

### 28.1 Reflection

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1 Peter Hopkinson boosts class interest using this zany demonstration of standing astride a large mirror as he lifts his right leg while his unseen left leg provides support behind the mirror. 2 Why do the legs of the duck, but not its feet, show in the reflected view? 3 Physics teacher Fred Myers stands between parallel mirrors and takes a photo of his daughter McKenzie, a design engineer. 4 How many mirrors produce these multiple reflections of Texas physics teacher Karen Jo Matsler?

French lawyer and mathematician Pierre de Fermat (pronounced fer-mah) was born in 1601. He attended the University of Toulouse before moving to Bordeaux in his twenties. He was fluent in Latin, Greek, Italian, and Spanish and was well recognized for his written verse in several languages.
 In 1629 , he produced important mathematical work on the ideas of maxima and minima, which turned out to be useful to Newton, as well as to Leibniz, when they independently developed calculus. Through his correspondence with Blaise Pascal in 1654, Fermat helped lay the fundamental groundwork for the theory of probability.

To mathematicians, Fermat is best remembered for his famous "Last Theorem," a special case of which states that the sum of two cubes of whole numbers cannot be the cube of another whole number. For more than 300 years, mathematicians were tantalized by a marginal note in Latin in one of Fermat's books, which is translated as "I have a truly marvelous proof of this proposition which this margin is too narrow to contain." Not until 1994 was the theorem proved (by Andrew Wiles of Princeton University) using methods unavailable to Fermat, so it seems unlikely that Fermat really did have a proof. This doesn't diminish the genius that he showed in many other ways.

Fermat had a unique way of looking at paths of light. He stated that of all the possible paths that light can travel from one point to another, it travels the path that requires the least time. Reflection and refraction, the chief topics of this chapter, are nicely understood with this principle.


FIGURE 28.1 Light interacts with atoms as sound interacts with tuning forks.

- B
A.


## Mirror

FIGURE 28.2


### 28.1 Reflection

Most of the things we see around us do not emit light of their own. They are visible because they reemit light that reaches their surface from a primary source, such as the Sun or a lamp, or from a secondary source, such as the illuminated sky. When light falls on the surface of a material, it is either reemitted without change in frequency or absorbed into the material and converted to heat. ${ }^{1}$ We say that light is reflected when it is returned into the medium from which it came-this process is reflection.

When sunlight or lamplight illuminates print on a paper page, electrons in the atoms of the paper and ink vibrate more energetically in response to the oscillating electric fields of the illuminating light. The energized electrons reemit the light by which you see the page. When the page is illuminated by white light, the paper appears white, which reveals that the electrons reemit all the visible frequencies. Very little absorption occurs. The ink is a different story. Except for a bit of reflection, it absorbs all the visible frequencies and therefore appears black.

## Principle of Least Time ${ }^{2}$

The idea that light takes the quickest path in going from one place to another was formulated by Pierre de Fermat. His idea is now called Fermat's principle of least time.

We can understand reflection with Fermat's principle. Consider the following situation: In Figure 28.2, we see two points, A and B, and an ordinary plane mirror beneath. How can we get from A to B most quickly-that is, in the shortest time? The answer is simple enough: Go straight from A to B! But, if we add the condition that the light must strike the mirror in going from A to B in the shortest

[^0]time, the answer is not so easy. One way would be to go as quickly as possible to the mirror and then to B, as shown by the solid lines in Figure 28.3. This gives us a short path to the mirror but a very long path from the mirror to B. If we instead consider a point on the mirror a little to the right, we slightly increase the first distance, but we considerably decrease the second distance, so the total path length shown by the dashed lines-and therefore the travel time-is less. How can we find the exact point on the mirror for which the time is least? We can find it very nicely by a geometric trick.

We construct, on the opposite side of the mirror, an artificial point, $\mathrm{B}^{\prime}$, which is the same distance "through" and below the mirror as point B is above the mirror (Figure 28.4). The shortest distance between A and this artificial point $\mathrm{B}^{\prime}$ is simple enough to determine: It's a straight line. Now this straight line intersects the mirror at a point C , the precise point of reflection for the shortest path and hence the path of least time for the passage of light from A to B. Inspection will show that the distance from C to B equals the distance from C to $\mathrm{B}^{\prime}$. We see that the length of the path from $A$ to $\mathrm{B}^{\prime}$ through C is equal to the length of the path from A to B bouncing off point C along the way.

Inspection of Figures 28.4 and 28.5 and a little geometric reasoning will show that the angle of incident light from A to C is equal to the angle of reflection from C to B.


### 28.2 Law of Reflection

As Fermat showed, the angle of incident light will be the same as the angle of reflected light. This is the law of reflection, and it holds for all angles (Figure 28.5):

## The angle of incidence equals the angle of reflection.

The law of reflection is illustrated with arrows representing light rays in Figure 28.6. Instead of measuring the angles of incident and reflected rays from the reflecting surface, it is customary to measure them from a line perpendicular to the plane of the reflecting surface. This imaginary line is called the normal. The incident ray, the normal, and the reflected ray all lie in the same plane. Such reflection from a smooth surface is called specular reflection. Mirrors produce excellent specular reflections.


White coatings on roofs reflect up to $85 \%$ of incident light, which on hot summer days greatly reduces air conditioning costs and carbon emissions. On cold winter days where heat is desirable, however, this is not such a good idea. So, for regions with hot summers and mild winters, paint your rooftops white! (As mentioned in Chapter 27, there are also new brown paints that aid cooling by reflecting infrared light.)


SCREENCAST: Reflection

FIGURE 28.6 INTERACTIVE FIGURE


The law of reflection.


FIGURE 28.7
A virtual image is formed behind the mirror and is located at the position where the extended reflected rays (dashed lines) converge.

FIGURE 28.8
Marjorie's image is as far behind the mirror as she is in front. Note that she and her image have the same color of clothing-evidence that light doesn't change frequency upon reflection. Interestingly, her leftright axis is no more reversed than her up-down axis. The axis that is reversed, as shown to the right, is front-back. That's why it seems like her left hand faces the right hand of her image.

## CHECK POINT

The construction of artificial point $B^{\prime}$ in Figures 28.4 and 28.5 shows how light encounters point $C$ in reflecting from $A$ to $B$. By similar construction, show that light originating from $B$ and reflecting to $A$ also encounters the same point C .

## CHECK YOUR ANSWER

Construct an artificial point $A^{\prime}$ as far below the mirror as $A$ is above; then draw a straight line from $B$ to $A^{\prime}$ to find $C$, as shown at the left. Both constructions superimposed, at right, show that $C$ is common to both. We see that light will follow the same path if it goes in the opposite direction. Whenever you see somebody else's eyes in a mirror, be assured that the person can also see your eyes.


## Plane Mirrors

Suppose a candle flame is placed in front of a plane mirror. Rays of light radiate from the flame in all directions. Figure 28.7 shows only four of the infinite number of rays leaving one of the infinite number of points on the candle. These rays diverge from the candle flame and encounter the mirror, where they are reflected at angles equal to their angles of incidence. The rays diverge from the mirror and appear to emanate from a particular point behind the mirror (where the dashed lines intersect). An observer sees an image of the flame at this point. The light rays do not actually originate from this point, so the image is called a virtual image. The image is as far behind the mirror as the object is in front of the mirror, and image and object have the same size. When you view yourself in a mirror, for example, the size of your image is the same as the size your twin would appear if located the same distance behind the mirror as you are in front-as long as the mirror is flat (we call a flat mirror a plane mirror).

When the mirror is curved, the sizes and distances of object and image are no longer equal. We will not discuss curved mirrors in this text, except to say that

the law of reflection still applies. A curved mirror behaves as a succession of flat mirrors, each at a slightly different angular orientation from the one next to it. At each point, the angle of incidence is equal to the angle of reflection (Figure 28.9). Note that, in a curved mirror, unlike in a plane mirror, the normals (shown by the dashed black lines to the left of the mirror) at different points on the surface are not parallel to one another.


