EngrD 2190 – Lecture 37

Concept: Dimensional Analysis and Dynamic Scaling

Context: Designing a Dynamically Similar Model

Defining Question: Restaurants display models of food items. Why don't fashion boutiques display clothes on dolls?

EngrD 2190: Professional Development Assignment - LinkedIn

Assigned November 10, 2021

Due: December 1, 2021

1. If you have not already done so, enroll in the Cornell Career Services "Career Development Toolkit" available via https://scl.cornell.edu/news-events/news/new-career-development-toolkit
2. Complete the LinkedIn module located within the "Networking" tab within the Toolkit.
3. Based on the guidance provided within the LinkedIn education module, create a LinkedIn account/page for yourself that is comprehensive, engaging, and effective.
4. Using the guidance provided in the education module, along with your own experience in creating your account, create a LinkedIn grading rubric and use it to assess the LinkedIn page of <u>two</u> of your class peers. Refer to the grading rubrics for homework and prelim 1 as examples.
5. Send to Professor Woltornist (aw499) and Professor Duncan (tmd10) –

    A link to your LinkedIn page, and

    Completed grading rubrics for the LinkedIn pages of two peers.

   by Wednesday, December 1.

Three skills in dimensional analysis / dynamic scaling

1. Derive a set of dimensionless groups given a list of parameters.

    Lectures 33, 34, and 35 last week:

        Pendulums Swinging, People Walking, and Spheres Falling.

    Practice Exercises: 5.8, 5.15, 5.20, and 5.22.

2. Use a Universal Correlation of Dimensionless Groups.

    Lecture 36.

    Practice Exercises: 5.28, 5.29, and 5.30.

3. Design a Dynamically Similar Model by Scaling.

    Lectures 37 and 38

    Practice Exercises: 5.33, 5.34, and 5.39.

Solutions to practice exercises are posted at the EngrD 2190 homepage.

Designing a Dynamically Similar Model

Assume the system is described by n Π groups: $\Pi_1, \Pi_2, \ldots, \Pi_n$

1. Set $(\Pi_i)_{\text{model}} = (\Pi_i)_{\text{actual}}$ for $i = 1$ to $n-1$.

2. Use the model to measure Π_n.

3. Use Π_n to calculate parameters for the actual system.

Example 1: Design a model to predict the pressure drop in an artificial kidney.

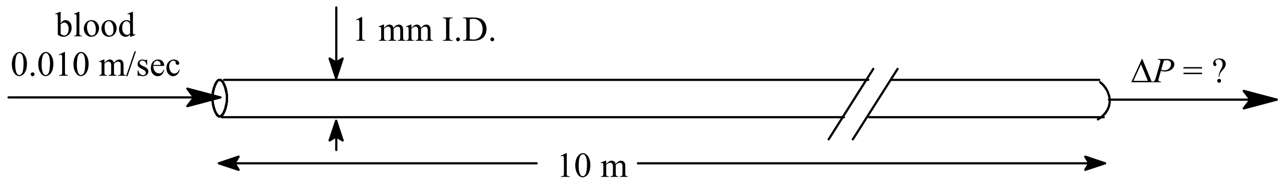


Table 5.8. The parameters of fluid flow through a pipe.

parameter	symbol	dimensions
pressure drop	ΔP	M/LT^2
pipe length	ℓ	L
pipe diameter	d	L
fluid velocity	v	L/T
fluid viscosity	μ	M/LT
fluid density	ρ_{fluid}	M/L^3

3 core variables: ΔP, ℓ, and μ.

our goal

$$\Pi_1 = \frac{\Delta P}{v^2 \rho_{fluid}} = \text{Eu, the Euler Number, } \frac{\text{friction force}}{\text{inertial force}}$$

use ×10 in model

$$\Pi_2 = \frac{\ell}{d} = \text{reduced pipe length, } \frac{\text{pipe length}}{\text{pipe diameter}}$$

$$\Pi_3 = \frac{\rho_{\text{fluid}} v d}{\mu} = \text{Re, the Reynolds Number, } \frac{\text{inertial force}}{\text{viscous force}}$$

need ×10 in model

Use to design model

Example 1: Design a model to predict the pressure drop in an artificial kidney.

