Comparing Fractions: Same Numerator

Materials:
- **Comparing Fractions**
- **Number Lines for Runners** - optional

Prompt students to complete **Comparing Fractions**. Provide students a copy of **Number Lines for Runners** if needed.

Debriefing Questions:
- How did you determine who had the longest distance to run?
- What do you notice about the numerators of the fractions? What does that tell us about the relationship among the three fractions?
- How does the denominator help determine who has the longest distance to run when the numerators are the same?
- When comparing fractions with the same numerator, what determines the greater or lesser fraction?
- How did you determine who had run the longest distance?
- How does the distance left to run relate to the distance that the runner has completed?

Comparing Fractions

Natalie, Lamar, and Andrew were running a mile in gym class. After 15 minutes, each student had a different distance left to run.

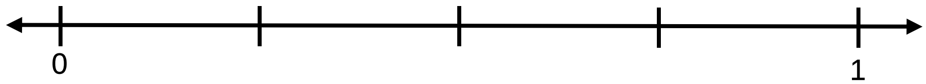
- Natalie had $\frac{1}{4}$ mile left to run.
- Lamar had $\frac{1}{6}$ mile left to run.
- Andrew had $\frac{1}{8}$ mile left to run.

1 Who had the longest distance left to run? How do you know?

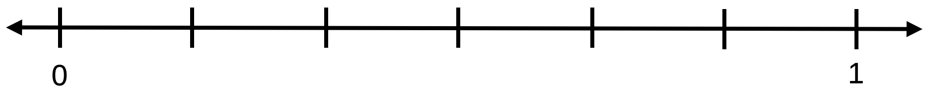
2 Who has run the longest distance? How do you know?

Number Lines for Runners

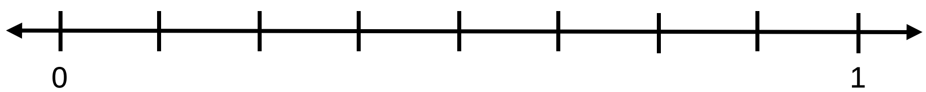
Natalie



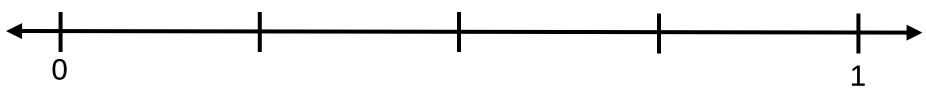
Lamar



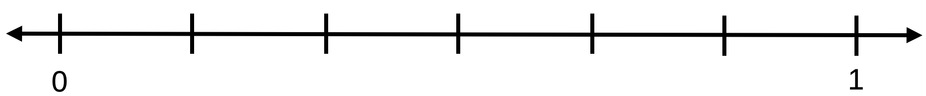
Andrew



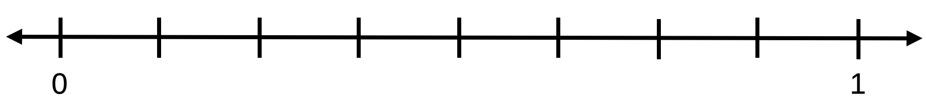
Natalie



Lamar



Andrew



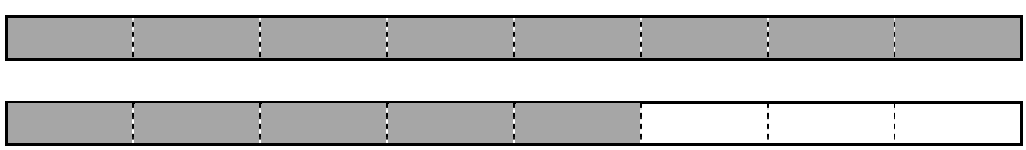
Pre-Assessment: Counting and Naming Fractional Parts

For all of the questions below, one whole equals:

1 What fraction is represented by the shaded parts below? _______________________

2 What fraction is represented by the shaded parts below? ______________________

3 What fraction is represented by the shaded parts below? _______________________



Partitioning Sets of Objects

Materials:

- **Sharing Brownies**

Prompt students to complete **Sharing Brownies**.

