Computer Organization & Assembly Languages

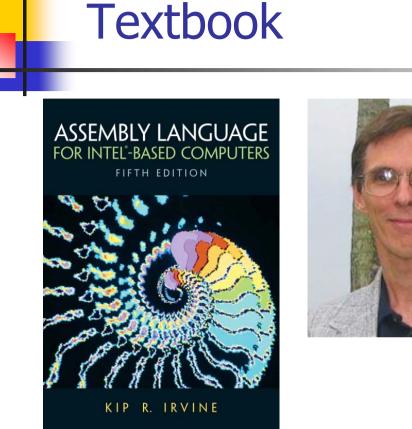
Introduction

Pu-Jen Cheng 2008/09/15

Course Administration

- Instructor: Pu-Jen Cheng (CSIE R323) *pjcheng@csie.ntu.edu.tw http://www.csie.ntu.edu.tw/~pjcheng*
- Class Hours: 2:00pm-5:00pm, Monday
- Classroom: CSIE R102
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- Course Information:

Announce: http://www.csie.ntu.edu.tw/~pjcheng/course/asm2008/ Q&A: bbs://ptt.cc → CSIE_ASM



- Assembly Language for Intel-Based Computers, 5th Edition, by Kip Irvine, Prentice-Hall, 2006
- http://www.asmirvine.com

Assembly Language for Intel-Based Computers, 5th Edition



by Kip Irvine, Florida International University ISBN: 0-13-238310 - 1 Published by: Prentice-Hall Visit the Web site for the Fourth Edition...

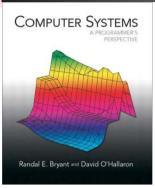


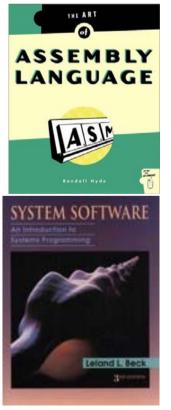
Printable Chapters

- TOC and Preface
- Chapter 1
- Chapter 2
- Chapter 3

Buy the book at Amazon.com Find it on Bookfinder.com Official Prentice-Hall Web site Microsoft's MASM Reference Intel IA-32 Architecture Manuals Information and Help Getting started Bug reports Link libraries and example programs Where is the CDROM? Supplemental files Chapter objectives Assembly language workbook Supplemental articles Help file for the book's link library **Contacts** Instructor resources Discussion group (Y-International reader

References





Computer Systems: A Programmer's Perspective By Randal E. Bryant and David R. O'Hallaron, Prentice Hall

http://csapp.cs.cmu.edu/

The Art of Assembly Language

By Randy Hyde,

http://webster.cs.ucr.edu/AoA/Windows/PDFs/0_P DFIndexWin.html

System Software: An Introduction to Systems Programming

By Leland L. Beck

Addison-Wesley



 Experiences in writing programs in a high-level language such as C, C++, and Java

Course Grading (tentative)

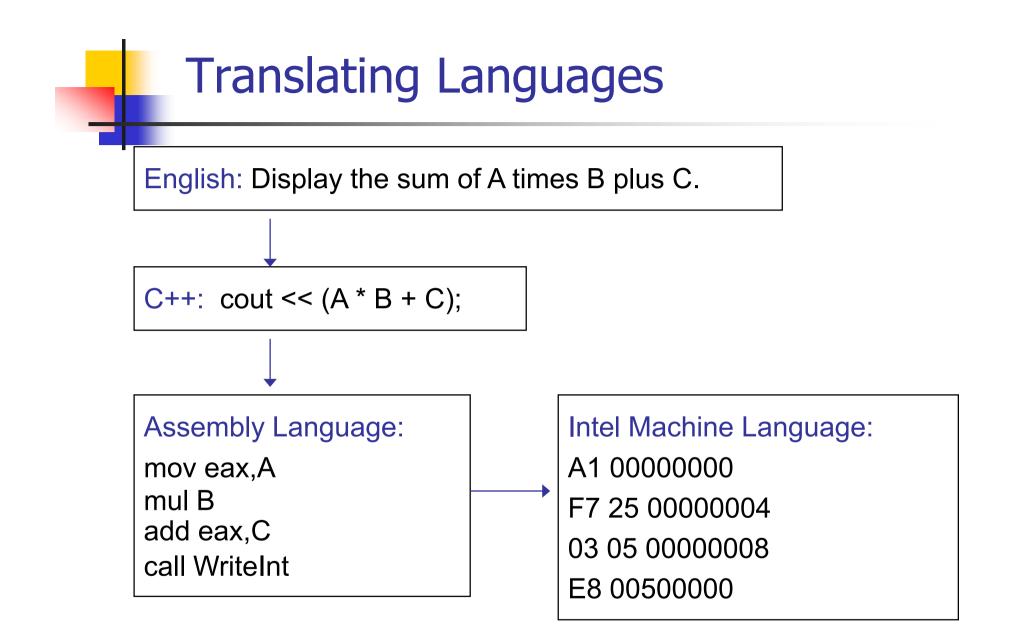
- Assignments (55%)
- Class participation (5%)
- Midterm exam (20%)
- Final exam (20%)

