# Comparison between Back-to-Back and Matrix Converters Based on Thermal Stress of the Switches 

D. Casadei, Member, IEEE ,G. Grandi, Member, IEEE ,C. Rossi, A. Trentin, L. Zarri<br>Dipartimento di Ingegneria Elettrica, Università di Bologna, Viale Risorgimento 2, 40136 Bologna, Italy<br>e-mail:(domenico.casadei, gabriele.grandi, claudio.rossi, andrew.trentin, luca.zarri)@mail.ing.unibo.it


#### Abstract

A comparison between a matrix converter and a back-to-back converter feeding a passive load is presented in this paper, with the aim of determining the converter topology which yields the highest output power per switches number.

The comparison has been performed for different values of the output frequency. For each output frequency the load power has been increased until one of the switching devices reaches the maximum thermal stress, so defining the maximum output power of the converter. For this purpose, a simplified thermal model has been used to evaluate the junction temperature of the switches on the basis of the switch losses.

An accurate computer model of both converters has been implemented taking into account the modulation laws and the real characteristics of the switching devices.

Simulation results are presented showing the different behaviour of the two converters as a function of the output frequency. It has been verified that matrix converters perform better than back-to-back converters at low output frequencies.


Index Terms - Matrix converter, Back-to-Back converter, Switch thermal stress, Switch losses .

## I. InTRODUCTION

Three-phase matrix converters provide bi-directional power flow, sinusoidal input/output waveforms, and controllable input power factor. For these reasons matrix converters have received considerable attention in the last years, and they may become a good alternative to back-toback converters. Furthermore, the matrix converter allows a compact design, due to the lack of dc-link capacitors [1][3].

The matrix converter has been already compared with the back-to-back converter obtaining some important but not conclusive results. The comparison is extremely difficult due to the high number of system parameters (i.e. input filter and load parameters, switching frequency, output frequency, modulation strategies, etc.) and to the inherent differences between the two converter topologies, such as the maximum voltage transfer ratio.

By means of proper control algorithms, the matrix converter is able to generate balanced and sinusoidal output voltages, whose amplitude can be regulated from zero to approximately $87 \%$ of the input voltage amplitude [4]. The output voltage of the back-to-back converter is related to the DC-link voltage, and can be equal or greater than the input voltage [5].

The switching frequencies of the two converters are related to the adopted modulation strategies and should be chosen with care in order to make a fair comparison. Fur-
thermore, both converters need an input filter for reducing the input current harmonics, and the filter parameters are strictly related to the switching frequency.

In [5]-[8] the comparison is performed in terms of total switch losses, evaluating the converter efficiency for given operating conditions. On the other hand, in [9] it has been clearly emphasized that in matrix converters the switch losses are not equally shared among the switches, being the distribution related to the output frequency. Thus, considering only the total switch losses as the key-parameter for the comparison may be misleading.

In this paper the comparison between matrix and back-to-back converters is performed by evaluating the maximum output power that each converter is able to deliver to the load for different output frequency. The comparison is carried out assuming the same types of IGBTs and diodes. The maximum output power is determined taking into account the thermal limits for each switch on the basis of thermal model proposed in [10].

Then, the output power of both converters has been increased step by step, evaluating the total losses of each switch on the basis of the current and voltage waveforms achieved in steady-state conditions. Monitoring the thermal behaviour of all switches, the maximum output power has been determined as one of the switching devices reaches the maximum thermal stress. The maximum output power has been divided by the number of switches in order to define a quantity representing the utilisation degree of the switches, particularly useful for the comparison.

An accurate computer model of both converters, taking into account the modulation laws and the real characteristics of the switching devices, has been implemented in order to emphasize how the behaviour of the two converters changes as a function of the output frequency. With reference to the matrix converter, an analytical approach is also presented in Appendix, to give a qualitative explanation of some phenomena occurring at particular values of the output frequency.

It has been shown that matrix converters perform better than back-to-back converters in the low frequency range and that matrix converters are in general more suitable for drive systems which require high start-up currents.

