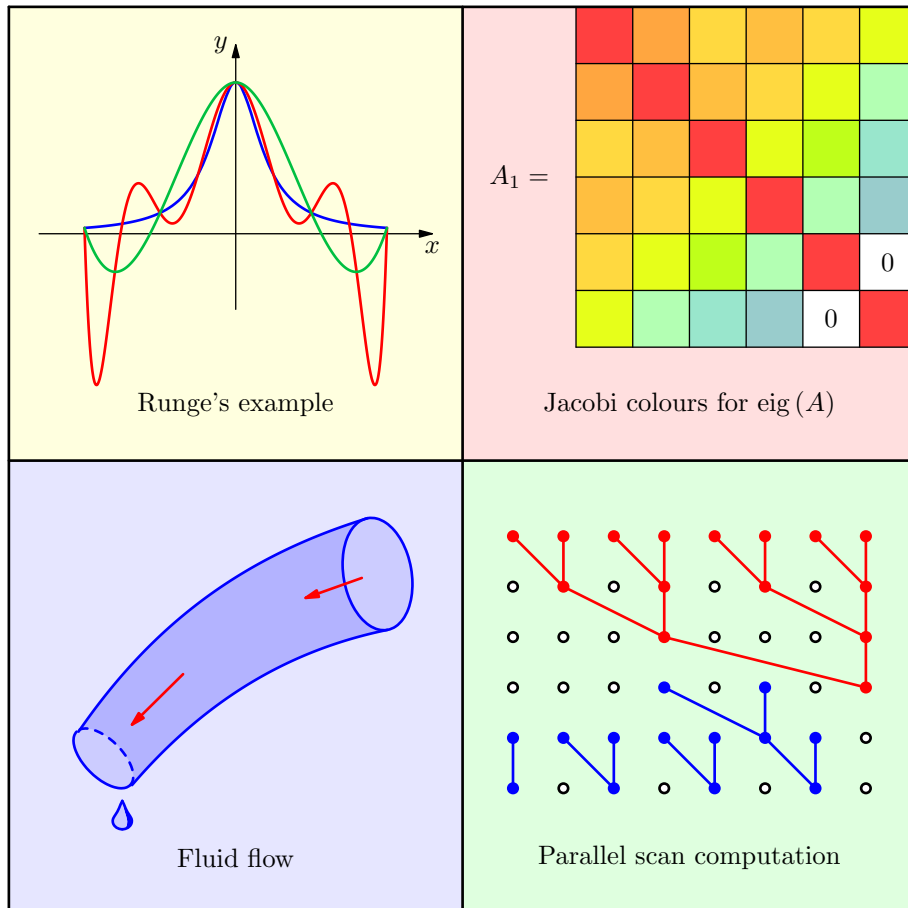


Conference on  
**APPLIED MATHEMATICS  
 AND  
 SCIENTIFIC COMPUTING**

Trogir, Croatia  
 June 13–17, 2011



**SCIENTIFIC PROGRAM**



## Monday, June 13 – Morning Session

8:30–8:45	<b>Conference opening – Miljenko Marušić</b>
-----------	--

Time	Chair: Miljenko Marušić
8:45–9:30	Tom Lyche Quadratic C1 splines on triangulations, <a href="#">page 10</a>

Time	Chair: Tom Lyche
9:50–10:15	<u>Gašper Jaklič</u> , Jernej Kozak, Marjetka Krajnc, Vito Vitrih and Emil Žagar Geometric Hermite interpolation by rational Bézier spatial curves, <a href="#">page 32</a>
10:15–10:40	Jernej Kozak, Marjeta Krajnc, Mladen Rogina and <u>Vito Vitrih</u> Pythagorean-hodograph cycloidal curves, <a href="#">page 38</a>
10:40–11:05	<u>Tina Bosner</u> and Mladen Rogina Greville points for Chebyshev splines, <a href="#">page 21</a>

Time	Chair: Takeshi Ogita
11:25–11:50	<u>Nela Bosner</u> , Zvonimir Bujanović, and Zlatko Drmač Efficient algorithms for solving shifted systems with multiple right-hand sides, <a href="#">page 20</a>
11:50–12:15	Zvonimir Bujanović On the Ritz values of normal matrices, <a href="#">page 21</a>
12:15–12:40	Ivica Nakić Generalized eigenvalue problems with prespecified eigenvalues, <a href="#">page 48</a>

### Monday, June 13 – Afternoon Session

Time	Chair: Emil Žagar
14:45–15:30	Larry L. Schumaker Splines on meshes with hanging vertices, <a href="#">page 14</a>

Time	Chair: Rudolf Scitovski
15:50–16:15	Vjeran Hari Convergence to diagonal form of block Jacobi-type processes, <a href="#">page 27</a>
16:15–16:40	Ivan Slapničar Jacobi-type algorithms for the Hamiltonian eigenvalue problem, <a href="#">page 54</a>
16:40–17:05	Nevena Jakovčević Stor Accurate eigenvalue decomposition of arrowhead matrices and applications, <a href="#">page 33</a>

Time	Chair: Vjeran Hari
17:25–17:50	<u>Vedran Novaković</u> , Saša Stanko, and Sanja Singer Parallel hybrid CPU–GPU full-block Jacobi algorithm for hyperbolic SVD, <a href="#">page 49</a>
17:50–18:15	Vedran Šego One decomposition of hyperbolic unitary matrices, <a href="#">page 54</a>
18:15–18:40	Vjeran Hari, <u>Sanja Singer</u> , and Saša Singer Computation of the hyperbolic CS decomposition, <a href="#">page 28</a>

**Tuesday, June 14 – Morning Session**

Time	Chair: Larry L. Schumaker
8:45–9:30	Carla Manni Generalized B-splines in isogeometric analysis, <a href="#">page 11</a>

Time	Chair: Carla Manni
9:50–10:15	Mario Berljafa, Sara Muhvić, <a href="#">Melkior Ornik</a> and Saša Singer Gaussian integration for compression splines, <a href="#">page 18</a>
10:15–10:40	Arne Lakså Problems connected to smoothness of surfaces on triangular structures, <a href="#">page 40</a>
10:40–11:05	Gašper Jaklič and <a href="#">Tadej Kanduč</a> Willmore energy minimizing PN triangles, <a href="#">page 31</a>

Time	Chair: Arne Lakså
11:25–11:50	Gašper Jaklič, Tadej Kanduč, Selena Praprotnik and <a href="#">Emil Žagar</a> Energy minimizing mountain ascent, <a href="#">page 30</a>
11:50–12:15	<a href="#">Bojan Crnković</a> and Nelida Črnjarić-Žic Polynomial weighted essentially non-oscillatory approximation, <a href="#">page 22</a>
12:15–12:40	Daniel Delahaye, <a href="#">Christophe Rabut</a> , and Stéphane Puechmorel Wind field evaluation by using onboard aircraft measurements and vector spline interpolation, <a href="#">page 23</a>

### Tuesday, June 14 – Afternoon Session

Time	Chair: Mladen Rogina
14:45–15:30	Lawrence J. Crane and <u>John J. H. Miller</u> Mathematical modelling of wave energy devices – an introduction, <a href="#">page 12</a>

Time	Chair: Ivan Slapničar
15:50–16:15	Martina Manhart Componentwise backward error bounds of relaxed forms of fast Bunch–Parlett and bounded Bunch–Kaufman algorithm, <a href="#">page 45</a>
16:15–16:40	Takeshi Ogita Robust singular value decomposition, <a href="#">page 50</a>
16:40–17:05	<u>Katsuhisa Ozaki</u> and Takeshi Ogita General matrix multiplication with guaranteed accuracy, <a href="#">page 51</a>

Time	Chair: John J. H. Miller
17:25–17:50	<u>Kristian Sabo</u> , Rudolf Scitovski and Ivan Vazler Center-based $l_1$ -clustering method, <a href="#">page 53</a>
17:50–18:15	Roderick Melnik Mathematical models for electronic structures of low dimensional nanostructures and their numerical approximations: Quantum-continuum coupling, <a href="#">page 46</a>
18:15–18:40	Lana Horvat Dmitrović Fractal analysis of Hopf bifurcation for maps, <a href="#">page 29</a>

### Wednesday, June 15 – Excursion to National park Krka

### Thursday, June 16 – Morning Session

Time	Chair: Nenad Anđonić
8:45–9:30	Grégoire Allaire Optimal design of low-contrast two phase structures for the wave equation, <a href="#">page 9</a>

Time	Chair: Grégoire Allaire
9:50–10:15	Igor Pažanin Effective flow of micropolar fluid through a thin or long pipe, <a href="#">page 52</a>
10:15–10:40	Ivan Dražić and Nermina Mujaković 3D model for compressible viscous heat conducting micropolar fluid with symmetry and free boundary: a global existence theorem, <a href="#">page 24</a>
10:40–11:05	Brahim Amaziane, Mladen Jurak and <a href="#">Anja Vrbaški</a> Homogenization of immiscible compressible two-phase flow in porous media by the concept of global pressure, <a href="#">page 15</a>

Time	Chair: Zvonimir Tutek
11:25–11:50	<a href="#">Boris Muha</a> and Zvonimir Tutek On the evolutionary free piston problem for the Stokes equations with slip boundary conditions, <a href="#">page 47</a>
11:50–12:15	<a href="#">Ivan Ivec</a> and Nenad Anđonić Boundedness of pseudodifferential operators on mixed-norm spaces, <a href="#">page 30</a>
12:15–12:40	<a href="#">Jadranka Kraljević</a> and Darko Žubrinić Quasilinear elliptic equations with positive exponent on the gradient, <a href="#">page 39</a>

**Thursday, June 16 – Afternoon Session**

Time	Chair: Ibrahim Aganović
14:45–15:30	Maria Giovanna Mora The time-dependent von Kármán plate equation as a limit of 3d nonlinear elastodynamics, <a href="#">page 13</a>

Time	Chair: Maria Giovanna Mora
15:50–16:15	Igor Velčić Periodically wrinkled plate of the Föppl-von Kármán type, <a href="#">page 57</a>
16:15–16:40	<u>Martin Lazar</u> and Nenad Antičić Application of parabolic H-measures to the vibrating plate equation, <a href="#">page 41</a>
16:40–17:05	<u>Maroje Marohnić</u> and Josip Tambača Derivation of a model of the prestressed elastic string, <a href="#">page 45</a>

Time	Chair: Senka Maćešić
17:25–17:50	<u>Vahidin Hadžiabdić</u> and Alma Omerspahić Retraction method in dynamics of some quadratics nonlinear system of differential equations, <a href="#">page 26</a>
17:50–18:15	Enes Duvnjaković and <u>Samir Karasuljić</u> Uniformly convergent difference scheme for semilinear reaction-diffusion problem, <a href="#">page 25</a>
18:15–18:40	<u>Louis Kavitha</u> , Narayanan Akila, Somasundram Bhuvaneshwari and Dhanaraj Gopi Perturbed soliton like excitations for dipolar interaction in the ferromagnetic spin systems, <a href="#">page 35</a>



### Friday, June 17 – Morning Session

Time	Chair: Josip Tambača
8:45–9:30	Boris Mordukhovich New trends and applications of variational analysis, <a href="#">page 13</a>

Time	Chair: Boris Mordukhovich
9:50–10:15	Nedžad Limić and <u>Maja Starčević</u> Dimension reduction for models of transport with sedimentation, <a href="#">page 44</a>
10:15–10:40	<u>Nedžad Limić</u> , Sunčana Geček, and Tarzan Legović Concepts related to residence time in transport models, <a href="#">page 42</a>
10:40–11:05	David L. Valentine, Igor Mezić, <u>Senka Maćešić</u> , Nelida Črnjarić-Žic, Stefan Ivić, Patrick Hogan, Vladimir A. Fonoberov, Sophie Loire Bacterial hydrocarbon uptake due to the oil spill in the Gulf of Mexico, <a href="#">page 55</a>

Time	Chair: Nelida Črnjarić-Žic
11:25–11:50	Sándor Baran Calibrating forecast ensembles of the LAMEPS system of the Hungarian Meteorological Service using Bayesian Model Averaging, <a href="#">page 16</a>
11:50–12:15	Maja Karaga Uniform asymptotic stability of dynamic equations with multiple delays, <a href="#">page 35</a>
12:15–12:40	Anett Bekéné Rácz Methods for increasing computational efficiency and stability in solving optimization problems, <a href="#">page 17</a>

**Friday, June 17 – Afternoon Session**

<b>Time</b>	<b>Chair: Nedžad Limić</b>
14:45–15:10	Wojciech M. Kempa On transient queue-size distribution in the batch arrival system with $N$ -policy and setup times, <a href="#">page 37</a>
15:10–15:35	<u>Slobodan Jelić</u> and Domagoj Matijević The relation of Connected Set Cover and Group Steiner Tree, <a href="#">page 33</a>
15:35–15:45	<b>Conference closing – Miljenko Marušić</b>

## INVITED LECTURES

THURSDAY, 8:45–9:30

### Optimal design of low-contrast two phase structures for the wave equation

Grégoire Allaire

**Abstract.** In this joint work with A. Kelly [1] we study the following optimal design problem: find the distribution of two phases in a given domain that minimizes an objective function evaluated through the solution of a wave equation. This type of optimization problem is known to be ill-posed in the sense that it generically does not admit a minimizer among classical admissible designs. Its relaxation could be found, in principle, through homogenization theory but, unfortunately, it is not always explicit, in particular for objective functions depending on the solution gradient. To circumvent this difficulty we make the simplifying assumption that the two phases have a low contrast. Then, a second order asymptotic expansion with respect to the small amplitude of the phase coefficients yields a simplified optimal design problem which is amenable to relaxation by means of  $H$ -measures [3, 4]. We prove a general existence theorem in a larger class of composite materials and propose a numerical algorithm to compute minimizers in this context. As in the case of an elliptic state equation, the optimal composites are shown to be rank one laminates. However the proof that relaxation and small amplitude limit commute is more delicate than in the elliptic case [2].

#### References

- [1] G. ALLAIRE, A. KELLY, *Optimal design of low-contrast two phase composites for wave propagation*, in press, Math. Models Methods Appl. Sci. (2011).
- [2] G. ALLAIRE, S. GUTIERREZ, *Optimal design in small amplitude homogenization*, M2AN Math. Model. Numer. Anal., **41** (2007) 543–574.
- [3] P. GÉRARD, *Microlocal defect measures*, Comm. Partial Differential Equations, **16** (1991) 1761–1794.

- [4] L. TARTAR, *H-measures, a new approach for studying homogenisation, oscillations, and concentration effects in partial differential equations*, Proc. Roy. Soc. Edinburgh Sect. A, **115A** (1990) 193–230.