Whether the mirror is plane or curved, the eye-brain system cannot ordinarily differentiate between an object and its reflected image. So the illusion that an object exists behind a mirror (or in some cases in front of a concave mirror) is merely due to the fact that the light from the object enters the eye in exactly the same manner, physically, as it would have entered if the object really were at the image location.

Only part of the light that strikes a surface is reflected. On a surface of clear glass, for example, and for normal incidence (light perpendicular to the surface), only about $4 \%$ is reflected from each surface. On a clean and polished aluminum or silver surface, however, about $90 \%$ of incident light is reflected.

## CHECK POINT

1. What evidence can you cite to support the claim that the frequency of light does not change upon reflection?
2. If you wish to take a picture of your image while standing 5 m in front of a plane mirror, for what distance should you set your camera to provide the sharpest focus?

## CHECK YOUR ANSWERS

1. The color of an image is identical to the color of the object that forms the image. When you look at yourself in a mirror, for example, the color of your eyes doesn't change.
2. Set your camera for 10 m ; the situation is equivalent to standing 5 m in front of an open window and viewing your twin standing 5 m beyond the window.

## Diffuse Reflection

When light is incident on a rough or granular surface, it is reflected in many directions. This is called diffuse reflection (Figure 28.10). If the surface is so smooth that the distances between successive elevations on the surface are less than about one-eighth the wavelength of the light, there is very little diffuse reflection, and the surface is said to be polished. A surface, therefore, may be polished for radiation of a long wavelength but not polished for light of a short wavelength. The wire-mesh "dish" shown in Figure 28.11 is very rough for light waves and so is hardly mirrorlike, but for long-wavelength radio waves, it is


VIDEO: Image Formation in a Mirror

FIGURE 28.9
(a) The virtual image formed by a convex mirror (a mirror that curves outward) is smaller and closer to the mirror than the object.
(b) When the object is close to a concave mirror (a mirror that curves inward like a "cave"), the virtual image is larger and farther away than the object. In either case, the law of reflection applies to each ray.


FIGURE 28.10
Diffuse reflection. Although each ray obeys the law of reflection, the many different surface angles that light rays encounter in striking a rough surface cause reflection in many directions.

FIGURE 28.11
The open-mesh parabolic dish is a diffuse reflector for short-wavelength light but a polished reflector for longwavelength radio waves.


FIGURE 28.12
A magnified view of the surface of ordinary paper.


FIGURE 28.13
Refraction.


FIGURE 28.14 INTERACTIVE FIGURE $_{k}$ MP
Refraction.

"polished" and therefore an excellent reflector. Reflection off the walls of your room is a good example of diffuse reflection. The light reflects back into the room but produces no mirror images. Unlike specular reflection, diffuse reflection does not produce a mirror image.

Light reflecting from this page is diffuse. The page may be smooth to a radio wave, but it is rough to a light wave. Rays of light that strike this page encounter millions of tiny flat surfaces facing in all directions. The incident light therefore is reflected in all directions, which enables us to see the page or other objects from any direction. You can see the road ahead of your car at night, for instance, because of diffuse reflection by the road surface. When the road is wet, diffuse reflection is less, and it is more difficult to see. Most of our environment is seen by diffuse reflection.

## CHECK POINT

How can the surface of water in a lake exhibit both specular and diffuse reflection?

## CHECK YOUR ANSWER

Where the water is very still and the surface smooth, reflected images occur. This is specular reflection. Where the water is rough and doesn't show reflected images, the reflection is diffuse.

### 28.3 Refraction

Recall from Chapter 26 that the average speed of light is lower through glass and other transparent materials than through empty space. Light travels at different speeds in different materials. It travels at $300,000 \mathrm{~km} / \mathrm{s}$ in a vacuum, at a slightly lower speed in air, and at about three-fourths that speed in water. In a diamond, light travels at about $40 \%$ of its speed in a vacuum. When light bends in passing obliquely from one medium to another, we call the process refraction. It is a common observation that a ray of light bends and takes a longer path when it encounters glass or water at an oblique angle. But the longer path taken is nonetheless the path that requires the least time. A straight-line path would take a longer time. We can illustrate this with the following situation.

Imagine that you are a lifeguard at a beach and you spot a person in distress in the water. We show the relative positions of you, the shoreline, and the person in distress in Figure 28.13. You are at point A, and the person is at point B. You can run faster than you can swim. Should you travel in a straight line to get to B? A little thought will show that a straight-line path would not be the best choice because, if you instead spent a little bit more time traveling farther on land, you would save a lot more time in swimming a shorter distance in the water. The path of least time is shown by the dashed line, which clearly is not the path of the shortest distance. The amount of bending at the shoreline depends, of course, on how much faster you can run than swim. The situation is similar for a ray of light incident upon a body of water, as shown in Figure 28.14. The angle of incidence is larger than the angle of refraction by an amount that depends on the relative speeds of light in air and in water.

Consider the pane of thick window glass in Figure 28.15. When light goes from point A through the glass to point B , it will go in a straight-line path. In this case,
light encounters the glass perpendicularly, and we see that the shortest distance through both air and glass corresponds to the shortest time. But what about light that goes from point A to point C ? Will it travel in the straight-line path shown by the dashed line? The answer is no, because if it did so it would be spending more time inside the glass, where light travels more slowly than in air. The light will instead take a less-inclined path through the glass. The time saved by taking the resulting shorter path through the glass more than compensates for the added time required to travel the slightly longer path through the air. The overall path is the path of least time-the quickest path. The result is a parallel displacement of the light beam because the angles in and out are the same. You'll notice this displacement when you look through a thick pane of glass at an angle. The more your angle of viewing differs from perpendicular, the more pronounced the displacement.

Another example of interest is the prism, in which opposite faces of the glass are not parallel (Figure 28.16). Light that goes from point A to point B will not follow the straight-line path shown by the dashed line because too much time would be spent in the glass. Instead, the light will follow the path shown by the solid line-a path that is a bit longer through the air-and pass through a thinner section of the glass to make its trip to point B. By this reasoning, one might think that the light should take a path closer to the upper vertex of the prism and seek the minimum thickness of glass. But if it did, the extra distance through the air would result in an overall longer time of travel. The quickest path followed is the path of least time.


FIGURE 28.16
A prism.


FIGURE 28.17
A "curved prism."


FIGURE 28.18
A converging lens.

It is interesting to note that a "properly curved prism" will provide many paths of equal time from a point $A$ on one side to a point $B$ on the opposite side (Figure 28.17). The curve decreases the thickness of the glass correctly to compensate for the extra distances light travels to points higher on the surface. For appropriate positions of A and B and for the appropriate curve on the surfaces of this modified prism, all light paths are of exactly equal time. In this case, all the light from A that is incident on the glass surface is focused on point B . We see that this shape is simply the upper half of a converging lens (Figure 28.18, and treated in more detail later in this chapter).

Whenever we watch a sunset, we see the Sun for several minutes after it has sunk below the horizon. Earth's atmosphere is thin at the top and dense at the bottom. Since light travels faster in thin air than it does in dense air, light from the Sun can reach us more quickly if, instead of traveling in a straight line, it avoids the denser air by taking a higher and longer path to penetrate the atmosphere at a steeper tilt (Figure 28.19). Since the density of the atmosphere changes gradually,



FIGURE 28.15
Refraction through glass. Although dashed line AC is the shortest path, light takes a slightly longer path through the air from A to $a$, then a shorter path through the glass to c , and then to C. The emerging light is displaced but parallel to the incident light.


SCREENCAST: Refraction

FIGURE 28.19
Because of atmospheric refraction, when the Sun is near the horizon, it appears to be higher in the sky.


FIGURE 28.20
The Sun's shape is distorted by differential refraction.


FIGURE 28.21
Refraction relates to indices of refraction in accord with Snell's law.
the light path bends gradually to produce a curved path. Interestingly, this path of least time provides us with a slightly longer period of daylight than if the light traveled without bending. Furthermore, when the Sun (or Moon) is near the horizon, the rays from the lower edge are bent more than the rays from the upper edge. This produces a shortening of the vertical diameter, causing the Sun to appear pumpkin shaped (Figure 28.20).

