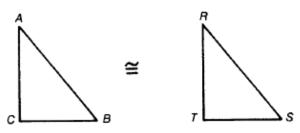
Proving Triangles Congruent



Topic	Pages in Packet	Assignment: (Honors TXTBK)
Angles in Triangles/Definition of Congruent Triangles	Pages 2-6	HOLT TXTBK: Page 227#9-14,19-22,41-42,45,49
Identifying Congruent Triangles	Pages 7- 13	This Packet pages 14- 15
Congruent Triangles Proofs	Pages 16-21	This Packet pages 22-24
C.P.C.T.C.	Pages 25-29	Pages 127-129 #'s 6,12,13,18,21
C.P.C.T.C. and BEYOND	Pages 30 - 33	Pages 135 #'s #2, 5, 7-11, 15
Isosceles Triangle	Pages 34 - 37	Page 155 #'s 20,21, 23, 24, 25 Page 160 # 16
Proving Triangles Congruent with hy.leg	Pages 38-43	Page 158 #'s 5, 12, 17
Right Angle Theorem & Equidistance Theorems	Pages 44-50	Pgs 182-183 #'s 4, 9, 14 Pg 189-190 #'s 14,15,16, 17, 20
Detour Proofs	Page 51- 57	Pages 174 – 175 #'s 11,13,14,17 Page 141 #4
Missing Diagram Proofs	Pages 58- 62	Page 179 #'s 8, 11, 12, 14
Answer Keys Start on page 63		

Day 1

SWBAT: Use properties of congruent triangles. Prove triangles congruent by using the definition of congruence.

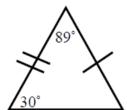
Vocabulary Review: Describe how to classify triangles by sides or angles. Draw a diagram for each.

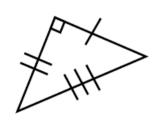
By Angles	By Sides
Acute	Scalene
Right	Isosceles
Obtuse	Equilateral
Equilateral	

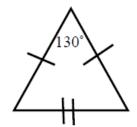
Theorem Review: Describe each theorem and include a diagram

Theorem	Diagram
Triangle Sum Theorem	
Exterior Angle Theorem	
Third Angles Theorem	

1. Classify the triangles based on their side lengths and angle measures.



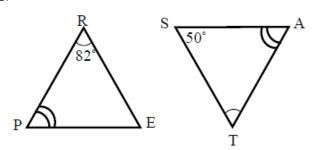






For #2-5, solve for the variable and then find missing angle or side lengths.

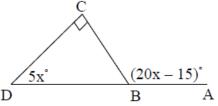
If $m \angle A = (5x - 7)^{\circ}$, find x, $m \angle A$ and $m \angle E$.



2. x = $m \measuredangle A = \underline{\hspace{1cm}}$ *m*∠*E* = _____

3. In \triangle WIN $m \angle W = (2y + 7)^{\circ}$, $m \angle I = (6y)^{\circ}$, $m \angle N = (8y + 13)^{\circ}$. Find y. (Hint: Draw a diagram.)

4.

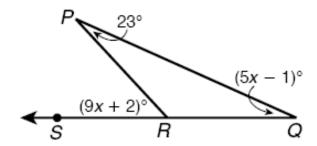


m∠*ABC* = ____

5. In right triangle ABC, $m\angle C = 3y - 10$, $m\angle B = y + 40$, and $m\angle A = 90$. What type of right triangle is triangle ABC?

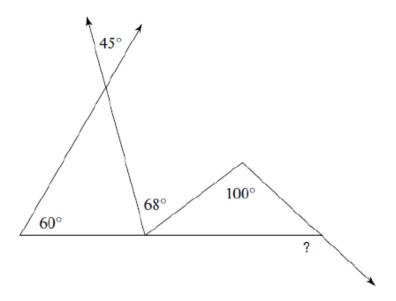
6. The angle measures of a triangle are in the ratio of 5:6:7. Find the angle measures of the triangle.

7. Solve for m*≯PRS*



<u>Challenge</u>

Find the measure of the angle indicated.



SUMMARY

Classification	Description	Example
acute triangle	triangle that has three acute angles	78° 69° 33°
equiangular triangle	triangle that has three congruent acute angles	
right triangle	triangle that has <i>one</i> right angle	
obtuse triangle	triangle that has <i>one</i> obtuse angle	120°
equilateral triangle	triangle with <i>three</i> congruent sides	\triangle
isosceles triangle	triangle that has at least two congruent sides	
scalene triangle	triangle that has <i>no</i> congruent sides	3 5

Exit Ticket

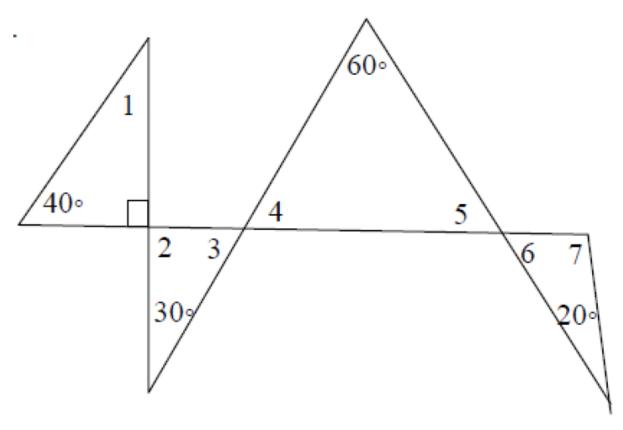
- 1. In $\triangle ABC$, $m\angle A = x$, $m\angle B = 2x + 2$, and $m\angle C = 3x + 4$. What is the value of x?
 - 1) 29
 - 2) 31
 - 3) 59
 - 4) 61

- 2. Triangle PQR has angles in the ratio of 2:3:5. Which type of triangle is △PQR?
 - 1) acute
 - 2) isosceles
 - 3) obtuse
 - 4) right

Day 2 - Identifying Congruent Triangles

Warm – Up

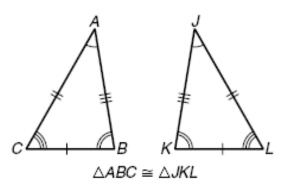
Find the measure of the missing angles



$$m \measuredangle 4 =$$

Geometric figures are congruent if they are the same size and shape. Corresponding angles and			
<u>corresponding sides</u> are in the same in polygons with an equal number of			
Two polygons are	polygons if and only if their	sides are	
1 , 0	ne same size and shape are congruent.		

Ex 1: Name all the corresponding sides and angles below if the polygons are congruent.



Corresponding Sides

Corresponding Angles

Ex 2:

Given $\triangle GEO \cong \triangle FUN$. Let $m \angle E = (3x-4)^\circ$, $m \angle F = 2x^\circ$, $m \angle N = (20-x)^\circ$.

- a. Draw and label a diagram.
- b. List all six pairs of congruent parts
- c. Solve for x

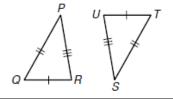
c. x = ____

Identifying Congruent Triangles

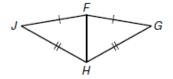
Side-Side-Side (SSS) Congruence Postulate

If three sides of one triangle are congruent to three sides of another triangle, then the triangles are congruent.

 $\overline{QR} \cong \overline{TU}$, $\overline{RP} \cong \overline{US}$, and $\overline{PQ} \cong \overline{ST}$, so $\triangle PQR \cong \triangle STU$.

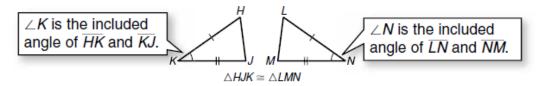


You can use SSS to explain why $\triangle FJH \cong \triangle FGH$. It is given that $\overline{FJ} \cong \overline{FG}$ and that $\overline{JH} \cong \overline{GH}$. By the Reflex. Prop. of \cong , $\overline{FH} \cong \overline{FH}$. So $\triangle FJH \cong \triangle FGH$ by SSS.



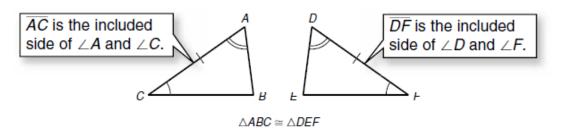
Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.



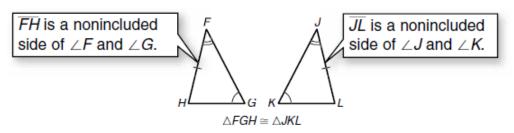
Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

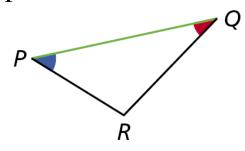


Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a nonincluded side of one triangle are congruent to the corresponding angles and nonincluded side of another triangle, then the triangles are congruent.

