3-2 Complex Numbers

Simplify.

1. $\sqrt{-81}$

SOLUTION:

$$\begin{aligned}\sqrt{-81} &= \sqrt{-1\cdot 9\cdot 9}\\ &= \sqrt{-1}\cdot\sqrt{9^2}\\ &= 9i\end{aligned}$$

2. $\sqrt{-32}$

SOLUTION:

$$\begin{aligned}\sqrt{-32} &= \sqrt{-1\cdot 2\cdot 2\cdot 2\cdot 2\cdot 2}\\ &= \sqrt{-1}\cdot\sqrt{2^2}\cdot\sqrt{2^2}\cdot\sqrt{2}\\ &= i\cdot 2\cdot 2\cdot\sqrt{2}\\ &= 4i\sqrt{2}\end{aligned}$$

3. $(4i)(-3i)$

SOLUTION:

$$\begin{aligned}(4i)(-3i) &= -12i^2\\ &= -12(-1)\\ &= 12\end{aligned}$$

4. $3\sqrt{-24}\cdot 2\sqrt{-18}$

SOLUTION:

$$\begin{aligned}&3\sqrt{-24}\cdot 2\sqrt{-18}\\ &= 3\cdot\sqrt{-1\cdot 2\cdot 2\cdot 2\cdot 3}\cdot 2\cdot\sqrt{-1\cdot 2\cdot 3\cdot 3}\\ &= 3\cdot i\cdot 2\cdot\sqrt{2}\cdot\sqrt{3}\cdot 2\cdot i\cdot\sqrt{2}\cdot 3\\ &= 72\cdot i^2\cdot\sqrt{3}\\ &= 72\cdot(-1)\cdot\sqrt{3}\\ &= -72\sqrt{3}\end{aligned}$$

5. i^{40}

SOLUTION:

$$\begin{aligned}i^{40} &= \left(i^2\right)^{20}\\ &= (-1)^{20}\\ &= 1\end{aligned}$$

6. i^{63}

SOLUTION:

$$\begin{aligned}i^{63} &= i^{62}\cdot i\\ &= \left(i^2\right)^{31}\cdot i\\ &= -1\cdot i\\ &= -i\end{aligned}$$

Solve each equation.

7. $4x^2+32=0$

SOLUTION:

$$\begin{aligned}4x^2+32&=0\\ 4x^2&=-32\\ x^2&=-8\\ x&=\pm\sqrt{-8}\\ x&=\pm\sqrt{-1\cdot 2\cdot 2\cdot 2}\\ x&=\pm 2i\sqrt{2}\end{aligned}$$

8. $x^2 + 1 = 0$

SOLUTION:

$$\begin{aligned} x^2 + 1 &= 0 \\ x^2 &= -1 \\ x &= \pm\sqrt{-1} \\ x &= \pm i\sqrt{1} \\ x &= \pm i \end{aligned}$$

Find the values of *a* and *b* that make each equation true.

9. $3a + (4b + 2)i = 9 - 6i$

SOLUTION:

Set the real parts equal to each other.

$$\begin{aligned} 3a &= 9 \\ a &= 3 \end{aligned}$$

Set the imaginary parts equal to each other.

$$\begin{aligned} 4b + 2 &= -6 \\ 4b &= -8 \\ b &= -2 \end{aligned}$$

10. $4b - 5 + (-a - 3)i = 7 - 8i$

SOLUTION:

Set the real parts equal to each other.

$$\begin{aligned} 4b - 5 &= 7 \\ 4b &= 12 \\ b &= 3 \end{aligned}$$

Set the imaginary parts equal to each other.

$$\begin{aligned} -a - 3 &= -8 \\ -a &= -5 \\ a &= 5 \end{aligned}$$

Simplify.