GRÉGOIRE ALLAIRE, Centre de Mathématiques Appliquées, École Polytechnique,  
Palaiseau, 91128 France.  
e-mail: [gregoire.allaire@polytechnique.fr](mailto:gregoire.allaire@polytechnique.fr)

MONDAY, 8:45–9:30

## Quadratic $C^1$ splines on triangulations

Tom Lyche

**Abstract.** Surfaces defined over triangulations have widespread application in many areas ranging from finite element analysis and physics and engineering applications to the entertainment industry. For many of these applications piecewise linear surfaces do not offer sufficient smoothness. To obtain  $C^1$  smoothness, one must either use quintic polynomials with 21 degree of freedom over each triangle or use lower degree macro-elements that subdivide each triangle into a number of subtriangles. Thus far, the second approach has largely been based on using the Bernstein–Bézier basis on each subtriangle, manually enforcing the smoothness internal to each triangle and solving the resulting constrained system. In this talk we consider using Simplex splines on triangulations.

TOM LYCHE, CMA and Department of Informatics, University of Oslo, P.O. Box 1053, Blindern, 0316 Oslo, Norway.

e-mail: [tom@ifi.uio.no](mailto:tom@ifi.uio.no)

<http://home.ifi.uio.no/tom/>

TUESDAY, 8:45–9:30

## Generalized B-splines in isogeometric analysis

Carla Manni

**Abstract.** In many application problems governed by partial differential equations, such as solids, structures and fluids the standard analysis methods, Finite Elements Methods (FEM), are based on quite crude approximations of the involved geometry. On the other hand, the geometric approximation inherent in the mesh can lead to accuracy problems.

In recent years an analysis framework based on functions capable of exact representation of the geometry was developed, giving rise to Isogeometric Analysis. The term *isogeometric* is due to the fact that the solution space for dependent variables is represented in terms of the same functions which represent the geometry. Isogeometric Analysis developed so far is mainly based on NURBS, and has revealed to be an effective tool as in this way it is possible to fit exact geometries at the coarsest level of discretization and eliminate geometry errors from the very beginning. Moreover, the approach can also manage approximating functions with high order smoothness. This is also regarded as a benefit.

Nevertheless rational representations present several drawbacks. This motivated the introduction in the context of CAGD of some possible alternatives to the rational model. Among the others, generalized splines have been introduced for their ability to exactly describe some profiles of salient interest in applications as conic sections, helices, cycloids, etc. Generalized splines are piecewise functions with sections in spaces of the form

$$\langle 1, x, \dots, x^{n-2}, u(x), v(x) \rangle,$$

instead of algebraic polynomials. For suitable choices of functions  $u$  and  $v$  they possess all the interesting properties of classical polynomial splines, and admit a representation in term of basis functions which are a natural extension of polynomial B-splines, the so called *generalized B-splines*.

On the other hand, the interest for generalized B-splines passes over the CAGD environment. For proper selections of  $u$  and  $v$ , they produce spaces widely used in constrained approximation and interpolation, and can be generated by (nonstationary) subdivision schemes.

Recently, generalized B-splines involving trigonometric and exponential functions have been profitably used as basis functions in Isogeometric Analysis as a possible alternative to NURBS.

In this talk we review the main properties of generalized B-splines and present some recent results concerning their application in Isogeometric Analysis.

## References

- [1] T. J. R. HUGHES, J. A. COTTRELL AND Y. BAZILEVS, *Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement*, Comput. Methods Appl. Mech. Engrg., **194** (2005), 4135–4195.

CARLA MANNI, Department of Mathematics, University of Roma “ Tor Vergata”,  
Via della Ricerca Scientifica, 00133 Roma, Italy.

e-mail: [manni@mat.uniroma2.it](mailto:manni@mat.uniroma2.it)

<http://www.mat.uniroma2.it/~manni>

TUESDAY, 14:45–15:30

## Mathematical modelling of wave energy devices – an introduction

Lawrence J. Crane and John J. H. Miller

**Abstract.** The system of equations for several different types of floating wave energy device are presented. The basic hydrodynamic equations and boundary conditions for ocean waves are then derived and linearized. The concepts of added mass and impedance matching are introduced and the Froude–Krylov hypothesis is described. Simple mathematical models of some generic devices are constructed and the results of numerical computations shown. Brief descriptions of some contemporary wave energy conversion devices are also given.

LAWRENCE J. CRANE, Department of Mathematics, Trinity College, Dublin 2,  
Ireland.

e-mail: [crane@maths.tcd.ie](mailto:crane@maths.tcd.ie)

JOHN J. H. MILLER, Institute for Numerical Computation and Analysis (INCA),  
7–9 Dame Court, Dublin 2, Ireland.

e-mail: [jm@incaireland.org](mailto:jm@incaireland.org)

<http://www.incaireland.org>

THURSDAY, 14:45–15:30

## The time-dependent von Kármán plate equation as a limit of 3d nonlinear elastodynamics

Maria Giovanna Mora

**Abstract.** In this talk we discuss the asymptotic behaviour of solutions of three-dimensional nonlinear elastodynamics in a thin plate, as the thickness of the plate tends to zero. Under appropriate scaling and smallness assumptions on the applied force and the initial values, we prove existence of strong solutions for large times and sufficiently small thickness and we show their convergence to solutions of the time-dependent von Kármán plate equation.

This is a joint work with Helmut Abels and Stefan Müller.

HELMUT ABELS, NWF I – Mathematik, Universität Regensburg, 93040 Regensburg, Germany.

e-mail: [helmut.abels@mathematik.uni-regensburg.de](mailto:helmut.abels@mathematik.uni-regensburg.de)

MARIA GIOVANNA MORA, Scuola Internazionale Superiore di Studi Avanzati (SISSA), via Bonomea 265, 34136 Trieste, Italy.

e-mail: [mora@sissa.it](mailto:mora@sissa.it)

<http://people.sissa.it/~mora/>

STEFAN MÜLLER, Hausdorff Center for Mathematics, Institute for Applied Mathematics, Universität Bonn, Endenicher Allee 60, 53115 Bonn, Germany.

e-mail: [stefan.mueller@hcm.uni-bonn.de](mailto:stefan.mueller@hcm.uni-bonn.de)

FRIDAY, 8:45–9:30

## New trends and applications of variational analysis

Boris Mordukhovich

**Abstract.** Variational analysis has been well recognized as a rapidly growing and fruitful area in mathematics, particularly in the fields of nonlinear analysis, optimization, equilibria, and control. Being primarily motivated by problems arising in the aforementioned fields and their applications, modern variational analysis develops powerful principles and techniques to study and solving broad classes of

problems that may be not of a variational nature. One of the most characteristic features of modern variational analysis is the intrinsic presence of nonsmoothness, which naturally enters not only through the initial data of the problems under consideration but largely via variational principles and perturbation techniques applied to a variety of problems with even smooth data. Nonlinear dynamics and variational systems in applied sciences also give rise to nonsmooth structures and motivate the development of new forms of analysis that rely on generalized differentiation.

In this talk we discuss some new trends and developments in variational analysis and its numerous applications in both finite-dimensional and infinite-dimensional spaces, emphasizing those to optimization, equilibrium, robust stability, and optimal control of ordinary differential and partial differential equations.

BORIS MORDUKHOVICH, Department of Mathematics, Wayne State University,  
1237 Faculty/Administration Bldg, 656 West Kirby Avenue, Detroit, Michigan 48202, USA.

e-mail: [boris@math.wayne.edu](mailto:boris@math.wayne.edu)

<http://www.math.wayne.edu/~boris/>

MONDAY, 14:45–15:30

## Splines on meshes with hanging vertices

Larry L. Schumaker

**Abstract.** A constructive theory is developed for polynomial spline spaces defined on mixed meshes consisting of triangles and rectangles. These meshes include triangulations with hanging vertices as well as T-meshes. Such meshes are useful in the FEM method. In addition to dimension formulae, explicit basis functions are constructed, and their support and stability properties are discussed. The approximation power of the spaces is also treated.

LARRY L. SCHUMAKER, Vanderbilt University, Stevenson Center 1532,  
37240 Nashville, Tennessee, USA.

e-mail: [larry.schumaker@gmail.com](mailto:larry.schumaker@gmail.com)

<http://www.math.vanderbilt.edu/~schumake/>



## CONTRIBUTED TALKS

THURSDAY, 10:40–11:05

### Homogenization of immiscible compressible two phase flow in porous media by the concept of global pressure

Brahim Amaziane, Mladen Jurak and Anja Vrbaški

**Abstract.** We consider an immiscible, compressible two-phase flow through highly heterogeneous porous media with a periodic microstructure. The mathematical model is formulated using the global pressure and the saturation as primary unknowns, as in [1]. The resulting equations are written in a fractional flow formulation and lead to a coupled system which consists of a nonlinear parabolic equation (the global pressure equation) and a nonlinear diffusion-convection one (the saturation equation). We obtain the homogenized model and prove convergence results using the two-scale convergence technique.

#### Acknowledgments

This research was partially supported by the AUF “Agence Universitaire de la Francophonie”, Projet de Coopération Scientifique Inter-Universitaire, entitled “Modélisation d’Écoulements d’Eau dans les Sols non Saturés: Aspects Mathématiques et Logiciels de Simulation Numérique”, and the GnR MoMaS (PACEN/CNRS, ANDRA, BRGM, CEA, EDF, IRSN) France, their supports are gratefully acknowledged.

#### References

- [1] B. AMAZIANE, M. JURAK AND A. ŽGALJIĆ KEKO, *An existence result for a coupled system modeling a fully equivalent global pressure formulation for immiscible compressible two-phase flow in porous media*, J. Differential Equations, **250**, no. 3 (2011), 1685–1718.

BRAHIM AMAZIANE, Université de Pau, Laboratoire de Mathématiques et de leurs Applications, CNRS-UMR 5142, Av. de l'Université, 64000 Pau, France.  
e-mail: brahim.amaziane@univ-pau.fr

MLADEN JURAK, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: jurak@math.hr

ANJA VRBAŠKI, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: avrbaski@math.hr

FRIDAY, 11:25–11:50

## Calibrating forecast ensembles of the LAMEPS system of the Hungarian Meteorological Service using Bayesian Model Averaging<sup>1</sup>

Sándor Baran

**Abstract.** Bayesian Model Averaging (BMA) is a statistical postprocessing technique which produces calibrated probability density functions (PDF) of the predictable meteorological quantities from ensembles of forecasts. The obtained PDFs are convex combinations of PDFs corresponding to the bias corrected ensemble members, and the weights express the predictive skill of an ensemble member over a training period [1]. The BMA method is highly computer intensive, to estimate the parameters of the model the EM algorithm is applied.

In the present work we apply BMA post-processing to temperature data obtained from the Hungarian Meteorological Service. The data file contains 11 member ensembles of 42 hour forecasts for temperature for the city of Debrecen obtained by LAMEPS ensemble prediction system of the HMS [2] and corresponding validating observations. An ensemble consists of 10 exchangeable forecasts started from perturbed initial conditions and one control member started from the unperturbed analysis. We determine the appropriate training period for our data and using appropriate scoring rules we demonstrate the advantage of the BMA post-processed forecasts [3] compared to the predictions calculated from the raw ensembles.

---

<sup>1</sup>Research has been supported by the Hungarian Scientific Research Fund under Grant No. OTKA T079128/2009 and partially supported by TÁMOP 4.2.1./B-09/1/KONV-2010-0007/IK/IT project. The project is implemented through the New Hungary Development Plan co-financed by the European Social Fund, and the European Regional Development Fund.

## References

- [1] T. GNEITING AND A. E. RAFTERY, *Weather forecasting with ensemble methods*, *Science*, **310** (2005), 248–249.
- [2] E. HÁGEL, *The quasi-operational LAMEPS system of the Hungarian Meteorological Service*, *Időjárás*, **114** (2010), 121–133.
- [3] C. FRALEY, A. E. RAFTERY AND T. GNEITING, *Calibrating multimodel forecast ensembles with exchangeable and missing members using Bayesian model averaging*, *Mon. Wea. Rev.*, **138** (2010), 190–202.

SÁNDOR BARAN, Faculty of Informatics, University of Debrecen, Egyetem tér 1,  
H-4032 Debrecen, Hungary.  
e-mail: [baran.sandor@inf.unideb.hu](mailto:baran.sandor@inf.unideb.hu)  
<http://www.inf.unideb.hu/~barans>

FRIDAY, 12:15–12:40

## Methods for increasing computational efficiency and stability in solving optimization problems

Anett Bekéné Rácz

**Abstract.** The real-world LP models tend to be large in size. In practice large scale problems are not solved in the form they are generally constructed. There are several operations called preprocessing that can be used to increase the reliability and efficiency. There are two parts of preprocessing, namely scaling and presolve algorithms.

To solve large scale problems hundreds of thousands to millions of floating-point arithmetic calculations are needed. Because of the finite precision inherent in computer arithmetic, small numerical errors occur in these calculations. These errors typically have a cumulative effect, that often leads to a numerically unstable problem and possibly large errors in the “solution” obtained. Scaling attempts to make the magnitudes of all the data as close as possible. We investigated the efficiency of different scaling methods and the needed re-scaling operation which are used for obtaining the optimal solution of the original problem from the solution of the scaled problem.

Presolve algorithms are also a part of preprocessing and they are used for reducing the size by eliminations, detecting infeasibility or unboundedness, fixing as

many variables as possible without solving the problem. They also attempt to realize the weaknesses of a model, such as empty rows/columns or redundant constraints which can be removed in order to reduce the size.

These operations can be different in case of linear programming (LP), quadratic programming (QP) or linear-fractional programming (LFP) problems. Most of these methods have already been implemented in case of LP [1] or QP [2], our aim was to adapt them into LFP problems.

## References

- [1] E. D. ANDERSEN AND K. D. ANDERSEN, *Presolving in linear programming*, Math. Program., **71** (1995), 221–245.
- [2] Cs. MÉSZÁROS AND U. H. SUHL, *Advanced preprocessing techniques for linear and quadratic programming*, OR Spectrum, **25** (2003), 575–595.

ANETT BEKÉNE RÁCZ, Faculty of Informatics, University of Debrecen, Egyetem tér 1, 4032 Debrecen, Hungary.  
 e-mail: [racz.anett@inf.unideb.hu](mailto:racz.anett@inf.unideb.hu)  
<http://www.inf.unideb.hu/valseg/dolgozok/anett.racz/anettracz.html>

TUESDAY, 9:50–10:15

## Gaussian integration for compression splines

Mario Berljafa, Sara Muhvić, Melkior Ornik and Saša Singer

**Abstract.** Compression splines are piecewise generated by ordinary powers  $x^k$ , up to a certain degree, and trigonometric functions  $\sin px$ ,  $\cos px$ , that depend on a given “compression” parameter  $p > 0$ . A recent algorithm for computing the compression B-splines requires the exact integrals of such splines. They can be computed efficiently by using the Gaussian quadrature formula that exactly integrates a compression spline on each subinterval of the spline mesh. However, since  $p$  can have different values on different subintervals, it is impossible to precalculate the nodes and weights for the Gaussian integration. They have to be computed “on-demand”, for each value of  $p$ . Moreover, the calculated results have to be highly accurate (ideally, to the full machine precision) to ensure the exact integration. The problem can always be reduced to the standard interval  $[-1, 1]$ , with  $0 < p < \pi$ .