Debriefing Questions:

- How did you share the brownies?
- Does everyone get the same amount? How do you know you shared the four brownies equally among the given number of people?
- What is the name of each of these parts?
- How can we write the name of this part?

Sharing Brownies

Three students are sharing 4 brownies equally with each other. How many brownies will each student receive?

Sharing Brownies

Three students are sharing 4 brownies equally with each other. How many brownies will each student receive?

Sharing Brownies: Anticipated Student Responses

- Model four different ways you anticipate students will answer the prompt. Possible misconceptions may be included.
- What understandings does each response communicate?

There are 3 students sharing 4 brownies equally with each other. How many brownies will each student receive?

Anticipated Student Response 1

Anticipated Student Response 2

Anticipated Student Response 3

Anticipated Student Response 4

Teacher: ________________________________ Class: ______________________

Date: ______________________

Foundations for Fractions: Student Interviews

	Sharing without considering the number of sharers or the amount to be shared		Partitioning each object by the number of sharers Sharing one part at a time		Considering how to partition the entire amount Distributing the appropriate amount to each sharer		Understanding the relationship between the number of sharers and the amount being shared	
Student	Model	Fraction Label	Model	Fraction Label	Model	Fraction Label	Model	Fraction Label

Using Iterations

Materials:

- **Sharing Licorice Recording Sheet**

One table set will provide materials for four students
Table Set A:

- 8.5 inch strips of paper (white) — 4 strips
- 11 inch strips of paper (colored) — 4-6 strips
- **Fraction Tasks Card**

Table Set B:

- 11 inch strips of paper (white) — 4 strips
- 14 inch strips of paper (colored) — 4-6 strips
- **Fraction Tasks Card**

Table Set C:

- 14 inch strips of paper (white) — 4 strips
- 8.5 inch strips of paper (colored) — 4-6 strips
- **Fraction Tasks Card**

1. Distribute **Sharing Licorice Recording Sheet** to each student.
2. Divide class into groups of four students. Table sets may be repeated as needed.
3. Distribute a different table set to each group of students.
4. Prompt students to complete **Fraction Tasks Card** and **Sharing Licorice Recording Sheet**.

Task 1 Debrief:

- What is the name of each part you created?
- How can you prove these are halves?
- How do you write one half using fraction notation?
- What does the numerator, the number above the fraction bar, tell you?
- What does the denominator, the number below the fraction bar, tell you?
- How many $\frac{1}{2}$s does it take to decompose the whole?
- Compare your strips of paper to a table with strips of a different length. What do you notice?
- Why can parts of different lengths all be called one-half?

Task 2 and Task 3 Debrief:

- How did you know if your first guess was too long or too short?
- How did you adjust your next guess if it was too long? Too short?
- How was this process of sharing different than when you folded the paper to make halves?
- What is the name of each part you created?
- How do you write one third using fraction notation? One-fourth? One-sixth? One-eighth? Why?
- How many $\frac{1}{3}$s did it take to compose the whole? $\frac{1}{4}$s? $\frac{1}{6}$s? $\frac{1}{8}$s?
- Compare your strips of paper to a table with strips of a different length. What do you notice?
- Why can parts of different lengths all be called one-third? One-fourth? One-sixth? One-eighth?
- What happened to the size of one part as we shared with more people?
- What connections do you see between the denominator and the number of parts of size $\frac{1}{b}$ needed to compose the whole?

Fraction Tasks

Task 1: Sharing with Two People

- One strip of paper represents one licorice stick.
- Fold your paper to show how you can share one licorice stick equally between two people.
- Draw a line on the fold to show each share.
- Use your strip of paper to answer the question on **Sharing Licorice Recording Sheet**.
- Label each share of the strip of paper and attach it to the back of **Sharing Licorice Recording Sheet**.