## II. MATRIX CONVERTER SCHEME

The schematic circuit of a matrix converter feeding a passive load is shown in Fig. 1. The system is composed by a voltage supply system, a line impedance, an input filter, a


Fig. 2 - Schematic drawing of the matrix converter.
matrix converter and a load impedance.
The input filter is generally needed to smooth the input currents and to satisfy the EMI requirements.

Several control techniques for matrix converters have been proposed in literature [1]-[4]. Among these, the most simple is the one based on detecting the zero-crossing of one input voltage for synchronising the input current.

This control technique performs correctly assuming an ideal power supply, but in presence of input voltage disturbances these are reflected on the output side determining low order voltage harmonics. It is possible to compensate these effects calculating the duty-cycles necessary to generate balanced and sinusoidal output voltages on the basis of the instantaneous values of the input voltages, as shown in Fig. 1.

In this paper, the output voltage is synthesized by means of the symmetrical Space Vector Modulation (SVM) technique, that uses only one zero vector for switching period, thus determining 8 commutations within a cycle period.

The commutation strategy is performed in 3 steps, according to the method proposed in [11], which requires the sign measurement of both output current and input voltage.

## III. BaCK-to-back Converter Scheme

The back-to-back converter includes two inverters: the first operates as an active rectifier, the second as a traditional voltage source inverter (VSI), as represented in Fig. 2. It is assumed that both converters are controlled with a symmetrical SVM technique with one zero vector for cycle period (two-phase modulation), and with commutation dead-time compensation [12]. This modulation strategy requires the minimum number of switch commutations per cycle period, and it is widely used in practical applications.

The system includes a voltage supply system with a line impedance, an input filter, a back-to-back converter and a load impedance.

## IV. COMPARISON BASIS

The matrix converter requires 18 IGBTs and 18 diodes, whereas the back-to-back converter requires 12 IGBTs and 12 diodes. The comparison is carried out assuming for the two converters the same IGBTs and diodes.

The aim of the comparison is to evaluate the utilisation degree of the semiconductor devices, determining the maximum output power per switch corresponding to the maximum thermal stress that the switches can withstand.

The thermal stress of the switches has been continuously monitored during the operation, while increasing the output power. As one switch of the converter reaches the thermal


Fig. 1 - Schematic drawing of the back-to-back converter.
stress limit, the corresponding power delivered to the load has been considered as the maximum output power of the converter.

As far as the input filter is concerned, the matrix converter uses a L-C filter topology, differently from the back-to-back converter, that uses a C-L filter topology. In fact, the matrix converter can be considered at input side as a current source, whereas the back-to-back converter appears as a voltage source.

A reactive current flows through the input filter capacitor of the matrix converter, which results in a reduced power factor especially at low output power. As a consequence, the capacitor has been chosen in order to assure at least a power factor of 0.8 with $10 \%$ of the rated output power. After the selection of the capacitor, the input filter inductance of the matrix converter has been chosen in order to satisfy the IEEE Recommended Practices and Requirements for Harmonic Control in Electrical Power Systems (IEEE Std. 519-1992).

The inductance of the back-to-back input filter has been designed such that the corresponding voltage drop at the line frequency is about 0.05 p.u.. After the choice of the inductor, the input filter capacitor has been selected in order to satisfy the IEEE recommended standards.

It is also known that the design of the input filters is strictly related to the switching frequency. With reference to matrix converter a switching frequency of 8 kHz has been chosen. This frequency corresponds to a cycle period of 125 $\mu$ s that is sufficient for the digital implementation of the control algorithm.

The back-to-back converter needs the definition of two switching frequencies, for the input and the output stage respectively. The switching frequency of the output stage has been increased up to 16 kHz in order to achieve the same number of switch commutations per second, and then a similar THD of the output voltage. The switching frequency of the input stage, instead, has been decreased to 6.6 kHz with the aim to reduce the switching losses. In fact, higher switching frequencies for the input stage are not necessary to comply with the input current harmonic standards, using common values of inductance and capacitance.

