Comprehension Strategies

Comprehension Strategies applied to Mathematics

This document is the seventh in a series of support materials. It contains a synthesis of material from a variety of on-line and printed sources. It has been designed to support the Northern Adelaide Region Comprehension focus 2010-2013

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Comprehension and Mathematics: Comprehension Strategies applied to Mathematics

In order for students to be successful in the maths classroom they must be able to find the meaning of a maths problem and look for approaches to a possible solution. Students must analyse and make conjectures about information. They need to analyse situations to make connections and plan solutions. Reading comprehension and writing strategies are parallel to strategies students need to be mathematically proficient.

Much like literacy, students need to self-monitor, evaluate their progress and ask questions when necessary. They need to be flexible in using different properties of math operations. They need to move freely and fluently between equations, verbal descriptions, tables, graphs, etc. Students need to verify their answers to math problem solving pieces just as students need to monitor for meaning while reading. They continually need to ask themselves, does this make sense? Asking questions is at the heart of a thoughtful reader and it is also at the heart of a good mathematician.

As with literacy, students need to clearly share their thinking and understand that there are many approaches to solving complex problems. Middle level math students need to be able to transform math problems into algebraic expressions representing a problem symbolically. Students need to be able to make justifications and support mathematical arguments. They need to make conjectures and build a logical progression of ideas to support them. They need to communicate concisely and use precise vocabulary and symbols to justify their conclusions.

Students often get confused because words and phrases that mean one thing in the world of mathematics mean another in every day context. For example, the word “similar” means “alike” in everyday usage, whereas in mathematics similar has to have proportionality. For example “similar” figures must have a relationship where corresponding sides of two shapes are proportional and corresponding angles are equal. “Similar” in mathematics, as with many other vocabulary words, has a much more profound meaning than in every day usage.

In addition to vocabulary, math has specialised symbols and technical language that students find confusing. Math operations have a variety of ways they can be represented. Symbols may be confusing because they look alike. For example the division ⁄ and square root symbols √ are visually similar but have very different meanings. Different representations may be used to describe the same process such as, 2•3, 2*3, (2)(3) and 2×3 all have the same implications for multiplication. In literacy students need to be cognisant of the fact that homophones and homonyms have different spellings and meanings. Likewise, students need to be aware of confusing mathematical terms and symbols and have the strategies to deal with them when being a mathematician.

For this reason, math classroom environments need to provide rich text, print and mathematical representations. Word walls are a technique that many classroom teachers use to help student become fluent with the language of mathematics. It is vital that vocabulary be taught as part of a lesson and not be taught as a separate activity.

As with literacy strategies, modeling is an essential and significant step for teaching math strategies. People who teach math must be mathematicians. Teachers must show students that math is not always easy for them and model how they genuinely struggle with problems. Students often struggle to understand the meaning of content area text. Teachers must give students the strategies and tools to tackle challenging mathematics. Students must be aware that struggling with content is necessary and a vital part of the learning process. Modeling helps teachers to build confidence and trust in their students giving them strategies to grapple with challenging material.

Visualising is an especially helpful strategy for the math student as it is for the literacy student. In order to solve the math problems, students should be urged to diagram how they interpret the math text. The students’ diagrams can also be used as formative assessment. The teacher can identify misconceptions the students may have around the math content and use the information to intervene.

Students need to be aware that the strategies are very much the same and can be used across content areas. Students often times feel that they are learning everything in isolation. We as educators need to help students see that there are connections between content areas and help them make these connections on a daily basis.

Effective Comprehension Strategies of Proficient Math & Science Learners

Strategies to Use Before Reading	Strategies to Use During Reading	Strategies to Use After Reading
• Activating Prior Knowledge • Making Connections • Predicting • Questioning	• Making Connections • Predicting • Questioning • Visualizing • Determining What is Important • Inferring • Synthesizing • Monitoring Comprehension	• Evaluate Predictions • Questioning • Visualizing • Determining What was Important • Inferring • Synthesizing

Sources

Beers, K. **When Kids Can't Read, What Teachers Can Do.** Portsmouth, NH: Heinemann, 2003.
Fountas, I. and Pinnell, G. **Guding Readers & Writers (Grades 3-6): Teaching Comprehension, Genre, & Content Literacy.** Portsmouth, NH: Heinemann, 2001.
Keene, E. and S. Zimmermann. **Mosaic of Thought.** Portsmouth, NH: Heinemann, 1997.
Robb, L. **Teachign Reading in Middle School.** Broadway, NY: Scholastic, 2000.
Tovani, C. **I Read It, But I Don't Get It: Comprehension Strategies for Adolescent Readers.** Portland, ME: Stenhouse, 2000.

Monitoring Comprehension

Once you look at a "word problem," the reading connection is obvious. If a child is not a fluent reader and has to figure out the words in slow, often inaccurate, manner, there is little or no chance for the problem to be understood. But the connection goes deeper than this.

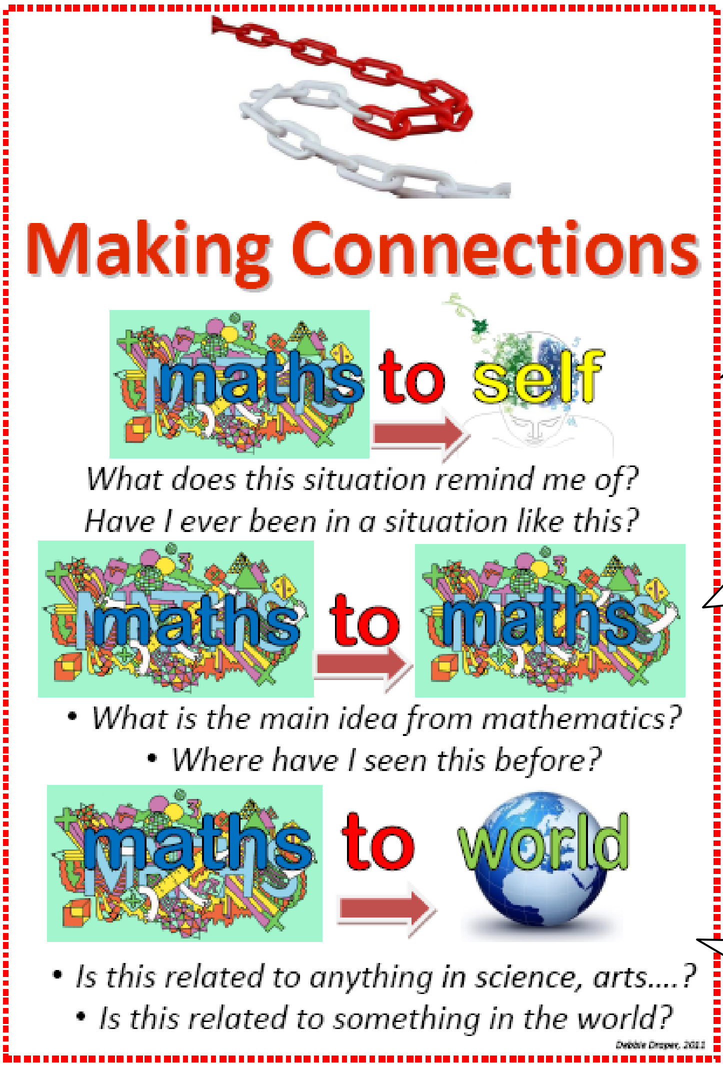
In order for students to be successful in the math classroom they must be able to find the meaning of a math problem and look for approaches to a possible solution. Students must analyse and make conjectures about information. They need to analyse situations to make connections and plan solutions. Reading comprehension and writing strategies are parallel to strategies students need to be mathematically proficient.

Much like literacy, students need to self-monitor, evaluate their progress and ask questions when necessary. They need to be flexible in using different properties of math operations. They need to move freely and fluently between equations, verbal descriptions, tables, graphs, etc.

Students need to verify their answers to math problem solving pieces just as students need to monitor for meaning while reading. They continually need to ask themselves, does this make sense? Asking questions is at the heart of a thoughtful reader and it is also at the heart of a good mathematician.

Making Connections

Reading teachers encourage students to make connections with stories, either text to self, text to text, or text to world. When we adapt these connections to mathematics, "we ask students to look for connections that are math-to-self (connecting math concepts to prior knowledge and experience); math-to-world (connecting math concepts to real-world situations, science, and social studies); and math-to-math (connecting math concepts within and between the branches of mathematics or connecting concepts and procedures."



Students need to be able to make connections between mathematics and their own lives.

Making connections across mathematical topics is important for developing conceptual understanding. For example, the topics of fractions, decimals, percentages, and proportions, while learning areas in their own right, can usefully be linked through exploration of differing representations (e.g., ½ = 50%) or through problems involving everyday contexts (e.g., determining fuel costs for a car trip).

