Presenter's Notes
Comparing Fractions
Principles to Actions: Effective Mathematics Teaching Practices

Slide 1	Facilitator should welcome participants and introduce him/herself to the audience.
Slide 2	https://www.freeimages.com/search/that-me/3 Warm-up Activity: Ask participants to stand if: • **If you love (summer, winter, spring, fall - select current season)** • **If you love mathematics** • **If you love teaching students** **If you are standing – this workshop if for YOU!** Share with participants that this session will provide valuable insights for all – regardless of assigned grade level assignment!
Slide 3	Source: NCTM www.nctm.org Review what has happened in the reform of mathematics education in the US: • **1989: Curriculum and Evaluation Standards for School Mathematics** In 1989, the National Council of Teachers of Mathematics (NCTM) released a document of major importance for improving the quality of mathematics education in grades K-12. This document, "Curriculum and Evaluation Standards for School Mathematics," contains a set of standards for judging mathematics curricula and for evaluating the quality of the curriculum and student achievement. It represents the consensus of NCTM's members about the fundamental content that should be included in the school mathematics curriculum, establishing a framework to guide reform in school mathematics. Inherent in the STANDARDS is the belief that all students need to learn more, and often different, mathematics. • **2000 Principles and Standards for School** A comprehensive and coherent set of mathematics standards for each and every student from prekindergarten through grade 12, *Principles and Standards* is the first set of rigorous, college and career readiness standards for the 21st century. *Principles and Standards for School Mathematics* outlines the essential components of a high-quality school mathematics program. It emphasizes the

	need for well-prepared and well-supported teachers and administrators, and it acknowledges the importance of a carefully organized system for assessing students' learning and a program's effectiveness. *Principles and Standards* calls for all partners—students, teachers, administrators, community leaders, and parents—to contribute to building a high-quality mathematics program for each and every student. • **2006 Curriculum Focal Points** Curriculum Focal Points are the most important mathematical topics for each grade level. They comprise related ideas, concepts, skills, and procedures that form the foundation for understanding and using mathematics and lasting learning. Curriculum Focal Points have been integral in the revision of many state math standards for Pre-K through grade 8. • **2010 Focus in High School Mathematics** Focus in High School Mathematics: Reasoning and Sense Making is a conceptual framework to guide the development of future publications and tools related to grades 9–12 mathematics curriculum and instruction. It suggests practical changes to the high school mathematics curriculum to refocus learning on reasoning and sense making. This shift constitutes a substantial rethinking of the high school math curriculum, advocating for more and better mathematics.
Slide 4	Source: NCTM *Principles to Actions* (www.nctm.org/principlestoactions) Continuing its tradition of mathematics education leadership, NCTM has defined and described the principles and actions, including specific teaching practices, that are essential for a high-quality mathematics education for all students.
Slide 5	The development of the standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the mathematics standards. However, the Standards do not describe or prescribe the teacher practices or actions that will ensure all students will be successful and mathematically literate.
Slide 6	Standards have contributed to higher achievement, but challenges remain. • In 2019, the National Assessment of Educational Progress (NAEP) mathematics assessment was administered to representative samples of

	fourth- and eighth-grade students in the nation, states, the District of Columbia, Department of Defense schools, and 27 participating large urban districts. The assessment was delivered on digital devices and assessed students' knowledge and skills in mathematics and their ability to solve problems in mathematical and real-world contexts. Students also answered survey questions asking about their opportunities to learn about and engage in mathematics inside and outside of school. • Mathematical performance, for PISA, measures the mathematical literacy of a 15-year-old student to formulate, employ and interpret mathematics in a variety of contexts to describe, predict and explain phenomena, recognizing the role that mathematics plays in the world. The mean score is the measure. A mathematically literate student recognizes the role that mathematics plays in the world in order to make well-founded judgments and decisions needed by constructive, engaged and reflective citizens. • 2019 NAEP: Lower-, middle-, and higher-performing students at grades 4 and 8 made gains compared to the early 1990s and 2000; **no significant progress was made at both grades for lower-performing students compared to a decade ago.**
Slide 7	Source: NCTM *Principles to Actions* (www.nctm.org/principlestoactions) Source: NCTM (www.nctm.org/principlestoactions) Summarize the Teaching and Learning Principle, noting the strong emphasis on promoting students' ability to make sense of mathematical ideas and to reason mathematically. Ask the participants to keep this Principle in mind throughout the session and in particular, as they watch the WV Classroom Video.
Slide 8	**Prior to the workshop** and based on the expected number of participants, prepare packets of the Beliefs About Teaching and Learning Mathematics cards. Cut the cards apart, shuffle the cards and place them in an envelope. Each pair or group of participants will need one packet of cards. Also, prepare a packet of cards for you to use during the discussion of the activity. During the activity, circulate among the pairs or groups of participants as they work to sort the belief cards. After all pairs or groups have completed the sorting task, ask the participants if there were any belief cards they found difficult to classify as either Productive or

