CD-404028

Helping Students Understand

Algebra II

GRADES 7+

WORKTEXT

STEP BY STEP

NCTM STANDARDS BASED

$$7b = 5(6 + 1)$$

$$7b = 5(7)$$

$$\frac{7b}{7} = \frac{35}{7}$$

$$b = 5$$

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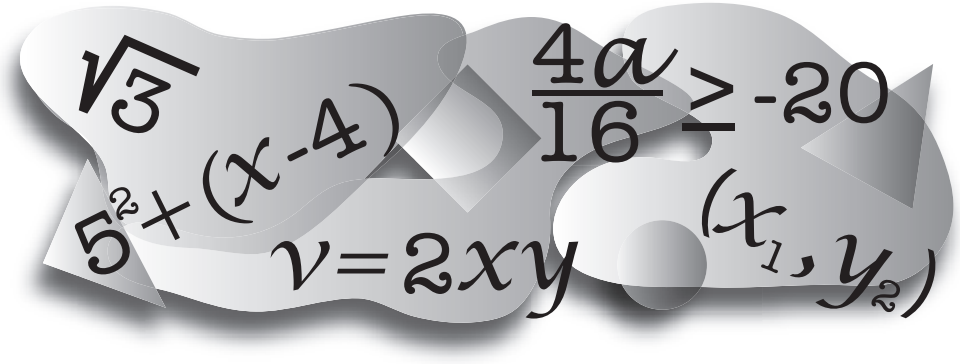
Introduction

The *Helping Students Understand Math series* will introduce students in middle school and high school to the topics of Pre-Algebra, Algebra, and Algebra II. Geometry will also be included in this series. All of the worktexts will be aligned with the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics.*

This series is written for classroom teachers, parents, families, and students. The worktexts in this series can be used as a full unit of study or as individual lessons to supplement textbooks or curriculum programs. Parents and students can use this series as an enhancement to what is being done in the classroom or as a tutorial at home. All students will benefit from these activities, but the series was designed with the struggling math student in mind. The **concepts** and **explanations** for the concepts are described in simple **step-by-step instructions** with **examples** in the introduction of each lesson. Students will be given practice problems using the concepts introduced and descriptions of real-life applications of the concepts.

According to the Mathematics Education Trust and NCTM, new technologies require the fundamentals of algebra, and algebraic thinking should be a part of the background for all citizens. These technologies also provide opportunities to generate numerical examples, graph data, analyze patterns, and make generalizations. An understanding of algebra is also important because business and industry require higher levels of thinking and problem solving.

NCTM Standards suggest content and vocabulary are necessary but of equal importance are the processes of mathematics. The process skills described in the *Standards* include problem solving, reasoning, communication, and connections. The worktexts in this series will address both the content and processes of algebra and algebraic thinking. This worktext, *Helping Students Understand Algebra II,* will help students transition from Algebra to Algebra II.



Teacher Note: For the purposes of this subject, it is not necessary for students to convert improper fractions to mixed numbers.

Chapter 1: Solving Equations and Problems

Introduction to Solving Equations and Problems

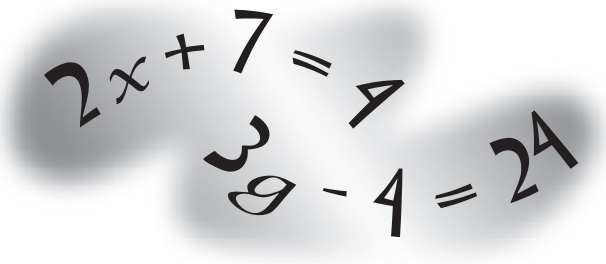
Algebra uses expressions and equations. A numerical expression or numeral is a symbol or group of symbols used to represent a number. The value of a numerical expression is the number represented by the expression.

Simplify an expression by replacing it with the simplest or most common symbol having the same value. In order to simplify expressions, it is necessary to review some of the basic algebraic concepts.