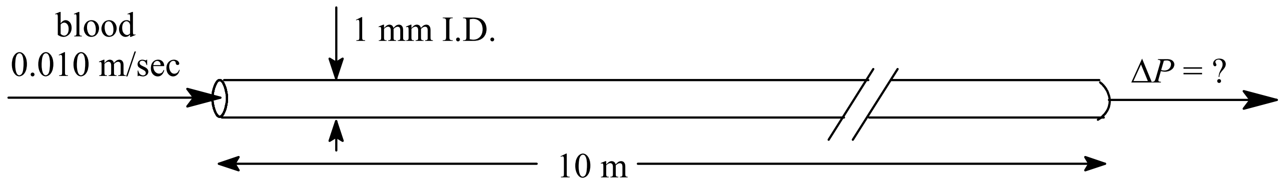


Table 5.8. The parameters of fluid flow through a pipe.

parameter	symbol	dimensions	artificial kidney	model
pressure drop	ΔP	M/LT^2	?	(to be measured)
pipe length	ℓ	L	10 m	Use Π_2 to calculate
pipe diameter	d	L	0.001 m	0.010 m first guess
fluid velocity	v	L/T	0.010 m/s	Use Π_3 to calculate
fluid viscosity	μ	M/LT	0.0040 Pa·sec	0.050 } vegetable oil
fluid density	ρ_{fluid}	M/L^3	1025 kg/m^3	930 } vegetable oil

fluid	density (kg/m^3)	viscosity (Pa·s)
water	1000.	0.0010
blood	1025.	0.0040
vegetable oil	930.	0.050
glycerin	1250.	1.2

~×10 Use vegetable oil in the model.

Example 1: Design a model to predict the pressure drop in an artificial kidney.

$$\Pi_1 = \frac{\Delta P}{v^2 \rho_{fluid}} = \text{Eu, the Euler Number, } \frac{\text{friction force}}{\text{inertial force}}$$

$$\Pi_2 = \frac{\ell}{d} = \text{reduced pipe length, } \frac{\text{pipe length}}{\text{pipe diameter}}$$

$$\Pi_3 = \frac{\rho_{\text{fluid}} v d}{\mu} = \text{Re, the Reynolds Number, } \frac{\text{inertial force}}{\text{viscous force}}$$

Use Π_2 to calculate the pipe length (d) in our model.

$$\left(\frac{\ell}{d}\right)_{\text{model}} = \left(\frac{\ell}{d}\right)_{\text{kidney}}$$

$$\ell_{\text{model}} = \left(\frac{\ell}{d}\right)_{\text{kidney}} d_{\text{model}} = \left(\frac{10\text{ m}}{0.001\text{ m}}\right) 0.01\text{ m} = 100\text{ m of tube}$$

Use Re (Π_3) to calculate the fluid velocity (v) in our model.

$$\left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{kidney}}$$

Re = 2.6

$$v_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{kidney}} \left(\frac{\mu}{\rho_{\text{fluid}} d}\right)_{\text{model}} = \left(\frac{1025 \times 0.01 \times 0.001}{0.004}\right)\left(\frac{0.05}{930 \times 0.01}\right)$$

$$v_{\text{model}} = 0.014\text{ m/sec} = 1.4\text{ cm/sec}$$ reasonable

If we used water in the model, $v_{\text{model}} = 0.03$ cm/sec. Too slow. If glycerin, $v_{\text{model}} = 25$ cm/sec. Too fast.

Example 1: Design a model to predict the pressure drop in an artificial kidney.

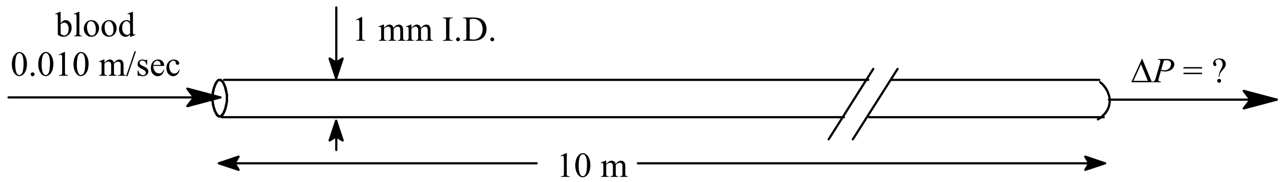


Table 5.8. The parameters of fluid flow through a pipe.

parameter	symbol	dimensions	artificial kidney	model	
pressure drop	ΔP	M/LT^2	?	(to be measured)	
pipe length	ℓ	L	10 m	100 m	from Π_2
pipe diameter	d	L	0.001 m	0.010 m	first guess
fluid velocity	v	L/T	0.010 m/s	0.014 m/sec	from Re
fluid viscosity	μ	M/LT	0.0040 Pa·sec	0.050	vegetable oil
fluid density	ρ_{fluid}	M/L^3	1025 kg/m^3	930	vegetable oil

We build the model and measure $\Delta P = 9.0 \times 10^5$ Pa = 8.9 atm = 130 psi