	Unproductive. Ask why they found it difficult to assign the belief card to a category. Select a belief card from your packet and read the card to the participants. Ask the participants how they classified the belief and why. Repeat selecting cards and engaging the participants until 3 to 5 cards have been discussed. Be sure to select both Productive and Unproductive beliefs. At the end of the discussion, move to the next slide so that participants may see the correct sorting. Source: *Principles to Actions, Ensuring Mathematical Success for All*, (NCTM, 2014) pg. 11.
Slide 9	Source: *Principles to Actions, Ensuring Mathematical Success for All*, (NCTM, 2014) pg. 11.
Slide 10	Source: NCTM *Principles to Action* (www.nctm.org/principlestoactions)
Slide 11	Source: NCTM *Principles to Actions* (www.nctm.org/principlestoactions) These are the practices at the heart of the work of teaching. According to the research of D. Ball and F.M. Forzani they are the practices that are most likely to affect student learning. Give participants a few minutes to review the list of the eight, research-based, Mathematics Teaching Practices identified by NCTM as highly effective for student learning of mathematics. Ask the participants to identify the most significant noun within each of the eight practices. 1. Establish mathematics **goals** to focus learning. 2. Implement **tasks** that promote reasoning and problem solving. 3. Use and connect mathematical **representations** 4. Facilitate meaningful mathematical **discourse**. 5. Pose purposeful **questions**. 6. Build procedural **fluency** from conceptual understanding. 7. Support productive **struggle** in learning mathematics. 8. Elicit and use **evidence** of student thinking.
Slide 12	Source: Creating a Road Map, Corwin Publishing https://us.corwin.com/sites/default/files/upm-binaries/92312_Chapter_2__Implementing_Effective_Teaching.pdf

	Give each participant the handout, Effective Teaching Look Fors. Discuss the "Look Fors" for each Teaching Practice. Make sure all participants have the same understanding of what each "Look For."
Slide 13	Explain to the participants that they are about to view a video featuring a WV teacher and her students. Before we watch the video, it is essential that we note that the teacher complete the task prior to classroom implementation to understand how the task works and clearly appreciate the mathematics embedded in the task. This also provides the opportunity to identify specific questions that may arise and better understand the variety of opportunities to embedded in the task to deepen student understanding of comparing fractions. This task allows the teacher to differentiate the lesson by having two types of cards available: fraction cards with pictures and fraction cards with no pictures, as well as, differentiated fraction mats. During the task, the teacher to check for understanding of students mastery of comparing fractions by circulating around the pairs to see if they need scaffold support with the materials. Some students will still need to have the visuals to see the fraction. Students understanding of fractions begins with the introduction to what a fraction is in third grade. Students use models such as number lines or bar models to compare two fractions with the same denominators or the same numerator. In Grade 4, students extend the use of these models to compare fractions with different numerators and denominators using several strategies: find equivalent fractions, if the denominator is the same – compare the numerator, if the numerator is the same – compare the denominators, and using a benchmark of ½ and 1. Vocabulary is another essential tool during this activity. Students should be familiar with numerator, denominator, equivalent, and the symbols to compare: >, <. =. The teacher should create a list of questions to support and scaffold the task for struggling students. These may include: • Is the numerator larger than the denominator? • Is the numerator smaller than the denominator? • What is half of the denominator? Is half the denominator greater than, less than, or equal to the numerator?