Materials

- Some materials used in this course are adapted from
 - The slides prepared by Kip Irvine for the book, Assembly Language for Intel-Based Computers, 5th Ed.
 - The slides prepared by S. Dandamudi for the book, Introduction to Assembly Language Programming, 2nd Ed.
 - Introduction to Computer Systems, CMU (http://www.cs.cmu.edu/afs/cs.cmu.edu/academic/class/ 15213-f05/www/)
 - Assembly Language & Computer Organization, NTU (http://www.csie.ntu.edu.tw/~cyy/courses/assembly/06fall/news//) (http://www.csie.ntu.edu.tw/~acpang/course/asm_2004)

What is Assembly Language

First Glance at Assembly Language



A Simple Example in VC++

🥙 helloworld - Microsoft Yisual C++ [break] - [helloworld.c] File Edit View Insert Project Debug Tools Window Help 管 🚅 🖬 🕼 🐰 🗈 💼 💼 🗠 - 🗠 - 🖪 🖪 😤 🙀 batch_readline_init - **1** [All global members - I main ▼ 🌂 🔹 ಿ 🏙 🐇 🗜 🗈 🕚 (Globals) #include <stdio.h> int main(void) { **int** a, b, c, d; a = 1; b = 2;c = 3;d = a * b + c;printf("a * b + c = %d\n", d); return 0; } | € ≚ Name × Value Context: main() Ŧ Value Name -858993460 а b -858993460 -858993460 С d -858993460

View/Debug Windows/Disassembly

🦇 helloworld - Microsoft Visual C++ [break] - [Disassembly]

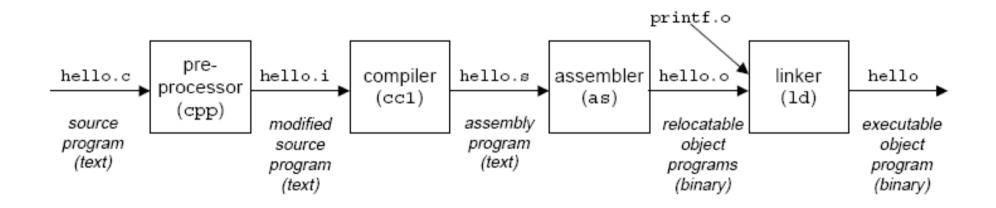
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(Globals) 💽 (All global members 🗨 🗞 main 💽 🔨 👻 🎬 👗 ! 🖹 🖑						
3: int main(void)						
4: {	· ·····,					
00401010	push	ebp	Registers 🔀			
00401011	, mov	ebp,esp	EAX = 00000005 EBX = 7FFDA000			
00401013	sub	esp,50h	ECX = 00000000 EDX = 00430DA0			
00401016	push	ebx	ESI = 00000000 EDI = 0012FF80			
00401017	push	esi	EIP = 0040104A ESP = 0012FF24			
00401018	push	edi	EBP = 0012FF80 EFL = 00000206 CS = 001B			
00401019	lea	edi,[ebp-50h]	DS = 0023 ES = 0023 SS = 0023 FS = 003B			
0040101C	mov	ecx,14h	GS = 0000 OV=0 UP=0 EI=1 PL=0 ZR=0 AC=0			
00401021	mov	eax,0CCCCCCCh	PE=1 CY=0			
00401026	rep stos	dword ptr [edi]				
5:	int a, b,	c, d;	0012FF70 = 00000005			
6:						
7:	a = 1;		ST0 = +0.000000000000000000000000000000000			
00401028	mov	dword ptr [ebp-4],1	ST1 = +0.0000000000000000000000000000000000			
8:	b = 2;		ST2 = +0.0000000000000000000000000000000000			
0040102F	mov	dword ptr [ebp-8],2	ST3 = +0.0000000000000000000000000000000000			
9:	c = 3;		ST4 = +0.0000000000000000000000000000000000			
00401036	mov	dword ptr [ebp-0Ch],3	ST6 = +0.0000000000000000000000000000000000			
10:	d = a * b	-	STO = +0.0000000000000000000000000000000000			
0040103D	mov	eax,dword ptr [ebp-4]	CTRL = 027F STAT = 0000 TAGS = FFFF			
00401040	imul	eax,dword ptr [ebp-8]	EIP = 00000000 CS = 0000 DS = 0000			
00401044	add	eax,dword ptr [ebp-0Ch]	EDO = 00000000			
00401047	mov	dword ptr [ebp-10h],eax	LD0 - 0000000			
11:	•	* b + c = %d\n", d);				
🕂 0040104A	mov	ecx,dword ptr [ebp-10h]				
0040104D	push	ecx				
0040104E	push	offset string "a * b + c = %d\n" (0042001c)				
00401053	call	printf (00401090)				
00401058	add	esp,8				
12:	return 0;					