Although the design of the input filter is an important issue, the input filter parameters are not crucial for the determination of the maximum output power of the two converters compatible with the switches thermal stress. Therefore, a more detailed analysis concerning the input filter is not necessary for the aim of the paper.

As an example, in Tab. I the input current harmonics of the two converters operating at rated power are compared with the recommended limits, when the ratio between the

TABLE I - CURRENT DISTORTION LIMITS FOR GENERAL DISTRIBUTION SYSTEMS IN PERCENT OF THE MAXIMUM DEMAND LOAD CURRENT

|  | Individual harmonic order (odd harmonics) |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Order | $\mathrm{h}<11$ | $11 \leq \mathrm{h}<17$ | $17 \leq \mathrm{h}<23$ | $23 \leq \mathrm{h}<35$ | $35 \leq \mathrm{h}$ | TDD |
| Limits (\%) | 12 | 5.5 | 5.0 | 2 | 1 | 15 |
| Matrix (\%) | 1.08 | 0.57 | 1.54 | 1.59 | 0.51 | 3.57 |
| B-to-B (\%) | 1.24 | 0.32 | 0.15 | 0.12 | 0.71 | 2.14 |

maximum short-circuit current in the point of common coupling and the maximum demand load current is between 100 and 1000. All the system parameters are shown in Tab. II. It is possible to verify that using the filter parameters given in Tab. II, both converters comply with the IEEE recommended standards.

It is known that in matrix converters the total losses of the switching devices are not equally shared among the switches, and the losses distribution is more or less uniform depending on the output frequency. As a consequence, the comparison between the two converters has been carried out assuming the output frequency as a parameter.

The performance of the two converter topologies has been tested for different values of the output frequency in the range $0-150 \mathrm{~Hz}$. The voltage has been changed with the frequency according to the well-known constant V/Hz law, as in induction motor drives. Thus, the output voltage is varied proportionally to the frequency until 50 Hz . For higher frequencies the phase to phase output voltage is kept constant, i.e. 330 V and 380 V for the matrix converter and the back-to-back converter respectively. At low frequencies, the output voltage has been changed in order to compensate the voltage drop on the stator winding resistance.

## V. Thermal Model of the Switches

Under the assumptions made, the maximum output power delivered by the converters to the load has been determined taking into account the maximum thermal stress of the switches

The junction temperature of the switching devices has been monitored by means of the dynamic thermal model of a single IGBT shown in Fig 3 [10].

The model represents the junction temperature $\theta_{\mathrm{j}}$ in terms of the static junction-to-case thermal resistance $R_{j c}$ and the thermal time constant $\tau=R_{j c} \cdot C_{j}$. Although this model is simplified and it does not take into account the non-uniform internal device temperature, it points out that the device thermal transfer function $H(s)$ is a low-pass filter with a cut-off frequency $f_{\theta}=1 / 2 \pi \tau$, which is comparable with the converter output fundamental frequency $\left(f_{\text {out }}\right)$, but much lower than the switching frequency ( $f_{s w}$ ).

As a consequence, the junction temperature is strictly dependent on the output frequency


Fig. 3 - Simplified thermal model for the IGBT.

