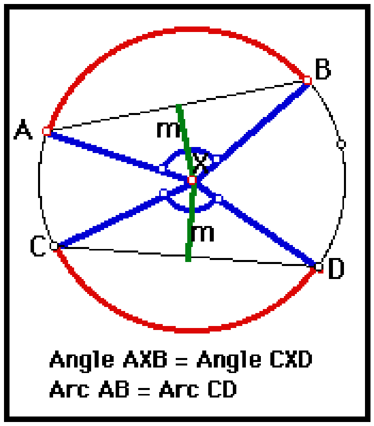
Congruent Chords Conjecture



Explanation:

A **chord** is a line segment with endpoints on the circle. We want to know when two chords in a circle are congruent. This conjecture tells us that the central angles determined by the congruent chords are equal in measure, which implies that the **intercepted arcs** are congruent. This conjectures also tells us that the distances from the centre of the circle to two congruent chords are equal.

The precise statement of the conjecture is:

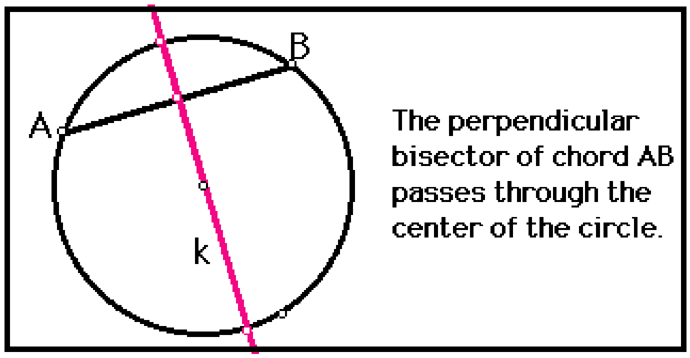
Conjecture (*Congruent Chords*):

If two chords of a circle are congruent, then they determine central angles which are equal in measure.

If two chords of a circle are congruent, then their intercepted arcs are congruent.

Two congruent chords in a circle are equal in distance from the centre.

Perpendicular Bisector of a Chord Conjecture



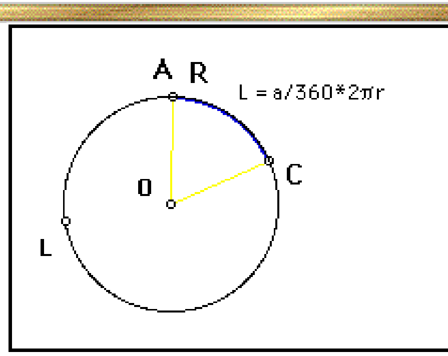
Explanation:

The **cord** of a circle is a segment whose endpoints are on the circle. This conjecture states that the perpendicular bisector of any chord passes through the centre of the circle. This could give us a good way to find the centre of any circle!

The precise statement of the conjecture is:

Conjecture (*Perpendicular Bisector of a Chord*): The perpendicular bisector of a chord in a circle passes through the centre of the circle

Arc Length Conjecture



Explanation:

The **arc length** of a circle is the distance from one point on the circumference to another point on the circumference, "traveling" along the edge of the circle. Because we know that the measure of the central angle is equal to the arc it intercepts, [(Inscribed angle conjectures)] by dividing that measure by 360 degrees, we find out what fraction of the circumference that the arc covers.

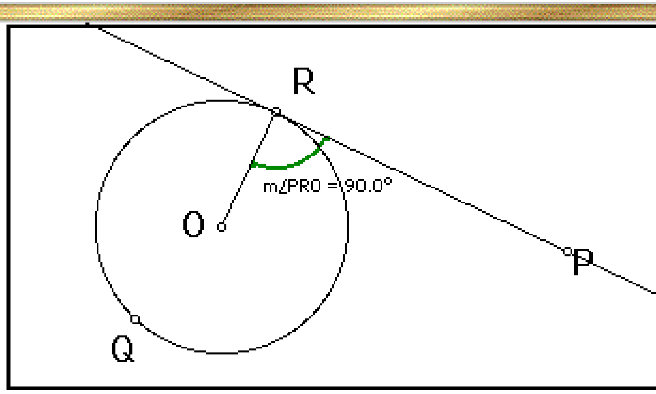
The precise statement of the conjecture is:

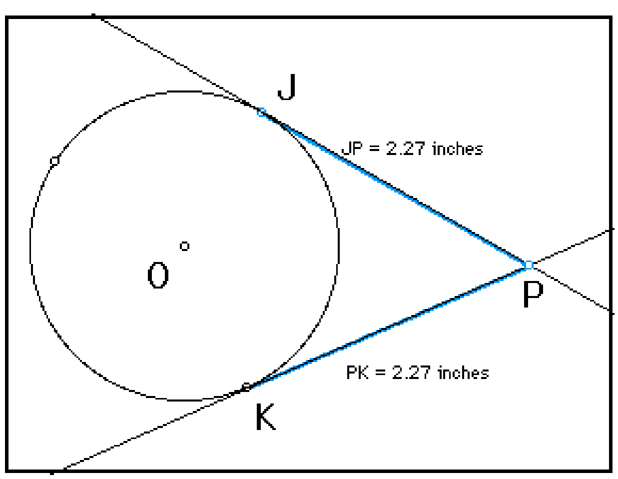
Conjecture (*Arc Length*): The arc length of an arc on a circle is given by

L = m/360 degrees * circumference

where m is the measure of the central angle.