Algebra uses equations to solve problems using numbers and/or variables. One of the major goals of algebra is to find unknown numbers. An equation is a mathematical sentence that uses numbers and variables to describe the relationship between two or more quantities. An equation is a mathematical statement that includes an equals sign. For example: 35 + 12 = 47 is an equation. Variables are letters used to identify an unknown number. An example of an equation using variables is $2x + 7 = 4$. There are many kinds of equations, but the most commonly used in algebra are linear and quadratic equations. This section covers strategies for simplifying algebraic expressions, algebraic equations with one variable, changing word problems into algebraic symbols, and using algebraic equations to solve problems.

Concepts of Solving Equations and Problems

1. Simplifying Expressions
2. Solving Equations With One Variable
3. Words Into Symbols
4. Problem Solving With Equations



Explanation of the Concepts of Solving Equations and Problems

1 Simplifying Expressions

A mathematical sentence or algebraic expression is a symbol or group of symbols used to represent a number. The value of an expression is the number represented by the expression when the value of the variables is known.

Algebraic expressions can be simplified by using the rules for order of operation. Mathematicians have a specific order to be used in solving mathematical problems.

Chapter 1: Solving Equations and Problems (cont.)

The order of operation is:

Step 1: Do what is inside the parentheses.
Step 2: Compute the value of the exponents.
Step 3: Multiply and/or divide.
Step 4: Add and/or subtract.

Example of Simplifying Algebraic Expressions:

Problem:	3(4 + 3)	
Step 1:	Do what is inside the parentheses.	(4 + 3) = 7
Step 2:	Substitute the 7 for (4 + 3).	3(7)
Step 3:	Compute the value of the exponents.	No exponents
Step 4:	Multiply and/or divide.	3(7) = 21

2 Solving Equations With One Variable

An equation is a mathematical sentence that uses numbers and/or variables to describe the relationship between two or more quantities. Sometimes you can look at an equation and guess what the number is. Sometimes you may have to solve the equation by using the principles of algebra. Equations can be solved by: 1) Simplifying each side of the equation, 2) Using addition and/or subtraction to solve equations, and 3) Using multiplication and/or division.

Examples of Simplifying Each Side of the Equation:

Problem: $7b = 5(6 + 1)$

Step 1: Simplify each side of the equation.
The left side is already simplified.
The right side can be simplified. (6 + 1) = 7
Substitute 7 for (6 + 1).
Multiply 5(7) = 35.
Substitute 35 for 5(6 + 1). $7b = 35$
The equation is simplified, so it can now be solved.

Chapter 1: Solving Equations and Problems (cont.)

Problem: $4x + 2x - 7 + 9 + x = 5x - x$

Step 1: Simplify each side of the equation.
Simplify the left side.
Combine all terms with the same variable. $4x + 2x + x = 7x$
Combine all numbers on the left side. $-7 + 9 = 2$
Simplified, the left side is $7x + 2$.

Step 2: Simplify the right side.
Combine the terms with the same variables. $5x - x = 4x$
Simplified, the right side is $4x$.

Step 3: Substitute the simplified left and right sides. $7x + 2 = 4x$
The equation is simplified, so it can be solved.

To solve the simplified equations with one variable by addition and subtraction, first move the variables to one side of the equation and the numbers to the other side of the equation by adding or subtracting. If an equation is a true sentence, then the left side is equal to the right side of the equation. If you add or subtract the same number to each side of the equation, the equation is still true.

Example of Simplifying Equations With One Variable by Addition and Subtraction:

Problem: $x - 4 = 9$

Step 1: Equation is simplified.

Step 2: Get the variable on one side and the numbers on the other.
Variables are on one side.
The number -4 must be moved to the other side.
Remember that as long as you do the same thing to both sides of the equation, it does not change the value of the equation. So if we add 4 to both sides of the equation, it will move the four to the other side.
$x - 4 + 4 = 9 + 4$

Answer: Combine the terms. $x = 13$

Chapter 1: Solving Equations and Problems (cont.)

Examples of Simplifying Equations With One Variable by Multiplication and Division:

Problem: $7b = 5(6 + 1)$

Step 1: Simplify each side of the equation. From a previous example, we know this equation can be simplified to $7b = 35$.