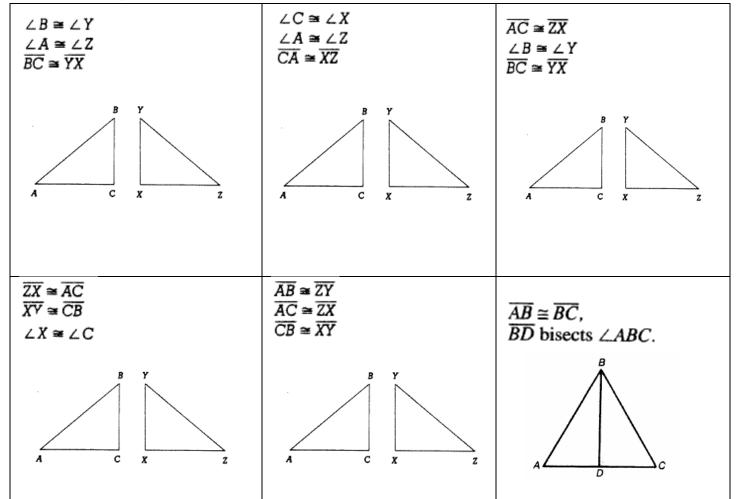


An <u>included side</u> is the common side of two consecutive angles in a polygon. The following postulate uses the idea of an included side.



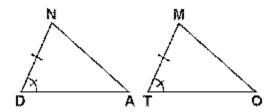
 \overline{PQ} is the included side of $\angle P$ and $\angle Q$.

Name the postulate or theorem you would use to prove $\triangle ACB \cong \triangle ZXY$ given following information. If there is not enough information, state none.



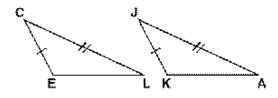
The pair of triangles below has two corresponding parts marked as congruent.

What additional information is needed for a SAS congruence correspondence?



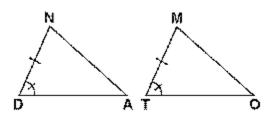
Answer: ____ ≅ ____

4. What additional information is needed for a SSS congruence correspondence?



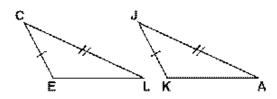
Answer: ____ ≅ ____

2. What additional information is needed for an ASA congruence correspondence?



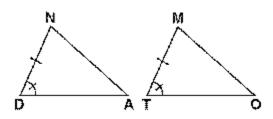
Answer: ____ ≅ ____

5. What additional information is needed for a SAS congruence correspondence.



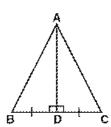
Answer: _____ ≅ ____

What additional information is needed for an AAS congruence correspondence?



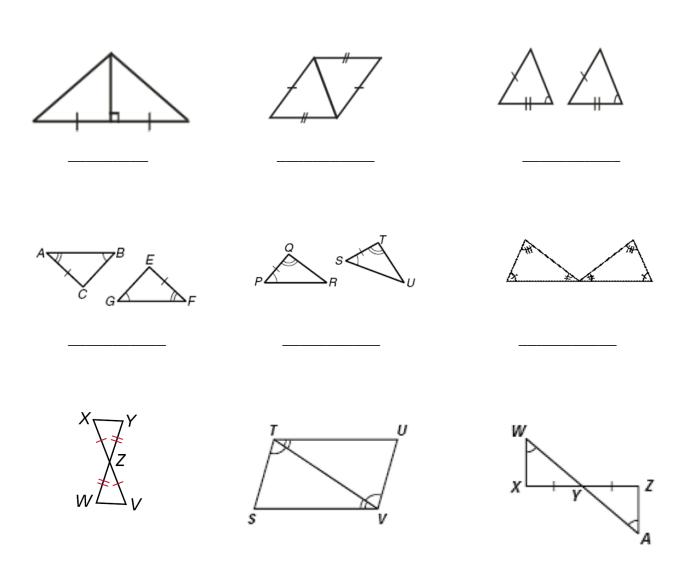
Answer: ____ ≅ ____

6. What additional information is needed for an ASA congruence correspondence?

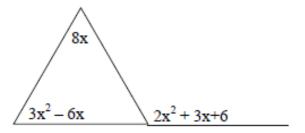


Answer: ____ ≅ ____

Using the tick marks for each pair of triangles, name the method {SSS, SAS, ASA, AAS} that can be used to prove the triangles congruent. If not, write *not possible*. (Hint: Remember to look for the <u>reflexive side</u> and vertical angles!!!!)

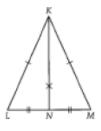


Challenge Solve for x.



SUMMARY

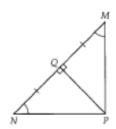
Side-Side-Side (S.S.S.)



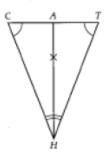
Side-Angle-Side (S.A.S.)



Angle-Side-Angle (A.S.A.)

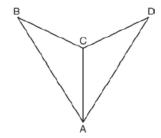


Angle-Angle-Side (A.A.S.)



Exit Ticket

As shown in the diagram below, \overline{AC} bisects $\angle BAD$ and $\angle B \cong \angle D$.



Which method could be used to prove

 $\triangle ABC \cong \triangle ADC$?

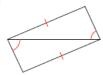
- 1) SSS
- 2) AAA
- 3) SAS
- 4) AAS

SSS, SAS, ASA, and AAS Congruence

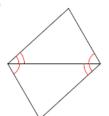
Date______ Period____

State if the two triangles are congruent. If they are, state how you know.

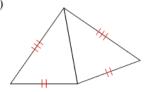
1)



2)



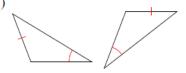
3)



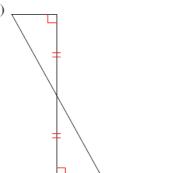
4)



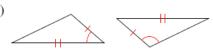
5



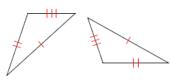
6)



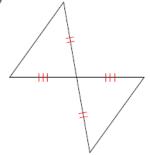
7)



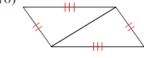
8)



9)

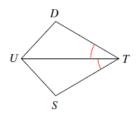


10)

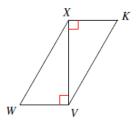


State what additional information is required in order to know that the triangles are congruent for the reason given.

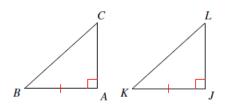
11) ASA



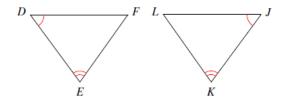
12) SAS



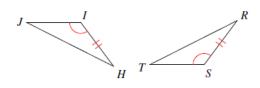
13) SAS



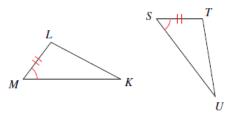
14) ASA



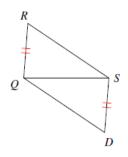
15) SAS



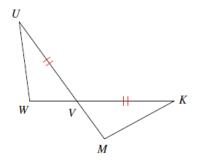
16) ASA



17) SSS



18) SAS

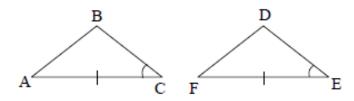


Day 3 – Proving Congruent Triangles

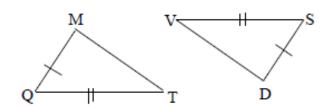
Warm - Up

Each pair of triangles below has two corresponding sides or angles marked congruent. Indicate the additional information needed to enable us to apply the specified congruence postulate.

1. For ASA ______
For SAS



For SSS



Congruent Triangle Proofs

Example 1: Proving Triangles Congruent

GIVEN: ∆ABC, $\overline{CD} \perp \overline{AB}$ D midpoint of \overline{AB} .

<u>/</u>*

PROVE: $\triangle ACD = \triangle BCD$

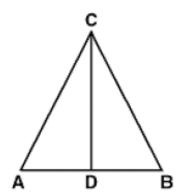
STATEMENTS	REASONS
1) CD⊥ AB D midpoint of AB.	1) Given
2) $\overline{AD} \stackrel{\sim}{=} \overline{DB}$ (s $\stackrel{\sim}{=}$ s)	A midpoint divides a segment into two congruent segments.
 ∠ ADC is a right angle ∠ BDC is a right angle. 	Perpendicular lines form right angles.
4) ∠ ADC ≅ ∠BDC (a ≅ a)	All right angles are congruent.
5) CD ≅ CD (s ≅ s)	5) Reflexive postulate.
6) Δ ACD ≅ Δ BCD	6) s.a.s. ≅ s.a.s.