We present an algorithm that, for a given value of  $p$ , computes the nodes and weights of the Gaussian quadrature formula of order  $n$  in only  $O(n^2)$  operations. This is accomplished by using the modified Golub–Welsch procedure for the ordinary Gaussian quadratures to compute an *approximate* formula that is always exact on the whole *polynomial* part of the space. By exploiting symmetry, only *one* defining nonlinear equation remains to be solved — the exact integration of  $\cos px$ . For this, we use the Brent–Dekker algorithm, as it requires no derivatives.

These “trigonometrically fitted” formulae are quite hard to calculate to high precision, especially for small values of  $p$ , since the ordinary Gaussian quadrature is already very accurate, but not exact. Therefore, the error of an approximate formula has to be computed with high relative precision — by using the truncated Legendre expansion of  $\cos px$ .

MARIO BERLJAJA, B.Sc. student, Faculty of Science, Department of Mathematics,  
University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [mberljaf@student.math.hr](mailto:mberljaf@student.math.hr)

SARA MUHVIĆ, B.Sc. student, Faculty of Science, Department of Mathematics,  
University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [smuhvic@student.math.hr](mailto:smuhvic@student.math.hr)

MELKIOR ORNIK, B.Sc. student, Faculty of Science, Department of Mathematics,  
University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [mornik@student.math.hr](mailto:mornik@student.math.hr)

SAŠA SINGER, Faculty of Science, Department of Mathematics, University of Za-  
greb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [singer@math.hr](mailto:singer@math.hr)  
<http://web.math.hr/~singer>

MONDAY, 11:25–11:50

## Efficient algorithms for solving shifted systems with multiple right-hand sides

Nela Bosner, Zvonimir Bujanović, and Zlatko Drmač

**Abstract.** We offer an efficient algorithm for solving shifted systems with multiple right sides, and for large number of shifts. This problems appears in control theory when frequency response matrix of a system is required, or when numerical solution of large linear system of ODE's is sought. The algorithm is based on two steps. The first step reduces matrix and right-hand side of the system to a suitable form, and this is done only once. This suitable form enables us to solve the system simply and quickly. The second step repeatedly solves the systems for different shifts. We distinguish two cases of the shifted systems with  $m$  right-hand sides.

- $(A - \sigma I)X = B$  where  $I$  is the identity matrix. In this case  $A$  is reduced to  $m$ -Hessenberg form and  $B$  is reduced to triangular form, and these reductions are done simultaneously in the first step.
- $(A - \sigma E)X = B$  where  $E$  is a general matrix. In this case  $A$  is reduced to  $m$ -Hessenberg form and  $B$  and  $E$  are reduced to triangular form. All these reductions are done simultaneously in the first step, too.

In both cases we will obtain a linear system with the  $m$ -Hessenberg system matrix and the triangular right-hand side, which is easily solved by annihilating  $m$  subdiagonals of the system matrix. For efficiency of the algorithm the triangular form of the right-hand side is very important when the number of shifts is large. All steps of the algorithm are properly blocked to produce optimal efficiency.

NELA BOSNER, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [nela@math.hr](mailto:nela@math.hr)

<http://http://www.math.hr/~nela>

ZVONIMIR BUJANOVIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [zbujanov@math.hr](mailto:zbujanov@math.hr)

<http://web.math.hr/~zbujanov>

ZLATKO DRMAČ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [drmac@math.hr](mailto:drmac@math.hr)

<http://http://www.math.hr/~drmac>

MONDAY, 10:40–11:05

## Greville points for Chebyshev splines

Tina Bosner and Mladen Rogina

**Abstract.** In the polynomial spline approximation theory, the Greville points, or knot averages, play an important role. The question arises whether similar points exist in the setting of Chebyshev splines. The answer is positive for very general spaces of Chebyshev splines which are based on Lebesgue–Stieltjes measures and possess the linear precision property. These points inherit the property of being convex combination of consecutive knots, thus enabling the construction of quasi-interpolants and the variation diminishing spline approximation. By means of dual functionals, we prove the convergence of such approximation, in terms of measures of subintervals in consideration, instead of their length. We end by providing examples of Greville points for a few important Chebyshev splines which are well known in various fields of numerical analysis and CAGD.

TINA BOSNER, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [tinab@math.hr](mailto:tinab@math.hr)

<http://web.math.hr/~tinab>

MLADEN ROGINA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [rogina@math.hr](mailto:rogina@math.hr)

<http://www.math.hr/~rogina>

MONDAY, 11:50–12:15

## On the Ritz values of normal matrices

Zvonimir Bujanović

**Abstract.** The implicitly restarted Arnoldi method (IRAM) introduced by Sorensen is a well-known algorithm for computing a few eigenpairs of a large, generally non-symmetric sparse matrix. The method is implemented in a freely available software package called ARPACK, and used successfully in a number of different applications.

The convergence of the algorithm has been a subject of intensive study. While Sorensen proved the convergence when the algorithm is used to compute the extreme eigenvalues of Hermitian matrices, the conditions for the convergence in the general case are still unknown. Embree constructed a class of matrices for which the algorithm fails to converge, even in the exact arithmetic: the desired eigenvector is deflated out of the search space. A key property that ensures the failure is the non-normality of the example matrices.

In our talk, we discuss the convergence of IRAM for normal matrices. We demonstrate the difficulty in keeping the monotonicity of the Ritz values, which was essential in Sorensen's proof. A simple condition for a set of complex numbers to appear as Ritz values of a normal matrix is given: it is necessary and sufficient that a certain Cauchy matrix has a positive vector in its kernel. This fact is then used to explore the more complex geometry of Ritz values in the normal case.

ZVONIMIR BUJANOVIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [zbujanov@math.hr](mailto:zbujanov@math.hr)  
<http://web.math.hr/~zbujanov>

TUESDAY, 11:50–12:15

## Polynomial weighted essentially non-oscillatory approximation

Bojan Crnković and Nelida Črnjarić-Žic

**Abstract.** Weighted essentially non-oscillatory (WENO) approximation has been mainly used in numerical schemes for solving hyperbolic partial differential equations. The typical WENO procedure constructs a rational function for approximating values of the function  $v(x)$  on cell boundaries from its known cell averages. This approximation is essentially non-oscillatory and high order accurate for smooth enough functions  $v(x)$ . However, the approximating rational function obtained by the classical WENO reconstruction contains poles. Therefore, if one needs to reconstruct the value of function  $v(x)$  in the interior of the numerical cell, the standard WENO procedure fails. In this paper we have developed a polynomial version of the WENO procedure that provides an elegant solution to this problem by constructing an approximating polynomial that is non-oscillatory and high order accurate not only at the cell boundaries but in the entire numerical cell. The proposed new algorithm is computationally comparable to the standard WENO procedure and



gives the same reconstruction values on the the cell boundaries. The obtained numerical results show that the newly proposed procedure performs very well on the considered test examples.

BOJAN CRNKOVIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58,  
51000 Rijeka, Croatia.

e-mail: [bojan.crnkovic@riteh.hr](mailto:bojan.crnkovic@riteh.hr)

NELIDA ČRNJARIĆ-ŽIC, Faculty of Engineering, University of Rijeka, Vukovarska  
58, 51000 Rijeka, Croatia.

e-mail: [nelida@riteh.hr](mailto:nelida@riteh.hr)

TUESDAY, 12:15–12:40

## Wind field evaluation by using onboard aircraft measurements and vector spline interpolation

Daniel Delahaye, Christophe Rabut, and Stéphane Puechmorel

**Abstract.** Accurate wind magnitude and direction estimation is essential for aircraft trajectory prediction. For instance, based on these data, one may compute entry and exit times from a sector or detect potential conflict between aircraft. Since the flight path has to be computed and updated on real time for such applications, wind information has to be available in real time too. The wind data which are currently available through meteorological service broadcast suffer from small measurement rate with respect to location and time. In this paper, a new wind estimation method based on radar track measures is proposed. When on board true air speed measures are available, a linear model is developed for which a Kalman filter is used to produce high quality wind estimate. When only aircraft position measures are available, an observability analysis shows that wind may be estimated only if trajectories have one or two turns depending of the number of aircraft located in a given area. Based on this observability conditions, closed forms of the wind has been developed for the one and two aircraft cases. By this mean, each aircraft can be seen as a wind sensor when it is turning. After performing these pointwise evaluations in realistic frameworks, our approach is now to estimate the wind vector field at any point. Based on those local wind estimates, a global space-time wind field estimation is derived by vector spline interpolation, or by solving Shallow–Water equations, which assumes geostrophic wind. The accuracy of this wind map is dependent of the number wind estimates in a given zone. Numerical tests show the efficiency of the method.

## References

- [1] R. E. COLE, *Wind prediction accuracy for air traffic management decision support tools*, in Proceedings of 3<sup>th</sup> USA–Europe ATM Seminar. FAA–Eurocontrol, 2000.
- [2] C. B. VREUGDENHIL, *Numerical Methods for Shallow–Water Flow*, Water Science and Technology Library, Vol. 13. Kluwer Academic Publishers, 1994.

DANIEL DELAHAYE, Applied Mathematics Laboratory, French Civil Aviation University, 7 Avenue Edouard Belin, 31055 Toulouse, France.  
e-mail: [delahaye@recherche.enac.fr](mailto:delahaye@recherche.enac.fr)

CHRISTOPHE RABUT, Department of Mathematics, Institute of Applied Science, 135, Avenue de Rangueil, 31077 Toulouse, France.  
e-mail: [rabut@insa-toulouse.fr](mailto:rabut@insa-toulouse.fr)

STÉPHANE PUECHMOREL, Applied Mathematics Laboratory, French Civil Aviation University, 7 Avenue Edouard Belin, 31055 Toulouse, France.  
e-mail: [puechmor@recherche.enac.fr](mailto:puechmor@recherche.enac.fr)

THURSDAY, 10:15–10:40

### 3D model for compressible viscous heat conducting micropolar fluid with symmetry and free boundary: a global existence theorem

Ivan Dražić and Nermina Mujaković

**Abstract.** We consider 3D compressible viscous micropolar fluid model with spherically symmetric initial data of large oscillation between a static solid core and a free boundary connected to a surrounding vacuum state. The fluid is thermodynamically perfect and polytropic.

First we describe the model and develop an effective difference scheme to construct approximate solutions. Then we obtain the bounded estimates of the sequence of the approximative solutions.

Analyzing the convergence of sequence of the approximative solutions we prove a global-in-time existence theorem.

IVAN DRAŽIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.

e-mail: [idrasic@riteh.hr](mailto:idrasic@riteh.hr)

NERMINA MUJAKOVIĆ, Department of Mathematics, University of Rijeka, Omladinska 14, 51000 Rijeka, Croatia.

e-mail: [mujakovic@inet.hr](mailto:mujakovic@inet.hr)

THURSDAY, 17:50–18:15

## Uniformly convergent difference scheme for semilinear reaction-diffusion problem

Enes Duvnjaković and Samir Karasuljić

**Abstract.** The talk deals with the discretization of nonlinear singularly perturbed boundary problem

$$\epsilon^2 y'' = f(x, y) \text{ on } (0, 1), \quad y(0) = y(1) = 0,$$

where  $0 < \epsilon < 1$ . It is assumed that the nonlinear function  $f(x, y)$  is continuously differentiable and that it has a strictly positive derivate in accordance with  $y$ , in other words

$$\frac{\partial f}{\partial y} = f_y \geq m > 0 \text{ on } [0, 1] \times \mathbb{R} \text{ (} m = \text{const.)}.$$

The perturbed parameter  $\epsilon$  causes the appearance of the boundary layers of the exponential type in the solution of problem. Through applying classical methods in order to obtain an approximate solution for the observed problem leads to a numerical solution, which has an unacceptably high error value, and therefore a discretizational approach in a nonstandard way. By using the spline method with a natural choice of basic functions within this problem, we obtained a new difference scheme on the piecewise equidistant mesh which contains  $O(N)$  points. On such a mesh the error of order  $N^{-2} \ln^2 N$  is shown in a discrete maximum norm. Numerical results which are presented, confirm this rate of convergence.

The talk has five parts. The first part is an introductory, and here we can find the previously mentioned problem and a small part about dealing with it in the past. In the second part a difference scheme is derived using Green's function  $G$  for the operator  $L_\epsilon := \epsilon y'' - \gamma y$ . Difference scheme has the form

$$r_i \bar{y}_{i-1} - (r_i + r_{i+1}) \bar{y}_i + r_{i+1} \bar{y}_{i+1} = \frac{\Delta d_i}{\gamma} \bar{f}_{i-1} + \frac{\Delta d_{i+1}}{\gamma} \bar{f}_i, \quad i = 1, 2, \dots, N-1,$$

where  $\bar{y}_k$ ,  $k \in \{i-1, i, i+1\}$  are values of approximate solution in the points of the mesh,  $\bar{f}_k$ ,  $k \in \{i-1, i\}$  approximation of  $f(x, y)$  on the segment  $[x_{k-1}, x_k]$ , while  $r_k$  and  $\Delta d_k$  coefficients were obtained through the construction of a difference scheme. There has also been stated and proved theorem about the existence and uniqueness of the nonlinear system, which had derived from a difference scheme. In the third part the construction of the piecewise equidistant mesh is described. Here the segment  $[0, 1]$  is divided into three parts and the transition points are  $-\frac{2\epsilon}{m_1} \ln \epsilon$  and  $1 + \frac{2\epsilon}{m_1} \ln \epsilon$ .

For given a positive integer  $N$ , we use an equidistant mesh on each of these subintervals with  $\frac{N}{4}$  points in each of  $[0, h_\epsilon]$ , and  $[1 - h_\epsilon, 1]$ , and the remaining points in  $[h_\epsilon, 1 - h_\epsilon]$ .

In the fourth part the rate of convergence is given and proven, where it is shown that it is the error  $\frac{\ln^2 N}{N^2}$ . Finally, in the last part numerical results that confirm the theoretical results are presented.

ENES DUVNJAKOVIĆ, Mathematics Department, Faculty of Natural Sciences, University of Tuzla, Univerzitetska ulica 1, 75000 Tuzla, Bosnia and Herzegovina.  
e-mail: enes.duvnjakovic@untz.ba

SAMIR KARASULJIĆ, Mathematics Department, Faculty of Natural Sciences, University of Tuzla, Univerzitetska ulica 1, 75000 Tuzla, Bosnia and Herzegovina.  
e-mail: samir.karasuljic@bih.net.ba

THURSDAY, 17:25–17:50

## Retraction method in dynamics of some quadratics nonlinear system of differential equations

Vahidin Hadžiabdić and Alma Omerspahić

**Abstract.** The quadratic nonlinear systems of differential equations are an algebraically simple, but they can generate complex dynamics within wide parameters ranges, including chaos. This talk deals with behavior, approximations and stability of solutions of some of this systems. Behavior of integral curves in neighborhoods of an arbitrary curve is considered. The qualitative analysis theory and topological retraction method are used.

