Complex Numbers

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September 9, 2009

The Initial Problem

Consider the equation:

$$x^2 + 1 = 0.$$

Try to solve by extracting square roots:

$$x^2 = -1$$

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The Imaginary Unit i

There is no **real** number x such that $x^2 = -1$.

So mathematicians invented such a number and called it i. Hence $i^2 = -1$.

The number i is called the **imaginary unit**. We will build the complex number system from the imaginary unit.

Other solutions to $x^2 = -1$?

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Why Consider Complex Numbers?

Complex Numbers have a range of applications from electrical engineering to fluid flows.

Complex numbers also have a great deal of mathematical applications. For example, we will be able find solutions to **any** quadratic equations.

Later in the course we will learn the *Fundamental Theorem of Algebra*, which says that **any** polynomial equation has a complex solution.

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Outline

1. The Complex Number System
2. Solving Quadratic Equations with Complex Numbers
3. Arithmetic with Complex Numbers

Imaginary Numbers

Consider the equation

$$x^2 + 4 = 0.$$

Extract square roots:

$$x^2 = -4$$

$$x = \pm\sqrt{-4}.$$

Let us use some rules of radicals:

$$\sqrt{-4} = \sqrt{4 \times -1} = (\sqrt{4})(\sqrt{-1}) = 2\mathrm{i}.$$

In general, it makes sense to have numbers $b\mathrm{i}$ for any real number b. These are called **imaginary numbers**.

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Principal Square Roots

Recall that the radical symbol $\sqrt{}$ means the **positive** square root when applied to a positive real number.

Similarly, if a is a positive real number, the symbol $\sqrt{-a}$ will denote

$$\sqrt{-a} = (\sqrt{a})\mathrm{i}.$$

This is called the **principal square root**.

For example, consider problem 41 on page 139. What is wrong with the following?

$$\sqrt{-6} \times \sqrt{-6} = \sqrt{(-6) \times (-6)} = \sqrt{36} = 6.$$

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A Quadratic Equation

Let us solve the quadratic equation

$$x^2 + 2x + 5 = 0.$$

We should use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This gives us

$$x = -1 \pm 2\mathrm{i}.$$

So it makes sense to add real numbers to imaginary numbers and consider $a + b\mathrm{i}$ for any real numbers a and b.

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Complex Numbers

The set of all numbers of the form $a + bi$ where a and b are real is called the set of **complex numbers**, written $\mathbb{C}$.

- When a complex number is written $a + bi$ with a and b real, it is said to be in **standard form**.
- For a complex number $a + bi$ written in standard form, a is called the **real part** and b is called the **imaginary part**.
- Are real numbers also complex?

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- Are real numbers also complex?

Conjugates

For our equation $x^2 + 2x + 5 = 0$, we had two solutions:

$$x_1 = -1 + 2\mathrm{i} \quad \text{and} \quad x_2 = -1 - 2\mathrm{i}.$$

What are the real and imaginary parts? A pair of complex numbers such as this are called **conjugates**.

In general, two complex numbers $a + b\mathrm{i}$ and $a - b\mathrm{i}$ (in standard form) are called **conjugates**.

For example, what is the conjugate of $4 - 3\mathrm{i}$?

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Solving a Quadratic with Complex Solutions I

Let us solve the equation

$$x^2 - 3x + 4 = 0.$$

Use the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

What are a, b and c in this problem? This gives us:

$$x = \frac{3 \pm \sqrt{9 - 16}}{2}$$

$$= \frac{3 \pm \sqrt{-7}}{2}$$

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Solving a Quadratic with Complex Solutions II

The solutions are

$$x_1 = \frac{3}{2} + \frac{\sqrt{7}}{2}\mathrm{i} \quad \text{and} \quad x_2 = \frac{3}{2} - \frac{\sqrt{7}}{2}.$$

Notice that the two solutions are conjugates!

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How Many Roots are there of 1?