11. $(-1 + 5i) + (-2 - 3i)$

SOLUTION:

$$\begin{aligned} (-1 + 5i) + (-2 - 3i) &= (-1 - 2) + (5i - 3i) \\ &= -3 + 2i \end{aligned}$$

12. $(7 + 4i) - (1 + 2i)$

SOLUTION:

$$\begin{aligned} (7 + 4i) - (1 + 2i) &= 7 + 4i - 1 - 2i \\ &= 6 + 2i \end{aligned}$$

13. $(6 - 8i)(9 + 2i)$

SOLUTION:

$$\begin{aligned} &(6 - 8i)(9 + 2i) \\ &= 6(9) + 6(2i) - 8i(9) - 8i(2i) \\ &= 54 + 12i - 72i - 16i^2 \\ &= 54 + 12i - 72i - 16(-1) \\ &= 54 + 12i - 72i + 16 \\ &= 70 - 60i \end{aligned}$$

14. $(3 + 2i)(-2 + 4i)$

SOLUTION:

$$\begin{aligned} &(3 + 2i)(-2 + 4i) \\ &= 3(-2) + 3(4i) + 2i(-2) + 2i(4i) \\ &= -6 + 12i - 4i + 8i^2 \\ &= -6 + 12i - 4i + 8(-1) \\ &= -6 + 12i - 4i - 8 \\ &= -14 + 8i \end{aligned}$$

3-2 Complex Numbers

15. $\frac{3-i}{4+2i}$

SOLUTION:

$$\begin{aligned}\frac{3-i}{4+2i} &= \frac{3-i}{4+2i}\cdot\frac{4-2i}{4-2i}\\ &= \frac{(3-i)(4-2i)}{(4+2i)(4-2i)}\\ &= \frac{12-6i-4i+2i^2}{16-4i^2}\\ &= \frac{12-6i-4i-2}{16-4(-1)}\\ &= \frac{10-10i}{20}\\ &= \frac{10(1-i)}{2\cdot 10}\\ &= \frac{1-i}{2}\\ &= \frac{1}{2}-\frac{1}{2}i\end{aligned}$$

16. $\frac{2+i}{5+6i}$

SOLUTION:

$$\begin{aligned}\frac{2+i}{5+6i} &= \frac{2+i}{5+6i}\cdot\frac{5-6i}{5-6i}\\ &= \frac{(2+i)(5-6i)}{(5+6i)(5-6i)}\\ &= \frac{10-12i+5i-6i^2}{25-36i^2}\\ &= \frac{10-12i+5i-6(-1)}{25-36(-1)}\\ &= \frac{10-12i+5i+6}{25+36}\\ &= \frac{16-7i}{61}\\ &= \frac{16}{61}-\frac{7}{61}i\end{aligned}$$

17. **ELECTRICITY** The current in one part of a series circuit is $5 - 3j$ amps. The current in another part of the circuit is $7 + 9j$ amps. Add these complex numbers to find the total current in the circuit.

SOLUTION:

$$\begin{aligned}\text{Total current} &= (5-3j)+(7+9j)\\ &= 5-3j+7+9j\\ &= 12+6j \text{ amps}\end{aligned}$$

CCSS STRUCTURE Simplify.

18. $\sqrt{-121}$

SOLUTION:

$$\begin{aligned}\sqrt{-121} &= \sqrt{-1\cdot 11\cdot 11}\\ &= \sqrt{-1}\cdot\sqrt{11^2}\\ &= 11i\end{aligned}$$

19. $\sqrt{-169}$

SOLUTION:

$$\begin{aligned}\sqrt{-169} &= \sqrt{-1\cdot 13\cdot 13}\\ &= \sqrt{-1}\cdot\sqrt{13^2}\\ &= 13i\end{aligned}$$

20. $\sqrt{-100}$

SOLUTION:

$$\begin{aligned}\sqrt{-100} &= \sqrt{-1\cdot 10\cdot 10}\\ &= \sqrt{-1}\cdot\sqrt{10^2}\\ &= 10i\end{aligned}$$

21. $\sqrt{-81}$

SOLUTION:

$$\begin{aligned}\sqrt{-81} &= \sqrt{-1\cdot 9\cdot 9}\\ &= \sqrt{-1}\cdot\sqrt{9^2}\\ &= 9i\end{aligned}$$

22. $(-3i)(-7i)(2i)$

SOLUTION:

$$\begin{aligned}(-3i)(-7i)(2i) &= (-3\cdot -7\cdot 2)(i\cdot i\cdot i)\\ &= (-3\cdot -7\cdot 2)(-1\cdot i)\\ &= -42i\end{aligned}$$

23. $4i(-6i)^2$

SOLUTION:

$$\begin{aligned}4i(-6i)^2 &= (4i)(36i^2)\\ &= (-144)(i)\\ &= -144i\end{aligned}$$

