Complete Solutions to Selected Problems

to accompany

MATERIALS SCIENCE AND ENGINEERING

AN INTRODUCTION

Sixth Edition

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John Wiley & Sons, Inc.

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PREFACE

This *Complete Solutions to Selected Problems* has been developed as a supplement to the sixth edition of *Materials Science and Engineering: An Introduction*. The author has endeavored to select problems that are representative of those that a student should be able to solve after having studied the related chapter topics. In some cases problem selection was on the basis of illustrating principles that were not detailed in the text discussion. Again, problems having solutions in this supplement have double asterisks by their numbers in both "Questions and Problems" sections at the end of each chapter, and in the "Answers to Selected Problems" section at the end of the printed text.

Most solutions begin with a reiteration of the problem statement. Furthermore, the author has sought to work each problem in a logical and systematic manner, and in sufficient detail that the student may clearly understand the procedure and principles that are involved in its solution; in all cases, references to equations in the text are cited. The student should also keep in mind that some problems may be correctly solved using methods other than those outlined.

Obviously, the course instructor has the option as to whether or not to assign problems whose solutions are provided here. Hopefully, for any of these solved problems, the student will consult the solution only as a check for correctness, or only after a reasonable and unsuccessful attempt has been made to solve the particular problem. This supplement also serves as a resource for students, to help them prepare for examinations, and, for the motivated student, to seek additional exploration of specific topics.

The author sincerely hopes that this solutions supplement to his text will be a useful learning aid for the student, and to assist him/her in gaining a better understanding of the principles of materials science and engineering. He welcomes any comments or suggestions from students and instructors as to how it can be improved.

William D. Callister, Jr.

CHAPTER 2

ATOMIC STRUCTURE AND INTERATOMIC BONDING

PROBLEM SOLUTIONS

2.3 (a) In order to determine the number of grams in one amu of material, appropriate manipulation of the amu/atom, g/mol, and atom/mol relationships is all that is necessary, as

$$\# \text{ g/amu} = \left(\frac{1 \text{ mol}}{6.023 \times 10^{23} \text{ atoms}}\right)\left(\frac{1 \text{ g/mol}}{1 \text{ amu/atom}}\right)$$

$$= 1.66 \times 10^{-24} \text{ g/amu}$$

(b) Since there are 453.6 g/lb$_m$,

$$1 \text{ lb-mol} = (453.6 \text{ g/lb}_m)(6.023 \times 10^{23} \text{ atoms/g-mol})$$

$$= 2.73 \times 10^{26} \text{ atoms/lb-mol}$$

2.14 (c) This portion of the problem asks that, using the solutions to Problem 2.13, we mathematically determine values of $\mathbf{r_o}$ and $\mathbf{E_o}$. From Equation (2.11) for $\mathbf{E_N}$

$A = 1.436$

$B = 7.32 \times 10^{-6}$

$n = 8$

Thus,

$$r_o = \left(\frac{A}{nB}\right)^{1/(1 - n)}$$

$$= \left[\frac{1.436}{(8)(7.32 \times 10^{-6})}\right]^{1/(1 - 8)} = 0.236 \text{ nm}$$

and

$$E_o = -\frac{1.436}{\left[\frac{1.436}{(8)\left(7.32 \text{ x } 10^{-6}\right)}\right]^{1/(1-8)}} + \frac{7.32 \text{ x } 10^{-6}}{\left[\frac{1.436}{(8)\left(7.32 \text{ x } 10^{-6}\right)}\right]^{8/(1-8)}}$$

$$= -\ 5.32 \text{ eV}$$

2.19 The percent ionic character is a function of the electron negativities of the ions $\mathbf{X_A}$ and $\mathbf{X_B}$ according to Equation (2.10). The electronegativities of the elements are found in Figure 2.7.

For MgO, $\mathbf{X_{Mg}}$ = 1.2 and $\mathbf{X_O}$ = 3.5, and therefore,

$$\%\text{IC} = \left[1 - \mathbf{e}^{(-0.25)(3.5-1.2)^2}\right] \text{x } 100 = 73.4\%$$

For CdS, $\mathbf{X_{Cd}}$ = 1.7 and $\mathbf{X_S}$ = 2.5, and therefore,

$$\%\text{IC} = \left[1 - \mathbf{e}^{(-0.25)(2.5-1.7)^2}\right] \text{x } 100 = 14.8\%$$

CHAPTER 3

THE STRUCTURE OF CRYSTALLINE SOLIDS

PROBLEM SOLUTIONS

3.3 For this problem, we are asked to calculate the volume of a unit cell of aluminum. Aluminum has an FCC crystal structure (Table 3.1). The FCC unit cell volume may be computed from Equation (3.4) as

$$V_C = 16R^3\sqrt{2} = (16)\left(0.143 \times 10^{-9}\ \text{m}\right)^3\sqrt{2} = 6.62 \times 10^{-29}\ \text{m}^3$$

3.12. (a) The volume of the Ti unit cell may be computed using Equation (3.5) as

$$V_C = \frac{nA_{Ti}}{\rho N_A}$$

Now, for HCP, **n** = 6 atoms/unit cell, and for Ti, $\mathbf{A_{Ti}}$ = 47.9 g/mol. Thus,

$$V_C = \frac{(6\ \text{atoms/unit cell})(47.9\ \text{g/mol})}{\left(4.51\ \text{g/cm}^3\right)\left(6.023 \times 10^{23}\ \text{atoms/mol}\right)}$$

$$= 1.058 \times 10^{-22}\ \text{cm}^3\text{/unit cell} = 1.058 \times 10^{-28}\ \text{m}^3\text{/unit cell}$$

(b) From the solution to Problem 3.7, since **a** = 2**R**, then, for HCP

$$V_C = \frac{3\sqrt{3}\,a^2c}{2}$$

but, since **c** = 1.58**a**

$$V_C = \frac{3\sqrt{3}\,(1.58)a^3}{2} = 1.058 \times 10^{-22}\ \text{cm}^3\text{/unit cell}$$

Now, solving for **a**

$$a = \left[\frac{(2)\left(1.058 \times 10^{-22}\ \text{cm}^3\right)}{(3)\left(\sqrt{3}\right)(1.58)}\right]^{1/3}$$

$$= 2.96 \times 10^{-8}\ \text{cm} = 0.296\ \text{nm}$$

And finally

$$c = 1.58a = (1.58)(0.296\ \text{nm}) = 0.468\ \text{nm}$$

3.17 (a) From the definition of the **APF**

$$\text{APF} = \frac{V_S}{V_C} = \frac{n\left(\frac{4}{3}\pi R^3\right)}{a^2 c}$$

we may solve for the number of atoms per unit cell, **n**, as

$$n = \frac{(\text{APF})a^2 c}{\frac{4}{3}\pi R^3}$$

$$= \frac{(0.693)(4.59)^2(4.95)\left(10^{-24}\ \text{cm}^3\right)}{\frac{4}{3}\pi\left(1.625 \times 10^{-8}\ \text{cm}\right)^3}$$

$$= 4.0\ \text{atoms/unit cell}$$

3.30 (a) We are asked for the indices of the two directions sketched in the figure. For direction **1**, the projection on the **x**-axis is zero (since it lies in the **y-z** plane), while projections on the **y**- and **z**-axes are **b**/2 and **c**, respectively. This is an [012] direction as indicated in the summary below

	x	y	z
Projections	0a	b/2	c
Projections in terms of **a**, **b**, and **c**	0	1/2	1

Reduction to integers	0	1	2
Enclosure		[012]	

(b) This part of the problem calls for the indices of the two planes which are drawn in the sketch. Plane **1** is an (020) plane. The determination of its indices is summarized below.

	x	y	z
Intercepts	∞ a	b/2	∞ c
Intercepts in terms of **a**, **b**, and **c**	∞	1/2	∞
Reciprocals of intercepts	0	2	0
Enclosure		(020)	

3.33 Direction **B** is a $[\bar{4}0\bar{3}]$ direction, which determination is summarized as follows. We first of all position the origin of the coordinate system at the tail of the direction vector; then in terms of this new coordinate system

	x	y	z
Projections	$-\frac{2a}{3}$	0b	$-\frac{c}{2}$
Projections in terms of **a**, **b**, and **c**	$-\frac{2}{3}$	0	$-\frac{1}{2}$
Reduction to integers	- 4	0	- 3
Enclosure		$[\bar{4}0\bar{3}]$	

Direction **D** is a $[\bar{1}1\bar{1}]$ direction, which determination is summarized as follows. We first of all position the origin of the coordinate system at the tail of the direction vector; then in terms of this new coordinate system

	x	y	z
Projections	$-\frac{a}{2}$	$\frac{b}{2}$	$-\frac{c}{2}$

Projections in terms of **a**, **b**, and **c**	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
Reduction to integers	- 1	1	- 1
Enclosure		$[\bar{1}1\bar{1}]$	

3.37 For plane **B** we will leave the origin of the unit cell as shown; thus, this is a (122) plane, as summarized below.

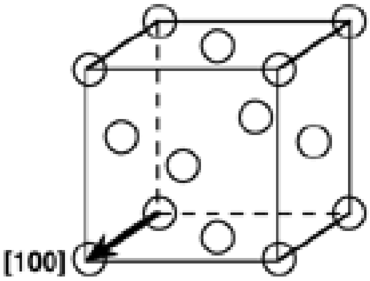
	x	y	z
Intercepts	a	$\frac{b}{2}$	$\frac{c}{2}$
Intercepts in terms of **a**, **b**, and **c**	1	$\frac{1}{2}$	$\frac{1}{2}$
Reciprocals of intercepts	1	2	2
Reduction		not necessary	
Enclosure		(122)	

3.40 (a) For this plane we will leave the origin of the coordinate system as shown; thus, this is a $(1\bar{2}11)$ plane, as summarized below.

	a_1	a_2	a_3	z
Intercepts	a	$-\frac{a}{2}$	a	c
Intercepts in terms of **a**'s and **c**	1	$-\frac{1}{2}$	1	1
Reciprocals of intercepts	1	- 2	1	1
Reduction		not necessary		
Enclosure		$(1\bar{2}11)$		

3.43 (a) The unit cell in Problem 3.21 is body-centered tetragonal. Only the (100) (front face) and $(0\bar{1}0)$ (left side face) planes are equivalent since the dimensions of these planes within the unit cell (and therefore the distances between adjacent atoms) are the same (namely 0.45 nm x 0.35 nm), which are different than the (001) (top face) plane (namely 0.35 nm x 0.35 nm).

3.45 (a) In the figure below is shown a [100] direction within an FCC unit cell.



For this [100] direction there is one atom at each of the two unit cell corners, and, thus, there is the equivalent of 1 atom that is centered on the direction vector. The length of this direction vector is just the unit cell edge length, $2\mathbf{R}\sqrt{2}$ [Equation (3.1)]. Therefore, the expression for the linear density of this plane is

$$LD_{100} = \frac{\text{number of atoms centered on [100] direction vector}}{\text{length of [100] direction vector}}$$

$$= \frac{1 \text{ atom}}{2R\sqrt{2}} = \frac{1}{2R\sqrt{2}}$$

3.54W We must first calculate the lattice parameter using Equation (3.3) and the value of **R** cited in Table 3.1 as

$$a = \frac{4R}{\sqrt{3}} = \frac{(4)(0.1249 \text{ nm})}{\sqrt{3}} = 0.2884 \text{ nm}$$

Next, the interplanar spacing may be determined using Equation (3.3W) according to

$$d_{310} = \frac{a}{\sqrt{(3)^2 + (1)^2 + (0)^2}} = \frac{0.2884 \text{ nm}}{\sqrt{10}} = 0.0912 \text{ nm}$$

And finally, employment of Equation (3.2W) yields

$$\sin\theta = \frac{n\lambda}{2d} = \frac{(1)(0.0711\ \text{nm})}{(2)(0.0912\ \text{nm})} = 0.390$$

$$\theta = \sin^{-1}(0.390) = 22.94°$$

And

$$2\theta = (2)(22.94°) = 45.88°$$

3.55W From the table, α-iron has a BCC crystal structure and an atomic radius of 0.1241 nm. Using Equation (3.3) the lattice parameter, **a**, may be computed as

$$a = \frac{4R}{\sqrt{3}} = \frac{(4)(0.1241\ \text{nm})}{\sqrt{3}} = 0.2866\ \text{nm}$$

Now, the $\mathbf{d_{111}}$ interplanar spacing may be determined using Equation (3.3W) as

$$d_{111} = \frac{a}{\sqrt{(1)^2 + (1)^2 + (1)^2}} = \frac{0.2866\ \text{nm}}{\sqrt{3}} = 0.1655\ \text{nm}$$

3.59W For each peak, in order to compute the interplanar spacing and the lattice parameter we must employ Equations (3.3W) and (3.2W), respectively. For the first peak which occurs at 31.3°

$$d_{111} = \frac{n\lambda}{2\sin\theta} = \frac{(1)(0.1542\ \text{nm})}{(2)\left(\sin\frac{31.3°}{2}\right)} = 0.2858\ \text{nm}$$

And

$$a = d_{hkl}\sqrt{(h)^2 + (k)^2 + (l)^2} = d_{111}\sqrt{(1)^2 + (1)^2 + (1)^2}$$

$$= (0.2858\ \text{nm})\sqrt{3} = 0.4950\ \text{nm}$$

Similar computations are made for the other peaks which results are tabulated below:

Peak Index	2θ	d_{hkl}(nm)	a (nm)
200	36.6	0.2455	0.4910
311	62.5	0.1486	0.4929

CHAPTER 4

IMPERFECTIONS IN SOLIDS

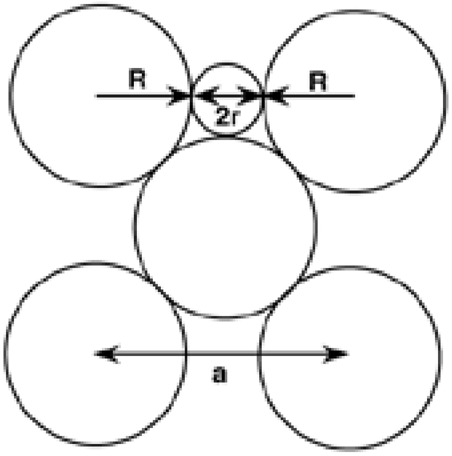
PROBLEM SOLUTIONS

4.1 In order to compute the fraction of atom sites that are vacant in lead at 600 K, we must employ Equation (4.1). As stated in the problem, $\mathbf{Q_V}$ = 0.55 eV/atom. Thus,

$$\frac{N_V}{N} = \exp\left(-\frac{Q_V}{kT}\right) = \exp\left[-\frac{0.55 \text{ eV/atom}}{\left(8.62 \text{ x } 10^{-5} \text{ eV/atom-K}\right)(600 \text{ K})}\right]$$

$$= 2.41 \text{ x } 10^{-5}$$

4.5 In the drawing below is shown the atoms on the (100) face of an FCC unit cell; the interstitial site is at the center of the edge.



The diameter of an atom that will just fit into this site (2**r**) is just the difference between the unit cell edge length (**a**) and the radii of the two host atoms that are located on either side of the site (**R**); that is

$$2r = a - 2R$$

However, for FCC **a** is related to **R** according to Equation (3.1) as **a** = $2\mathbf{R}\sqrt{2}$; therefore, solving for **r** gives

$$r = \frac{a - 2R}{2} = \frac{2R\sqrt{2} - 2R}{2} = 0.41R$$

4.10 The concentration of an element in an alloy, in atom percent, may be computed using Equation (4.5). With this problem, it first becomes necessary to compute the number of moles of both Cu and Zn, for which Equation (4.4) is employed. Thus, the number of moles of Cu is just

$$n_{m_{Cu}} = \frac{m'_{Cu}}{A_{Cu}} = \frac{33\ \text{g}}{63.55\ \text{g/mol}} = 0.519\ \text{mol}$$

Likewise, for Zn

$$n_{m_{Zn}} = \frac{47\ \text{g}}{65.39\ \text{g/mol}} = 0.719\ \text{mol}$$

Now, use of Equation (4.5) yields

$$C'_{Cu} = \frac{n_{m_{Cu}}}{n_{m_{Cu}} + n_{m_{Zn}}} \times 100$$

$$= \frac{0.519\ \text{mol}}{0.519\ \text{mol} + 0.719\ \text{mol}} \times 100 = 41.9\ \text{at\%}$$

Also,

$$C'_{Zn} = \frac{0.719\ \text{mol}}{0.519\ \text{mol} + 0.719\ \text{mol}} \times 100 = 58.1\ \text{at\%}$$

4.14 This problem calls for a determination of the number of atoms per cubic meter for aluminum. In order to solve this problem, one must employ Equation (4.2),

$$N = \frac{N_A \rho_{Al}}{A_{Al}}$$

The density of Al (from the table inside of the front cover) is 2.71 g/cm^3, while its atomic weight is 26.98 g/mol. Thus,

$$N = \frac{(6.023 \text{ x } 10^{23} \text{ atoms/mol})(2.71 \text{ g/cm}^3)}{26.98 \text{ g/mol}}$$

$$= 6.05 \text{ x } 10^{22} \text{ atoms/cm}^3 = 6.05 \text{ x } 10^{28} \text{ atoms/m}^3$$

4.22 This problem asks us to determine the weight percent of Ge that must be added to Si such that the resultant alloy will contain 2.43 x10^{21} Ge atoms per cubic centimeter. To solve this problem, employment of Equation (4.18) is necessary, using the following values:

$N_1 = N_{Ge} = 2.43 \text{ x } 10^{21}$ atoms/cm^3
$\rho_1 = \rho_{Ge} = 5.32$ g/cm^3
$\rho_2 = \rho_{Si} = 2.33$ g/cm^3
$A_1 = A_{Ge} = 72.59$ g/mol
$A_2 = A_{Si} = 28.09$ g/mol

Thus

$$C_{Ge} = \frac{100}{1 + \frac{N_A \rho_{Si}}{N_{Ge} A_{Ge}} - \frac{\rho_{Si}}{\rho_{Ge}}}$$

$$= \frac{100}{1 + \frac{(6.023 \text{ x}10^{23} \text{ atoms/mol})(2.33 \text{ g/cm}^3)}{(2.43 \text{ x}10^{21} \text{ atoms/cm}^3)(72.59 \text{ g/mol})} - \left(\frac{2.33 \text{ g/cm}^3}{5.32 \text{ g/cm}^3}\right)}$$

$$= 11.7 \text{ wt\%}$$

4.25 (a) The Burgers vector will point in that direction having the highest linear density. From Section 3.11, the linear density for the [110] direction in FCC is 1/2**R**, the maximum possible; therefore for FCC

$$\mathbf{b} = \frac{a}{2}[110]$$

(b) For Cu which has an FCC crystal structure, **R** = 0.1278 nm (Table 3.1) and **a** = $2\mathbf{R}\sqrt{2}$ = 0.3615 nm [Equation (3.1)]; therefore

$$\mathbf{b} = \frac{a}{2}\sqrt{h^2 + k^2 + l^2}$$

$$= \frac{0.3615 \text{ nm}}{2}\sqrt{(1)^2 + (1)^2 + (0)^2} = 0.2556 \text{ nm}$$

4.32 (a) This part of problem asks that we compute the number of grains per square inch for an ASTM grain size of 6 at a magnification of 100x. All we need do is solve for the parameter **N** in Equation 4.16, inasmuch as **n** = 6. Thus

$$N = 2^{n-1}$$

$$= 2^{6-1} = 32 \text{ grains/in.}^2$$

(b) Now it is necessary to compute the value of **N** for no magnification. In order to solve this problem it is necessary to use the following equation:

$$N_M\left(\frac{M}{100}\right)^2 = 2^{n-1}$$

where $\mathbf{N_M}$ = the number of grains per square inch at magnification **M**, and **n** is the ASTM grain size number. (The above equation makes use of the fact that, while magnification is a length parameter, area is expressed in terms of units of length squared. As a consequence, the number of grains per unit area increases with the square of the increase in magnification.) Without any magnification, **M** in the above equation is 1, and therefore,

$$N_1\left(\frac{1}{100}\right)^2 = 2^{6-1} = 32$$

And, solving for $\mathbf{N_1}$, $\mathbf{N_1}$ = 320,000 grains/in.^{2}.