Task 2: Sharing with Three People

- One strip of paper represents one licorice stick.
- Share one licorice stick equally among three people.
  - You may **NOT** fold or use a ruler to equally share the licorice stick.
  - You may use the colored paper, your fingers, or a pencil to help you equally share the licorice stick.
  - Draw a line to show each share.
  - Use your representation of a licorice stick to answer the question on **Sharing Licorice Recording Sheet**.
  - Label each share of the licorice stick and attach it to the back of **Sharing Licorice Recording Sheet**.

Task 3: Sharing with More People

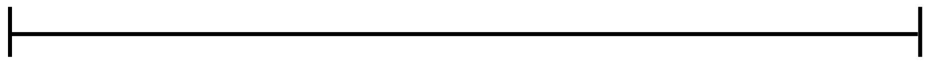
- One strip of paper represents one licorice stick.
- Look at the size of the part when shared among two people and three people. Do you think each part will be longer or shorter when shared between four people? Six people? Eight people? Why?

- Share 1 licorice stick equally among four, six, or eight people.
  - You may **NOT** fold or use a ruler to partition the licorice stick.
  - You may use the colored paper, your fingers, or a pencil to help you equally share the licorice stick.
  - Draw a line to show each share.
  - Use your representation of a licorice stick to answer the question on **Sharing Licorice Recording Sheet**.
  - Label each share of the licorice stick and attach it to the back of **Sharing Licorice Recording Sheet**.

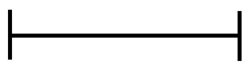
Sharing Licorice Recording Sheet

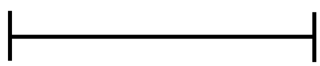
	Task 1: Sharing with Two People	Task 2: Sharing with Three People	Task 3: Sharing with More People
What is the size of each share called?			
Complete the statements	The size is “one-________” because the whole is equally divided into ________ equal parts. The whole is ________ times as large as each share. It takes exactly _____ parts of size “one-________” to compose the whole.	The size is “one-________” because the whole is equally divided into ________ equal parts. The whole is ________ times as large as each share. It takes exactly _____ parts of size “one-________” to compose the whole.	The size is “one-________” because the whole is equally divided into ________ equal parts. The whole is ________ times as large as each share. It takes exactly _____ parts of size “one-________” to compose the whole.

Check Point: Using Iterations



1 Which rod appears to be $\frac{1}{4}$ of the rod shown above?





2 Complete the statement to describe the rod chosen in question 1.

The size of the shorter rod is “one-________” because the whole is ________ times as large as each share. It takes exactly _____ parts to compose the whole.

Paper Strips: 8.5 inches Long

Cut along the dotted lines. Each strip is 8.5 inches long and 1 inch wide.

Paper Strips: 14 inches Long

Cut along the dotted lines. Each strip is 14 inches long and 1 inch wide.

Building a Number Line

Materials:

- **Building a Number Line**
- Cuisenaire® Rods
- Highlighters (optional)

1. Prompt students to form groups of two.
2. Prompt students to work with their partner to complete **Building a Number Line**.
3. Debrief the activity.
   Note: The intent of the debriefing discussion is to transition students from iterating on a number line to writing an equation representing a fraction as a sum of unit fractions.

Iterate to build a whole on a number line.	→	Label the number line using fraction notation.	→	Identify fractions as sums of unit fractions.	→	Write an equation representing a fraction as a sum of unit fractions.