Tangent Conjectures



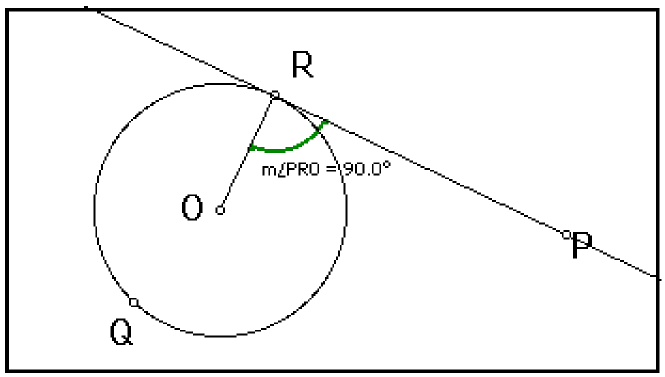


Explanation:

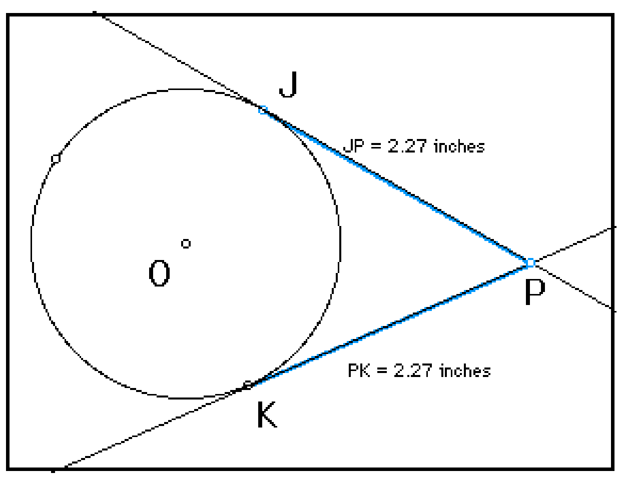
A **tangent line** to a circle is any line which intersects the circle in exactly one point. You can think of a tangent line as "just touching" the circle, without ever traveling "inside". A line which intersects a circle in two points is called a **secant line**. Chords of a circle will lie on secant lines.

The precise statement of the conjecture is:

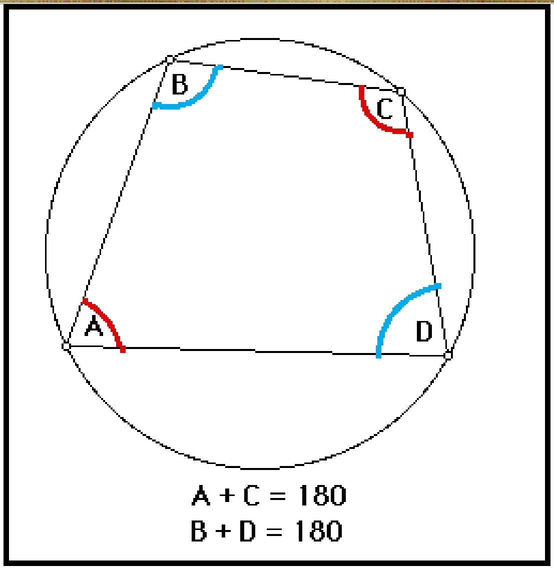
Conjecture (*Tangent Conjecture I*): Any tangent line to a circle is perpendicular to the radius drawn to the point of tangency.



Conjecture (*Tangent Conjecture II*): Tangent segments to a circle from a point outside the circle are equal in length.



Inscribed Quadrilateral Conjecture



Explanation:

An **inscribed quadrilateral** is any four sided figure whose vertices all lie on a circle. (The sides are therefore chords in the circle!) This conjecture give a relation between the opposite angles of such a quadrilateral. It says that these opposite angles are in fact supplements for each other. In other words, the sum of their measures is 180 degrees.

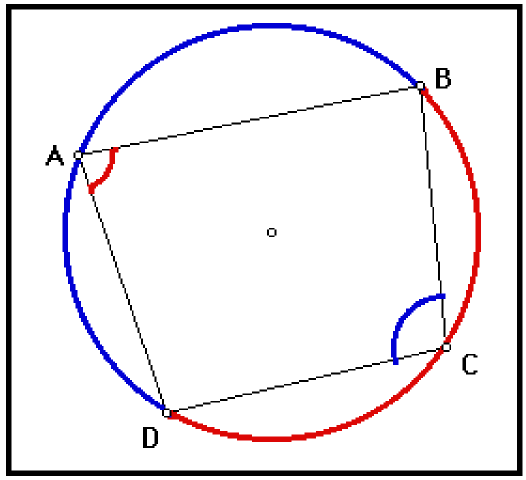
For the quadrilateral shown above, this conjecture says that:

AngleA + AngleC = 180
AngleB + AngleD = 180

The precise statement of the conjecture is:

Conjecture (*Quadrilateral Sum*): Opposite angles in any quadrilateral inscribed in a circle are supplements of each other. (Their measures add up to 180 degrees.)

Proof: We use ideas from the Inscribed Angles Conjecture to see why this conjecture is true. The main result we need is that an inscribed angle has half the measure of the intercepted arc. Here, the intercepted arc for Angle(A) is the red Arc(BCD) and for Angle(C) is the blue Arc(DAB).



Arc(BCD)=2*Angle(A) **and** Arc(DAB)=2*Angle(C)

Arc(BCD)+Arc(DAB) = 360

2*Angle(A)+2*Angle(C) = 2*[Angle(A)+Angle(C)] = 360

Angle(A)+Angle(C) = 180

The proof is complete!