gcc -s prog.c

🚰 ken.csie.ntu.edu.tw - PuTTY

```
.file
                 "prog1.c"
        .section
                         .rodata
LCO:
        .string "a * b + c = d\n"
        .text
globl main
        .type
                main, @function
main:
        leal
                4(%esp), %ecx
        andl
                $-16, %esp
                -4(%ecx)
        pushl
        pushl
                %ebp
                %esp, %ebp
        movl
        pushl
                *ecx
        subl
                $36, %esp
                $1, -20(\ensuremath{\$ebp})
        movl
        movl
               $2, -16(\ensuremath{\$ebp})
        movl
                $3, -12(%ebp)
        movl
                -20(%ebp), %eax
        imull
              -16(%ebp), %eax
        addl
                -12(%ebp), %eax
        movl
                %eax, -8(%ebp)
                -8(%ebp), %eax
        movl
        movl
                %eax, 4(%esp)
        movl
                $.LCO, (%esp)
        call
                printf
        movl
                $0, %eax
        addl
                $36, %esp
        popl
                *ecx
        popl
                %ebp
                -4(\text{secx}), \text{sesp}
        leal
        ret
        .size
                main, -main
                 "GCC: (GNU) 4.1.2 20060901 (prerelease) (Debian 4.1.1-13)"
        .ident
                         .note.GNU-stack,"",@progbits
        .section
```





First Glance at Assembly Language

- Low-level language
 - Each instruction performs a much lower-level task compared to a high-level language instruction
 - Most high-level language instructions need more than one assembly instruction
- One-to-one correspondence between assembly language and machine language instructions
 - For most assembly language instructions, there is a machine language equivalent
- Directly influenced by the instruction set and architecture of the processor (CPU)

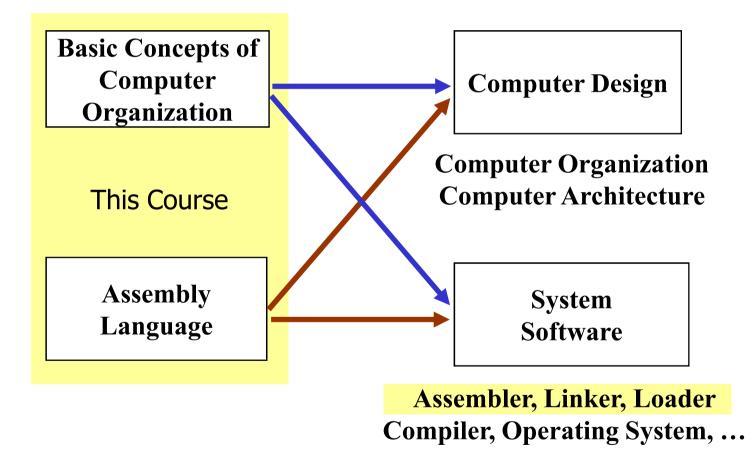
Comparisons with High-level Languages

- Advantages of Assembly Languages
 - Space-efficiency
 - (e.g. hand-held device softwares, etc)
 - Time-efficiency
 - (e.g. Real-time applications, etc)
 - Accessibility to system hardwares
 - (e.g., Network interfaces, device drivers, video games, etc)
- Advantages of High-level Languages
 - Development
 - Maintenance (Readability)
 - Portability (compiler, virtual machine)

Comparisons with High-level Languages (cont.)

Type of Application	High-Level Languages	Assembly Language	
Business application soft- ware, written for single platform, medium to large size.	Formal structures make it easy to organize and maintain large sec- tions of code.	Minimal formal structure, so one must be imposed by program- mers who have varying levels of experience. This leads to difficul- ties maintaining existing code.	
Hardware device driver.	Language may not provide for direct hardware access. Even if it does, awkward coding techniques must often be used, resulting in maintenance difficulties.	Hardware access is straightfor- ward and simple. Easy to main- tain when programs are short and well documented.	
Business application written for multiple platforms (dif- ferent operating systems).	Usually very portable. The source code can be recompiled on each target operating system with mini- mal changes.	Must be recoded separately for each platform, often using an assembler with a different syn- tax. Difficult to maintain.	
Embedded systems and computer games requiring direct hardware access.	Produces too much executable code, and may not run efficiently.	Ideal, because the executable code is small and runs quickly.	

Why Taking the Course?



"I really don't think that you can write a book for serious computer programmers unless you are able to discuss low-level details."

Donald Knuth (高徳納) The Art of Computer Programming



http://en.wikipedia.org/wiki/Donald_Knuth

Course Coverage

- Basic Concepts
- IA-32 Processor Architecture
- Assembly Language Fundamentals
- Data Transfers, Addressing, and Arithmetic
- Procedures
- Conditional Processing
- Integer Arithmetic
- Advanced Procedures
- Strings and Arrays
- Structures and Macros
- High-Level Language Interface
- Assembler, Linker, and Loader
- Other Advanced Topics (optional)

What You Will Learn

- Basic principles of computer architecture
- IA-32 processors and memory management
- Basic assembly programming skills
- How high-level language is translated to assembly
- How assembly is translated to machine code
- How application program communicates with OS
- Interface between assembly to high-level language