TABLE II - SySTEM PARAMETERS

| Simulation Parameters |  |  |
| :---: | :---: | :---: |
|  | Back to Back | Matrix Converter |
| $V_{I N}$ | 380 V(RMS), 50 Hz | 380 V(RMS), 50 Hz |
| $R_{\text {line }}$ | $0.11 \Omega$ | $0.11 \Omega$ |
| $L_{\text {line }}$ | 0.167 mH | 0.167 mH |
| $V_{D C}$ | 600 V | - |
| $C_{D C}$ | $200 \mu \mathrm{~F}$ | - |
| $C_{f}$ | $25 \mu \mathrm{~F}$ (Y) | $40 \mu \mathrm{~F}(\mathrm{Y})$ |
| $L_{f}$ | 1.00 mH | 0.35 mH |
| $R_{\text {jc }}$ | $0.64{ }^{\circ} \mathrm{C} / \mathrm{W}$ | $0.64{ }^{\circ} \mathrm{C} / \mathrm{W}$ |
| $C_{j}$ | $31.2 \mathrm{~mJ} /{ }^{\circ} \mathrm{C}$ | $31.2 \mathrm{~mJ} /{ }^{\circ} \mathrm{C}$ |
| $\theta_{\text {case }}$ | $70^{\circ} \mathrm{C}$ | $70^{\circ} \mathrm{C}$ |
| $f_{s w}$ | 6.6 kHz (ac-dc), 16 kHz (dc-ac) | 8 kHz |
| $\cos \varphi_{\text {load }}$ | 0.8 | 0.8 |
| $V_{\text {out }}$ | $\begin{aligned} & f_{\text {out }}<50 \mathrm{~Hz}, \text { const. } \mathrm{V} / \mathrm{Hz} \\ & f_{\text {out }}>50 \mathrm{~Hz}, V_{\text {out }}=380 \mathrm{~V} \end{aligned}$ | $\begin{aligned} & f_{\text {out }}<50 \mathrm{~Hz}, \text { const. } \mathrm{V} / \mathrm{Hz} \\ & f_{\text {out }}>50 \mathrm{~Hz}, V_{\text {out }}=330 \mathrm{~V} \end{aligned}$ |
| Diodes | HFA16PB120 | HFA16PB120 |
| IGBTs | IRG4PH50U | IRG4PH50U |

For both converters the maximum junction temperature and the case temperature have been assumed $150^{\circ} \mathrm{C}$ and $70^{\circ} \mathrm{C}$ respectively. Then, the maximum junction-to-case temperature is $\theta_{j c, \text { max }}=80^{\circ} \mathrm{C}$.

## VI. Simulation Results

The behaviour of the two converters has been analysed using the electronic circuit analysis program MICROCAP 7.0, which provides accurate models of the power switches, improving the simulation reliability.

Firstly, the behaviour of both converters has been verified with reference to the rated output frequency of 50 Hz .

Fig. 4 shows the junction-to-case temperature behaviour for all the 18 matrix IGBTs corresponding to the maximum output power. As can be seen, the thermal stress of the switches is not equal, since 6 IGBTs are more stressed than the others. In fact, when the output frequency is equal to the input frequency, the output current is not shared uniformly among all the switches. The portion of the output current that flows through each switch depends on the displacement angle between the input and output voltage vectors, and on the load power factor [7]. The current waveform and the junction-to-case temperature behaviour of one the most stressed switches are shown in details in Fig. 5.

The junction-to-case IGBTs temperature behaviour of the back-to-back converter is represented in Fig. 6. The upper traces are related to the output stage, whereas the lower traces correspond to the input one. It is evident that the thermal stress for input stage is lower than that of the output one. The converse occurs for regenerative operation of the back-to-back converter. Fig. 7 shows in details the current waveform and the junction-to-case temperature of one switch.

It can be seen in Fig. 6 that the thermal stress of the back-to-back converter output stage switches is more uniform in comparison with the thermal stress of the matrix converter switches. As a consequence, the maximum output power per switch that can be achieved with a matrix converter at $50 \mathrm{~Hz}\left(P_{\text {matrix }} / 18\right)$ is expected to be lower than that of the back-to-back converter ( $P_{\mathrm{b} \text {-to-b }} / 12$ ).

Fig. 8 shows the junction-to-case temperature waveforms


Fig. 4 - Junction-to-case temperature waveforms of the matrix converter switches for the maximum output power corresponding to an output frequency of 50 Hz .


Fig. 6 - Junction-to-case temperature waveforms of the back-to-back converter switches for the maximum output power corresponding to an output frequency of 50 Hz .
of the matrix converter IGBTs corresponding to the maximum output power and with an output frequency of 60 Hz . The current waveform and the junction-to-case temperature behaviour of one of the most stressed switches are shown in Fig. 9.