## CHECK POINT

Suppose the lifeguard in our earlier example were a seal instead of a human being. How would its path of least time from $A$ to $B$ differ?

## CHECK YOUR ANSWER

The seal can swim faster than it can run, and its path would bend as shown. Likewise with light emerging from the bottom of a piece of glass into air.


## Index of Refraction

Light slows when it enters a transparent medium. How much the speed of light differs from its speed in a vacuum is given by the index of refraction, $n$, of the material:

$$
n=\frac{\text { speed of light in vacuum }}{\text { speed of light in material }}
$$

For example, the speed of light in a diamond is $124,000 \mathrm{~km} / \mathrm{s}$, so the index of refraction for diamond is

$$
n=\frac{300,000 \mathrm{~km} / \mathrm{s}}{124,000 \mathrm{~km} / \mathrm{s}}=2.42
$$

For a vacuum, $n=1$.
For optical crown glass, common in eyeglasses, $n$ is 1.52 , which means light slows from speed $c$ in air to speed $c / n=c / 1.52=0.66 c$. The greater the $n$, the greater the bending of light in the lens, which translates into less lens thickness. The $n$ of high-index plastic lenses reaches 1.76 , so light slows more and bends more, and lenses can be made thinner-good news for nearsighted people who want thinner eyeglasses. And what about the speed of light when it exits a lens? That's right—back up to nearly $c$, light's usual speed in air.

The quantitative law of refraction, called Snell's law, is credited to W. Snell, a 17th-century Dutch astronomer and mathematician:

$$
n_{1} \sin \theta_{1}=n_{2} \sin \theta_{2}
$$

where $n_{1}$ and $n_{2}$ are the indices of refraction of the media on either side of the surface and $\theta_{1}$ and $\theta_{2}$ are the respective angles of incidence and refraction. If three of these values are known, the fourth can be calculated from this relationship. Perhaps in the lab part of your course you'll make use of Snell's law.

## Mirage

We are all familiar with the mirage we sometimes see while driving on a hot road. The distant road appears to be wet, but when we get there, the road is dry. Why is this so? The air is very hot just above the road surface and cooler above. Light travels faster through the thinner hot air than through the denser cool air above. So light, instead of coming to us from the sky in straight lines, also has least-time paths by which it curves down into the hotter region near the road for a while before it

reaches our eyes (Figure 28.22). Where we are seeing "wetness," we are really seeing the sky. A mirage is not, as many people mistakenly believe, a "trick of the mind." A mirage is formed by real light and can be photographed, as shown in Figure 28.23.

When we look at an object over a hot stove or over hot pavement, we see a wavy, shimmering effect. This is due to the various least-time paths of light as it passes through varying temperatures and therefore varying densities of air. The twinkling of stars results from similar phenomena in the sky, where light passes through unstable layers in the atmosphere.

In the foregoing examples, how does light seemingly "know" what conditions exist and what compensations a least-time path requires? When approaching window glass, a prism, or a lens at an angle, how does light know to travel a bit farther in air to save time in taking a shorter path through the glass? How does light from the Sun know to travel above the atmosphere an extra distance before taking a shortcut through the denser air to save time? How does sky light above know that it can reach us in minimum time if it dips toward a hot road before tilting upward to our eyes? The principle of least time appears to be noncausal that light has a mind of its own and can "sense" all the possible paths, calculate the time for each, and choose the one that requires the least time. Is this the case? As intriguing as all this may seem, there is a simpler explanation that doesn't assign foresight to light: Refraction is simply a consequence of light having different average speeds in different media.

## CHECK POINT

If the speed of light were the same in air of various temperatures and densities, would there still be slightly longer periods of daylight, twinkling stars at night, mirages, and slightly squashed Suns at sunset?

## CHECK YOUR ANSWER

No, because then no refraction would occur.

### 28.4 Cause of Refraction

Refraction occurs when the average speed of light changes in going from one transparent medium to another. We can understand this by considering the action of a pair of toy cart wheels connected to an axle as the wheels roll gently downhill from a smooth sidewalk onto a grass lawn. If the wheels meet the grass at an angle, as Figure 28.23 shows, they are deflected from their straight-line course. Note that the left wheel slows first when it interacts with the grass on the lawn. The higherspeed right wheel on the sidewalk then pivots about the slower-moving left wheel. The direction of the rolling wheels is bent toward the normal (the black dashed line perpendicular to the grass-sidewalk border in Figure 28.24).

A light wave bends in a similar way, as shown in Figure 28.25. Note the direction of light, indicated by the blue arrow (the light ray), and also note the wavefronts (red) drawn at right angles to the ray. (Recall that a wavefront is the crest, trough, or any continuous portion of a wave.) In the figure the wave meets the water surface at an angle. This means that the left portion of the wave slows down

FIGURE 28.22
Light from the sky picks up speed in the air near the ground because that air is hotter and less dense than the air above. When the light grazes the surface and bends upward, the observer sees a mirage.


FIGURE 28.23
A mirage. The apparent wetness of the road is not reflection of the sky by water but, rather, refraction of sky light through the hotter and lessdense air near the road surface.


FIGURE 28.24
The direction of the rolling wheels changes when one wheel slows down before the other one does.


FIGURE 28.25
The direction of the light waves changes when one part of each wave slows down before the other part.


FIGURE 28.27
When light slows down in going from one medium to another, such as going from air to water, it refracts toward the normal. When it speeds up in traveling from one medium to another, such as going from water to air, it refracts away from the normal.


FIGURE 28.29
Because of refraction, the full rootbeer mug appears to hold more root beer than it actually does.
in the water while the remainder in the air travels at speed $c$. The light ray remains perpendicular to the wavefront and therefore bends at the surface. It bends like the wheels bend when they roll from the sidewalk into the grass. In both cases, the bending is a consequence of a change in speed.

The changeable speed of light provides a wave explanation for mirages. Sample wavefronts coming from the top of a tree on a hot day are shown in Figure 28.26. If the temperature of the air were uniform, the average speed of light would be the same in all parts of the air; light traveling toward the ground would meet the ground. But the air is warmer and less dense near the ground, and the wavefronts gain speed as they travel downward, making them bend upward. So, when the observer looks downward, he sees the top of the tree-this is a mirage.

FIGURE 28.26
A wave explanation of a mirage. Wavefronts of light travel faster in the hot air near the ground and bend upward.


Refraction accounts for many illusions. A common one is the apparent bending of a stick that is partially in water. The submerged part seems closer to the surface than it really is. Likewise when you view a fish in the water, the fish appears nearer to the surface and closer than it really is (Figure 28.28). If we look straight down into water, an object submerged 4 m beneath the surface will appear to be only 3 m deep. Because of refraction, submerged objects appear to be closer, so they seem bigger.

FIGURE 28.28
Because of refraction, a submerged object seems to be nearer to the surface than it actually is.


We see that we can interpret the bending of light at the surface of the water in at least two ways. We can say that the light that leaves the fish and reaches the observer's eye does so in the least time by taking a shorter path upward toward the surface of the water and a correspondingly longer path through the air. In this view, least time dictates the path taken. Or we can say that the waves of light directed upward at an angle toward the surface are bent off-kilter as they speed up when emerging into the air reaching the observer's eye. In this view, the change in speed from water to air dictates the path taken, and this path turns out to be a least-time path. Whichever view we choose, the results are the same.

## CHECK POINT

If the speed of light were the same in all media, would refraction still occur when light passes from one medium to another?

## CHECK YOUR ANSWER

No.