Here are four related questions:

1. How many square roots of 1 are there?
2. How many cube roots of 1 are there?
3. How many fourth roots of 1 are there?
4. How many sixth roots of 1 are there?

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Roots of 1 as Solutions to Equations

The square roots of 1 are solutions to the equation $x^2 = 1$, which is equivalent to $x^2 - 1 = 0$.

By factoring, $x^2 - 1 = (x - 1)(x + 1)$, so the square roots of 1 are the solutions $x = 1$ and $x = -1$.

Similarly: cube roots of 1 are solutions to $x^3 - 1 = 0$,

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Exercise

Use the formulae on page 37 for factoring the difference of squares, difference of cubes and sum of cubes together with the quadratic formula to completely factor the left hand sides and solve the following equations:

1. $x^3 - 1 = 0$
2. $x^4 - 1 = 0$
3. $x^6 - 1 = 0.$

The solutions are:

1. $x^3 - 1 = 0$, $x = 1, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$;
2. $x^4 - 1 = 0$, $x = \pm 1, \pm \mathrm{i}$; and
3. $x^6 - 1 = 0$, $x = \pm 1, \frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}, -\frac{1}{2} \pm \frac{\sqrt{3}}{2}\mathrm{i}$.

In general, for any positive whole number n, there are n distinct n^{th} roots of 1.

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In general, for any positive whole number n, there are n distinct n^{th} roots of 1.

Apples and Oranges

Suppose you have 2 apples and 5 oranges. Then somebody gives you 3 applies and 4 oranges. How many apples and how many oranges do you have?

Suppose you have 2 apples and 5 oranges. Now somebody steals 1 apple and 2 oranges. How many apples and oranges do you have left?

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Addition and Subtraction of Complex Numbers

Adding and subtracting complex numbers is like adding baskets of apples and oranges. Treat the real part like apples and the imaginary part like oranges.

Thus $(2 + 5i) + (3 + 4i) = 5 + 9i$.

Additionally, $(2 + 5i) - (1 + 2i) = 1 + 3i$.

In general, to add or subtract complex numbers, you simply add or subtract the real and imaginary parts separately.

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Examples

Perform the indicated operations and write in standard form:

1. $(2 - 7i) + (-1 + 3i)$.
2. $(-1 - 4i) + (-1 + 10i)$.
3. $(6 - 7i) - (2 + 2i)$.
4. $(-4 + i) - 3i$.

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1. $(2-7i)+(-1+3i)$.
2. $(-1-4i)+(-1+10i)$.
3. $(6-7i)-(2+2i)$.
4. $(-4+i)-3i$.

Distributive Property

To multiply two complex numbers, we use the distributive property (you may use FOIL) and the fact that $i^2 = -1$.

For example, to multiply $2 + \mathrm{i}$ by $3 + 2\mathrm{i}$:

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Examples

Perform the indicated operations and write in standard form:

1. $(2 - 7i)(-1 + 3i)$.
2. $(-1 - 4i)(-1 + 10i)$.
3. $(6 - 7i)(2 + 2i)$.
4. $3i(-4 + i)$
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Multiplying by the Conjugate

The product of a complex number and its conjugate is real. For example:

- $(1 + 2\mathrm{i})(1 - 2\mathrm{i}) = 1 + 2\mathrm{i} - 2\mathrm{i} - 4\mathrm{i}^2 = 1 + 4 = 5.$
- $(3 + 4\mathrm{i})(3 - 4\mathrm{i}) = 9 + 12\mathrm{i} - 12\mathrm{i} - 16\mathrm{i}^2 = 25.$

Do exercise 91 on page 140 if you want to convince yourself that this is true in general.

Note that this gives us a factorization for the **sum of squares**: $a^2 + b^2 = (a + b\mathrm{i})(a - b\mathrm{i})$.

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Assignment

Section 1.5, pages 139 - 140, problems 5 - 11(o), 17 - 23 (o), 27 - 39 (o), 65 - 71(o). Due Friday, Sept. 11.