Comparing Volatility Forecasts of Univariate and Multivariate GARCH Models: Evidence from the Asian Stock Markets

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Abstract: This paper compares the forecasting performance of univariate (EGARCH) and multivariate GARCH models for the volatilities of stock market index returns of Japan, India, Indonesia and Pakistan each paired with the US stock market. We also investigate the role of Global Financial Crisis (GFC) of 2007-2009 in affecting forecasting performance. We investigate whether incorporation of the linkage with the US stock market in a multivariate GARCH framework helps in improving the volatility forecasts of Asian stock markets. The daily stock returns from July 3, 1997 to November 12, 2012 are employed. Forecasts are evaluated using three measures namely, R^2 (coefficient of determination), Mean Absolute Percentage Error (MAPE) and Median Absolute Percentage Error (MdAPE). The results show that correlation with the US helps in improving the accuracy of volatility forecast of Asian stock markets i.e. performance of multivariate GARCH is found to be better than the EGARCH for all the countries considered while including GFC dummy does not result in improved forecast of stock market volatility forecast.

Keywords: GARCH, Multivariate, Univariate, Volatility JEL Classification: F37, F47, C58

1. Introduction

Stock market volatility plays a prominent role in many financial decision making cases. Stock market volatility is reflected in large stock price movements that often occur in bunch in response to news that are expected to affect firm's cash flows. Volatility forecasts are employed in several financial activities e.g. in risk management and options pricing. An option trader makes his decsion about the future pay off of the contract through the expected volatility of the underlying asset. Volatility forecast are also used in hedging, portfolio selection, market making and timing etc (Engle and Patton, 2001). Conditional volatility is an important ingredients in being a part in the computation of important financial measures value-at-risk (VaR), conditional asset pricing and option pricing so obtaining reliable volatility forecast has been an important area of interest for academics and practitioners.

The international trade and finance between the economies and deregulation and liberalization of both emerging and developed stock markets have increased the integration of the markets of developed and developing countries in terms of increase in correlation. A question of interest is whether and to what extent this integration improves the volatility forecast of the markets. The US is perhaps the most important financial market and the development in the US financial, economic, and political conditions are expected to drive the world financial markets. So it is interesting to investigate whether linkage with the US help improve the volatility forecast. In this paper we investigate this possibility for some Asian stock markets namely, Japan, India, Indonesia and Pakistan.

There are empirical evidences that correlation between the stock market is increased during financial crisis, see for example Hartmann et al. (2004), Jang and Sul (2002) and Lin et al. (1994). We therefore also consider the forecast performance of volatility models when the dummy variable capturing the global financial crises of 2007-2009 is included in the models. To attain this objective we compare the performance of volatility forecast of univariate GARCH model and multivariate GARCH model for four Asian markets namely Japan, India, Indonesia and Pakistan, with the US market pairs.

2. Literature Review

Iqbal and Javed (2012) investigate d the role of local and global local macroeconomic variables on the forecasting performance of Pakistani stock market volatility through EGARCH model. They used monthly data on stock returns and macroeconomic variables from January 1990 to December 2010. One step ahead forecasts for last 190 months through rolling window method was performed. Using the Mean Absolute Percentage Error (MAPE) and the Median Absolute Percentage Error (MdAPE) they found that the local macroeconomic variables improve the forecast of Pakistani stock market more than the global variables.

Angabeni et al. (2011) compared the forecasting performance of GARCH, EGARCH and GJR-GARCH models with and without global financial crisis 2007-2008 consideration for daily data on the Malaysian stock markets. The forecasting is performed for both the normal periods of March 16, 2010 to September 16, 2010 and for the crises period of January 1, 2008 to July 1, 2008. Using the RMSE, MAE, MAPE and Thiel's Inequality Coefficient (TIC) they found that GARCH (1,1) model performed better than the competing models.

Wei (2002) considered the weekly returns of two Chinese stock markets namely Shanghai and Shenzhen stock exchange to test the forecasting performance of their volatilities through the GARCH, Quadratic GARCH, GJR and Random Walk models. Using the data from the period 1992 to 1998 one step ahead rolling forecast from 1997 to 1998 was computed. To compare the performance of the volatility forecast, he used Mean Square Error (MSE) and Median Square Error (MedSE) techniques. He provided evidence in favor of QGARCH model and that the GJR model is not recommended to forecast.