Debriefing

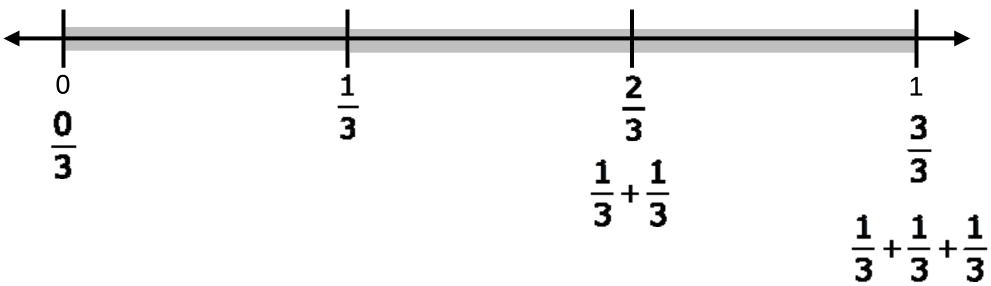
Note: The debriefing sequence below is based on thirds as an example.

1. Partition a number line by iteration.
   - How did you use the Cuisenaire® Rods to partition your number line?
   - How did you label each tick mark of your number line?
   - How do you know that you correctly identified the one-third piece?

   Example:

2. Use the number line and unit fractions to compose a whole.
   - Prompt students to shade the length between zero and one-third on the number line.
   - Prompt students to shade the length between one-third and two-thirds using a different color.
   - Label two-thirds on the number line using an expression representing the fraction as a sum of unit fractions, as shown below.
   - Repeat for three-thirds.

   Example:



   - How many distances of $\frac{1}{3}$ does it take to compose the whole?
   - How many thirds does it take to compose a whole?
   - How can we record this mathematically as a number sentence?
     $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{3}{3}=1$

Building a Number Line

1 Halves on a number line

a) Determine the Cuisenaire® Rod that is one-half of the whole represented below.

b) Use the one-half rod to partition the number line below into halves. Label the partition on the number line.

2 Fourths on a number line

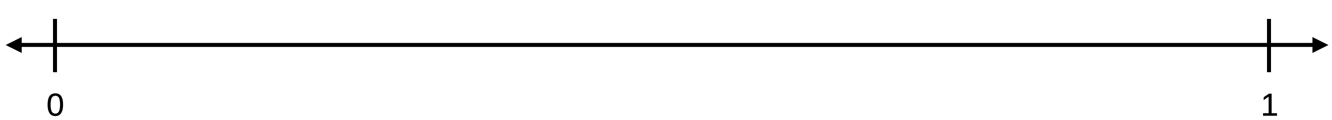
a) Determine the Cuisenaire® Rod that is one-fourth of the whole represented below.

b) Use the one-fourth rod to partition the number line below into fourths. Label the partitions on the number line.

3 Eighths on a number line

a) Determine the Cuisenaire® Rod that is one-eighth of the whole represented below.

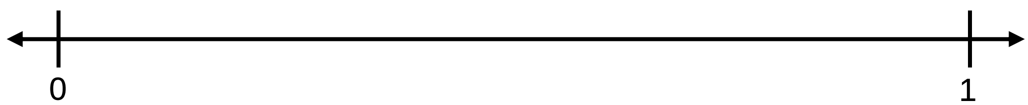
b) Use the one-eighth rod to partition the number line below into eighths. Label the partitions on the number line.



4 Thirds on a number line

a) Determine the Cuisenaire® Rod that is one-third of the whole represented below.

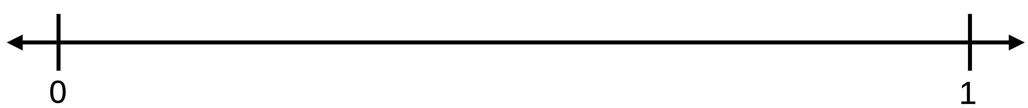
b) Use the one-third fraction rod to partition the number line below into thirds. Label the partitions on the number line.



5 Sixths on a number line

a) Determine the Cuisenaire® Rod that is one-sixth of the whole represented below.

b) Use the one-sixth fraction rod to partition the number line below into sixths. Label the partitions on the number line.