Performance: Multiword Arithmetic

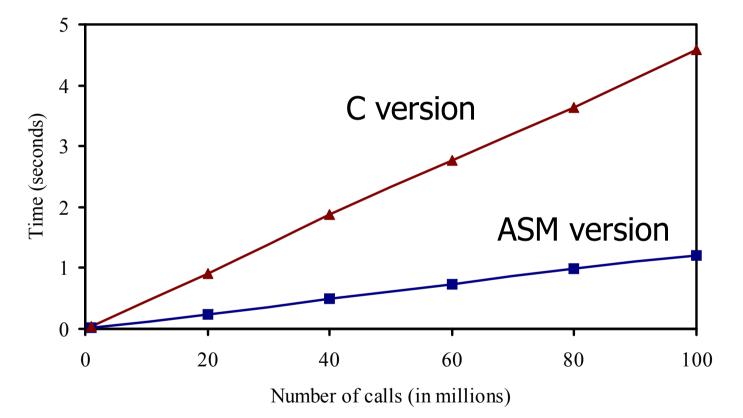
- Longhand multiplication
 - Final 128-bit result in P:A
 - P := 0; count := 64
 - A := multiplier; B := multiplicand
 - while (count > 0)
 - <u>if</u> (LSB of A = 1)
 - <u>then</u> P := P+B
 - CF := carry generated by P+B
 - <u>else</u> CF := 0
 - end if
 - shift right CF:P:A by one bit position
 - count := count-1
 - end while

Example

- $A = 1101_2$ (13)
- $B = 0101_2$ (5)

	After P+B			After the shift			
	CF	Р	А		CF	Р	А
Initial state	?	0000	1101				
Iteration 1	0	0101	1101		?	0010	1110
Iteration 2	0	0010	1110		?	0001	0111
Iteration 3	0	0110	0111		?	0011	0011
Iteration 4	0	1000	0011		?	0100	0001





Multiplication time comparison on a 2.4-GHz Pentium 4 system

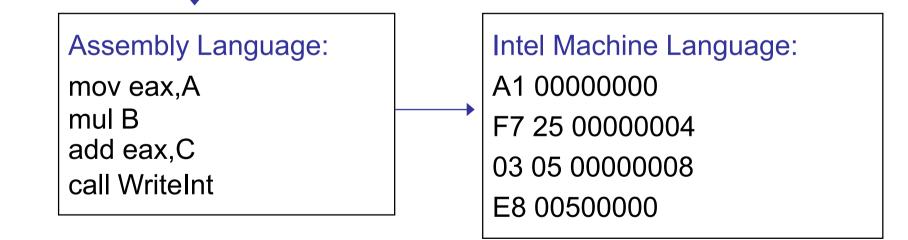
Chapter 1: Basic Concept

- Virtual Machine Concept
- Data Representation
- Boolean Operations



English: Display the sum of A times B plus C.

C++: cout << (A * B + C);





Abstractions for computers

High-Level Language	Level 5	Machine-independent
Assembly Language	Level 4	↓ Machine-specific
Operating System	Level 3	
Instruction Set Architecture	Level 2	
Microarchitecture	Level 1	
Digital Logic	Level 0	

High-Level Language

- Level 5
- Application-oriented languages
 - > C++, Java, Pascal, Visual Basic . . .
- Programs compile into assembly language (Level 4)

Assembly Language

- Level 4
- Instruction mnemonics that have a oneto-one correspondence to machine language
- Calls functions written at the operating system level (Level 3)
- Programs are translated into machine language (Level 2)

Operating System

- Level 3
- Provides services to Level 4 programs
- Translated and run at the instruction set architecture level (Level 2)

Instruction Set Architecture

- Level 2
- Also known as conventional machine language
- Executed by Level 1 (microarchitecture) program

Microarchitecture

- Level 1
- Interprets conventional machine instructions (Level 2)
- Executed by digital hardware (Level 0)

Digital Logic

- Level 0
- CPU, constructed from digital logic gates
- System bus
- Memory

next: Data Representation

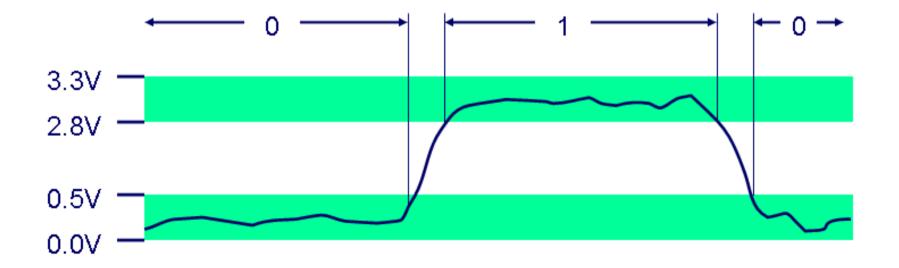
Data Representation

- Binary Numbers
 - Translating between binary and decimal
- Binary Addition
- Integer Storage Sizes
- Hexadecimal Integers
 - Translating between decimal and hexadecimal
 - Hexadecimal subtraction
- Signed Integers
 - Binary subtraction
- Fractional Binary Numbers
- Character Storage
- Machine Words



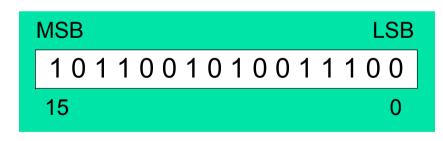
Electronic Implementation

- Easy to store with bistable elements
- Reliably transmitted on noisy and inaccurate wires



Binary Numbers

- Digits are 1 and 0
 - 1 = true
 - > 0 = false
- MSB most significant bit
- LSB least significant bit
- Bit numbering:





- Each digit (bit) is either 1 or 0
- Each bit represents a power of 2:

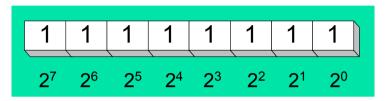


Table 1-3 Binary Bit Position Values.