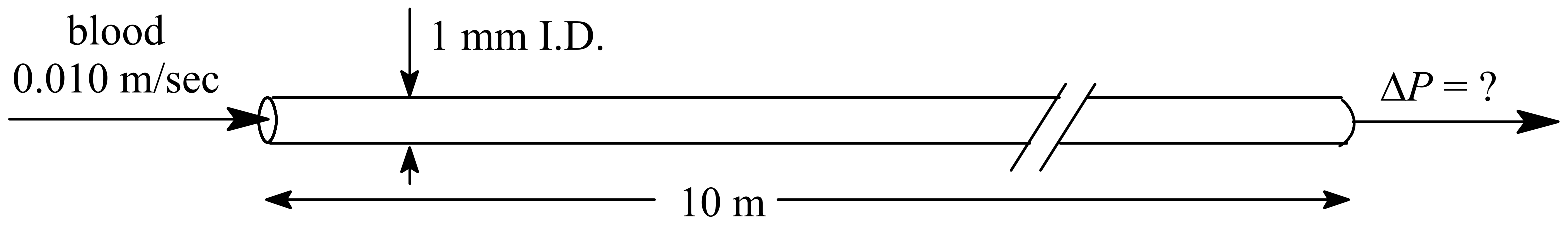
Use Π_1 to calculate ΔP in our artificial kidney.

$$\left(\frac{\Delta P}{v^2 \rho_{\text{fluid}}}\right)_{\text{kidney}} = \left(\frac{\Delta P}{v^2 \rho_{\text{fluid}}}\right)_{\text{model}}$$

$$(\Delta P)_{\text{kidney}} = \left(\frac{\Delta P}{v^2 \rho_{\text{fluid}}}\right)_{\text{model}} \left(v^2 \rho_{\text{fluid}}\right)_{\text{kidney}} = \left(\frac{9.0\times10^5}{(0.014)^2 \times 930}\right)\left((0.01)^2 \times 1025\right)$$

$$(\Delta P)_{\text{kidney}} = 9.0\times10^5 \text{ Pa} = 5.0 \text{ atm} = 74 \text{ psi}$$

Example 1: Design a model to predict the pressure drop in an artificial kidney.



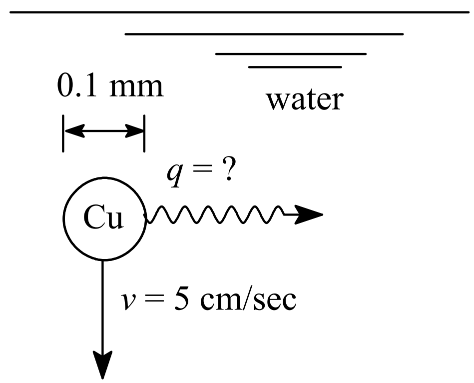
$$(\Delta P)_{\text{kidney}} = 9.0 \times 10^5 \text{ Pa} = 5.0 \text{ atm} = 74 \text{ psi}$$

Blood pressure is measured in units of mm Hg. 80/160 ~ 1.5 psi/3.1 psi.

Pressure drop is too large. What to do?

Use a hundred 100-cm tubes in parallel.
Total length = 10 m, $\Delta P = 0.74$ psi

Example 2: Hot microspheres descending and cooling in a liquid.



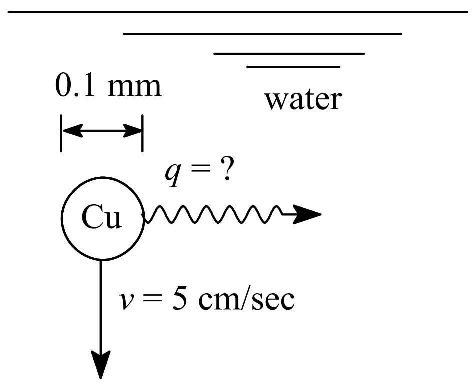
We need the sphere's heat transfer coefficient, h.

$$h \equiv \frac{q}{(\text{area})(\Delta T)}$$

$$q = h(\text{sphere surface area})(T_{\text{sphere}} - T_{\text{fluid}})$$

Design a more convenient model with larger spheres.

Example 2: Hot microspheres descending and cooling in a liquid.



	parameter	symbol	dimensions
the sphere	diameter	d	L
the fluid	fluid viscosity	μ	M/LT
	fluid density	ρ_{fluid}	M/L^3
	fluid heat capacity	C_P	L$^2/\Theta$T^2
	fluid thermal conductivity	k	ML/ΘT^3
dynamics	sphere velocity	v	L/T
	heat transfer coefficient	h	M/ΘT^3

our goal

7 parameters – 4 dimensions = 3 core variables: h, C_P, and v.

$$\Pi_1 = \frac{hd}{k} = \text{Nu, the Nusselt Number, } \frac{\text{convective heat transfer}}{\text{conductive heat transfer}}$$

$$\Pi_2 = \frac{C_P\mu}{k} = \text{Pr, the Prandtl Number, } \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}}$$

$$\Pi_3 = \frac{\rho_{fluid} v d}{\mu} = \text{Re, the Reynolds Number, } \frac{\text{inertial force}}{\text{viscous force}}$$

Use to design model

The Prandtl number comprises fluid properties only.