Comparing Fractions on a Number Line: Same Denominator

Materials:

- **Comparing Fractions on a Number Line Same Denominator**

Prompt students to complete **Comparing Fractions on a Number Line Same Denominator**

Debriefing Questions:

- How did you label the tick marks on each of the number lines? Why?
- How did you determine who ran the longest and shortest portions of the race?
- What do you notice about the denominators of each of the fractions? What does that tell us about the relationship among the three fractions?
- How does the numerator help determine who ran most/least when the denominators are the same? Why?
- When comparing fractions with the same denominator, what determines the greater or lesser fraction? Why?

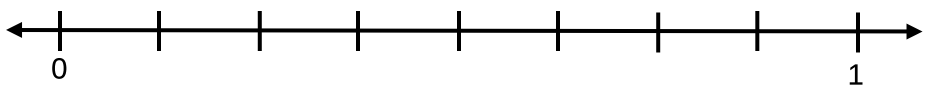
Comparing Fractions on a Number Line
Same Denominator

Kyle, Lorena, and Vanessa were competing in a running race.
After 10 minutes, each runner had completed a different fraction of the race.

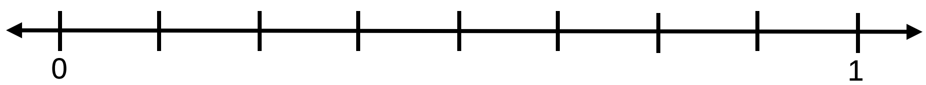
- Kyle completed $\frac{6}{8}$ of the race.
- Lorena completed $\frac{3}{8}$ of the race.
- Vanessa completed $\frac{7}{8}$ of the race.

Label each number line below.
Shade each number line to represent the fraction of the race each runner had completed after 10 minutes.

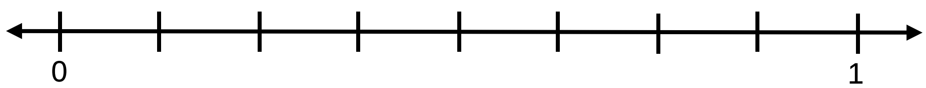
Kyle



Lorena



Vanessa



1 Who completed the greatest distance after 10 minutes? How do you know?

____

____

____

2 Who completed the shortest distance after 10 minutes? How do you know?

____

____

____

Comparing Fractions on a Number Line:
Same Numerator

Materials:

- **Comparing Fractions on a Number Line Same Numerator**

Prompt students to complete **Comparing Fractions on a Number Line Same Numerator**

Debriefing Questions:

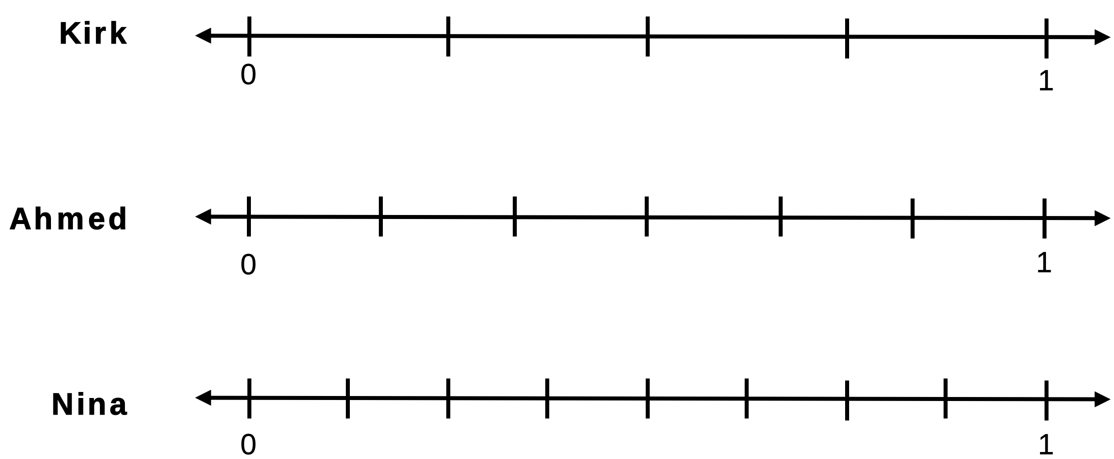
- How did you label the tick marks on each of the number lines? Why?
- How did you determine who ran the longest and shortest portions of the race?
- What did you notice about the numerators of each of the fractions? What does that tell you about the relationship among the three fractions?
- How does the denominator help determine who ran the most/least when the numerators are the same? Why?
- When comparing fractions with the same numerator, what determines the greater or lesser fraction? Why?