VAHIDIN HADŽIABDIĆ, Faculty of Mechanical Engineering, University of Sarajevo, Vilsonovo šetalište 9, 71000 Sarajevo, Bosnia and Herzegovina.  
e-mail: hadziabdic@mef.unsa.ba

ALMA OMERSPAHIĆ, Faculty of Mechanical Engineering, University of Sarajevo,  
 Vilsonovo šetalšte 9, 71000 Sarajevo, Bosnia and Herzegovina.  
 e-mail: [alma.omerspahic@mef.unsa.ba](mailto:alma.omerspahic@mef.unsa.ba)

MONDAY, 15:50–16:15

## Convergence to diagonal form of block Jacobi-type processes

Vjeran Hari

**Abstract.** We provide sufficient conditions for the general block Jacobi-type process to converge to diagonal form. We skip considering the convergence of the diagonal elements because such results are generally dependant on the perturbation theory specific for the given problem. The theory considers periodic pivot strategies, but stronger results refer to some classes of cyclic and quasi-cyclic strategies. In particular for the the class of strategies that are weakly equivalent to the column-cyclic strategy.

Given a block-matrix partition  $(A_{ij})$  of a square matrix  $A$ , we analyze the iterative process of the form  $A^{(k+1)} = P_k^* A^{(k)} Q_k$ ,  $k \geq 0$ ,  $A^{(0)} = A$ , where  $P_k$  and  $Q_k$  are nonsingular elementary block matrices which differ from the identity matrix in four blocks, two diagonal and the two corresponding off-diagonal blocks. The proof uses a new tool, the theory of block Jacobi operators. Jacobi operators were introduced by Henrici and Zimmermann in 1968 and in 2009 we generalized them to cope with block methods. The whole convergence theory is based on several modules. Hence replacing the module which deals with one class of pivot strategies by the one which deals with another class, enables to obtain new convergence results.

The main applications of the new results lie in proving the global convergence of block Jacobi-type methods for solving simple and generalized eigenvalue and singular value problems. Block diagonalization methods are typically implemented as BLAS3 algorithms, which are lately in the center of interest for their efficiency and relative accuracy. As model examples, we consider the block Jacobi method for Hermitian matrices and the block Kogbetliantz method. As a nontrivial example, we consider the latest result for the block  $J$ -Jacobi method.

VJERAN HARI, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
 e-mail: [hari@math.hr](mailto:hari@math.hr)  
<http://www.math.hr/~hari>

MONDAY, 18:15–18:40

## Computation of the hyperbolic CS decomposition

Vjeran Hari, Sanja Singer, and Saša Singer

**Abstract.** Accurate computation of the cosine-sine (CS) decomposition can be a step forward to make the one-sided block-Jacobi method faster.

The same is valid for the hyperbolic cosine-sine (JCS) decomposition, as a part of the corresponding hyperbolic Jacobi algorithm. The JCS is especially efficient in the case of postmultiplication of tall and skinny matrix by a  $J$ -unitary matrix. Such problem arises in the parallelization of the Jacobi algorithm, where each process holds only two block-columns.

There are two possible accurate approaches for the calculation of the JCS. One depends on the SVD of the diagonal blocks of  $Q$ , while the other uses the SVD of the off-diagonal blocks of  $Q$ .

VJERAN HARI, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [hari@math.hr](mailto:hari@math.hr)

<http://web.math.hr/~hari>

SANJA SINGER, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lučića 5, 10000 Zagreb, Croatia.

e-mail: [ssinger@fsb.hr](mailto:ssinger@fsb.hr)

<http://www.fsb.hr/ssinger>

SAŠA SINGER, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [singer@math.hr](mailto:singer@math.hr)

<http://web.math.hr/~singer>

TUESDAY, 18:15–18:40

## Fractal analysis of Hopf bifurcation for maps

Lana Horvat Dmitrović

**Abstract.** In this paper we study a box dimension of the orbits of two-dimensional discrete dynamical systems and their bifurcations in a nonhyperbolic fixed point. It is already known that there is a connection between the bifurcations in the nonhyperbolic fixed point of one-dimensional maps, and the box dimension of the orbits near that point. Now we will prove that connection in the case of the bifurcations in the nonhyperbolic fixed point of the planar maps as well. We will show that the value of box dimension changes at the bifurcation point, from zero to positive value. First, we study the two-dimensional discrete dynamical systems with only one multiplier on the unit circle, and get the result for box dimension of the orbit on the center manifold. Then we consider planar discrete systems undergoing a Hopf bifurcation for maps (Neimark–Sacker bifurcation). Fractal analysis of Hopf bifurcation for maps is especially interesting because of the difference between the rational and irrational case. Namely, we show that the box dimension around a nonhyperbolic fixed point with the multiplier  $e^{\pm i\Theta_0}$ , where  $\Theta_0 = 2\pi\beta$ ,  $\beta \in \mathbb{Q}$  (rational case), is equal to the box dimension of the one-dimensional discrete system generated by the map of radial component, which is  $\frac{2}{3}$ . But in the irrational case ( $\beta \in \mathbb{R} \setminus \mathbb{Q}$ ) we get that the box dimension of an orbit equals the box dimension of a spiral trajectory of the corresponding Hopf bifurcation for vector fields, which is  $\frac{4}{3}$ . In order to prove this result, we use the connection between the Hopf bifurcation for maps and the Hopf bifurcation for vector fields by the unit-time map. We also apply the result to the  $n$ -dimensional discrete dynamical systems with the two-dimensional center manifold on which Hopf bifurcation occurs. At the end, we apply the previous results to the Hopf bifurcation of limit cycles for the continuous dynamical systems in  $\mathbb{R}^3$ .

LANA HORVAT DMITROVIĆ, Department of Applied Mathematics, Faculty of Electrical Engineering and Computing, University of Zagreb, Unska 3, 10000 Zagreb, Croatia.

e-mail: [lanea.horvat@fer.hr](mailto:lanea.horvat@fer.hr)

THURSDAY, 11:50–12:15

## Boundedness of pseudodifferential operators on mixed-norm spaces

Ivan Ivec and Nenad Antonić

**Abstract.** Pseudodifferential operators have nice boundedness properties on Sobolev spaces. These properties are consequences of  $L^p$  boundedness of zeroth order operators, a classical result improving Hörmander–Mihlin theorem.

We prove the extension of the above result to mixed-norm  $L^p$  spaces (introduced by Benedek & Panzone in 1961), by applying the Calderon–Zygmund decomposition and generalised Marcinkiewicz interpolation theorem.

Finally, we show some extensions to boundedness of classical pseudodifferential operators (of any order) on Sobolev spaces, having possible application to the theory of partial differential equations.

IVAN IVEC, Gymnasium A. G. Matoša, Andrije Hebranga 26, 10430 Samobor, Croatia.  
e-mail: [ivan.ivec@gmail.com](mailto:ivan.ivec@gmail.com)

NENAD ANTONIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [nenad@math.hr](mailto:nenad@math.hr)

TUESDAY, 11:25–11:50

## Energy minimizing mountain ascent

Gašper Jaklič, Tadej Kanduč, Selena Praprotnik and Emil Žagar

**Abstract.** When moving from one point to another, an energy consumption is very important for choosing an optimal path. In this article, an optimal mountain ascent will be studied as a particular problem of a human being walking over a rugged terrain. For given discrete terrain data, a smooth relief description is constructed by using particular smooth splines known as macroelements. A functional, which measures energy consumption based on terrain inclination and on path length, is defined. Thus the problem simplifies to a discrete problem of finding the shortest



path on a mesh of Bézier curves. Numerical results on real-life data indicate that computed paths are a good approximation of hiking paths in nature.

The presented method can be easily generalized to similar problems in theory and applications.

GAŠPER JAKLIČ, FMF, University of Ljubljana, Jadranska 19, Ljubljana, Slovenia, IMFM, Jadranska 19, Ljubljana, Slovenia, and PINT, University of Primorska, Muzejski trg 2, Koper, Slovenia.

e-mail: [gasper.jaklic@fmf.uni-lj.si](mailto:gasper.jaklic@fmf.uni-lj.si)

<http://www.fmf.uni-lj.si/~jaklicg/>

TADEJ KANDUČ, Turboinštitut, Rovšnikova 7, Ljubljana, Slovenia.

e-mail: [tadej.kanduc@student.fmf.uni-lj.si](mailto:tadej.kanduc@student.fmf.uni-lj.si)

SELENA PRAPROTNIK, IMFM, Jadranska 19, Ljubljana, Slovenia.

e-mail: [selena.praprotnik@fmf.uni-lj.si](mailto:selena.praprotnik@fmf.uni-lj.si)

EMIL ŽAGAR, FMF, University of Ljubljana, Jadranska 19, Ljubljana, Slovenia, and IMFM, Jadranska 19, Ljubljana, Slovenia.

e-mail: [emil.zagar@fmf.uni-lj.si](mailto:emil.zagar@fmf.uni-lj.si)

<http://valjhun.fmf.uni-lj.si/~emil/>

TUESDAY, 10:40–11:05

## Willmore energy minimizing PN triangles

Gašper Jaklič and Tadej Kanduč

**Abstract.** PN triangle is a cubic Bézier surface that interpolates given spatial points and tangent planes at triangle vertices. Splines made from PN triangles are widely used in computer graphics due to their simple construction and desirable shape properties.

In this talk I will show some modifications of PN triangles that minimize Willmore energy. Optimization will be done in two separate steps. First, control points of every boundary curve will be set to minimize its bending energy. Then the internal control point will be optimized regarding discrete Willmore energy of a triangle. The construction will depend on local data only. The optimization problem will be easily solved and will have an unique solution at mild presumptions. We shall also see that classical PN triangles and optimized ones produce the same boundary curves. In the end, some examples of surface reconstruction and restorations will be shown.

GAŠPER JAKLIČ, FMF and IMFM, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia.

e-mail: [gasper.jaklic@fmf.uni-lj.si](mailto:gasper.jaklic@fmf.uni-lj.si)

<http://www.fmf.uni-lj.si/~jaklicg>

TADEJ KANDUČ, Turboinštitut d.d., Rovšnikova 7, 1210 Ljubljana, Slovenia.

e-mail: [tadej.kanduc@student.fmf.uni-lj.si](mailto:tadej.kanduc@student.fmf.uni-lj.si)

MONDAY, 9:50–10:15

## Geometric Hermite interpolation by rational Bézier spatial curves

Gašper Jaklič, Jernej Kozak, Marjetka Krajnc, Vito Vitrih and Emil Žagar

**Abstract.** Geometric interpolation by parametric polynomial curves became one of the standard techniques for interpolation of geometric data. A natural generalization leads to rational geometric interpolation schemes. The aim of this talk is to present a general framework for Hermite geometric interpolation by rational Bézier spatial curves. In particular, the cubic  $G^2$  and quartic  $G^3$  interpolation are analyzed in detail. Systems of nonlinear equations are derived and the analysis of the existence of admissible solutions is studied. For the cubic case the solution is obtained in a closed form and geometric conditions on its existence are given. The quartic case transforms into solving a univariate quartic equation. The asymptotic analysis is done.

GAŠPER JAKLIČ, FMF and IMFM, University of Ljubljana, PINT, University of Primorska, Jadranska 19, SI-1000 Ljubljana, Slovenia.

e-mail: [gasper.jaklic@fmf.uni-lj.si](mailto:gasper.jaklic@fmf.uni-lj.si)

JERNEJ KOZAK, FMF and IMFM, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia.

e-mail: [jernej.kozak@fmf.uni-lj.si](mailto:jernej.kozak@fmf.uni-lj.si)

MARJETKA KRAJNC, FMF and IMFM, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia.

e-mail: [marjetka.krajnc@fmf.uni-lj.si](mailto:marjetka.krajnc@fmf.uni-lj.si)

VITO VITRIH, FAMNIT and PINT, University of Primorska, Muzejski trg 2, SI-6000 Koper, Slovenia.

e-mail: [vito.vitrih@upr.si](mailto:vito.vitrih@upr.si)

EMIL ŽAGAR, FMF and IMFM, University of Ljubljana, Jadranska 19, SI-1000  
Ljubljana, Slovenia.  
e-mail: [emil.zagar@fmf.uni-lj.si](mailto:emil.zagar@fmf.uni-lj.si)

MONDAY, 16:40–17:05

## Accurate eigenvalue decomposition of arrowhead matrices and applications

Nevena Jakovčević Stor

**Abstract.** We present a new algorithm for solving an eigenvalue problem of real symmetric arrowhead matrix. Under certain conditions the algorithm computes all eigenvalues and all components of the corresponding eigenvectors with high relative accuracy in  $O(n^2)$  operations. The algorithm is based on shift-and-invert technique and limited use of double precision arithmetic when necessary. Each eigenvalue and the corresponding eigenvector can be computed separately, which makes the algorithm adaptable for parallel computing.

We also present perturbation theory, applications to Hermitian arrowhead matrices and symmetric tridiagonal matrices, and numerical examples.

NEVENA JAKOVČEVIĆ STOR, University of Split, Faculty of Electrical Engineering,  
Mechanical Engineering and Naval Architecture, R. Boškovića 32, 21000 Split,  
Croatia.  
e-mail: [nevena@fesb.hr](mailto:nevena@fesb.hr)

FRIDAY, 15:10–15:35

## The relation of Connected Set Cover and Group Steiner Tree

Slobodan Jelić and Domagoj Matijević

**Abstract.** Let  $U$  be the universe of elements which have to be covered,  $\mathcal{S}$  family of subsets of  $U$  and  $G = (\mathcal{S}, E)$  connected graph on vertex set  $\mathcal{S}$ . We say that subfamily  $\mathcal{R} \subseteq \mathcal{S}$  is *connected set cover* (CSC) if every  $u \in U$  is covered by at least one set from  $\mathcal{R}$  and subgraph  $G[\mathcal{R}]$  induced by  $\mathcal{R}$  is connected. The problem is to find connected set cover with respect to  $(U, \mathcal{S}, G)$  with minimum number of sets

(vertices). On the other hand, suppose that we are given a graph  $G$  with edge weight function  $w : E(G) \rightarrow \mathbb{R}^+$  and family of subsets of vertices  $\mathcal{G} = \{g_1, g_2, \dots, g_k\}$ ,  $g_i \subset V$  which will be called groups. The well known and well studied *Group Steiner Tree* (GST) is to find a subtree  $T$  that minimizes the weight function  $\sum_{e \in E(T)} w(e)$  such that  $V(T) \cap g_i \neq \emptyset$  for all  $i \in \{1, \dots, k\}$ . We showed in our work that CSC is equivalent to the variant of GST when all edge weights equal to 1. Hence, all algorithms for GST immediately apply for CSC problem as well. As a result, we obtain an approximation algorithm for CSC problem with approximation ratio  $O(\log^2 m \log \log m \log n)$  where  $n$  is the size of universe  $U$  and  $m$  is the size of family  $\mathcal{S}$ . This is the first polylogarithmic approximation algorithm for CSC problem.