4.D1 This problem calls for us to compute the concentration of lithium (in wt%) that, when added to aluminum, will yield an alloy having a density of 2.55 g/cm^3. Solution of this problem requires the use of Equation (4.10a), which takes the form

$$\rho_{\mathrm{ave}} = \frac{100}{\dfrac{C_{\mathrm{Li}}}{\rho_{\mathrm{Li}}} + \dfrac{100 - C_{\mathrm{Li}}}{\rho_{\mathrm{Al}}}}$$

inasmuch as $\mathbf{C_{Li}} + \mathbf{C_{Al}} = 100$. According to the table inside the front cover, the respective densities of Li and Al are 0.534 and 2.71 g/cm^3. Upon solving for $\mathbf{C_{Li}}$ from the above equation

$$C_{\mathrm{Li}} = \frac{100\,\rho_{\mathrm{Li}}\left(\rho_{\mathrm{Al}} - \rho_{\mathrm{ave}}\right)}{\rho_{\mathrm{ave}}\left(\rho_{\mathrm{Al}} - \rho_{\mathrm{Li}}\right)}$$

$$= \frac{(100)\left(0.534\ \mathrm{g/cm^3}\right)\left(2.71\ \mathrm{g/cm^3} - 2.55\ \mathrm{g/cm^3}\right)}{2.55\ \mathrm{g/cm^3}\left(2.71\ \mathrm{g/cm^3} - 0.534\ \mathrm{g/cm^3}\right)}$$

$$= 1.537\ \mathrm{wt\%}$$

CHAPTER 5

DIFFUSION

PROBLEM SOLUTIONS

5.8 This problem calls for computation of the diffusion coefficient for a steady-state diffusion situation. Let us first convert the carbon concentrations from wt% to kg C/m^3 using Equation (4.9a). For 0.012 wt% C

$$C''_C = \frac{C_C}{\dfrac{C_C}{\rho_C} + \dfrac{C_{Fe}}{\rho_{Fe}}} \times 10^3$$

$$= \frac{0.012}{\dfrac{0.012}{2.25 \ \mathrm{g/cm^3}} + \dfrac{99.988}{7.87 \ \mathrm{g/cm^3}}} \times 10^3$$

0.944 kg C/m^3

Similarly, for 0.0075 wt% C

$$C''_C = \frac{0.0075}{\dfrac{0.0075}{2.25 \ \mathrm{g/cm^3}} + \dfrac{99.9925}{7.87 \ \mathrm{g/cm^3}}} \times 10^3$$

= 0.590 kg C/m^3

Now, using a form of Equation (5.3)

$$D = -J\left[\frac{x_A - x_B}{C_A - C_B}\right]$$

$$= -\left(1.40 \times 10^{-8} \ \mathrm{kg/m^2\text{-}s}\right)\left[\frac{-10^{-3} \ \mathrm{m}}{0.944 \ \mathrm{kg/m^3} - 0.590 \ \mathrm{kg/m^3}}\right]$$

$$= 3.95 \times 10^{-11}\ \mathrm{m^2/s}$$

5.15 This problem calls for an estimate of the time necessary to achieve a carbon concentration of 0.45 wt% at a point 5 mm from the surface. From Equation (5.6b),

$$\frac{x^2}{Dt} = \text{constant}$$

But since the temperature is constant, so also is **D** constant, and

$$\frac{x^2}{t} = \text{constant}$$

or

$$\frac{x_1^2}{t_1} = \frac{x_2^2}{t_2}$$

Thus,

$$\frac{(2.5\ \mathrm{mm})^2}{10\ \mathrm{h}} = \frac{(5.0\ \mathrm{mm})^2}{t_2}$$

from which

$$t_2 = 40\ \mathrm{h}$$

5.21 (a) Using Equation (5.9a), we set up two simultaneous equations with $\mathbf{Q_d}$ and $\mathbf{D_o}$ as unknowns. Solving for $\mathbf{Q_d}$ in terms of temperatures $\mathbf{T_1}$ and $\mathbf{T_2}$ (1273 K and 1473 K) and $\mathbf{D_1}$ and $\mathbf{D_2}$ (9.4×10^{-16} and $2.4 \times 10^{-14}\ \mathrm{m^2/s}$), we get

$$Q_d = -R\,\frac{\ln D_1 - \ln D_2}{\dfrac{1}{T_1} - \dfrac{1}{T_2}}$$

$$= -\frac{(8.31\ \text{J/mol-K})\left[\ln\left(9.4 \times 10^{-16}\right) - \ln\left(2.4 \times 10^{-14}\right)\right]}{\dfrac{1}{1273\ \text{K}} - \dfrac{1}{1473\ \text{K}}}$$

$$= 252{,}400\ \text{J/mol}$$

Now, solving for $\mathbf{D_o}$ from Equation (5.8)

$$D_o = D_1 \exp\left(\frac{Q_d}{RT_1}\right)$$

$$= \left(9.4 \times 10^{-16}\ \text{m}^2/\text{s}\right)\exp\left[\frac{252{,}400\ \text{J/mol}}{(8.31\ \text{J/mol-K})(1273\ \text{K})}\right]$$

$$= 2.2 \times 10^{-5}\ \text{m}^2/\text{s}$$

(b) Using these values of $\mathbf{D_o}$ and $\mathbf{Q_d}$, **D** at 1373 K is just

$$D = \left(2.2 \times 10^{-5}\ \text{m}^2/\text{s}\right)\exp\left[-\frac{252{,}400\ \text{J/mol}}{(8.31\ \text{J/mol-K})(1373\ \text{K})}\right]$$

$$= 5.4 \times 10^{-15}\ \text{m}^2/\text{s}$$

5.29 In order to determine the position within the diffusion couple at which the concentration of A in B is 2.5 wt%, we must employ Equation (5.6b) with **t** constant. That is

$$\frac{x^2}{D} = \text{constant}$$

Or

$$\frac{x_{800}^2}{D_{800}} = \frac{x_{1000}^2}{D_{1000}}$$

It is necessary to compute both $\mathbf{D_{800}}$ and $\mathbf{D_{1000}}$ using Equation (5.8), as follows:

$$D_{800} = \left(1.5 \times 10^{-4}\ \text{m}^2/\text{s}\right)\exp\left[-\frac{125{,}000\ \text{J/mol}}{(8.31\ \text{J/mol-K})(1073\ \text{K})}\right]$$

$$= 1.22 \times 10^{-10}\ \mathrm{m^2/s}$$

$$D_{1000} = \left(1.5 \times 10^{-4}\ \mathrm{m^2/s}\right)\exp\left[-\frac{125{,}000\ \mathrm{J/mol}}{(8.31\ \mathrm{J/mol\text{-}K})(1273\ \mathrm{K})}\right]$$

$$= 1.11 \times 10^{-9}\ \mathrm{m^2/s}$$

Now, solving for $\mathbf{x_{1000}}$ yields

$$x_{1000} = x_{800}\sqrt{\frac{D_{1000}}{D_{800}}}$$

$$= (5\ \mathrm{mm})\sqrt{\frac{1.11 \times 10^{-9}\ \mathrm{m^2/s}}{1.22 \times 10^{-10}\ \mathrm{m^2/s}}}$$

$$= 15.1\ \mathrm{mm}$$

CHAPTER 6

MECHANICAL PROPERTIES OF METALS

PROBLEM SOLUTIONS

6.4 We are asked to compute the maximum length of a cylindrical titanium alloy specimen that is deformed elastically in tension. For a cylindrical specimen

$$A_o = \pi\left(\frac{d_o}{2}\right)^2$$

where $\mathbf{d_o}$ is the original diameter. Combining Equations (6.1), (6.2), and (6.5) and solving for $\mathbf{l_o}$ leads to

$$l_o = \frac{E\pi d_o^2 \Delta l}{4F}$$

$$= \frac{\left(107 \text{ x } 10^9 \text{ N/m}^2\right)(\pi)\left(3.8 \text{ x } 10^{-3} \text{ m}\right)^2\left(0.42 \text{ x } 10^{-3} \text{ m}\right)}{(4)(2000 \text{ N})}$$

$$= 0.25 \text{ m} = 250 \text{ mm (10 in.)}$$

6.9 This problem asks that we calculate the elongation •l of a specimen of steel the stress-strain behavior of which is shown in Figure 6.24. First it becomes necessary to compute the stress when a load of 65,250 N is applied as

$$\sigma = \frac{F}{A_o} = \frac{F}{\pi\left(\frac{d_o}{2}\right)^2} = \frac{65{,}250 \text{ N}}{\pi\left(\frac{8.5 \text{ x } 10^{-3} \text{ m}}{2}\right)^2} = 1150 \text{ MPa} \ \ (170{,}000 \text{ psi})$$

Referring to Figure 6.24, at this stress level we are in the elastic region on the stress-strain curve, which corresponds to a strain of 0.0054. Now, utilization of Equation (6.2) yields

$$\Delta l = \varepsilon l_o = (0.0054)(80 \text{ mm}) = 0.43 \text{ mm} \ (0.017 \text{ in.})$$

6.14 (a) We are asked, in this portion of the problem, to determine the elongation of a cylindrical specimen of aluminum. Using Equations (6.1), (6.2), and (6.5)

$$\frac{F}{\pi\left(\dfrac{d_o^2}{4}\right)} = E\,\frac{\Delta l}{l_o}$$

Or

$$\Delta l = \frac{4Fl_o}{\pi d_o^2 E}$$

$$= \frac{(4)(48{,}800 \text{ N})\left(200 \times 10^{-3} \text{m}\right)}{(\pi)\left(19 \times 10^{-3} \text{m}\right)^2\left(69 \times 10^9 \text{ N/m}^2\right)} = 0.50 \text{ mm} \ (0.02 \text{ in.})$$

(b) We are now called upon to determine the change in diameter, **Δd**. Using Equation (6.8)

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d/d_o}{\Delta l/l_o}$$

From Table 6.1, for Al, **ν** = 0.33. Now, solving for **•d** yields

$$\Delta d = -\frac{\nu \Delta l d_o}{l_o} = -\frac{(0.33)(0.50 \text{ mm})(19 \text{ mm})}{200 \text{ mm}}$$

$$= -1.57 \times 10^{-2} \text{ mm} \ (-6.2 \times 10^{-4} \text{ in.})$$

The diameter will decrease.

6.16 This problem asks that we compute Poisson's ratio for the metal alloy. From Equations (6.5) and (6.1)

$$\varepsilon_z = \frac{\sigma}{E} = \frac{F/A_o}{E} = \frac{F}{\pi\left(\dfrac{d_o}{2}\right)^2 E} = \frac{4F}{\pi d_o^2 E}$$

Since the transverse strain $\boldsymbol{\varepsilon_x}$ is just

$$\varepsilon_x = \frac{\Delta d}{d_o}$$

and Poisson's ratio is defined by Equation (6.8) then

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\Delta d / d_o}{\left(\dfrac{4F}{\pi d_o^2 E}\right)} = -\frac{d_o \Delta d \, \pi E}{4F}$$

$$= -\frac{\left(10 \times 10^{-3}\ \text{m}\right)\left(-7 \times 10^{-6}\ \text{m}\right)(\pi)\left(100 \times 10^{9}\ \text{N/m}^2\right)}{(4)(15{,}000\ \text{N})} = 0.367$$

6.21 (a) This portion of the problem asks that we compute the elongation of the brass specimen. The first calculation necessary is that of the applied stress using Equation (6.1), as

$$\sigma = \frac{F}{A_o} = \frac{F}{\pi\left(\dfrac{d_o}{2}\right)^2} = \frac{10{,}000\ \text{N}}{\pi\left(\dfrac{10 \times 10^{-3}\ \text{m}}{2}\right)^2} = 127\ \text{MPa}\ \ (17{,}900\ \text{psi})$$

From the stress-strain plot in Figure 6.12, this stress corresponds to a strain of about 1.5 x 10^{-3}. From the definition of strain, Equation (6.2)

$$\Delta l = \varepsilon l_o = \left(1.5 \times 10^{-3}\right)(101.6\ \text{mm}) = 0.15\ \text{mm}\ \ \ (6.0 \times 10^{-3}\ \text{in.})$$

(b) In order to determine the reduction in diameter **•d**, it is necessary to use Equation (6.8) and the definition of lateral strain (i.e., $\boldsymbol{\varepsilon_x}$ = **•d/d$_o$**) as follows

$$\Delta d = d_o \varepsilon_x = -d_o \nu \varepsilon_z = -(10\ \text{mm})(0.35)\left(1.5 \times 10^{-3}\right)$$

$$= -5.25 \times 10^{-3} \text{ mm } (-2.05 \times 10^{-4} \text{ in.})$$

6.27 This problem asks us to determine the deformation characteristics of a steel specimen, the stress-strain behavior of which is shown in Figure 6.24.

(a) In order to ascertain whether the deformation is elastic or plastic, we must first compute the stress, then locate it on the stress-strain curve, and, finally, note whether this point is on the elastic or plastic region. Thus,

$$\sigma = \frac{F}{A_o} = \frac{140{,}000 \text{ N}}{\pi\left(\frac{10 \times 10^{-3} \text{ m}}{2}\right)^2} = 1782 \text{ MPa} \quad (250{,}000 \text{ psi})$$

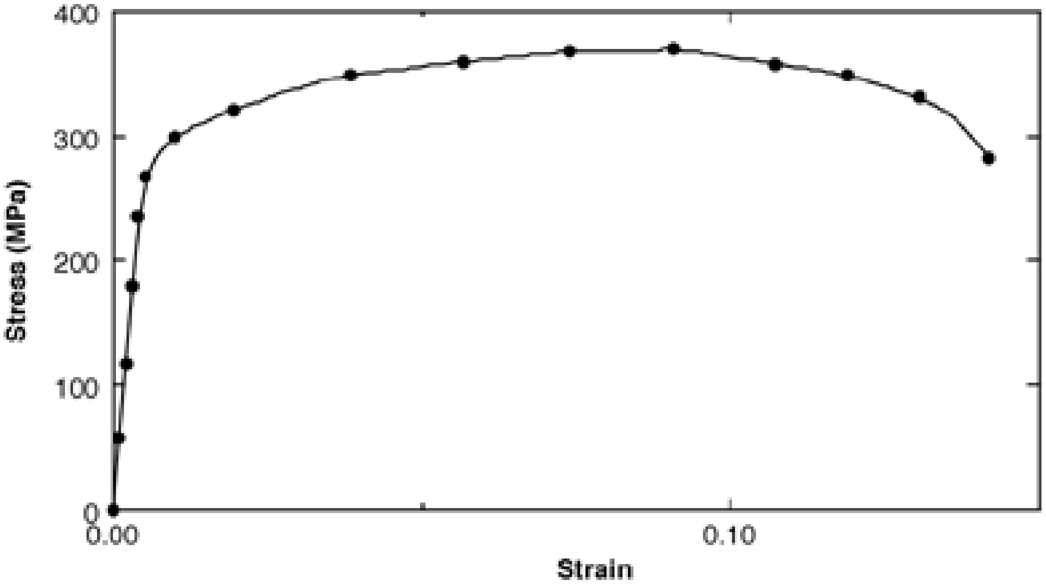
The 1782 MPa point is past the linear portion of the curve, and, therefore, the deformation will be both elastic and plastic.

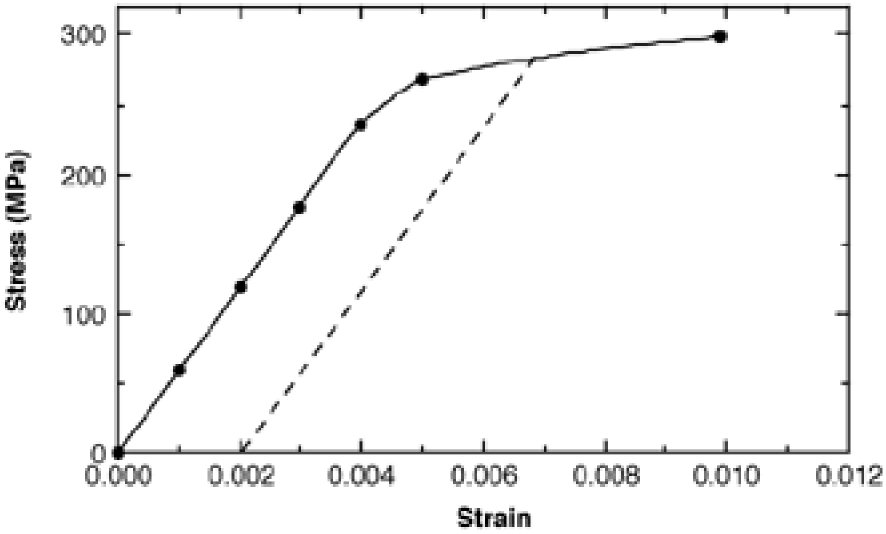
(b) This portion of the problem asks us to compute the increase in specimen length. From the stress-strain curve, the strain at 1782 MPa is approximately 0.017. Thus, from Equation (6.2)

$$\Delta l = \varepsilon l_o = (0.017)(500 \text{ mm}) = 8.5 \text{ mm} \quad (0.34 \text{ in.})$$

6.29 This problem calls for us to make a stress-strain plot for aluminum, given its tensile load-length data, and then to determine some of its mechanical characteristics.

(a) The data are plotted below on two plots: the first corresponds to the entire stress-strain curve, while for the second, the curve extends just beyond the elastic region of deformation.





(b) The elastic modulus is the slope in the linear elastic region as

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} = \frac{200\ \text{MPa} - 0\ \text{MPa}}{0.0032 - 0} = 62.5 \times 10^3\ \text{MPa} = 62.5\ \text{GPa} \quad \left(9.1 \times 10^6\ \text{psi}\right)$$

(c) For the yield strength, the 0.002 strain offset line is drawn dashed. It intersects the stress-strain curve at approximately 285 MPa (41,000 psi).

(d) The tensile strength is approximately 370 MPa (53,500 psi), corresponding to the maximum stress on the complete stress-strain plot.

(e) The ductility, in percent elongation, is just the plastic strain at fracture, multiplied by one-hundred. The total fracture strain at fracture is 0.165; subtracting out the elastic strain (which is about 0.005) leaves a plastic strain of 0.160. Thus, the ductility is about 16%EL.

(f) From Equation (6.14), the modulus of resilience is just

$$U_r = \frac{\sigma_y^2}{2E}$$

which, using data computed in the problem, yields a value of

$$U_r = \frac{(285\ \text{MPa})^2}{(2)\left(62.5 \times 10^3\ \text{MPa}\right)} = 6.5 \times 10^5\ \text{J/m}^3 \quad \left(93.8\ \text{in.-lb}_f/\text{in.}^3\right)$$

6.32 This problem asks us to calculate the moduli of resilience for the materials having the stress-strain behaviors shown in Figures 6.12 and 6.24. According to Equation (6.14), the modulus of resilience $\mathbf{U_r}$ is a function of the yield strength and the modulus of elasticity as

$$U_r = \frac{\sigma_y^2}{2E}$$

The values for $\boldsymbol{\sigma_y}$ and **E** for the brass in Figure 6.12 are 250 MPa (36,000 psi) and 93.9 GPa (13.6 x 10^6 psi), respectively. Thus

$$U_r = \frac{(250 \text{ MPa})^2}{(2)\left(93.9 \text{ x } 10^3 \text{ MPa}\right)} = 3.32 \text{ x } 10^5 \text{ J/m}^3 \quad \left(47.6 \text{ in.-lb}_f/\text{in.}^3\right)$$

6.41 For this problem, we are given two values of $\boldsymbol{\varepsilon_T}$ and $\boldsymbol{\sigma_T}$, from which we are asked to calculate the true stress which produces a true plastic strain of 0.25. After taking logarithms of Equation (6.19), we may set up two simultaneous equations with two unknowns (the unknowns being **K** and **n**), as

$$\log (50{,}000 \text{ psi}) = \log K + n \log (0.10)$$

$$\log (60{,}000 \text{ psi}) = \log K + n \log (0.20)$$

From these two expressions,

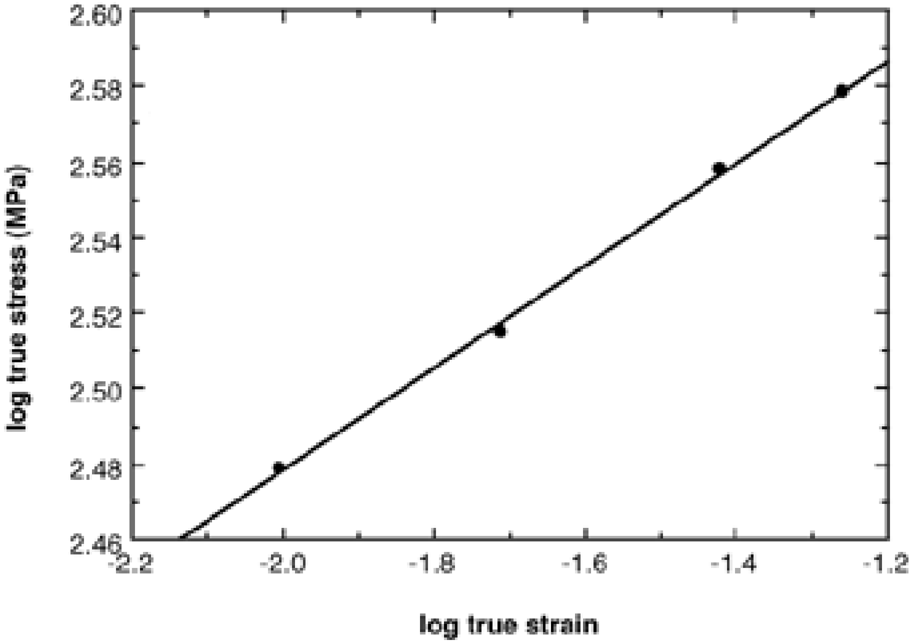
$$n = \frac{\log (50{,}000) - \log (60{,}000)}{\log (0.1) - \log (0.2)} = 0.263$$

$$\log K = 4.96 \text{ or } K = 91{,}623 \text{ psi}$$

Thus, for $\boldsymbol{\varepsilon_T}$ = 0.25

$$\sigma_T = K\left(\varepsilon_T\right)^{0.263} = (91{,}623 \text{ psi})(0.25)^{0.263} = 63{,}700 \text{ psi} \quad (440 \text{ MPa})$$

6.45 This problem calls for us to utilize the appropriate data from Problem 6.29 in order to determine the values of **n** and **K** for this material. From Equation (6.38) the slope and intercept of a log $\boldsymbol{\sigma_T}$ versus log ε_T plot will yield values for **n** and log **K**, respectively. However, Equation (6.19) is only valid in the region of plastic deformation to the point of necking; thus, only the 7th, 8th, 9th, and 10th data points may be utilized. The log-log plot with these data points is given below.