Model Problem #1

Given: $\overline{AC} \cong \overline{BC}$

D is the midpoint of \overline{AB}

Prove: $\triangle ACD \cong \triangle BCD$



Statements

2

☐ 4 _____ ≅ ____

 $5 \Delta ACD \cong \Delta BCD$



- 1 Given
- 2 Given

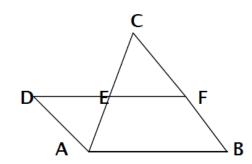
3 →

4

5

2) **Given:** \overline{AC} and \overline{DF} bisect each other at E

Prove: $\triangle DEA \cong \triangle FEC$



Statements

2

1

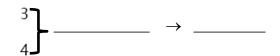
4 ____ ≅ ____

5 _____ ≅ ____

6 $\Delta DEA \cong \Delta FEC$

Reasons

- 1 Given
- 2 Seg bisector → _____

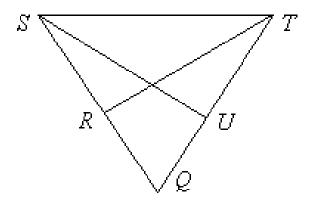


5

6

3) Given: $\overline{SR} \perp \overline{RT}$, $\overline{TU} \perp \overline{US}$, $\angle STR \cong \angle TSU$

Prove: $\triangle TRS \cong \triangle SUT$



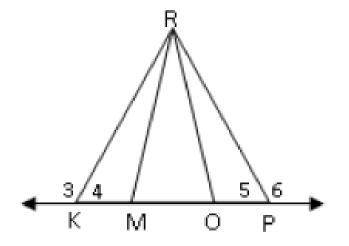
Statements	Reasons
1	1 Given
2	2 Given
3	3 →
4 ≅	4
5 ≅	5
6 $\Delta TRS \cong \Delta SUT$	6

LEVEL B

4) Given: $\angle 3 \cong \angle 6$, $\overline{KR} \cong \overline{PR}$,

 $\angle KRO \cong \angle PRM$

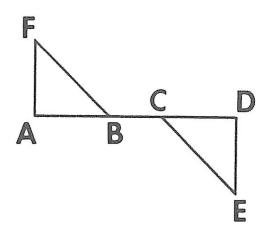
Prove: $\triangle KRM \cong \triangle PRO$



• , •	1
Statements	Reasons

5. Given: $\overline{FA} \perp \overline{AD}$, $\overline{ED} \perp \overline{AD}$, $\overline{AC} \cong \overline{DB}$, $\not \preceq F \cong \not \preceq E$

Prove: $\triangle ABF \cong \triangle DCE$.



Statements	Reasons

SUMMARY

Example 1: Proving Triangles Congruent

GIVEN: ∆ABC, $\overline{CD} \bot \overline{AB}$

D midpoint of AB.

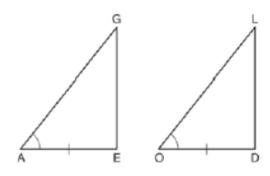
A D

PROVE: $\triangle ACD = \triangle BCD$

STATEMENTS		REASONS
1) CD⊥ AB D midpoint of AB.	1)	Given
2) ĀD ≅ ŪB (s ≅ s)	2)	A midpoint divides a segment into two congruent segments.
3) ∠ ADC is a right angle∠ BDC is a right angle.	3)	Perpendicular lines form right angles.
4) ∠ ADC ≅ ∠BDC (a ≅ a)	4)	All right angles are congruent.
5) CD ≅ CD (s ≅ s)	5)	Reflexive postulate.
6) ∆ ACD ≅ ∆ BCD	6)	s.a.s. = s.a.s.

Exit Ticket

In the diagram below of $\triangle AGE$ and $\triangle OLD$, $\angle GAE \cong \angle LOD$, and $\overline{AE} \cong \overline{OD}$.



To prove that $\triangle AGE$ and $\triangle OLD$ are congruent by SAS, what other information is needed?

- 1) $\overline{GE} \cong \overline{LD}$
- 2) $\overline{AG} \cong \overline{OL}$
- ∠AGE ≅ ∠OLD
- 4) $\angle AEG \cong \angle ODL$

Practice with Congruent Triangles

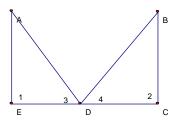
1. Given: $\overline{AE} \perp \overline{ED}$

 $\overline{BC} \perp \overline{CD}$

D is the midpoint of \overline{EC} .

∡3 ≅ **∡4**

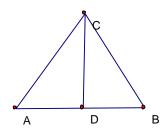
Prove: $\triangle AED \cong \triangle BCD$



2. Given: $\overline{AC} \cong \overline{CB}$

 \overline{CD} Bisects \overline{AB}

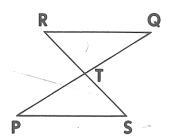
Prove: $\triangle ADC \cong \triangle BDC$



3.

Given: \overline{RS} bisects \overline{PQ} at T, \overline{PQ} bisects \overline{RS} at T.

Prove: $\triangle PTS \cong \triangle QTR$.

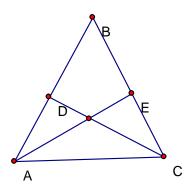


4. Given: ∡BAC ≅ ∡BCA

CD bisects ≰BCA

AE bisects ≰BAC

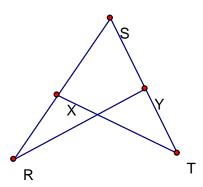
Prove: $\triangle ADC \cong \triangle CEA$



5. Given: \overline{SR} and \overline{ST} are straight lines.

 $\frac{\overline{SX}}{\overline{XR}} \cong \frac{\overline{SY}}{\overline{YT}}$

Prove: $\Delta RSY \cong \Delta TSX$

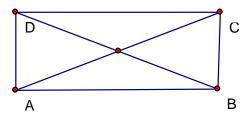


6. Given: $\overline{DA} \cong \overline{CB}$

 $\overline{DA} \perp \overline{AB}$

 $\overline{\textit{CB}} \perp \overline{\textit{AB}}$

Prove: Δ **DAB** \cong Δ **CBA**

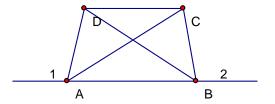


7. Given: \overline{LM} is a straight line

 $\overline{\textit{CB}} \cong \overline{\textit{DA}}$

∡1 ≅ **∡2**

Prove: $\triangle ABC \cong \triangle BAD$



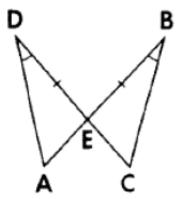
Day 4 - CPCTC

SWBAT: To use triangle congruence and CPCTC to prove that parts of two triangles are congruent.

Warm-Up

What additional information would you need to prove these triangles congruent by ASA?

____ ≅ ____



With SSS, SAS, ASA, and AAS, you know how to use three parts of triangles to show that the triangles are congruent. Once you have triangles congruent, you can make conclusions about their other parts because, by definition, corresponding parts of congruent triangles are congruent. You can abbreviate this as **CPCTC.**

CPCTC Proofs

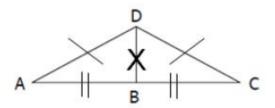
Corresponding Parts of Congruent Triangles are Congruent (CPCTC)

To use CPCTC, you must <u>first</u> show the triangles are congruent!!!

Example 2: USING CPCTC

Given: B is the midpoint of \overline{AC} , $\overline{AD} \cong \overline{CD}$

Prove: $\angle DAB \cong \angle DCB$



Statements	Reasons
(s) 1) B is the midpoint of \overline{AC} , $\overline{AD} \cong \overline{CD}$	1) Given
(s) 2) $\overline{AB} \cong \overline{CB}$	2) Midpoint \rightarrow 2 \cong segments
(s) 3) $\overline{DB} \cong \overline{DB}$	3) Reflexive property of segments
 △ADB ≅△CDB. 	4) <u>SSS</u>

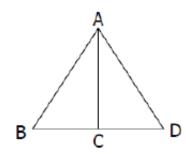
5) CPCTC

You Try It!

5) ∠DAB ≅ ∠DCB

Given: C is the midpoint of \overline{BD} , $\overline{AC} \perp \overline{BD}$

Prove: $\angle BAC \cong \angle DAC$

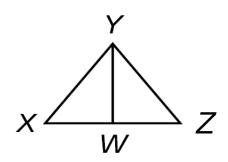


Statements	Reasons
1)	1)
2) $\overline{BC} \cong \overline{DC}$	2)
3) ∠ACB & ∠ACD are right angles	3)
4) $\angle ACB \cong \angle ACD$	4)
5)	5) Reflexive Property
6) △ACB ≅△ACD	6)
7)	7)
	I .

Example 1:

Given: W is the midpoint of \overline{XZ} , $\overline{XY} \cong \overline{ZY}$

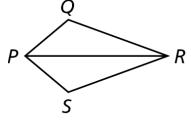
Prove: \(\times XYW \cong \times ZYYW\)



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6

Example 2: Given: \overline{PR} bisects $\angle QPS$ and $\angle QRS$.

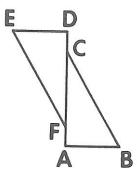
Prove: $\overline{PQ} \cong \overline{PS}$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6

3. Given: \overline{AFCD} , $\overline{ED} \perp \overline{DA}$, $\overline{BA} \perp \overline{DA}$, $\overline{DC} \cong \overline{AF}$, and $\angle E \cong \angle B$.