Yu (2002) employed nine different models including univariate ARCH and GARCH model and stochastic volatility (SV) model to evaluate their monthly volatility forecasting performance for New Zealand stock markets. The sample period considered range from January 1, 1980 to December 31, 1998. Forecasting accuracy was computed using the Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Theil-U statistics and the LINEX loss function. One step ahead monthly volatility rolling forecast are performed for January 1994 to December. The results suggest that stochastic volatility model improves the forecasting compared to the other models while the performance of ARCH models depends on the type of the models used.

monthly FTA-All Share (1984 to 1996) and FTSE100 (1969 to 1996) stock index volatility from the UK stock market and used the symmetric and asymmetric loss functions for forecasting comparison. Forecasting period was considered from 1995 to 1996. The results suggest that the GARCH, moving average and exponential smoothing models gave better daily volatility forecast than others models under the symmetric loss function. Moreover, the GARCH and moving average models are revealed as the most consistent forecasting performance provider for all frequencies.

Gokcan (2000) compare the linear and non-linear univariate GARCH versions i.e. GARCH (1,1) and EGARCH(1,1) respectively, using the monthly returns of the seven developing stock markets using data from February 1988 to December 1996. One-step ahead rolling method to forecast monthly volatility is performed for the period July 1997 to November 1997. Using the Mean Square Error (MSE) to evaluate forecast accuracy he found support for the GARCH (1,1) model instead of EGARCH (1,1) model.

Chong et al. (1999) used five different daily stock market indices of the Malaysian stock market to compare the volatility forecasting performance of the univariate GARCH, unconstrained GARCH, non-negative GARCH, EGRACH, IGARCH and GARCH-in mean models. The data was taken from January 1, 1989 to December 31, 1990. Based on the Mean Square Error (MSE) to evaluate the one step ahead forecasting for last 50 observations they found that the EGARCH was the best for out of sample forecasting.

Fransis and van Dijk (1996) analyze the performance of univariate GARCH, Quadratic GARCH and GJR-GARCH models to forecast the weekly volatility of five European stock markets. They employed weekly data from 1986 to 1994 and used the Median Square Error (MedSE) to evaluate forecasting. One step ahead forecasting is performed for four years i.e. 1990 to 1994. The study suggested that EGARCH model improves the volatility forecast than other models.

Tse and Tung (1992) compared three methods i.e. a naive method, exponentially weighted moving average (EWMA) method and the univariate GARCH model to examine the out-of-sample forecast of the monthly volatility using five different daily indices of Singapore stock market. They considered the period 1975 to 1988 and used the Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) as the forecasting accuracy measures. Their results supported the EWMA as the best model among selected.

Tse (1991) focused the daily stock returns volatility of Japanese stock market from 1986 to 1989 and compared the forecasting performance of univariate ARCH/GARCH models with naive historical variance exponential weighted moving average (EWMA) model. He identified that the EWMA provided the best forecast.

Akgiray (1989) employed the Center for Research and Security Prices (CRSP) value-weighted and equal-weighted indices from January 1963 to December 1986. Several out of sample forecasts for monthly return variances are computed from historical estimate, EWMA and ARCH/GARCH

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model. He evaluated the forecasting performance through ME, RMSE, MAE and MAPE measures and concluded the GARCH (1, 1) performed the best.

It can be seen that all of these studies employed only univariate GARCH models in forecasting stock market volatility. Brooks and Persand (2003) was among very few studies which compared the forecasting ability of univariate and multivariate GARCH models. They employed several linear and GARCH-type models including GARCH to forecast the daily stock volatility considering the VaRs and equally weighted portfolio of three key financial time series of UK namely, FTSE ALL Share Total Return Index, the FTA British Government Bond Index and the Reuter Commodities Price Index. They used daily data was collected from the period 1980 to 1999 and employed forecasting evaluation measures of Mean Square Error (MSE), Mean Absolute Error (MAE) and Percentage of over-predictions. One, two to twenty steps ahead forecasts were computed then aggregated to calculate the volatility forecast over the next 5, 10, and 20 days. Their findings suggest that the performance of Random Walk in volatility, EGARCH and the EWMA models was poor.

As evident from the literature very studies have compared the forecasting performance of univariate and multivariate GARCH models. Also no previous study, to the best of our knowledge, has investigated the role of stock market linkages and of the impact of financial crises on performance of volatility forecast using a multivariate framework. Brooks and Persand (2003) did not consider the correlation with any other market e.g. the US market to investigate the improvement in forecasting. Accordingly the aim of this paper is to investigate the forecasting performance of univariate and multivariate GARCH models in forecasting volatilities of some Asian stock markets using the linkages with the US market and also to examine whether using global financial crises period improves the forecasting performance.