Dynamic similarity requirement $(\text{Pr})_{model} = (\text{Pr})_{actual}$ demands model system and actual system use same fluid: water.

Example 2: Hot microspheres descending and cooling in a liquid.

	parameter	symbol	dimensions	micro spheres	model
the sphere	diameter	d	L	1×10^{-4} m	0.010 m
the fluid	fluid viscosity	μ	M/LT	10^{-3} Pa·sec	use Prandtl number; model must use water
	fluid density	ρ_{fluid}	M/L^3	1000 kg/m^3	
	fluid heat capacity	C_P	L^2/ΘT^2	4200 J/(kg·K)	
	fluid thermal conductivity	k	ML/ΘT^3	0.59 J/(m·sec·K)	
dynamics	sphere velocity	v	L/T	0.05 m/s	use Reynolds number
	heat transfer coefficient	h	M/ΘT^3	?	measure

Calculate v_{model}.

$$\mathrm{Re}_{\text{model}} = \mathrm{Re}_{\text{microsphere}}$$

$$\left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{microsphere}}$$

$$v_{\text{model}} = \left(\frac{\rho_{\text{fluid}} v d}{\mu}\right)_{\text{microsphere}} \left(\frac{\mu}{\rho_{\text{fluid}} d}\right)_{\text{model}}$$

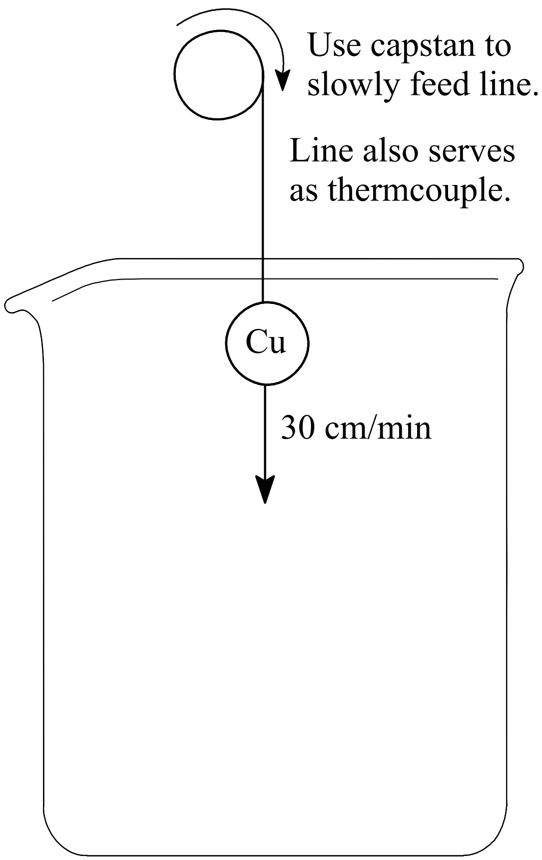
$$v_{\text{model}} = (vd)_{\text{microsphere}} \left(\frac{1}{d}\right)_{\text{model}} = \frac{0.05\ \text{m/sec} \times 10^{-4}\ \text{m}}{0.01\ \text{m}} = 5 \times 10^{-4}\ \text{m/sec}$$

v_{model} = 0.5 mm/sec = 30 cm/min

Too slow for a free-falling 1-cm copper sphere in water.

Example 2: Hot microspheres descending and cooling in a liquid.

$$v_{\text{model}} = 0.5 \text{ mm/sec} = 30 \text{ cm/min}$$



Conduct experiments, measure $h_{\text{model}} = 240 \text{ J/(m}^2\cdot\text{sec}\cdot\text{K)}$

Calculate the Nusselt Number -

$$\text{Nu} = \frac{hd}{k} = \frac{(240 \text{ J/(m}^2\cdot\text{sec}\cdot\text{K}))(0.01\text{ m})}{0.59 \text{ J/(m}\cdot\text{sec}\cdot\text{K})} = 4.1$$