Teachers can also help students to make connections to real experiences. When students find they can use mathematics as a tool for solving significant problems in their everyday lives, they begin to view the subject as relevant and interesting.

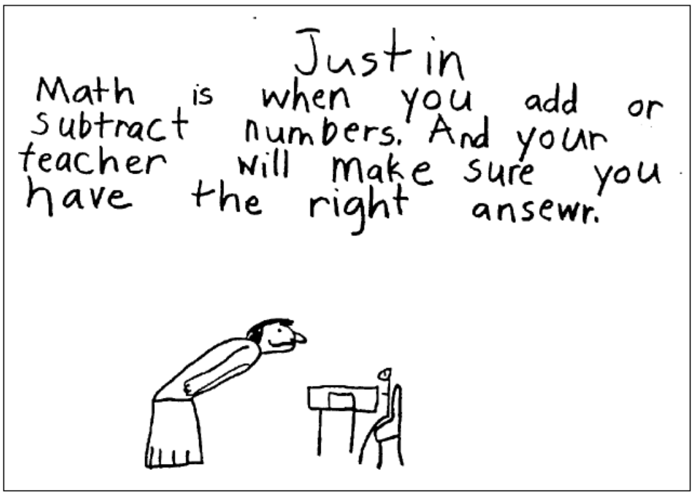
How is this relevant to my life?

One of the most common phrases that a maths teacher is likely to hear is the classic, "Why are we bothering to learn this, I will never use any of this in real life!" The simple answer to that question is "While a great deal of mathematics you learn may not be explicitly used later in life for most of you, the truth is that you learn it primarily as a means of education to the ends of exercising your brain.

This means your brain is better prepared to problem solve, and can you think of any areas in life where problem-solving ability might come in handy?" Besides the mental exercise aspect, it is no small fact that our entire world runs on numbers, applied though it may be. It is the language of the universe, of our cosmos.

Consider the checkout at the supermarket to the scale in your bathroom to the taxes you do every year to buying petrol to the receipt for anything you purchase to your phone number to your favourite team's sports statistics to weather predictions to how much food to buy for dinner to playing video games to anytime you count, measure, compare values to channel surfing to your address, geographic or digital IP to your watch to the calendar on the wall to ∞ and beyond!

Consider asking student to draw what mathematics is – in other words, draw their current “connections” to mathematics. Most students seem to see mathematics as calculation, something you do in school and do not make connections to their own life.



The importance of numbers – Give each pair of students a single page from a magazine and get them to work out how often on that page (both sides) numbers are written, mentioned, used in any way (highlighter pens could be used). You could leave it at this, with a brief discussion of how this demonstrates how frequently we refer to and need to use numbers. Or it could extended – make a class graph and do some whole class or group analysis appropriate to the age and ability of the students. (e.g. average number of numbers on each page, how many numbers in total in the magazine, which numbers occur most often? Are more of them written in words or in digits, etc?)

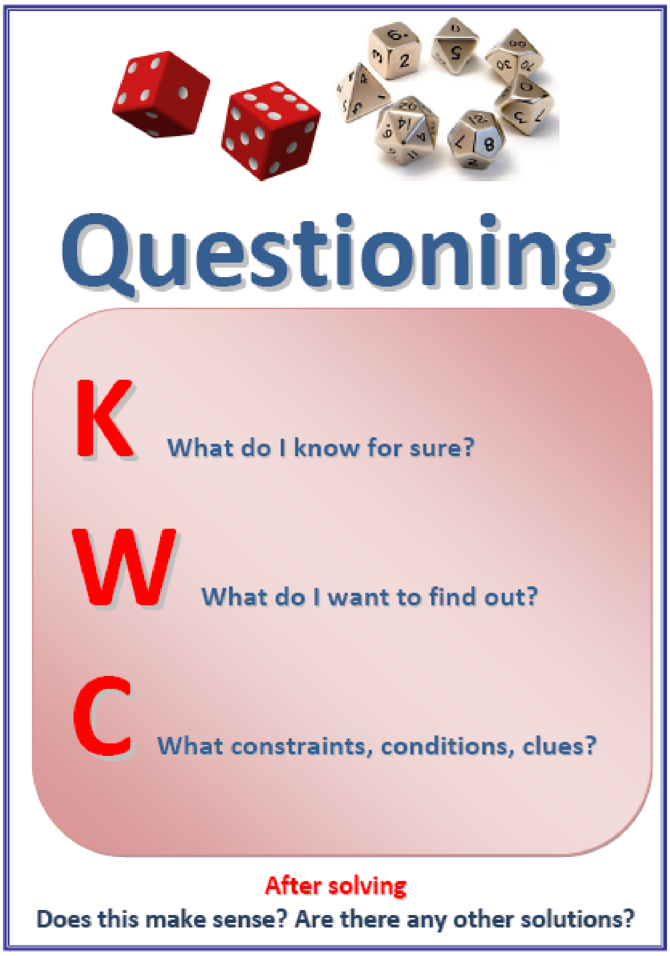
- K-W-L

The K-W-L strategy in reading helps to activate prior knowledge and peak interest in what's to come by asking "What do I know?", What do I want to learn more about?", and "What did I learn?" . Applied to math instruction, the K-W-L can be modified to K-W-C. Here the K stands for what is **k**nown, the W represents **w**hat is to be determined, and the C cautions the learner to look for special **c**onditions. This structure helps activate students' prior knowledge about mathematics and how it is used.

Using Maths in Real Life

http://www.ehow.com/how_7902055_use-math-everyday-situations.html
http://www.ehow.com/how_5894887_use-math-science-everyday-life.html
http://www.ehow.com/how_4966735_use-math-health-care-careers.html
http://www.ehow.com/how_8120188_use-math-medical-assisting.html
http://www.ehow.com/how_7426419_use-math-measure-beauty-face.html
http://www.ehow.com/how_8679431_use-math-create-dance-movements.html
http://www.ehow.com/info_8538732_ideas-games-having-do-jobs.html

Questioning



Questioning
The questions posed in mathematics classrooms are often low order, recall type questions that result in low levels of intellectual quality. To shift to a higher level of thinking, questions that foster deeper knowledge and access deeper understandings are required.

Questioning Strategies

The art of teaching is based on effective questioning strategies. Asking good questions is an informative process that needs development, refinement, and practice. Teaching through questioning is interactive and engages students by providing them with opportunities to share their thinking. The classroom should be a community of collaborative learners whose voices and ideas are valued.

In order to obtain more information from students during classroom discourse, we need to develop an open-ended questioning technique and use a more inquiring form of response, encouraging students to defend or explain both correct and incorrect responses. Here is an example of closed and open questioning for the same situation:

> Closed—**What** unit should be used to measure this room? (limiting)
> Open—**How** could we measure the length of this room? What choices of units do we have? **Why** would some units seem more appropriate than others? (probing—encourages students to think about several related ideas)

Good questioning involves responding to students in a manner that helps them think and lets you see what they are thinking. Response techniques involve:

- Waiting. Time is a critical component. An immediate judgment of a response stops any further pondering or reflection on the part of the students.
- Requesting a rationale for answers and or solutions. Students will utimately accept this procedure as an expected norm.
- Eliciting alternative ideas and approaches
- Posing questions and tasks that elicit, engage, and challenge each student's thinking;
- Asking students to justify their ideas orally and in writing.

Levels of Questioning

- Category 1 questions focus on helping students work together to make sense of mathematics.
  "Do you agree? Disagree?"
  "Does anyone have the same answer but a different way to explain it?"

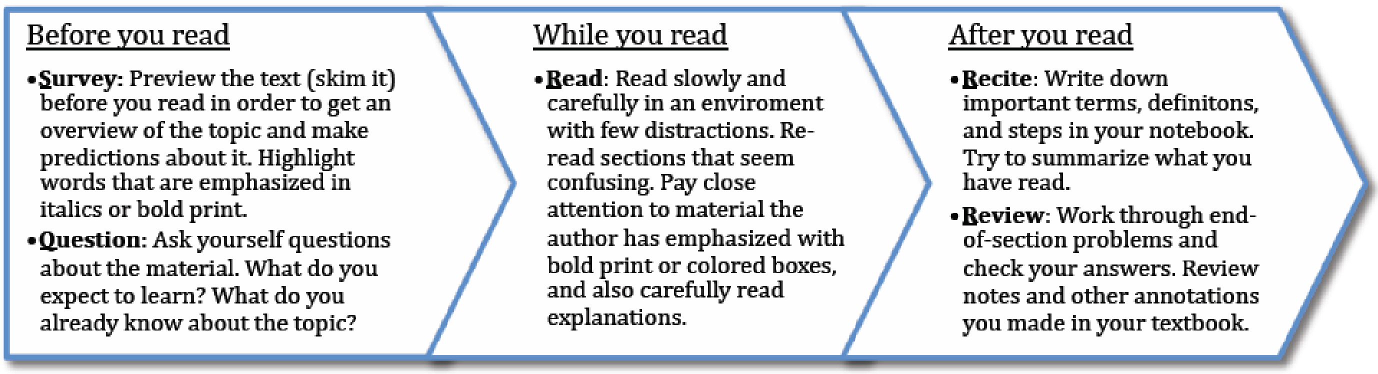
- Category 2 contains questions that help students rely more on themselves to determine whether something is mathematically correct.
  "Does that make sense?"
  "What model shows that?"