The remaining paper is organized as follows. Section 3 describes the methodology used in this paper. The data is explained in section 4. Results and discussion and conclusion are provided in section 5 and 6 respectively.

3. Methodology

In this paper we first estimate the EGARCH model for the stock market of Pakistan, India, Indonesia and Japan and subsequently the bivariate GARCH model by taking the US market with each market i.e. i.e. the Japan-US, India-US, Indonesia-US and Pakistan-US pairs. We also performed the forecasting using the GFC dummy in both univariate and bivariate model for each market. We compute the one step ahead recursive forecast from both models and compare their performance through some forecasting evaluation methods.

3.1 The Models of Volatility Forecasting

The EGARCH Model

The volatility equation of the EGARCH (1,1) model proposed by Nelson (1991) is expressed as:

$$\ln(h_{t}) = \alpha_{0} + \alpha_{1} \left| \frac{u_{t-1}}{h_{t-1}} \right| + \gamma \frac{u_{t-1}}{h_{t-1}} + \beta \ln(h_{t-1})$$
(1)

Where h_t and u_{t-1} represent conditional variance and error term of stock returns respectively. Moreover, the inclusion of γ captures the asymmetric effect on volatility of returns. In order to investigate the role of global financial cries we use the GFC dummy, in the EGARCH (1,1) as:

$$\ln(h_{t}) = \alpha_{0} + \alpha_{1} \left| \frac{u_{t-1}}{h_{t-1}} \right| + \gamma \frac{u_{t-1}}{h_{t-1}} + \beta \ln(h_{t-1}) + \delta D_{t}$$
(2)

Where D_t is the GFC dummy variable assuming value 1 for the crisis period i.e. September 15, 2008 to March 31, 2009 and zero otherwise.

The MGARCH Model

We consider the BEKK specification proposed by Engle and Kroner (1995) of multivariate GARCH model (MGARCH). A bivariate MGARCH (1,1)-BEKK modelallowing the asymmetric effect is represented as follows:

$$H_{t} = \Gamma'\Gamma + \Theta'u_{t-1}u_{t-1}\Theta + \Phi'H_{t-1}\Phi + A'\xi_{t-1}\xi_{t-1}A$$
(3)

In order to forecast the volatility with GFC dummy, we consider bivariate BEKK as:

$$H_{t} = \Gamma'\Gamma + \Theta'u_{t-1}u_{t-1}\Theta + \Phi'H_{t-1}\Phi + A'\xi_{t-1}\xi_{t-1}A + G'DG$$
(4)

Where, the residual vector is explained by $\mathbf{u}_t = \begin{bmatrix} \mathbf{u}_{1,t} & \mathbf{u}_{2,t} \end{bmatrix}'$ and the conditional variancecovariance matrix $\mathbf{H}_t = \begin{bmatrix} \mathbf{h}_{ij,t} \end{bmatrix}_{i,j=1,2}$. ξ_t is defined as \mathbf{u}_t if \mathbf{u}_t is negative and zero otherwise. Note that here D is a diagonal matrix containing the global financial crisis dummy variables as defined above on its main diagonal. The set of given information available at time (t-1) is expressed by \mathbf{I}_{t-1} . The parameter matrices of the volatility equations (3) and (4) are denoted as $\Gamma = \begin{bmatrix} \gamma_{ij} \end{bmatrix}_{i,j=1,2}$ which is an upper triangular matrix while $\Theta = \begin{bmatrix} \Theta_{ij} \end{bmatrix}_{i,j=1,2}$ and $\Phi = \begin{bmatrix} \Phi_{ij} \end{bmatrix}_{i,j=1,2}$ are the restriction free ARCH and GARCH coefficient matrices respectively. Whereas, $\mathbf{A} = \begin{bmatrix} \mathbf{a}_{ij} \end{bmatrix}_{i,j=1,2}$ is also the restriction free coefficient matrix of asymmetric response of volatility. The matrix $\mathbf{g} = \begin{bmatrix} g_{ij} \end{bmatrix}_{i,j=1,2}$ is used as the coefficients of financial crisis dummies.

3.1 Evaluation of Volatility Forecast

Realized Volatility Proxy

The unobservability of the volatility creates difficulty in comparison of volatility forecast. To find a proxy of observed volatility is a challanging question for reserachers. In literarture many studies emply realized current period squared returns as a proxy of current period obseved volatility. However to expressing the forecast error in a more interpretable way we consider the observed value of current period absolute return as the proxy of observed volatility to be compared with one day ahead forecast of conditioal standard deviation.