Prove: $\overline{EF} \cong \overline{BC}$.	
Statements	Reasons
1	1
2	2
3	3



 5
 5

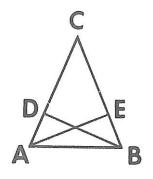
 6
 6

 7
 7

 8
 8

4

4. Given: In $\triangle ACB$, $\overline{AC} \cong \overline{BC}$ and $\angle ADB \cong \angle BEA$. Prove: $\overline{AE} \cong \overline{BD}$.



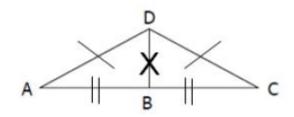
Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

SUMMARY

Given: B is the midpoint of \overline{AC} , $\overline{AD} \cong \overline{CD}$

Prove: $\angle DAB \cong \angle DCB$

Statements



795 170								
(0)	**	n .	41	3 * · · · · · · · · · · · · · · · · · ·		10	40	00
(5)	1)	B 15	the	midpoin	t of	AC.	AU:	= UD

(s) 2)
$$\overline{AB} \cong \overline{CB}$$

(s) 3) $\overline{DB} \cong \overline{DB}$

5) ∠DAB ≅ ∠DCB

Reasons

1) Given

2) Midpoint \rightarrow 2 \cong segments

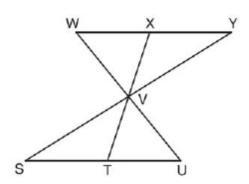
3) Reflexive property of segments

4) <u>SSS</u>

5) CPCTC

Warm - Up

In the diagram below, $\triangle XYV \cong \triangle TSV$.



Which statement can not be proven?

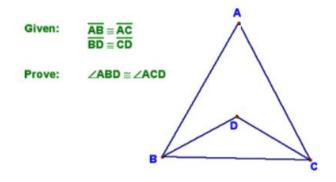
- ∠XVY ≅ ∠TVS
- ∠VYX ≅ ∠VUT
- 3) $XY \cong TS$
- $YV \cong SV$

C.P.C.T.C. and BEYOND

Auxiliary Lines

A diagram in a proof sometimes requires lines, rays, or segments that do not appear in the original figure. These additions to diagrams are auxiliary lines.

Ex 1: Consider the following problem.



This proof would be easy if_____

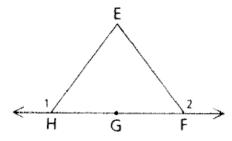
Theorem:

Ex 2:

Given: G is the midpt. of FH.

 $\overline{EF}\cong\overline{EH}$

Prove: $\angle 1 \cong \angle 2$



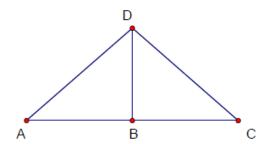
Statements Reasons

Ex 3: CPCTC and Beyond

Given: $\overline{AD} \cong \overline{CD}$

 $\angle ADB \cong \angle CDB$

Prove: \overline{DB} is the median to \overline{AC}



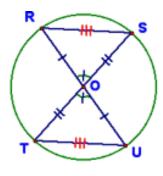
	Statements	Reasons
1. $\overline{AD} \cong \overline{CD}$		1. Given
2. $\angle ADB \cong \angle C$	CDB	2. Given
3.4. △ABD ≅△CE	BD	3. 4.
5.		5. CPCTC
6.		6.
7. \overline{DB} is the m	dedian to \overline{AC}	7.

Defn: A circle is the set of all points in a plane that are a given distance from a point located at its center. This distance is called the radius. (plural - radii). A circle consists only of of a "rim" but is named by the point in its center -- even though the center is not an element of the circle.

Theorem: All radii of a circle are congruent!

Given: ⊙o

Prove: RS ≅ UT



Statements

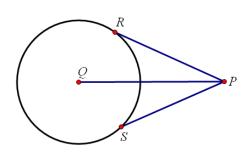
Reasons

- 1. ⊙0
- \$ 2. RO ≅ UO
- S 3. TO ≃ SO
- A 4. ∠ROS≅∠UOT
 - 5. △ROS ≅ △UOT
 - 6. RS ≅ UT

- 1. Given
- All radii of a ⊙ are ≅
- 3. Same as 2
- 4. Vertical ∠s are ≅ (VAT)
- 5. SAS (Steps 2, 4, 3)
- CPCTC

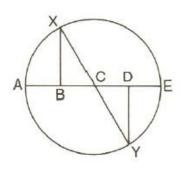
Example 4: Given: \bigcirc Q, $\overline{RP} \cong \overline{SP}$

Prove: \overrightarrow{PQ} bisects $\angle RPS$



Statements Reasons

Example 5: Given: $\bigcirc C$, $\angle ABX \cong \angle EDY$ Prove: C is the midpoint of \overline{BD}



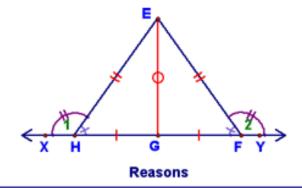
Statements Reasons

SUMMARY

Given: G is the midpoint of HF

EF ≅ EH

Prove: ∠1 ≅ ∠2



Statements

- 1. G is the midpoint of HF
- S 2. HG ≅ FG
- S 3. EH ≃ EF
 - 4. Draw EG
- S 5. EG ≅ EG
 - 6. \triangle EGH \cong \triangle EGF
 - 7. ∠EHG ≅ ∠EFG
 - 8. ∠1 is supp to ∠EHG
 - 9. . ∠2 is supp to ∠EFG
 - 10. ∠1 ≅ ∠2

- 1. Given
- 2. Defn. of midpoint
- 3. Given
- 4. Auxiliary Lines
- 5. Reflexive Property
- 6. SSS (Steps 2, 3, 5)
- 7. CPCTC
- 8. Linear Pair Thm.
- Linear Pair Thm.
- 10. Supps of ≅ ∠s are ≅

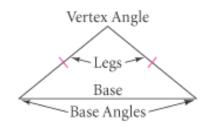
Exit Ticket

If $\Delta JKL \cong \Delta MNO$, which statement is always true?

- ∠KLJ ≅ ∠NMO
- ∠KJL ≅ ∠MON
- JL ≅ MO
- JK ≅ ON

Day 6 - Isosceles Triangle Proofs

Isosceles triangles are common in the real world. You can find them in structures such as bridges and buildings. The congruent sides of an isosceles triangle are its <u>legs</u>. The third side is the <u>base</u>. The two congruent sides form the <u>vertex angle</u>. The other two angles are the <u>base angles</u>.



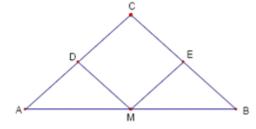
Theorem	Examples
Isosceles Triangle Theorem If two sides of a triangle are congruent, then the angles opposite the sides are congruent. (If A, then A.)	If $\overline{RT}\cong\overline{RS}$, then $\angle T\cong \angle S$.
Converse of Isosceles Triangle Theorem If two angles of a triangle are congruent, then the sides opposite those angles are congruent. (If \(\Delta \), then \(\Delta \).)	If $\angle N \cong \angle M$, then $\overline{LN} \cong \overline{LM}$.

Practice Problems:

Given: Isosceles triangle ABC with base AB
 M is the midpoint of AB

 $\overline{AD} \cong \overline{BE}$

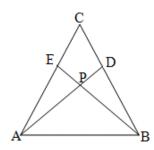
Prove: $\overline{DM} \cong \overline{ME}$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

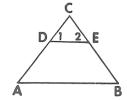
Given: $\overline{CA} \cong \overline{CB}$ $\angle PAB \cong \angle PBA$

Prove: $\triangle EPA \cong \triangle DPB$



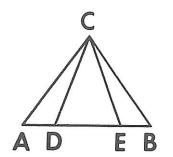
Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

If $\overline{CA} \cong \overline{CB}$, and $\overline{DA} \cong \overline{EB}$, prove that $\angle 1 \cong \angle 2$.