- Category 3 questions seek to help students learn to reason mathematically.
  "Does that always work?"
  "How could we prove that?"

- Category 4 questions focus on helping students learn to conjecture, invent, and solve problems.
  "What would happen if...?"
  "What would happen if not...?"
  "What pattern do you see?"

- Category 5 questions relate to helping students connect mathematics, its ideas, and its applications.
  "Have we solved a problem that is similar to this one?"
  "How does this relate to ...?"

Through modelling of investigative questioning, the teacher should help students learn to conjecture, invent, and solve problems.



http://teswww.tes.tp.edu.tw/cmsimages/bi/documents/MathsDictionary.pdf

Types of question

Recalling facts

- What is 3 add 7?
- How many days are there in a week?
- How many centimetres are there in a metre?
- Is 31 a prime number?

Applying facts

- Tell me two numbers that have a difference of 12.
- What unit would you choose to measure the width of the table?
- What are the factors of 42?

Hypothesising or predicting

- Estimate the number of marbles in this jar.
- If we did our survey again on Friday, how likely is it that our graph would be the same?
- Roughly, what is 51 times 47?
- How many rectangles in the next diagram?
- And the next?

Designing and comparing procedures

- How might we count this pile of sticks?

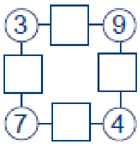
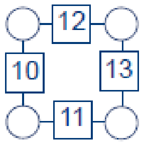
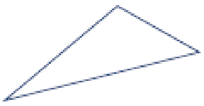
- How could you subtract 37 from 82?
- How could we test a number to see if it is divisible by 6?
- How could we find the 20th triangular number?
- Are there other ways of doing it?

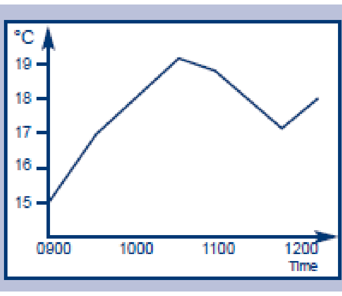
Interpreting results

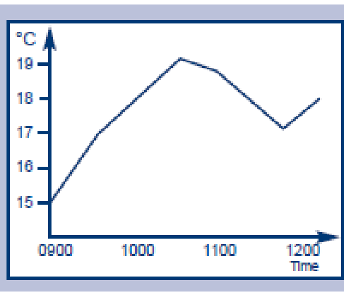
- So what does that tell us about numbers that end in 5 or 0?
- What does the graph tell us about the most common shoe size?
- So what can we say about the sum of the angles in a triangle?

Applying reasoning

- The seven coins in my purse total $2.35. What could they be?
- In how many different ways can four children sit at a round table?
- Why is the sum of two odd numbers always even?

<table>
<tr><th>Closed Questioning</th><th>Open Questioning</th></tr>
<tr><td>What is 6 – 4?</td><td>Tell me two numbers with a difference of 2.</td></tr>
<tr><td>What is 2 + 6 – 3?</td><td>What numbers can you make with 2, 3 and 6?</td></tr>
<tr><td>Is 16 an even number?</td><td>What even numbers lie between 10 and 20?</td></tr>
<tr><td>Write a number in each box so that it equals the sum of the two numbers on each side of it.

</td><td>Write a number in each circle so that the number in each box equals the sum of the two numbers on each side of it. Find different ways of doing it.

</td></tr>
<tr><td>Copy and complete this addition table.<table><tr><td>+</td><td>4</td><td>7</td></tr><tr><td>2</td><td></td><td></td></tr><tr><td>6</td><td></td><td></td></tr></table></td><td>Find different ways of completing this table.<table><tr><td></td><td></td><td></td></tr><tr><td></td><td>3</td><td>4</td></tr><tr><td></td><td>7</td><td></td></tr></table></td></tr>
<tr><td>What are four threes?</td><td>Tell me two numbers with a product of 12.</td></tr>
<tr><td>What is 7 x 6?</td><td>If 7 x 6 = 42, what else can you work out?</td></tr>
<tr><td>How many centimetres are there in a metre?</td><td>Tell me two lengths that together make 1 metre.</td></tr>
<tr><td>Continue this sequence: 1, 2, 4…</td><td>Find different ways of continuing this sequence: 1, 2, 4…</td></tr>
<tr><td>What is one fifth add four fifths?</td><td>Write eight different ways of adding two numbers to make 1.</td></tr>
<tr><td>What is 10% of 300?</td><td>Find ways of completing: …% of … = 30</td></tr>
<tr><td>What is this shape called?
</td><td>Sketch some different triangles.</td></tr>
<tr><td>This graph shows room temperature on 19 May.

What was the temperature at 10.00 am?

</td><td>This graph shows room temperature on 19 May.

Can you explain it?

</td></tr>
</table>

Ask children who are getting started with a piece of work:

- How are you going to tackle this?
- What information do you have? What do you need to find out or do?
- What operation/s are you going to use?
- Will you do it mentally, with pencil and paper, using a number line, with a calculator…? Why?
- What method are you going to use? Why?
- What equipment will you need?
- What questions will you need to ask?
- How are you going to record what you are doing?
- What do you think the answer or result will be?
- Can you estimate or predict?

Make positive interventions to check progress while children are working, by asking:

- Can you explain what you have done so far?
- What else is there to do?
- Why did you decide to use this method or do it this way?
- Can you think of another method that might have worked?
- Could there be a quicker way of doing this?
- What do you mean by…?
- What did you notice when…?
- Why did you decide to organise your results like that?
- Are you beginning to see a pattern or a rule?
- Do you think that this would work with other numbers?
- Have you thought of all the possibilities? How can you be sure?

Questions that can help to extend children's thinking

Ask children who are stuck:

- Can you describe the problem in your own words?
- Can you talk me through what you have doneso far?
- What did you do last time? What is different this time?
- Is there something that you already know that might help?
- Could you try it with simpler numbers… fewer numbers… using a number line…?
- What about putting things in order?
- Would a table help, or a picture/diagram/graph?
- Why not make a guess and check if it works?
- Have you compared your work with anyone else's?

During the plenary session of a lesson ask:

- How did you get your answer?
- Can you describe your method/pattern/rule to us all? Can you explain why it works?
- What could you try next?
- Would it work with different numbers?
- What if you had started with… rather than…?
- What if you could only use…?
- Is it a reasonable answer/result? What makes you say so?
- How did you check it?
- What have you learned or found out today?
- If you were doing it again, what would you do differently?
- Having done this, when could you use this method/information/idea again?
- Did you use any new words today? What do they mean? How do you spell them?
- What are the key points or ideas that you need to remember for the next lesson?

QAR: Question Answer Relationships

Students in a Summer Bridge course were also taught a method for reading word problems based on strategies recommended by Polya (1957). These students ranked instruction in reading word problems second in importance of the six strategies they were taught. They indicated that they wanted more strategies and instruction for reading word problems. In response, Lou Ann Pate refined and further developed Question Answer Relationship Activities based on strategies developed by Polya (1957), Raphael and Gavelek (1988), as well as McIntosh and Draper (1995).

The QAR strategy was designed to enable students to understand where basic mathematical concepts apply to the real world and how they connect to more sophisticated mathematical concepts. This strategy begins with

- "Right There Questions" which are based on information that is right there in the problem.
- "Think and Search Questions" require students to identify relationships among the givens and the unknowns and require students to perform calculations using them.
- "Author and You Questions" provide an extension of basic concepts used in "Think and Search."

The three types of questions all require students to become aware of the different kinds of information provided in the story problem that they can use to answer the different kinds of questions. Finally, students learn to answer "On Your Own Questions." They are taught how to identify prior knowledge or additional information needed to solve the problem.

KNWS Strategy

Students read the problem and record what facts they know, what information is not needed, what the problem is asking them to find, and what strategy they will use to solve the problem. Ask students to read a word problem, and model charting the information in the proper columns.