The slope yields a value of 0.136 for **n**, whereas the intercept gives a value of 2.7497 for log **K**, and thus **K** = 562 MPa.

6.49 This problem calls for estimations of Brinell and Rockwell hardnesses.

(a) For the brass specimen, the stress-strain behavior for which is shown in Figure 6.12, the tensile strength is 450 MPa (65,000 psi). From Figure 6.19, the hardness for brass corresponding to this tensile strength is about 125 HB or 70 HRB.

6.54 The working stresses for the two alloys the stress-strain behaviors of which are shown in Figures 6.12 and 6.24 are calculated by dividing the yield strength by a factor of safety, which we will take to be 2. For the brass alloy (Figure 6.12), since σ_y = 250 MPa (36,000 psi), the working stress is 125 MPa (18,000 psi).

6.D6 (a) This portion of the problem asks that we derive a performance index expression for strength analogous to Equation (6.33) for a cylindrical cantilever beam that is stressed in the manner shown in the accompanying figure. The stress on the unfixed end, **σ**, for an imposed force, **F**, is given by the expression [Equation (6.42) in the textbook]

$$\sigma = \frac{FLr}{I} \tag{6.D1}$$

where **L** and **r** are the rod length and radius, respectively, and **I** is the moment of inertia; for a cylinder the expression for **I** is provided in Figure 12.29:

$$I = \frac{\pi r^4}{4} \tag{6.D2}$$

Substitution for **I** into Equation (6.D1) leads to

$$\sigma = \frac{4FL}{\pi r^3} \tag{6.D3}$$

Now, the mass **m** of some given quantity of material is the product of its density (**ρ**) and volume. Inasmuch as the volume of a cylinder is just **$\pi r^2 L$**, then

$$m = \pi r^2 L\rho \tag{6.D4}$$

From this expression, the radius is just

$$r = \sqrt{\frac{m}{\pi L\rho}} \tag{6.D5}$$

Inclusion of Equation (6.D5) into Equation (6.D3) yields

$$\sigma = \frac{4F\pi^{1/2}L^{5/2}\rho^{3/2}}{m^{3/2}} \tag{6.D6}$$

And solving for the mass gives

$$m = \left(16\pi F^2 L^5\right)^{1/3} \frac{\rho}{\sigma^{2/3}} \tag{6.D7}$$

To ensure that the beam will not fail, we replace stress in Equation (6.D7) with the yield strength (**σ_y**) divided by a factor of safety (**N**) as

$$m = \left(16\pi F^2 L^5 N^2\right)^{1/3} \frac{\rho}{\sigma_y^{2/3}} \tag{6.D8}$$

Thus, the best materials to be used for this cylindrical cantilever beam when strength is a consideration are those having low $\frac{\boldsymbol{\rho}}{\boldsymbol{\sigma}_\mathbf{y}^{2/3}}$ ratios. Furthermore, the strength performance index, **P**, is just the reciprocal of this ratio, or

$$P = \frac{\sigma_y^{2/3}}{\rho} \tag{6.D9}$$

The second portion of the problem asks for an expression for the stiffness performance index. Let us begin by consideration of Equation (6.43) which relates **δ**, the elastic deflection at the unfixed end, to the force (**F**), beam length (**L**), the modulus of elasticity (**E**), and moment of inertia (**I**) as

$$\delta = \frac{FL^3}{3EI} \tag{6.43}$$

Again, Equation (6.D2) gives an expression for **I** for a cylinder, which when substituted into Equation (6.43) yields

$$\delta = \frac{4FL^3}{3\pi E r^4} \tag{6.D10}$$

And, substitution of the expression for **r** [Equation (6.D5)] into Equation (6.D10), leads to

$$\delta = \frac{4FL^3}{3\pi E\left(\sqrt{\frac{m}{\pi L\rho}}\right)^4}$$

$$= \frac{4FL^5\pi\rho^2}{3Em^2} \tag{6.D11}$$

Now solving this expression for the mass **m** yields

$$m = \left(\frac{4FL^5\pi}{3\delta}\right)^{1/2} \frac{\rho}{\sqrt{E}} \tag{6.D12}$$

Or, for this cantilever situation, the mass of material experiencing a given deflection produced by a specific force is proportional to the $\frac{\rho}{\sqrt{E}}$ ratio for that material. And, finally, the stiffness performance index, **P**, is just the reciprocal of this ratio, or

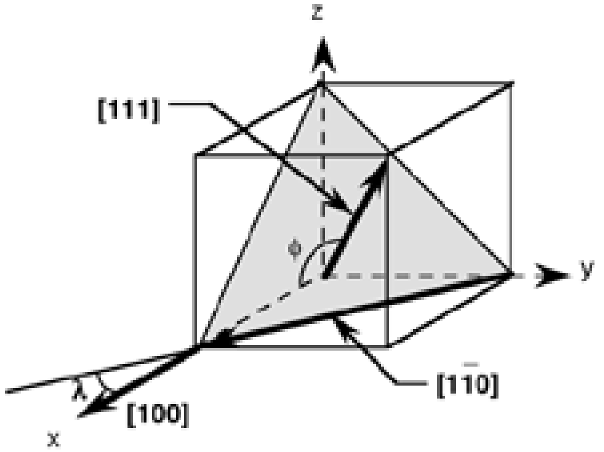
$$P = \frac{\sqrt{E}}{\rho} \tag{6.D13}$$

CHAPTER 7

DISLOCATIONS AND STRENGTHENING MECHANISMS

PROBLEM SOLUTIONS

7.10 We are asked to compute the **Schmid factor** for an FCC crystal oriented with its [100] direction parallel to the loading axis. With this scheme, slip may occur on the (111) plane and in the $[1\bar{1}0]$ direction as noted in the figure below.



The angle between the [100] and $[1\bar{1}0]$ directions, λ, is 45°. For the (111) plane, the angle between its normal (which is the [111] direction) and the [100] direction, ϕ, is $\tan^{-1}\left(\frac{a\sqrt{2}}{a}\right) = 54.74°$, therefore

$$\cos\lambda\cos\phi = \cos(45°)\cos(54.74°) = 0.408$$

7.22 We are asked to determine the grain diameter for an iron which will give a yield strength of 205 MPa (30,000 psi). The best way to solve this problem is to first establish two simultaneous expressions of Equation (7.5), solve for σ_o and k_y, and finally determine the value of **d** when σ_y = 205 MPa. The data pertaining to this problem may be tabulated as follows:

σ_y	d (mm)	$d^{-1/2}$ $(mm)^{-1/2}$
135 MPa	5×10^{-2}	4.47
260 MPa	8×10^{-3}	11.18

The two equations thus become

$$135 \text{ MPa} = \sigma_o + (4.47)\,k_y$$
$$260 \text{ MPa} = \sigma_o + (11.18)\,k_y$$

Which yield the values, $\boldsymbol{\sigma_o}$ = 51.7 MPa and $\mathbf{k_y}$ = 18.63 MPa(mm)$^{1/2}$. At a yield strength of 205 MPa

$$205 \text{ MPa} = 51.7 \text{ MPa} + \left[18.63 \text{ MPa}(\text{mm})^{1/2}\right]d^{-1/2}$$

or $\mathbf{d}^{-1/2}$ = 8.23 (mm)$^{-1/2}$, which gives $\mathbf{d}$ = 1.48 x 10^{-2} mm.

7.27 In order for these two cylindrical specimens to have the same deformed hardness, they must be deformed to the same percent cold work. For the first specimen

$$\%CW = \frac{A_o - A_d}{A_o} \text{ x } 100 = \frac{\pi r_o^2 - \pi r_d^2}{\pi r_o^2} \text{ x } 100$$

$$= \frac{\pi(15 \text{ mm})^2 - \pi(12 \text{ mm})^2}{\pi(15 \text{ mm})^2} \text{ x } 100 = 36\%CW$$

For the second specimen, the deformed radius is computed using the above equation and solving for $\mathbf{r_d}$ as

$$r_d = r_o\sqrt{1 - \frac{\%CW}{100}}$$

$$= (11 \text{ mm})\sqrt{1 - \frac{36\%CW}{100}} = 8.80 \text{ mm}$$

7.29 This problem calls for us to calculate the precold-worked radius of a cylindrical specimen of copper that has a cold-worked ductility of 25%EL. From Figure 7.17(c), copper that has a ductility of 25%EL will have experienced a deformation of about 11%CW. For a cylindrical specimen, Equation (7.6) becomes

$$\%CW = \left[\frac{\pi r_o^2 - \pi r_d^2}{\pi r_o^2}\right] \text{ x } 100$$

Since $\mathbf{r_d}$ = 10 mm (0.40 in.), solving for $\mathbf{r_o}$ yields

$$r_o = \frac{r_d}{\sqrt{1 - \frac{\%CW}{100}}} = \frac{10 \text{ mm}}{\sqrt{1 - \frac{11.0}{100}}} = 10.6 \text{ mm} \quad (0.424 \text{ in.})$$

7.37 In this problem, we are asked for the length of time required for the average grain size of a brass material to increase a specified amount using Figure 7.23.

(a) At 500°C, the time necessary for the average grain diameter to increase from 0.01 to 0.1 mm is approximately 3500 min.

7.D1 This problem calls for us to determine whether or not it is possible to cold work steel so as to give a minimum Brinell hardness of 240 and a ductility of at least 15%EL. According to Figure 6.19, a Brinell hardness of 240 corresponds to a tensile strength of 800 MPa (116,000 psi). Furthermore, from Figure 7.17(b), in order to achieve a tensile strength of 800 MPa, deformation of at least 13%CW is necessary. Finally, if we cold work the steel to 13%CW, then the ductility is 15%EL from Figure 7.17(c). Therefore, it **is possible** to meet both of these criteria by plastically deforming the steel.

7.D6 Let us first calculate the percent cold work and attendant yield strength and ductility if the drawing is carried out without interruption. From Equation (7.6)

$$\%CW = \frac{\pi\left(\frac{d_o}{2}\right)^2 - \pi\left(\frac{d_d}{2}\right)^2}{\pi\left(\frac{d_o}{2}\right)^2} \text{ x } 100$$

$$= \frac{\pi\left(\frac{10.2\ \text{mm}}{2}\right)^2 - \pi\left(\frac{7.6\ \text{mm}}{2}\right)^2}{\pi\left(\frac{10.2\ \text{mm}}{2}\right)^2} \times 100 = 44.5\%\text{CW}$$

At 44.5%CW, the brass will have a yield strength on the order of 420 MPa (61,000 psi), Figure 7.17(a), which is adequate; however, the ductility will be about 5%EL, Figure 7.17(c), which is insufficient.

Instead of performing the drawing in a single operation, let us initially draw some fraction of the total deformation, then anneal to recrystallize, and, finally, cold work the material a second time in order to achieve the final diameter, yield strength, and ductility.

Reference to Figure 7.17(a) indicates that 26%CW is necessary to give a yield strength of 380 MPa. Similarly, a maximum of 27.5%CW is possible for 15%EL [Figure 7.17(c)]. The average of these two values is 26.8%CW, which we will use in the calculations. If the final diameter after the first drawing is $\mathbf{d'_o}$, then

$$26.8\%\text{CW} = \frac{\pi\left(\frac{d'_o}{2}\right)^2 - \pi\left(\frac{7.6\ \text{mm}}{2}\right)^2}{\pi\left(\frac{d'_o}{2}\right)^2} \times 100$$

And, solving for $\mathbf{d'_o}$ yields $\mathbf{d'_o}$ = 9.4 mm (0.37 in.).

CHAPTER 8

FAILURE

PROBLEM SOLUTIONS

8.6 We may determine the critical stress required for the propagation of an internal crack in aluminum oxide using Equation (8.3); taking the value of 393 GPa (Table 12.5) as the modulus of elasticity, we get

$$\sigma_c = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

$$= \sqrt{\frac{(2)\left(393 \times 10^9 \text{ N/m}^2\right)(0.90 \text{ N/m})}{(\pi)\left(\frac{4 \times 10^{-4} \text{ m}}{2}\right)}} = 33.6 \times 10^6 \text{ N/m}^2 = 33.6 \text{ MPa}$$

8.8W This problem calls for us to calculate the normal $\boldsymbol{\sigma_x}$ and $\boldsymbol{\sigma_y}$ stresses in front on a surface crack of length 2.0 mm at various positions when a tensile stress of 100 MPa is applied. Substitution for **K** = $\boldsymbol{\sigma\sqrt{\pi a}}$ into Equations (8.9aW) and (8.9bW) leads to

$$\sigma_x = \sigma f_x(\theta)\sqrt{\frac{a}{2r}}$$

$$\sigma_y = \sigma f_y(\theta)\sqrt{\frac{a}{2r}}$$

where $\boldsymbol{f_x}(\boldsymbol{\theta})$ and $\boldsymbol{f_y}(\boldsymbol{\theta})$ are defined in the accompanying footnote 2. For $\boldsymbol{\theta}$ = 0°, $\boldsymbol{f_x}(\boldsymbol{\theta})$ = 1.0 and $\boldsymbol{f_y}(\boldsymbol{\theta})$ = 1.0, whereas for $\boldsymbol{\theta}$ = 45°, $\boldsymbol{f_x}(\boldsymbol{\theta})$ = 0.60 and $\boldsymbol{f_y}(\boldsymbol{\theta})$ = 1.25.

(a) For **r** = 0.1 mm and $\boldsymbol{\theta}$ = 0°,

$$\sigma_x = \sigma_y = \sigma(1.0)\sqrt{\frac{a}{2r}} = (100 \text{ MPa})\sqrt{\frac{2.0 \text{ mm}}{(2)(0.1 \text{ mm})}} = 316 \text{ MPa} \quad (45{,}800 \text{ psi})$$

(d) For **r** = 0.5 mm and **θ** = 45°,

$$\sigma_x = \sigma(0.6)\sqrt{\frac{a}{2r}} = (100 \text{ MPa})(0.6)\sqrt{\frac{2.0 \text{ mm}}{(2)(0.5 \text{ mm})}} = 84.8 \text{ MPa} \quad (12,300 \text{ psi})$$

$$\sigma_y = \sigma(1.25)\sqrt{\frac{a}{2r}} = (100 \text{ MPa})(1.25)\sqrt{\frac{2.0 \text{ mm}}{(2)(0.5 \text{ mm})}} = 177 \text{ MPa} \quad (25,600 \text{ psi})$$

8.10W (a) In this portion of the problem it is necessary to compute the stress at point **P** when the applied stress is 140 MPa (20,000 psi). In order to determine the stress concentration it is necessary to consult Figure 8.2cW. From the geometry of the specimen, **w**/**h** = (40 mm)/(20 mm) = 2.0; furthermore, the **r**/**h** ratio is (4 mm)/(20 mm) = 0.20. Using the **w**/**h** = 2.0 curve in Figure 8.2cW, the $\mathbf{K_t}$ value at **r**/**h** = 0.20 is 1.8. And since $\mathbf{K_t} = \frac{\boldsymbol{\sigma_m}}{\boldsymbol{\sigma_o}}$, then

$$\sigma_m = K_t\sigma_o = (1.8)(140 \text{ MPa}) = 252 \text{ MPa } (36,000 \text{ psi})$$

8.13W This problem calls for us to determine the value of **B**, the minimum component thickness for which the condition of plane strain is valid using Equation (8.14W), for the metal alloys listed in Table 8.1.

For the 2024-T3 aluminum alloy

$$B = 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2 = (2.5)\left(\frac{44 \text{ MPa}\sqrt{m}}{345 \text{ MPa}}\right)^2 = 0.0406 \text{ m} = 40.6 \text{ mm} \quad (1.60 \text{ in.})$$

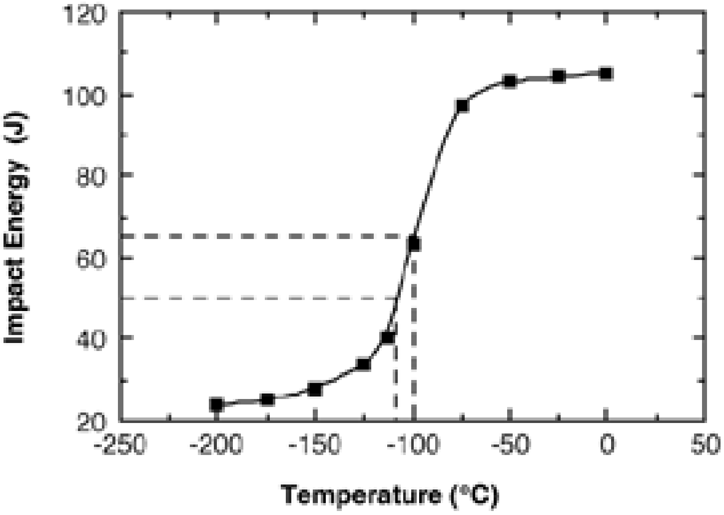
For the 4340 alloy steel tempered at 260°C

$$B = (2.5)\left(\frac{50 \text{ MPa}\sqrt{m}}{1640 \text{ MPa}}\right)^2 = 0.0023 \text{ m} = 2.3 \text{ mm} \quad (0.09 \text{ in.})$$

8.17 For this problem, we are given values of $\mathbf{K_{Ic}}$, **σ**, and **Y** for a large plate and are asked to determine the minimum length of a surface crack that will lead to fracture. All we need do is to solve for $\mathbf{a_c}$ using Equation (8.7); therefore

$$a_c = \frac{1}{\pi}\left(\frac{K_{Ic}}{Y\sigma}\right)^2 = \frac{1}{\pi}\left[\frac{82.4 \text{ MPa}\sqrt{\text{m}}}{(1)(345 \text{ MPa})}\right]^2 = 0.0182 \text{ m} = 18.2 \text{ mm} \quad (0.72 \text{ in.})$$

8.22 (a) The plot of impact energy versus temperature is shown below.



(b) The average of the maximum and minimum impact energies from the data is

$$\text{Average} = \frac{105 \text{ J} + 24 \text{ J}}{2} = 64.5 \text{ J}$$

As indicated on the plot by the one set of dashed lines, the ductile-to-brittle transition temperature according to this criterion is about -100°C.

(c) Also, as noted on the plot by the other set of dashed lines, the ductile-to-brittle transition temperature for an impact energy of 50 J is about -110°C.

8.27 We are asked to determine the fatigue life for a cylindrical 2014-T6 aluminum rod given its diameter (6.4 mm) and the maximum tensile and compressive loads (+5340 N and -5340 N, respectively). The first thing that is necessary is to calculate values of σ_{max} and σ_{min} using Equation (6.1). Thus

$$\sigma_{max} = \frac{F_{max}}{A_o} = \frac{F_{max}}{\pi\left(\frac{d_o}{2}\right)^2}$$

$$= \frac{5340\ \text{N}}{(\pi)\left(\dfrac{6.4\ \text{x}\ 10^{-3}\ \text{m}}{2}\right)^2} = 166\ \text{x}\ 10^6\ \text{N/m}^2 = 166\ \text{MPa} \quad (24{,}400\ \text{psi})$$

$$\sigma_{\text{min}} = \frac{F_{\text{min}}}{\pi\left(\dfrac{d_o}{2}\right)^2}$$

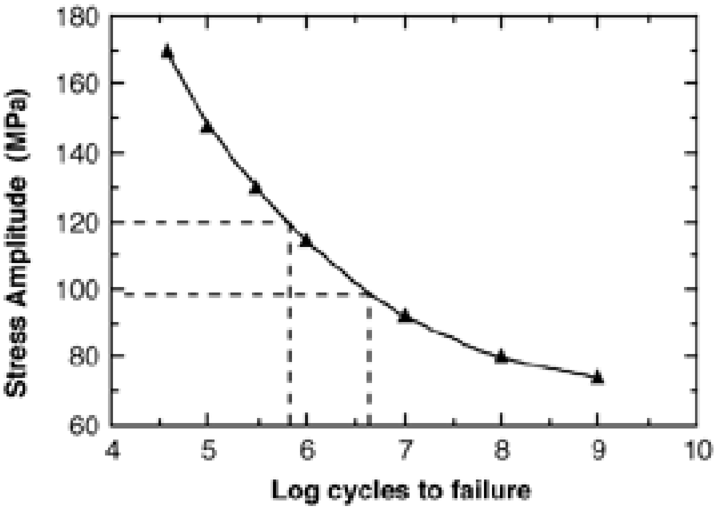
$$= \frac{-5340\ \text{N}}{(\pi)\left(\dfrac{6.4\ \text{x}\ 10^{-3}\ \text{m}}{2}\right)^2} = -166\ \text{x}\ 10^6\ \text{N/m}^2 = -166\ \text{MPa} \quad (-24{,}400\ \text{psi})$$

Now it becomes necessary to compute the stress amplitude using Equation (8.16) as

$$\sigma_a = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} = \frac{166\ \text{MPa} - (-166\ \text{MPa})}{2} = 166\ \text{MPa} \quad (24{,}400\ \text{psi})$$

From Figure 8.44, for the 2014-T6 aluminum, the number of cycles to failure at this stress amplitude is about 1 x 10^7 cycles.