Calculate $h_{\text{microsphere}}$

$$\text{Nu}_{\text{microsphere}} = \text{Nu}_{\text{model}}$$

$$\left(\frac{hd}{k}\right)_{\text{microsphere}} = 4.1$$

$$h_{\text{microsphere}} = 4.1\left(\frac{k}{d}\right)_{\text{microsphere}}$$

$$h_{\text{microsphere}} = 4.1\left(\frac{0.59 \text{ J/(m}\cdot\text{sec}\cdot\text{K})}{0.0001\text{ m}}\right) = 2.4\times10^{4} \text{ J/(m}^2\cdot\text{sec}\cdot\text{K)}$$

recap on Designing a Dynamically Similar Model by Scaling

Assume the system is described by n Π groups: $\Pi_1, \Pi_2, \ldots, \Pi_n$

1. Set $(\Pi_i)_{\text{model}} = (\Pi_i)_{\text{actual}}$ for $i = 1$ to $n-1$.
2. Use the model to measure Π_n.
3. Use Π_n to calculate parameters for the actual system.

If your model has a different length and/or mass dimension - your model is smaller or less massive - your model must also change something so there is a different time dimension.

Usually, use a different fluid for the model.

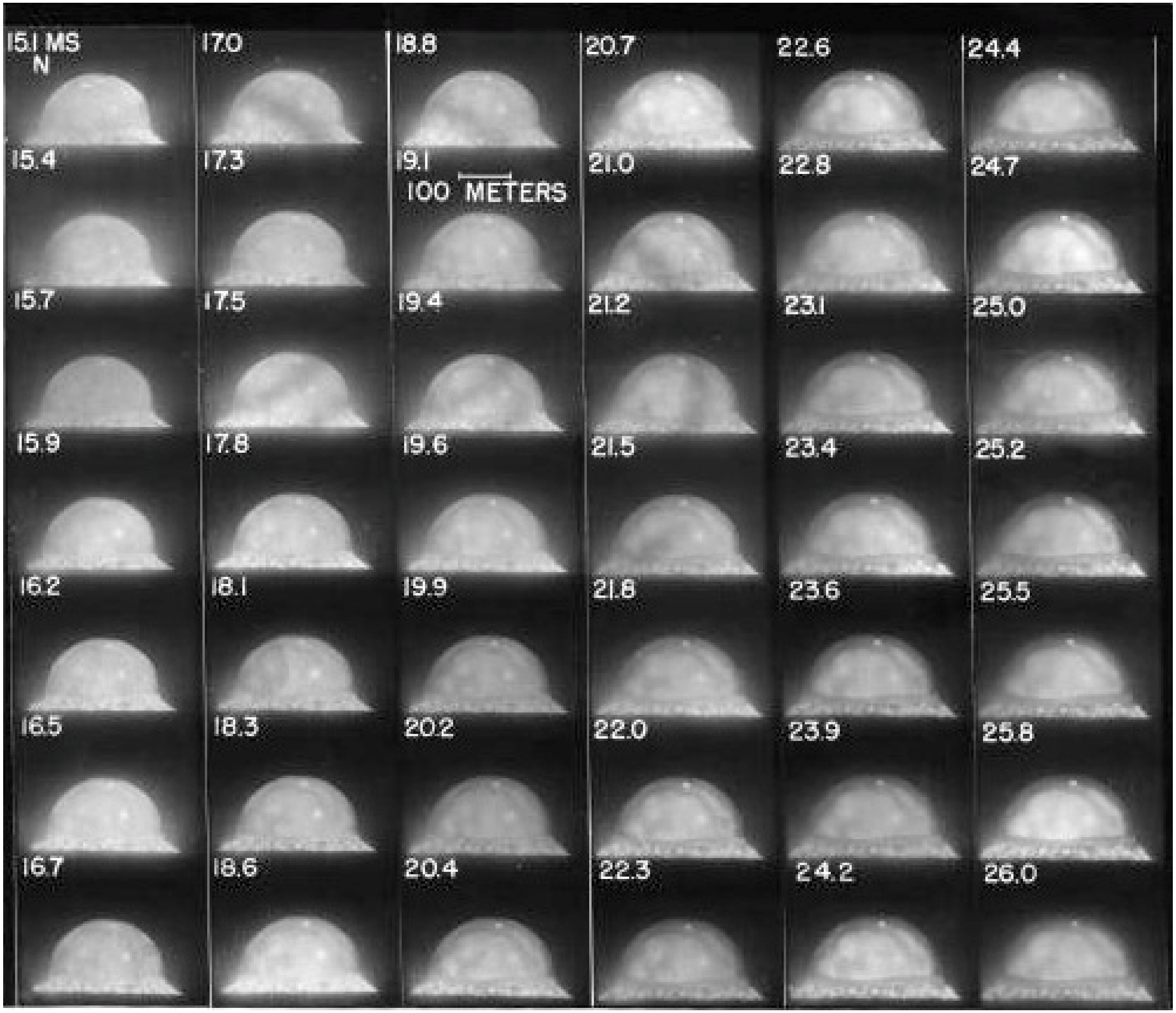
Fluid parameters: viscosity: $[\mu] = \dfrac{\text{M}}{\text{LT}}$