Step 2: Solve the equation. Multiply and/or divide. Remember that as long as you do the same thing to both sides of the equation, it does not change the value of the equation. Divide both sides by 7.

$$\frac{7b}{7} = \frac{35}{7}$$

Answer: $b = 5$

Problem: $4x + 2x - 7 + 9 + x = 5x - x$

Step 1: Simplify each side of the equation. From a previous example, we know this equation can be simplified to $7x + 2 = 4x$.

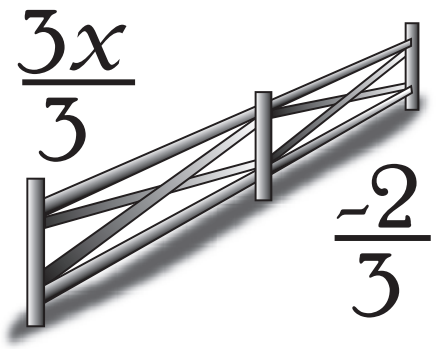
Step 2: Get the variables on one side of the equation and the numbers on the other side. Add and/or subtract.

$7x + 2 - 2 = 4x - 2$
$7x - 4x = 4x - 2 - 4x$
$3x = -2$

Step 3: Multiply and/or divide. Divide both sides by 3.

$$\frac{3x}{3} = \frac{-2}{3}$$

Answer: $x = -\frac{2}{3}$



Chapter 1: Solving Equations and Problems (cont.)

3 Words Into Symbols

Word problems describe relationships between numbers. If you change the relationships of the numbers into an equation, you can solve the problem by solving the equation. Word problems can be solved by changing some of the words in the phrase or sentence into numbers and/or symbols and putting them into algebraic expressions.

Example of Changing Word Phrases Into Algebraic Expressions:

"five more than three times the number"

Use n for the variable.
"five more" is represented by $+ 5$.
"three times the number" is represented by $3 \times n$ or $3n$.
So the expression representing "five more than three times the number" is $3n + 5$.

Example of Changing Sentences Into Algebraic Expressions:

"If a runner runs at x miles/hour, what would her speed be if she ran 2 miles per hour slower?"

"speed" is represented by x.
"2 miles an hour slower" is represented by $- 2$.
So the runner's speed could be represented by the algebraic expression $x - 2$.

Example of Using a Variable as an Unknown and Writing an Equation to Represent the Situation:

Your class has 23 students. The number of boys is 4 less than two times the girls. How would you write an algebraic equation to represent this?

Choose a variable to represent the unknown, the number of girls. Girls = g
Write an expression for the number of boys in terms of the variable.
Boys are represented by $2g - 4$.
Write the equation that represents the situation. There are 23 students in the class.
$g + (2g - 4) = 23$ or $3g - 4 = 23$

Chapter 1: Solving Equations and Problems (cont.)

4 Problem Solving With Equations

In solving problems with equations, you need to translate the relationship described in the word problem into an equation. Start by reading the problem carefully and asking what you are trying to find out. Choose a variable and use it with the given facts to represent the numbers in the problem. Write out a word equation to represent the relationship between the numbers. This should give you an algebraic equation that can be solved. Check your results.

Example of Problem Solving With Equations:

Tickets to a major league game were $100 for a box seat and $50 for a bleacher seat. There were 1,000 more bleacher seats sold than box seats. If the total ticket sales were $62,000, how many of each ticket were sold?

Step 1: What are you trying to find out? "How many of each ticket were sold?"

Step 2: Choose a variable to represent the relationship. Let b represent the number of box seats.

Step 3: Write a word equation to represent the relationship.
Money from the Box Tickets Sold + Money from the Bleacher Tickets Sold = Total Sales

Step 4: Write it out as an algebraic equation. $\$100 \times b + \$50(b + 1{,}000) = \$62{,}000$

Step 5: Simplify. $100b + 50b + 50{,}000 = 62{,}000$

Step 6: Combine the same variables. $150b + 50{,}000 = 62{,}000$

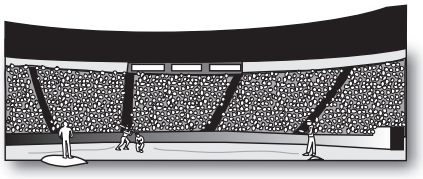
Step 7: Solve the equation by subtraction to get the variables on one side and the numbers on the other side.