Natural generalization of CSC problem is to associate the nonnegative weight function with sets in  $\mathcal{S}$ . Weighted CSC problem assumes finding of connected set cover that minimizes the total weight of subfamily  $\mathcal{R}$ . We showed that this problem can be solved by reduction to the fault-tolerant version of Group Steiner problems for which  $O(\sqrt{m} \log m)$  approximation algorithm is known. We also consider generalization of CSC problem where each element  $u$  from universe has requirement  $r_u$  on number of sets covering element  $u$  associated. We showed the reduction of this problem to the variant of GST problem with requirements associated to the groups for which  $O(\log^2 m \log \log m \log(R \cdot n))$  approximation algorithm is known, where  $R$  denotes the largest requirement.

SLOBODAN JELIĆ, University of Josip Juraj Strossmayer, Department of mathematics, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia.

e-mail: [sjelic@mathos.hr](mailto:sjelic@mathos.hr)

<http://www.mathos.hr/~sjelic>

DOMAGOJ MATIJEVIĆ, University of Josip Juraj Strossmayer, Department of mathematics, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia.

e-mail: [domagoj@mathos.hr](mailto:domagoj@mathos.hr)

<http://www.mathos.hr/~domagoj>

FRIDAY, 11:50–12:15

## Uniform asymptotic stability of dynamic equations with multiple delays

Maja Karaga

**Abstract.** Many real life phenomena (problems in physics, biology, medicine, economy, engineering) are modeled using delay differential equations. Often, to obtain more realistic model, we use equations with multiple delays. The theory of time scales unifies continuous and discrete analysis, differential and difference equations into dynamic equations. It can model systems which display combination of discrete and continuous dynamics or discrete dynamics with nonuniform stepsize. In this talk some new results concerning uniform and uniform asymptotic stability of dynamic equations with multiple delays will be given.

MAJA KARAGA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [karaga@math.hr](mailto:karaga@math.hr)

<http://www.math.hr/~karaga>

THURSDAY, 18:15–18:40

## Perturbed soliton like excitations for dipolar interaction in the ferromagnetic spin systems

Louis Kavitha, Narayanan Akila, Somasundram Bhuvaneshwari and Dhanaraj Gopi

**Abstract.** A realistic theoretical model of low dimensional magnetic system must include the exchange interaction of dipolar interaction [1]. Despite its smallness the long range or short range dipolar interaction plays a pivotal role in one dimensional ferromagnetic system. The influence of soliton like excitations due to the dipolar interaction has been increasing at tremendous rate in magnetic recording media. To study the effect of dipolar interaction on soliton nature of an integrable higher order nonlinear Schrödinger equation

$$\begin{aligned}
 H = & -\frac{J}{2} \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - g\mu_B H_0 \sum_{\langle i,j \rangle} S_i^z - D \sum_{\langle i,j \rangle} (S_i^z)^2 - \frac{1}{2} J\alpha \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j)^2 \\
 & - J_1 \sum_{\langle i,j \rangle} (\mathbf{S}_i \cdot \mathbf{S}_j) (\mathbf{S}_j \cdot \hat{k})^2 + \frac{1}{2} (g\mu_B)^2 \sum_{\langle i,j \rangle} \frac{\mathbf{S}_i \cdot \mathbf{S}_j}{r_{ij}^3} - \frac{3(\mathbf{S}_i \cdot \mathbf{r}_{ij})(\mathbf{S}_j \cdot \mathbf{r}_{ij})}{r_{ij}^5}.
 \end{aligned}$$

through the technique of multiple scale perturbation in the corresponding continuum limit of semiclassical approximation. The evolution equation for the amplitude and velocity of the perturbed soliton will also be constructed and will be solved using fourth order Runge-kutta method. Based on the localized nature of the soliton solution, we also attempt to solve the nonlinear evolution equation using some of the algebraic methods such as modified extended tangent hyperbolic function method [2], sine-cosine function method [3] and Jacobi elliptic function method [4] which has been developed in recent years coupled with symbolic computation. Among these methods, the sine cosine method provides an effective and direct algebraic method for solving nonlinear evolution equations. By using this method, novel types of soliton solutions have been found for the higher order nonlinear Schrödinger equation which governs the dynamics of the ferromagnetic spin system.

## References

- [1] J. P. NGUENANG AND T. C. KOFANÉ, *Solitary waves in spin chains with biquadratic exchange and dipole dipole interactions*, Phys. Scr., **55** (1997), 367–377.
- [2] S. A. EL-WAKIL AND M. A. ABDU, *New exact travelling wave solutions using modified extended tanh function method*, Chaos Solitons Fractals, **31** (2007), 840–852.
- [3] L. KAVITHA, P. SATHISH KUMAR, T. NATHIYAA AND D. GOPI, *Cusp-like singular soliton solutions of Jaulent–Miodek equation using symbolic computation*, Phys. Scr., **79** (2009), 035403.
- [4] E. YOMBA, *The extended Fan’s sub-equation method and its application to KdV–mKdV BKK and variant Boussinesq equation*, Phys. Lett., A **336** (2005), 463–476.

LOUIS KAVITHA, Department of Physics, Periyar University, Salem–636 011, India.  
e-mail: kavithalouis@yahoo.com

NARAYANAN AKILA, Department of Physics, Periyar University, Salem–636 011, India.  
e-mail: agilanarayanan@gmail.com

SOMASUNDRAM BHUVANESWARI, Department of Physics, Periyar University, Salem–636 011, India.  
e-mail: bhuvi\_physics@yahoo.co.in

DHANARAJ GOPI, Department of Chemistry, Periyar University, Salem–636 011,  
India.

e-mail: dhanaraj\_gopi@yahoo.com

FRIDAY, 14:45–15:10

## On transient queue-size distribution in the batch arrival system with $N$ -policy and setup times

Wojciech M. Kempa

**Abstract.** Queueing systems with a temporarily unavailable server occur in telecommunication and computer networks, manufacturing etc. In practice time duration of the vacation period can depend on the arrival process. An example of such service policy is the so called  $N$ -policy in that each time when the system empties the server waits until exactly  $N$  customers are accumulated, and next starts the service. A kind of extension of the classical  $N$ -policy model is a system with setup times, in that the server needs a random time for the activation after the vacation.

In the talk we study the transient queue-size distribution in the infinite-buffer queueing system with  $N$ -policy and setup times, in that customers arrive in batches according to a Poisson process and are served individually. Such a characteristic was investigated in [1] for the general-type system without vacation. Other transient characteristics of systems with different-type vacations were studied in [2] and [3].

The representation for the Laplace transform of the queue-size distribution at time  $t$  is obtained in terms of transforms of “input” distributions of the system and components of certain Wiener–Hopf factorization identity related to them. The approach consists of two main steps: firstly we consider a simplified system that starts working being empty and waits for customers, and next we obtain the general results using the renewal-theory approach.

### References

- [1] M. BRATIICHUK, W. M. KEMPA, *Explicit formulae for the queue length distribution of batch arrival systems*, Stoch. Models, **20**, no. 4, (2004), 457–472.
- [2] W. M. KEMPA, *GI/G/1/∞ batch arrival queueing system with a single exponential vacation*, Math. Methods Oper. Res., **69**, no. 1, (2009), 81–97.
- [3] —, *Some new results for departure process in the  $M^X/G/1$  queueing system with a single vacation and exhaustive service*, Stoch. Anal. Appl., **28**, no. 1 (2010), 26–43.

WOJCIECH M. KEMPA, Silesian University of Technology, Institute of Mathematics,  
ul. Kaszubska 23, 44-100 Gliwice, Poland.  
e-mail: wojciech.kempa@polsl.pl

MONDAY, 10:15-10:40

## Pythagorean-hodograph cycloidal curves

Jernej Kozak, Marjeta Krajnc, Mladen Rogina and Vito Vitrih

**Abstract.** Polynomial Pythagorean-hodograph (PH) curves play significant role in the theory as well as in practical applications of polynomial curves. They are characterized by the property that the Euclidean norm of their hodograph, called the parametric speed, is also a polynomial. As a consequence, they have rational tangent, rational offsets, polynomial arc-length, etc. These properties are of a particular importance in developing algorithms that guide CNC (computerized numerical control) machines. As an example, suppose that a part of the machine should move at a given constant distance from the manufactured shape. A rational offset reduces this task to elementary numerical operations in algorithms. CNC machines in general are capable of linear and circular motions, and the programming is usually done in either G-code or M-code describing such motions. But the instruction set is usually extended by additional elementary operations such as sine or cosine evaluation (e.g., G107, G108).

This property makes the use of non-polynomial parametric curves attractive from the practical standpoint as well as from the more general mathematical one. In particular, it may be required that parametric curve class selected should reproduce exactly geometric objects that appear often in practice, such as segments, circular arcs, cycloids, helices, etc. With CNC application in mind, it makes sense to extend the PH property from polynomial parametric curves to the more general ones.

In this talk, Pythagorean-hodograph cycloidal curves as an extension of PH cubics will be introduced. Their properties will be examined and a constructive geometric characterization will be established. Further, PHC curves will be applied in the Hermite interpolation and the closed form solutions will be determined. The asymptotic approximation order analysis will also be presented, which clearly indicates which interpolatory curve solution should be selected in practice.

JERNEJ KOZAK, FMF and IMFM, University of Ljubljana, Jadranska 19, SI-1000  
Ljubljana, Slovenia.  
e-mail: jernej.kozak@fmf.uni-lj.si



MARJETKA KRAJNC, FMF and IMFM, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia.

e-mail: [marjetka.krajnc@fmf.uni-lj.si](mailto:marjetka.krajnc@fmf.uni-lj.si)

MLADEN ROGINA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [rogina@math.hr](mailto:rogina@math.hr)

<http://www.math.hr/~rogina>

VITO VITRIH, FAMNIT and PINT, University of Primorska, Glagoljaška 8, SI-6000 Koper, Slovenia.

e-mail: [vito.vitrih@upr.si](mailto:vito.vitrih@upr.si)

THURSDAY, 12:15–12:40

## Quasilinear elliptic equations with positive exponent on the gradient

Jadranka Kraljević and Darko Žubrinić

**Abstract.** We study problems of existence, nonexistence and regularity of positive, spherically symmetric solutions of next quasilinear elliptic equations involving  $p$ -Laplacian, with arbitrary positive growth rate  $e_0$  on the gradient on the right-hand side:

$$\begin{cases} -\Delta_p u = \tilde{g}_0 |x|^m + \tilde{f}_0 |\nabla u|^{e_0} & \text{in } B \setminus \{0\}, \\ u = 0 & \text{on } \partial B, \\ u(x) \text{ spherically symmetric and decreasing,} \end{cases}$$

Here  $B$  is an open ball of radius  $R$  centered at the origin in  $\mathbb{R}^N$ ,  $1 < p < \infty$ ,  $\tilde{f}_0, \tilde{g}_0$  are positive real numbers, and  $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$  is  $p$ -Laplacian.

We show that  $e_0 = p - 1$  is the critical exponent: for  $e_0 < p - 1$  there exists a strong solution for any choice of positive coefficients  $\tilde{f}_0, \tilde{g}_0$ , while for  $e_0 > p - 1$  we have existence-nonexistence splitting of coefficients. The elliptic problem is studied by relating it to the corresponding singular ODE of the first order. We study regularity of strong and weak solutions and we proved a result that gives the conditions for the strong solution of the quasilinear elliptic problem to be a classical solution.

JADRANKA KRALJEVIĆ, University of Zagreb, Faculty of Economics, Department of Mathematics, Kennedyev trg 6, 10000 Zagreb, Croatia.

e-mail: jkraljevic@efzg.hr

DARKO ŽUBRINIĆ, University of Zagreb, Faculty of Electrical Engineering, Department of Applied Mathematics, Unska 3, 10000 Zagreb, Croatia.  
e-mail: darko.zubrinic@gmail.com

TUESDAY, 10:15–10:40

## Problems connected to smoothness of surfaces on triangular structures

Arne Lakså

**Abstract.** In the construction of continuous and smooth surfaces that are composed of multiple triangular surfaces, it is often used triangular Bezier surfaces. One problem is that the requirements for smoothness makes such a composition very stiff. That is, the possibilities of shaping is limited.

Another way to construct such surfaces is to blend triangular parts of three surfaces, called local patches, into one triangular patch.

Two triangular patches share an edge, and a set of patches share a vertex. All the patches that are sharing a vertex has a local patch connected to this vertex. All these local patches are triangular subsurfaces of a common surface and they are also sharing edges.

This structure makes that changes in a vertex only affects the closest neighborhood of this vertex. The structure of such a surface, which consists of a coherent set of triangular surfaces, are as follows:

- There is a set of vertices. They are ordered such that we know which vertices that are neighbors to a vertex. The neighboring vertices are connected by an edge.
- Three connected vertices defines a triangular surface.
- To each vertex there is a surface (patch) covering the first neighborhood of the vertex. It is that a patch connected to a vertex also must cover all neighboring vertices to this vertex.
- Each of the patches associated with a vertex splits into triangular subsurfaces covering the domain of a triangular surface in the main structure.

Three triangular subsurfaces, one connected to each vertex, is then blended into one triangular patch. The blending function must have the following properties,

- (i) they must fulfill partition of unity, i.e. sum up to one,
- (ii) the value must be one at their respective vertex. That is the vertex they are connected to,

(iii) all directional derivatives must be zero at their respective vertex and at the opposite edge.

The resulting surface is smooth in all vertices, but in general only continuous over the edges. We will discuss problems connected to the smoothness of the edges, and also look at possible ways to get smooth edges.

ARNE LAKSÅ, Faculty of Technology, Narvik University College, Lodve Langes gt.  
2, N-8505 Narvik, Norway.  
e-mail: ala@hin.no

THURSDAY, 16:15–16:40

## Application of parabolic H-measures to the vibrating plate equation

Martin Lazar and Nenad Antonić

**Abstract.** The theory of a new variant of H-measures, called the parabolic H-measures, has been recently developed, aimed at the study of non-hyperbolic problems. Here we present its application to the vibrating plate equation, based on the localisation and propagation property of the new variant. Relation between the microlocal energy density and initial data, as well as directions of propagation and the equipartition of energy will be discussed.