24. i^{11}

SOLUTION:

$$\begin{aligned}i^{11} &= i^{10}\cdot i\\ &= (i^2)^5\cdot i\\ &= -1\cdot i\\ &= -i\end{aligned}$$

25. i^{25}

SOLUTION:

$$\begin{aligned}i^{25} &= i^{24}\cdot i\\ &= (i^2)^{12}\cdot i\\ &= 1\cdot i\\ &= i\end{aligned}$$

26. $(10 - 7i) + (6 + 9i)$

SOLUTION:

$$\begin{aligned}(10-7i)+(6+9i) &= (10+6)+(-7i+9i)\\ &= 16+2i\end{aligned}$$

27. $(-3 + i) + (-4 - i)$

SOLUTION:

$$\begin{aligned}(-3+i)+(-4-i) &= (-3-4)+(i-i)\\ &= -7\end{aligned}$$

28. $(12 + 5i) - (9 - 2i)$

SOLUTION:

$$\begin{aligned}(12+5i)-(9-2i) &= 12+5i-9+2i\\ &= 3+7i\end{aligned}$$

29. $(11 - 8i) - (2 - 8i)$

SOLUTION:

$$\begin{aligned}(11-8i)-(2-8i) &= 11-8i-2+8i\\ &= 9\end{aligned}$$

30. $(1 + 2i)(1 - 2i)$

SOLUTION:

$$\begin{aligned}(1+2i)(1-2i) &= 1(1)+1(-2i)+2i(1)+2i(-2i)\\ &= 1-2i+2i-4i^2\\ &= 1-2i+2i-4(-1)\\ &= 1+4\\ &= 5\end{aligned}$$

31. $(3 + 5i)(5 - 3i)$

SOLUTION:

$$\begin{aligned}(3+5i)(5-3i) &= 3(5)+3(-3i)+5i(5)+5i(-3i)\\ &= 15-9i+25i-15i^2\\ &= 15-9i+25i+15\\ &= 30+16i\end{aligned}$$

32. $(4 - i)(6 - 6i)$

SOLUTION:

$$\begin{aligned}(4-i)(6-6i) &= 4(6)+4(-6i)-i(6)-i(-6i)\\ &= 24-24i-6i+6i^2\\ &= 24-24i-6i-6\\ &= 18-30i\end{aligned}$$

33. $\frac{2i}{1+i}$

SOLUTION:

$$\begin{aligned}\frac{2i}{1+i} &= \frac{2i}{1+i}\cdot\frac{1-i}{1-i}\\ &= \frac{2i(1-i)}{(1+i)(1-i)}\\ &= \frac{2i-2i^2}{1-i^2}\\ &= \frac{2i+2}{1+1}\\ &= \frac{2i+2}{2}\\ &= 1+i\end{aligned}$$

34. $\frac{5}{2+4i}$

SOLUTION:

$$\begin{aligned}\frac{5}{2+4i} &= \frac{5}{2+4i}\cdot\frac{2-4i}{2-4i}\\ &= \frac{5(2-4i)}{(2+4i)(2-4i)}\\ &= \frac{10-20i}{4-16i^2}\\ &= \frac{10-20i}{4+16}\\ &= \frac{10-20i}{20}\\ &= \frac{1}{2}-i\end{aligned}$$

35. $\frac{5+i}{3i}$

SOLUTION:

$$\begin{aligned}\frac{5+i}{3i} &= \frac{5+i}{3i}\cdot\frac{3i}{3i}\\ &= \frac{3i(5+i)}{9i^2}\\ &= \frac{15i+3i^2}{9i^2}\\ &= \frac{15i+3(-1)}{9(-1)}\\ &= \frac{15i-3}{-9}\\ &= \frac{1}{3}-\frac{5}{3}i\end{aligned}$$

Solve each equation.