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

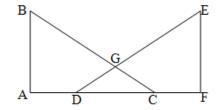
Given $\overline{AD}\cong \overline{BE}, \ \overline{CD}\cong \overline{CE}, \ \text{and} \ \overline{ADEB}, \ \text{prove that} \ \overline{AC}\cong \overline{BC}.$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

Given: $\overline{\overline{DG}} \cong \overline{\overline{CG}}$, $\overline{\overline{AD}} \cong \overline{\overline{FC}}$ $\overline{\overline{BC}} \cong \overline{\overline{ED}}$

 $\text{Prove: } \angle B \cong \ \angle E$



	Statements	Reasons	
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
7		7	
8		8	

Summary of Isosceles Triangles

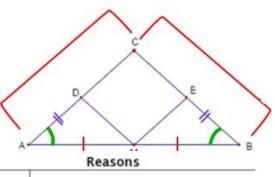
Given: Isosceles triangle ABC with base AB

M is the midpoint of AB

 $\overline{AD} \cong \overline{BE}$

Prove: $\overline{DM} \cong \overline{ME}$

Plan: $\triangle ADM \cong \triangle BEM$ by SAS



Statements

1 Isosceles triangle ABC with base AB

M is the midpoint of AB

$$4 \overline{CA} \cong \overline{CB}$$

$$8 \ \overline{DM} \cong \overline{ME}$$

4 Isosceles $\Delta \rightarrow 2 legs \cong$

5 If A, then A

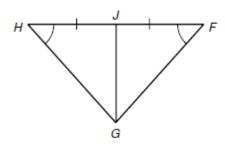
6 Midpoint → 2 \cong segments

7 SAS (3, 5, 6)

8 CPCTC

Exit Ticket

Use the figure for Exercises 1 and 2.



1. What postulate or theorem proves

 $\overline{HG} \cong \overline{FG}$?

A Isosceles Triangle Theorem (If \triangle , then \triangle .)

B Converse of Isosceles Triangle (If \triangle , then \triangle .) Theorem

2. If $\triangle FGJ \cong \triangle HGJ$, what reason justifies the statement $\angle HGJ \cong \angle FGJ$?

A ASA

B Reflex. Prop. of \cong

C Def. of bisects

D CPCTC

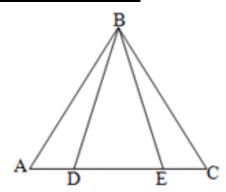
Day 7 - Hy-Leg

$\underline{Warm - Up}$

Given: ∠BDE ≅ ∠BED

 $\angle ABE \cong \angle CBD$

Prove: AABC is Isosceles



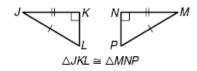
Statements	Reasons

PROVING RIGHT TRIANGLES CONGRUENT BY HYPOTENUSE, LEG

If the hypotenuse and a leg of one triangle are congruent to the corresponding parts of the other, then the two right triangles are congruent. (HL)

Hypotenuse-Leg (HL) Congruence Theorem

If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

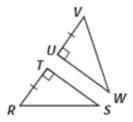


To use the HL Theorem, you must show that these three conditions are met:

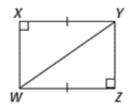
- There are two right triangles
- □ There is one pair of \cong hypotenuses
- \Box There is one pair of \cong legs

What additional information would you need to prove the triangles congruent by the HL Theorem?

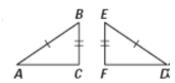
1.



2

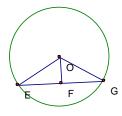


3.



1. Given: \overline{OF} is an altitude in Circle O.

Prove: $\overline{EF} \cong \overline{FG}$



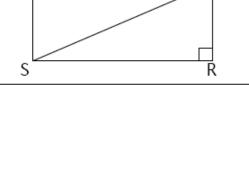
Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

2. Given: $\angle P, \angle R$ are ri	ight angles
--	-------------

 $\overline{PS}\cong \overline{QR}$

Prove: $\triangle PQS \cong \triangle RSQ$

Statements



1		
2		
3		
4		
5		

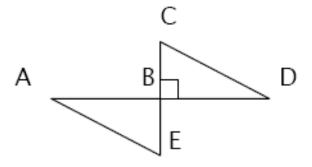
2	
3	
4	
5	

1

3. Given: \overline{AD} is the \bot bisector of \overline{EC}

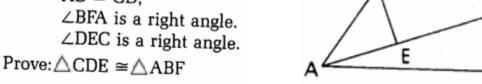
 $\overline{CD}\cong \overline{EA}$

Prove: $\triangle CDB \cong \triangle EBA$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

Given: AE ≅ CF,
 AB ≅ CD;
 ∠BFA is a right angle.
 ∠DEC is a right angle.



F

Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

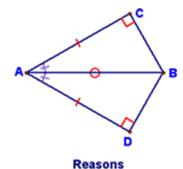
SUMMARY

Given:

BC ⊥AC BD ⊥AD AC ≅ AD

Prove:

→ AB bisects ∠CAD



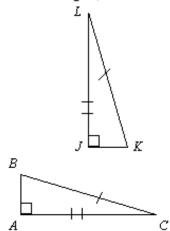
Statements

- 1. BC \perp AC
- R 2. ∠ACB is a right ∠
 - 3. BD \perp AD
- R 4. ∠ADB is a right ∠
- L 5. AC ≅ AD
- H 6. AB ≃ AB
 - 7. △ACB ≅ △ADB
 - 8. ∠CAB ≅ ∠DAB
 - 9. AB bisects ∠CAD

- 1. Given
- 2. Definition of \bot Segments
- 3. Given
- 4. Same as 2
- 5. Given
- 6. Reflexive Property
- 7. HL (2, 4, 6, 5)
- 8. CPCTC
- 9. Definition of ∠ Bisector

Exit Ticket

For these triangles, select the triangle congruence statement and the postulate or theorem that supports it.



- 1) $\triangle ABC \cong \triangle JLK$, HL
- 2) $\triangle ABC \cong \triangle JKL$, HL

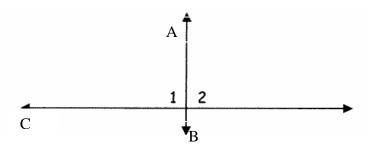
- 3) $\triangle ABC \cong \triangle JLK$, SAS
- 4) $\triangle ABC \cong \triangle JKL$, SAS

<u>Day 8 –</u> <u>Right Angle Theorems & Equidistance Theorem</u>

Theorem: If two angles are both supplementary and congruent, then they are right angles.

 $(\not a's \cong \& Suppl. \rightarrow right angles)$

Given: $\angle 1 \cong \angle 2$



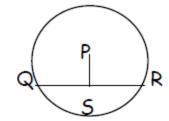
Conclusion: _____

*** Proving that lines are perpendicular depends on you proving that they form ______.

1. Given: $\odot P$

S is the midpoint of \overline{QR} .

Prove: $\overline{PS} \perp \overline{QR}$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

EQUIDISTANCE THEOREM

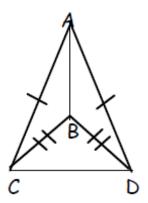
Definition: The distance between two objects is the length of the shortest path joining them.

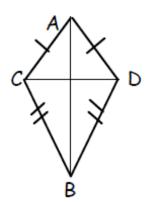
Postulate: A line segment is the shortest path between two points.

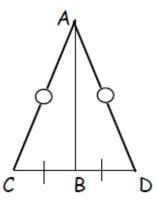
If two points P and Q are the same distance from a third point, X, they are said to be equidistant from X.

Picture:

Statement	Means
1. $\overline{AC} \cong \overline{BC}$	
$2. \ \overline{MQ} \cong \overline{NQ}$	
3. $\overline{DF} \cong \overline{GF}$, and $\overline{HF} \cong \overline{EF}$	







(Please highlight segment CD and put a circle around points A and B.)

These diagrams have something in common. In each, both points A and B are equidistant from the endpoints ____ and ___ of segment___.

You can prove that line AB is the <u>perpendicular bisector</u> of segment CD.

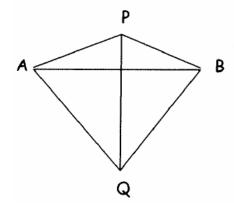
Definition: The perpendicular bisector of a segment is the line that bisects and is perpendicular to the segment.

Equidistance Theorem –

If two points are each equidistant from the endpoints of a segment, then the two points determine the <u>perpendicular bisector</u> of that segment.

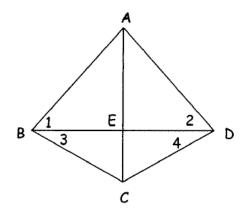
Given: $\overline{PA} \cong \overline{PB}, \overline{QA} \cong \overline{QB}$

Conclusion: _____



2. Given: $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$

Prove: $\overrightarrow{AE} \perp \text{bis. } \overrightarrow{BD}$

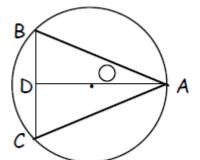


Statements	Reasons
1	1
2	2
3	3

3. Given: $\odot o$

 $\triangle ABC$ isosceles,

Prove: $\overline{AD} \perp bis. \overline{BC}$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

WHY the Equidistance Theorem?