8.29 (a) The fatigue data for this alloy are plotted below.



(b) As indicated by one set of dashed lines on the plot, the fatigue strength at 4 x 10^6 cycles [log (4 x 10^6) = 6.6] is about 100 MPa.

(c) As noted by the other set of dashed lines, the fatigue life for 120 MPa is about 6 x 10^5 cycles (i.e., the log of the lifetime is about 5.8).

8.30 We are asked to compute the maximum torsional stress amplitude possible at each of several fatigue lifetimes for the brass alloy the fatigue behavior of which is given in Problem 8.29. For each lifetime, first compute the number of cycles, and then read the corresponding fatigue strength from the above plot.

(a) Fatigue lifetime = (1 yr)(365 days/year)(24 h/day)(60 min/h)(1800 cycles/min) = 9.5 x 10^8 cycles. The stress amplitude corresponding to this lifetime is about 74 MPa.

(c) Fatigue lifetime = (24 h)(60 min/h)(1800 cycles/min) = 2.6 x 10^6 cycles. The stress amplitude corresponding to this lifetime is about 103 MPa.

8.43 This problem asks that we determine the total elongation of a low carbon-nickel alloy that is exposed to a tensile stress of 70 MPa (10,000 psi) at 427°C for 10,000 h; the instantaneous and primary creep elongations are 1.3 mm (0.05 in.).

From the 427°C line in Figure 8.29, the steady state creep rate, $\dot{\varepsilon}_s$, is about 0.035 %/1000 h (or 3.5 x 10^{-5} %/h) at 70 MPa. The steady state creep strain, ε_s, therefore, is just the product of $\dot{\varepsilon}_s$ and time as

$$\varepsilon_s = \dot{\varepsilon}_s \times (\text{time})$$

$$= \left(3.5 \times 10^{-5}\ \%/\text{h}\right)(10{,}000\ \text{h}) = 0.35\ \% = 3.5 \times 10^{-3}$$

Strain and elongation are related as in Equation (6.2); solving for the steady state elongation, Δl_s, leads to

$$\Delta l_s = l_o \varepsilon_s = (1015\ \text{mm})\left(3.5 \times 10^{-3}\right) = 3.6\ \text{mm} \quad (0.14\ \text{in.})$$

Finally, the total elongation is just the sum of this Δl_s and the total of both instantaneous and primary creep elongations [i.e., 1.3 mm (0.05 in.)]. Therefore, the total elongation is 4.9 mm (0.19 in.).

8.47 The slope of the line from a log $\dot{\varepsilon}_s$ versus log σ plot yields the value of **n** in Equation (8.19); that is

$$n = \frac{\Delta \log \dot{\varepsilon}_s}{\Delta \log \sigma}$$

We are asked to determine the values of **n** for the creep data at the three temperatures in Figure 8.29. This is accomplished by taking ratios of the differences between two log $\dot{\varepsilon}_s$ and log σ values. Thus for 427°C

$$n = \frac{\Delta \log \dot{\varepsilon}_s}{\Delta \log \sigma} = \frac{\log\left(10^{-1}\right) - \log\left(10^{-2}\right)}{\log(85\ \text{MPa}) - \log(55\ \text{MPa})} = 5.3$$

8.50 This problem gives $\dot{\varepsilon}_s$ values at two different temperatures and 140 MPa (20,000 psi), and the stress exponent **n** = 8.5, and asks that we determine the steady-state creep rate at a stress of 83 MPa (12,000 psi) and 1300 K.

Taking the natural logarithm of Equation (8.20) yields

$$\ln \dot{\varepsilon}_s = \ln K_2 + n \ln \sigma - \frac{Q_c}{RT}$$

With the given data there are two unknowns in this equation--namely $\mathbf{K_2}$ and $\mathbf{Q_c}$. Using the data provided in the problem we can set up two independent equations as follows:

$$\ln\left[6.6 \times 10^{-4}\,(\text{h})^{-1}\right] = \ln K_2 + (8.5)\ln(140\ \text{MPa}) - \frac{Q_c}{(8.31\ \text{J/mol-K})(1090\ \text{K})}$$

$$\ln\left[8.8 \times 10^{-2}\,(\text{h})^{-1}\right] = \ln K_2 + (8.5)\ln(140\ \text{MPa}) - \frac{Q_c}{(8.31\ \text{J/mol-K})(1200\ \text{K})}$$

Now, solving simultaneously for $\mathbf{K_2}$ and $\mathbf{Q_c}$ leads to $\mathbf{K_2}$ = 57.5 $(\text{h})^{-1}$ and $\mathbf{Q_c}$ = 483,500 J/mol. Thus, it is now possible to solve for $\dot{\varepsilon}_s$ at 83 MPa and 1300 K using Equation (8.20) as

$$\dot{\varepsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

$$= \left[57.5\ (h)^{-1}\right](83\ \text{MPa})^{8.5} \exp\left[-\frac{483{,}500\ \text{J/mol}}{(8.31\ \text{J/mol-K})(1300\ \text{K})}\right]$$

$$4.31 \times 10^{-2}\ (h)^{-1}$$

8.D1W This problem asks us to calculate the minimum $\mathbf{K_{Ic}}$ necessary to ensure that failure will not occur for a flat plate given the following: an expression from which **Y**(**a**/**W**) may be determined, the internal crack length, 2**a** (25 mm), the plate width, **W** (100 mm), and the value of $\boldsymbol{\sigma}$ (415 MPa). First we compute the value of **Y(a**/**W**) using Equation (8.12W), as follows:

$$Y(a/W) = \left[\frac{W}{\pi a} \tan \frac{\pi a}{W}\right]^{1/2}$$

$$= \left[\frac{100\ \text{mm}}{(\pi)(12.5\ \text{mm})} \tan \frac{(\pi)(12.5\ \text{mm})}{100\ \text{mm}}\right]^{1/2} = 1.027$$

Now, using Equation (8.13W) [or Equation (8.5)] it is possible to determine $\mathbf{K_{Ic}}$; thus

$$K_{Ic} = Y(a/W)\sigma\sqrt{\pi a}$$

$$= (1.027)(415\ \text{MPa})\sqrt{(\pi)\left(12.5 \times 10^{-3}\ \text{m}\right)} = 84.5\ \text{MPa}\sqrt{\text{m}} \quad (77.2\ \text{ksi}\sqrt{\text{in.}})$$

8.D7W We are asked in this problem to estimate the maximum tensile stress that will yield a fatigue life of 3.2 x 10^5 cycles, given values of $\mathbf{a_o}$, $\mathbf{a_c}$, **m**, **A**, and **Y**. Since **Y** is independent of crack length we may utilize Equation (8.29W) which, upon integration, takes the form

$$N_f = \frac{1}{A\pi^{m/2}(\Delta\sigma)^m Y^m} \int_{a_o}^{a_c} a^{-m/2}\, da$$

And for **m** = 4

$$N_f = \frac{1}{A\pi^2(\Delta\sigma)^4Y^4}\int_{a_o}^{a_c} a^{-2}\,da$$

$$= -\frac{1}{A\pi^2(\Delta\sigma)^4Y^4}\left[\frac{1}{a_c} - \frac{1}{a_o}\right]$$

Now, solving for **Δσ** from this expression yields

$$\Delta\sigma = \left[\frac{\dfrac{1}{a_o} - \dfrac{1}{a_c}}{N_f A\pi^2 Y^4}\right]^{1/4}$$

$$= \left[\frac{\dfrac{1}{2.5 \times 10^{-4}\ \text{m}} - \dfrac{1}{5 \times 10^{-3}\ \text{m}}}{\left(3.2 \times 10^5\ \text{cycles}\right)\left(5 \times 10^{-15}\right)(\pi)^2(2)^4}\right]^{1/4}$$

$$= 350\ \text{MPa}$$

This 350 MPa will be the maximum tensile stress since we can show that the minimum stress is a compressive one--when $\boldsymbol{\sigma_{min}}$ is negative, **Δσ** is taken to be $\boldsymbol{\sigma_{max}}$. If we take $\boldsymbol{\sigma_{max}}$ = 350 MPa, and since $\boldsymbol{\sigma_m}$ is stipulated in the problem to have a value of 25 MPa, then from Equation (8.14)

$$\sigma_{min} = 2\sigma_m - \sigma_{max} = 2(25\ \text{MPa}) - 350\ \text{MPa} = -300\ \text{MPa}$$

Therefore $\boldsymbol{\sigma_{min}}$ is negative and we are justified in taking $\boldsymbol{\sigma_{max}}$ to be 350 MPa.

8.D14W (a) This portion of the problem asks that we compute the maximum tensile load that may be applied to a spring constructed of a $\frac{1}{4}$ hard 304 stainless steel such that the total deflection is less than 5 mm; there are 10 coils in the spring, whereas, its center-to-center diameter is 15 mm, and the wire diameter is 2.0 mm. The total spring deflection $\boldsymbol{\delta_s}$ may be determined by combining Equations (8.32W) and (8.33W); solving for the load **F** from the combined equation leads to

$$F = \frac{\delta_s d^4 G}{8N_c D^3}$$

However, it is also necessary to determine the value of the shear modulus **G**. This is possible using Equation (6.9) and values of the modulus of elasticity (193 GPa) and Poisson's ratio (0.30) as taken from Tables B.2 and B.3 in Appendix B. Thus

$$G = \frac{E}{2(1 + \nu)}$$

$$= \frac{193 \text{ GPa}}{2(1 + 0.30)} = 74.2 \text{ GPa}$$

Substitution of this value and values of the other parameters into the above equation for **F** leads to

$$F = \frac{\left(5 \times 10^{-3} \text{ m}\right)\left(2 \times 10^{-3} \text{ m}\right)^4 \left(74.2 \times 10^9 \text{ N/m}^2\right)}{(8)(10 \text{ coils})\left(15 \times 10^{-3} \text{ m}\right)^3}$$

$$= 22.0 \text{ N} \ \ (5.1 \text{ lb}_f)$$

(b) We are now asked to compute the maximum tensile load that may be applied without any permanent deformation of the spring wire. This requires that we combine Equations (8.30W) and (8.31W), and then solve for **F**. However, it is first necessary to calculate the shear yield strength and substitute it for τ in Equation (8.30W). The problem statement stipulates that $\tau_y = 0.6\,\sigma_y$. From Table B.4 in Appendix B, we note that the tensile yield strength for this alloy in the 1/4 hardened state is 515 MPa; thus τ_y = 309 MPa. Now, solving for **F** as outlined above

$$F = \frac{\pi \tau_y d^3}{(1.6)(8)(D)\left(\frac{D}{d}\right)^{-0.140}}$$

$$= \frac{\pi\left(309 \times 10^6 \text{ N/m}^2\right)\left(2 \times 10^{-3} \text{ m}\right)^3}{(1.6)(8)\left(15 \times 10^{-3} \text{ m}\right)\left(\frac{15 \times 10^{-3} \text{ m}}{2 \times 10^{-3} \text{ m}}\right)^{-0.140}}$$

$$= 53.6 \text{ N} \ \ (12.5 \text{ lb}_f)$$

8.D21W We are asked in this problem to calculate the stress levels at which the rupture lifetime will be 1 year and 15 years when an 18-8 Mo stainless steel component is subjected to a temperature of 650°C (923 K). It first becomes necessary to calculate the value of the Larson-Miller parameter for each time. The values of $\mathbf{t_r}$ corresponding to 1 and 15 years are 8.76×10^3 h and 1.31×10^5 h, respectively. Hence, for a lifetime of 1 year

$$T\left(20 + \log t_r\right) = 923\left[20 + \log\left(8.76 \times 10^3\right)\right] = 22.10 \times 10^3$$

Using the curve shown in Figure 8.45, the stress value corresponding to the one-year lifetime is approximately 110 MPa (16,000 psi).

CHAPTER 9

PHASE DIAGRAMS

PROBLEM SOLUTIONS

9.5 This problem asks that we cite the phase or phases present for several alloys at specified temperatures.

(a) For an alloy composed of 15 wt% Sn-85 wt% Pb and at 100°C, from Figure 9.7, α and β phases are present, and

$$C_\alpha = 5 \text{ wt\% Sn-95 wt\% Pb}$$
$$C_\beta = 98 \text{ wt\% Sn-2 wt\% Pb}$$

(c) For an alloy composed of 85 wt% Ag-15 wt% Cu and at 800°C, from Figure 9.6, β and liquid phases are present, and

$$C_\beta = 92 \text{ wt\% Ag-8 wt\% Cu}$$
$$C_L = 77 \text{ wt\% Ag-23 wt\% Cu}$$

9.7 This problem asks that we determine the phase mass fractions for the alloys and temperatures in Problem 9.5.

(a)

$$W_\alpha = \frac{C_\beta - C_o}{C_\beta - C_\alpha} = \frac{98 - 15}{98 - 5} = 0.89$$

$$W_\beta = \frac{C_o - C_\alpha}{C_\beta - C_\alpha} = \frac{15 - 5}{98 - 5} = 0.11$$

(c)

$$W_\beta = \frac{C_o - C_L}{C_\beta - C_L} = \frac{85 - 77}{92 - 77} = 0.53$$

$$W_L = \frac{C_\beta - C_o}{C_\beta - C_L} = \frac{92 - 85}{92 - 77} = 0.47$$

9.9 This problem asks that we determine the phase volume fractions for the alloys and temperatures in Problems 9.5a, b, and d. This is accomplished by using the technique illustrated in Example Problem 9.3, and the results of Problems 9.5 and 9.7.

(a) This is a Sn-Pb alloy at 100°C, wherein

C_α = 5 wt% Sn-95 wt% Pb
C_β = 98 wt% Sn-2 wt% Pb
$W_\alpha = 0.89$
$W_\beta = 0.11$
$\rho_{Sn} = 7.29\ \text{g/cm}^3$
$\rho_{Pb} = 11.27\ \text{g/cm}^3$

Using these data it is first necessary to compute the densities of the **α** and **β** phases using Equation (4.10a). Thus

$$\rho_\alpha = \frac{100}{\dfrac{C_{Sn(\alpha)}}{\rho_{Sn}} + \dfrac{C_{Pb(\alpha)}}{\rho_{Pb}}}$$

$$= \frac{100}{\dfrac{5}{7.29\ \text{g/cm}^3} + \dfrac{95}{11.27\ \text{g/cm}^3}} = 10.97\ \text{g/cm}^3$$

$$\rho_\beta = \frac{100}{\dfrac{C_{Sn(\beta)}}{\rho_{Sn}} + \dfrac{C_{Pb(\beta)}}{\rho_{Pb}}}$$

$$= \frac{100}{\dfrac{98}{7.29\ \text{g/cm}^3} + \dfrac{2}{11.27\ \text{g/cm}^3}} = 7.34\ \text{g/cm}^3$$

Now we may determine the $\mathbf{V}_\alpha$ and $\mathbf{V}_\beta$ values using Equation 9.6. Thus,

$$V_\alpha = \frac{\dfrac{W_\alpha}{\rho_\alpha}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_\beta}{\rho_\beta}}$$

$$= \frac{\dfrac{0.89}{10.97\ \text{g/cm}^3}}{\dfrac{0.89}{10.97\ \text{g/cm}^3} + \dfrac{0.11}{7.34\ \text{g/cm}^3}} = 0.84$$

$$V_\beta = \frac{\dfrac{W_\beta}{\rho_\beta}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_\beta}{\rho_\beta}}$$

$$= \frac{\dfrac{0.11}{7.34\ \text{g/cm}^3}}{\dfrac{0.89}{10.97\ \text{g/cm}^3} + \dfrac{0.11}{7.34\ \text{g/cm}^3}} = 0.16$$

9.12 (a) We are asked to determine how much sugar will dissolve in 1000 g of water at 80°C. From the solubility limit curve in Figure 9.1, at 80°C the maximum concentration of sugar in the syrup is about 74 wt%. It is now possible to calculate the mass of sugar using Equation (4.3) as

$$C_{\text{sugar}}(\text{wt\%}) = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + m_{\text{water}}} \times 100$$

$$74\ \text{wt\%} = \frac{m_{\text{sugar}}}{m_{\text{sugar}} + 1000\ \text{g}} \times 100$$

Solving for $\mathbf{m}_{\textbf{sugar}}$ yields $\mathbf{m}_{\textbf{sugar}}$ = 2846 g

(b) Again using this same plot, at 20°C the solubility limit (or the concentration of the saturated solution) is about 64 wt% sugar.

(c) The mass of sugar in this saturated solution at 20°C ($\mathbf{m_{sugar}'}$) may also be calculated using Equation (4.3) as follows:

$$64 \text{ wt\%} = \frac{m_{sugar}'}{m_{sugar}' + 1000 \text{ g}} \times 100$$

which yields a value for $\mathbf{m_{sugar}'}$ of 1778 g. Subtracting the latter from the former of these sugar concentrations yields the amount of sugar that precipitated out of the solution upon cooling $\mathbf{m_{sugar}''}$; that is

$$m_{sugar}'' = m_{sugar} - m_{sugar}' = 2846 \text{ g} - 1778 \text{ g} = 1068 \text{ g}$$

9.21 Upon cooling a 50 wt% Ni-50 wt% Cu alloy from 1400°C and utilizing Figure 9.2a:

(a) The first solid phase forms at the temperature at which a vertical line at this composition intersects the **L**-(α + **L**) phase boundary--i.e., at about 1320°C;

(b) The composition of this solid phase corresponds to the intersection with the **L**-(α + **L**) phase boundary, of a tie line constructed across the α + **L** phase region at 1320°C--i.e., $\mathbf{C_\alpha}$ = 62 wt% Ni-38 wt% Cu;

(c) Complete solidification of the alloy occurs at the intersection of this same vertical line at 50 wt% Ni with the (α + **L**)-α phase boundary--i.e., at about 1270°C;

(d) The composition of the last liquid phase remaining prior to complete solidification corresponds to the intersection with the **L-**(α + **L**) boundary, of the tie line constructed across the α + **L** phase region at 1270°C--i.e., $\mathbf{C_L}$ is about 37 wt% Ni-63 wt% Cu.

9.24 (a) We are given that the mass fractions of α and liquid phases are both 0.5 for a 40 wt% Pb-60 wt% Mg alloy and asked to estimate the temperature of the alloy. Using the appropriate phase diagram, Figure 9.18, by trial and error with a ruler, a tie line within the α + **L** phase region that is divided in half for an alloy of this composition exists at about 540°C.

(b) We are now asked to determine the compositions of the two phases. This is accomplished by noting the intersections of this tie line with both the solidus and liquidus lines. From these intersections, $\mathbf{C_\alpha}$ = 26 wt% Pb, and $\mathbf{C_L}$ = 54 wt% Pb.

9.27 Yes, it is possible to have a Cu-Ag alloy of composition 20 wt% Ag-80 wt% Cu which consists of mass fractions $\mathbf{W_\alpha}$ = 0.80 and $\mathbf{W_L}$ = 0.20. Using the appropriate phase diagram, Figure 9.6, by trial and error with a ruler, the tie-line segments within the $\alpha + \mathbf{L}$ phase region are proportioned such that

$$W_\alpha = 0.8 = \frac{C_L - C_o}{C_L - C_\alpha}$$

for $\mathbf{C_o}$ = 20 wt% Ag. This occurs at about 800°C.

9.34 This problem asks that we determine the composition of a Cu-Ag alloy at 775°C given that $\mathbf{W_{\alpha'}}$ = 0.73 and $\mathbf{W_{eutectic}}$ = 0.27. Since there is a primary α microconstituent present, we know that the alloy composition, $\mathbf{C_o}$ is between 8.0 and 71.9 wt% Ag (Figure 9.6). Furthermore, this figure also indicates that $\mathbf{C_\alpha}$ = 8.0 wt% Ag and $\mathbf{C_{eutectic}}$ = 71.9 wt% Ag. Applying the appropriate lever rule expression for $\mathbf{W_{\alpha'}}$

$$W_{\alpha'} = \frac{C_{eutectic} - C_o}{C_{eutectic} - C_\alpha} = \frac{71.9 - C_o}{71.9 - 8.0} = 0.73$$

and solving for $\mathbf{C_o}$ yields $\mathbf{C_o}$ = 25.2 wt% Ag.

9.44W We are asked to specify the value of **F** for Gibbs phase rule at points A, B, and C on the pressure-temperature diagram for H_2O. Gibbs phase rule in general form is

$$P + F = C + N$$

For this system, the number of components, **C**, is 1, whereas **N**, the number of noncompositional variables, is 2--viz. temperature and pressure. Thus, the phase rule now becomes

$$P + F = 1 + 2 = 3$$

Or

$$F = 3 - P$$

where **P** is the number of phases present at equilibrium.

At point A, only a single (liquid) phase is present (i.e., **P** = 1), or

$$F = 3 - P = 3 - 1 = 2$$

which means that both temperature and pressure are necessary to define the system.