K-N-W-S Worksheet

K	N	W	S
What facts do I KNOW form the information in the problem?	Which information do I NOT need?	WHAT does the problem ask me to find?	What STRATEGY / operations / tools will I use to solve the problem?

The Braid Model of Problem Solving

Understanding the problem / reading the story

Visualization

Do I see pictures in my mind? How do they help me understand the situation?
Imagine the **SITUATION.** What is going on here.?

Asking Questions (and Discussing the problem in small groups)

K: What do I know for sure?
W: What do I want to figure out, find out, or do?
C: Are there any special conditions, rules or tricks I have to watch out for?

Making Connections

Math to Self

What does this situation remind me of?
Have I ever been in any situation like this?

Math to World

Is this related to anything I've seen in social studies or science, the arts? or related to things I've seen anywhere?

Math to Math

What is the main idea from mathematics that is happening here?
Where have I seen that idea before?
What are some other math ideas that are related to this one?
Can I use them to help me with this problem?

Infer What inferences have I made? For each connection, what is its significance?

Look back at my notes on K and C. Which are facts and which are inferences?
Are my inferences accurate?

Planning how to solve the problem

What **REPRESENTATIONS** can I use to help me solve the problem?
Which problem-solving strategy will help me the most in this situation?

Make a model	**Draw a picture**	**Make an organized list**
Act it out	**Make a table**	Write an equation
Find a pattern	Use logical reasoning	Draw a diagram
Work backward	Solve a simpler problem	Predict and test

Carrying out the plan / Solving the problem

Work on the problem using a strategy.

Does this strategy show me something I didn't see before now?
Should I try another strategy?
Am I able to **infer** any **PATTERNS**?
Am I able to **predict** based on this inferred pattern?

Looking back / Checking

Does my answer make sense for the problem?
Is there a pattern that makes the answer reasonable?
What **CONNECTIONS link** this problem and answer to the big ideas of mathematics I am learning ?
Is there another way to do this? Have I made an assumption?

Newman's Prompts can be used to question students and to determine where their understanding breaks down. The Australian educator Anne Newman (1977) suggested five significant prompts to help determine where errors may occur in students' attempts to solve written problems. She asked students the following questions as they attempted problems.

1. *Please read the question to me. If you don't know a word, leave it out.*
2. *Tell me what the question is asking you to do.*
3. *Tell me how you are going to find the answer.*
4. *Show me what to do to get the answer. "Talk aloud" as you do it, so that I can understand how you are thinking.*
5. *Now, write down your answer to the question*.

These five questions can be used to determine why students make mistakes with written mathematics questions. A student wishing to solve a written mathematics problem typically has to work through five basic steps:

1.Reading the problem	Reading
2. Comprehending what is read	Comprehension
3. Carrying out a transformation from the words of the problem to the selection of an appropriate mathematical strategy	Transformation
4. Applying the process skills demanded by the selected strategy	Process skills
5. Encoding the answer in an acceptable written form	Encoding

The five questions the teacher asks clearly link to the five processes involved in solving a written mathematics problem.

If when reworking a question using the Newman analysis the student is able to correctly answer the question, the original error is classified as a careless error.

Research using Newman's error analysis has shown that over 50% of errors occur before students get to use their process skills. Yet many attempts at remediation in mathematics have in the past over-emphasised the revision of standard algorithms and basic facts.

How can teachers assist their students

Teaching ideas for addressing the first three hurdles:

- Reading
- Comprehension
- Transformation

Addressing reading

Natalie paddled 402 km of the Murray River in her canoe over 6 days. She paddled the same distance each day. How far did Natalie paddle each day?

What can a teacher do in the mathematics classroom with a student who has difficulty with reading mathematics problems?

The task for the teacher in the mathematics classroom is to teach the student to read the particular text under consideration.

Provide an orientation

Students who have difficulty with reading find it hard to establish a context for a particular text, predict its grammatical structure, predict the meaning of the text and anticipate words that are likely to occur within it. To assist these students, the teacher can provide an orientation to the text before they read the problem. The aim of the orientation is to make the students aware of:

1. the story in which the problem is embedded,
2. the context of the problem,
3. unusual language, likely to cause difficulties for the students,
4. mathematical words in the text of the problem.

'This is a problem about a girl who goes on a canoe trip on the Murray River' is a possible orientation to this problem, providing a context to it and enabling students to access unusual words that might be a stumbling block.

It is important that teachers do not read out the problem for the students, that they do not simplify the language of the problem or present an orientation that provides too much guidance to solve the problem.

Addressing comprehension

Natalie paddled 402 km of the Murray River in her canoe over 6 days. She paddled the same distance each day. How far did Natalie paddle each day?

What can a teacher do in the mathematics classroom with a student who has difficulty with comprehending mathematics problems?

Focus on language features
Students need to be familiar with a range of mathematical texts and understand the language, features and grammar of these texts. For example, knowing that what needs to be worked out often appears as a question at the end of the problem may assist students to read and understand the problem.

Discuss Cloze passages
While being of limited benefit when attempted individually, Cloze passages can be used for a guided discussion, in which students identify how different words change the meaning of a problem. To be able to maintain meaning while reading a text, a student needs to be able to read over 90% of it, therefore blanking out more than 10% of the words in a Cloze passage turns it into an illegible text for many students. This means that in a problem such as the one quoted above, no more than three words should be blanked out. Generally, the blanked out words should be **prepositions** and **conjunctions**, rather than nouns, as they have a greater effect on the meaning of the text.

Reassemble texts
Another useful strategy is to present to the students the text of a problem cut up into separate strips of paper and have the students order these to reconstruct the text. For example:

Natalie

This strip can later be used to discuss with students that, because this is someone's name, it is not necessary, in the context of the problem, to be able to read it.

The next three strips can later be used to discuss how to represent each one of the terms of the problem.

paddled 402 km of the Murray River

'Murray River' can also be identified as a noun that can be understood without needing to be able to read it.

in her canoe over 6 days

She paddled the same distance each day.

How far did Natalie paddle each day?

This strip can be used later to discuss the location of a question in a mathematics word problem.

Addressing transformation

Natalie paddled 402 km of the Murray River in her canoe over 6 days. She paddled the same distance each day. How far did Natalie paddle each day?

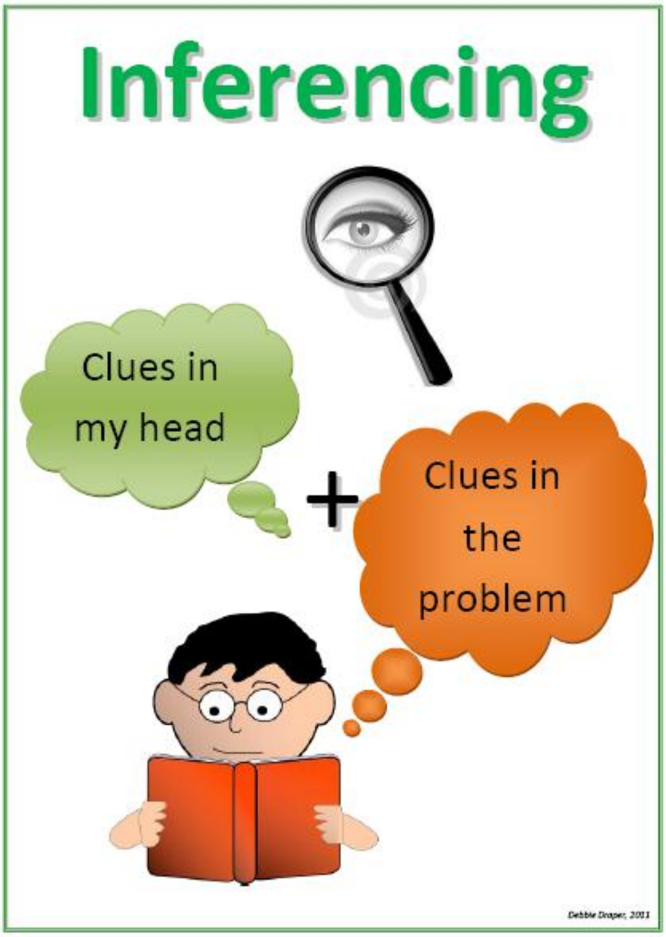
What can a teacher do in the mathematics classroom with a student who has difficulty with transforming mathematics problems?

Focus on solving problems
Teachers can build the ability of students to transform mathematical texts into mathematical processes by creating classrooms where learning to read mathematics problems occurs frequently and where solving problems is the focus of mathematics lessons.

Teach students to represent problems
Through discussion, a class could identify that some effective ways of representing the above problem would be to act it out, to draw a table or to draw a series of pictures. Different groups of students could solve the problem using one of these representations and present their solutions to the class, for discussion.