Slide 14	Explain this video clip features a WV Teacher, Ms. Danielle Irby, and her students. The activity in our video, Comparing Fractions Using Benchmark Fractions, is designed for pairs of students to strengthen their conceptual understanding of fractions. The teacher introduces the task and incorporates a short review of the concept. The activity uses a set of cards and a benchmark fraction mat. The goal is to place each fraction in the appropriate category on the mat. The students use fraction cards to place on a benchmark mat. The benchmark mats are differentiated for the variety of students' knowledge of fractions. Each time the student places a fraction, he/she will explain their reasoning for the placement of the fraction between the benchmark fractions. The introduction of the lesson is one of the most crucial parts to engage the students in the math learning time. Video clip (Video 1- 0:00 to 3:20 and 12:16 to 13:15), The teacher, Danielle Irby, is focusing her fourth grade class upon the learning activity. Note: You will hear Ms. Irby state the activity as a game. However, this task does not have a winner or loser nor does the activity keep any points. So, the task is not a game but simply a learning activity.) She introduced the learning activity with her class. She has arranged the students in groups of two students except one group had 3. Her beginning to the task reviewed the concepts of fractions to spark the students' critical thinking skills while playing the activity. You will see how the teacher, Danielle Irby, focuses the students to make sense of the task, outline what concepts or vocabulary they need to know and do in order to position the fraction pieces on the benchmark mat. Other options a teacher could use to spark students' interest and engage their interest: 1) You could play a benchmark fraction video to capture their attention: https://youtu.be/vJXkxu7JIOo 2) Music such as Equivalent Fractions is another way to spark the students' interest: https://youtu.be/vKXqzpz-G0s
Slide 15	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM , 2014, p. 12.

	One of the research-based, teaching practices identified by NCTM is the importance of establishing clear mathematics goals to focus student learning and to guide teacher decisions. The mathematical purpose of a lesson should not be a mystery to students. Classrooms in which students understand the learning expectations for their work perform at higher levels than classrooms where the expectations are unclear (Haystead and Marzano 2009; Hattie 2009). Although daily goals need not be posted, it is important that students understand the mathematical purpose of a lesson and how the activities contribute to and support their mathematics learning. Goals or essential questions motivate learning when students perceive the goals as challenging but attainable (Marzano 2003; McTighe and Wiggins 2013). Teachers can discuss student-friendly versions of the mathematics goals as appropriate during the lesson so that students see value in and understand the purpose of their work (Black and William 1998a; Marzano 2009). When teachers refer to the goals during instruction, students become more focused and better able to perform self-assessment and monitor their own learning (Clarke, Timperley, and Hattie 2004; Zimmerman 2001).
Slide 16	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM , 2014, p. 12.
Slide 17	Source: YouTube https://www.youtube.com/watch?v=EcZBUFqFLxc
Slide 18	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM , 2014, p. 16. Compare the actions of teachers versus the actions of students when establishing mathematics goals to focus learning. Ask the participants if they believe the teacher actions above are routine in their schools. If yes, ask how they know. If no, ask what is needed for the teacher actions above to become routine in their school.
Slide 19	The WV College and Career Readiness Standards (WVCCR) in this lesson address students' ability to understand and use fractions with benchmark fractions. The teacher might use an "I Can" statement for the student to understand the focus. "I can use benchmark fractions to compare other fractions."

	I can convert a mixed number to an improper fraction for comparison of fractions." I can convert different fractions to see if they equivalent fractions for comparison." The students are using their knowledge of fractions in this activity: • If the denominators are the same, then the **fraction** with the greater numerator is the greater **fraction**. • The **fraction** with the lesser numerator is the lesser **fraction**. • And, as noted above, if the numerators are equal, the **fractions** are equivalent. Use < or > to **compare** the two **fractions**. Since equivalent fractions do not always have the same numerator and denominator, one way to determine if two fractions are equivalent is to find a common denominator and rewrite each fraction with that denominator. Once the two fractions have the same denominator, can check to see if the numerators are equal. If they are equal, then the two fractions are equal as well.
Slide 20	The learning goals of this lesson could be written in student friendly language. Students need to understand what they are learning. The first goal of "explain the strategies and reasoning of their placement of the fraction" would be 'Tell us where you decided to place the card and why you chose this position on the benchmark mat." The second learning goal would be " do both students agree on the where the fraction is placed. If not, why do you disagree? Talk about it until you have a decision." The third one could be stated as "when we finish our activity, we are going to talk about what ways you used to compare the fractions." The fourth goal and the most important one to this activity, " How did you decide the fraction was greater, less or equal to the benchmark fractions?" Learning goals need to be have clear verbs, small and specific, in a checklist sequence that will give students the needed boost to feel successful.
Slide 21	THINK, PAIR AND SHARE Ask participants to respond to the questions. Remind them to utilize their completed *Effective Teaching Look Fors* form. Bring the group back together to summarize their thoughts. 1. Students will apply the understandings of knowledge of fractions on a number line with benchmark fractions.