9.51 This problem asks that we compute the carbon concentration of an iron-carbon alloy for which the fraction of total ferrite is 0.94. Application of the lever rule [of the form of Equation (9.12)] yields

$$W_\alpha = 0.94 = \frac{C_{Fe_3C} - C'_o}{C_{Fe_3C} - C_\alpha} = \frac{6.70 - C'_o}{6.70 - 0.022}$$

and solving for $\mathbf{C'_o}$

$$C'_o = 0.42 \text{ wt\% C}$$

9.56 This problem asks that we determine the carbon concentration in an iron-carbon alloy, given the mass fractions of proeutectoid ferrite and pearlite (0.286 and 0.714, respectively). From Equation (9.18)

$$W_p = 0.714 = \frac{C'_o - 0.022}{0.74}$$

which yields $\mathbf{C'_o} = 0.55$ wt% C.

9.61 This problem asks if it is possible to have an iron-carbon alloy for which $\mathbf{W_{Fe_3C}} = 0.057$ and $\mathbf{W_{\alpha'}} =$ 0.36. In order to make this determination, it is necessary to set up lever rule expressions for these two mass fractions in terms of the alloy composition, then to solve for the alloy composition of each; if both alloy composition values are equal, then such an alloy is possible. The expression for the mass fraction of total cementite is

$$W_{Fe_3C} = \frac{C_o - C_\alpha}{C_{Fe_3C} - C_\alpha} = \frac{C_o - 0.022}{6.70 - 0.022} = 0.057$$

Solving for this $\mathbf{C_o}$ yields $\mathbf{C_o}$ = 0.40 wt% C. Now for $\mathbf{W_{\alpha'}}$ we utilize Equation (9.19) as

$$W_{\alpha'} = \frac{0.76 - C_o'}{0.74} = 0.36$$

This expression leads to $\mathbf{C_o'}$ = 0.49 wt% C. And, since $\mathbf{C_o}$ and $\mathbf{C_o'}$ are different this alloy is **not** possible.

9.67 This problem asks that we determine the approximate Brinell hardness of a 99.8 wt% Fe-0.2 wt% C alloy. First, we compute the mass fractions of pearlite and proeutectoid ferrite using Equations (9.18) and (9.19), as

$$W_p = \frac{C_o' - 0.022}{0.74} = \frac{0.20 - 0.022}{0.74} = 0.24$$

$$W_{\alpha'} = \frac{0.76 - C_o'}{0.74} = \frac{0.76 - 0.20}{0.74} = 0.76$$

Now, we compute the Brinell hardness of the alloy as

$$HB_{alloy} = HB_{\alpha'}W_{\alpha'} + HB_p W_p$$

$$= (80)(0.76) + (280)(0.24) = 128$$

9.70 We are asked to consider a steel alloy of composition 93.8 wt% Fe, 6.0 wt% Ni, and 0.2 wt% C.

(a) From Figure 9.31, the eutectoid temperature for 6 wt% Ni is approximately 650°C (1200°F).

(b) From Figure 9.32, the eutectoid composition is approximately 0.62 wt% C. Since the carbon concentration in the alloy (0.2 wt%) is less than the eutectoid, the proeutectoid phase is ferrite.

(c) Assume that the α-(α + Fe_3C) phase boundary is at a negligible carbon concentration. Modifying Equation (9.19) leads to

$$W_{\alpha'} = \frac{0.62 - C_o'}{0.62 - 0} = \frac{0.62 - 0.20}{0.62} = 0.68$$

Likewise, using a modified Equation (9.18)

$$W_p = \frac{C_o' - 0}{0.62 - 0} = \frac{0.20}{0.62} = 0.32$$

CHAPTER 10

PHASE TRANSFORMATIONS IN METALS

PROBLEM SOLUTIONS

10.4 This problem gives us the value of **y** (0.30) at some time **t** (100 min), and also the value of **n** (5.0) for the recrystallization of an alloy at some temperature, and then asks that we determine the rate of recrystallization at this same temperature. It is first necessary to calculate the value of **k** in Equation (10.1) as

$$k = -\frac{\ln(1 - y)}{t^n}$$

$$= -\frac{\ln(1 - 0.3)}{(100 \text{ min})^5} = 3.57 \times 10^{-11}$$

At this point we want to compute $\mathbf{t_{0.5}}$, the value of **t** for **y** = 0.5, also using Equation (10.1). Thus

$$t_{0.5} = \left[-\frac{\ln(1 - 0.5)}{k}\right]^{1/n}$$

$$= \left[-\frac{\ln(1 - 0.5)}{3.57 \text{ x } 10^{-11}}\right]^{1/5} = 114.2 \text{ min}$$

And, therefore, from Equation (10.2), the rate is just

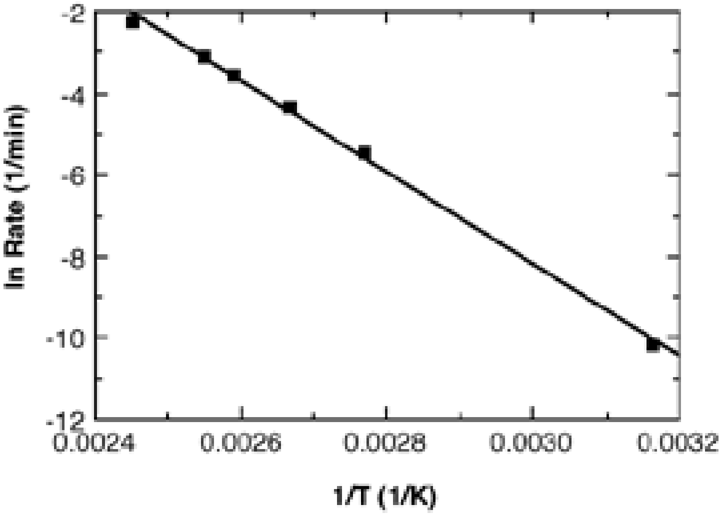
$$\text{rate} = \frac{1}{t_{0.5}} = \frac{1}{114.2 \text{ min}} = 8.76 \text{ x } 10^{-3} \text{ (min)}^{-1}$$

10.7 This problem asks us to consider the percent recrystallized versus logarithm of time curves for copper shown in Figure 10.2.

(a) The rates at the different temperatures are determined using Equation (10.2), which rates are tabulated below:

Temperature (°C)	Rate $(\text{min})^{-1}$
135	0.105
119	4.4×10^{-2}
113	2.9×10^{-2}
102	1.25×10^{-2}
88	4.2×10^{-3}
43	3.8×10^{-5}

(b) These data are plotted below.



The activation energy, **Q**, is related to the slope of the line drawn through the data points as

$$Q = -\,\text{Slope}\,(R)$$

where **R** is the gas constant. The slope of this line is -1.126×10^{4} K, and thus

$$Q = -\left(-1.126 \times 10^{4}\ \text{K}\right)(8.31\ \text{J/mol-K})$$

$$= 93{,}600\ \text{J/mol}$$

(c) At room temperature (20°C), $1/\mathbf{T} = 3.41 \times 10^{-3}\ \text{K}^{-1}$. Extrapolation of the data in the plot to this $1/\mathbf{T}$ value gives

$$\ln(\text{rate}) \cong -12.8$$

which leads to

$$\text{rate} \cong \exp(-12.8) = 2.76 \times 10^{-6}\ (\text{min})^{-1}$$

But since

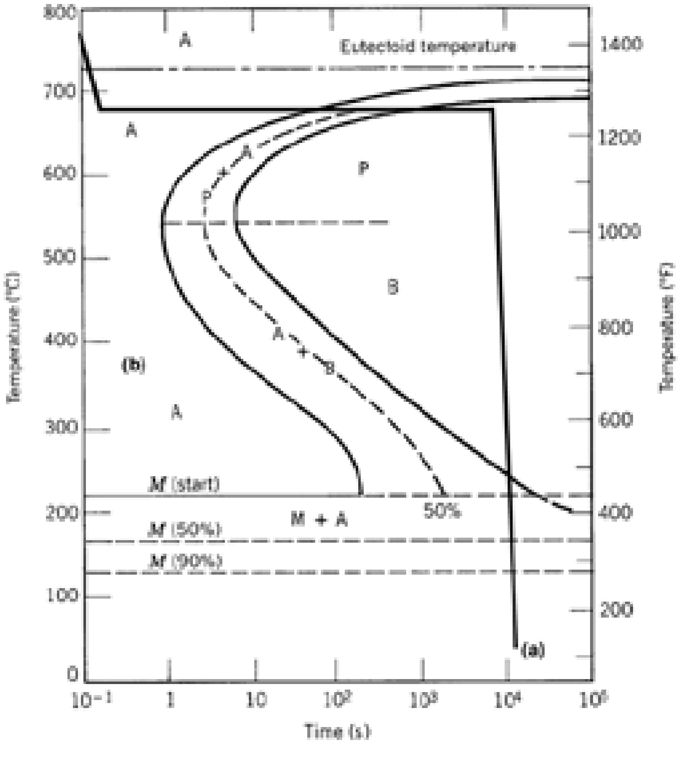
$$\text{rate} = \frac{1}{t_{0.5}}$$

then

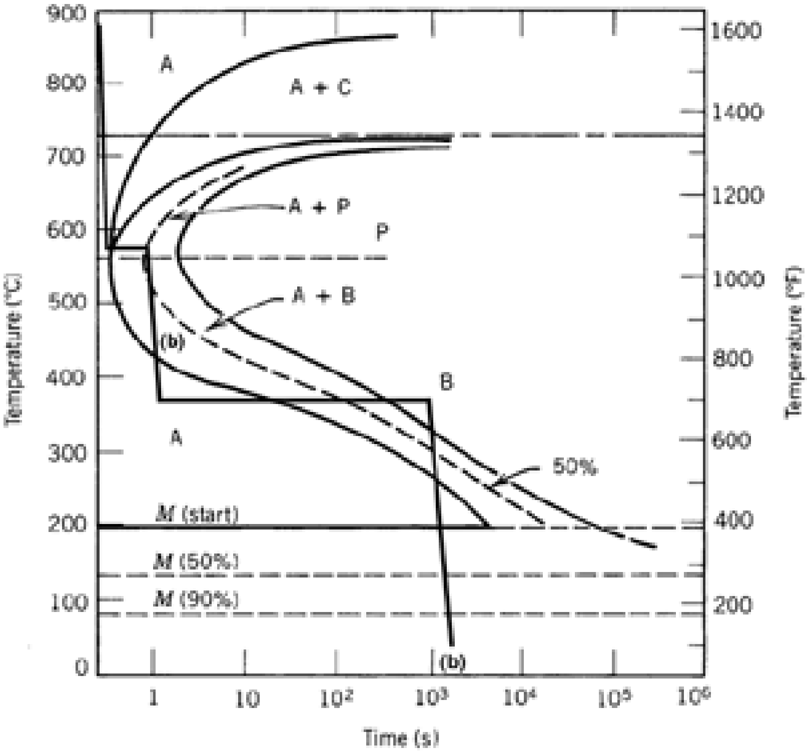
$$t_{0.5} = \frac{1}{\text{rate}} = \frac{1}{2.76 \times 10^{-6}\ (\text{min})^{-1}}$$

$$= 3.62 \times 10^{5}\ \text{min} = 250\ \text{days}$$

10.15 Below is shown the isothermal transformation diagram for a eutectoid iron-carbon alloy, with a time-temperature path that will yield (a) 100% coarse pearlite.



10.18 Below is shown an isothermal transformation diagram for a 1.13 wt% C iron-carbon alloy, with a time-temperature path that will produce (b) 50% fine pearlite and 50% bainite.



10.34 This problem asks for Rockwell hardness values for specimens of an iron-carbon alloy of eutectoid composition that have been subjected to some of the heat treatments described in Problem 10.14.

(b) The microstructural product of this heat treatment is 100% spheroidite. According to Figure 10.21(a) the hardness of a 0.76 wt% C alloy with spheroidite is about 87 HRB (180 HB).

(g) The microstructural product of this heat treatment is 100% fine pearlite. According to Figure 10.21(a), the hardness of a 0.76 wt% C alloy consisting of fine pearlite is about 27 HRC (270 HB).

10.37 For this problem we are asked to describe isothermal heat treatments required to yield specimens having several Brinell hardnesses.

(b) This portion of the problem asks for a hardness of 220 HB. According to Figure 10.21(a), for an alloy of this composition to have this hardness, the microstructure would have to be intermediate between coarse and fine pearlite—that is, medium pearlite. Thus, an isothermal heat treatment is necessary at a temperature in between those at which fine and coarse pearlites form—for example, about 630°C. At this temperature, an isothermal heat treatment for at least 25 s is required.

10.D1 This problem inquires as to the possibility of producing an iron-carbon alloy of eutectoid composition that has a minimum hardness of 200 HB and a minimum ductility of 25%RA. If the alloy is possible, then the continuous cooling heat treatment is to be stipulated.

According to Figures 10.21(a) and (b), the following is a tabulation of Brinell hardnesses and percents reduction of area for fine and coarse pearlites and spheroidite for a 0.76 wt% C alloy.

Microstructure	HB	%RA
Fine pearlite	270	22
Coarse pearlite	205	29
Spheroidite	180	68

Therefore, coarse pearlite meets both of these criteria. The continuous cooling heat treatment which will produce coarse pearlite for an alloy of eutectoid composition is indicated in Figure 10.18. The cooling rate would need to be considerably less than 35°C/s, probably on the order of 0.1°C/s.

CHAPTER 11

APPLICATIONS AND PROCESSING OF METAL ALLOYS

PROBLEM SOLUTIONS

11.5 We are asked to compute the volume percent graphite in a 3.5 wt% C cast iron. It first becomes necessary to compute mass fractions using the lever rule. From the iron-carbon phase diagram (Figure 11.2), the tie-line in the α and graphite phase field extends from essentially 0 wt% C to 100 wt% C. Thus, for a 3.5 wt% C cast iron

$$W_\alpha = \frac{C_{Gr} - C_o}{C_{Gr} - C_\alpha} = \frac{100 - 3.5}{100 - 0} = 0.965$$

$$W_{Gr} = \frac{C_o - C_\alpha}{C_{Gr} - C_\alpha} = \frac{3.5 - 0}{100 - 0} = 0.035$$

Conversion from weight fraction to volume fraction of graphite is possible using Equation (9.6a) as

$$V_{Gr} = \frac{\dfrac{W_{Gr}}{\rho_{Gr}}}{\dfrac{W_\alpha}{\rho_\alpha} + \dfrac{W_{Gr}}{\rho_{Gr}}}$$

$$= \frac{\dfrac{0.035}{2.3\ \text{g/cm}^3}}{\dfrac{0.965}{7.9\ \text{g/cm}^3} + \dfrac{0.035}{2.3\ \text{g/cm}^3}}$$

$$= 0.111 \text{ or } 11.1 \text{ vol\%}$$

11.D14 We are to determine, for a cylindrical piece of 8660 steel, the minimum allowable diameter possible in order yield a surface hardness of 58 HRC, when the quenching is carried out in moderately agitated oil.

From Figure 11.14, the equivalent distance from the quenched end of an 8660 steel to give a hardness of 58 HRC is about 18 mm (3/4 in.). Thus, the quenching rate at the surface of the specimen should correspond to this equivalent distance. Using Figure 11.16(b), the surface specimen curve takes on a value of 18 mm equivalent distance at a diameter of about 95 mm (3.75 in.).

CHAPTER 12

STRUCTURES AND PROPERTIES OF CERAMICS

PROBLEM SOLUTIONS

12.5 This problem calls for us to predict crystal structures for several ceramic materials on the basis of ionic charge and ionic radii.

(a) For CsI, from Table 12.3

$$\frac{r_{Cs^+}}{r_{I^-}} = \frac{0.170 \text{ nm}}{0.220 \text{ nm}} = 0.773$$

Now, from Table 12.2, the coordination number for each cation (Cs^+) is eight, and, using Table 12.4, the predicted crystal structure is cesium chloride.

(c) For KI, from Table 12.3

$$\frac{r_{K^+}}{r_{I^-}} = \frac{0.138 \text{ nm}}{0.220 \text{ nm}} = 0.627$$

The coordination number is six (Table 12.2), and the predicted crystal structure is sodium chloride (Table 12.4).

12.9 This question is concerned with the zinc blende crystal structure in terms of close-packed planes of anions.

(a) The stacking sequence of close-packed planes of anions for the zinc blende crystal structure will be the same as FCC (and not HCP) because the anion packing is FCC (Table 12.4).

(b) The cations will fill tetrahedral positions since the coordination number for cations is four (Table 12.4).

(c) Only one-half of the tetrahedral positions will be occupied because there are two tetrahedral sites per anion, and yet only one cation per anion.

12.19 (a) We are asked to compute the theoretical density of CsCl. Modifying the result of Problem 3.4, we get

$$a = \frac{2r_{Cs^+} + 2r_{Cl^-}}{\sqrt{3}} = \frac{2(0.170\ \text{nm}) + 2(0.181\ \text{nm})}{\sqrt{3}}$$

$$= 0.405\ \text{nm} = 4.05 \times 10^{-8}\ \text{cm}$$

From Equation (12.1)

$$\rho = \frac{n'\left(A_{Cs} + A_{Cl}\right)}{V_C N_A} = \frac{n'\left(A_{Cs} + A_{Cl}\right)}{a^3 N_A}$$

For the CsCl crystal structure, **n'** = 1 formula unit/unit cell, and thus

$$\rho = \frac{(1\ \text{formula unit/unit cell})(132.91\ \text{g/mol} + 35.45\ \text{g/mol})}{\left[\left(4.05 \times 10^{-8}\ \text{cm}\right)^3/\text{unit cell}\right]\left(6.023 \times 10^{23}\ \text{formula units/mol}\right)}$$

$$= 4.20\ \text{g/cm}^3$$

12.25 We are asked in this problem to compute the atomic packing factor for the CsCl crystal structure. This requires that we take the ratio of the sphere volume within the unit cell and the total unit cell volume. From Figure 12.3 there is the equivalent of one Cs and one Cl ion per unit cell; the ionic radii of these two ions are 0.170 nm and 0.181 nm, respectively (Table 12.3). Thus, the sphere volume, $\mathbf{V_S}$, is just

$$V_S = \frac{4}{3}(\pi)\left[(0.170\ \text{nm})^3 + (0.181\ \text{nm})^3\right] = 0.0454\ \text{nm}^3$$

Using a modified form of the result of Problem 3.4, for CsCl we may express the unit cell edge length, **a**, in terms of the atomic radii as

$$a = \frac{2r_{Cs^+} + 2r_{Cl^-}}{\sqrt{3}} = \frac{2(0.170\ \text{nm}) + 2(0.181\ \text{nm})}{\sqrt{3}}$$

$$= 0.405 \text{ nm}$$

Since $\mathbf{V_C} = \mathbf{a}^3$

$$V_C = (0.405 \text{ nm})^3 = 0.0664 \text{ nm}^3$$

And, finally the atomic packing factor is just

$$\text{APF} = \frac{V_S}{V_C} = \frac{0.0454 \text{ nm}^3}{0.0664 \text{ nm}^3} = 0.684$$

12.33 (a) For Li^+ substituting for Ca^{2+} in CaO, oxygen vacancies would be created. For each Li^+ substituting for Ca^{2+}, one positive charge is removed; in order to maintain charge neutrality, a single negative charge may be removed. Negative charges are eliminated by creating oxygen vacancies, and for every two Li^+ ions added, a single oxygen vacancy is formed.

12.38 We are asked for the critical crack tip radius for an Al_2O_3 material. From Equation (8.1)

$$\sigma_m = 2\sigma_o\left(\frac{a}{\rho_t}\right)^{1/2}$$

Fracture will occur when $\boldsymbol{\sigma_m}$ reaches the fracture strength of the material, which is given as **E**/10; thus

$$\frac{E}{10} = 2\sigma_o\left(\frac{a}{\rho_t}\right)^{1/2}$$

Or, solving for $\boldsymbol{\rho_t}$

$$\rho_t = \frac{400\, a\sigma_o^2}{E^2}$$

From Table 12.5, **E** = 393 GPa, and thus,

$$\rho_t = \frac{(400)\left(2 \times 10^{-3} \text{ mm}\right)(275 \text{ MPa})^2}{\left(393 \times 10^3 \text{ MPa}\right)^2}$$

$$= 3.9 \times 10^{-7} \text{ mm} = 0.39 \text{ nm}$$

12.42 For this problem, the load is given at which a circular specimen of aluminum oxide fractures when subjected to a three-point bending test; we are then are asked to determine the load at which a specimen of the same material having a square cross-section fractures. It is first necessary to compute the flexural strength of the alumina using Equation (12.3b), and then, using this value, we may calculate the value of $\mathbf{F_f}$ in Equation (12.3a). From Equation (12.3b)

$$\sigma_{fs} = \frac{F_f L}{\pi R^3}$$

$$= \frac{(3000 \text{ N})\left(40 \times 10^{-3} \text{ m}\right)}{(\pi)\left(5.0 \times 10^{-3} \text{ m}\right)^3} = 306 \times 10^6 \text{ N/m}^2 = 306 \text{ MPa} \quad (42{,}970 \text{ psi})$$

Now, solving for $\mathbf{F_f}$ from Equation (12.3a), realizing that **b** = **d** = 12 mm, yields

$$F_f = \frac{2\sigma_{fs} d^3}{3L}$$

$$= \frac{(2)\left(306 \times 10^6 \text{ N/m}^2\right)\left(15 \times 10^{-3} \text{m}\right)^3}{(3)\left(40 \times 10^{-3} \text{ m}\right)} = 17{,}200 \text{ N} \quad (3870 \text{ lb}_f)$$

12.47 (a) This part of the problem asks us to determine the flexural strength of nonporous MgO assuming that the value of **n** in Equation (12.6) is 3.75. Taking natural logarithms of both sides of Equation (12.6) yields

$$\ln \sigma_{fs} = \ln \sigma_o - nP$$

In Table 12.5 it is noted that for **P** = 0.05, $\boldsymbol{\sigma_{fs}}$ = 105 MPa. For the nonporous material **P** = 0 and, ln $\boldsymbol{\sigma_o}$ = ln $\boldsymbol{\sigma_{fs}}$. Solving for ln $\boldsymbol{\sigma_o}$ from the above equation and using these data gives

$$\ln \sigma_o = \ln \sigma_{fs} + nP$$

$$= \ln (105 \text{ MPa}) + (3.75)(0.05) = 4.841$$

or $\boldsymbol{\sigma_o} = e^{4.841}$ = 127 MPa (18,100 psi)

(b) Now we are asked to compute the volume percent porosity to yield a $\boldsymbol{\sigma_{fs}}$ of 74 MPa (10,700 psi). Taking the natural logarithm of Equation (12.6) and solving for **P** leads to

$$P = \frac{\ln \sigma_o - \ln \sigma_{fs}}{n}$$

$$= \frac{\ln (127 \text{ MPa}) - \ln (74 \text{ MPa})}{3.75}$$

$$= 0.144 \text{ or } 14.4 \text{ vol\%}$$

CHAPTER 13

APPLICATIONS AND PROCESSING OF CERAMICS

PROBLEM SOLUTIONS

13.5 (a) From Figure 12.25, the maximum temperature without a liquid phase corresponds to the temperature at the MgO(ss)-[MgO(ss) + Liquid] boundary at this composition, which is approximately 2240°C (4060°F).