Given: $\overline{AB} \cong \overline{AD}$ $BC \cong \overline{CD}$

Prove: \overline{AC} is the \bot bisector of \overline{BD}

Statements

- 1. $\overline{AB} \cong \overline{AD}$ $\overline{BC} \cong \overline{DC}$
- 2. <u>AC</u> ≅ <u>AC</u>
- 3. △ ABC ≅ △ ADC
- 4. ∠BAC≅∠DAC
- 5. $\overline{AE} \cong \overline{AE}$
- 6. △BAE≅ △DAE
- 7. ∠AEB≅∠AED
- 8. AEB is suppl. to AED
- 9. ∠ AEB and ∠ AED are right angles
- 10. AC ⊥ BD
- 11 $\overline{BE} \cong \overline{DE}$
- 12. E is the midpoint of \overline{BD}
- 13. \overline{AC} is the \perp bisector of \overline{BD}

Reasons

- 1. Given
- 2. Reflexive
- 3. SSS
- 4. CPCTC
- 5. Reflexive
- 6. SAS
- 7. CPCTC
- 8. L.P.'s form suppl. \(\sigma s
- 9. $4's \cong \& Suppl. \rightarrow right angles$
- 10. $right \not \leq s \rightarrow \bot$
- 11 CPCTC
- 12. Definition of Midpoint
- 13. Definition of a L bisector



Given: $\overline{AB} \cong \overline{AD}$

 $\overline{BC} \cong \overline{CD}$

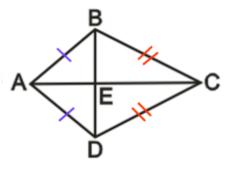
Statements

Prove: \overline{AC} is the \bot bisector of \overline{BD}

Reasons

- $1 \ \overline{AB} \cong \overline{AD}$
- $2 \overline{BC} \cong \overline{CD}$
- 3 \overline{AC} is the \perp bisector of \overline{BD}

- 1 Given
- 2 Given
- 3 Equidistance Thm(1, 2)

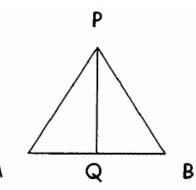


Converse of the Equidistance Theorem –

If a point is on the perpendicular bisector a segment, then it is equidistant from the endpoints of that segment.

Given: \overrightarrow{PQ} is the \bot bisector of \overrightarrow{AB}

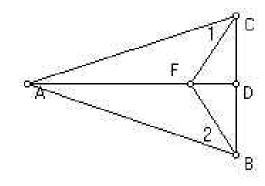
Conclusion: _____



4.

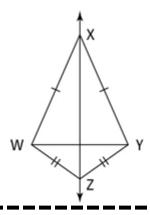
Given: $\overline{AD} \perp bis. \overline{BC}$

Prove: $41 \cong 42$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6

SUMMARY

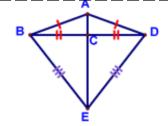


If you know that $\overline{XW} \cong \overline{XY}$ and $\overline{ZW} \cong \overline{ZY}$, then you can conclude that \overline{XZ} is the perpendicular bisector of \overline{WY} .

Given: AB ≈ AD

BC ≈ CD

Prove: BE ≈ ED



Statements

- 1. AB ≈ AD
- 2. BC ≈ CD
- 3. \overline{AE} is the \bot bisector of \overline{BD}

4. BE ≈ ED

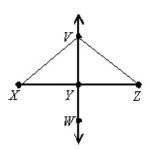
Reasons

- Given
- 2. Given
- ET (1,2) (If two points are each equidistant from the endpoints of a segment, the the two points determine the perpendicular bisector of that segment).
- Converse of ET (If a point is on the

 bisector of a segment, then it is
 equidistant from the endpoints of the
 segment).

Exit Ticket

Given: VW is the perpendicular bisector of \overline{XZ} . Which statement is true?



- A. Y is the midpoint of \overline{VW} .
- B. $\overline{XY} \cong \overline{YV}$
- C. ∠YVZ is a right angle.
- $\overline{\mathsf{D}}.\ \overline{XY}\cong\overline{YZ}$

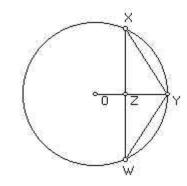
Day 9 - Detour Proofs

Warm - Up

Given: $\odot 0$

 $\not\preceq YXZ \cong \not\preceq YWZ$

Prove: $\overline{OY} \perp bis. \overline{XW}$



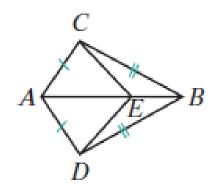
Statements	Reasons
1	1
2	2
3	3
4	4
5	5

Sometimes, it is impossible to use the *given* in order to prove immediately that a particular pair of triangles is congruent. In such cases, the *given* may contain enough information to first prove another pair of triangles congruent. Then, corresponding congruent parts in these congruent triangles may be used to prove the original pair of triangles congruent. See how this is done in the following example.

Example 1:

Given: \overline{AEB} , $\overline{AC} \cong \overline{AD}$, and $\overline{CB} \cong \overline{DB}$

Prove: $\triangle ACE \cong \triangle ADE$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

Whenever you are asked to prove that triangles or parts of triangles are congruent and you suspect a detour may be needed, use the following procedures.

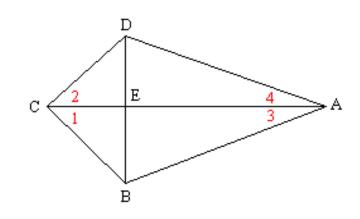
Procedure for Detour Proofs

- 1. Determine which triangles you must prove congruent to reach the desired conclusion
- 2. Attempt to prove those triangles congruent if you cannot due to a lack of information it's time to take a detour...
- 3. Find a different pair of triangles congruent based on the given information
- 4. Get something congruent by CPCTC
- 5. Use the CPCTC step to now prove the triangles you wanted congruent.

Example 2:

Given: $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$

Prove: $\triangle CDE \cong \triangle CBE$



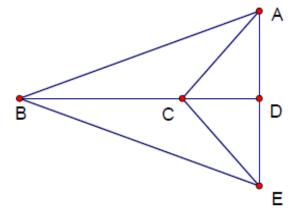
Statements	Reasons
1 ≰1 ≅ ≰2, ≰3 ≅ ≰4	1 Given
2	² Reflexive Property
$^{3}\Delta CBA \cong \Delta CDA$	3
4	4 CPCTC
5	5 Reflexive Property
6 $\Delta CDE \cong \Delta CBE$	6

Example 3:

Given: $\triangle ABC \cong \triangle EBC$

 $\overline{\text{DB}}$ is a median to $\overline{\text{AE}}$

Prove: $\triangle ACD \cong \triangle ECD$



Statements	Reasons
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8

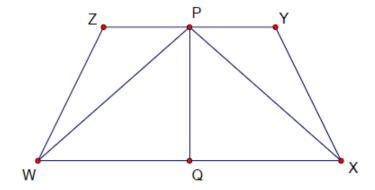
Example 4:

Given: \overline{PQ} bisects \overline{YZ}

Q is the midpoint of $\,\overline{WX}\,$

$$\frac{\angle Y \cong \angle Z}{WZ} \cong \overline{XY}$$

Prove: $\angle WQP \cong \angle XQP$



Statements	Reasons

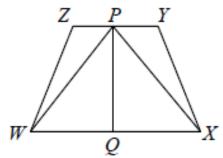
SUMMARY

Given: \overrightarrow{PQ} bisects \overline{YZ} .

Q is the midpt. of \overline{WX} .

$$\angle Y \cong \angle Z, \ \overline{WZ} \cong \overline{XY}$$

Prove: $\angle WQP \cong \angle XQP$



Statements

Reasons

- PQ bisects YZ.
- 2 P is the midpoint of ZY

$$S_1$$
 3. $\overline{ZP} \cong \overline{PY}$

$$A_1$$
 4. $\angle Z \cong \angle Y$

$$S_1$$
 5. $\overline{WZ} \cong \overline{XY}$

6.
$$\Delta ZWP \cong \Delta YXP$$

$$S_2$$
 7. $\overline{WP} \cong \overline{PX}$

8. Q is the midpt. of \overline{WX} .

$$S_2 g$$
, $\overline{WQ} \cong \overline{QX}$

$$S_2$$
 10. $\overline{PQ} \cong \overline{PQ}$

11.
$$\Delta WQP \cong \Delta XQP$$

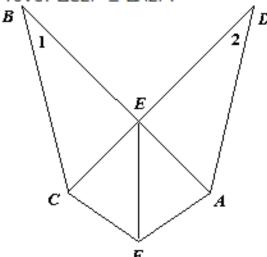
- Given
- 2. Def. of segment bis.
- Def. of midpt.
- 4. Given
- 5. Given
- 6. **SAS** (3,4,5)
- CPCTC
- Given
- 9. Def. of midpt.
- 10. Reflexive Property
- 11. SSS (7,9,10)
- 12. CPCTC

Exit Ticket

Complete the proof.

Given: $\overline{BC} \cong \overline{DA}$, $\angle 1 \cong \angle 2$, and $\overline{CF} \cong \overline{AF}$.