Teach students to write problems
Having worked in this way to solve the focus problem, students could be asked to write a problem about a bike trek, the solution for which can be obtained by dividing 402 by 6. This provides students with an opportunity to transfer their understanding to a very similar context. When the students are successful at doing this, a very different context could be provided, for example, "Write a problem about $402, where the solution will be obtained by dividing 402 by 6." Adding distractors to these student-written problems and having the students exchange and solve the problems that they have written can assist in improving the students' ability to deal with problems at the transformation stage.

Rewrite the problem
In the case of the problem outlined above, a large number of students thought that 402 × 6 would lead to the correct answer. Teachers could guide their students in a joint rewriting of the problem, so that the solution can be obtained by doing 402 × 6. This should be followed up by a discussion of the changes that needed to be made to the text for this to happen.



Inferring in mathematics involves determining patterns. Considering almost all mathematics involves the science of patterns, inferring is essential to developing conceptual understandings in mathematics.

Inference is used to

- Recognise patterns and relationships
- Determine the meaning of unknown words in context – looking at diagrams, using schema and through discussion
- Deduce appropriate operations to solve problems
- Judge the reasonableness of an answer
- Predict possible alternatives
- Confirm / adjust predictions

When problems with only one solution are presented, students are discouraged from thinking inferentially. It is better to deconstruct, discuss and evaluate one problem 20 ways than it is to use the same process on 20 different problems.

Students need lots of practice and repetition to develop inferencing skills in mathematics. Discussion and conferencing can help teachers understand the inferences students are making.

Many students develop misconceptions based on reasonable inference processes.
Multiplication always results in a larger number

This is true when working with positive whole numbers. However not true when working with fractions and negative numbers. Students latch on to this misconception because of earlier experiences with positive whole numbers.

- Instead of using "one half times eight," try using "one half of eight." The use of the word "of" when multiplying a fraction times a whole number informs students the answer will be less than eight.

In Fractions the Largest Denominator is the Largest Fraction

Students assume this is always true because they learned that a 6 is larger than a 3 for example.

- The best way to eliminate this misconception is to allow students to work with math manipulatives when beginning work with fractions. This allows students to visualize denominators and numerators broken down into their basic parts.

Geometric Shapes are not Recognized Unless Held Upright

This is typically an inadvertent misconception passed on by teachers. If geometric shapes, such as triangles or rectangles, are held in one direction all the time students will not recognize it when viewed in a different direction.

- Students can only find a diamond shape if pointed in the right direction. In reality there is no such thing as a diamond shape, it is either a square or a rhombus.
- The best ways to eliminate this misconception is to allow students to draw geometric shapes in any direction, provide examples of shapes in a variety of directions, and rearrange displays of geometric shapes to point in different directions regularly.

To Multiply by 10 Just Add Zero

This is a common misconception students fall into because this is something they hear all the time from parents, siblings, students, and others.

- Students learn this from working with positive and negative whole numbers. However it is not true when working with decimals and fractions.

- The best way to eliminate this misconception is to have students work out problems with decimals and fractions being multiplied by 10. When they work out the problems themselves, they will internalize that multiplying by 10 does not always mean just add zero.

The Tallest Container Always has the Greatest Volume

This a misconception caused by visual perception. Also they learn this from eating in fast food restaurants and similar locations that display cup sizes. The tallest cup always holds more, because of the way they are displayed.

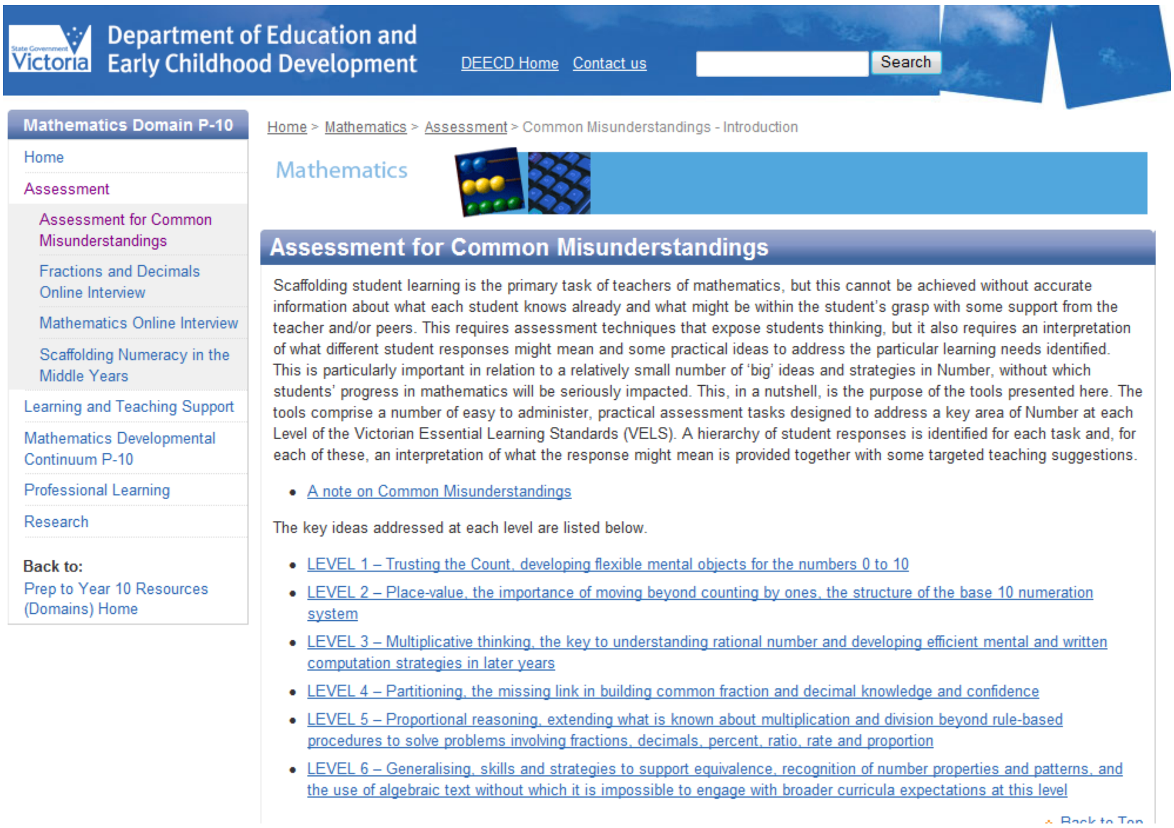
- The best way to eliminate this problem is to have students fill tall containers with water and then pour the water into a shorter container which has the same volume. This is a difficult misconception to break and even adults have issues with this misconception.

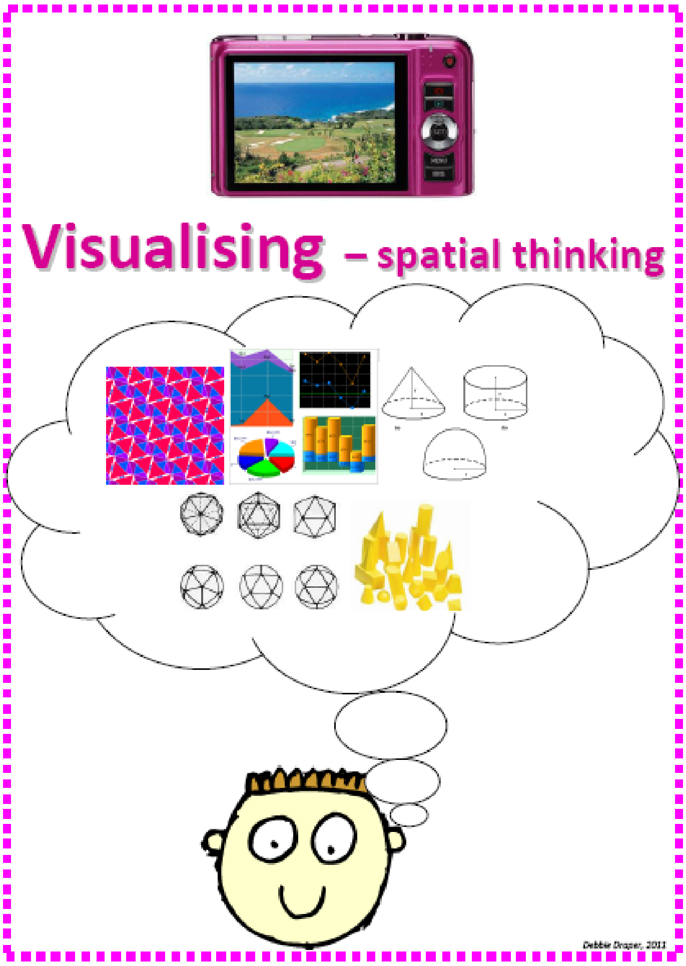
The most effective method of eliminating math misconceptions is to address them immediately when observed. This is imperative, so students do not carry these misconceptions any further and develop a better understanding of mathematics.