36. $4x^2+4=0$

SOLUTION:

$$\begin{aligned}4x^2+4&=0\\4x^2&=-4\\x^2&=-1\\x&=\pm\sqrt{-1}\\x&=\pm i\end{aligned}$$

37. $3x^2+48=0$

SOLUTION:

$$\begin{aligned}3x^2+48&=0\\3x^2&=-48\\x^2&=-16\\x&=\pm\sqrt{-16}\\x&=\pm 4i\end{aligned}$$

38. $2x^2+50=0$

SOLUTION:

$$\begin{aligned}2x^2+50&=0\\2x^2&=-50\\x^2&=-25\\x&=\pm\sqrt{-25}\\x&=\pm 5i\end{aligned}$$

39. $2x^2+10=0$

SOLUTION:

$$\begin{aligned}2x^2+10&=0\\2x^2&=-10\\x^2&=-5\\x&=\pm\sqrt{-5}\\x&=\pm i\sqrt{5}\end{aligned}$$

40. $6x^2+108=0$

SOLUTION:

$$\begin{aligned}6x^2+108&=0\\6x^2&=-108\\x^2&=-18\\x&=\pm\sqrt{-18}\\x&=\pm 3i\sqrt{2}\end{aligned}$$

41. $8x^2+128=0$

SOLUTION:

$$\begin{aligned}8x^2+128&=0\\8x^2&=-128\\x^2&=-16\\x&=\pm\sqrt{-16}\\x&=\pm 4i\end{aligned}$$

Find the values of x and y that make each equation true.

42. $9 + 12i = 3x + 4yi$

SOLUTION:

Set the real parts equal to each other.

$$9 = 3x$$
$$3 = x$$

Set the imaginary parts equal to each other.

$$12 = 4y$$
$$3 = y$$

43. $x + 1 + 2yi = 3 - 6i$

SOLUTION:

Set the real parts equal to each other.

$$x + 1 = 3$$
$$x = 3 - 1$$
$$x = 2$$

Set the imaginary parts equal to each other.

$$2y = -6$$
$$y = -3$$

44. $2x + 7 + (3 - y)i = -4 + 6i$

SOLUTION:

Set the real parts equal to each other.

$2x + 7 = -4$
$2x + 7 - 7 = -4 - 7$
$2x = -11$

$$x = -\frac{11}{2}$$

Set the imaginary parts equal to each other.

$3 - y = 6$
$y = -3$

45. $5 + y + (3x - 7)i = 9 - 3i$

SOLUTION:

Set the real parts equal to each other.

$$5 + y = 9$$
$$y = 4$$

Set the imaginary parts equal to each other.

$$3x - 7 = -3$$
$$3x - 7 + 7 = -3 + 7$$
$$3x = 4$$
$$x = \frac{4}{3}$$

46. $a + 3b + (3a - b)i = 6 + 6i$

SOLUTION:

Set the real parts equal to each other.

$a + 3b = 6 \rightarrow (1)$

Set the imaginary parts equal to each other.

$3a - b = 6 \rightarrow (2)$

Multiply the second equation by 3 and add the resulting equation to (1).

$$\begin{aligned} a + 3b &= 6 \\ \underline{9a - 3b = 18} &\quad (+) \\ 10a \quad &= 24 \\ a &= \frac{24}{10} \\ a &= \frac{12}{5} \end{aligned}$$

Substitute $a = \frac{12}{5}$ in (1).

$$\begin{aligned} \frac{12}{5} + 3b &= 6 \\ \frac{12 + 15b}{5} &= 6 \\ \frac{12 + 15b}{5} \cdot 5 &= 6 \cdot 5 \\ 12 + 15b &= 30 \\ 15b &= 18 \\ b &= \frac{18}{15} \\ b &= \frac{6}{5} \end{aligned}$$

47. $(2a - 4b)i + a + 5b = 15 + 58i$

SOLUTION:

Set the real parts equal to each other.

$a + 5b = 15 \rightarrow (1)$

Set the imaginary parts equal to each other.

$2a - 4b = 58 \rightarrow (2)$

Multiply the first equation by 2 and subtract the second equation from the resulting equation.

$$\begin{aligned} 2a + 10b &= 30 \\ \underline{2a - 4b = 58} &\quad (-) \\ 14b &= -28 \\ b &= -2 \end{aligned}$$

Substitute $b = -2$ in (1).

$$\begin{aligned} a + 5(-2) &= 15 \\ a - 10 &= 15 \\ a &= 25 \end{aligned}$$

Simplify.