Every binary number is a sum of powers of 2

2 ⁿ	Decimal Value	2 ⁿ	Decimal Value
2 ⁰	1	2 ⁸	256
2 ¹	2	2 ⁹	512
2 ²	4	2 ¹⁰	1024
2 ³	8	2 ¹¹	2048
24	16	2 ¹²	4096
2 ⁵	32	2 ¹³	8192
2 ⁶	64	2 ¹⁴	16384
27	128	2 ¹⁵	32768

Translating Binary to Decimal

Weighted positional notation shows how to calculate the decimal value of each binary bit: $dec = (D_{n-1} \times 2^{n-1}) + (D_{n-2} \times 2^{n-2}) + \dots + (D_1 \times 2^1) + (D_0 \times 2^0)$

D = binary digit

binary 00001001 = decimal 9: $(1 \times 2^3) + (1 \times 2^0) = 9$

Translating Unsigned Decimal to Binary

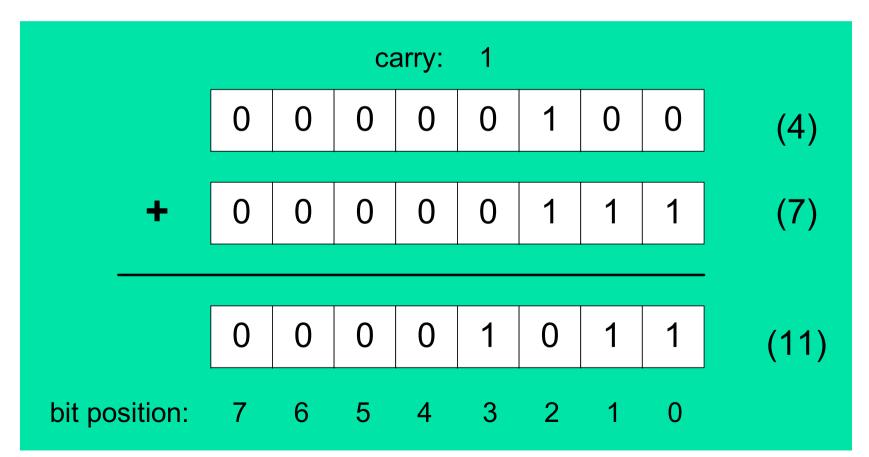
- Repeatedly divide the decimal integer by 2.
- Each remainder is a binary digit in the translated value:

Division	Quotient	Remainder
37 / 2	18	1
18 / 2	9	0
9/2	4	1
4/2	2	0
2/2	1	0
1/2	0	1

37 = 100101

Binary Addition

 Starting with the LSB, add each pair of digits, include the carry if present.



Integer Storage Sizes

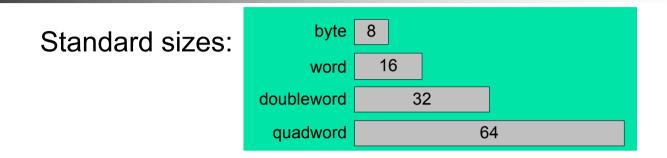


Table 1-4Ranges of Unsigned Integers.

Storage Type	Range (low–high)	Powers of 2
Unsigned byte	0 to 255	0 to $(2^8 - 1)$
Unsigned word	0 to 65,535	0 to $(2^{16} - 1)$
Unsigned doubleword	0 to 4,294,967,295	0 to $(2^{32} - 1)$
Unsigned quadword	0 to 18,446,744,073,709,551,615	0 to (2 ⁶⁴ – 1)

What is the largest unsigned integer that may be stored in 20 bits?

Large Measurements

- Kilobyte (KB), 2¹⁰ bytes
- Megabyte (MB), 2²⁰ bytes
- Gigabyte (GB), 2³⁰ bytes
- Terabyte (TB), 2⁴⁰ bytes
- Petabyte, 2⁵⁰ bytes
- Exabyte, 2⁶⁰ bytes
- Zettabyte, 2⁷⁰ bytes
- Yottabyte, 2⁸⁰ bytes
- Googol, 10¹⁰⁰

Hexadecimal Integers

Binary values are represented in hexadecimal.

Table 1-5 Binary, Decimal, and Hexadecimal Equivalents.

Binary	Decimal	Hexadecimal	Binary	Decimal	Hexadecimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	10	А
0011	3	3	1011	11	В
0100	4	4	1100	12	С
0101	5	5	1101	13	D
0110	6	6	1110	14	Е
0111	7	7	1111	15	F

Translating Binary to Hexadecimal

- Each hexadecimal digit corresponds to 4 binary bits.
- Example: Translate the binary integer 000101101010011110010100 to hexadecimal:

1	6	А	7	9	4
0001	0110	1010	0111	1001	0100

Converting Hexadecimal to Decimal

- Multiply each digit by its corresponding power of 16: $dec = (D_3 \times 16^3) + (D_2 \times 16^2) + (D_1 \times 16^1) + (D_0 \times 16^0)$
- Hex 1234 equals (1 × 16³) + (2 × 16²) + (3 × 16¹) + (4 × 16⁰), or decimal 4,660.
- Hex 3BA4 equals (3 × 16³) + (11 * 16²) + (10 × 16¹) + (4 × 16⁰), or decimal 15,268.