	2. Students' familiarity with fractions and methods to determine their relationship to other fractions. 3. Students' developed skills and understandings about a variety of methods of determining how to compare fractions by using the numbers, area, bar model, or see if the fractions are equivalent fractions. In what ways did the math goals focus the teacher's interactions with the students throughout the lesson? The key to making these students' learning experience worthwhile is to focus the planning and interactions on math expectations and goals or phrased in terms of desired student outcomes—the knowledge, skills, attitudes, values, and mathematical dispositions that the teacher wants to develop in the students. In this learning task, the teacher's interactions are aimed to providing feedback specific to math expectations and goals. The teacher's feedback and open-ended questions as he/she walks around and checks on the groups' progress helps her students improve their performance and solidify their understanding. The math expectations, learning goals and providing feedback work in tandem. As the students work in groups, the teacher is listening, checking to see if progress is being made, and providing thought provoking comments to redirect any groups who are frustrated beyond productive math struggle. Providing feedback is an ongoing process for the teacher. She monitors they are continually working toward the math goals. The teacher. Danielle Irby, assists by communicating in the interactions to the students prompts . She provides thought-provoking comments that helps them better understand what they are to learn and what changes are necessary to improve their learning or completion of this mathematical activity. One point you will notice is as the teacher she does not give the students the answers to where the fractions are located on the benchmark mat but guides the students by questioning.
Slide 22	Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving such as this learning activity. Effective teaching with this kind of mathematical problem engages students in solving and discussing real world applications of the math concepts. In this video, Danielle Irby, the teacher is encouraging the students to reason and promote use of the students' prior knowledge to apply this thinking in this task.

Slide 23	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM , 2014, p. 17. There is **no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks** with which the teacher engages students in studying mathematics. Tasks should provide opportunities for students to think and make sense of mathematics. Having **multiple entry points is very important** because of the **impact on equity**. If students can make a table, create a drawing, or use manipulatives, the math becomes more accessible to students who might not immediately know how to solve the problem.
Slide 24	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM , 2014, p. 17. Tasks should provide opportunities for students to make sense of mathematics. Rich mathematical tasks engage students in sense-making through deeper learning that require high levels of thinking, reasoning, and problem solving.
Slide 25	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM , 2014, p. 18. THINK,PAIR AND SHARE: Ask participants if the math tasks provided in students' texts have all the characteristics of a GOOD math task. If NO, discuss what traits are usually missing in textbook provided math tasks. Ask participants to share how they find GOOD math tasks (outside of the adopted textbook) for their students.
Slide 26	THINK, PAIR AND SHARE Ask participants to respond to the question. Remind them to utilize their completed *Effective Teaching Look Fors* form. Give the participants time to communicate and discussion this question. When the students were comparing fractions, there were multiple ways they could use to determine the placement of the fractions in relationship to the benchmark fractions. The comparing of fractions did not have to be determined by one certain method.

	Sample responses of their reasoning might include: • A fraction is bigger than one if the numerator is larger than the denominator (e.g. 7/6) and a fraction is less than one if the numerator is smaller than the denominator (e.g. 2/5). A fraction is equal to one if the numerator is equal to the denominator (e.g. 3/3). • A fraction is bigger than 1/2 if the denominator is less than two times the numerator. A fraction is equal to 1/2 if the denominator is equal to two times the numerator. A fraction is less than 1/2 if the denominator is more than twice the numerator. • Students can draw pictures of fractions to compare them with 1/2 and 1. • Students can plot fractions on the number line to compare them with 1/2 and 1 .
Slide 27	In this video clip, you will see the teacher, Ms. Irby, and her students' discussion a problem with the cards. It leads to different representation of the mat. Video clip: (Video 1: 18:36 to 19:44) Lead a discussion about how this arrangement of the fractions assist the students will the task.
Slide 28	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p. 24 Ask participants what is meant by representations of mathematical ideas.
Slide 29	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p. 24

Slide 30	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p. 25
	Engage participants in a discussion of the graphic. Some probing questions you might ask include:
	• What is the distinction between physical and visual representations?
	    • Describe some physical representations that might support students' thinking.
	    • Describe some visual representations you would expect students to produce.
	    • In the diagram, why do the arrows go both ways?