surface tension: $[\gamma] = \dfrac{\text{M}}{\text{T}^2}$

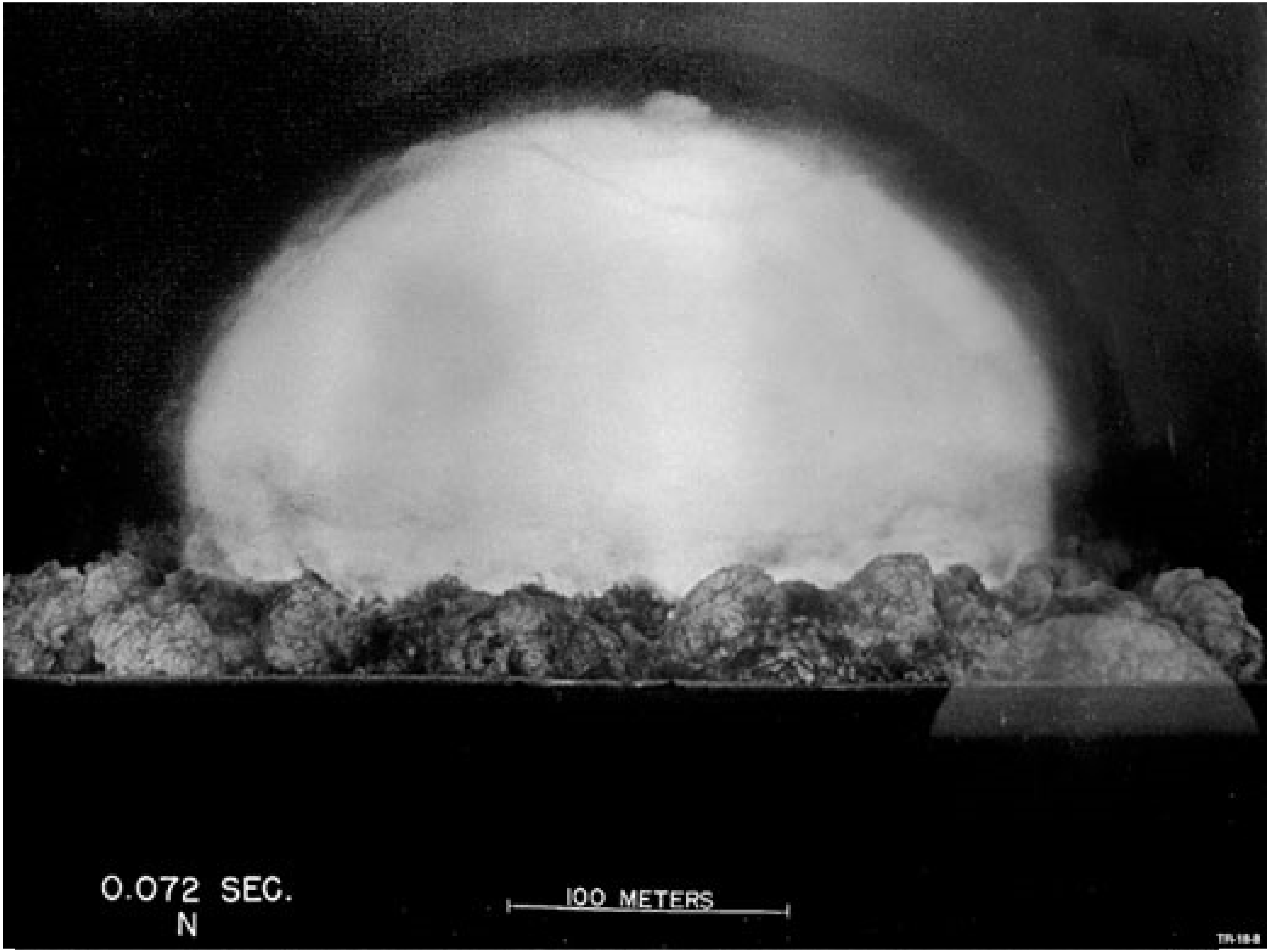
heat capacity: $[C_\text{P}] = \dfrac{\text{L}^2}{\text{T}^2\Theta}$

thermal conductivity: $[k] = \dfrac{\text{ML}}{\text{T}^3\Theta}$

Example 3: A Universal Correlation for an Explosive Shock Wave.



Example 3: A Universal Correlation for an Explosive Shock Wave.