Powers of 16

Used when calculating hexadecimal values up to 8 digits long:

16 ⁿ	Decimal Value	16 ⁿ	Decimal Value
16 ⁰	1	16 ⁴	65,536
16 ¹	16	16 ⁵	1,048,576
16 ²	256	16 ⁶	16,777,216
16 ³	4096	16 ⁷	268,435,456

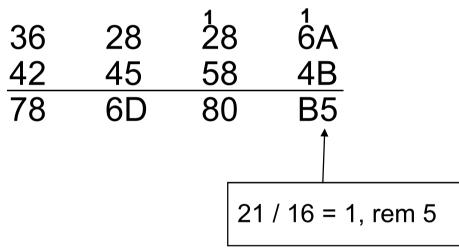
Converting Decimal to Hexadecimal

Division	Quotient	Remainder
422 / 16	26	6
26 / 16	1	А
1 / 16	0	1

decimal 422 = 1A6 hexadecimal

Hexadecimal Addition

- Divide the sum of two digits by the number base (16).
- The quotient becomes the carry value, and the remainder is the sum digit.



Important skill: Programmers frequently add and subtract the addresses of variables and instructions.

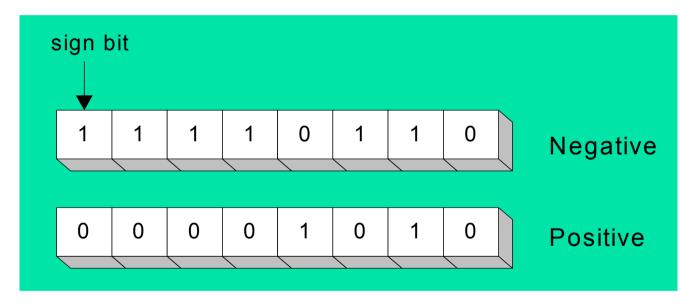


 When a borrow is required from the digit to the left, add 16 (decimal) to the current digit's value:

Practice: The address of var1 is 00400020. The address of the next variable after var1 is 0040006A. How many bytes are used by var1?

Signed Integers

- The highest bit indicates the sign.
- 1 = negative, 0 = positive



If the highest digit of a hexadecimal integer is > 7, the value is negative. Examples: 8A, C5, A2, 9D

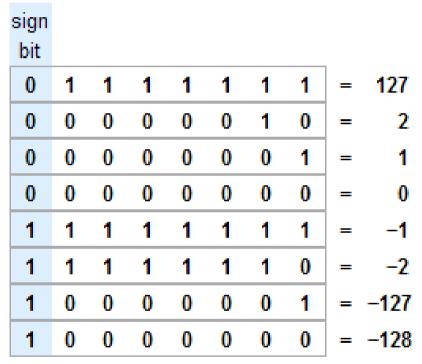
Forming the Two's Complement

Bitwise NOT of the number and add 1

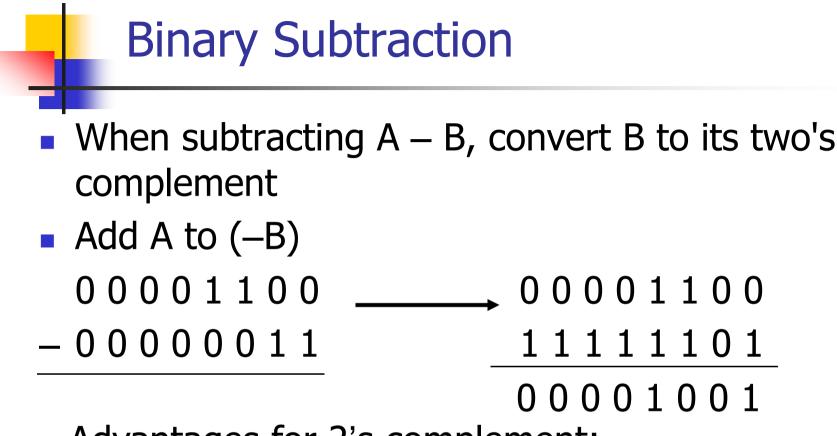
Starting value	0000001
Step 1: reverse the bits	11111110
Step 2: add 1 to the value from Step 1	11111110 +00000001
Sum: two's complement representation	11111111