Recursive Estimation Method

We use a recursive window estimation to compute the volatility forecasts. For daily data, we estimate the volatility models using the first 3090 observations and obtain one day ahead forecast conditional standard deviation to be compared with absolute return observation of the day 3091. Keeping the first observation and including obsevarion for day 3091 in the sample we estimate the volatility model and make forecast for the day 3092. We repeat this process for the entire available data sample. This process yields a series of one period ahead forecast for 25 days which corresponds roughly to month of trading. We also computed the one step ahead recursive forecast for 130 days and 260 days which correspond to half year and one year forecast horizons respectively.

Out of Sample Forecast Evaluation

To evaluate forecast out of sample, several measures are employed in the literature. We consider MAPE, MdAPE and the coefficient of determination R^2 . Median absolute percentage error provides a better outlier resistant evaluation measure.

Mean Absolute Percentage Error (MAPE)

MAPE is given by:

MAPE = Mean of
$$\left| \frac{\sigma_{t} \sqrt{\hat{h}_{t}}}{\sigma_{t}} \right| \times 100$$
 (5)

Where, σ_t day t is realized standard deviation obtained as the absolute day t return and \hat{h}_t is the forecast variance for day t obtained from the volatility model.

Median Absolute Percentage Error (MdAPE)

MdAPE is given by:

$$MdAPE = Median of \left| \frac{\sigma_{t} \sqrt{\hat{h}_{t}}}{\sigma_{t}} \right| \times 100$$
(6)

R² (Coefficient of determination)

The following regression is estimated and the coefficient of determination R^2 is obtained.

$$\log(|\mathbf{r}_{t}|) = \alpha + \beta \log\left(\sqrt{\hat{\mathbf{h}}_{t}}\right) + \epsilon_{t}$$
(7)

4. Data

We obtained the daily closing index prices on S&P-500 (New York Stock Exchange), NIKKIE-225 (Tokyo Stock Exchange), BSE-SENSEX-30 (Bombay Stock Exchange), JSX-Composite (Jakarta Stock Exchange) and KSE-100 (Karachi Stock Exchange) to represent the stock markets of the US, Japan, India and Pakistan respectively. The data for each country consist of 3115 value weighted index observations of closing prices adjusted for dividends and splits from July 3, 1997 to November 13, 2012. We delete all same date observations of each market when the observation of any of the market is found absent on account of no trading. In fact, we consider the observations of those particular dates when all the markets were open. The percentage daily log returns for given indices are employed by taking the first difference of log indices and multiplying by 100, i.e. $r_t = (\ln P_t - \ln P_{t-1}) \times 100$. All the data are obtained from the Datastream.

In case of GFC we split the data into pre, during and post crisis period. The observations of pre, during and post crisis are from July 3, 1997 to Sept 14, 2008 (total 2275 observations), September 15, 2008 to March 31, 2009 (total 73 observations) and April 1, 2009 to November 13, 2012 (total 767 observations) respectively.

5. Results and Discussion

Our analysis is based on the results presented in Table 1 and 2. Table 1 reports the three forecast evaluation measures i.e. R², MAPE and MdAPE for the one step ahead recursive volatility forecast for 25 days, 130 days and 260 days through EGARCH(1,1) and multivariate asymmetric GARCH(1,1)-BEKK models. Table 2 shows the similar results with GFC dummy variable. Constant mean for Japan, MA(1) for India and Indonesia and ARMA(1,1) for Pakistan are taken as the mean equations for univariate while VAR(1) for multivariate GARCH models. This was determined using the information criteria. The t-distribution of error is assumed for both models. The univariate and multivariate Ljung-Box Q statistic on linear and squared standardized residuals are also performed. This test shows no serial dependence in the linear and squared standardized residuals, indicating the appropriateness of the fitted mean and variance-covariance equations.

According to the results presented in Table 1, the MAPE and MdAPE based on the multivariate asymmetric GARCH (1,1) model is smaller as compared to the corresponding measures for the univariate EGARCH(1,1) model indicating the superior forecasting performance of the multivariate GARCH model for all the three forecast horizons and for all the four stock markets considered. Similarly the R^2 is found to be higher for the multivariate GARCH model when the linkage of the Asian markets with the US is considered. It is concluded that the correlation with US helps to improve the volatility forecast of each of the market considered.