### 28.5 Dispersion and Rainbows

We know that the average speed of light is less than $c$ in a transparent medium; how much less depends on the nature of the medium and on the frequency of light. The speed of light in a transparent medium depends on its frequency. Recall from Chapter 26 that light with frequencies that match the natural or resonant frequencies of the electron oscillators in the atoms and molecules of the transparent medium is absorbed, and light with frequencies near the resonant frequencies is not absorbed but interacts more often than light of lower frequencies in the absorption-reemission sequence. Since the natural or resonant frequency of most transparent materials is in the ultraviolet part of the spectrum, higher-frequency light travels more slowly than lowerfrequency light. Violet light travels about $1 \%$ slower in ordinary glass than red light. Light waves with colors between red and violet travel at their own intermediate speeds.

Because light of different frequencies travels at different speeds in transparent materials, it refracts by different amounts. When white light is refracted twice, as in a prism, the separation of the different colors of light is quite noticeable. This separation of light into colors arranged according to frequency is called dispersion (Figure 28.30). It is what enabled Isaac Newton to form a spectrum when he held a glass prism in sunlight.

A most spectacular illustration of dispersion is a rainbow. For a rainbow to be seen, the Sun must be shining in one part of the sky and water drops in a cloud or in falling rain must be present in the opposite part of the sky. When we turn our backs toward the Sun, we see the spectrum of colors in a bow. Seen from an airplane near midday, the bow forms a complete circle (with the shadow of a low-flying airplane seen in the center). All rainbows would be completely round if the ground were not in the way.

The beautiful colors of rainbows are dispersed from the sunlight by millions of tiny spherical water droplets that act like prisms. We can better understand this by considering an individual raindrop, as shown in Figure 28.31. Follow the ray of sunlight as it enters the drop near its top surface. Some of the light here is reflected (not shown), and the remainder is refracted into the water. At this first refraction, the light is dispersed into its spectrum colors, violet being deviated the most and red the least. Reaching the opposite side of the drop, each color is partly refracted out into the air (not shown) and partly reflected back into the water. Arriving at the lower surface of the drop, each color is again reflected (not shown) and refracted into the air. This second refraction is similar to that of a prism, where refraction at the second surface increases the dispersion already produced at the first surface.

Two refractions and a reflection can actually result in the angle between the incoming and outgoing rays being anything between $0^{\circ}$ and about $42^{\circ}\left(0^{\circ}\right.$ corresponds to a full $180^{\circ}$ reversal of the light). There is a strong concentration of light intensity, however, near the maximum angle of $42^{\circ}$. That is what is shown in Figure 28.31.

Although each drop disperses a full spectrum of colors, an observer is in a position to see the concentrated light of only a single color from any one drop (Figure 28.32). If violet light from a single drop reaches the eye of an observer, red light from the same drop is incident elsewhere below the eyes. To see red light, one must look to a drop higher in the sky. The color red will be seen when the angle between a beam of sunlight and the light sent back by a drop is $42^{\circ}$. The color violet is seen when the angle between the sunbeams and deflected light is $40^{\circ}$.


FIGURE 28.32
Sunlight incident on two sample raindrops, as shown, emerges from them as dispersed light. The observer sees the red light from the upper drop and the violet light from the lower drop. Millions of drops produce the whole spectrum of visible light.


VIDEO: The Rainbow

FIGURE 28.33
When your eye is located between the Sun (not shown off to the left) and a water drop region, the rainbow you see is the edge of a three-dimensional cone that extends through the water drop region. (Innumerable layers of drops form innumerable two-dimensional arcs like the four suggested here.)


FIGURE 28.34
Only raindrops along the dashed line disperse red light to the observer at a $42^{\circ}$ angle; hence, the light forms a bow.

Why does the light dispersed by the raindrops form a bow? The answer to this involves a little geometric reasoning. First of all, a rainbow is not the flat twodimensional arc it appears to be. It appears flat for the same reason a spherical burst of fireworks high in the sky appears as a disc-because of a lack of distance cues. The rainbow you see is actually a three-dimensional cone with the tip (apex) at your eye (Figure 28.33). Consider a glass cone, the shape of those paper cones you sometimes see at drinking fountains. If you held the tip of such a glass cone against your eye, what would you see? You'd see the glass as a circle. Likewise with a rainbow. All the drops that disperse the rainbow's light toward you lie in the shape of a cone-a cone of different layers with drops that disperse red to your eye on the outside, orange beneath the red, yellow beneath the orange, and so on, all the way to violet on the inner conical surface. The thicker the region containing water drops, the thicker the conical edge you look through, and the more vivid the rainbow.


To further understand this, consider only the deflection of red light. You see red when the angle between the incident rays of sunlight and dispersed rays is $42^{\circ}$. Of course, beams are dispersed $42^{\circ}$ from drops all over the sky in all directions-up, down, and sideways. But the only red light you see is from drops that lie on a cone with a side-to-axis angle of $42^{\circ}$. Your eye is at the apex of this cone, as shown in Figure 28.34. To see violet, you look $40^{\circ}$ from the conical axis (so the thickness of glass in the cone of the preceding paragraph is tapered-very thin at the tip and thicker with increased distance from the tip). Your cone of vision that intersects the cloud of drops that creates your rainbow is different from that of a person next to you. So when a friend says, "Look at the pretty rainbow," you can reply, "Okay, move aside so I can see it too." Everybody sees his or her own personal rainbow.

Another fact about rainbows: A rainbow always faces you squarely because of the lack of distance cues mentioned earlier. When you move, your rainbow moves with you. So you can never approach the side of a rainbow, or see it nearly end-on as in the exaggerated view of Figure 28.33. You can't get to its end. Therefore the expression "looking for the pot of gold at the end of the rainbow" means pursuing something you can never reach.

Often a larger secondary bow with colors reversed can be seen arching at a larger angle around the primary bow. We won't treat this secondary bow except to say that it is formed by similar circumstances and is a result of double reflection within the raindrops (Figure 28.36). Because of this extra reflection (and extra refraction loss), the secondary bow is much dimmer and its colors are reversed.


FIGURE 28.35
Two refractions and a reflection in water droplets produce light at all angles up to about $42^{\circ}$, with the intensity concentrated where we see the rainbow at $40^{\circ}$ to $42^{\circ}$. No light emerges from the water droplet at angles larger than $42^{\circ}$ unless it undergoes two or more reflections inside the drop. So the sky is brighter inside the rainbow than outside it. Notice the weak secondary rainbow to the right of the primary.

## CHECK POINT

1. If you point to a wall with your arm extended to make about a $42^{\circ}$ angle to the normal to the wall and then rotate your arm in a full circle while keeping the same angle, what shape does your arm describe? What shape on the wall does your finger sweep out?

2. If light traveled at the same speed in raindrops as it does in air, would we still have rainbows?

## CHECK YOUR ANSWERS

1. Your arm describes a cone, and your finger sweeps out a circle—likewise with rainbows.
2. No.

### 28.6 Total Internal Reflection

Some Saturday night when you're taking your bath, fill the tub extra deep and bring a waterproof flashlight into the tub with you. Switch off the bathroom light. Shine the submerged light straight up and then slowly tip it away from the surface. Note how the intensity of the emerging beam diminishes and how more light is reflected from the surface of the water to the bottom of the tub. At a certain angle, called the critical angle, you'll notice that the beam no longer emerges into the air above the surface. The intensity of the emerging beam reduces to zero where it tends to graze the surface. The critical angle is the minimum angle of incidence inside a medium at which a light ray is totally reflected. When the



FIGURE 28.36
Double reflection in a drop produces a secondary bow.

FIGURE 28.37

Light emitted in the water is partly refracted and partly reflected at the surface. The blue dashed lines show the direction of light and the lengths of the arrows indicate the proportions refracted and reflected. Beyond the critical angle, the beam is totally internally reflected.


FIGURE 28.38
An observer underwater sees a circle of light at the still surface. Beyond a cone of $96^{\circ}$ (twice the critical angle), an observer sees a reflection of the water interior or bottom.

FIGURE 28.39
Total internal reflection in a prism. The prism changes the direction of the light beam (a) by $90^{\circ}$, (b) by $180^{\circ}$, and (c) not at all. Note that, in each case, the orientation of the image is different from the orientation of the object.