	The point is not for students to use different representations just for the sake of it. What's crucial is that students are using/connecting representations as TOOLS to solve problems and to build understanding of concepts. The depth of understanding is related to the strength of connections among mathematical representations that students have internalized (Pape and Tchoshanov 2001; Webb, Boswinkel, and Dekker 2008). For example, students develop understanding of the meaning of the fraction 7/4 (symbolic form) when they can see it as the quantity formed by "7 parts of size one-fourth" with a tape diagram or on a number line (visual form), or measure a string that has a length of 7-fourths yards (physical form).
Slide 31	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p. 25
Slide 32	Source: Virginia Department of Education Mathematics Institutes 2019 Professional Development Resources http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2019/index.shtml Provide participants with a copy of the Rich Mathematical Task Rubric. Allow time for participants to read the rubric. Discuss the Task Levels and the Descriptions for **Representations and Connections**.
Slide 33	THINK, PAIR AND SHARE In pairs, ask participants to respond to the questions. Remind them to utilize their completed *Effective Teaching Look Fors* form.
Slide 34	Video clip: Video 2 the times on the clip are 0:41 to 12:27 Guide the discussion about how Danielle Irby, the teacher in this clip is having effective conversations. She is teaching and engaging her students in discourse to advance the mathematical learning of the students. This mathematical discourse includes the purposeful exchange of ideas through classroom discussion, as well as through other forms of verbal, visual, and written communication.
Slide 35	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM, 2014, p. 29 Ask participants what their concerns are about facilitating meaningful mathematical discourse.

Slide 36	Source: *Asking Questions and Promoting Discourse*, NCTM https://www.nctm.org/Conferences-and-Professional-Development/Tips-for-Teachers/Asking-Questions-and-Promoting-Discourse/ Mathematical discourse is a powerful sense-making tool, but it doesn't just *happen*. Students must develop both the inclination and habit of attending to each other's mathematical ideas, and they must have the time and space to make sense of, critique, and develop the ideas. Teacher talk moves are crucial supports for developing students' capacity to engage in productive mathematical discussions (Kazemi and Hintz, 2014; Chapin, O'Connor, and Anderson, 2009).
Slide 37	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM , 2014, p.35. Ask participants about student interest in engaging in mathematical discourse. Why do some students try to avoid participating in mathematical discourse? How can the teacher help students to become more active participants in mathematical discourse?
Slide 38	Source: Virginia Department of Education Mathematics Institutes 2019 Professional Development Resources Grades 6-8 Institute PowerPoint http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2019/index.shtml The chart is from the work of Dr. John Hattie. He analyzed the impact of student, teacher, home, curriculum and community actions on student learning. Based on data, each action scored an Effect Rate for the ability to affect student achievement. The values range from a negative .20 to a positive 1.20. The higher the score, the greater the positive impact of the action on student achievement. Classroom discussion has an effect size of 0.82, which is more than twice what we need to know that a specific strategy will make a difference in learning. Classroom discussion is defined as "a method of teaching that involves the entire class in a discussion. Classroom discussion is a critical area with a huge effect size. Classroom discussions provide the opportunity for students to communicate with one another for a variety of functions including to activate prior knowledge, to explore new topics, to learn from others, and to demonstrate their learning. This is an engagement strategy which provides all students the chance to participate, especially when structured in a way that extends beyond a teacher-student question and answer sequence.

	Consider what visiblelearning.org, asserts regarding what the most effective classrooms discussions should include: • creating a series of questions for the students to think about • allocating enough time in the lesson for an elaborate discussion • making sure that students can freely express their opinion without being laughed at or ridiculed
Slide 39	THINK, PAIR AND SHARE In pairs, ask participants to respond to the questions. Remind them to utilize their completed *Effective Teaching Look Fors* form.
Slide 40	Video Clip: Video 1 Video 1 times are 5:10 to 7:53 and 24:38 to 25:19 In this clip, what did you notice about posing purposeful questions? Points or comments that might be forthcoming from the audience: 1. The teacher, Danielle Irby, was crafting *questions* that help *students* deepen their *thinking* rather telling the students what to do. 2. You will notice her comments with *questions* are on the video is guiding the students to the right track: Posing *purposeful questions* is necessary to keep students working productively. 3. Danielle Irby used questions like these in the video. You might want a few general questions in your "back pocket" to *guide students* stumped by *any math problem*. 4. Teachers can also support productive struggle by *asking questions* that make student think about they are doing or solving. Here are some purposeful questions you want to keep handy to use with any math problem: • What do you already know? What do you need to know? • What do you understand so far? • What parts of this problem make sense to you? • What in this problem doesn't yet make sense to you? • How might we decide which approach makes more sense? • What are some math ideas we've worked on before that could help you with this new idea? • Purposeful Questions to this activity: Is the numerator larger than the denominator? Is the numerator smaller than the denominator?

