. Sarah's hours are more spread out, and evenly distributed on either side of the median, while DeShawn's hours are more clustered, and are overall greater.
  - c. Sarah's data are symmetric about the median, so she was as likely to work more than 8 hours as she was less than 8 hours in any week. DeShawn's hours are clustered about the median, showing that he more often worked about the same number of hours each week.
4.
  - a. Overall, students sent more texts during the school year than during the summer.
  - b. summer median = 40; school-year median = 60;  $60 - 40 = 20$
  - c. The IQR is probably greater during the summer, because the data appear more spread out in that dot plot.