48. $\sqrt{-10} \cdot \sqrt{-24}$

SOLUTION:

$$\begin{aligned} \sqrt{-10} \cdot \sqrt{-24} &= \sqrt{-1 \cdot 2 \cdot 5} \cdot \sqrt{-2 \cdot 2 \cdot 2 \cdot 3} \\ &= \sqrt{-1} \cdot \sqrt{2} \cdot \sqrt{5} \cdot \sqrt{-1} \cdot 2 \cdot \sqrt{2} \cdot \sqrt{3} \\ &= i \cdot 2 \cdot \sqrt{15} \cdot i \cdot 2 \\ &= -4\sqrt{15} \end{aligned}$$

49. $4i\left(\frac{1}{2}i\right)^2(-2i)^2$

SOLUTION:

$$\begin{aligned} 4i\left(\frac{1}{2}i\right)^2(-2i)^2 &= 4i\left(\frac{1}{2}\right)^2 i^2(-2)^2 i^2 \\ &= 4i\left(\frac{1}{4}\right)(-1)(4)(-1) \\ &= 4i \end{aligned}$$

50. i^{41}

SOLUTION:

$$\begin{aligned} i^{41} &= i^{40} \cdot i \\ &= \left(i^2\right)^{20} \cdot i \\ &= 1 \cdot i \\ &= i \end{aligned}$$

51. (4 – 6*i*) + (4 + 6*i*)

SOLUTION:

(4 – 6*i*) + (4 + 6*i*) = 4 + 4 – 6*i* + – 6*i*
= 8

52. (8 – 5*i*) – (7 + *i*)

SOLUTION:

(8 – 5*i*) – (7 + *i*) = 8 – 5*i* – 7 – *i*
= 1 – 6*i*

53. (–6 – *i*)(3 – 3*i*)

SOLUTION:

$$\begin{aligned} (-6-i)(3-3i) &= -6(3) - 6(-3i) - i(3) - i(-3i) \\ &= -18 + 18i - 3i - 3 \\ &= -21 + 15i \end{aligned}$$

54. $\dfrac{(5+i)^2}{3-i}$

SOLUTION:

$$\begin{aligned} \frac{(5+i)^2}{3-i} &= \frac{(5+i)^2}{3-i} \cdot \frac{3+i}{3+i} \\ &= \frac{(5+i)^2(3+i)}{(3-i)(3+i)} \\ &= \frac{(25-1+10i)(3+i)}{9+1} \\ &= \frac{(24+10i)(3+i)}{10} \\ &= \frac{72+30i+24i+10i^2}{10} \\ &= \frac{72+30i+24i-10}{10} \\ &= \frac{62+54i}{10} \\ &= \frac{31}{5} + \frac{27}{5}i \end{aligned}$$

55. $\dfrac{6-i}{2-3i}$

SOLUTION:

$$\begin{aligned} \frac{6-i}{2-3i} &= \frac{6-i}{2-3i} \cdot \frac{2+3i}{2+3i} \\ &= \frac{(6-i)(2+3i)}{(2-3i)(2+3i)} \\ &= \frac{12+18i-2i-3i^2}{4+9} \\ &= \frac{12+18i-2i+3}{13} \\ &= \frac{15+16i}{13} \\ &= \frac{15}{13} + \frac{16}{13}i \end{aligned}$$

3-2 Complex Numbers

56. $(-4 + 6i)(2 - i)(3 + 7i)$

SOLUTION:

$$\begin{aligned}&(-4+6i)(2-i)(3+7i)\\&=(-4(2)-4(-i)+6i(2)+6i(-i))(3+7i)\\&=(-8+4i+12i+6)(3+7i)\\&=(-2+16i)(3+7i)\\&=-2(3)-2(7i)+16i(3)+16i(7i)\\&=-6-14i+48i-112\\&=-118+34i\end{aligned}$$

57. $(1 + i)(2 + 3i)(4 - 3i)$

SOLUTION:

$$\begin{aligned}&(1+i)(2+3i)(4-3i)\\&=(1(2)+1(3i)+i(2)+i(3i))(4-3i)\\&=(2+3i+2i-3)(4-3i)\\&=(-1+5i)(4-3i)\\&=-1(4)-1(-3i)+5i(4)+5i(-3i)\\&=-4+3i+20i+15\\&=11+23i\end{aligned}$$