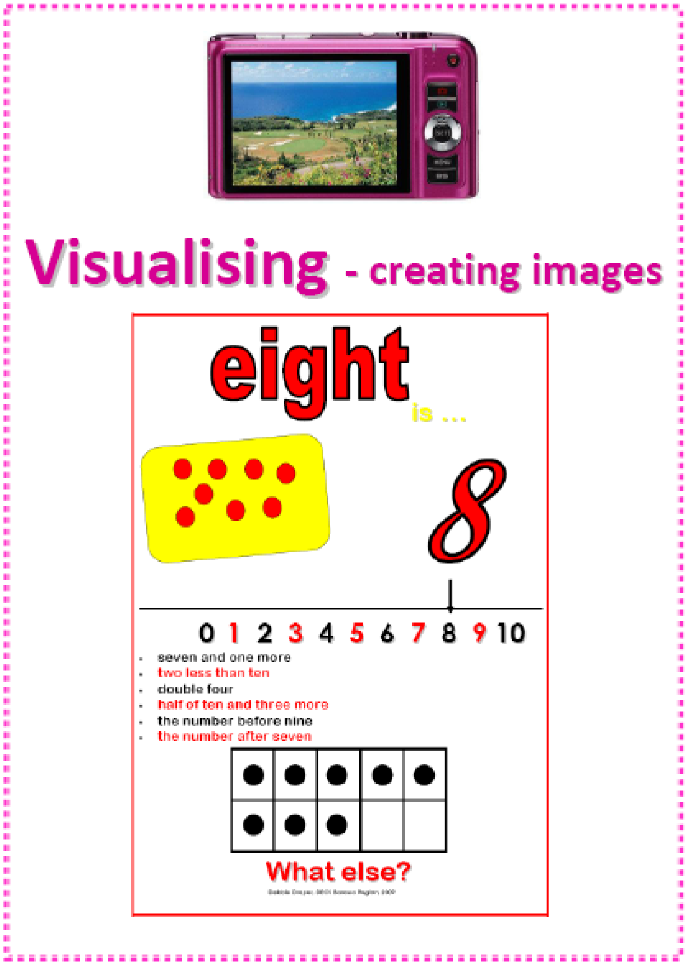
For more on assessing students' misconceptions and strategies to address these see:

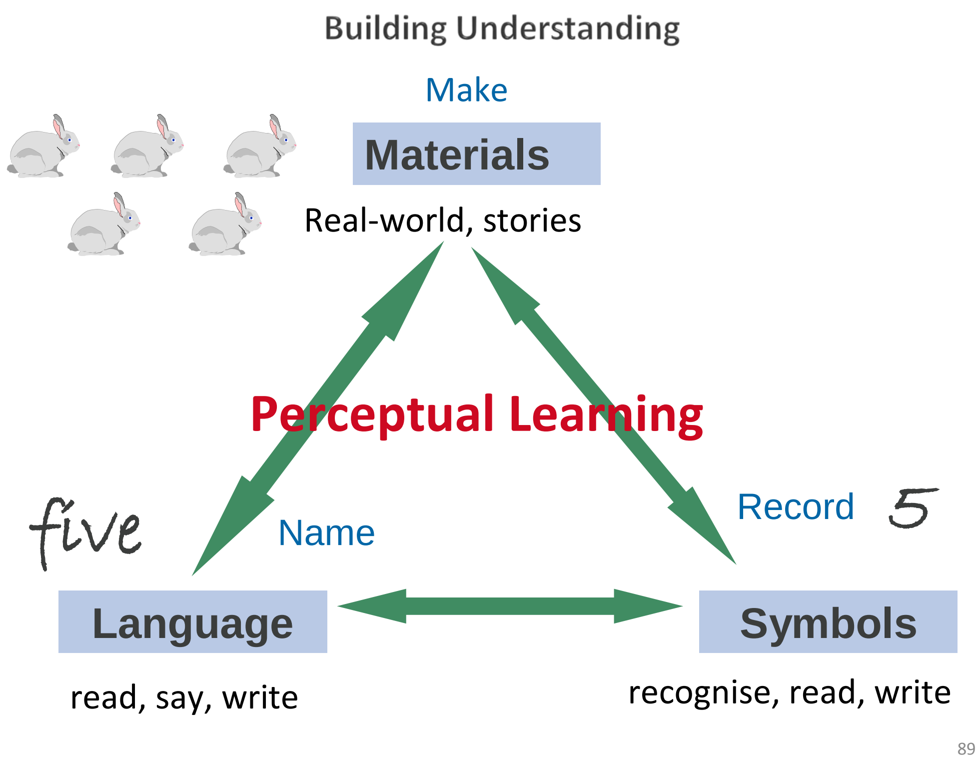
http://www.counton.org/resources/misconceptions/

http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm









In reading, we suggest that students create mind movies to help them visualise the text. This can be applied to math as well by drawing pictures and making tables. Due to the compact nature of word problems, students can also elaborate to help them get the full meaning of the problem. Draw pictures of word problems. Using this visualising strategy from reading helps illustrate the information given.

Determining Importance

Which parts of the word problem are essential to working it out?

K. What do I know for sure? What information is relevant / irrelevant?

W. What do I want to find out? What do I need to ignore?

C. What constraints, conditions, clues help me to work determine the important information?

What is the problem about? What are you trying to find?

What information do you have and need to solve the problem?

What information is shared that you really do not need to solve the problem?

What steps will you use to solve the problem?

What is the answer to the problem?

What evidence do you have that your answer is correct?

Determining Importance in mathematics involves asking questions about:

- the nature of the problem and what it is asking
- which information is relevant to problem solving
- which information is irrelevant
- what mathematical processes are relevant
- what steps are required to solve the problem
- whether the answer makes sense to the original problem

Determining importance is a great strategy to use when reading a *math textbook* or even *word problems*. There are several ways to approach the importance of what you are reading.

OVERVIEW

This is a type of skimming or scanning the text before you actually read it in depth. This can help you in the following ways:

1. It can help you make connections.
2. It can help you determine the type of operation you need to use to solve the problem or to determine what the lesson is about.
3. It can help you to determine what you need to pay careful attention to.
4. It can help you determine what to ignore (some text can get windy with their examples or there could be extra information you don't need in word problems).
5. It can help you determine to quit reading if the text has no relevancy to what you are learning.
6. It can help you determine if the text is worth reading or if skimming will to the job.

HIGHLIGHTING

To effectively highlight the text, you need to read the text, think about it, and make a conscious decision on what you need to remember and learn.

1. Carefully look at the first and last line of each paragraph (especially in word problems).
2. Highlight only the words and phrases that are necessary.

3. Make notes in the margins (or on a separate piece of paper!) to emphasize the words or phrases that are important.
4. Pay attention to surprising information — it means you might have learned something new!
5. Visualize what the text is actually saying and what it means.
6. Look to see that you did not highlight the entire paragraph. About only one-third of the text should be highlighted.

ORGANIZATION OF TEXTBOOKS

Math textbooks are all formatted pretty much the same way with an opening paragraph; sample problems; drawings, graphs, or diagrams; and practice exercises.

OPENING PARAGRAPH

1. Has the explanation of what you are going to learn about, vocabulary, and rules.
2. This is the material that needs to be understood but you may not use right away.
3. It gives some general information on how to complete the task.
4. It may include information that will help with making connections, questioning, and visualization.

SAMPLE PROBLEMS

1. It shows how to do computations in simple problems or more complex ones.
2. You may have to practice the idea as it is introduced or apply it to solve some other task.

DRAWINGS, GRAPHS, AND DIAGRAMS

1. These help with the visualization of the problem.
2. They aid in solving the actual problem.

EXERCISES

1. These are problems that relate to the work done in the sample problems.

All publishes of textbooks have certain signals —- fonts, graphics, aids, textboxes — that help you through the page. There are headings, subheadings, bullets, arrows, and such that you should be aware of when reading any math textbook.

NOTE TAKING

Following the above strategies will help you become an effective note taker. When working with math vocabulary it is EXTREMELY important you understand what the word means because maths builds upon what you already know. Many times these words will be used over and over again. Sometimes they may be used several times in one year and then not heard of again for awhile. Then a few years down the road you will have to know what it means again.

Math vocabulary can be tricky. There are three categories or types or words used in math. There are words that are used in everyday real life and math, words that only mean something in math class, and then there are words that have different meanings in everyday real life and math.