Would you like to get rich? Be the first to invent a surface that reflects $100 \%$ of the external light incident upon it.


FIGURE 28.40
Total internal reflection in a pair of prisms, common in binoculars.
flashlight is tipped beyond the critical angle ( $48^{\circ}$ from the normal for water), you'll notice that all the light is reflected back into the tub. This is total internal reflection. The light striking the air-water surface obeys the law of reflection: The angle of incidence is equal to the angle of reflection. The only light emerging from the surface of the water is the light that is diffusely reflected from the bottom of the bathtub. This procedure is shown in Figure 28.37. The proportions of light refracted and light internally reflected are indicated by the relative lengths of the arrows.

Total internal reflection occurs in materials in which the speed of light is less than the speed of light outside. The speed of light is less in water than in air, so all light rays in water that reach the surface at more than an incident angle of $48^{\circ}$ are reflected back into the water. So your pet goldfish in its aquarium looks up to see a reflected view of the sides and bottom of the aquarium. Directly above, it sees a compressed view of the outside world (Figure 28.38). The outside $180^{\circ}$ view from horizon to opposite horizon is seen through an angle of $96^{\circ}$-twice the critical angle. A lens that similarly compresses a wide view, called a fisheye lens, is used for special-effect photography.

Total internal reflection occurs in glass surrounded by air because the speed of light in glass is lower than in air. The critical angle for glass is about $43^{\circ}$ depending on the type of glass. So light in the glass that is incident at angles larger than $43^{\circ}$ to the surface is totally internally reflected. No light escapes beyond this angle; instead, all of it is reflected back into the glass, even if the outside surface is marred by dirt or dust-hence the usefulness of glass prisms (Figure 28.39). A little light is lost by reflection before it enters the prism, but once the light is inside, reflection from the $45^{\circ}$ slanted face is total- $100 \%$. In contrast, silvered or aluminized mirrors reflect only about $90 \%$ of incident light. This is the reason for the use of prisms instead of mirrors in many optical instruments.


A pair of prisms, each reflecting light through $180^{\circ}$, is shown in Figure 28.40. Binoculars use pairs of prisms to lengthen the light path between lenses and thus eliminate the need for long barrels. So a compact set of binoculars is as effective as a longer telescope. Another advantage of prisms is that whereas the image in a straight telescope is upside down, reflection by the prisms in binoculars reinverts the image, so things are seen right-side up.


The critical angle for a diamond is about $24.5^{\circ}$, smaller than for any other common substance. The critical angle varies slightly for different colors because the speed of light varies slightly for different colors. Once light enters a diamond gemstone, most is incident on the sloped backsides at angles larger than $24.5^{\circ}$ and is totally internally reflected (Figure 28.41). Because of the great slowdown in speed as light enters a diamond, refraction is pronounced, and because of the frequency-dependence of the speed, there is great dispersion. Further dispersion occurs as the light exits through the many facets at its face. Hence we see unexpected flashes of a wide array of colors. Interestingly, when these flashes are narrow enough to be seen by only one eye at a time, the diamond "sparkles."


Total internal reflection also underlies the operation of optical fibers, or light pipes (Figure 28.42). An optical fiber "pipes" light from one place to another by a series of total internal reflections, much as a bullet ricochets down a steel pipe. Light rays bounce along the inner walls, following the twists and turns of the fiber. Bundles of optical fibers are used to see what is going on in inaccessible places, such as the interior of a motor or a patient's stomach. The fibers can be made small enough to snake through blood vessels or through narrow canals in the body, such as the urethra. Light shines down some of the fibers to illuminate the scene and is reflected back along others.

Fiber-optic cables are also important in communications because they offer a practical alternative to copper wires and cables. Thin glass fibers now replace thick, bulky, expensive copper cables to carry thousands of simultaneous telephone messages among the major switching centers and across the ocean floor. Control signals are fed in aircraft from the pilot to the control surfaces by means of fiber optics. Signals are carried in the modulations of laser light. Unlike electricity, light is indifferent to temperature and fluctuations in surrounding magnetic fields, and so the signal is clearer. Also, it is much less likely to be tapped by eavesdroppers.

### 28.7 Lenses

A very practical case of refraction occurs in lenses. We can understand a lens by analyzing equal-time paths as we did earlier, or we can assume that a lens consists of several matched prisms and blocks of glass arranged in the order shown in Figure 28.43. The prisms and blocks refract incoming parallel light rays so that they converge to (or diverge from) a point. The arrangement shown in Figure 28.43a converges the light, and we call such a lens a converging lens. Note that it is thicker in the middle and thinner at the edges.

(a)


FIGURE 28.41
Paths of light in a diamond. Rays that strike the inner surface at angles larger than the critical angle are internally reflected and exit via refraction at the top surface.


FIGURE 28.42
The light is "piped" from below by a succession of total internal reflections until it emerges at the top ends.

Learning about lenses is a handson activity. Not fiddling with lenses while learning about them is like taking swimming lessons away from water.

FIGURE 28.43
A lens may be thought of as a set of blocks and prisms: (a) a converging lens and (b) a diverging lens.

FIGURE 28.44
Wavefronts travel more slowly in glass than in air. (a) The waves are retarded more through the center of the lens, and convergence results. (b) The waves are retarded more at the edges, and divergence results.

FIGURE 28.45
Key features of a converging lens.

FIGURE 28.46
The moving patterns of bright and dark areas at the bottom of the pool result from the uneven surface of the water, which behaves like a blanket of undulating lenses. Just as we see the pool bottom shimmering, a fish looking upward at the Sun would see it shimmering too. Because of similar irregularities in the atmosphere, we see the stars twinkle.


SCREENCAST: Pinhole Images

The arrangement in Figure 28.43b is different. The middle is thinner than the edges, and it diverges the light; such a lens is called a diverging lens. Note that the prisms diverge the incident rays in a way that makes them appear to come from a single point in front of the lens. In both lenses, the greatest deviation of rays occurs at the outermost prisms because they have the largest angle between the two refracting surfaces. No deviation occurs exactly in the middle because in that region the glass faces are parallel to each other. Real lenses are not made of prisms, of course, as is indicated in Figure 28.43; they are made of a solid piece of glass with surfaces that are ground usually to a spherical curve. In Figure 28.44, we see how smooth lenses refract waves.


Some key features in lens description are shown for a converging lens in Figure 28.45. The principal axis of a lens is the line that joins the centers of curvatures of its surfaces. The focal point is the point where a beam of light parallel to the principal axis converges. Incident parallel beams that are not parallel to the principal axis focus at points above or below the focal point. All such possible points make up a focal plane. A lens has two focal points and two focal planes. When the lens of a camera is set for distant objects, the photosensitive surface is very nearly in the focal plane behind the lens in the camera. The focal length of the lens is the distance between the center of the lens and either focal point.


## Image Formation by a Lens

At this moment, light is reflecting from your face onto this page. Light that reflects from your forehead, for example, strikes every part of the page. Likewise for light that reflects from your chin. Every part of the page is illuminated with reflected light from your forehead, your nose, your chin, and every other part of your face. You don't see an image of your face on the page because there is too much overlapping of light. But put a barrier with a pinhole in it between your face and the page, and the

## PRACTICING PHYSICS

Make a pinhole camera. Cut out one end of a small cardboard box, and cover the end with semitransparent tracing or tissue paper. Make a clean-cut pinhole at the other end. (If the cardboard is thick, you can make the pinhole through a piece of tinfoil placed over a larger opening in the cardboard.) Aim the camera at a bright object in a darkened room, and you will see an upside-down image on the tracing paper. The tinier the pinhole, the dimmer and sharper the image. If you are in a dark room, replace the tracing paper with unexposed photographic film, cover the back so that it is light-tight, and cover the pinhole with a removable flap. You're ready to take a picture. Exposure times differ depending principally on the kind of film and the amount of light. Try different exposure times,
starting with about 3 seconds. Also try boxes of various lengths.