Note that 0000001 + 11111111 = 00000000

8-bit Two's Complement Integers



8-bit two's complement integers



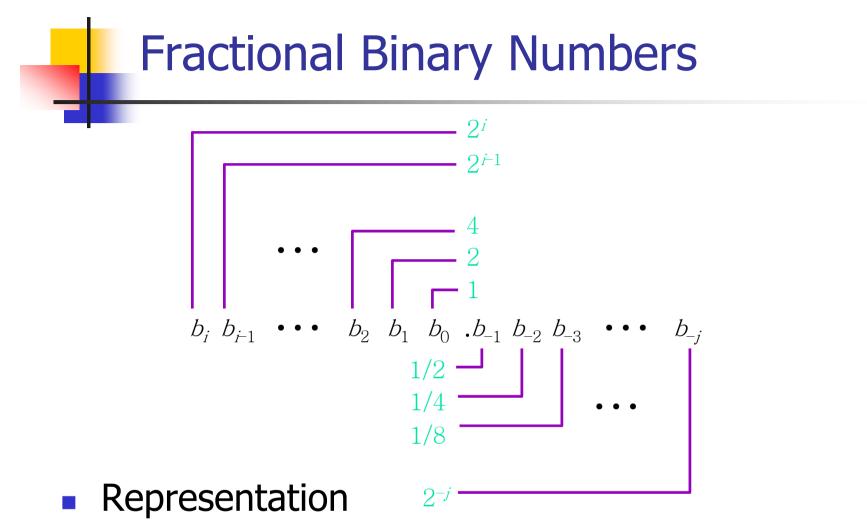
Advantages for 2's complement:

- No two 0's
- Sign bit
- Remove the need for separate circuits for add and sub

Ranges of Signed Integers

The highest bit is reserved for the sign. This limits the range:

Storage Type	Range (low–high)	Powers of 2
Signed byte	-128 to +127	-2^7 to $(2^7 - 1)$
Signed word	-32,768 to +32,767	-2^{15} to $(2^{15} - 1)$
Signed doubleword	-2,147,483,648 to 2,147,483,647	-2^{31} to $(2^{31} - 1)$
Signed quadword	-9,223,372,036,854,775,808 to +9,223,372,036,854,775,807	-2^{63} to $(2^{63} - 1)$



- Bits to right of "binary point" represent fractional powers of 2
- Represents rational number:

$$\sum_{k=-j}^{l} b_k \cdot 2^k$$

Examples of Fractional Binary Numbers

Value	Representation
5-3/4	101.112
2-7/8	10.1112
63/64	0.1111112

Observations

- Divide by 2 by shifting right
- Multiply by 2 by shifting left
- Numbers of form 0.1111111...2 just below 1.0
 - $1/2 + 1/4 + 1/8 + ... + 1/2^{i} + ... \rightarrow 1.0$

• Use notation 1.0 – ϵ

Representable Numbers

- Limitation
 - > Can only exactly represent numbers of the form $x \times 2^y$
 - Other numbers have repeating bit representations
- Value Representation

 1/3
 0.01010101[01]...2
 - **1/5** 0.00110011[0011]...₂
 - **1/10** 0.000110011[0011]...₂

Converting Real Numbers

Binary real to decimal real

 $110.011_2 = 4 + 2 + 0.25 + 0.125 = 6.375$

- Decimal real to binary real
 - $0.5625 \times 2 = 1.125$ first bit = 1
 - $0.125 \times 2 = 0.25$ second bit = 0
 - $0.25 \times 2 = 0.5$ third bit = 0
 - $0.5 \times 2 = 1.0$ fourth bit = 1

 $4.5625 = 100.1001_2$

True or False

- If x > 0 then x + 1 > 0
- If x < 0 then x * 2 < 0
- If x > y then -x < -y
- If $x \ge 0$ then $-x \le 0$
- If x < 0 then -x > 0
- If $x \ge 0$ then ((|x 1| & x) = x
- If x < 0 && y > 0 then x * y < 0
- If x < 0 then $((x \land x >> 31) + 1) > 0$

Character Storage

- Character sets
 - Standard ASCII (0 − 127)
 - ▶ Extended ASCII (0 255)
 - ► ANSI (0 255)
 - ▶ Unicode (0 65,535)
- Null-terminated String
 - Array of characters followed by a null byte
- Using the ASCII table
 - back inside cover of book

Machine Words

- Machine Has "Word Size"
 - Nominal size of integer-valued data
 - Including addresses
 - Most current machines use 32 bits (4 bytes) words
 - Limits addresses to 4GB
 - Users can access 3GB
 - Becoming too small for memory-intensive applications
 - High-end systems use 64 bits (8 bytes) words
 - Potential address space \approx 1.8 X 10¹⁹ bytes
 - x86-64 machines support 48-bit addresses: 256 Terabytes
 - Machines support multiple data formats
 - Fractions or multiples of word size
 - Always integral number of bytes

Word-Oriented Memory Organization 32-bit 64-bit Bytes Addr. Words Words 0000 Addr 0001 = Addresses Specify Byte 0002 0000 Addr 0003 Locations = 0004 0000 Addr Address of first byte in 0005 = 0006 0004 word 0007 > Addresses of successive 0008 Addr words differ by 4 (32-bit) 0009 = or 8 (64-bit) 0010 0008 Addr 0011 = 0008 0012 Addr 0013 = 0014 0012 0015

Data Representations

Sizes of C Objects (in Bytes)

- C Data Type Typical 32-bit Intel IA32 x86-64
 - unsigned 4 4
 - int 4 4
 - long int 4 4

2

4

8

8

4

8

4

• char 1 1

4

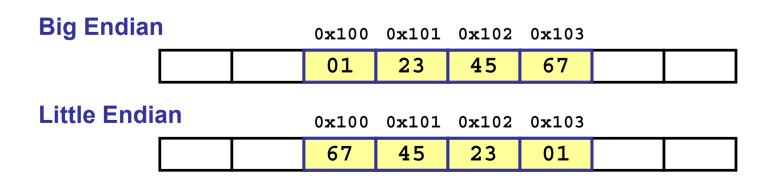
- short 2 2
- float 4
- double 8
- char *
 - Or any other pointer