The formation of a blast wave by a very intense explosion. II. The atomic explosion of 1945

By Sir Geoffrey Taylor, F.R.S.

(*Received* 10 *November* 1949)

[Plates 7 to 9]

Photographs by J. E. Mack of the first atomic explosion in New Mexico were measured, and the radius, R, of the luminous globe or 'ball of fire' which spread out from the centre was determined for a large range of values of t, the time measured from the start of the explosion. The relationship predicted in part I, namely, that $R^{\frac{5}{2}}$ would be proportional to t, is surprisingly accurately verified over a range from $R=20$ to 185 m. The value of $R^{\frac{5}{2}}t^{-1}$ so found was used in conjunction with the formulae of part I to estimate the energy E which was generated in the explosion. The amount of this estimate depends on what value is assumed for γ, the ratio of the specific heats of air.

Two estimates are given in terms of the number of tons of the chemical explosive T.N.T. which would release the same energy. The first is probably the more accurate and is 16,800 tons. The second, which is 23,700 tons, probably overestimates the energy, but is included to show the amount of error which might be expected if the effect of radiation were neglected and that of high temperature on the specific heat of air were taken into account. Reasons are given for believing that these two effects neutralize one another.

After the explosion a hemispherical volume of very hot gas is left behind and Mack's photographs were used to measure the velocity of rise of the glowing centre of the heated volume. This velocity was found to be 35 m./sec.

Until the hot air suffers turbulent mixing with the surrounding cold air it may be expected to rise like a large bubble in water. The radius of the 'equivalent bubble' is calculated and found to be 293 m. The vertical velocity of a bubble of this radius is $\frac{2}{3}\sqrt{(g\,29300)}$ or 35·7 m./sec. The agreement with the measured value, 35 m./sec., is better than the nature of the measurements permits one to expect.

Example 3: A Universal Correlation for an Explosive Shock Wave.

The parameters of the shock wave from an explosion.

parameter	symbol	dimensions
explosive energy	E	ML^2/T^2
shock wave radius	r	L
time after explosion	t	T
fluid density	ρ_{fluid}	M/L^3

4 parameters – 3 dimensions = 1 Π group

$$\Pi = E^a r^b t^c \rho^d$$

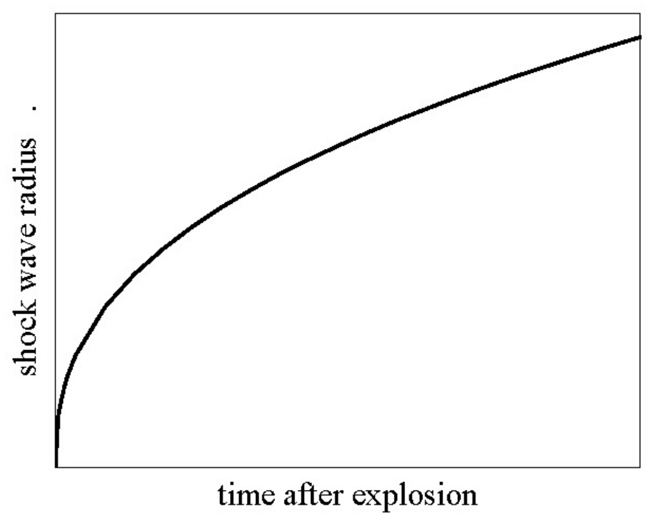
$$[\Pi] = \left[E^a r^b t^c \rho^d\right] = \left(\frac{ML}{T^2}\right)^a L^b T^c \left(\frac{M}{L^3}\right)^d = M^{a+d} L^{2a+b-3d} T^{-2a+c}$$

Set $a = 1$

$$\Pi = \left(\frac{Et^2}{\rho r^5}\right)^{1/5}$$

From experiments, $\Pi = 1$

Thus $r = \left(\frac{Et^2}{\rho}\right)^{1/5}$



From the declassified photos, Geoffrey Taylor calculated the energy of the Trinity explosion:

E = 28 kilotons of TNT

Dimensional Analysis and Dynamic Scaling

Trucks, Power Shovels, and Barbie Dolls

Dynamic Similarity in Trucks

Dynamic Similarity in Trucks

Dynamic Similarity in Trucks

Dynamic Similarity in Trucks

Height to Load Bed

Taylor Dunn ET-3000

Terex TR-100

3.5 ft $\xrightarrow{\times 4.3}$ 15 ft

Dynamic Similarity in Trucks

Vehicle Weight

Taylor Dunn ET-3000

Terex TR-100

0.95 ton $\xrightarrow{\times(4.3)^3}$ 76 ton

Dynamic Similarity in Trucks

Load Capacity

Taylor Dunn ET-3000

Terex TR-100

$$3000\ \text{lb} = 1.5\ \text{ton} \xrightarrow{\times(4.1)^3} 100\ \text{ton}$$

Dynamic Similarity in Trucks

Engine Power

Taylor Dunn ET-3000

Terex TR-100

15 hp $\xrightarrow{\times(4.1)^3}$ 1050 hp

Dynamic Similarity in Trucks

Tires

Taylor Dunn ET-3000

Terex TR-100

20.5×8×10
20 inches = 1.7 ft

$\xrightarrow{\times(4.3)^{3/2}}$

59/80R63
14 ft

How Tires Carry the Load

Tire Contact Area Scales as Height Squared

How to Scale Tire Height?