In most of the cases it is found that the longer horizon of 260 days results in more accurate forecast for both types of modes as seen from the generally lower values of the MAPE and MdAPE and higher values of R^2 . Also the forecast are overall better for Japan as compared to the developing countries as seen by the forecast evaluation criteria especially for longer horizon of one year. It is also noted that the linkages with the US improves the volatility forecast of Pakistani market for the short horizon of one month but for India the forecast is more accurate for longer horizon of one year.

When the GFC dummy variables is incorporated in the volatility equation (Table 2), it is found that generally the improvement in forecast if any, is very minimal as compared to when the GFC dummy is not used (Table 1). Only the long horizon (one year) forecast of volatility in case of India is found to improve as results of considering GFC dummy.

According to the results presented in Table 2, the MdAPE for the multivariate asymmetric GARCH (1,1) model is lower compared to the EGARCH(1,1) model for all the cases except for short horizon case of Indonesia. Similar results are also observed using the MAPE criteria where the multivariate GARCH appears to improve the forecast performance. The results with R² are not very conclusive. It is concluded that the correlation with the US helps to improve the volatility forecast of each market.

6. Conclusion

This paper compares the forecasting performance of univariate EGARCH(1,1) and multivariate asymmetric GARCH-BEKK (1,1) models for the stock markets of Japan, India, Indonesia and Pakistan. We consideed daily data from these mareksta and performed volatility forecast from both the univariate and multivariate GARCH models using recurisive out of sample forecasts obtained for 25 days, 130 days and 260 days. The results show that using correlation with US helps improving accuracy of volatility forecast i.e. performance of multivariate GARCH is found to improve as compared to the univariate EGARCH model for all countries and for all the three forecast horizons considered. However the use of global financial crises dummy GFC does not result in any important improvement in accuracy of volatility forecast of the univariate and multivariate GARCH models.

Endnotes

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Table 1.	Recursive	Volatility	Forecast	Comparison	of U	U nivariate	and	Multivariate	GARCH
Model									

Country	Forecast Length	EGARCH(1,1)			Multivariate Asymmetric GARCH(1,1)-BEKK			
		R ²	MAPE(%)	MdAPE(%)	R ²	MAPE(%)	MdAPE(%)	
Japan	25 days	0.016	449.797	100.517	0.046	403.984	90.104	
	130 days	0.001	772.193	61.629	0.000	631.799	51.036	
	260 days	0.020	680.325	59.720	0.035	599.445	52.925	
India	25 days	0.070	252.203	134.331	0.111	247.150	113.899	
	130 days	0.008	540.085	122.569	0.012	521.094	116.656	
	260 days	0.076	374.995	101.534	0.078	357.624	90.308	
Indonesia	25 days	0.006	1056.621	175.974	0.005	996.996	205.478	
	130 days	0.030	784.053	132.686	0.021	730.028	128.776	
	260 days	0.045	932.981	119.601	0.047	855.482	99.864	
Pakistan	25 days	0.073	210.157	94.890	0.081	203.923	84.241	
	130 days	0.002	624.148	104.488	0.002	590.265	97.340	
	260 days	0.006	610.990	102.676	0.009	570.814	94.211	

Country	Forecast horizon	EGARCH(1,1)			Multivariate Asymmetric GARCH(1,1)-BEKK			
		R ²	MAPE(%)	MdAPE(%)	R ²	MAPE(%)	MdAPE(%)	
Japan	25 days	0.016	449.066	100.077	0.053	408.338	91.683	
	130 days	0.001	769.964	60.664	0.003	630.727	51.569	
	260 days	0.020	678.214	59.569	0.039	604.940	53.519	
India	25 days	0.071	255.857	136.286	0.006	292.850	118.044	
	130 days	0.009	542.668	123.738	0.000	532.740	119.539	
	260 days	0.075	374.952	99.2778	0.018	347.825	64.857	
Indonesia	25 days	0.006	1055.602	176.614	0.004	987.688	196.664	
	130 days	0.030	783.405	132.401	0.024	723.871	126.739	
	260 days	0.045	931.714	119.750	0.046	855.126	100.154	
Pakistan	25 days	0.073	209.697	94.369	0.056	202.955	84.152	
	130 days	0.002	622.965	104.147	0.002	590.352	101.229	
	260 days	0.006	609.380	102.376	0.009	571.170	97.248	

Table 2. Recursive Volatility Forecast Comparison of Univariate and Multivariate GARCHModel with Impact of GFC Dummy