MARTIN LAZAR, University of Dubrovnik, Ćira Carića 4, 20000 Dubrovnik, Croatia.  
e-mail: mlazar@unidu.hr

NENAD ANTONIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: nenad@math.hr

FRIDAY, 10:15–10:40

## Concepts related to residence time in transport models

Nedžad Limić, Sunčana Geček, and Tarzan Legović

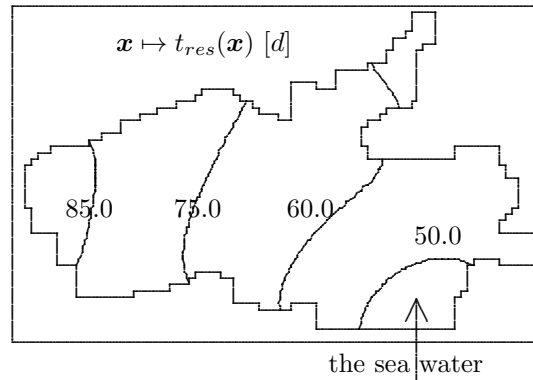
**Abstract.** A transport model of substance in a basin by waterflow is standardly defined by an initial value problem (IVP) for a parabolic differential equation of the second order. There exists an equivalent definition in terms of a stochastic process. The two equivalent definitions are called the approach in terms of a concentration field and a pathwise approach. The stochastic process  $X(\cdot)$  of the pathwise approach is related to the diffusion for which the forward Kolmogorov equation and the IVP for the concentration field have the same forms. We consider a problem of the transport by velocity field and sedimentation. Thus the substance can exit the basin either through a part of boundary  $S_{bt}$  with the adjacent sea or by sedimentation into the bottom  $S_{bt}$ .

The process  $X(\cdot)$  has continuous sample paths  $t \mapsto \boldsymbol{\omega}(t)$  and each sample paths ends at a point of  $S_{op}$  or  $S_{bt}$ , i.e.,  $\boldsymbol{\omega}(t_\omega) \in S_{op} \cup S_{bt}$  for some exit time  $t_\omega$ . The times  $t_\omega$  are outcomes of the exit time  $\theta$ . One of the most important characteristics of the transport is the expectation  $\mathbf{E}[\theta]$  that is called the residence time of substance. The set of possible exits  $S_{op} \cup S_{bt}$  is denoted by  $S$ . We have shown the existence of the representation:

$$\mathbf{E}[\theta] = \int_S p(\mathbf{x}) t_{\text{res}}(\mathbf{x}) d\mathbf{x},$$

where  $p, t_{\text{res}}$  are functions on  $S_{op} \cup S_{bt}$  with the following interpretation. The function  $p$  is the probability density of exit at  $\mathbf{x} \in S_{op} \cup S_{bt}$  and  $t_{\text{res}}$  is the conditional residence time, assuming that the exit has occurred at  $\mathbf{x}$ . For the case of a steady state transport the functions  $p, t$  are solutions to some boundary value problems and can be easily computed. Results are applied to two problems in situ.

**Flush out of lead from the water body of the Punat bay:** The Punat bay on the island Krk in the northern Adriatic sea has the area equal to 2.4 km<sup>2</sup> and an average depth equal to 3.2 m. The bay communicates with the rest of the sea through a narrow gate  $S_{op}$  of 200 m width. A large number of boats is anchored in a marina, causing a permanent pollution of water by heavy metals, such as lead, that is attributed to the disintegrated paints of anchored boats in the marina. Some of lead in the water column is removed by tidal current, but the most of lead is



removed by chemical processes resulting in the sedimentation into the bottom  $S_{bt}$ . The probabilities of exits and the corresponding conditional residence times for the Punat bay are computed:  $p_{op} = 0.18$   $t_{op} = 7.36$  [d]  $p_{bt} = 0.82$   $t_{bt} = 7.89$  [d].

**Entrainment of the sea water in the stratified flow of the Prokljan lake:**

The Prokljan lake is a part of the Krka river estuary with its mouth in the Adriatic sea. The river flow is stratified, the flow in the surface layer has the direction towards the estuary mouth, while the flow in the bottom layer has the opposite direction. Due to the shear the sea water is permanently entraining from the bottom layer into the surface layer. We computed and illustrated (Figure) the function  $x \mapsto t_{res}(x)$  of the entrained water from the bottom layer into the upper layer of the Prokljan lake.

NEDŽAD LIMIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [nlimic@math.hr](mailto:nlimic@math.hr)

SUNČANA GEČEK, Ruđer Bošković Institute, Division for Marine and Environmental Research, POB 180, Bijenička cesta 54, 10000 Zagreb, Croatia.  
e-mail: [suncana@irb.hr](mailto:suncana@irb.hr)

TARZAN LEGOVIĆ, Ruđer Bošković Institute, Division for Marine and Environmental Research, POB 180, Bijenička cesta 54, 10000 Zagreb, Croatia.  
e-mail: [legovic@irb.hr](mailto:legovic@irb.hr)

FRIDAY, 9:50–10:15

## Dimension reduction for models of transport with sedimentation

Nedžad Limić and Maja Starčević

**Abstract.** In the water management with a dissolved pollutant the estimation of the concentration field of pollutant and associated parameters are usually obtained from the transport model of substance. The model is acceptable if the estimated parameters are reliable far from the source of pollutant. One of the basic indicators of the acceptability of model is the mass balance equation of substance. If  $S$  is a bounded domain around the source of substance, then the values of fluxes of exiting substance through different parts of the boundary  $\partial S$  must agree with available data. It is expected that the agreement with data is better, farther the boundary  $\partial S$  is from the source.

In many cases an estimation of parameters is not necessary to carry out with the original  $3D$ -model. Reliable estimations can be obtained by using  $2D$  or  $1D$  approximations. For this purpose we analyzed the basic problem of dimension reduction of a  $3D$ -model into a  $2D$ -model. The analysis is performed for the original and reduced models that describe a steady state transport of substance with the sedimentation. We justify the criterion of the acceptability for the reduced model.

NEDŽAD LIMIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: nlimic@math.hr

MAJA STARČEVIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: mstarcev@math.hr

TUESDAY, 15:50–16:15

## Componentwise backward error bounds of relaxed forms of fast Bunch–Parlett and bounded Bunch–Kaufman algorithm

Martina Manhart

**Abstract.** A nonsingular symmetric matrix  $A$  can be factored as

$$PAP^t = LDL^t,$$

where  $P$  is a permutation matrix,  $L$  is unit lower triangular, and  $D$  is block diagonal matrix with diagonal blocks of order 1 or 2.

For two families of pivoting strategies, so-called relaxed forms of the bounded Bunch-Kaufman and the fast Bunch-Parlett strategy, we derive the componentwise error bounds.

MARTINA MANHART, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [martina.manhart@math.hr](mailto:martina.manhart@math.hr)

THURSDAY, 16:40–17:05

## Derivation of a model of the prestressed elastic string

Maroje Marohnić and Josip Tambača

**Abstract.** We derive an one-dimensional model for displacement and torsion of the elastic string starting from the cylindrical three-dimensional prestressed elastic body with small thickness  $\varepsilon$ . We assume that the stress in the body is due to a prior elastic deformation  $\varphi$  of isotropic, homogenous, elastic body for which constitutive equations are known. By formal asymptotic expansion we deduce the scaling of forces. Then we prove that the solutions of three-dimensional problems converge to the solutions of the string equations via singular perturbation techniques. Coefficients of the string model depend on three-dimensional elasticity coefficients and tension produced by deformation  $\varphi$ .

MAROJE MAROHNIC, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [maroje.marohnic@math.hr](mailto:maroje.marohnic@math.hr)

JOSIP TAMBAČA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.

e-mail: [tambaca@math.hr](mailto:tambaca@math.hr)

TUESDAY, 17:50–18:15

## Mathematical models for electronic structures of low dimensional nanostructures and their numerical approximations: Quantum-continuum coupling

Roderick Melnik

**Abstract.** Low-dimensional semiconductor nanostructures (LDSNs) are receiving increasing attention as key components of semiconductor lasers, semiconductor optical amplifiers, and other optoelectronic devices. They are challenging objects to study from a mathematical point of view, and the number of their practical applications continues to grow. Quantum dots (QDs) – LDSNs in which the motion of electrons is confined from all three spatial dimensions – can be used as biological tags in clinical research, DNA analysis, as well as in other bio-technological applications, while the idea of using a spin confined to a QD as a qubit promises imminent breakthrough in quantum information processing.

Despite a wide range of current and potential applications, properties of QDs are still frequently analyzed with simplified mathematical models, incapable to account correctly for many effects that are coming from other than quantum mechanical scales, including the continuum mechanics scales (e.g., strain, piezoelectric coupling, thermal and other important effects).

In this contribution we will first provide a survey of the existing mathematical models for electronic bandstructure calculations focusing on coupled and nonlinear effects. We will show how to construct such models where the coupling between quantum and continuum mechanical models is essential. We will then focus on the new coupled mathematical models capable to demonstrate that *coupled* electro-mechanical effects can lead to pronounced contributions in electronic bandstructure calculations of LDSNs such as quantum dots, wires, and even wells (see references at the webpage below).



Finally, by using the fully coupled mathematical model of thermoelectroelasticity, we will show how to build on these previous results in developing a new theory, while analyzing the influence of the mechanical, electric, and thermal effects on optoelectronic properties of quantum dots. Results will be reported for two different groups of semiconductors. Details of numerical approximations will also be discussed on several examples to illustrate the theory.

RODERICK MELNIK, Department of Mathematics and M<sup>2</sup>NeT Laboratory, Wilfrid Laurier University, 75 University Avenue West, Waterloo, ON, Canada.  
e-mail: [rmelnik@wlu.ca](mailto:rmelnik@wlu.ca)  
<http://www.m2netlab.wlu.ca>

THURSDAY, 11:25–11:50

## On the evolutionary free piston problem for the Stokes equations with slip boundary conditions

Boris Muha and Zvonimir Tutek

**Abstract.** The problem considered is the evolutionary Stokes flow through a system of two pipes in a gravity field; inside vertical pipe there is a heavy rigid body (piston) which can freely move along the vertical pipe. Although the Stokes equations are linear, the full problem in unknowns fluid pressure, fluid velocity and motion of the piston, is nonlinear. We formulate corresponding initial-boundary value problem and prove the existence result. We compare obtained results with corresponding results for potential ideal fluid flow.

BORIS MUHA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [borism@math.hr](mailto:borism@math.hr)  
<http://web.math.hr/~borism/>

ZVONIMIR TUTEK, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [tutek@math.hr](mailto:tutek@math.hr)

MONDAY, 12:15–12:40

## Generalized eigenvalue problems with prespecified eigenvalues

Ivica Nakić

**Abstract.** We consider the distance from a pencil (square or rectangular) to the nearest pencil in 2-norm that has a subset of specified eigenvalues. A singular value optimization characterization is derived for this problem. This yields a singular value formula to determine the nearest pencil whose eigenvalues lie in a region in the complex plane, which for instance makes the numerical computation of the nearest stable descriptor system possible. Secondly this partially solves the problem posed in [1] regarding the distance from a rectangular pencil to the nearest pencil with a full set of eigenvalues. The derived singular value optimization problems are solved by means of BFGS and Lipschitz-based global optimization algorithms.

This is a joint work with Daniel Kressner, Emre Mengi and Ninoslav Truhar.

### References

- [1] G. H. GOLUB, G. BOUTRY, M. ELAD AND P. MILANFAR, *The generalized eigenvalue problem for nonsquare pencils using a minimal perturbation approach*, SIAM J. Matrix Anal. Appl., **27** (2005), 582–600.

IVICA NAKIĆ, Faculty of Science, Department of Mathematics, University of Zagreb,  
Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: nakic@math.hr

MONDAY, 17:25–17:50

## Parallel hybrid CPU–GPU full-block Jacobi algorithm for hyperbolic SVD

Vedran Novaković, Saša Stanko, and Sanja Singer

**Abstract.** In the recent years, graphics processing units (GPUs) have become an important platform for scientific computing in general, with algorithms of numerical linear algebra being no exception. Various diagonalization methods have been tailored to the specific execution paradigm of the GPUs. Among them, the Jacobi algorithm stands out for its simplicity in both CPU and GPU parallelization, and for high relative accuracy, when the matrices permit.

The family of CPU and GPU parallel Jacobi-like hyperbolic SVD algorithms has already been developed. The CPU algorithms exploit the blocking techniques for both parallelization and efficient cache usage. In this talk the blend of two architectures will be presented. More specifically, the hyperbolic SVD for many-core shared memory machines will be described, which utilizes the GPU algorithm at the block level.

VEDRAN NOVAKOVIĆ, Faculty of Mechanical Engineering and Naval Architecture,  
University of Zagreb, Ivana Lučića 5, 10000 Zagreb, Croatia.  
e-mail: [venovako@fsb.hr](mailto:venovako@fsb.hr)  
<http://web.math.hr/~venovako>

SAŠA STANKO, M.S. student, Faculty of Science, Department of Mathematics, Uni-  
versity of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [sstanko@student.math.hr](mailto:sstanko@student.math.hr)

SANJA SINGER, Faculty of Mechanical Engineering and Naval Architecture, Uni-  
versity of Zagreb, Ivana Lučića 5, 10000 Zagreb, Croatia.  
e-mail: [ssinger@fsb.hr](mailto:ssinger@fsb.hr)  
<http://www.fsb.hr/ssinger>

TUESDAY, 16:15–16:40

## Robust singular value decomposition

Takeshi Ogita

**Abstract.** In this talk, an algorithm for accurately computing singular values of matrices is proposed. The proposed algorithm can treat the case where the matrices are extremely ill-conditioned, i.e., the condition numbers of the matrices are allowed to go far beyond the bounds of base precision such as IEEE standard 754 double precision (binary64).

The algorithm requires a standard (backward stable) numerical algorithm for singular value decomposition (SVD), which is commonly implemented in several numerical libraries such as LAPACK, and an accurate algorithm for matrix multiplication. Therefore, the algorithm is scalable and has high program portability. The algorithm is also adaptive, i.e., the algorithm adaptively changes the computational precision corresponding to the ill-conditionedness of the given problem which is determined by the condition number of the input matrix. It has the same structure as the previous works on inversion of extremely ill-conditioned matrices by Rump [4] and robust matrix factorizations by the author [1, 2]. Recently, Ozaki et al. [3] have significantly accelerated the algorithm for accurate matrix multiplication using BLAS level-3 operations. The proposed algorithm for SVD can significantly benefit from it in terms of computational speed. Moreover, based on the algorithm for SVD, an algorithm for computing accurate eigenvalues of symmetric matrices is also proposed. Numerical results are presented, which illustrate the performance of the proposed algorithms.

### References

- [1] T. OGITA, *Accurate matrix factorization: Inverse LU and inverse QR factorizations*, SIAM J. Matrix Anal. Appl., **31** no. 5 (2010), 2477–2497.
- [2] T. OGITA AND S. OISHI, *Accurate and robust inverse Cholesky factorization*, Nonlinear Theory and Its Applications, IEICE, accepted for publication.
- [3] K. OZAKI, T. OGITA AND S. OISHI, *Tight and efficient enclosure of matrix multiplication by using optimized BLAS*, Numer. Linear Algebra Appl., **18**, no. 2 (2011), 237–248.
- [4] S. M. RUMP, *Inversion of extremely ill-conditioned matrices in floating-point*, Japan J. Indust. Appl. Math., **26**, (2009), 249–277.