Prove: $\triangle CEF \cong \triangle AEF$.



 $\angle BEC \cong \angle DEA$ by vertical angles. $\triangle BEC \cong \triangle DEA$ by (a)____. Then by CPCTC, $\overline{CE} \cong \overline{AE}$. $\overline{EF} \cong \overline{EF}$ by the Reflexive Property. So $\triangle CEF \cong \triangle AEF$ by (b)____.

- A. a. SAS; b. SAS
- B. a. AAS; b. SSS
- C. a. ASA; b. SSS
- D. a. AAS; b. HL

Day 10 - Missing Diagram Proofs

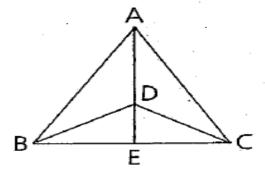
Warm - Up

Given: $\overline{AB} \cong \overline{AC}$;

BD bisects ∠ABE.

 \overrightarrow{CD} bisects $\angle ACE$.

Conclusion: \overline{AE} bisects \overline{BC} .



	1
Statements	Reasons

Many proofs we encounter will not always be accompanied by a diagram or any given information. It is up to us to find the important information, set up the problem, and draw the diagram all by ourselves!!!

Procedure for Missing Diagram Proofs

- 1. Draw the shape, label everything.
 - 2. The "if" part of the statement is the "given."
- 3. The "then" part of the statement is the "prove."
 - 4. Write the givens and what you want to prove.

<u>Example 1</u>: If two altitudes of a triangle are congruent, then the triangle is isosceles.

Given:		
Prove:		

<u>Example 2</u> :	ine medians of a triangle are congruent if the triangle is equilatera
Given:	
Prove:	

Example 3:	the altitude	to the	base of a	an isosceles	triangle
bisects the	vertex angle	-			

Given:			
Prove:			

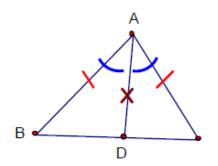
SUMMARY

Example: Prove that the bisector of the vertex angle of an isosceles triangle is also the median to the base.



AD bisects ∠BAC

Prove: AD is a median to base BC.



Statements Reasons

1 ABC is isosceles with base BC.

$$5\overline{AD} \cong \overline{AD}$$

$$6 \Delta BAD \cong \Delta CAD$$

⁸ D is the midpoint of BC.

9 AD is a median to base BC.

Given

² Def. of Isosceles Δ

3 Given

4 Def. of ∠ bisector

5 Reflexive Prop

6 SAS (2, 4, 5)

7 CPCTC

8 Def of midpoint

9. Def'n of a median

Exit Ticket

In $\triangle BAT$ and $\triangle CRE$, $\angle A \cong \angle R$ and $\overline{BA} \cong \overline{CR}$. Write *one* additional statement that could be used to prove that the two triangles are congruent. State the method that would be used to prove that the triangles are congruent.

ANSWER KEYS

EXERCISES, PAGES 227-230

9.
$$m \angle M + m \angle N = m \angle NPQ$$

 $3y + 1 + 2y + 2 = 48$
 $5y + 3 = 48$
 $5y = 45$
 $y = 9$
 $m \angle M = 3y + 1 = 3(9) + 1 = 28^{\circ}$

10.
$$m \angle K + m \angle L = m \angle HJL$$

 $7x + 6x - 1 = 90$
 $13x = 91$
 $x = 7$
 $m \angle L = 6x - 1 = 6(7) - 1 = 41^{\circ}$

11.
$$m\angle A + m\angle B = 117$$

 $65 + m\angle B = 117$
 $65 + m\angle B = 117$
 $m\angle B = 52^{\circ}$
 $m\angle A + m\angle B + m\angle BCA = 180$
 $117 + m\angle BCA = 180$
 $m\angle BCA = 63^{\circ}$

12.
$$\angle C \cong \angle F$$
 13. $\angle S \cong \angle U$ $m\angle C = m\angle F$ $m\angle S = m\angle U$ $S = m\angle U$ $S = m\angle U$ $S = m\angle C = m\angle C = 100^\circ$ $S = m\angle C = 100^\circ$

14.
$$\angle C \cong \angle Z$$

 $m \angle C = m \angle Z$
 $4x + 7 = 3(x + 5)$
 $4x + 7 = 3x + 15$
 $x = 8$
 $m \angle C = 4x + 7 = 4(8) + 7 = 39$
 $m \angle Z = m \angle C = 39^{\circ}$

19. Think: Use Ext.
$$\angle$$
 Thm.
 $m \angle W + m \angle X = m \angle XYZ$
 $5x + 2 + 8x + 4 = 15x - 18$
 $13x + 6 = 15x - 18$
 $24 = 2x$
 $x = 12$
 $m \angle XYZ = 15x - 18$
 $= 15(12) - 18 = 162^{\circ}$

20. Think: Use Ext.
$$\angle$$
 Thm and subst. $m\angle C = m\angle D$. $m\angle C + m\angle D = m\angle ABD$ $2m\angle D = m\angle ABD$ $2(6x - 5) = 11x + 1$ $12x - 10 = 11x + 1$ $x = 11$ $m\angle C = m\angle D$ $= 6x - 5$ $= 6(11) - 5 = 61^{\circ}$

21. Think: Use Third & Thm.

$$\angle N \cong \angle P$$

 $m \angle N = m \angle P$
 $3y^2 = 12y^2 - 144$
 $-9y^2 = -144$
 $y^2 = 16$
 $m \angle N = 3y^2 = 3(16) = 48^\circ$
 $m \angle P = m \angle N = 48^\circ$

22. Think: Use Third & Thm.

41. C

$$128 = 71 + x$$

 $x = 57$

42. F
$$(2s + 10) + 58 + 66 = 180$$

$$2s + 134 = 180$$

$$2s = 46$$

$$s = 23$$

45.
$$117 = (2y^2 + 7) + (61 - y^2)$$

 $117 = y^2 + 68$
 $49 = y^2$
 $y = 7 \text{ or } -7$

49. Let
$$m \angle A = x^{\circ}$$
.
 $m \angle B = 1\frac{1}{2}(x) - 5$
 $m \angle C = 2\frac{1}{2}(x) - 5$
 $m \angle A + m \angle B + m \angle C = 180$
 $x + 1\frac{1}{2}(x) - 5 + 2\frac{1}{2}(x) - 5 = 180$
 $5x - 10 = 180$
 $5x = 190$
 $x = 38$
 $m \angle A = x^{\circ} = 38^{\circ}$

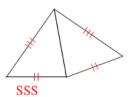
Day 2 Answers

State if the two triangles are congruent. If they are, state how you know.

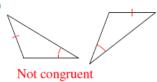
1)



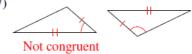
3)



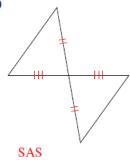
5)



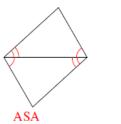
7)



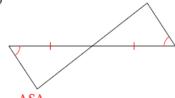
9)



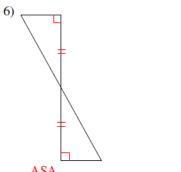
2)



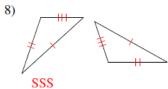
4)

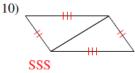


ASA



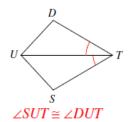
ASA



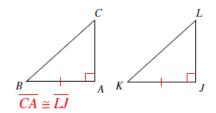


State what additional information is required in order to know that the triangles are congruent for the reason given.

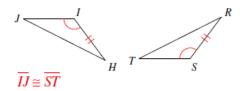
11) ASA



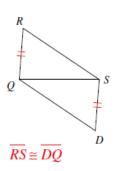
13) SAS



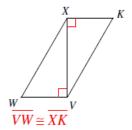
15) SAS



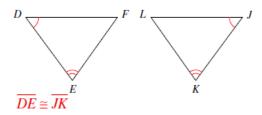
17) SSS



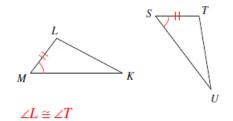
12) SAS



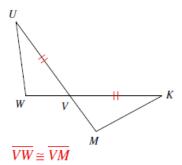
14) ASA



16) ASA



18) SAS



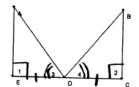
Day 3 – Answers

Practice with Congruent Triangles

1. Given: $\overline{AE} \perp \overline{ED}$

D is the midpoint of \overline{EC} .

∡3 ≅ **∡4** Prove: △AED ≅ △BCD



FELED, BC 1 CD 71 and 22 arei

right & y

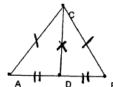
41242 (A)

D is the mapt of EC

Ogiven

3) all rt x's =

Given: $\overline{AC} \cong \overline{CB}$ \overline{CD} Bisects \overline{AB} Prove: $\triangle ADC \cong \triangle BDC$



Statements

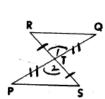
D bixeti AB

bixeti AB Dgiven the mapt of AB 3 def. of xg bixedar

\AD(≅∆BD(

Reasons

3. Given: \overline{RS} bisects \overline{PQ} at T, \overline{PQ} bisects \overline{RS} at T. Prove: $\triangle PTS \cong \triangle QTR$.