Comparing Fractions on a Number Line
Same Numerator

Kirk, Ahmed, and Nina were competing in a running race. After 10 minutes, each runner had completed a different fraction of the race.

- Kirk completed $\frac{2}{4}$ of the race.
- Ahmed completed $\frac{2}{6}$ of the race.
- Nina completed $\frac{2}{8}$ of the race.

Label each of the number lines below.
Shade each number line to represent the fraction of the race each runner had completed after 10 minutes.



1 Who ran the greatest distance after 10 minutes? How do you know?

____

____

____

2 Who ran the shortest distance after 10 minutes? How do you know?

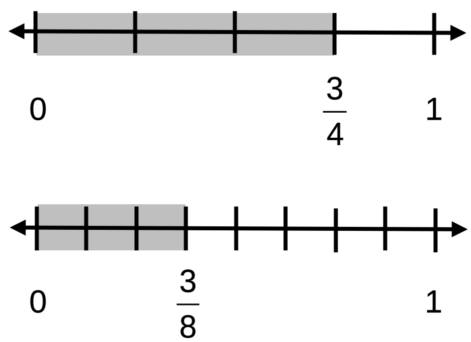
____

____

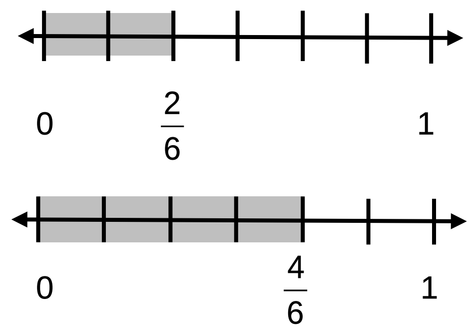
____

Check Point: Comparing Fractions on a Number Line

Common Numerators



Common Denominators



1 When comparing fractions with common numerators, I need to think about . . .

____

____

____

____

____

____

____

____

____

____

____

2 When comparing fractions with common denominators, I need to think about . . .

____

____

____

____

____

____

____

____

____

____

____

Representing Equivalent Fractions

Materials:

- **Representing Equivalent Fractions: Examples**
- **Representing Equivalent Fractions**

- Prompt students to review the **Representing Equivalent Fractions: Examples**.
- Prompt students to use the examples to complete **Representing Equivalent Fractions.**

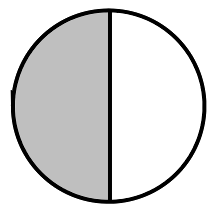
Debriefing Questions:

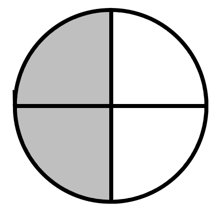
- What does it mean for two fractions to be equivalent?
- How did you determine which portions of the circle models to shade?
- How did you determine which portions of the area models to shade?
- How did you determine which points to label on the number line?
- How did you determine which portions to shade on the number line?
- How can you explain to a friend how to determine if two fractions are equivalent?

Representing Equivalent Fractions: Examples

The following examples represent pairs of equivalent fractions.