Weight scales as $(\text{Height})^3$

Increase Tire Height as $(\text{Height})^1$:

$\Rightarrow$ Tire Area increases as $(\text{Height})^2$

Not Enough

Increase Tire Height as $(\text{Height})^{3/2}$:

$\Rightarrow$ Tire Area increases as $(\text{Height}^{3/2})^2 = (\text{Height})^3$

Enough!

Dynamic Similarity in Trucks

Tires

Taylor Dunn ET-3000

Terex TR-100

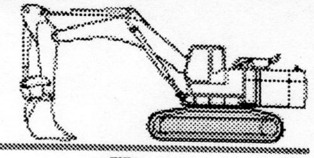
20.5×8×10
20 inches = 1.4 ft

$\xrightarrow{\times(4.3)^{3/2}}$

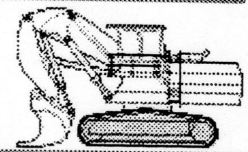
59/80R63
14 ft


RH 400
11-37
CATERPILLAR
797
797
500

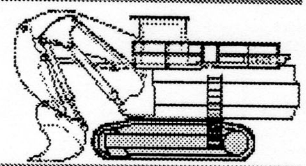
	RH 30 E	RH 30 F
Operating weight	80 t	90 t
Shovel	6,3 m³	
Backhoe		5,1 m³



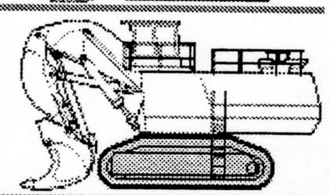
	RH 40 E
Operating weight	100 t
Shovel	8,1 m³
Backhoe	7,0 m³



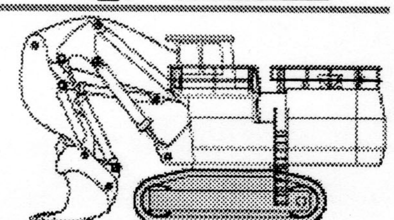
	RH 90 C
Operating weight	163 t
Shovel	12,0 m³
Backhoe	10,0 m³



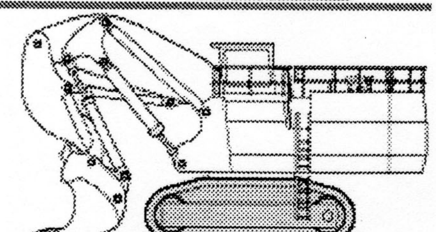
	RH 120 E
Operating weight	265 t
Shovel	17,0 m³
Backhoe	17,0 m³



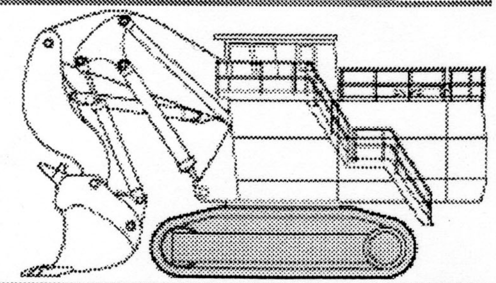
	RH 170
Operating weight	360 t
Shovel	21,0 m³
Backhoe	20,0 m³



	RH 200
Operating weight	480 t
Shovel	30,5 m³
Backhoe	26,0 m³



	RH 400
Operating weight	900 t
Shovel	46,0 - 52,0 m³



Why does this garment display poorly?

What scaling was ignored?

Thread count!

The cloth used for this garment is the same cloth used at full scale.

The scaled equivalent is burlap at full scale, which has a thread count of about 10-12 per inch.

Burlap garments hang poorly.

Prelim 3 2021 Statistics

Mean: 65 / 120 (54%)
Std. Deviation: 20

A - L: Lucy (Front of room)
M - Z: Kelsey (Back of room)

Solution is posted.

Problem 1: 19 ± 7 / 30 (62%)
Problem 2: 13 ± 5 / 25 (52%)
Problem 3: 13 ± 7 / 25 (52%)
Problem 4: 20 ± 12 / 40 (50%)