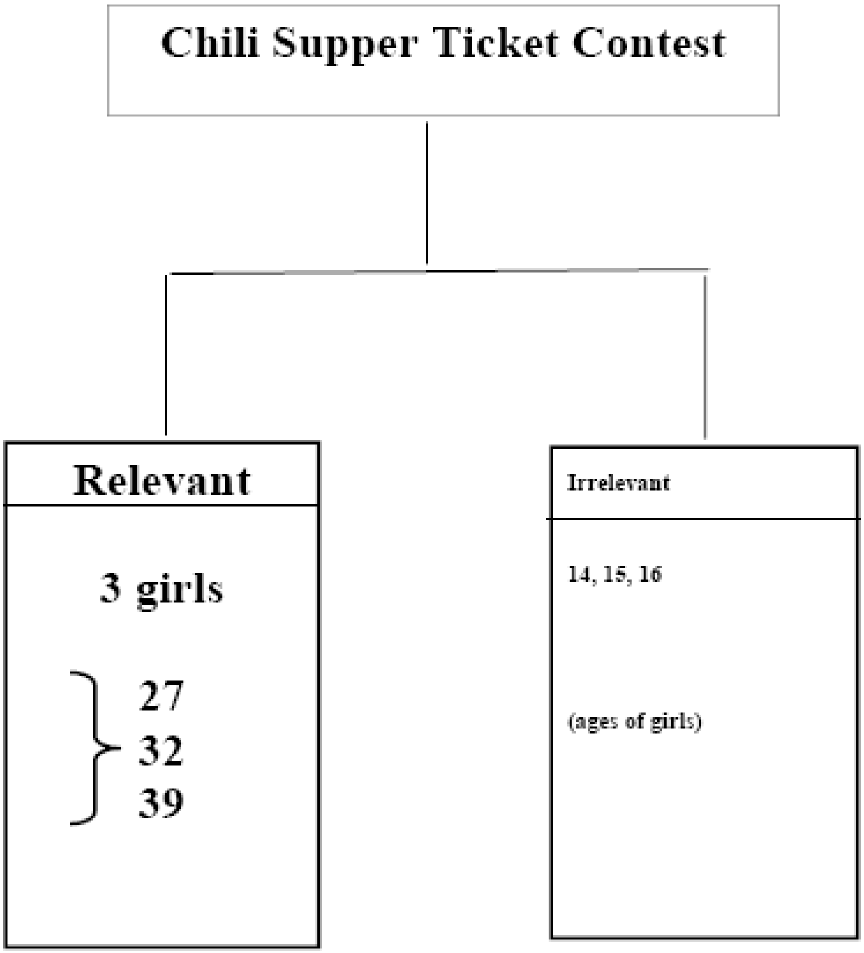
1. Words that have the same meaning in math as they do in everyday life. EXAMPLES: dollars, cents, because, driving, apple
2. Words that have meaning only in math. EXAMPLES: hypotenuse, numerator, coefficient, mixed number, cosine
3. Words that have different meaning in math and everyday life. EXAMPLES: difference, multiple, factor, average, similar

To help you remember these words you may want to make *FLASHCARDS*, keep a *VOCABULARY JOURNAL*, or *DRAW A SIMPLE SKETCH* of what the word means. Sometimes writing the definition in you own words can help you remember what it means.

Here is a great strategy to help you with the ever important vocabulary words in math. Divide a piece of notebook paper in half. You can label the left-hand side “KEY TERMS” and make the right-hand side “EXAMPLES.” Make sure that when you put down your examples you try to make up your own examples rather than using ones from the book.

<u>Description</u>: In mathematics students can be asked to determine information as relevant or irrelevant in an effort to focus on necessary information to solve the problem.

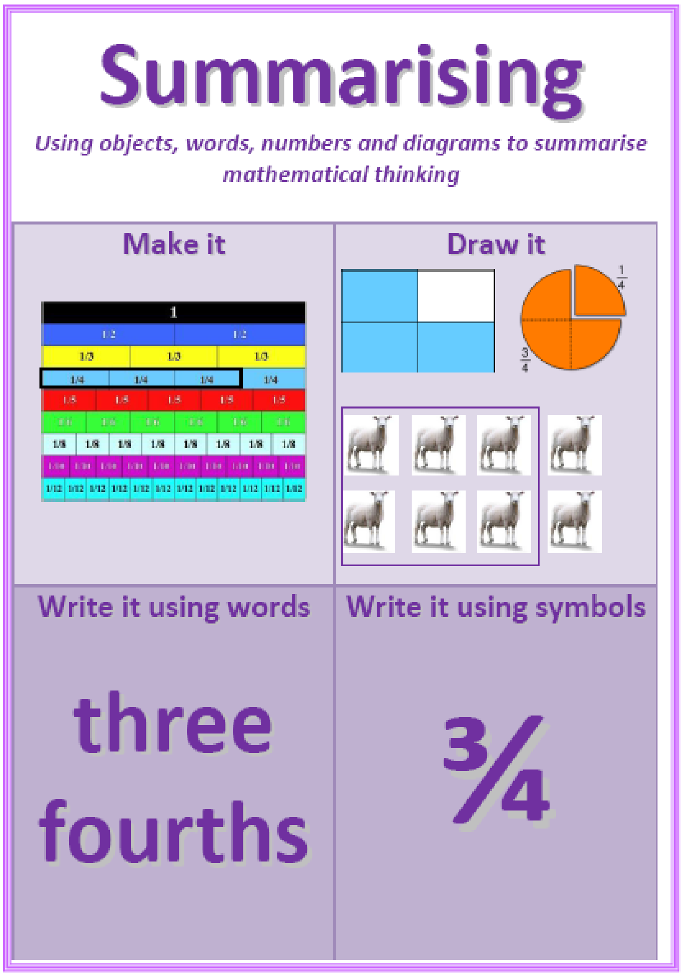
Lisa was 14. Diane was 15. Sara was 16. The three girls reported their classes' results for the Chili Supper tickets drive. Lisa's class sold 27 tickets. Diane's class sold 32 tickets and Sarah's class sold 39 tickets. What was the average number of items collected for the Chili Supper contest?



Using **RIDGES** to Solve Word Problems

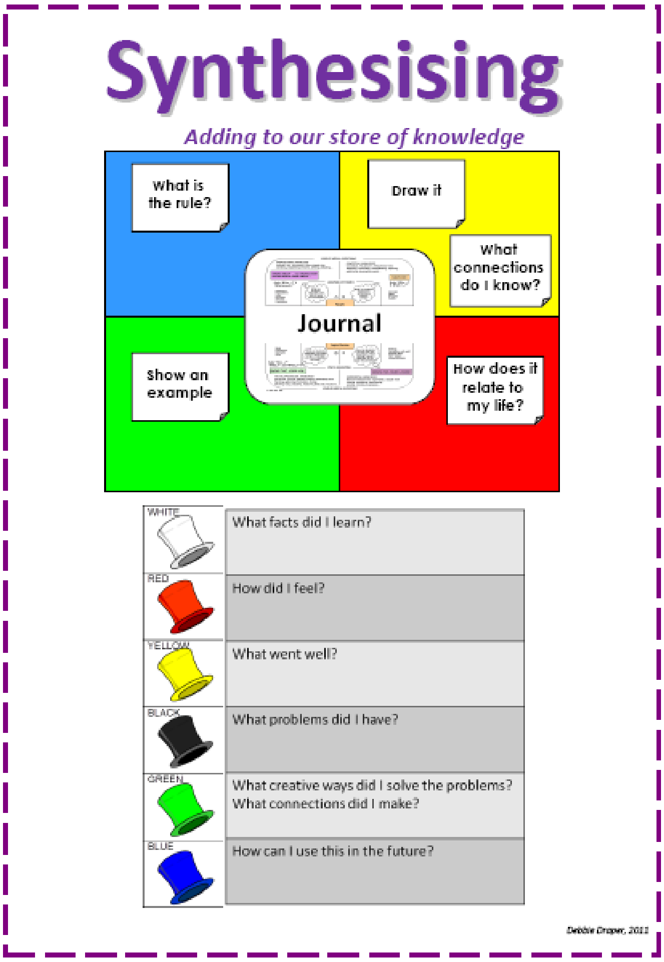
(1) **Read** the problem. If the problem is not understood, re-read it.
(2) **Identify** all of the information given in the word problem. List the information separately. After listing all of the information, circle the information that is needed to solve the problem.
(3) **Draw** a picture- Draw a picture of the information in the problem. This may help a student pick out the relevant information.
(4) **Goal** Statements. The student should express, in his or her own words, the question the problem is asking.
(5) **Equation** development- The student will write an equation to the problem. (i.e. length + width + length + width = distance around the field)
(6) **Solve** the equation- The given information is plugged into the equation (i.e. 10+6+10+6=distance around the field)