	What is half of the denominator? Is half the denominator greater than, less than, or equal to the numerator? These questions can help students continue building their background knowledge of how to compare fractions.
Slide 41	Ask participants how posing purposeful questions can be used to inform instruction and assess student understanding? Ask participants how posing purposeful questions can promote equitable learning opportunities for all students? **Identify, in advance, the big ideas that your lesson examines and the mathematical outcomes that students should achieve.** Take time to brainstorm the multiple approaches that could be taken to work through similar problems and the misconceptions that students might have. Make sure that you prepare questions that address these multiple approaches and misconceptions, prompting a discussion about when particular approaches are better than others and how to explain why each misconception is faulty. Close each lesson with a summarizing question that reiterates the big ideas.
Slide 42	Source: Principles to Action: Ensuring Mathematical Success for All, NCTM, 2014 Effective mathematics teaching relies on questions that encourage students to explain and reflect on their thinking as an essential component of meaningful mathematical discourse. Purposeful questions allow teachers to discern what students know and adapt lessons to meet varied levels of understanding, help students make important mathematical connections, and support students in posing their own questions. However, merely asking questions is not enough to ensure that students make sense of mathematics and advance their reasoning.
Slide 43	Source: Virginia Department of Education Mathematics Institutes 2019 Professional Development Resources Grades 6-8 Institute PowerPoint http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2019/index.shtml The chart is from the work of Dr. John Hattie. He analyzed the impact of student, teacher, home, curriculum and community actions on student learning. Based on data, each action scored an Effect Rate for the ability to affect student achievement. The values range from a negative .20 to a positive 1.20. The higher the score, the greater the positive impact of the action on student achievement.

	Ask participants why Self-verbalization and Self-questioning have a greater impact on student achievement. So much of classroom time is spent with teachers questioning the students. Cotton (1989), for example, reviewed the evidence and found questioning was the second most dominant teaching method (after teacher talk), with teachers spending between 35–50 percent of teaching time posing questioning (e.g., Long & Sato, 1983; van Lier, 1998)— that is about 100 questions per hour (Mohr, 1998)—and the responses from the teacher to the students' answers to these questions was some form of judgment or correction, primarily reinforcing in nature, affirming, restating, and consolidating student responses. Brualdi (1998) claimed that teachers asked 300 to 400 questions per day, and the majority of these were low-level cognitive questions—60 percent recall facts and 20 percent are procedural in nature (Wilen, 1991) These are not open, inquiry questions, as students understand that the teacher already knows the answer (they are "display" questions; 82 percent are of this nature: Cotton, 1989). The reason for so much questioning relates to the conceptions of teaching and learning held by many teachers—that is, their role is to impart knowledge and information about a subject, and student learning is the acquisition of this information through processes of repetition, memorization, and recall: hence the need for much questioning to check that they have recalled this information. The overall effects of questioning vary, and the major moderator is the type of question asked—surface questions can enhance surface knowing and higher-order questions can enhance deeper understanding.
Slide 44	Adapted from Smith, M. S., et al. (2017). *Taking Action: Implementing Effective Mathematics Teaching Practices, National* Council of Teachers of Mathematics, p.102 Researchers have created a variety of frameworks to categorize the types of questions that teachers ask (e.g., Boaler and Brodie 2004; Chapin and O'Connor 2007). Though the categories differ across frameworks, commonalities exist among the types of questions. For example, the frameworks generally include questions that ask students to recall information, as well as questions that ask students to explain their reasoning. The chart above displays a set of question types that synthesizes key aspects of these frameworks that are particularly important for mathematics teaching. Although the question types differ with respect to the level of thinking required in a response, all of the question types are