13.7 This problem calls for us to compute the mass fractions of liquid for two fireclay refractory materials at 1600°C. In order to solve this problem it is necessary that we use the SiO_2-Al_2O_3 phase diagram (Figure 12.27). The mass fraction of liquid, $\mathbf{W_L}$, as determined using the lever rule and tie line at 1600°C, is just

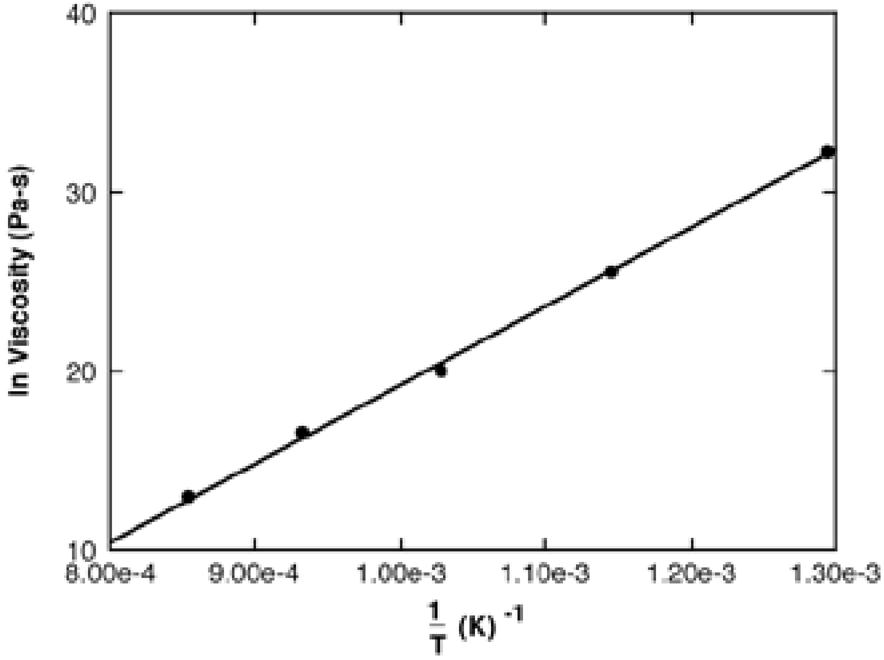
$$W_L = \frac{C_{mullite} - C_o}{C_{mullite} - C_L}$$

where $\mathbf{C_{mullite}}$ = 72 wt% Al_2O_3 and $\mathbf{C_L}$ = 8 wt% Al_2O_3, as determined using the tie-line; also, $\mathbf{C_o}$ is the composition (in weight percent Al_2O_3) of the refractory material.

(a) For the 25 wt% Al_2O_3- 75 wt% SiO_2 composition, $\mathbf{C_o}$ = 25 wt% Al_2O_3, and

$$W_L = \frac{72 - 25}{72 - 8} = 0.73$$

13.16 (a) Below is shown the logarithm viscosity versus reciprocal of temperature plot for the borosilicate glass, using the data in Figure 13.6.



(b) The activation energy, $\mathbf{Q_{vis}}$, may be computed from this plot according to

$$Q_{vis} = R\left[\frac{\Delta \ln \eta}{\Delta\left(\frac{1}{T}\right)}\right]$$

where **R** is the gas constant, and Δln **η**/Δ(1/**T**) is the slope of the line that has been constructed. The value of this slope is 4.36×10^4 K. Therefore,

$$Q_{vis} = (8.31 \text{ J/mol-K})\left(4.36 \times 10^4 \text{ K}\right)$$

$$= 362{,}000 \text{ J/mol}$$

CHAPTER 14

POLYMER STRUCTURES

PROBLEM SOLUTIONS

14.4 We are asked to compute the number-average degree of polymerization for polypropylene, given that the number-average molecular weight is 1,000,000 g/mol. The mer molecular weight of polypropylene is just

$$m = 3(A_C) + 6(A_H)$$

$$= (3)(12.01\text{ g/mol}) + (6)(1.008\text{ g/mol}) = 42.08\text{ g/mol}$$

If we let $\mathbf{n_n}$ represent the number-average degree of polymerization, then from Equation (14.4a)

$$n_n = \frac{\overline{M}_n}{m} = \frac{10^6\text{ g/mol}}{42.08\text{ g/mol}} = 23{,}700$$

14.6 (a) From the tabulated data, we are asked to compute $\mathbf{\overline{M}_n}$, the number-average molecular weight. This is carried out below.

Molecular wt Range	Mean M_i	x_i	x_iM_i
8,000-16,000	12,000	0.05	600
16,000-24,000	20,000	0.16	3200
24,000-32,000	28,000	0.24	6720
32,000-40,000	36,000	0.28	10,080
40,000-48,000	44,000	0.20	8800
48,000-56,000	52,000	0.07	3640

$$\overline{M}_n = \sum x_i M_i = 33{,}040\text{ g/mol}$$

(c) Now we are asked to compute $\mathbf{n_n}$ (the number-average degree of polymerization), using the Equation (14.4a). For polypropylene,

$$m = 3(A_C) + 6(A_H)$$

$$= (3)(12.01 \text{ g/mol}) + (6)(1.008 \text{ g/mol}) = 42.08 \text{ g/mol}$$

And

$$n_n = \frac{\overline{M}_n}{m} = \frac{33,040 \text{ g/mol}}{42.08 \text{ g/mol}} = 785$$

14.11 This problem first of all asks for us to calculate, using Equation (14.11), the average total chain length, **L**, for a linear polyethylene polymer having a number-average molecular weight of 300,000 g/mol. It is necessary to calculate the number-average degree of polymerization, $\mathbf{n_n}$, using Equation (14.4a). For polyethylene, from Table 14.3, each mer unit has two carbons and four hydrogens. Thus,

$$m = 2(A_C) + 4(A_H)$$

$$= (2)(12.01 \text{ g/mol}) + (4)(1.008 \text{ g/mol}) = 28.05 \text{ g/mol}$$

and

$$n_n = \frac{\overline{M}_n}{m} = \frac{300,000 \text{ g/mol}}{28.05 \text{ g/mol}} = 10,695$$

which is the number of mer units along an average chain. Since there are two carbon atoms per mer unit, there are two C--C chain bonds per mer, which means that the total number of chain bonds in the molecule, **N**, is just (2)(10,695) = 21,390 bonds. Furthermore, assume that for single carbon-carbon bonds, **d** = 0.154 nm and **θ** = 109° (Section 14.4); therefore, from Equation (14.11)

$$L = Nd \sin\left(\frac{\theta}{2}\right)$$

$$= (21,390)(0.154 \text{ nm}) \left[\sin\left(\frac{109°}{2}\right)\right] = 2682 \text{ nm}$$

It is now possible to calculate the average chain end-to-end distance, **r**, using Equation (14.12) as

$$r = d\sqrt{N} = (0.154 \text{ nm})\sqrt{21,390} = 22.5 \text{ nm}$$

14.28 Given that polyethylene has an orthorhombic unit cell with two equivalent mer units, we are asked to compute the density of totally crystalline polyethylene. In order to solve this problem it is necessary to employ Equation (3.5), in which **n** represents the number of mer units within the unit cell (**n** = 2), and **A** is the mer molecular weight, which for polyethylene is just

$$A = 2(A_C) + 4(A_H)$$

$$= (2)(12.01 \text{ g/mol}) + (4)(1.008 \text{ g/mol}) = 28.05 \text{ g/mol}$$

Also, $\mathbf{V_C}$ is the unit cell volume, which is just the product of the three unit cell edge lengths as shown in Figure 14.10. Thus,

$$\rho = \frac{nA}{V_C N_A}$$

$$= \frac{(2 \text{ mers/uc})(28.05 \text{ g/mol})}{\left[\left(7.41 \times 10^{-8} \text{ cm}\right)\left(4.94 \times 10^{-8} \text{ cm}\right)\left(2.55 \times 10^{-8} \text{ cm}\right)/\text{uc}\right]\left(6.023 \times 10^{23} \text{ mers/mol}\right)}$$

$$= 0.998 \text{ g/cm}^3$$

CHAPTER 15

CHARACTERISTICS, APPLICATIONS, AND PROCESSING OF POLYMERS

PROBLEM SOLUTIONS

15.15 This problem gives us the tensile strengths and associated number-average molecular weights for two polymethyl methacrylate materials and then asks that we estimate the tensile strength for $\overline{\mathbf{M}}_{\mathbf{n}}$ = 30,000 g/mol. Equation (15.3) provides the dependence of the tensile strength on $\overline{\mathbf{M}}_{\mathbf{n}}$. Thus, using the data provided in the problem, we may set up two simultaneous equations from which it is possible to solve for the two constants $\mathbf{TS}_{\bullet}$ and **A**. These equations are as follows:

$$107 \text{ MPa} = \mathrm{TS}_{\infty} - \frac{\mathrm{A}}{40{,}000 \text{ g/mol}}$$

$$170 \text{ MPa} = \mathrm{TS}_{\infty} - \frac{\mathrm{A}}{60{,}000 \text{ g/mol}}$$

Thus, the values of the two constants are: $\mathbf{TS}_{\bullet}$ = 296 MPa and **A** = 7.56 x 10^6 MPa-g/mol. Substituting these values into the equation for which $\overline{\mathbf{M}}_{\mathbf{n}}$ = 30,000 g/mol leads to

$$\mathrm{TS} = \mathrm{TS}_{\infty} - \frac{\mathrm{A}}{30{,}000 \text{ g/mol}}$$

$$= 296 \text{ MPa} - \frac{7.56 \times 10^6 \text{ MPa - g/mol}}{30{,}000 \text{ g/mol}}$$

$$= 44 \text{ MPa}$$

15.24 This problem asks that we compute the fraction of possible crosslink sites in 10 kg of polybutadiene when 4.8 kg of S is added, assuming that, on the average, 4.5 sulfur atoms participate in each crosslink bond. Given the butadiene mer unit in Table 14.5, we may calculate its molecular weight as follows:

$$A(\text{butadiene}) = 4(A_C) + 6(A_H)$$

$$= (4)(12.01\text{ g/mol}) + 6(1.008\text{ g/mol}) = 54.09\text{ g/mol}$$

Which means that in 10 kg of butadiene there are $\frac{10{,}000\text{ g}}{54.09\text{ g/mol}}$ = 184.9 mol.

For the vulcanization of polybutadiene, there are two possible crosslink sites per mer--one for each of the two carbon atoms that are doubly bonded. Furthermore, each of these crosslinks forms a bridge between two mers. Therefore, we can say that there is the equivalent of one crosslink per mer. Therefore, let us now calculate the number of moles of sulfur ($\mathbf{n_{sulfur}}$) that react with the butadiene, by taking the mole ratio of sulfur to butadiene, and then dividing this ratio by 4.5 atoms per crosslink; this yields the fraction of possible sites that are crosslinked. Thus

$$n_{\text{sulfur}} = \frac{4800\text{ g}}{32.06\text{ g/mol}} = 149.7\text{ mol}$$

And

$$\text{fraction sites crosslinked} = \frac{\frac{149.7\text{ mol}}{184.9\text{ mol}}}{4.5} = 0.180$$

15.42 (a) This problem asks that we determine how much ethylene glycol must be added to 20.0 kg of adipic acid to produce a linear chain structure of polyester according to Equation 15.9. Since the chemical formulas are provided in this equation we may calculate the molecular weights of each of these materials as follows:

$$A(\text{adipic}) = 6(A_C) + 10(A_H) + 4(A_O)$$

$$= 6(12.01\text{ g/mol}) + 10(1.008\text{ g/mol}) + 4(16.00\text{ g/mol}) = 146.14\text{ g/mol}$$

$$A(\text{glycol}) = 2(A_C) + 6(A_H) + 2(A_O)$$

$$= 2(12.01\text{ g/mol}) + 6(1.008\text{ g/mol}) + 2(16.00\text{ g/mol}) = 62.07\text{ g/mol}$$

The 20.0 kg mass of adipic acid equals 20,000 g or $\frac{20,000 \text{ g}}{146.14 \text{ g/mol}}$ = 136.86 mol. Since, according to Equation (15.9), each mole of adipic acid used requires one mole of ethylene glycol, which is equivalent to (136.86 mol)(62.07 g/mol) = 8495 g = 8.495 kg.

(b) Now we are asked for the mass of the resulting polyester. Inasmuch as one mole of water is given off for every mer unit produced, this corresponds to 136.86 moles or (136.86 mol)(18.02 g/mol) = 2466 g or 2.466 kg since the molecular weight of water is 18.02 g/mol. The mass of polyester is just the sum of the masses of the two reactant materials [as computed in part (a)] minus the mass of water released, or

$$\text{mass(polyester)} = 20.0 \text{ kg} + 8.495 \text{ kg} - 2.466 \text{ kg} = 26.03 \text{ kg}$$

CHAPTER 16

COMPOSITES

PROBLEM SOLUTIONS

16.4 This problem asks for the maximum and minimum thermal conductivity values for a TiC-Co cermet. Using a modified form of Equation (16.1) the maximum thermal conductivity **k_{max}** is calculated as

$$k_{max} = k_m V_m + k_p V_p = k_{Co} V_{Co} + k_{TiC} V_{TiC}$$

$$= (69 \text{ W/m-K})(0.15) + (27 \text{ W/m-K})(0.85) = 33.3 \text{ W/m-K}$$

And, from a modified form of Equation (16.2), the minimum thermal conductivity **k_{min}** is

$$k_{min} = \frac{k_{Co} k_{TiC}}{V_{Co} k_{TiC} + V_{TiC} k_{Co}}$$

$$= \frac{(69 \text{ W/m-K})(27 \text{ W/m-K})}{(0.15)(27 \text{ W/m-K}) + (0.85)(69 \text{ W/m-K})}$$

$$= 29.7 \text{ W/m-K}$$

16.12 This problem asks for us to determine if it is possible to produce a continuous and oriented aramid fiber-epoxy matrix composite having longitudinal and transverse moduli of elasticity of 35 GPa and 5.17 GPa, respectively, given that the modulus of elasticity for the epoxy is 3.4 GPa. Also, from Table 16.4 the value of **E** for aramid fibers is 131 GPa. The approach to solving this problem is to calculate two values of **V_f** using the data and Equations (16.10b) and (16.16); if they are the same then this composite is possible.

For the longitudinal modulus **E_{cl}**,

$$E_{cl} = E_m\left(1 - V_{fl}\right) + E_f V_{fl}$$

$$35 \text{ GPa} = (3.4 \text{ GPa})\left(1 - V_{fl}\right) + (131 \text{ GPa})V_{fl}$$

Solving this expression for $\mathbf{V_{fl}}$ yields $\mathbf{V_{fl}} = 0.248$.

Now, repeating this procedure for the transverse modulus $\mathbf{E_{ct}}$

$$E_{ct} = \frac{E_m E_f}{(1 - V_{ft})E_f + V_{ft}E_m}$$

$$5.17 \text{ GPa} = \frac{(3.4 \text{ GPa})(131 \text{ GPa})}{(1 - V_{ft})(131 \text{ GPa}) + V_{ft}(3.4 \text{ GPa})}$$

Solving this expression for $\mathbf{V_{ft}}$ leads to $\mathbf{V_{ft}} = 0.351$. Thus, since $\mathbf{V_{fl}}$ and $\mathbf{V_{ft}}$ are not equal, the proposed composite is **not possible**.

16.17 This problem stipulates that the cross-sectional area of a composite, $\mathbf{A_c}$, is 320 mm^2 (0.50 $in.^2$), and that the longitudinal load, $\mathbf{F_c}$, is 44,500 N (10,000 lb_f) for the composite described in Problem 16.11.

(a) First, we are asked to calculate the $\mathbf{F_f/F_m}$ ratio. According to Equation (16.11)

$$\frac{F_f}{F_m} = \frac{E_f V_f}{E_m V_m} = \frac{(131 \text{ GPa})(0.30)}{(2.4 \text{ GPa})(0.70)} = 23.4$$

Or, $\mathbf{F_f} = 23.4\mathbf{F_m}$

(b) Now, the actual loads carried by both phases are called for. Since

$$F_f + F_m = F_c = 44{,}500 \text{ N}$$

$$23.4F_m + F_m = 44{,}500 \text{ N}$$

which leads to

$$F_m = 1824 \text{ N} \quad (410 \text{ lb}_f)$$

$$F_f = 44{,}500 \text{ N} - 1824 \text{ N} = 42{,}676 \text{ N} \quad (9590 \text{ lb}_f)$$

(c) To compute the stress on each of the phases, it is first necessary to know the cross-sectional areas of both fiber and matrix. These are determined as

$$A_f = V_f A_c = (0.30)(320\ \text{mm}^2) = 96\ \text{mm}^2\ \ (0.15\ \text{in.}^2)$$

$$A_m = V_m A_c = (0.70)(320\ \text{mm}^2) = 224\ \text{mm}^2\ \ (0.35\ \text{in.}^2)$$

Now, for the stresses,

$$\sigma_f = \frac{F_f}{A_f} = \frac{42{,}676\ \text{N}}{96\ \text{mm}^2} = \frac{42{,}676\ \text{N}}{96 \times 10^{-6}\ \text{m}^2} = 445\ \text{MPa}\ \ (63{,}930\ \text{psi})$$

$$\sigma_m = \frac{F_m}{A_m} = \frac{1824\ \text{N}}{224\ \text{mm}^2} = \frac{1824\ \text{N}}{224 \times 10^{-6}\ \text{m}^2} = 8.14\ \text{MPa}\ \ (1170\ \text{psi})$$

(d) The strain on the composite is the same as the strain on each of the matrix and fiber phases, as

$$\varepsilon_m = \frac{\sigma_m}{E_m} = \frac{8.14\ \text{MPa}}{2.4 \times 10^3\ \text{MPa}} = 3.39 \times 10^{-3}$$

$$\varepsilon_f = \frac{\sigma_f}{E_f} = \frac{445\ \text{MPa}}{131 \times 10^3\ \text{MPa}} = 3.39 \times 10^{-3}$$

16.21 In this problem, for an aligned glass fiber-epoxy matrix composite, we are asked to compute the longitudinal tensile strength given the following: the average fiber diameter (0.015 mm), the average fiber length (2.0 mm), the volume fraction of fibers (0.25), the fiber fracture strength (3500 MPa), the fiber-matrix bond strength (100 MPa), and the matrix stress at composite failure (5.5 MPa). It is first necessary to compute the value of the critical fiber length using Equation (16.3). If the fiber length is much greater than $\mathbf{l_c}$, then we may determine $\boldsymbol{\sigma^*_{cl}}$ using Equation (16.17), otherwise, use of either Equation (16.18) or (16.19) is necessary. Thus,

$$l_c = \frac{\sigma_f^* d}{2\tau_c} = \frac{(3500\ \text{MPa})(0.015\ \text{mm})}{2(100\ \text{MPa})} = 0.263\ \text{mm}\ \ (0.010\ \text{in.})$$

Inasmuch as $\mathbf{l} > \mathbf{l_c}$ (2.0 mm > 0.263 mm), but $\mathbf{l}$ is not much greater than $\mathbf{l_c}$, then use of Equation (16.18) is necessary. Therefore,

$$\sigma^*_{cd} = \sigma^*_f V_f \left(1 - \frac{l_c}{2l}\right) + \sigma'_m (1 - V_f)$$

$$= (3500\ \text{MPa})(0.25)\left[1 - \frac{0.263\ \text{mm}}{(2)(2.0\ \text{mm})}\right] + (5.5\ \text{MPa})(1 - 0.25)$$

$$= 822\ \text{MPa}\ \ (117{,}800\ \text{psi})$$

16.D1 In order to solve this problem, we want to make longitudinal elastic modulus and tensile strength computations assuming 50 vol% fibers for all three fiber materials, in order to see which meet the stipulated criteria [i.e., a minimum elastic modulus of 50 GPa (7.3 x 10^6 psi), and a minimum tensile strength of 1300 MPa (189,000 psi)]. Thus, it becomes necessary to use Equations (16.10b) and (16.17) with $\mathbf{V_m}$ = 0.5 and $\mathbf{V_f}$ = 0.5, $\mathbf{E_m}$ = 3.1 GPa, and $\boldsymbol{\sigma^*_m}$ = 75 MPa.