58. $\dfrac{4-i\sqrt{2}}{4+i\sqrt{2}}$

SOLUTION:

$$\begin{aligned}\frac{4-i\sqrt{2}}{4+i\sqrt{2}}&=\frac{4-i\sqrt{2}}{4+i\sqrt{2}}\cdot\frac{4-i\sqrt{2}}{4-i\sqrt{2}}\\&=\frac{\left(4-i\sqrt{2}\right)\left(4-i\sqrt{2}\right)}{\left(4+i\sqrt{2}\right)\left(4-i\sqrt{2}\right)}\\&=\frac{\left(16-2-8i\sqrt{2}\right)}{16+2}\\&=\frac{14-8i\sqrt{2}}{18}\\&=\frac{7}{9}-\frac{4i\sqrt{2}}{9}\end{aligned}$$

59. $\dfrac{2-i\sqrt{3}}{2+i\sqrt{3}}$

SOLUTION:

$$\begin{aligned}\frac{2-i\sqrt{3}}{2+i\sqrt{3}}&=\frac{2-i\sqrt{3}}{2+i\sqrt{3}}\cdot\frac{2-i\sqrt{3}}{2-i\sqrt{3}}\\&=\frac{\left(2-i\sqrt{3}\right)\left(2-i\sqrt{3}\right)}{\left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}\right)}\\&=\frac{\left(4-3-4i\sqrt{3}\right)}{4+3}\\&=\frac{1-4i\sqrt{3}}{7}\\&=\frac{1}{7}-\frac{4i\sqrt{3}}{7}\end{aligned}$$

60. **ELECTRICITY** The impedance in one part of a series circuit is $7 + 8j$ ohms, and the impedance in another part of the circuit is $13 - 4j$ ohms. Add these complex numbers to find the total impedance in the circuit.

SOLUTION:

Total impedance = $7 + 8j + 13 - 4j$
= $20 + 4j$ ohms

ELECTRICITY Use the formula $V = C \cdot I$.

61. The current in a circuit is $3 + 6j$ amps, and the impedance is $5 - j$ ohms. What is the voltage?

SOLUTION:
We know that voltage can be calculated by $V = C \cdot I$.
V = Voltage
C = current
I = impedance

$$V = (3+6j)(5-j)$$
$$= 15 - 3j + 30j + 6$$
$$= 21 + 27j$$

Therefore, the voltage is $21 + 27j$ Volts.

62. The voltage in a circuit is $20 - 12j$ volts, and the impedance is $6 - 4j$ ohms. What is the current?

SOLUTION:
We know that voltage can be calculated by $V = C \cdot I$.
V = Voltage
C = current
I = impedance

$$20 - 12j = I(6 - 4j)$$
$$I = \frac{20-12j}{6-4j}$$
$$= \frac{20-12j}{6-4j} \cdot \frac{6+4j}{6+4j}$$
$$= \frac{(20-12j)(6+4j)}{(6-4j)(6+4j)}$$
$$= \frac{120+80j-72j+48}{36+16}$$
$$= \frac{168+8j}{52}$$
$$= \frac{42}{13} + \frac{2}{13}j$$

Therefore, the current is $\frac{42}{13} + \frac{2}{13}j$ Amps.

63. Find the sum of $ix^2 - (4 + 5i)x + 7$ and $3x^2 + (2 + 6i)x - 8i$.

SOLUTION:
$ix^2 - (4 + 5i)x + 7 + 3x^2 + (2 + 6i)x - 8i$
$= (3 + i)x^2 - 5ix - 4x + 2x + 6ix + 7 - 8i$
$= (3 + i)x^2 + ix - 2x + 7 - 8i$
$= (3 + i)x^2 + (-2 + i)x + 7 - 8i$

64. Simplify $[(2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 + (5 - 5i)x - 6]$.

SOLUTION:
$[(2 + i)x^2 - ix + 5 + i] - [(-3 + 4i)x^2 + (5 - 5i)x - 6]$
$= [(2 + i)x^2 - ix + 5 + i] - (-3 + 4i)x^2 - (5 - 5i)x + 6$
$= 2x^2 + ix^2 - ix + 5 + i + 3x^2 - 4ix^2 - 5x + 5ix + 6$
$= 5x^2 - 3ix^2 + i - 5x + 4ix + 11$
$= (5 - 3i)x^2 + (-5 + 4i)x + i + 11$

65. **MULTIPLE REPRESENTATIONS** In this problem, you will explore quadratic equations that have complex roots. Use a graphing calculator.

a. Algebraic Write a quadratic equation in standard form with 3i and -3i as its roots.

b. Graphical Graph the quadratic equation found in part **a** by graphing its related function.