$150b + 50{,}000 - 50{,}000 = 62{,}000 - 50{,}000$

$150b = 12{,}000$

$$\frac{150b}{150} = \frac{12{,}000}{150}$$

$b = 80$ box seats were sold.



Answer: There were 80 box seats sold. The number of bleacher seats is 1,000 more than the box seats. Bleacher seats = $b + 1{,}000$
The number of bleacher seats is $80 + 1{,}000 = 1{,}080$.

Check your answer: Substitute the numbers in the original equation.
Money from the Box Tickets Sold + Money from the Bleacher Tickets Sold = Total Sales. $\$100 \times b + \$50(b + 1{,}000) = \$62{,}000$
Substitute the 80 for number of box seats. $\$150(80) + \$50(80 + 1{,}000) = \$62{,}000$

Name: _____ Date: _____

Chapter 1: Solving Equations and Problems (cont.)

Practice: Solving Equations and Problems

Simplifying Expressions
Simplify the following expressions.

1. $4 \div (2 + 2) - 1$

2. $(5 - 3)^2(2 - 1)$

3. $5x - (6 - 3) + 7$

Simplify each side of the equation.

4. $4a + 5a - 2 = 5 + 3 - 1$

5. $3x - 2x + x - 4 + 3 - 2 = 0$

6. $3(x + 5) = 0$

7. $3x - 2x = 6 - 9$

8. $3x + 4x = 6(2x + 1)$

Solving Equations With One Variable
Simplify and solve the equations in questions 9–13.

9. $4a + 5a - 2 = 5 + 3 - 1$

10. $3x - 2x + x - 4 + 3 - 2 = 0$

11. $3(x + 5) = 0$

12. $3x - 2x = 6 - 9$

13. $3x + 4x = 6(2x + 1)$

Name: _____ Date: _____

Chapter 1: Solving Equations and Problems (cont.)

Words Into Symbols

For questions 14–19, express each phrase or sentence in symbols in the simplest form in terms of the given variable.

14. The cube of a number decreased by that number _____

15. Three times the sum of a number and 7 _____

16. What is the area of a tabletop that is twice as long as it is wide? _____

17. What is the price of a pencil if the price of 24 pencils is $1.00? _____

18. Yuki has *n* DVDs. If she has 13 fewer than Frieda, and Frieda has 80 DVDs, then how many does Yuki have?

_____

19. If a car traveled for 3 hours at *r* mph and increased the speed by 10 mph and traveled for 1 more hour, how far did the car travel?

_____

For questions 20–21, choose a variable to represent the unknown number and write an equation to describe a given situation.

20. The eighth grade class has 75 students. The number of girls is 6 less than twice the number of boys.

_____

21. Ian bought a pair of jeans at the regular price. When the jeans went on sale, he purchased 3 more pairs that were $5 off of the regular price. He spent a total of $50 for 4 pairs of jeans.

_____

Problem Solving With Equations

Change this word problem into a mathematical equation and solve.

22. Tara receives an annual return of $332 from $5,000 invested at a simple interest. How much is the interest rate? (Interest = amount invested × interest rate).

_____

Chapter 1: Solving Equations and Problems (cont.)

Summary of Solving Equations and Problems

To simplify expressions, use the following order for operations:

1. Simplify the expression within each grouping symbol, working outward from the innermost group.
2. Simplify the powers.
3. Perform multiplication and division from left to right.
4. Perform addition and subtraction in order from left to right.

Simplify an expression by replacing it by the simplest or most common symbol having the same value. To simplify an algebraic expression, replace each variable in an expression by a given value and simplify the results.

To solve the simplified equations with one variable by addition and subtraction, first move the variables to one side of the equation and the numbers to the other side of the equation by adding or subtracting.