The junction-to-case IGBTs temperature behaviour of the back-to-back output inverter, corresponding to the maximum output power at 60 Hz , is represented in Fig. 10. Fig. 11 shows in details the current and the junction-to-case temperature waveforms of one switch.

Comparing Fig. 6 and Fig. 10, both referring to back-toback converter, it is evident that the thermal stress at 60 Hz is quite similar to that at 50 Hz , and all switches are equally stressed.

On the contrary, with reference to matrix converter the comparison between Fig. 4 and Fig. 8 emphasises that the output frequency has a relevant effect on the thermal stress


Fig. 8 - Junction-to-case temperature waveforms of the matrix converter switches for the maximum output power corresponding to an output frequency of 60 Hz .


Fig. 10 - Junction-to-case temperature waveforms of switches of the back-to-back converter output stage for the maximum output power corresponding to an output frequency of 60 Hz .


Fig. 5 - Junction-to-case temperature and current waveforms of the one of the most stressed matrix converter switches, for the same operating conditions of Fig. 4.


Fig. 7 - Junction-to-case temperature and current waveforms of one of the most stressed back-to-back converter switch, for the same operating conditions of Fig. 6.
of the switches. At 50 Hz the IGBTs can be divided into three groups, having a quite different behaviour in terms of thermal stress, whereas at 60 Hz the 18 IGBTs are in turn equally stressed.

The maximum output power achievable by the two converters as a function of the output frequency is summarised in Fig. 12. It is evident that the output power of matrix converter is always higher than that of back-to-back converter, showing a decrease around the frequency values of 50 and 100 Hz .

Fig. 13 shows the load current corresponding to the maximum output power as a function of the output frequency, for the matrix and the back-to-back converters. In this figure it is evident the better performance of the matrix converter in terms of maximum output current, especially in the low output frequency range. The reason of so low values of the output current for the back-to back converter


Fig. 9 - Junction-to-case temperature and current waveforms of the most stressed matrix converter switches, for the same operating conditions of Fig. 8.


Fig. 11 - Junction-to-case temperature and current waveform of one switch of the back-to-back converter output stage, for the same operating conditions of Fig. 10 .
is that, at low frequency, the output current is not equally shared among the six switches. This situation is similar to the one occurring when the matrix converter operates at 50 Hz . On the contrary, the matrix converter is able to deliver high currents at low frequency because these currents are equally shared among the 18 switches.

Furthermore, it can be noted that in Fig. 13, as in Fig. 12, the output current of the matrix converter shows more or less evident decreases at the frequency values of 25,50 and 100 Hz . More details about these aspects, that are strictly related to the quasi-periodic operation of matrix converters, are given in Appendix.

A further comparison between the two types of converter is shown in Fig. 14, where the operating area in the I-V plane is represented. This figure clearly emphasises that matrix converters are more suitable for drive systems that in general require high start-up currents.

In order to make a fair comparison, one should take into account that the two converter topologies are realized with a different number of switches (i.e. 18 for the matrix converter and 12 for the back-to-back converter). For this purpose two more significant quantities have been introduced, which represent the maximum output power per switch and the corresponding output current per switch. These new quantities are represented in Figs. 15 and 16 respectively.

Fig. 15 shows that the output power per switch of the matrix converter is always lower than that of back-to-back converter, except for frequency values ranging from 0 to about 30 Hz . On the other hand, it can be seen from Fig. 16 that the load current per switch of the matrix converter is always higher than that of the back-to-back converter, ex-


Fig. 12 -Maximum output power as a function of the output frequency.

Fig. 13 -Maximum output current as a function of the output frequency.


Fig. 14 - I-V operating area for matrix converter and back-to-back converter.


Fig. 15 -Maximum output power per switch as a function of the output frequency.


Fig. 16 - Maximum output current per switch as a function of the output frequency.
cept for frequency values around 50 Hz .
From Figs. 15 and 16 it can be concluded that in terms of maximum output power per switch the two converter topologies show practically a similar performance, whereas in terms of output current per switch the matrix converter should be preferred to back-to-back converter particularly in the low output frequency range. These results could be usefully employed in the choice of the converter topology for drive systems, once the operating conditions and the overload capability were specified in details.