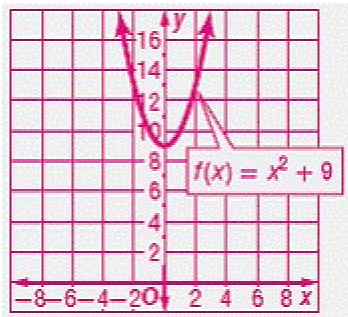
c. Algebraic Write a quadratic equation in standard form with $2 + i$ and $2 - i$ as its roots.

d. Graphical Graph the related function of the quadratic equation you found in part **c**. Use the graph to find the roots if possible. Explain.

e. Analytical How do you know when a quadratic equation will have only complex solutions?

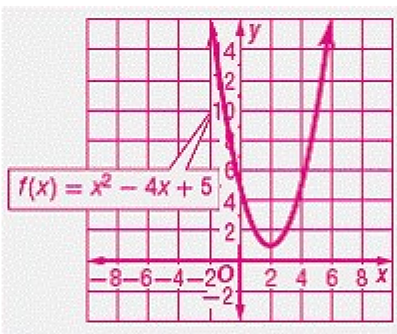
SOLUTION:
a. Sample answer: $x^2 + 9 = 0$

b.



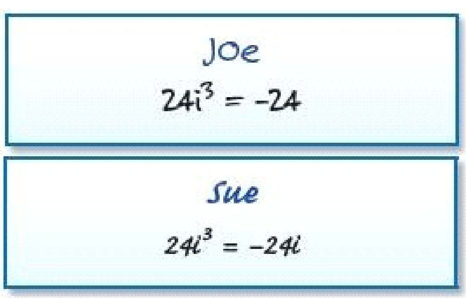
c. Sample answer: $x^2 - 4x + 5 = 0$

d.



e. Sample answer: A quadratic equation will have only complex solutions when the graph of the related function has no *x*-intercepts.

66. **CCSS CRITIQUE** Joe and Sue are simplifying $(2i)(3i)(4i)$. Is either of them correct? Explain your reasoning.



SOLUTION:

Sue; $i^3 = -i$, not -1.

67. **CHALLENGE** Simplify $(1 + 2i)^3$.

SOLUTION:

$$\begin{aligned}(1+2i)^3 &= (1+2i)(1+2i)(1+2i)\\ &= (1-4+4i)(1+2i)\\ &= (-3+4i)(1+2i)\\ &= -3-6i+4i-8\\ &= -11-2i\end{aligned}$$

68. **REASONING** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain your reasoning.

Every complex number has both a real part and an imaginary part.

SOLUTION:

Sample answer: Always. The value of 5 can be represented by $5 + 0i$, and the value of 3i can be represented by $0 + 3i$.

69. **OPEN ENDED** Write two complex numbers with a product of 20.

SOLUTION:

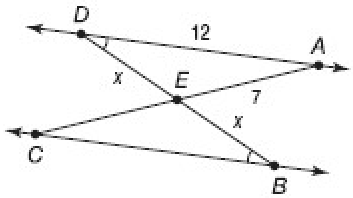
Sample answer: $(4 + 2i)(4 - 2i)$

70. **WRITING IN MATH** Explain how complex numbers are related to quadratic equations.

SOLUTION:

Some quadratic equations have complex solutions and cannot be solved using only the real numbers.

71. **EXTENDED RESPONSE** Refer to the figure to answer the following.



a. Name two congruent triangles with vertices in correct order.

b. Explain why the triangles are congruent.

c. What is the length of $\overline{EC}$? Explain your procedure.

SOLUTION:

a. $\Delta CBE \cong \Delta ADE$

b. $\angle AED \cong \angle CEB$ (Vertical angles)
$\overline{DE} \cong \overline{BE}$ (Both have length *x*.)
$\angle ADE \cong \angle CBE$ (Given)
Consecutive angles and the included side are all congruent, so the triangles are congruent by the ASA Property.

c. $\overline{EC} \cong \overline{EA}$ by CPCTC (corresponding parts of congruent triangles are congruent.) *EA* = 7, so *EC* = 7.

72. $(3 + 6)^2 =$

A $2 \times 3 + 2 \times 6$

B 9^2

C $3^2 + 6^2$

D $3^2 \times 6^2$

SOLUTION:

$(3 + 6)^2 = 9^2$

So, the correct option is B.