Rather than viewing a candle, as the sketch sug-
 gests, point your box skyward toward the Sun. The solar image on the tracing paper is clear and bright. Pinhole images of the Sun are also evident on the ground beneath a tree on a sunny day. When openings between leaves in the tree are small compared with the height of the tree, the openings behave as pinholes and cast circles of light, many overlapping, on the ground. Recall the opening photos of Chapters 1 and 27 that show what occurs at the time of a partial solar eclipse.
light that reaches the page from your forehead does not overlap the light from your chin. Likewise for the rest of your face. Without this overlapping, an image of your face is formed on the page. It will be very dim because very little light reflected from your face gets through the pinhole. To see the image, you'd have to shield the page from other light sources. The same is true of the vase and flowers in Figure 28.47b. ${ }^{3}$


(b)

(c)

FIGURE 28.47

## INTERACTIVE FIGURE $_{k}$ MP

Image formation. (a) No image appears on the wall because rays from all parts of the object overlap all parts of the wall. (b) A single small opening in a barrier prevents overlapping rays from reaching the wall; a dim upside-down image is formed. (c) A lens converges the rays upon the wall without overlapping; more light makes a brighter image.

The first cameras had no lenses and admitted light through a small pinhole. You can see why the image is upside down by the sample rays in Figures 28.47b. Long exposure times were required because of the small amount of light admitted by the pinhole. A somewhat larger hole would admit more light, but overlapping rays would produce a blurry image. Too large a hole would allow too much overlapping and no image would be discernible. That's where a converging lens comes in (Figure 28.47 c ). The lens converges light onto the screen without the unwanted overlapping of rays. Whereas the first pinhole cameras were useful for only still objects because of the long exposure time required, moving objects can be photographed with the lens camera because of the short exposure time-which is why photographs taken with lens cameras came to be called snapshots.
${ }^{3}$ A quantitative way of relating object distances to image distances is given by the thin-lens equation:

$$
\frac{1}{d_{o}}+\frac{1}{d_{i}}=\frac{1}{f} \quad \text { or } \quad d_{i}=\frac{d_{o} f}{d_{o}-f}
$$

where $d_{o}$ is the distance of the object from the lens, $d_{i}$ is the distance from the lens to the image, and $f$ is the focal length of the lens.


Can you see why the image in Figure 28.47 b is upside down? And, is it true that, when your photographs are processed and printed, they're all upside down?
$-1$


FIGURE 28.48
Viewing.

## SCREENCAST: Lenses

### 28.8 Lens Defects

No lens provides a perfect image. A distortion in an image is called an aberration. By combining lenses in certain ways, aberrations can be minimized. For this reason, most optical instruments use compound lenses, each consisting of several simple lenses instead of single lenses.

Spherical aberration results from light that passes through the edges of a lens and focuses at a slightly different place from where light passing near the center of the lens focuses (Figure 28.52). This can be remedied by covering the edges of a lens, as with a diaphragm in a camera. Spherical aberration is corrected in good optical instruments by a combination of lenses.


Chromatic aberration is the result of light of different colors having different speeds and hence different refractions in the lens (Figure 28.54). In a simple lens (as in a prism), different colors of light do not come to focus in the same place. Achromatic lenses, which combine simple lenses of different kinds of glass, correct this defect. (Interestingly, Isaac Newton replaced the objective lens in a telescope with a parabolic mirror to avoid chromatic aberration.)

The pupil of the eye changes in size to regulate the amount of light that enters. Vision is sharpest when the pupil is smallest because light then passes through only the central part of the eye's lens, where spherical and chromatic aberrations are minimal. Also, the eye then acts more like a pinhole camera, so minimum focusing is required for a sharp image. You see better in bright light because in such light your pupils are smaller.

Astigmatism of the eye is a defect that results when the cornea is curved more in one direction than the other, somewhat like the side of a barrel. Because of this defect, the eye does not form sharp images. The remedy is eyeglasses with cylindrical lenses that have more curvature in one direction than in another.

Today, an option for those with poor sight is wearing eyeglasses. The advent of eyeglasses probably occurred in China and in Italy in the late 1200s. (Curiously enough, the telescope wasn't invented until some 300 years later. If, in the meantime, anybody viewed objects through a pair of lenses separated along their axes, such as lenses fixed at the ends of a tube, there is no record of it.) An alternative to eyeglasses is contact lenses. A further alternative is LASIK (an acronym for laserassisted in-situ keratomileusis), in which pulses of laser light reshape the cornea and produce normal vision. Another procedure, PRK (photorefractive keratectomy), corrects all three common defects in vision. IntraLase, implantable contact lenses, and newer procedures continue to be developed. It's safe to say that wearing eyeglasses and contact lenses may soon be a thing of the past. We really do live in a rapidly changing world. And that can be nice.


FIGURE 28.52
Spherical aberration.

FIGURE 28.53
How is vision affected when both eye views are deflected to the side by prisms? After a few minutes the eye, brain, and muscles all adapt to the change!


FIGURE 28.54
Chromatic aberration.


- Now there are inexpensive eyeglasses with water-filled lenses. Adding or removing water between two polycarbonate membranes can correct both nearsighted and farsighted vision. A small pump regulates the amount of water between the membranes. Water can produce a convex lens for farsighted vision, or less water can produce a concave lens for nearsighted vision. Once each lens is optimized, the user locks the water setting in place. The pump and tube assembly is removed and left intact for later adjustments. Check the Internet for Self-Adjusting Eyeglasses for the World's Poor.

If you wear glasses and have ever misplaced them, or if you find it difficult to read small print, try squinting or, even better, try holding a pinhole (in a piece of paper or whatever) in front of your eye, close to the page. You'll see the print clearly and, because you're close to it, it is magnified. Try it and see!


## CHECK POINT

1. If light traveled at the same speed in both glass and air, would glass lenses alter the direction of light rays?
2. Why is there chromatic aberration in light that passes through a lens but none in light that reflects from a mirror?

## CHECK YOUR ANSWERS

1. No.
2. Different frequencies travel at different speeds in a transparent medium and therefore refract at different angles, which produces chromatic aberration. The angles at which light reflects, however, have nothing to do with its frequency. One color reflects the same as any other color. In telescopes, therefore, mirrors are preferable to lenses because there is no chromatic aberration.

## SUMMARY OF TERMS (KNOWLEDGE)

Reflection The return of light rays from a surface.
Fermat's principle of least time Light takes the path that requires the least time when it goes from one place to another.
Law of reflection The angle of reflection equals the angle of incidence.
Refraction The bending of an oblique ray of light when it passes from one transparent medium to another.
Critical angle The minimum angle of incidence inside a medium at which a light ray is totally reflected.
Total internal reflection The total reflection of light traveling within a denser medium when it strikes the boundary with a less dense medium at an angle larger than the critical angle.

Converging lens A lens that is thicker in the middle than at the edges and that refracts parallel rays to a focus.
Diverging lens $A$ lens that is thinner in the middle than at the edges, causing parallel rays to diverge as if from a point.
Virtual image An image formed by light rays that do not converge at the location of the image.
Real image An image formed by light rays that converge at the location of the image. A real image, unlike a virtual image, can be displayed on a screen.
Aberration Distortion in an image produced by a lens, which to some degree is present in all optical systems.

## READING CHECK QUESTIONS (COMPREHENSION)

### 28.1 Reflection

1. How does incident light that falls on an object affect the motion of electrons in the atoms of the object?
2. What do the electrons affected by illumination do when they are made to vibrate with greater energy?
3. What is Fermat's principle of least time?

### 28.2 Law of Reflection

4. Cite the law of reflection.
5. Relative to the distance of an object in front of a plane mirror, how far behind the mirror is the image?
6. What fraction of the light shining straight at a piece of clear glass is reflected from the first surface?
7. Can a surface be considered polished for some waves and not for others? Give an example.