## VII. Conclusion

In this paper a comparison between matrix and back-toback converters has been carried out taking into account the thermal limits of the switching devices and the different number of switches required in the two topologies. The same IGBTs and diodes have been used for the comparison. Therefore, the maximum output power per switch and the corresponding output current per switch have been regarded as the most representative quantities for a fair comparison. The analysis made it possible to emphasise the different behaviour of the two converters as the output frequency changes. The most important difference between the two topologies has been found in the low frequency range, where the matrix converter switches are equally stressed, differently from the back-to-back converter. This results in the possibility to better exploit the current capability of the switches for drive systems, which usually require high overload capability in the low frequency range.

## VIII. APPENDIX

An analytical approach is presented, demonstrating that the highest temperature of the switches of a matrix converter correspond to the frequency values of 25, 50 and 100 Hz . In general, the worst operating condition occurs when the output frequency is equal to the input frequency.

In the following, the phase angles of the input voltage
and output voltage vector are denoted with $\alpha_{i}$ and $\alpha_{0}$, respectively.

In steady state operation, for given load and voltage transfer ratio, the mean power loss $P$ of a single switch in a cycle period $T_{c}$ depends on $\alpha_{\mathrm{i}}$ and $\alpha_{0}$. This dependence can be expressed as follows

$$
\begin{equation*}
P=P\left(\alpha_{\mathrm{i}}, \alpha_{\mathrm{o}}\right) \tag{1}
\end{equation*}
$$

The function $P$ is periodic with respect $\alpha_{\mathrm{i}}$ and $\alpha_{0}$, i.e.

$$
\begin{equation*}
P\left(\alpha_{\mathrm{i}}, \alpha_{\mathrm{o}}\right)=P\left(\alpha_{\mathrm{i}}+2 \pi, \alpha_{\mathrm{o}}\right)=P\left(\alpha_{\mathrm{i}}, \alpha_{\mathrm{o}}+2 \pi\right) \tag{2}
\end{equation*}
$$

As shown in (2), $P$ is a quasi-periodic function and it can be expressed by means of a double Fourier series as follows [13]:

$$
\begin{equation*}
P\left(\alpha_{i}, \alpha_{o}\right)=\operatorname{Re}\left\{\sum_{k=0}^{\infty} \sum_{h=-\infty}^{\infty} \bar{P}_{k, h} e^{J\left(k \alpha_{i}+h \alpha_{o}\right)}\right\} \tag{3}
\end{equation*}
$$

where $\bar{P}_{k, h}$ is the Fourier coefficient of index $h$ and $k$.
In steady-state condition $\alpha_{\mathrm{i}}$ and $\alpha_{\mathrm{o}}$ can be expressed as a function of time as follows

$$
\begin{equation*}
\alpha_{\mathrm{i}}=\omega_{i} t \tag{5}
\end{equation*}
$$

where $\omega_{\mathrm{i}}$ and $\omega_{\mathrm{o}}$ are the input and output angular frequency, respectively.

Substituting (4) and (5) in (3) leads to the following form of the power losses:

$$
\begin{equation*}
P(t)=\operatorname{Re}\left\{\sum_{k=0}^{\infty} \sum_{h=-\infty}^{\infty} \bar{P}_{k, h} e^{J\left(k \omega_{i}+h \omega_{o}\right) t}\right\} \tag{6}
\end{equation*}
$$

The junction-to-case temperature of the switch can be found through the thermal model of the IGBT represented in Fig.3. It can be derived from the thermal model that the transfer function $\bar{H}(\omega)$ between the junction-to-case temperature and the power losses is a low pass filter, expressed as follows:

$$
\begin{equation*}
\bar{H}(\omega)=\frac{R_{j c}}{1+j \omega \tau} \tag{7}
\end{equation*}
$$