Byte Ordering

- How should bytes within multi-byte word be ordered in memory?
- Conventions
 - > Big Endian: Sun, PPC Mac
 - Least significant byte has highest address
 - Little Endian: x86
 - Least significant byte has lowest address

Byte Ordering Example

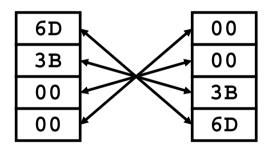
- Big Endian
 - Least significant byte has highest address
- Little Endian
 - Least significant byte has lowest address
- Example
 - Variable x has 4-byte representation 0x01234567
 - Address given by &x is 0x100



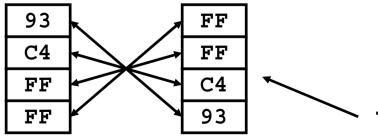
Representing Integers

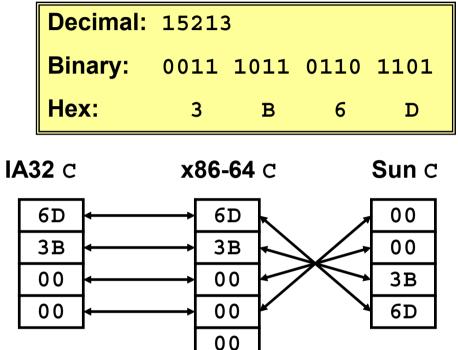
- int A = 15213;
- int B = -15213;
- long int C = 15213;











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Two's complement representation

Representing Strings

Strings in C

```
char S[6] = "15213";
```

- Represented by array of characters
- Each character encoded in ASCII format
 - Standard 7-bit encoding of character set
 - Character "0" has code 0x30
 - Digit *i* has code 0x30+*i*
- String should be null-terminated
 - Final character = 0
- Compatibility
 - Byte ordering not an issue

Linux/Alpha s Sun s

31	←──→	31
35	← →	35
32	← →	32
31	← →	31
33	← →	33
00	←	00

Boolean Operations

- NOT
- AND
- OR
- Operator Precedence
- Truth Tables

Boolean Algebra

- Based on symbolic logic, designed by George Boole
- Boolean expressions created from:
 - > NOT, AND, OR

Expression	Description
\neg_X	NOT X
$X \wedge Y$	X AND Y
$X \lor Y$	X OR Y
$\neg X \lor Y$	(NOT X) OR Y
$\neg(X \land Y)$	NOT (X AND Y)
$X \land \neg Y$	X AND (NOT Y)



- Inverts (reverses) a boolean value
- Truth table for Boolean NOT operator:

X	¬ X	
F	Т	
Т	F	

Digital gate diagram for NOT:

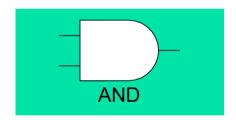
NOT



Truth table for Boolean AND operator:

X	Y	$\mathbf{X} \wedge \mathbf{Y}$		
F	F	F		
F	Т	F		
Т	F	F		
Т	Т	Т		

Digital gate diagram for AND:

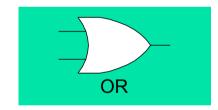


Truth table for Boolean OR operator:

х	Y	$\mathbf{X} \lor \mathbf{Y}$	
F	F	F	
F	Т	Т	
Т	F	Т	
Т	Т	Т	

OR

Digital gate diagram for OR:



Operator Precedence

- NOT > AND > OR
- Examples showing the order of operations:

Expression	Order of Operations		
$\neg X \lor Y$	NOT, then OR		
$\neg(X \lor Y)$	OR, then NOT		
$X \lor \ (Y \land Z)$	AND, then OR		

Use parentheses to avoid ambiguity

Truth Tables (1 of 3)

- A Boolean function has one or more Boolean inputs, and returns a single Boolean output.
- A truth table shows all the inputs and outputs of a Boolean function

Example: $\neg X \lor Y$

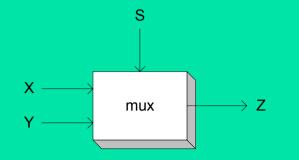
X
$$\neg$$
XY \neg X \lor YFTFTFTTTTFFFTFTT



• Example: $X \land \neg Y$

Х	Y	$\neg Y$	X∧¬Y
F	F	Т	F
F	Т	F	F
Т	F	Т	Т
Т	Т	F	F

Truth Tables (3 of 3)



■ Example: (Y ∧ S) ∨ (X ∧ ¬S)

Two-input multiplexer

X	Y	S	$Y \wedge S$	$\neg_{\mathbf{S}}$	X∧¬S	$(\mathbf{Y} \wedge \mathbf{S}) \lor (\mathbf{X} \wedge \neg \mathbf{S})$
F	F	F	F	Т	F	F
F	Т	F	F	Т	F	F
Т	F	F	F	Т	Т	Т
Т	Т	F	F	Т	Т	Т
F	F	Т	F	F	F	F
F	Т	Т	Т	F	F	Т
Т	F	Т	F	F	F	F
Т	Т	Т	Т	F	F	Т