	necessary in the interactions among teachers and students. For example, questions that gather information are needed to establish what students know, while questions that encourage reflection and justification are essential to reveal student reasoning.
Slide 45	THINK, PAIR AND SHARE With participants grouped in pairs, ask participants to respond to the questions. Remind them to utilize their completed *Effective Teaching Look Fors* form.
Slide 46	Video clip: Video 1 times are 13:03 to 13:39 and 13:44 to 14:12. Video 2 times are 4:49 to 6;47 The last clip is from the class demonstrating their conceptual understanding by selecting a fraction and laying it on a large yarn benchmark number line. The benchmarks may be sticky notes with 0, ½ and 1 printed on them. Video 3: 00:05 to 2:11 and 3:21 to 5:28. There are many important lessons to be learned, reviewed or applied from comparing fractions to benchmark fractions activity, including: • The denominator tells how many pieces my whole is. When the whole is partition into more pieces, the pieces are smaller (this is why 1/3 is less than 1/2). • The numerator tells me how many equal sized pieces I have. So 6/10 is more than 5/10 because I have one extra piece. • Fractions are built from the unit fractions so it is important to understand and be able to represent the unit fractions. • If using the fraction cards with pictures, equal sized wholes are important when comparing fractions. • In a picture of equivalent fractions, the whole is broken into different sized pieces, but the same total amount is shaded. • The relationship between the numerator and denominator can help the student think about the value of the fraction. • Fractions equivalent to 1/2 have a numerator that is half of the denominator. • Fractions equivalent to 1 whole have the same numerator and denominator. • When the numerator is a bigger number than the denominator, the fraction is greater than one whole.

	• When doing mathematics, patterns emerge. These patterns support students in making conjectures, supporting their reasoning, and proving mathematical claims. Overall, this math activity provides students with opportunities use of the conceptual understanding with fractions.
Slide 47	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014 Ask participants to share their thoughts on the importance of fluency with procedures to student success in mathematics. Discuss with the participants the significance of the quote: *A rush to fluency undermines students' confidence and interest in mathematics and is considered a cause of mathematics anxiety.*
Slide 48	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014 Paraphrase the bullets on the slide. Fluency is not intended as the main or sole target of instruction. Problem Solving and Reasoning that are the focus of the second teaching practice need to co-exist with procedural fluency. This occurs when students first have opportunities to develop conceptual understanding. Computational fluency is strongly related to number sense and involves so much more than the conventional view of it encompasses. Developing students' computational fluency extends far beyond having students memorize facts or a series of steps unconnected to understanding (Baroody 2006; Griffin 2005) Early work with reasoning strategies is related to algebraic reasoning. As students learn how quantities can be taken apart and put back together in different ways (i.e., decomposition and composition of numbers), they establish a basis for understanding properties of the operations. Students need this early foundation for meaningful learning of more formal algebraic concepts and procedures throughout elementary school and into middle and high school.
Slide 49	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p. 42. Discuss why is it important to build procedures from conceptual understanding. Ask participants: What kind of "bizarre results' could happen if students are taught mechanical execution of procedures without understanding the mathematical basis?

	Fluency is not a simple idea. Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently. Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems. This sequence is beneficial whether students are building toward fluency with single- and multi-digit computation with whole numbers or fluency with, for example, fraction operations, proportional relationships, measurement formulas, or algebraic procedures.
Slide 50	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p. 47 Ask participants: What visual models have they used to build procedural fluency? Is it important for students to be able to explain why the procedures they used worked? Why?
Slide 51	THINK, PAIR AND SHARE With participants grouped in pairs, ask participants to respond to the questions. Remind them to utilize their completed *Effective Teaching Look Fors* form.
Slide 52	In this clip, Video clip: Video 1 times are 9:44 to 10:42 What did you notice in the clip? Danielle Irby is teaching that struggling is part of learning math. She wants to encourage creativity and build authentic engagement and perseverance in her students. She guiding their thoughts with the direction that the placement of the fraction on the benchmark mat is going to make sense.
Slide 53	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014 Discuss with the participants the importance for students to be able to figure things out for themselves. It is through this process of figuring things out on their own that they will develop authority and ownership of their own learning.