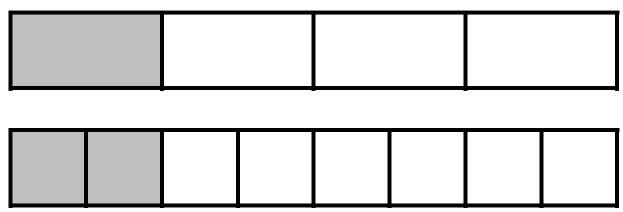
Example 1





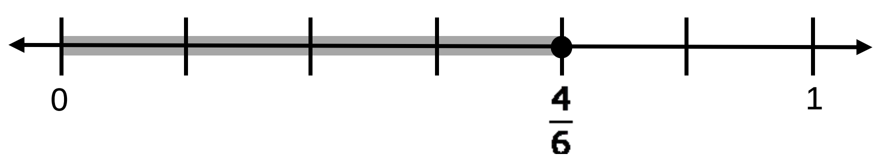
As shown in this example, $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent fractions. They both represent the same portion of the same sized whole.

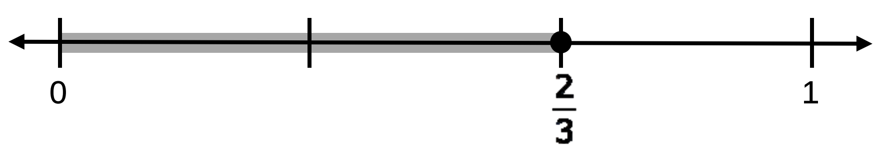
Example 2



As shown in this example, $\frac{1}{4}$ and $\frac{2}{8}$ are equivalent fractions. They both represent the same portion of the same sized whole.

Example 3

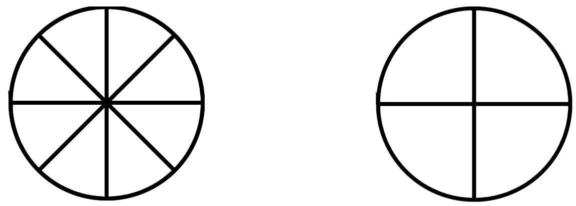




As shown in this example, $\frac{4}{6}$ and $\frac{2}{3}$ are equivalent fractions. They both represent the same distance from zero because the distance from zero to one is the same on both number lines.

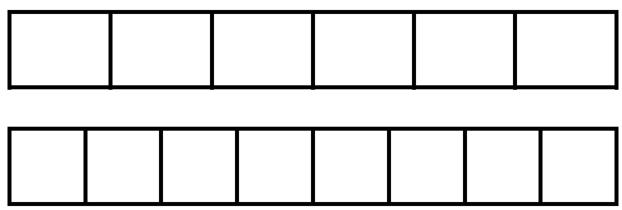
Representing Equivalent Fractions

Use the given models to represent the pairs of equivalent fractions. Write an explanation that describes why the fractions are equivalent.



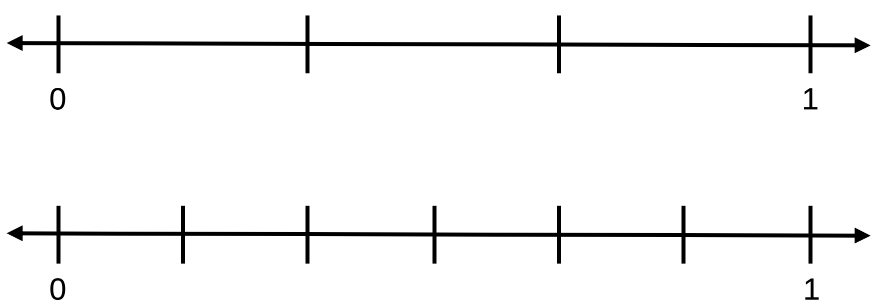
The fractions $\frac{6}{8}$ and $\frac{3}{4}$ are equivalent fractions because_________________

____ .



The fractions $\frac{3}{6}$ and $\frac{4}{8}$ are equivalent fractions because_________________