TAKESHI OGITA, Division of Mathematical Sciences, School of Arts and Sciences,  
Tokyo Woman's Christian University, 2-6-1 Zempukuji, Suginami-ku, Tokyo  
167-8585, Japan.  
e-mail: ogita@lab.twcu.ac.jp

TUESDAY, 16:40–17:05

## General matrix multiplication with guaranteed accuracy

Katsuhisa Ozaki and Takeshi Ogita

**Abstract.** Matrix multiplication is one of basic computations in numerical linear algebra. It is supported in BLAS (Basic Linear Algebra Subprograms). Let  $\mathbb{F}$  be a set of floating-point numbers. In BLAS, there are so-called ‘gemm’ functions, here gemm means ‘general matrix multiply’ such that  $\tilde{C} \leftarrow \alpha AB + \beta C$ . In the expression,  $A$ ,  $B$  and  $C$  denote  $A \in \mathbb{F}^{m \times n}$ ,  $B \in \mathbb{F}^{n \times p}$  and  $C \in \mathbb{F}^{m \times p}$ , respectively. The constants  $\alpha$  and  $\beta$  are represented by floating-point numbers, respectively. The expression is evaluated by floating-point arithmetic defined by the IEEE Standard for Floating-Point Arithmetic (IEEE 754). Since precision of floating-point numbers is finite, so that rounding errors accumulate. In the worst case, computed results may be inaccurate. From an a priori error analysis [1], we have the following error bound for  $\tilde{C}_1 \leftarrow AB$

$$|\tilde{C}_1 - AB| \leq \frac{n\mathbf{u}}{1 - n\mathbf{u}} |A||B|,$$

where  $\mathbf{u}$  is the roundoff unit ( $\mathbf{u} = 2^{-53}$  in binary64).

We propose an algorithm which outputs an accurate result for  $\tilde{C} \leftarrow \alpha AB + \beta C$  based on [2]. The algorithm in [2] depends on routines in BLAS. There are several BLAS implementations which are well optimized for specified architectures, for example, GotoBLAS2, Intel Math Kernel Library, ATLAS and so forth. Our algorithm uses gemm in BLAS. Therefore, our algorithm benefits from high performance BLAS. An error bound for a computed result by our algorithm satisfies

$$|\alpha AB + \beta C - \tilde{C}| \leq 2\mathbf{u}|\alpha AB + \beta C|.$$

### References

- [1] N. J. HIGHAM, *Accuracy and Stability of Numerical Algorithms*, SIAM, Philadelphia, 1996.
- [2] K. OZAKI, T. OGITA AND S. OISHI, *Tight and efficient enclosure of matrix multiplication by using optimized BLAS*, Numer. Linear Algebra Appl., **18**, no. 2 (2011), 237–248.

KATSUHISA OZAKI, Department of Mathematical Sciences, Shibaura Institute of Technology, Fukasaku 307, Minama-ku, Saitama-shi, Saitama 337-8570, Japan.  
e-mail: [ozaki@sic.shibaura-it.ac.jp](mailto:ozaki@sic.shibaura-it.ac.jp)

TAKESHI OGITA, Division of Mathematical Sciences, Tokyo Woman's Christian University, 2-6-1 Zempukuji, Suginami-ku, Tokyo 167-8585, Japan.  
e-mail: [ogita@lab.twcu.ac.jp](mailto:ogita@lab.twcu.ac.jp)

THURSDAY, 9:50–10:15

## Effective flow of micropolar fluid through a thin or long pipe

Igor Pažanin

**Abstract.** The Navier–Stokes model of classical hydrodynamics has one important limitation: it does not take into account the microstructure of the fluid. One of the best-established theories of fluids with microstructure is the theory of micropolar fluids, introduced by Eringen. The mathematical model of micropolar fluid enables us to study many physical phenomena arising from the local structure and micro-motions of the fluid particles. It describes the behavior of numerous real fluids (such as polymeric suspensions, liquid crystals, muddy fluids, animal blood, etc.) better than the classical Navier–Stokes model, especially when the characteristic dimensions of the flow (e.g., diameter of the pipe) become small.

The aim of this talk is to present recent results about asymptotic approximation of the micropolar fluid flow through a very thin (or a very long) pipe with circular cross-section. First, we consider an incompressible micropolar fluid flowing through an undeformed straight pipe and then extend our analysis to the case of curved pipe with an arbitrary (smooth) central line. The effective behavior of the flow is found via rigorous asymptotic analysis with respect to the small parameter, being the ratio between pipe's thickness and its length. In both cases, we derive the explicit formulae for the approximation showing the effects of the microstructure and pipe's distortion on the flow. We prove the corresponding error estimates rigorously justifying the obtained asymptotic models.

IGOR PAŽANIN, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [pazanin@math.hr](mailto:pazanin@math.hr)  
<http://web.math.hr/~pazanin>

TUESDAY, 17:25–17:50

## Center-based $l_1$ -clustering method

Kristian Sabo, Rudolf Scitovski and Ivan Vazler

**Abstract.** We consider a clustering problem for data-points set  $\mathcal{A} = \{a^i \in \mathbb{R}^n : i = 1, \dots, m\}$  into  $k$  disjoint subsets  $\pi_1, \dots, \pi_k$ ,  $1 \leq k \leq m$ . If outliers are expected among the data, usage of the Least Absolute Deviations-optimality criterion is proposed.

In that case, the objective function does not have to be either convex or differentiable and generally it may have many local minima. Therefore, it becomes a complex global optimization problem.

An efficient method for searching stationary points is proposed and convergence of the corresponding iterative process is proved. Given is also a corresponding algorithm, which in only few steps gives a stationary point and corresponding partition.

The method is illustrated and visualized on the example of looking for an optimal partition with two clusters, where we check all stationary points of the corresponding minimizing functional. Also, the method is tested on the basis of large numbers of data points and clusters and compared with the method for solving center-based Least Squares – clustering problem.

KRISTIAN SABO, Department of Mathematics, University of Osijek, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia.  
e-mail: [ksabo@mathos.hr](mailto:ksabo@mathos.hr)  
<http://www.mathos.hr/~ksabo>

RUDOLF SCITOVSKI, Department of Mathematics, University of Osijek, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia.  
e-mail: [scitowsk@mathos.hr](mailto:scitowsk@mathos.hr)  
<http://www.mathos.hr/~scitowsk>

IVAN VAZLER, Department of Mathematics, University of Osijek, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia.  
e-mail: [ivazler@mathos.hr](mailto:ivazler@mathos.hr)  
<http://www.mathos.hr/~ivazler>

MONDAY, 16:15–16:40

## Jacobi-type algorithms for the Hamiltonian eigenvalue problem

Ivan Slapničar

**Abstract.** We present recent results on Jacobi-type algorithms for real Hamiltonian matrices. We describe both, real and complex algorithms. The algorithms use orthogonal (unitary) and non-orthogonal shear transformations. Convergence and accuracy properties of the algorithms are discussed.

IVAN SLAPNIČAR, University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, R. Boškovića 32, 21000 Split, Croatia.  
e-mail: [Ivan.Slapnicar@fesb.hr](mailto:Ivan.Slapnicar@fesb.hr)  
<http://www.fesb.hr/~slap>

MONDAY, 17:50–18:15

## One decomposition of hyperbolic unitary matrices

Vedran Šego

**Abstract.** Zakrajšek and Vidav have proposed a decomposition of an arbitrary orthogonal matrix  $Q = R\Delta$ , where  $R$  is a block-rotation and  $\Delta$  is a block-diagonal orthogonal matrix.

An extension of this result to the complex spaces equipped with an arbitrary hyperbolic scalar product will be shown, using the hyperbolic block-rotations introduced by Veselić.

Such decomposition can be used for speeding up hyperbolic versions of various algorithms based on block-rotations, such as block-Jacobi algorithms, block-QR factorizations, etc.

VEDRAN ŠEGO, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [vsego@math.hr](mailto:vsego@math.hr)



FRIDAY, 10:40–11:05

## Bacterial hydrocarbon uptake due to the oil spill in the Gulf of Mexico

David L. Valentine, Igor Mezić, Senka Maćešić, Nelida Črnjarić-Žic, Stefan Ivić, Patrick Hogan, Vladimir A. Fonoberov, Sophie Loire

**Abstract.** After the oil spill in the Gulf of Mexico Mezić et al. in [1] introduced a new mixing diagnostic based on the finite time theory for two dimensional incompressible fluid flows with arbitrary time dependence and applied it successfully on prediction of oil spill movements in the Gulf of Mexico. Kessler, Valentine et al. in [2] discovered a persistent oxygen anomaly in the deep Gulf of Mexico and suggested that a vigorous deepwater bacterial bloom was the cause of it. Now we present an ongoing research whose goal is to prove the scenario in [2] with the use of mathematical tools introduced in [1].

The Hybrid Coordinate Ocean Model (HYCOM) and Navy Coupled Ocean Data Assimilation (NCODA) system provide us with the velocity fields which we use to compute fluid particle trajectories in any two dimensional layer in the deep Gulf of Mexico. Particularly for the depths of interest, we compute the trajectories over the positions of measurements backwards, i.e., starting at the time of the measurements in [2] and ending at the time before the oil spill. Then, along the computed trajectories we integrate the equations for the growth of bacterial populations and for the evolution of chemical compounds. Actually, there are 26 primary chemical compounds (hydrocarbons), each of which is consumed by a separate bacterium species. Finally, from the obtained results we evaluate the oxygen depletion and compare it with the measured oxygen anomaly.

Naturally, the proposed approach will be a valuable model for bacterial hydrocarbon uptake due to oil spill in general.

### References

- [1] I. MEZIĆ, S. LOIRE, V. A. FONOBEROV, P. HOGAN, *A new mixing diagnostic and Gulf oil spill movement*, Science, **330** (2010), 486–489.
- [2] J. D. KESSLER, D. L. VALENTINE, ET AL., *A persistent oxygen anomaly reveals the fate of spilled methane in the deep Gulf of Mexico*, Science, 2011, DOI: 10.1126/science.1199697

DAVID L. VALENTINE, Department of Earth Science and Marine Science Institute,  
University of California, Santa Barbara CA 93106, USA.

e-mail: [valentine@geol.ucsb.edu](mailto:valentine@geol.ucsb.edu)

IGOR MEZIĆ, Center for Control, Dynamical Systems and Computation, and Department of Mechanical Engineering, University of California, Santa Barbara, CA 93106, USA.

e-mail: [mezic@engineering.ucsb.edu](mailto:mezic@engineering.ucsb.edu)

SENKA MAČEŠIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.

e-mail: [senka.macesic@riteh.hr](mailto:senka.macesic@riteh.hr)

NELIDA ČRNJARIĆ-ŽIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.

e-mail: [nelida.crnjaric@riteh.hr](mailto:nelida.crnjaric@riteh.hr)

STEFAN IVIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.

e-mail: [stefan.ivic@riteh.hr](mailto:stefan.ivic@riteh.hr)

PATRICK HOGAN, Naval Research Laboratory, Stennis Space Center, MS 39529, USA.

e-mail: [patrick.hogan@nrlssc.navy.mil](mailto:patrick.hogan@nrlssc.navy.mil)

VLADIMIR A. FONOBEROV, Aimdyn, Santa Barbara, CA 93101, USA.

e-mail: [vfonoberov@aymdin.com](mailto:vfonoberov@aymdin.com)

SOPHIE LOIRE, Center for Control, Dynamical Systems and Computation, and Department of Mechanical Engineering, University of California, Santa Barbara, CA 93106, USA.

e-mail: [sloire@engineering.ucsb.edu](mailto:sloire@engineering.ucsb.edu)

THURSDAY, 15:50–16:15

## Periodically wrinkled plate of the Föppl-von Kármán type

Igor Velčić

**Abstract.** Starting from 3D nonlinear elasticity we derive the model of periodically wrinkled plate. Föppl-von Kármán type means that we assume a specific relation between the thickness of the plate and the order of the energy of the plate. We derive the model by means of  $\Gamma$ -convergence.

IGOR VELČIĆ, Faculty of Electrical Engineering and Computing, University of Zagreb, Unska 3, 10000 Zagreb, Croatia.  
e-mail: igor.velcic@fer.hr

## LIST OF PARTICIPANTS

RAPHAEL ADENIYI, Department of Mathematics, Faculty of Science, University of Ilorin, 240003 Ilorin, Nigeria  
e-mail: [raphade@unilorin.edu.ng](mailto:raphade@unilorin.edu.ng)

IBRAHIM AGANOVIĆ, professor emeritus, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [aga@math.hr](mailto:aga@math.hr)

GRÉGOIRE ALLAIRE, Centre de Mathématiques Appliquées, École Polytechnique, Palaiseau, 91128 France  
e-mail: [gregoire.allaire@polytechnique.fr](mailto:gregoire.allaire@polytechnique.fr)  
page(s) 9

NENAD ANTONIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [nenad@math.hr](mailto:nenad@math.hr)  
page(s) 30, 41

SÁNDOR BARAN, Faculty of Informatics, University of Debrecen, Egyetem tér 1, H-4032 Debrecen, Hungary  
e-mail: [baran.sandor@inf.unideb.hu](mailto:baran.sandor@inf.unideb.hu)  
<http://www.inf.unideb.hu/~barans>  
page(s) 16

ANETT BEKÉNÉ RÁCZ, Faculty of Informatics, University of Debrecen, Egyetem tér 1, 4032 Debrecen, Hungary  
e-mail: [racz.anett@inf.unideb.hu](mailto:racz.anett@inf.unideb.hu)  
<http://www.inf.unideb.hu/valseg/dolgozok/anett.racz/anettracz.html>  
page(s) 17

MARIO BERLJAJA, B.Sc. student, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [mberljaf@student.math.hr](mailto:mberljaf@student.math.hr)  
page(s) 18

MARCEL BLIEM, University of Stuttgart, Paffenwaldring 57, 70569 Stuttgart, Germany  
e-mail: [bliem@mathematik.uni-stuttgart.de](mailto:bliem@mathematik.uni-stuttgart.de)

NELA BOSNER, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [nela@math.hr](mailto:nela@math.hr)  
<http://http://www.math.hr/~nela>  
page(s) 20

TINA BOSNER, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [tinab@math.hr](mailto:tinab@math.hr)  
<http://www.math.hr/~tinab>  
page(s) 21

ZVONIMIR BUJANOVIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [zbujanov@math.hr](mailto:zbujanov@math.hr)  
<http://web.math.hr/~zbujanov>  
page(s) 20, 21

BOJAN CRNKOVIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia  
e-mail: [bojan.crnkovic@riteh.hr](mailto:bojan.crnkovic@riteh.hr)  
page(s) 22