### 28.3 Refraction

8. How does the angle at which a ray of light strikes a pane of window glass compare with the angle at which the light passes out the other side?
9. When is the angle at which a ray of light strikes glass not the same as the angle at which it exits?
10. In which medium does light travel faster: thin air or dense air? How does this affect the period of daylight?
11. Does the law of reflection hold for curved mirrors? Explain.
12. Is a mirage the result of reflection or refraction?

### 28.4 Cause of Refraction

13. When the wheel of a cart rolls from a smooth sidewalk onto a plot of grass, the interaction of the wheel with blades of grass slows the wheel. What slows light when it passes from air into glass or water?
14. What is the angle between a light ray and its wavefront?
15. What is the relationship between refraction and the speed of light?
16. Are eyeglasses made with "high index of refraction" materials thin or thick?
17. Does the refraction of light make a swimming pool appear deeper or shallower than it really is?

### 28.5 Dispersion and Rainbows

18. Which travels more slowly in glass: red light or violet light?
19. Does a single raindrop illuminated by sunlight deflect light of a single color, or does it disperse a spectrum of colors?
20. Does a viewer see a single color or a spectrum of colors coming from a single faraway drop?
21. Why is a secondary rainbow dimmer than a primary bow?

### 28.6 Total Internal Reflection

22. What is meant by "critical angle"?
23. At what angle inside glass is light totally internally reflected? At what angle inside a diamond is light totally internally reflected?
24. Light normally travels in straight lines, but it "bends" in an optical fiber. Explain.

### 28.7 Lenses

25. Distinguish between a converging lens and a diverging lens.
26. What is the focal length of a lens?
27. Distinguish between a virtual image and a real image.
28. What kind of lens can be used to produce a real image? A virtual image?

### 28.8 Lens Defects

29. Why is vision sharpest when the pupils of the eye are very small?
30. What is astigmatism, and how can it be corrected?

## THINK AND DO (HANDS-ON APPLICATION)

31. Text Grandma and convince her that in order to see her full-length image in a mirror, the mirror need be only half her height. Discuss also the intriguing role of distance in a mirror being half size. Perhaps rough sketches to accompany your explanations will help.
32. You can produce a spectrum by placing a tray of water in bright sunlight. Lean a pocket mirror against the inside edge of the tray and adjust it until a spectrum appears on the wall or ceiling. Aha! You've produced a spectrum without a prism.

33. Set up two pocket mirrors at right angles and place a coin between them. You'll see four coins. Change the angle of the mirrors and see how many images of the coin you can see. With the mirrors at right angles, look at your face. Then wink. Do you see anything unusual? Hold a printed page up to the double mirrors and contrast its appearance with the reflection from a single mirror.

34. Look at yourself in a pair of mirrors at right angles to each other. You see yourself as others see you. Rotate the mirrors, still at right angles to each other. Does your image rotate also? Now place the mirrors $60^{\circ}$ apart so
you again see your face. Again rotate the mirrors and see whether your image rotates also. Amazing?

35. Determine the magnifying power of a lens by focusing on the lines of a ruled piece of paper. Count the spaces between the lines that fit into one magnified space, and you have the magnifying power of the lens. You can do the same with binoculars and a distant brick wall. Hold the binoculars so that only one eye looks at the bricks through the eyepiece while the other eye looks directly at the bricks. The number of bricks seen with the unaided eye that will fit into one magnified brick gives the magnification of the instrument.

36. Poke a hole in a piece of paper, hold it in sunlight so that the solar image is the same size as a coin on the ground, and then determine how many coins would fit between the ground and the pinhole. That's the same number of solar diameters that would fit in the distance from Earth to the Sun. (Do you remember this exercise from Chapter 1?)

## THINK AND SOLVE (MATHEMATICAL APPLICATION)

37. Suppose you walk toward a mirror at $2 \mathrm{~m} / \mathrm{s}$. How fast do you and your image approach each other? (The answer is not $2 \mathrm{~m} / \mathrm{s}$.)
38. Show with a simple diagram that when a mirror with a fixed beam incident upon it is rotated through a certain angle, the reflected beam is rotated through an angle twice as large. (This doubling of displacement makes irregularities in ordinary window glass more evident.)
39. A butterfly at eye level is 20 cm in front of a plane mirror. You are behind the butterfly, 50 cm from the mirror. What is the distance between your eye and the image of the butterfly in the mirror?
40. When light strikes glass perpendicularly, about $4 \%$ is reflected at each surface. Show that $92 \%$ of light is transmitted through a pane of window glass.
41. No glass is perfectly transparent. Mainly because of reflections, about $92 \%$ of light passes through an average sheet of clear windowpane. The $8 \%$ loss is not noticed through a single sheet, but through several sheets, the loss is apparent. How much light is transmitted by a double-paned window (one with two sheets of glass)?
42. The diameter of the Sun makes an angle of $0.53^{\circ}$ from Earth. How many minutes does it take the Sun to move 1 solar diameter in an overhead sky? (Remember that it takes 24 hours, or 1440 minutes, for the Sun to move through $360^{\circ}$.) How does your answer compare with the time it takes the Sun to disappear, once its lower edge meets the horizon at sunset? (Does refraction affect your answer?)

## THINK AND RANK (ANALYSIS)

43. She looks at her face in the handheld mirror. Rank the amounts of her face that she sees in the three locations, from greatest to least (or is it the same in all positions?).

44. Wheels from a toy cart are rolled from a concrete sidewalk onto the following surfaces: A, a paved driveway; B, a grass lawn; and C, close-cropped grass on a golf-course putting green. Due to slowing, each set of wheels bends at the boundary and is deflected from its initial straightline course. Rank the surfaces according to the amount each set of wheels bends at the boundary, from greatest amount of bending to least.

45. Identical rays of light enter three transparent blocks composed of different materials. Light slows down upon entering the blocks. Rank the blocks according to the speed light travels in each, from highest to lowest.

46. Identical rays of light in air are refracted upon entering three transparent materials: A, water, where the speed of light is $0.75 c$; B, ethyl alcohol (speed $0.7 c$ ); and C, crown glass (speed $0.6 c$ ). Rank the materials according to how much the light ray bends toward the normal, from most bending to least bending.


## THINK AND EXPLAIN (SYNTHESIS)

47. This chapter opened with a photo of physics instructor Peter Hopkinson seeming to hover above the table. He isn't. Explain how he creates this illusion.
48. In the opening photo of the duck standing on the rock, why aren't the duck's feet shown in the reflected view?
49. In the opening photo of physics teacher Fred Myers taking a photo of his daughter McKenzie, how many mirrors were involved? Explain.
50. In the multiple images of physics teacher Karen Jo Matsler in the opening photo, how many mirrors are involved?
51. Fermat's principle is of least time rather than of least distance. Would least distance apply as well for reflection? For refraction? Why are your answers different?
52. Her eye at point $P$ looks into the mirror. Which of the numbered cards can she see reflected in the mirror?

53. Cowboy Joe wishes to shoot his assailant by ricocheting a bullet off a mirrored metal plate. To do so, should he simply aim at the mirrored image of his assailant? Explain.
54. Why is the lettering on the front of some vehicles "backward"?

## ЭЈИАЈUЯМА

55. Trucks often have signs on their back ends that say, "If you can't see my mirrors, I can't see you." Explain the physics here.
56. When you look at yourself in the mirror and wave your right hand, your beautiful image waves the left hand. Then why don't the feet of your image wiggle when you shake your head?
57. Car mirrors are uncoated on the front surface and silvered on the back surface. When the mirror is properly adjusted, light from behind reflects from the silvered surface into the driver's eyes. Good. But this is not so good at nighttime with the glare of headlights behind. This problem is solved by the wedge shape of the mirror (see the sketch). When the mirror is tilted slightly upward to the "nighttime" position, glare is directed upward toward the ceiling, away from the driver's eyes. Yet the driver can still see cars behind in the mirror. Explain.