____ .



The fractions $\frac{1}{3}$ and $\frac{2}{6}$ are equivalent fractions because__________________

____ .

Equivalent Fractions

Materials:
- **Equivalent Fractions**
- **Number Line Cards**

Prompt students to use the **Number Line Cards** to complete **Equivalent Fractions.**

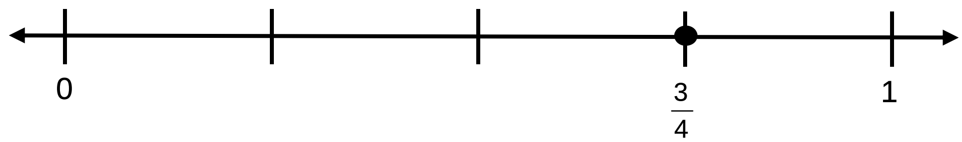
Debriefing Questions:
- How did you determine an example of an equivalent fraction?
- How did you determine a non-example of an equivalent fraction?

Equivalent Fractions

Use the **Number Line Cards** to find one example and one non-example of an equivalent fraction for each given point.

- Label the new fractions above the number line.
- Fill in the blanks with the new fractions to make each statement true.

1



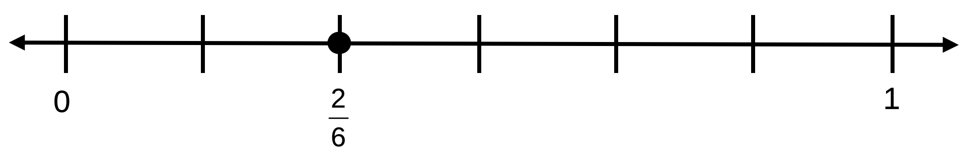
The fractions $\frac{3}{4}$ and ______ are equivalent fractions because _____________

____ .

The fractions $\frac{3}{4}$ and ______ are not equivalent fractions because __________

____ .

2



The fractions $\frac{2}{6}$ and ______ are equivalent fractions because _____________

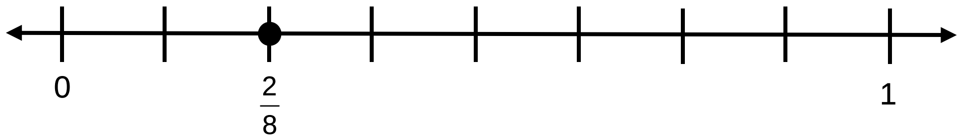
____ .

The fractions $\frac{2}{6}$ and ______ are not equivalent fractions because __________

____

____ .

3



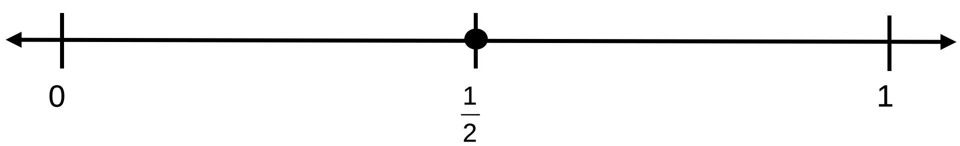
The fractions $\frac{2}{8}$ and ______ are equivalent fractions because ___________

____ .

The fractions $\frac{2}{8}$ and ______ are not equivalent fractions because ________

____ .

4



The fractions $\frac{1}{2}$ and ______ are equivalent fractions because ___________

____ .

The fractions $\frac{1}{2}$ and ______ are not equivalent fractions because ________

____ .

Check Point: Equivalent Fractions

1 **Fractions:** $\frac{4}{6}$ and $\frac{2}{3}$

Model:

Statement: $\frac{4}{6}$ and $\frac{2}{3}$ **are / are not** equivalent fractions (circle one)

because

---

2 **Fractions:** $\frac{2}{4}$ and $\frac{6}{8}$

Model:

Statement:

Number Line Cards

Fold along the dotted lines.