For glass, $\mathbf{E_f}$ = 72.5 GPa and $\boldsymbol{\sigma^*_f}$ = 3450 MPa. Therefore,

$$E_{cl} = E_m(1 - V_f) + E_f V_f$$

$$= (3.1\ \text{GPa})(1 - 0.5) + (72.5\ \text{GPa})(0.5) = 37.8\ \text{GPa}\ \ (5.48 \times 10^6\ \text{psi})$$

Since this is less than the specified minimum, glass is not an acceptable candidate.

For carbon (PAN standard-modulus), $\mathbf{E_f}$ = 230 GPa and $\boldsymbol{\sigma^*_f}$ = 4000 MPa (the average of the extreme values in Table B.4), thus

$$E_{cl} = (3.1\ \text{GPa})(0.5) + (230\ \text{GPa})(0.5) = 116.6\ \text{GPa}\ \ (16.9 \times 10^6\ \text{psi})$$

which is greater than the specified minimum. In addition, from Equation (16.17)

$$\sigma^*_{cl} = \sigma'_m(1 - V_f) + \sigma^*_f V_f$$

$$= (30\ \text{MPa})(0.5) + (4000\ \text{MPa})(0.5) = 2015\ \text{MPa}\ \ (292{,}200\ \text{psi})$$

which is also greater than the minimum. Thus, carbon (PAN standard-modulus) is a candidate.

For aramid, $\mathbf{E_f}$ = 131 GPa and $\boldsymbol{\sigma_f^*}$ = 3850 MPa (the average of the extreme values in Table B.4), thus

$$E_{cl} = (3.1 \text{ GPa})(0.5) + (131 \text{ GPa})(0.5) = 67.1 \text{ GPa} \quad (9.73 \times 10^6 \text{ psi})$$

which value is greater than the minimum. Also, from Equation (16.17)

$$\sigma_{cl}^* = \sigma_m'\left(1 - V_f\right) + \sigma_f^* V_f$$

$$= (50 \text{ MPa})(0.5) + (3850 \text{ MPa})(0.5) = 1950 \text{ MPa} \quad (283{,}600 \text{ psi})$$

which is also greater than the minimum strength value. Therefore, of the three fiber materials, both the carbon (PAN standard-modulus) and the aramid meet both minimum criteria.

CHAPTER 17

CORROSION AND DEGRADATION OF MATERIALS

PROBLEM SOLUTIONS

17.5 (a) We are asked to compute the voltage of a nonstandard Cd-Fe electrochemical cell. Since iron is lower in the emf series (Table 17.1), we begin by assuming that iron is oxidized and cadmium is reduced, as

$$\mathrm{Fe} + \mathrm{Cd}^{2+} \rightarrow \mathrm{Fe}^{2+} + \mathrm{Cd}$$

and

$$\Delta V = (V^{\circ}_{\mathrm{Cd}} - V^{\circ}_{\mathrm{Fe}}) - \frac{0.0592}{2} \log \frac{[\mathrm{Fe}^{2+}]}{[\mathrm{Cd}^{2+}]}$$

$$= \left[-0.403 \text{ V} - (-0.440 \text{ V})\right] - \frac{0.0592}{2} \log \left[\frac{0.40}{2 \times 10^{-3}}\right]$$

$$= -0.031 \text{ V}$$

(b) Since this **ΔV** is negative, the spontaneous cell direction is just the reverse of that above, or

$$\mathrm{Fe}^{2+} + \mathrm{Cd} \rightarrow \mathrm{Fe} + \mathrm{Cd}^{2+}$$

17.13 This problem calls for us to compute the time of submersion of a steel piece. In order to solve this problem, we must first rearrange Equation (17.23), as

$$t = \frac{KW}{\rho A (\mathrm{CPR})}$$

Thus,

$$t = \frac{(534)\left(2.6 \times 10^{6} \text{ mg}\right)}{\left(7.9 \text{ g/cm}^{3}\right)\left(10 \text{ in.}^{2}\right)(200 \text{ mpy})}$$

$$= 8.8 \times 10^{4} \text{ h} = 10 \text{ yr}$$

17.20W (a) This portion of the problem asks that we compute the rate of oxidation for Ni given that both the oxidation and reduction reactions are controlled by activation polarization, and also given the polarization data for both nickel oxidation and hydrogen reduction. The first thing necessary is to establish relationships of the form of Equation (17.1W) for the potentials of both oxidation and reduction reactions. Next we will set these expressions equal to one another, and then solve for the value of **i** which is really the corrosion current density, $\mathbf{i_c}$. Finally, the corrosion rate may be calculated using Equation (17.24). The two potential expressions are as follows:

For hydrogen reduction

$$V_H = V_{(H^+/H_2)} + \beta_H \log\left(\frac{i}{i_{o_H}}\right)$$

And for Ni oxidation

$$V_{Ni} = V_{(Ni/Ni^{2+})} + \beta_{Ni} \log\left(\frac{i}{i_{o_{Ni}}}\right)$$

Setting $\mathbf{V_H} = \mathbf{V_{Ni}}$ and solving for log **i** (log $\mathbf{i_c}$) leads to

$$\log i_c = \left(\frac{1}{\beta_{Ni} - \beta_H}\right)\left[V_{(H^+/H_2)} - V_{(Ni/Ni^{2+})} - \beta_H \log i_{o_H} + \beta_{Ni} \log i_{o_{Ni}}\right]$$

$$= \left[\frac{1}{0.12 - (-0.10)}\right]\left[0 - (-0.25) - (-0.10)\left\{\log\left(6 \times 10^{-7}\right)\right\} + (0.12)\left\{\log\left(10^{-8}\right)\right\}\right]$$

$$= -6.055$$

Or

$$i_c = 10^{-6.055} = 8.81 \times 10^{-7} \text{ A/cm}^2$$

And from Equation (17.24)

$$r = \frac{i_c}{nF}$$

$$= \frac{8.81 \times 10^{-7}\ \text{C/s-cm}^2}{(2)(96{,}500\ \text{C/mol})} = 4.56 \times 10^{-12}\ \text{mol/cm}^2\text{ - s}$$

(b) Now it becomes necessary to compute the value of the corrosion potential, $\mathbf{V_c}$. This is possible by using either of the above equations for $\mathbf{V_H}$ or $\mathbf{V_{Ni}}$ and substituting for **i** the value determined above for $\mathbf{i_c}$. Thus

$$V_c = V_{(H^+/H_2)} + \beta_H \log\left(\frac{i_c}{i_{o_H}}\right)$$

$$= 0 + (-0.10\ \text{V}) \log\left(\frac{8.81 \times 10^{-7}\ \text{A/cm}^2}{6 \times 10^{-7}\ \text{A/cm}^2}\right) = -0.0167\ \text{V}$$

17.34 For this problem we are given, for three metals, their densities, oxide chemical formulas, and oxide densities, and are asked to compute the Pilling-Bedworth ratios, and then to specify whether or not the oxide scales that form will be protective. The general form of the equation used to calculate this ratio is Equation (17.30) [or Equation (17.29)]. For magnesium, oxidation occurs by the reaction

$$\text{Mg} + \frac{1}{2}\text{O}_2 \rightarrow \text{MgO}$$

and therefore, from Equation (17.29)

$$\text{P-B ratio} = \frac{A_{MgO}\rho_{Mg}}{A_{Mg}\rho_{MgO}}$$

$$= \frac{(40.31\ \text{g/mol})(1.74\ \text{g/cm}^3)}{(24.31\ \text{g/mol})(3.58\ \text{g/cm}^3)} = 0.81$$

Thus, this would probably be a nonprotective oxide film since the P-B ratio is less than unity; to be protective, this ratio should be between one and two.

17.36 For this problem we are given weight gain-time data for the oxidation of Ni at an elevated temperature.

(a) We are first asked to determine whether the oxidation kinetics obey a parabolic, linear, or logarithmic rate expression, which expressions are represented by Equations (17.31), (17.32), and (17.33), respectively. One way to make this determination is by trial and error. Let us assume that the parabolic relationship is valid; that is from Equation (17.31)

$$W^2 = K_1 t + K_2$$

which means that we may establish three simultaneous equations using the three sets of given **W** and **t** values, then using two combinations of two pairs of equations, solve for $\mathbf{K_1}$ and $\mathbf{K_2}$; if $\mathbf{K_1}$ and $\mathbf{K_2}$ have the same values for both solutions, then the kinetics are parabolic. If the values are not identical then the other kinetic relationships need to be explored. Thus, the three equations are

$$(0.527)^2 = 0.278 = 10K_1 + K_2$$
$$(0.857)^2 = 0.734 = 30K_1 + K_2$$
$$(1.526)^2 = 2.329 = 100K_1 + K_2$$

From the first two equations $\mathbf{K_1}$ = 0.0228 and $\mathbf{K_2}$ = 0.050; these same two values are obtained using the last two equations. Hence, the oxidation rate law is parabolic.

(b) Since a parabolic relationship is valid, this portion of the problem calls for us to determine **W** after a total time of 600 min. Again, using Equation (17.31) and the values of $\mathbf{K_1}$ and $\mathbf{K_2}$

$$W^2 = K_1 t + K_2$$

$$= (0.0228)(600 \text{ min}) + 0.05 = 13.37$$

Or **W** = $\sqrt{13.73}$ = 3.70 mg/cm^2.

CHAPTER 18

ELECTRICAL PROPERTIES

PROBLEM SOLUTIONS

18.5 (a) In order to compute the resistance of this copper wire it is necessary to employ Equations (18.2) and (18.4). Solving for the resistance in terms of the conductivity,

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A}$$

From Table 18.1, the conductivity of copper is 6.0 x 10^7 $(\Omega\text{-m})^{-1}$, and

$$R = \frac{l}{\sigma A} = \frac{2 \text{ m}}{\left[6.0 \times 10^7 \ (\Omega - \text{m})^{-1}\right](\pi)\left(\frac{3 \times 10^{-3} \text{ m}}{2}\right)^2}$$

$$= 4.7 \times 10^{-3} \ \Omega$$

(b) If **V** = 0.05 V then, from Equation (18.1)

$$I = \frac{V}{R} = \frac{0.05 \text{ V}}{4.7 \times 10^{-3} \ \Omega} = 10.6 \text{ A}$$

(c) The current density is just

$$J = \frac{I}{A} = \frac{I}{\pi\left(\frac{d}{2}\right)^2} = \frac{10.6 \text{ A}}{\pi\left(\frac{3 \times 10^{-3} \text{ m}}{2}\right)^2} = 1.5 \times 10^6 \text{ A/m}^2$$

(d) The electric field is just

$$E = \frac{V}{l} = \frac{0.05 \text{ V}}{2 \text{ m}} = 2.5 \times 10^{-2} \text{ V/m}$$

18.13 (a) The number of free electrons per cubic meter for copper at room temperature may be computed using Equation (18.8) as

$$n = \frac{\sigma}{|e|\,\mu_e}$$

$$= \frac{6.0 \times 10^{7}\ (\Omega - \text{m})^{-1}}{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(0.0030\ \text{m}^2/\text{V-s}\right)}$$

$$= 1.25 \times 10^{29}\ \text{m}^{-3}$$

(b) In order to calculate the number of free electrons per copper atom, we must first determine the number of copper atoms per cubic meter, $\mathbf{N_{Cu}}$. From Equation (4.2)

$$N_{Cu} = \frac{N_A \rho}{A_{Cu}}$$

$$= \frac{\left(6.023 \times 10^{23}\ \text{atoms/mol}\right)\left(8.94\ \text{g/cm}^3\right)\left(10^6\ \text{cm}^3/\text{m}^3\right)}{63.55\ \text{g/mol}}$$

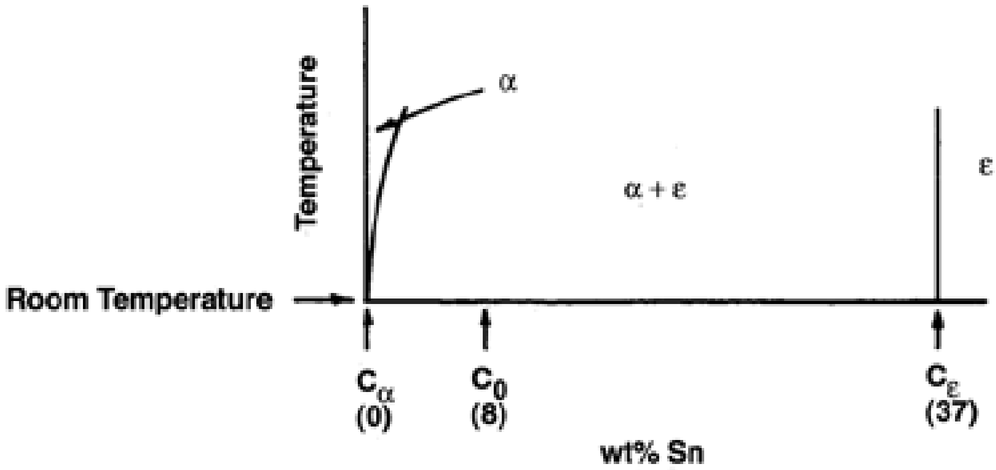
$$= 8.47 \times 10^{28}\ \text{m}^{-3}$$

The number of free electrons per copper atom is just

$$\frac{n}{N} = \frac{1.25 \times 10^{29}\ \text{m}^{-3}}{8.47 \times 10^{28}\ \text{m}^{-3}} = 1.48$$

18.18 This problem asks for us to compute the room-temperature conductivity of a two-phase Cu-Sn alloy. It is first necessary for us to determine the volume fractions of the α and ε phases, after which the resistivity (and subsequently, the conductivity) may be calculated using Equation (18.12). Weight fractions of the two phases are first calculated using the phase diagram information provided in the problem.

We might represent the phase diagram near room temperature as shown below.



Applying the lever rule to this situation

$$W_\alpha = \frac{C_\varepsilon - C_o}{C_\varepsilon - C_\alpha} = \frac{37 - 11}{37 - 0} = 0.703$$

$$W_\varepsilon = \frac{C_o - C_\alpha}{C_\varepsilon - C_\alpha} = \frac{11 - 0}{37 - 0} = 0.297$$

We must now convert these mass fractions into volume fractions using the phase densities given in the problem. (Note: in the following expressions, density is represented by **ρ'** in order to avoid confusion with resistivity which is designated by **ρ**.) Utilization of Equations (9.6a) and (9.6b) leads to

$$V_\alpha = \frac{\dfrac{W_\alpha}{\rho'_\alpha}}{\dfrac{W_\alpha}{\rho'_\alpha} + \dfrac{W_\varepsilon}{\rho'_\varepsilon}}$$

$$= \frac{\dfrac{0.703}{8.94\ \text{g/cm}^3}}{\dfrac{0.703}{8.94\ \text{g/cm}^3} + \dfrac{0.297}{8.25\ \text{g/cm}^3}}$$

$$= 0.686$$

$$V_\varepsilon = \frac{\dfrac{W_\varepsilon}{\rho'_\varepsilon}}{\dfrac{W_\alpha}{\rho'_\alpha} + \dfrac{W_\varepsilon}{\rho'_\varepsilon}}$$

$$= \frac{\dfrac{0.297}{8.25\ \mathrm{g/cm^3}}}{\dfrac{0.703}{8.94\ \mathrm{g/cm^3}} + \dfrac{0.297}{8.25\ \mathrm{g/cm^3}}}$$

$$= 0.314$$

Now, using Equation (18.12)

$$\rho = \rho_\alpha V_\alpha + \rho_\varepsilon V_\varepsilon$$

$$= \left(1.88 \times 10^{-8}\ \Omega\text{-m}\right)(0.686) + \left(5.32 \times 10^{-7}\ \Omega\text{-m}\right)(0.314)$$

$$= 1.80 \times 10^{-7}\ \Omega\text{-m}$$

Finally, for the conductivity [Equation (18.4)]

$$\sigma = \frac{1}{\rho} = \frac{1}{1.80 \times 10^{-7}\ \Omega-\mathrm{m}} = 5.56 \times 10^{6}\ (\Omega\text{-m})^{-1}$$

18.31 (a) In this problem, for a Si specimen, we are given **p** and **σ**, while μ_h and μ_e are included in Table 18.2. In order to solve for **n** we must use Equation (18.13), which, after rearrangement, leads to

$$n = \frac{\sigma - p|e|\mu_h}{|e|\mu_e}$$

$$= \frac{10^3\ (\Omega-\mathrm{m})^{-1} - \left(1.0 \times 10^{23}\ \mathrm{m^{-3}}\right)\left(1.602 \times 10^{-19}\ \mathrm{C}\right)\left(0.05\ \mathrm{m^2/V\text{-}s}\right)}{\left(1.602 \times 10^{-19}\ \mathrm{C}\right)\left(0.14\ \mathrm{m^2/V\text{-}s}\right)}$$

$$= 8.9 \times 10^{21}\ \mathrm{m}^{-3}$$

(b) This material is **p**-type extrinsic since **p** (1.0×10^{23} m^{-3}) is greater than **n** (8.9×10^{21} m^{-3}).

18.41 This problems asks that we determine the room-temperature electrical conductivity of silicon that has been doped with 10^{23} m^{-3} of arsenic atoms. Inasmuch as As is a group VA element in the periodic table (Figure 2.6) it acts as a donor in silicon. Thus, this material is **n**-type extrinsic, and it is necessary to use Equation (18.16), with **n** = 10^{23} m^{-3} since at room temperature all of the As donor impurities are ionized. The electron mobility, from Figure 18.17 at an impurity concentration of 10^{23} m^{-3}, is 0.065 m^2/V-s. Therefore, the conductivity is equal to

$$\sigma = n|e|\mu_e = \left(10^{23}\ \mathrm{m}^{-3}\right)\left(1.6 \times 10^{-16}\ \mathrm{C}\right)\left(0.065\ \mathrm{m}^2/\mathrm{V}-\mathrm{s}\right) = 1040\ (\Omega - \mathrm{m})^{-1}$$

18.47W In this problem we are asked to determine the magnetic field required to produce a Hall voltage of -3.5×10^{-7} V, given that $\boldsymbol{\sigma} = 1.2 \times 10^{7}$ $(\Omega\text{-m})^{-1}$, $\boldsymbol{\mu_e}$ = 0.0050 m^2/V-s, $\mathbf{I_x}$ = 40 A, and **d** = 35 mm. Combining Equations (18.1W) and (18.3bW), and after solving for $\mathbf{B_z}$, we get

$$B_z = \frac{|V_H|\sigma d}{I_x \mu_e}$$

$$= \frac{\left(\left|-3.5 \times 10^{-7}\ \mathrm{V}\right|\right)\left[1.2 \times 10^{7}\ (\Omega - \mathrm{m})^{-1}\right]\left(35 \times 10^{-3}\ \mathrm{m}\right)}{(40\ \mathrm{A})\left(0.0050\ \mathrm{m}^2/\mathrm{V\text{-}s}\right)}$$

$$= 0.74\ \text{tesla}$$

18.54W We want to compute the plate spacing of a parallel-plate capacitor as the dielectric constant is increased form 2.5 to 4.0, while maintaining the capacitance constant. Combining Equations (18.6W) and (18.7W) yields

$$C = \frac{\varepsilon_r \varepsilon_o A}{l}$$

Now, let us use the subscripts **1** and **2** to denote the initial and final states, respectively. Since $\mathbf{C_1} = \mathbf{C_2}$, then

$$\frac{\varepsilon_{r1}\varepsilon_o A}{l_1} = \frac{\varepsilon_{r2}\varepsilon_o A}{l_2}$$

And, solving for $\mathbf{l_2}$

$$l_2 = \frac{\varepsilon_{r2} l_1}{\varepsilon_{r1}} = \frac{(4.0)(1 \text{ mm})}{2.5} = 1.6 \text{ mm}$$