NELIDA ČRNJARIĆ-ŽIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia.  
e-mail: [nelida@riteh.hr](mailto:nelida@riteh.hr)  
page(s) 22, 55

ANDRIJANA ČURKOVIĆ, Faculty of Science, Department of Mathematics, University of Split, Teslina 12, 21000 Split, Croatia  
e-mail: [andrijana@pmfst.hr](mailto:andrijana@pmfst.hr)

IVAN DRAŽIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia  
e-mail: [idrazic@riteh.hr](mailto:idrazic@riteh.hr)  
page(s) 24

ENES DUVNJAKOVIĆ, Mathematics Department, Faculty of Natural Sciences, University of Tuzla, Univerzitetska ulica 1, 75000 Tuzla, Bosnia and Herzegovina  
e-mail: [enes.duvnjakovic@untz.ba](mailto:enes.duvnjakovic@untz.ba)  
page(s) 25

TOMISLAV FRATROVIĆ, Faculty of Traffic and Transport Sciences, University of Zagreb, ZUK Borongaj, Borongajska cesta 83A, pp. 170, Zagreb, Croatia  
e-mail: [tomislav.fratrovic@fpz.hr](mailto:tomislav.fratrovic@fpz.hr)

VAHIDIN HADŽIABDIĆ, Faculty of Mechanical Engineering, University of Sarajevo, Vilsonovo šetalište 9, 71000 Sarajevo, Bosnia and Herzegovina  
e-mail: [hadziabdic@mef.unsa.ba](mailto:hadziabdic@mef.unsa.ba)  
page(s) 26

VJERAN HARI, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [hari@math.hr](mailto:hari@math.hr)  
<http://www.math.hr/~hari>  
page(s) 27, 28

LANA HORVAT DMITROVIĆ, Department of Applied Mathematics, Faculty of Electrical Engineering and Computing, University of Zagreb, Unska 3, 10000 Zagreb, Croatia  
e-mail: [lane.horvat@fer.hr](mailto:lane.horvat@fer.hr)  
page(s) 29

IVAN IVEC, Gymnasium A. G. Matoša, Andrije Hebranga 26, 10430 Samobor, Croatia  
e-mail: [ivan.ivec@gmail.com](mailto:ivan.ivec@gmail.com)  
page(s) 30

GAŠPER JAKLIČ, FMF and IMFM, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia  
e-mail: [gasper.jaklic@fmf.uni-lj.si](mailto:gasper.jaklic@fmf.uni-lj.si)  
<http://www.fmf.uni-lj.si/~jaklicg>  
page(s) 30, 31, 32

NEVENA JAKOVČEVIĆ STOR, University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, R. Boškovića 32, 21000 Split, Croatia  
e-mail: [nevena@fesb.hr](mailto:nevena@fesb.hr)  
page(s) 33

SLOBODAN JELIĆ, University of Josip Juraj Strossmayer, Department of mathematics, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia  
e-mail: [sjelic@mathos.hr](mailto:sjelic@mathos.hr)  
<http://www.mathos.hr/~sjelic>  
page(s) 33

- TADEJ KANDUČ, Turboinštitut d.d., Rovšnikova 7, 1210 Ljubljana, Slovenia  
e-mail: [tadej.kanduc@student.fmf.uni-lj.si](mailto:tadej.kanduc@student.fmf.uni-lj.si)  
page(s) 30, 31
- MAJA KARAGA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [karaga@math.hr](mailto:karaga@math.hr)  
<http://www.math.hr/~karaga>  
page(s) 35
- SAMIR KARASULJIĆ, Mathematics Department, Faculty of Natural Sciences, University of Tuzla, Univerzitetska ulica 1, 75000 Tuzla, Bosnia and Herzegovina  
e-mail: [samir.karasuljic@bih.net.ba](mailto:samir.karasuljic@bih.net.ba)  
page(s) 25
- LOUIS KAVITHA, Department of Physics, Periyar University, Salem-636 011, India  
e-mail: [kavithalouis@yahoo.com](mailto:kavithalouis@yahoo.com)  
page(s) 35
- WOJCIECH M. KEMPA, Silesian University of Technology, Institute of Mathematics, ul. Kaszubska 23, 44-100 Gliwice, Poland  
e-mail: [wojciech.kempa@polsl.pl](mailto:wojciech.kempa@polsl.pl)  
page(s) 37
- JADRANKA KRALJEVIĆ, University of Zagreb, Faculty of Economics, Department of Mathematics, Kennedyev trg 6, 10000 Zagreb, Croatia  
e-mail: [jkraljevic@efzg.hr](mailto:jkraljevic@efzg.hr)  
page(s) 39
- ARNE LAKSÅ, Faculty of Technology, Narvik University College, Lodve Langes gt. 2, N-8505 Narvik, Norway  
e-mail: [ala@hin.no](mailto:ala@hin.no)  
page(s) 40
- MARTIN LAZAR, University of Dubrovnik, Ćira Carića 4, 20000 Dubrovnik, Croatia  
e-mail: [mlazar@unidu.hr](mailto:mlazar@unidu.hr)  
page(s) 41
- NEDŽAD LIMIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [nlimic@math.hr](mailto:nlimic@math.hr)  
page(s) 42, 44

TOM LYCHE, CMA and Department of Informatics, University of Oslo, P.O. Box 1053, Blindern, 0316 Oslo, Norway  
e-mail: [tom@ifi.uio.no](mailto:tom@ifi.uio.no)  
<http://home.ifi.uio.no/tom/>  
page(s) 10

SENKA MAĆEŠIĆ, Faculty of Engineering, University of Rijeka, Vukovarska 58, 51000 Rijeka, Croatia  
e-mail: [senka.macesic@riteh.hr](mailto:senka.macesic@riteh.hr)  
page(s) 55

MARTINA MANHART, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [martina.manhart@math.hr](mailto:martina.manhart@math.hr)  
page(s) 45

CARLA MANNI, Department of Mathematics, University of Roma “ Tor Vergata”, Via della Ricerca Scientifica, 00133 Roma, Italy  
e-mail: [manni@mat.uniroma2.it](mailto:manni@mat.uniroma2.it)  
<http://www.mat.uniroma2.it/~manni>  
page(s) 11

MAROJE MAROHNIC, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [maroje.marohnic@math.hr](mailto:maroje.marohnic@math.hr)  
page(s) 45

MILJENKO MARUŠIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [miljenko.marusic@math.hr](mailto:miljenko.marusic@math.hr)

RODERICK MELNIK, Department of Mathematics and M<sup>2</sup>NeT Laboratory, Wilfrid Laurier University, 75 University Avenue West, Waterloo, ON, N2L 3C5 Canada  
e-mail: [rmelnik@wlu.ca](mailto:rmelnik@wlu.ca)  
<http://www.m2netlab.wlu.ca>  
page(s) 46

JOHN J. H. MILLER, Institute for Numerical Computation and Analysis (INCA), 7–9 Dame Court, Dublin 2, Ireland.  
e-mail: [jm@incaireland.org](mailto:jm@incaireland.org)  
<http://www.incaireland.org>  
page(s) 12



IVANČICA MIROŠEVIĆ, University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, R. Boškovića 32, 21000 Split, Croatia  
e-mail: [ivancica.mirosevic@fesb.hr](mailto:ivancica.mirosevic@fesb.hr)

MARIA GIOVANNA MORA, Scuola Internazionale Superiore di Studi Avanzati (SISSA), via Bonomea 265, 34136 Trieste, Italy  
e-mail: [mora@sissa.it](mailto:mora@sissa.it)  
<http://people.sissa.it/~mora/>  
page(s) 13

BORIS MORDUKHOVICH, Department of Mathematics, Wayne State University, 1237 Faculty/Administration Bldg, 656 West Kirby Avenue, Detroit, Michigan 48202, USA  
e-mail: [boris@math.wayne.edu](mailto:boris@math.wayne.edu)  
<http://www.math.wayne.edu/~boris/>  
page(s) 13

BORIS MUHA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [borism@math.hr](mailto:borism@math.hr)  
<http://web.math.hr/~borism/>  
page(s) 47

SARA MUHVIĆ, B.Sc. student, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [smuhvic@student.math.hr](mailto:smuhvic@student.math.hr)  
page(s) 18

IVICA NAKIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [nakic@math.hr](mailto:nakic@math.hr)  
page(s) 48

VEDRAN NOVAKOVIĆ, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lučića 5, 10000 Zagreb, Croatia  
e-mail: [venovako@fsb.hr](mailto:venovako@fsb.hr)  
<http://web.math.hr/~venovako>  
page(s) 49

TAKESHI OGITA, Division of Mathematical Sciences, Tokyo Woman's Christian University, 2-6-1 Zempukuji, Suginami-ku, Tokyo 167-8585, Japan  
e-mail: [ogita@lab.twcu.ac.jp](mailto:ogita@lab.twcu.ac.jp)  
page(s) 50, 51

ALMA OMERSPAHIĆ, Faculty of Mechanical Engineering, University of Sarajevo,  
Vilsonovo šetalište 9, 71000 Sarajevo, Bosnia and Herzegovina  
e-mail: [alma.omerspahic@mef.unsa.ba](mailto:alma.omerspahic@mef.unsa.ba)  
page(s) 26

MELKIOR ORNIK, B.Sc. student, Faculty of Science, Department of Mathematics,  
University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [mornik@student.math.hr](mailto:mornik@student.math.hr)  
page(s) 18

KATSUHISA OZAKI, Department of Mathematical Sciences, Shibaura Institute of  
Technology, Fukasaku 307, Minama-ku, Saitama-shi, Saitama 337-8570, Japan  
e-mail: [ozaki@sic.shibaura-it.ac.jp](mailto:ozaki@sic.shibaura-it.ac.jp)  
page(s) 51

IGOR PAŽANIN, Faculty of Science, Department of Mathematics, University of Za-  
greb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [pazanin@math.hr](mailto:pazanin@math.hr)  
<http://web.math.hr/~pazanin>  
page(s) 52

TATJANA PECAK, Faculty of Civil Engineering, University of Rijeka, Viktora Cara  
Emina 5, 51000 Rijeka, Croatia  
e-mail: [tatjana.pecak@gradri.hr](mailto:tatjana.pecak@gradri.hr)

CHRISTOPHE RABUT, Department of Mathematics, Institute of Applied Science,  
135, Avenue de Ranguel, 31077 Toulouse, France  
e-mail: [rabut@insa-toulouse.fr](mailto:rabut@insa-toulouse.fr)  
page(s) 23

MLADEN ROGINA, Faculty of Science, Department of Mathematics, University of  
Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia.  
e-mail: [rogina@math.hr](mailto:rogina@math.hr)  
<http://www.math.hr/~rogina>  
page(s) 21, 38

LUCA RONDI, Università di Trieste, Dip. di Matematica e Informatica, via Valerio,  
12/1, 34127 Trieste, Italy  
e-mail: [rondi@units.it](mailto:rondi@units.it)  
<http://people.sissa.it/~mora/>

KRISTIAN SABO, Department of Mathematics, University of Osijek, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia  
e-mail: [ksabo@mathos.hr](mailto:ksabo@mathos.hr)  
<http://www.mathos.hr/~ksabo>  
page(s) 53

LARRY L. SCHUMAKER, Vanderbilt University, Stevenson Center 1532, 37240 Nashville, Tennessee, USA  
e-mail: [larry.schumaker@gmail.com](mailto:larry.schumaker@gmail.com)  
<http://www.math.vanderbilt.edu/~schumake/>  
page(s) 14

RUDOLF SCITOVSKI, Department of Mathematics, University of Osijek, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia  
e-mail: [scitowsk@mathos.hr](mailto:scitowsk@mathos.hr)  
<http://www.mathos.hr/~scitowsk>  
page(s) 53

SANJA SINGER, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lučića 5, 10000 Zagreb, Croatia  
e-mail: [ssinger@fsb.hr](mailto:ssinger@fsb.hr)  
<http://www.fsb.hr/ssinger>  
page(s) 28, 49

SAŠA SINGER, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [singer@math.hr](mailto:singer@math.hr)  
<http://web.math.hr/~singer>  
page(s) 18, 28

IVAN SLAPNIČAR, University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, R. Boškovića 32, 21000 Split, Croatia  
e-mail: [Ivan.Slapnicar@fesb.hr](mailto:Ivan.Slapnicar@fesb.hr)  
<http://www.fesb.hr/~slap>  
page(s) 54

SAŠA STANKO, M.S. student, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [sstanko@student.math.hr](mailto:sstanko@student.math.hr)  
page(s) 49

- MAJA STARČEVIĆ, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [mstarcev@math.hr](mailto:mstarcev@math.hr)  
page(s) 44
- NATAŠA STRABIĆ, Ph.D. student, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [nstrabic@gmail.com](mailto:nstrabic@gmail.com)
- VEDRAN ŠEGO, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [vsego@math.hr](mailto:vsego@math.hr)  
page(s) 54
- JOSIP TAMBAČA, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [tambaca@math.hr](mailto:tambaca@math.hr)  
page(s) 45
- ZVONIMIR TUTEK, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [tutek@math.hr](mailto:tutek@math.hr)  
page(s) 47
- IVAN VAZLER, Department of Mathematics, University of Osijek, Trg Ljudevita Gaja 6, 31000 Osijek, Croatia  
e-mail: [ivazler@mathos.hr](mailto:ivazler@mathos.hr)  
<http://www.mathos.hr/~ivazler>  
page(s) 53
- IGOR VELČIĆ, Faculty of Electrical Engineering and Computing, University of Zagreb, Unska 3, 10000 Zagreb, Croatia  
e-mail: [igor.velcic@fer.hr](mailto:igor.velcic@fer.hr)  
page(s) 57
- VITO VITRIH, FAMNIT and PINT, University of Primorska, Muzejski trg 2, SI-6000 Koper, Slovenia  
e-mail: [vito.vitrih@upr.si](mailto:vito.vitrih@upr.si)  
page(s) 32, 38
- ANJA VRBAŠKI, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia  
e-mail: [avrbaski@math.hr](mailto:avrbaski@math.hr)  
page(s) 15

MARKO VRDOLJAK, Faculty of Science, Department of Mathematics, University of Zagreb, Bijenička cesta 30, 10000 Zagreb, Croatia

e-mail: [marko@math.hr](mailto:marko@math.hr)

VIDA ZADELJ MARTIĆ, Faculty of Geodesy, University of Zagreb, Kačićeva 26, 10000 Zagreb, Croatia

e-mail: [vzadelj@geof.hr](mailto:vzadelj@geof.hr)

<http://www.geof.hr/~vzadelj>

EMIL ŽAGAR, FMF and IMFM, University of Ljubljana, Jadranska 19, SI-1000 Ljubljana, Slovenia

e-mail: [emil.zagar@fmf.uni-lj.si](mailto:emil.zagar@fmf.uni-lj.si)

<http://valjhun.fmf.uni-lj.si/~emil>

page(s) 30, 32