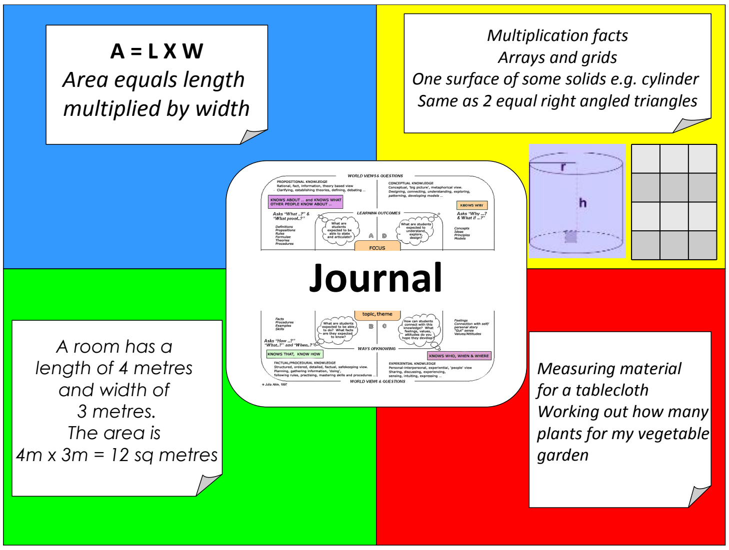
Source: Snyder, K. (1988) Ridges: A problem-solving math strategy. <u>Academic Therapy,</u> 230), 261-263.



Journaling gives students an opportunity to summarise and synthesise their learning of the lesson.

Use maths word wall words to scaffold journaling. Include words like “as a result”, “finally”, “therefore”, and “last” that denote synthesising for students to use in their writing. Or have them use sentence starters like ”I have learned that…”, “This gives me an idea that”, or “Now I understand that…”





Synthesising is all about reflecting on what has changed in our thinking.

What did I think before?

How has my thinking changed?

What do I now know that I didn’t know before?

Vocabulary

Vocabulary instruction is important to all content areas including math. Students enter the math classroom with vocabulary from other disciplines and everyday life. These definitions, however, are altered for mathematical purposes. For example, the word volume has an everyday meaning of a noise level, but in math it means the "amount of mass taken up by an object." Therefore, vocabulary must become part to of regular mathematics instruction to help students avoid confusion.

Angle the shape made by two rays extending from a common end point, the vertex. Measures of angles are described using the degree system. E

Mark on the line your knowledge of this word.

Never Heard of It — Somewhat Familiar — Use It All the Time

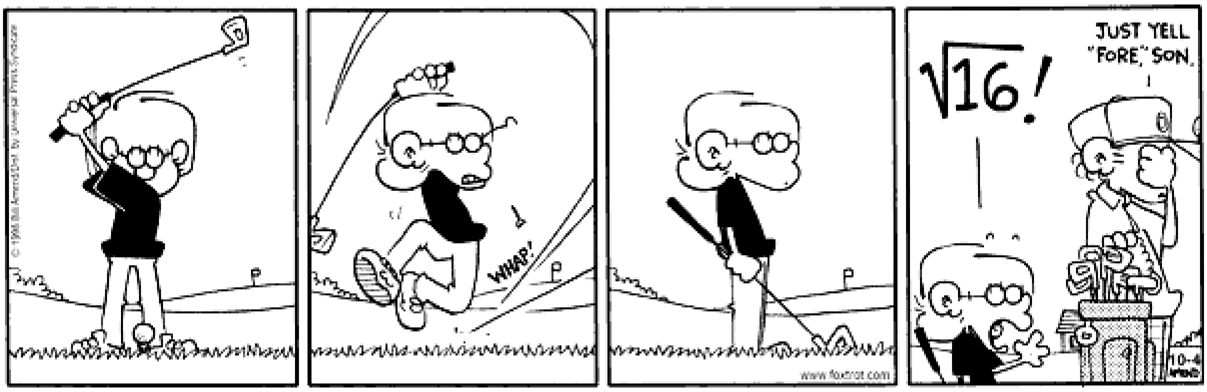
Explain in your own words	Example
Facts/Rules/Formulas	Picture or Graph

Composite number a whole number that has more than two factors. E

Mark on the line your knowledge of this word.

Never Heard of It — Somewhat Familiar — Use It All the Time

Explain in your own words	Example
Facts/Rules/Formulas	Picture or Graph



Useful Maths Vocabulary Sites

Interactive Maths Dictionary from Jenny Eather - http://www.amathsdictionaryforkids.com/dictionary.html

Mathematical vocabulary (more advanced) with examples - http://www.capitan.k12.nm.us/teachers/shearerk/vocabulary_abc.htm

Word Wall cards - http://www.doe.virginia.gov/instruction/mathematics/resources/vocab_cards/index.shtml

For a detailed look at the mathematical vocabulary used at different year levels see http://teswww.tes.tp.edu.tw/cmsimages/bi/documents/MathsDictionary.pdf

The booklet is based on the English numeracy strategy but is highly relevant to Australian educators

Why is the book needed?

There are three main ways in which children's failure to understand mathematical vocabulary may show itself: children do not respond to questions in lessons, they cannot do a task they are set and/or they do poorly in tests.

Their lack of response may be because:

they do not understand the spoken or written instructions,
such as 'draw a line between…', 'ring…' or 'find two different ways to…'

they are not familiar with the mathematical vocabulary,
that is, words such as 'difference', 'subtract', 'divide' or 'product'

they may be confused about mathematical terms,
such as 'odd' or 'table', which have different meanings in everyday English

they may be confused about other words,
like 'area' or 'divide', which are used in everyday English and have similar, though more precise, meanings in mathematics

There are, then, practical reasons why children need to acquire appropriate vocabulary so that they can participate in the activities, lessons and tests that are part of classroom life. There is, however, an even more important reason: mathematical language is crucial to children's development of thinking. If children don't have the vocabulary to talk about division, or perimeters, or numerical difference, they cannot make progress in understanding these areas of mathematical knowledge.

Terms used in Australian Curriculum – Mathematics F-10

Algebraic expression
Algebraic fraction
Algebraic term
Alternate
Angle
Angles of elevation and depression
Array
Associative
Back to back stem and leaf plot
Bi modal
Bivariate data
Box plot
Capacity
Cartesian coordinate system
Categorical variable
Census
Chord
Circle
Class interval Frequency
Cointerior angles
Column graph
Common factor
Commutative
Complementary events
Composite number
Compound interest
Congruence
Congruent triangles
Continuous variable
Cosine
Counting number
Counting on
Cylinder
Data
Data display
Decimal
Denominator
Dependent variable
Difference
Distributive
Divisible
Dot plot
Element
Enlargement (Dilation)
Equally Likely outcomes
Equation
Equivalent fractions
Estimate
Even number
Event
Expression
Factor
Factor and remainder theorem
Factorise
Fraction
Frequencies
Frequency table
Function
Gradient
Greatest common divisor
Histogram
Independent event
Independent variable
Index
Index law
Informal unit
Integer
Interquartile range
Interval
Irrational number
Irregular shape
Kite
Line segment (Interval)
Linear equation
Location (statistics)
Logarithm
Many to one correspondence
Mean
Median
Midpoint
Mode
Monic
Multiple
Multiplication
Net
Number
Number line
Numeral
Numerator
Numerical data
Odd and even number
One to one correspondence
Operation
Order of operations
Outlier
Parabola
Parallel box plots
Parallelogram
Partitioning
Percentage
Perimeter
Picture graphs
Place value
Point
Polynomial
Population
Prime number
Prism
Probability
Product
Proportion
Pyramid
Pythagoras' theorem
Quadratic equation
Quadratic expression
Quartile
Quotient
Random number
Range (statistics)
Rate
Ratio
Real numbers
Rectangle
Rectangular Hyperbola
Recurring decimal
Reflection
Related denominators
Remainder
Rhombus
Right Cone
Rotation
Rounding
Sample
Sample space
Scientific notation
Secondary data set
Shape (statistics)
Shapes (geometry)
Side by side column graph
Similar
Similarity
Simple interest
Sine
Square
Standard deviation
Stem and leaf plot
Subitising
Sum
Surd
Symmetrical
Tangent
Terminating decimal
Transformation
Translation
Transversal
Trapezium
Tree diagram
Triangular number
Trigonometric ratios
Unit fraction
Variable
Variable (algebra)
Variable (statistics)
Venn diagram
Vertically opposite angle
Volume
Whole number