73. **SAT/ACT** A store charges $49 for a pair of pants. This price is 40% more than the amount it costs the store to buy the pants. After a sale, any employee is allowed to purchase any remaining pairs of pants at 30% off the store's cost.
How much would it cost an employee to purchase the pants after the sale?

F $10.50

G $12.50

H $13.72

J $24.50

K $35.00

SOLUTION:

Let *x* be the original amount of the pants.

$\$49 = 40\%x + x$

$\$49 = 0.4x + x$

$\$49 = 1.4x$

$x = \$35$

$\$35 \cdot \frac{30}{100} = \$10.50$

$\$35 - \$10.50 = \$24.50$

So, the correct option is J.

74. What are the values of x and y when $(5 + 4i) - (x + yi) = (-1 - 3i)$?

A $x = 6, y = 7$

B $x = 4, y = i$

C $x = 6, y = i$

D $x = 4, y = 7$

SOLUTION:

Set the real parts equal to each other.

$5 - x = -1$

$x = 6$

Set the imaginary parts equal to each other.

$4 - y = -3$

$y = 7$

So, the correct option is A.

Solve each equation by factoring.

75. $2x^2 + 7x = 15$

SOLUTION:

Write the equation with right side equal to zero.

$2x^2 + 7x - 15 = 0$

Find factors of 2(–15) = –30 whose sum is 7.

10(–3) = –30 and 10 + (–3) = 7

$$2x^2 + 10x - 3x - 15 = 0$$
$$2x(x+5) - 3(x+5) = 0$$
$$(x+5)(2x-3) = 0$$
$$\Rightarrow x + 5 = 0 \quad \text{or } 2x - 3 = 0$$
$$\Rightarrow \quad x = -5 \quad \text{or} \quad x = \frac{3}{2}$$

Therefore, the roots are -5 and $\frac{3}{2}$.

76. $4x^2 - 12 = 22x$

SOLUTION:

Write the equation with right side equal to zero.

$4x^2 - 22x - 12 = 0$

Find factors of 4(–12) = –48 whose sum is –22.

–24(2) = –48 and 2 + (–24) = –22

$$4x^2 - 24x + 2x - 12 = 0$$
$$4x(x-6) + 2(x-6) = 0$$
$$(x-6)(4x+2) = 0$$
$$\Rightarrow x - 6 = 0 \text{ or } 4x + 2 = 0$$
$$\Rightarrow x = 6 \quad \text{or} \quad x = -\frac{1}{2}$$

Therefore, the roots are $-\frac{1}{2}$ and 6.

77. $6x^2 = 5x + 4$

SOLUTION:

Write the equation with right side equal to zero.

$6x^2 - 5x - 4 = 0$

Find factors of 6(–4) = –24 whose sum is –5.

–8(3) = –24 and 3+ (–8) = –5

$$6x^2 - 8x + 3x - 4 = 0$$
$$2x(3x-4) + 1(3x-4) = 0$$
$$(2x+1)(3x-4) = 0$$
$$\Rightarrow 2x + 1 = 0 \text{ or } 3x - 4 = 0$$
$$\Rightarrow x = -\frac{1}{2} \quad \text{or} \quad x = \frac{4}{3}$$

Therefore, the roots are $-\frac{1}{2}$ and $\frac{4}{3}$.

Determine whether each trinomial is a perfect square trinomial. Write *yes* or *no*.

78. $x^2 - 12x + 36$

SOLUTION:

$x^2 - 12x + 36$ can be written as $(x - 6)^2$.

So, $x^2 - 12x + 36$ is a perfect square trinomial. The answer is "yes".

79. $x^2 + 8x - 16$

SOLUTION:

We cannot write the given trinomial as the perfect square format. So, the answer is "no".

80. $x^2 - 14x - 49$

SOLUTION:

We cannot write the given trinomial as the perfect square format. So, the answer is "no".

81. $x^2 + x + 0.25$

SOLUTION:

$x^2 + x + 0.25$ can be written as $(x + 0.5)^2$.

So, $x^2 + x + 0.25$ is a perfect square trinomial. The answer is "yes".

82. $x^2 + 5x + 6.25$

SOLUTION:

$x^2 + 5x + 6.25$ can be written as $(x + 2.5)^2$.

So, $x^2 + 5x + 6.25$ is a perfect square trinomial. The answer is "yes".