Slide 54	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p.48. In comparisons of mathematics teaching in the United States and in high-achieving countries, U.S. mathematics instruction has been characterized as rarely asking students to think and reason with or about mathematical ideas (Banilower et al. 2006; Hiebert and Stigler 2004). Teachers sometimes perceive student frustration or lack of immediate success as indicators that they have somehow failed their students. As a result, they jump in to “rescue” students by breaking down the task and guiding students step by step through the difficulties. Although well intentioned, such “rescuing” undermines the efforts of students, lowers the cognitive demand of the task, and deprives students of opportunities to engage fully in making sense of the mathematics.
Slide 55	Source: Virginia Department of Education Mathematics Institutes 2019 Professional Development Resources Grades 6-8 Institute PowerPoint http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2019/index.shtml The chart is from the work of Dr. John Hattie. He analyzed the impact of student, teacher, home, curriculum and community actions on student learning. Based on data, each action scored an Effect Rate for the ability to affect student achievement. The values range from a negative .20 to a positive 1.20. The higher the score, the greater the positive impact of the action on student achievement. There is a high relationship between engagement and degree of concentration on tasks. One way of enhancing concentration is to mentally visualize the processes and strategies involved in a task. Students who mentally visualized various motor tasks were more effective compared to those that did not (d = 0.48). Teachers greatly influence how students perceive and approach struggle in the mathematics classroom. Even young students can learn to value struggle as an expected and natural part of learning, as demonstrated by the class motto of one first-grade math class: “If you are not struggling, you are not learning” (Carter 2008, p. 136). Teachers must accept that struggle is important to students’ learning of mathematics, convey this message to students, and provide time for them to try to work through their uncertainties. Unfortunately, this may not be enough, since some students will still simply shut down in the face of frustration, proclaim “I don’t know,” and give up. Dweck (2006) has shown that students with a fixed mindset— that is, those who believe that intelligence (especially math ability) is an innate trait—are more likely to give up when they encounter

	difficulties because they believe that learning mathematics should come naturally. By contrast, students with a growth mindset—that is, those who believe that intelligence can be developed through effort—are likely to persevere through a struggle because they see challenging work as an opportunity to learn and grow.
Slide 56	Source: *Principles to Action: Ensuring Mathematical Success for All*, NCTM, 2014, p.52. Use the chart to engage participants in a discussion of how they support students to persevere in problem solving.
Slide 57	THINK, PAIR AND SHARE With participants grouped in pairs, ask participants to respond to the questions. Remind them to utilize their completed *Effective Teaching Look Fors* form.
Slide 58	Set up: In this clip, you will see Danielle Irby eliciting and using evidence of the students' thinking about the activity. Play: Video 4 The times of this clip are 0:31 to 4:38. Follow-up: What did you notice about the teacher's conversation with the students? Danielle Irby, the teacher, was asking good questions to generate the types of thinking and discussion that are central to the learning goal. She is listening carefully to student thinking and make note of which types of fractions or misconceptions about fractions for reteaching and note the students who are progressing successfully. When you, the teacher, are eliciting and using evidence of student thinking, some questions you might use are: Who Disagrees? Who will explain why or why not? Who has the same answer, but a different way to explain it? Who has a different answer? You, the teacher, are listening carefully to students' ideas during the math learning activity as this task, Comparing Fractions to Benchmark Fractions provides.
Slide 59	Source: *Principles to Action: Ensuring Mathematical* Success for All, NCTM, 2014, p. 53 Ask participants: What can serve as evidence of student thinking when assessing progress toward understanding?

	How should the evidence affect what a teacher does instructionally for students?
Slide 60	Source: *Principles to Action: Ensuring Mathematical* Success for All, NCTM, 2014, p. 53 Ask participants: What is formative assessment? Is it possible to do formative assessment every day? If so, how?
Slide 61	Source: Virginia Department of Education Mathematics Institutes 2019 Professional Development Resources Grades 6-8 Institute PowerPoint http://www.doe.virginia.gov/instruction/mathematics/professional_development/institutes/2019/index.shtml The chart is from the work of Dr. John Hattie. He analyzed the impact of student, teacher, home, curriculum and community actions on student learning. Based on data, each action scored an Effect Rate for the ability to affect student achievement. The values range from a negative .20 to a positive 1.20. The higher the score, the greater the positive impact of the action on student achievement. Cognitive task analysis was among the highest scoring actions to affect student achievement. Ask participants to give an example of what a cognitive task might look like in the classroom and why it is so effective.
Slide 62	THINK, PAIR AND SHARE With participants grouped in pairs, ask participants to respond to the questions. Remind them to utilize their completed *Effective Teaching Look Fors* form.
Slide 63	THINK, PAIR AND SHARE – **CLOSURE ACTIVITY** With participants grouped in pairs, ask participants to reflect on what they have learned about the eight Effective Mathematics Teaching Practices. Ask each participant to select 1-2 Practices to study and to implement with students. In pairs, develop a list of actions they will need to take in the pursuit of mathematical success for all their students.
Slide 64	We wish to thank Danielle Irby and her students for providing these rich videos for our discussions today.