18.60W (a) We want to solve for the voltage when **Q** = 3.5 x 10^{-11} C, **A** = 160 mm^2, **l** = 3.5 mm, and $\boldsymbol{\varepsilon_r}$ = 5.0. Combining Equations (18.4W), (18.6W), and (18.7W) yields

$$\frac{Q}{V} = \varepsilon_r \varepsilon_o \frac{A}{l}$$

And, solving for **V**

$$V = \frac{Ql}{\varepsilon_r \varepsilon_o A}$$

$$= \frac{\left(3.5 \text{ x } 10^{-11} \text{ C}\right)\left(3.5 \text{ x } 10^{-3} \text{ m}\right)}{(5.0)\left(8.85 \text{ x } 10^{-12} \text{ F/m}\right)\left(160 \text{ mm}^2\right)\left(1 \text{ m}^2/10^6 \text{ mm}^2\right)}$$

$$= 17.3 \text{ V}$$

(b) For this same capacitor, if a vacuum is used

$$V = \frac{Ql}{\varepsilon_o A}$$

$$= \frac{\left(3.5 \text{ x } 10^{-11} \text{ C}\right)\left(3.5 \text{ x } 10^{-3} \text{ m}\right)}{\left(8.85 \text{ x } 10^{-12} \text{ F/m}\right)\left(160 \text{ x } 10^{-6} \text{ m}^2\right)}$$

$$= 86.5 \text{ V}$$

(e) The polarization is determined using Equations (18.12W) and (18.6) as

$$P = \varepsilon_o(\varepsilon_r - 1)\frac{V}{l}$$

$$= \frac{\left(8.85 \times 10^{-12}\,\mathrm{F/m}\right)(5.0 - 1)(17.3\,\mathrm{V})}{3.5 \times 10^{-3}\,\mathrm{m}}$$

$$= 1.75 \times 10^{-7}\ \mathrm{C/m^2}$$

18.D2 This problem asks that we determine the electrical conductivity of an 85 wt% Cu-15 wt% Zn alloy at -100°C using information contained in Figures 18.8 and 18.28. In order to solve this problem it is necessary to employ Equation (18.9) which is of the form

$$\rho_{\mathrm{total}} = \rho_t + \rho_i$$

since it is assumed that the alloy is undeformed. Let us first determine the value of $\boldsymbol{\rho_i}$ at room temperature (25°C) which value will be independent of temperature. From Figure (18.8), at 25°C and for pure Cu, $\boldsymbol{\rho_t(25)}$ = 1.75 x 10^{-8} Ω-m. Now, since it is assumed that the curve in Figure 18.28 was generated also at room temperature, we may take ρ as $\boldsymbol{\rho_{\mathrm{total}}(25)}$ at 85 wt% Cu-15 wt% Zn which has a value of 4.7 x 10^{-8} Ω-m. Thus

$$\rho_i = \rho_{\mathrm{total}}(25) - \rho_t(25)$$

$$= 4.7 \times 10^{-8}\ \Omega\text{-m} - 1.75 \times 10^{-8}\ \Omega\text{-m} = 2.95 \times 10^{-8}\ \Omega\text{-m}$$

Finally, we may determine the resistivity at -100°C, $\boldsymbol{\rho_{\mathrm{total}}(-100)}$, by taking the resistivity of pure Cu at -100°C from Figure 18.8, which gives us $\boldsymbol{\rho_t(-100)}$ = 0.90 x 10^{-8} Ω-m. Therefore

$$\rho_{\mathrm{total}}(-100) = \rho_i + \rho_t(-100)$$

$$= 2.95 \times 10^{-8}\ \Omega\text{-m} + 0.90 \times 10^{-8}\ \Omega\text{-m} = 3.85 \times 10^{-8}\ \Omega\text{-m}$$

And, using Equation (18.4) the conductivity is calculated as

$$\sigma = \frac{1}{\rho} = \frac{1}{3.85 \ \text{x} \ 10^{-8} \ \Omega - \text{m}} = 2.60 \ \text{x} \ 10^{7} \ (\Omega \text{ - m})^{-1}$$

CHAPTER 19

THERMAL PROPERTIES

PROBLEM SOLUTIONS

19.4 (a) For aluminum, $\mathbf{C_v}$ at 50 K may be approximated by Equation (19.2), since this temperature is significantly below the Debye temperature. The value of $\mathbf{C_v}$ at 30 K is given, and thus, we may compute the constant **A** as

$$A = \frac{C_v}{T^3} = \frac{0.81 \text{ J/mol-K}}{(30 \text{ K})^3} = 3 \times 10^{-5} \text{ J/mol-K}^4$$

Therefore, at 50 K

$$C_v = AT^3 = \left(3 \times 10^{-5} \text{ J/mol-K}^4\right)(50 \text{ K})^3 = 3.75 \text{ J/mol-K}$$

and

$$c_v = (3.75 \text{ J/mol-K})(1 \text{ mol}/26.98 \text{ g})(1000 \text{ g/kg}) = 139 \text{ J/kg-K}$$

19.14 This problem asks for us to determine the temperature to which a cylindrical rod of tungsten 15.025 mm in diameter must be heated in order for it of just fit into a 15.000 mm diameter circular hole in a plate of 1025 steel (which, of course, is also heated), assuming that the initial temperature is 25°C. This requires the use of Equation (19.3a), which is applied to the diameters of both the rod and hole. That is

$$\frac{d_f - d_o}{d_o} = \alpha_l \left(T_f - T_o\right)$$

Solving this expression for $\mathbf{d_f}$ yields

$$d_f = d_o\left[1 + \alpha_l \left(T_f - T_o\right)\right]$$

Now all we need do is to establish expressions for $\mathbf{d_f}$(steel) and $\mathbf{d_f}$(W), set them equal to one another, and solve for $\mathbf{T_f}$. According to Table 19.1, α_l(steel) = 12.0 x 10^{-6} $(°C)^{-1}$ and α_l(W) = 4.5 x 10^{-6} $(°C)^{-1}$. Thus

$$d_f(\text{steel}) = d_f(\text{W})$$

$$(15.000\ \text{mm})\left[1 + \left\{12.0 \times 10^{-6} (°\text{C})^{-1}\right\}\left(T_f - 25°\text{C}\right)\right]$$
$$= (15.025\ \text{mm})\left[1 + \left\{4.5 \times 10^{-6} (°\text{C})^{-1}\right\}\left(T_f - 25°\text{C}\right)\right]$$

Now solving for $\mathbf{T_f}$ gives $\mathbf{T_f}$ = 222.4°C

19.24 This problem asks that we treat a porous material as a composite wherein one of the phases is a pore phase, and that we estimate upper and lower limits for the room-temperature thermal conductivity of an aluminum oxide material having a 0.25 volume fraction of pores. The upper limit of **k** ($\mathbf{k_{upper}}$) may be determined using Equation (16.1) with thermal conductivity substituted for the elastic modulus, **E**. From Table 19.1, the value of **k** for Al_2O_3 is 39 W/m-K, while for still air in the pore phase, **k =** 0.02 W/m-K. Thus

$$k_{upper} = V_p k_{air} + V_{Al_2O_3} k_{Al_2O_3}$$

$$= (0.25)(0.02\ \text{W/m-K}) + (0.75)(39\ \text{W/m-K}) = 29.3\ \text{W/m-K}$$

19.29 We want to heat the copper wire in order to reduce the stress level from 70 MPa to 35 MPa; in doing so, we reduce the stress in the wire by 70 MPa - 35 MPa = 35 MPa, which stress will be a compressive one (i.e., σ = -35 MPa). Solving for $\mathbf{T_f}$ from Equation (19.8)

$$T_f = T_o - \frac{\sigma}{E\alpha_l}$$

$$= 20°\text{C} - \frac{-35\ \text{MPa}}{\left(110 \times 10^3\ \text{MPa}\right)\left[17 \times 10^{-6}\ (°\text{C})^{-1}\right]}$$

$$= 20°\text{C} + 19°\text{C} = 39°\text{C} \ \ (101°\text{F})$$

19.D1 For these railroad tracks, each end is allowed to expand one-half of the joint space distance, or the track may expand a total of this distance (4.6 mm). Equation (19.3a) is used to solve for $\mathbf{T_f}$, where $\boldsymbol{\alpha_l}$ for the 1025 steel is found in Table 19.1 [i.e., 12.0 x 10^{-6} $(°\text{C})^{-1}$]. Thus,

$$T_f = \frac{\Delta l}{\alpha_l l_o} + T_o$$

$$= \frac{4.6 \times 10^{-3} \text{ m}}{\left[12.0 \times 10^{-6} \ (°\text{C})^{-1}\right](11.9 \text{ m})} + 10°\text{C}$$

$$= 32.2°\text{C} + 10°\text{C} = 42.2°\text{C} \ \ (108°\text{F})$$

CHAPTER 20

MAGNETIC PROPERTIES

PROBLEM SOLUTIONS

20.1 (a) We may calculate the magnetic field strength generated by this coil using Equation (20.1) as

$$H = \frac{NI}{l}$$

$$= \frac{(200 \text{ turns})(10 \text{ A})}{0.2 \text{ m}} = 10{,}000 \text{ A-turns/m}$$

(b) In a vacuum, the flux density is determined from Equation (20.3). Thus,

$$B_o = \mu_o H$$

$$= \left(1.257 \times 10^{-6} \text{ H/m}\right)(10{,}000 \text{ A-turns/m}) = 1.257 \times 10^{-2} \text{ tesla}$$

(c) When a bar of titanium is positioned within the coil, we must use an expression that is a combination of Equations (20.5) and (20.6) in order to compute the flux density given the magnetic susceptibility. Inasmuch as $\chi_m = 1.81 \times 10^{-4}$ (Table 20.2), then

$$B = \mu_o H + \mu_o M = \mu_o H + \mu_o \chi_m H = \mu_o H\left(1 + \chi_m\right)$$

$$= \left(1.257 \times 10^{-6} \text{ H/m}\right)(10{,}000 \text{ A-turns/m})\left(1 + 1.81 \times 10^{-4}\right)$$

$$\cong 1.257 \times 10^{-2} \text{ tesla}$$

which is essentially the same result as part (b). This is to say that the influence of the titanium bar within the coil makes an imperceptible difference in the magnitude of the **B** field.

(d) The magnetization is computed from Equation (20.6):

$$M = \chi_m H = \left(1.81 \times 10^{-4}\right)(10{,}000 \text{ A-turns/m}) = 1.81 \text{ A/m}$$

20.6 (a) The magnetic permeability of this material may be determined according to Equation (20.2) as

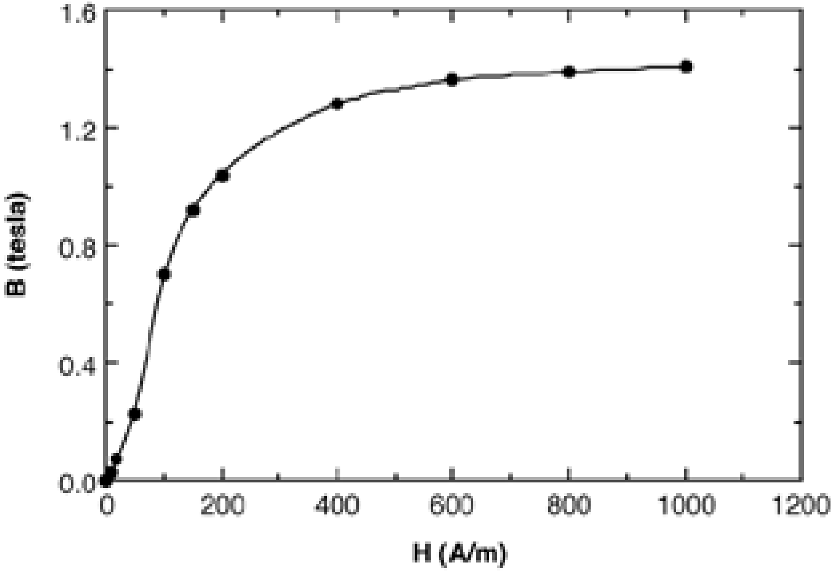
$$\mu = \frac{B}{H} = \frac{0.630 \text{ tesla}}{5 \times 10^5 \text{ A/m}} = 1.26 \times 10^{-6} \text{ H/m}$$

(b) The magnetic susceptibility is calculated as

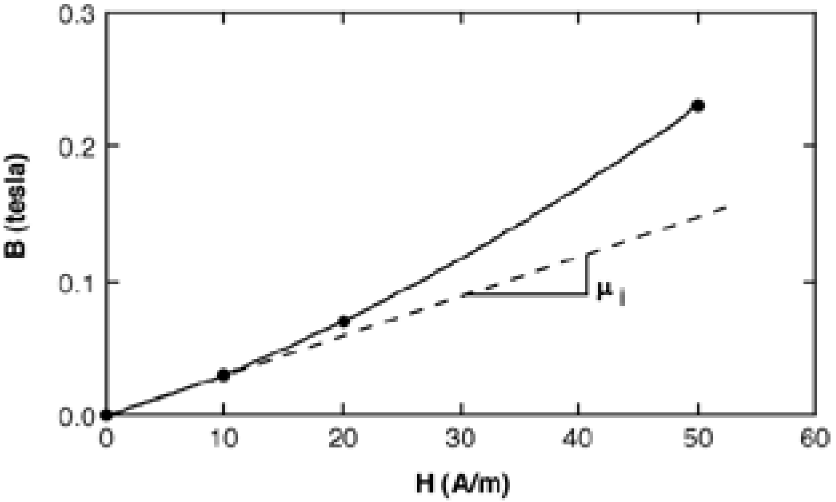
$$\chi_m = \frac{\mu}{\mu_o} - 1 = \frac{1.26 \times 10^{-6} \text{ H/m}}{1.257 \times 10^{-6} \text{ H/m}} - 1$$

$$= 2.39 \times 10^{-3}$$

20.27 (a) The **B**-**H** data provided in the problem are plotted below.



(b) The first four data points are plotted below.



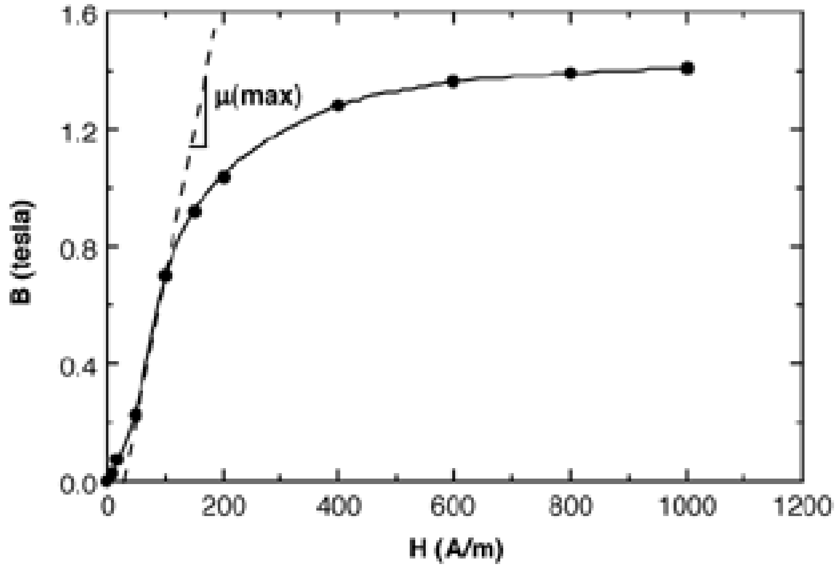
The slope of the initial portion of the curve is **μ_i** (as shown), is

$$\mu_i = \frac{\Delta B}{\Delta H} = \frac{(0.15 - 0)\ \text{tesla}}{(50 - 0)\ \text{A/m}} = 3.0 \times 10^{-3}\ \text{H/m}$$

Also, the initial relative permeability [Equation (20.4)] is just

$$\mu_{ri} = \frac{\mu_i}{\mu_o} = \frac{3.0 \times 10^{-3}\ \text{H/m}}{1.257 \times 10^{-6}\ \text{H/m}} = 2400$$

(c) The maximum permeability is the tangent to the **B-H** curve having the greatest slope; it is drawn on the plot below, and designated as **μ(max)**.



The value of **μ(max)** is

$$\mu(\max) = \frac{\Delta B}{\Delta H} = \frac{(1.3 - 0.3)\ \text{tesla}}{(160 - 45)\ \text{A-m}} = 8.70 \times 10^{-3}\ \text{H/m}$$

20.32 (a) Given Equation (20.12) and the data in Table 20.7, we are asked to calculate the critical magnetic fields for lead at 2.5 and 5.0 K. From the table, for Pb, $\mathbf{T_C}$ = 7.19 K and $\mathbf{B_C(0)}$ = 0.0803 tesla. Thus, from Equation (20.3)

$$H_C(0) = \frac{B_C(0)}{\mu_o}$$

$$= \frac{0.0803\ \text{tesla}}{1.257 \times 10^{-6}\ \text{H/m}} = 6.39 \times 10^4\ \text{A/m}$$

Now, solving for $\mathbf{H_C(2.5)}$ using Equation (20.12) yields

$$H_C(T) = H_C(0)\left[1 - \frac{T^2}{T_C^2}\right]$$

$$H_C(2.5) = \left(6.39 \times 10^4\ \text{A/m}\right)\left[1 - \frac{(2.5\ \text{K})^2}{(7.19\ \text{K})^2}\right] = 5.62 \times 10^4\ \text{A/m}$$

(b) Now we are to determine the temperature to which mercury must be cooled in a magnetic field of 15,000 A/m in order for it to be superconductive. The value of $\mathbf{H_C(0)}$ must first be determined using $\mathbf{B_C(0)}$ given in the table (i.e., 0.0411 tesla); thus from Equation (20.3)

$$H_C(0) = \frac{B_C(0)}{\mu_o} = \frac{0.0411\ \text{tesla}}{1.257 \times 10^{-6}\ \text{H/m}} = 3.27 \times 10^4\ \text{A/m}$$

Since $\mathbf{T_C}$ = 4.15 K we may solve for **T** using Equation (20.12) as

$$T = T_C\sqrt{1 - \frac{H_C(T)}{H_C(0)}}$$

$$= (4.15\ \text{K})\sqrt{1 - \frac{15{,}000\ \text{A/m}}{32{,}700\ \text{A/m}}} = 3.05\ \text{K}$$

CHAPTER 21

OPTICAL PROPERTIES

PROBLEM SOLUTIONS

21.9 We want to compute the velocity of light in diamond given that $\boldsymbol{\varepsilon_r}$ = 5.5 and $\boldsymbol{\chi_m}$ = -2.17 x 10^{-5}. The velocity is determined using Equation (21.8); but first, we must calculate the values of $\boldsymbol{\varepsilon}$ and $\boldsymbol{\mu}$ for diamond. According to Equation (18.7W)

$$\varepsilon = \varepsilon_r \varepsilon_o = (5.5)\left(8.85 \times 10^{-12} \text{ F/m}\right) = 4.87 \times 10^{-11} \text{ F/m}$$

Now, combining Equations (20.4) and (20.7)

$$\mu = \mu_o\left(\chi_m + 1\right)$$

$$= \left(1.257 \times 10^{-6} \text{ H/m}\right)\left(1 - 2.17 \times 10^{-5}\right) = 1.257 \times 10^{-6} \text{ H/m}$$

And, finally

$$v = \frac{1}{\sqrt{\varepsilon\mu}}$$

$$= \frac{1}{\sqrt{\left(4.87 \times 10^{-11} \text{ F/m}\right)\left(1.257 \times 10^{-6} \text{ H/m}\right)}}$$

$$= 1.28 \times 10^{8} \text{ m/s}$$

21.11W This problem asks for us, using data in Table 21.1, to estimate the dielectric constants for silica glass, soda-lime glass, PTFE, polyethylene, and polystyrene, and then to compare these values with those cited in Table 18.1W and briefly explain any discrepancies. From Equation (21.10)

$$\varepsilon_r = n^2$$

Thus, for fused silica, since **n** = 1.458

$$\varepsilon_r = (1.458)^2 = 2.13$$

For polyethylene

$$\varepsilon_r = (1.51)^2 = 2.28$$

When we compare the value of $\boldsymbol{\varepsilon_r}$ for PE with that in Table 18.1W at frequencies of 1 MHz, there is reasonable agreement (i.e., 2.28 versus 2.3). However, for fused silica there is a significant discrepancy (i.e., 2.13 versus 3.8). The reason for this discrepancy is that an ionic component to the dielectric constant is present at 1 MHz, but is absent at frequencies within the visible electromagnetic spectrum, which frequencies are on the order 10^9 MHz (10^{15} Hz). This effect may be noted in Figure 18.8W.

21.19 In this problem we are asked to calculate the fraction of nonreflected light transmitted through a 20 mm thickness of transparent material, given that the fraction transmitted through a 10 mm width is 0.90. From the fraction of nonreflected light transmitted, $\mathbf{I'_T/I'_0}$, and using a rearranged form of Equation (21.18), we may determine the value of $\boldsymbol{\beta}$ as

$$\beta = -\frac{1}{x}\ln\left(\frac{I'_T}{I'_0}\right)$$

$$= -\left(\frac{1}{10\ \text{mm}}\right)\ln(0.90) = 1.05 \times 10^{-2}\ \text{mm}^{-1}$$

Now, solving for $\mathbf{\frac{I'_T}{I'_0}}$ when **x** = 20 mm gives

$$\frac{I'_T}{I'_0} = \exp(-\beta x)$$

$$= \exp\left[-\left(1.05 \times 10^{-2}\ \text{mm}^{-1}\right)(20\ \text{mm})\right] = 0.81$$

21.30 This problem asks for the difference in energy between metastable and ground electron states for a ruby laser. The wavelength of the radiation emitted by an electron transition from the metastable to ground state is cited as 0.6943 μm. The difference in energy between these states, **ΔE**, may be determined from using a combined form of Equations (21.6) and (21.3), as

$$\Delta E = h\nu = \frac{hc}{\lambda}$$

$$= \frac{(4.13 \text{ x } 10^{-15} \text{ eV-s})(3 \text{ x } 10^{8} \text{ m/s})}{6.943 \text{ x } 10^{-7} \text{ m}}$$

$$= 1.78 \text{ eV}$$