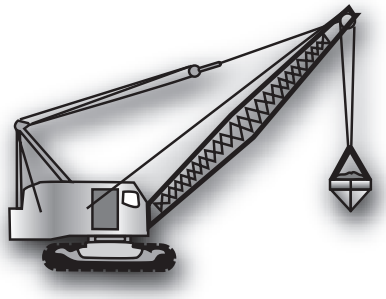
Word problems describe relationships between numbers. If you change the relationships of the numbers into an equation, you can solve the problem by solving the equation. Word problems can be solved by changing some of the words in the phrase or sentence into numbers and/or symbols and putting them into algebraic expressions.

In solving problems with equations, you need to translate the relationship described in the word problem into an equation. Start by reading the problem carefully and asking what you are trying to find out. Choose a variable and use it with the given facts to represent the numbers in the problem. Write out a word equation to represent the relationship between the numbers. Then write the word equation as an algebraic equation and solve it. Check your results.

Tips to Remember

Two hints will help change the word problems into mathematical statements. Usually the words *equals, is, are, was,* and *were* in sentences are changed to an equals sign (=). The second hint is to use a letter to represent an unknown number (a variable).

Real Life Applications of Solving Equations and Problems



Changing words into algebraic expressions can be used to solve problems in science. To find the amount of work done by a machine, the formula used is "work equals the force times the distance." If you represent the work with the variable *W,* force with *f,* and distance with *d,* the formula becomes $W = f \times d$. Solving problems with equations can also be used to determine interest rates on investments.

Answer Keys

Chapter 1: Practice: Solving Equations and Problems (pages 21–22)

Simplifying Expressions

1. $4 \div (2 + 2) - 1$
   $4 \div 4 - 1$
   $1 - 1 = 0$

2. $(5 - 3)^2(2 - 1)$
   $(2)^2(1)$
   $(4)(1) = 4$

3. $5x - (6 - 3) + 7$
   $5x - 3 + 7$
   $5x + 4$

Simplify each side of the equation

4. $4a + 5a - 2 = 5 + 3 - 1$
   $9a - 2 = 7$

5. $3x - 2x + x - 4 + 3 - 2 = 0$
   $2x - 3 = 0$

6. $3(x + 5) = 0$
   $3x + 15 = 0$
   This could also be simplified.
   $\frac{3(x + 5)}{3} = \frac{0}{3}$
   $x + 5 = 0$

7. $3x - 2x = 6 - 9$
   $x = -3$

8. $3x + 4x = 6(2x + 1)$
   $7x = 12x + 6$

Solving Equations with One Variable

9. $4a + 5a - 2 = 5 + 3 - 1$
   $9a - 2 = 7$
   $9a - 2 + 2 = 7 + 2$
   $9a = 9$
   $\frac{9a}{9} = \frac{9}{9}$
   $a = 1$

10. $3x - 2x + x - 4 + 3 - 2 = 0$
    $x = \frac{3}{2}$

11. $3(x + 5) = 0$
    $x = -5$

12. $3x - 2x = 6 - 9$
    $x = -3$

13. $3x + 4x = 6(2x + 1)$
    $x = -\frac{6}{5}$

Words Into Symbols

14. $n^3 - n$
15. $3(n + 7)$
16. $A = 2w(w)$ or $A = 2w^2$
17. $24p = \$1.00$
18. $n = 80 - 13$
19. $3r + 1(r + 10)$; $3r + 1r + 10$; $4r + 10$

Write an Equation

20. n = number of boys $(2n - 6) + n = 75$
21. r = regular price $r + 3(r - 5) = 50$

Problem Solving With Equations

22. $332 = 5000 \times i$ $\frac{332}{5000} = \frac{5000i}{5000}$ $0.0664 = i$ This could also be written as 6.64%.

Chapter 2: Practice: Inequalities (pages 33–35)

Inequalities

1. $3 > 7$ False
2. $6 \geq 8$ False
3. $6 \leq 6$ True
4. $0 < -6$ False
5. $-6 \leq -1$ True
6. $a < 5$; a is less than five; Numbers that make the inequality true: Any number less than five
7. $c \geq 2$; c is greater than or equal to two; Numbers that make the inequality true: Any number equal to two or greater than two
8. $x \leq 0$; x is less than or equal to 0; Numbers that make the inequality true: Any number equal to zero or less than zero

Graphing Inequalities

9. $n \leq 10$