58. A person in a dark room looking through a window can clearly see a person outside in the daylight, whereas the person outside cannot see the person inside. Explain.
59. What is the advantage of having matte (nonglossy) pages in this book rather than pages with a glossier surface?
60. What must be the minimum length of a plane mirror in order for you to see a full image of yourself?
61. What effect does your distance from the plane mirror have in your answer to the preceding exercise? (Try it and see!)
62. Hold a pocket mirror almost at arm's length from your face and note how much of your face you can see. To see more of your face, should you hold the mirror closer or farther away, or would you have to have a larger mirror? (Try it and see!)
63. On a steamy mirror, wipe away just enough to see your full face. How tall will the wiped area be compared with the vertical dimension of your face?
64. The sketch shows a person and her twin at equal distances on opposite sides of a thin wall. Suppose a window is to be cut into the wall so that each twin can see a complete view of the other. Show the size and location of the smallest window that can be cut into the wall to do the job. (Hint: Draw rays from the top of each twin's head to the other twin's eyes. Do the same from the feet of each to the eyes of the other.)

65. You can tell whether people are nearsighted or farsighted by looking at the size of their eyes through their glasses. When a person's eyes seem magnified, is the person nearsighted or farsighted?
66. If a nearsighted person wants thinner eyeglasses, is a higher or a lower index of refraction for the lenses recommended?
67. Your friend says that the wavelength of light waves is shorter in water than in air and cites Figure 28.25 as evidence. Do you agree or disagree, and why?
68. A pair of toy cart wheels is rolled obliquely from a smooth surface onto two plots of grass, a rectangular plot and a triangular plot, as shown. The ground is on a slight incline so that, after slowing down in the grass, the wheels will speed up again when emerging on the smooth surface. Finish each sketch by showing some positions of the wheels inside the plots and on the other sides, thereby indicating the direction of travel.

69. A pulse of red light and a pulse of blue light enter a glass block at the same time normal to its surface. Strictly speaking, after passing through the block, which pulse exits first?
70. During a lunar eclipse, the Moon is not completely dark but is usually deep red. Explain this in terms of the refraction of all the sunsets and sunrises around the world.
71. What accounts for the large shadows cast by the ends of the thin legs of the water strider?

72. When you stand with your back to the Sun, you see a rainbow as a circular arc. Could you move off to one side and then see the rainbow as the segment of an ellipse rather than the segment of a circle (as Figure 28.33 suggests)? Defend your answer.
73. Why will goggles allow a swimmer under water to focus more clearly on what he or she is looking at?
74. If a fish wore goggles above the water surface, why would the fish's vision be better if the goggles were filled with water? Explain.
75. Does a diamond under water sparkle more or less than in air? Defend your answer.
76. Cover the top half of a camera lens. What effect does this have on the pictures taken?
77. What will happen to the image projected onto a screen by a lens when you cover one-third of the lens with a red filter, one-third with a green filter, and one-third with a blue filter? (Try it and see!)
78. How could a converging lens be made for sound waves? (Such a lens, a spherical bag of gas, is a feature of San Francisco's Exploratorium.)
79. Would refracting telescopes and microscopes magnify if light had the same speed in glass as in air? Defend your answer.
80. There is less difference between the speed of light in glass and the speed of light in water than there is between the speed of light in glass and the speed of light in air. Does this mean that a magnifying glass will magnify more or magnify less when it is used under water rather than in air?
81. Waves don't overlap in the image of a pinhole camera. Does this feature contribute to a sharp image or to a blurry image?
82. Why doesn't the sharpness of the image in a pinhole camera depend on the position of the viewing screen?

83. Whereas pinholes provide sharp images, lenses with large apertures are advantageous for spy cameras of high-flying aircraft. Why?
84. If you point the pinhole camera of question 82 at the Sun, a clear and bright solar image will be seen on the viewing screen. How does this relate to the circular spots
of light that surround Lillian beneath the sunlit tree shown in the photo?

85. In terms of focal length, how far behind the camera lens is a photosensitive surface located when very distant objects are being photographed?
86. Why do you put slides into an old-fashioned slide projector upside down?
87. The image produced by a converging lens is upside down. Our eyes have converging lenses. Does this mean the images we see are upside down on our retinas? Explain.
88. The images produced by a converging camera lens are upside down. Does this mean the photographs taken with cameras are upside down?
89. Maps of the Moon are upside down. Why?
90. Why do older people who do not wear glasses read books farther away from their eyes than younger people do?
91. When Stephanie Hewitt dips a glass rod into vegetable oil, the submerged part of the rod is invisible. What does this say about the relative speeds of light in the glass and in the oil? Or asked another way, how do the indices of refraction, $n$, compare for the glass and oil?


## THINK AND DISCUSS (EVALUATION)

92. To reduce glare from the surroundings, the windows of some department stores, rather than being vertical, slant inward at the bottom. Discuss why this reduces glare.
93. Which kind of road surface is easier to see when driving at night: a pebbled, uneven surface or a mirror-smooth surface? Discuss why is it difficult to see the roadway in front of you when driving on a rainy night.
94. Why does reflected light from the Sun or Moon appear as a column in the body of water as shown? How would the reflected light appear if the water surface were perfectly smooth?

95. What exactly are you seeing when you observe a "water on the road" mirage?
96. What is wrong with the cartoon of the man looking at himself in the mirror? (Have a friend face a mirror as shown, and you'll see.)

97. A beam of light bends as shown in (a), while the edges of the immersed square bend as shown in (b). Do these pictures contradict each other? Explain.

98. When a flashlight submerged in water shines up into the air above, does the speed of light increase or decrease when the light passes from water into the air?
99. If, while standing on a riverbank, you wish to spear a fish beneath the water surface in front of you, should you aim above, below, or directly at the observed fish to make a direct hit? If, instead, you zap the fish with a laser, should you aim above, below, or directly at the observed fish? Defend your answers.
100. If the fish in the preceding exercise were small and blue and your laser light were red, what corrections should you make? Explain.
101. When a fish in a pond looks upward at an angle of $45^{\circ}$, does it see the sky above the water's surface or a reflection from the water-air boundary of the bottom of the pond? Defend your answer.
102. Rays of light moving upward through water toward the water-air boundary at angles larger than $48^{\circ}$ to the normal are totally reflected. No rays larger than $48^{\circ}$ refract outside. How about the reverse? Is there an angle at which light rays in air meeting the air-water boundary will totally reflect? Or will some light be refracted at all angles?
103. If you were to send a beam of laser light to a space station above the atmosphere that appears just above the horizon, would you aim the laser above, below, or at the visible space station? Defend your answer.
104. Two observers standing apart from each other do not see the "same" rainbow. Explain.
105. A rainbow viewed from an airplane may form a complete circle. Where will the shadow of the airplane appear? Explain.
106. How is a rainbow similar to the halo sometimes seen around the Moon on a frosty night? If you're stumped, check the Internet and see how rainbows and halos differ.
107. Transparent plastic swimming-pool covers called solar heat sheets have thousands of small air-filled bubbles that resemble lenses. The bubbles in these sheets are advertised to focus heat from the Sun into the water, thereby raising its temperature. Do you think these bubbles direct more solar energy into the water? Defend your answer.
108. Would the average intensity of sunlight measured by a light meter at the bottom of the pool in Figure 28.46 be different if the water were still?
109. When your eye is submerged in water, light rays bend only slightly when they pass from the water into your cornea. Why isn't the bending as pronounced as when light passes from air into your cornea? (How do the indices of refraction differ for your cornea, air, and water?)
110. When your eye is submerged in water, does the speed of light increase, decrease, or remain constant as it passes from the water into your cornea?
111. Two rays are shown in the sketch that accompanies footnote 3, repeated here. Discuss whether these two rays produce the image or merely locate where the image is in relation to the lens.


[^0]:    ${ }^{1}$ Another less common fate is absorption followed by reemission at lower frequencies-fluorescence (see Chapter 30).
    ${ }^{2}$ This material and many of the examples of least time are adapted from R. P. Feynman, R. B. Leighton, and M. Sands, The Feynman Lectures on Physics, Vol. I, Chap. 26 (Reading, MA: Addison-Wesley, 1963).