Applying (7) to (6) leads to the following expression of the junction-to-case temperature of the switch:

$$
\begin{equation*}
\vartheta_{j c}(t)=\operatorname{Re}\left\{\sum_{k=0}^{\infty} \sum_{h=-\infty}^{\infty} \bar{P}_{k h} \bar{H}\left(k \omega_{i}-h \omega_{o}\right) e^{j\left(k \omega_{i}+h \omega_{o}\right) t}\right\} \tag{8}
\end{equation*}
$$

It is worth noting that the amplitude of the harmonics of $P(\mathrm{t})$ decreases proportionally to the product of the harmonic indexes $k$ and $h$. In addition, the harmonics with frequency much higher that the cut-off frequency of the thermal filter give a negligible contribute to the temperature.

As a consequence, (8) can be approximated by a sum with a finite number of terms, as follows

$$
\begin{align*}
& \vartheta_{j c}(t)=P_{0} H(0)+2 P_{0,1} H\left(\omega_{0}\right) \cos \left(\omega_{0} t+\varphi_{0,1}\right)+ \\
& P_{1,-1} H\left(\omega_{i}-\omega_{o}\right) \cos \left(\left(\omega_{i}-\omega_{o}\right) t+\varphi_{1,-1}\right)+ \\
& P_{1,-2} H\left(\omega_{i}-2 \omega_{o}\right) \cos \left(\left(\omega_{i}-2 \omega_{o}\right) t+\varphi_{1,-2}\right)+ \\
& P_{2,-1} H\left(2 \omega_{i}-\omega_{o}\right) \cos \left(\left(2 \omega_{i}-\omega_{o}\right) t+\varphi_{2,-1}\right) \tag{9}
\end{align*}
$$

where $\varphi_{\mathrm{k}, \mathrm{h}}$ is the phase angle of $\bar{P}_{k, h}$.
An approximate estimate of the maximum junction-tocase temperature can be derived from (9), as follows:

$$
\begin{align*}
& \vartheta_{j c, \max }\left(\omega_{o}\right)=P_{0} H(0)+2 P_{0,1} H\left(\omega_{0}\right)+P_{1,-1} H\left(\omega_{i}-\omega_{o}\right)  \tag{10}\\
& +P_{1,-2} H\left(\omega_{i}-2 \omega_{o}\right)+P_{2,-1} H\left(2 \omega_{i}-\omega_{o}\right)
\end{align*}
$$

As can be seen from (10) the maximum junction-to-case
temperature depends on the output frequency.
In order to determinate the values of $\omega_{o}$ that make (10) to assume a relative maximum, the derivative of (10) is needed:

$$
\begin{align*}
& \frac{\partial \vartheta_{j c, \text { max }}}{\partial \omega_{o}}=\frac{\tau^{2}}{R_{j c}^{2}}\left[2 P_{0,1} H^{3}\left(\omega_{0}\right)\left(-\omega_{0}\right)+P_{1,-1} H^{3}\left(\omega_{i}-\omega_{o}\right)\left(\omega_{i}-\omega_{o}\right)+\right. \\
& \left.2 P_{1,-2} H^{3}\left(\omega_{i}-2 \omega_{o}\right)\left(\omega_{i}-2 \omega_{o}\right)+P_{2,-1} H^{3}\left(2 \omega_{i}-\omega_{o}\right)\left(2 \omega_{i}-\omega_{o}\right)\right] \tag{11}
\end{align*}
$$

It should be noted that for $\omega_{o} \cong \omega_{i}$ the dominant term in (11) is $P_{1,-1} H^{3}\left(\omega_{i}-\omega_{o}\right)\left(\omega_{i}-\omega_{o}\right)$, whereas the other terms, multiplied by the factor $H^{3}\left(\omega_{i}\right)$, are negligible. As a consequence, $\omega_{o} \cong \omega_{i}$ can be considered an approximate solution of (11), corresponding to a relative maximum of (10).

With similar procedure, the solutions $\omega_{o} \cong 2 \omega_{i}$, $\omega_{o} \cong \omega_{i} / 2$ can be found, which correspond to local maxima of lower importance than the previous one.

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