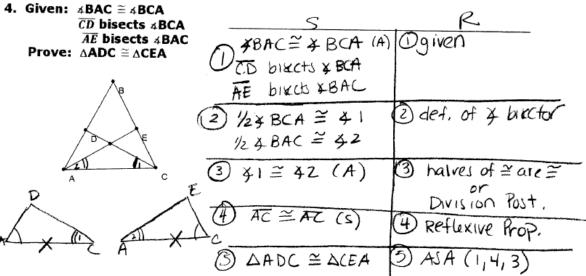
Statements

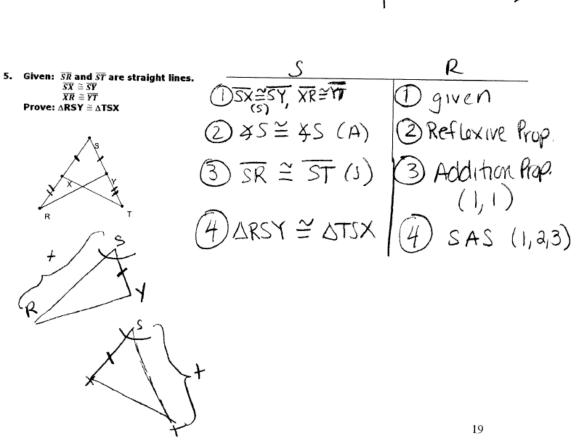
ORS bixets POATT

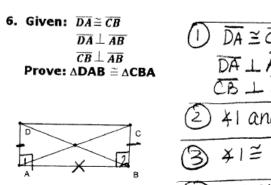
T is the mapt of PQ

- - 19 given

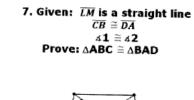
DPTS = DOTR

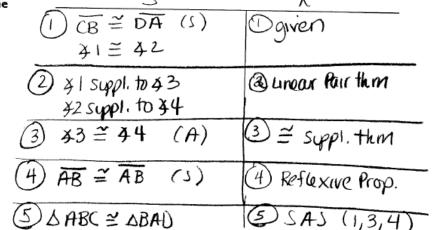




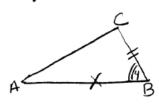


<u> </u>	R
1) DA \(\text{ZB}\) (S)	Ogiven
DALAB	
CB _ AB	
2 ×1 and ×2 rt x's	@def of 1 lines
3 x1= x2 (A)	3 all right 41 =
	4) Restuxive Prop.
5 DDAB ≅ DCBA	5 SAS (1, 3,4)





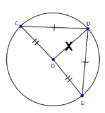




Answers to Day 4

6. Given:
$$\bigcirc$$
 O , \overline{CD} \cong \overline{DE}

Prove: $\angle COD \cong \angle DOE$



Statements

_						
R	Δ	2	c	O	n	¢
-	C	а	-	v		e

1. \bigcirc 0, \overline{C}	$\overline{D} \cong \overline{DE}$	(S)
---------------------------------	------------------------------------	-----

2.
$$\overline{CO} \cong \overline{EO}$$

3. $\overline{DO} \cong \overline{DO}$

4.
$$\triangle COD \cong \triangle EOD$$

2. All radii of a
$$⊙$$
 are $≅$

12 Given: H is the midpt. of
$$\overline{GJ}$$
.
M is the midpt. of \overline{OK} .

$$\overline{GO} \cong \overline{JK}$$

$$\overline{GJ} \cong \overline{OK}$$
.

$$\angle G \cong \angle K$$
,

$$OK = 27$$
,

$$m\angle GOH = x + 24$$
, $m\angle GHO = 2y - 7$,

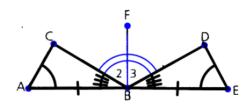
$$m \angle JMK = 3y - 23$$
, $m \angle MJK = 4x - 105$

Find: m∠GOH, m∠GHO, and GH

$$24 = 3x - 105$$

$$24-7=34-23$$

- 13 Given: $\angle A \cong \angle E$, $\overline{AB} \cong \overline{BE}$, $\overline{FB} \perp \overline{AE}$, $\angle 2 \cong \angle 3$
 - Prove: $\overline{CB} \cong \overline{DB}$

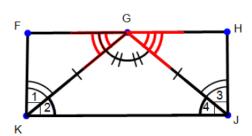


Statements

Reasons

- $\frac{1}{AB} \cong \angle E, (A)$ $\frac{1}{AB} \cong \overline{BE}, (S)$
 - $\overline{FB} \perp \overline{AE}$, (3)
 - $\angle 2 \cong \angle 3$
- 2. 4FBA and 4FBE are rt 4s
- **3.** ≰1 compl. ≰2 ≰3 compl. ≰4
- 4. ≰1 ≅ ≰4 (A)
- 5. $\triangle ACB \cong \triangle EDB$
- 6. $\overline{CB} \cong \overline{DB}$

- 1. Given
- 2. \perp lines \rightarrow right $\not \perp$
- 3. If 2 $\not 4s$ form a rt $\not 4$, $\rightarrow \not 4s$ compl.
- 4. Congruent Compl. Thm
- 5. ASA (1, 1, 4)
- 6. CPCTC
- 18 Given: $\overline{KG} \cong \overline{GJ}$, $\angle 2 \cong \angle 4$, $\angle 1$ is comp. to $\angle 2$. $\angle 3$ is comp. to $\angle 4$. $\angle FGJ \cong \angle HGK$
 - Conclusion: $\overline{FG} \cong \overline{HG}$



Statements

Reasons 1. Given

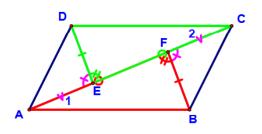
- 1. $\overline{KG} \cong \overline{GJ}$, (S)
 - $\angle 2 \cong \angle 4$
 - $\angle 1$ is comp. to $\angle 2$.
 - $\angle 3$ is comp. to $\angle 4$.
 - $\angle FGJ \cong \angle HGK$
- 2. $\angle 1 \cong \angle 3$ (A)
- 3. $\angle KGJ \cong \angle KGJ$
- 4. $\angle FGK \cong \angle HGJ$ (A)
- 5. $\triangle FGK \cong \triangle HGJ$
- 6. $\overline{FG} \cong \overline{HG}$

- 2. Congruent Compl. Thm
- 3. Reflexive Property
- 4. Subtraction Postulate (1, 3)
- 5. ASA (2, 1, 4)
- 6. CPCTC

21 Given: $\overline{AE} \cong \overline{FC}$, $\overline{FB} \cong \overline{DE}$,

 $\angle CFB \cong \angle AED$

Prove: $\angle 1 \cong \angle 2$



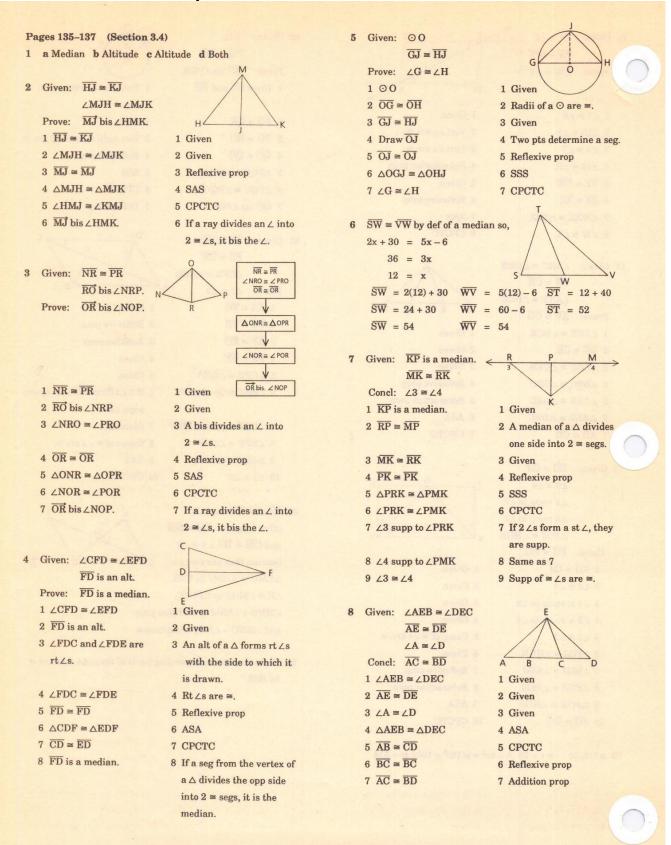
Statements

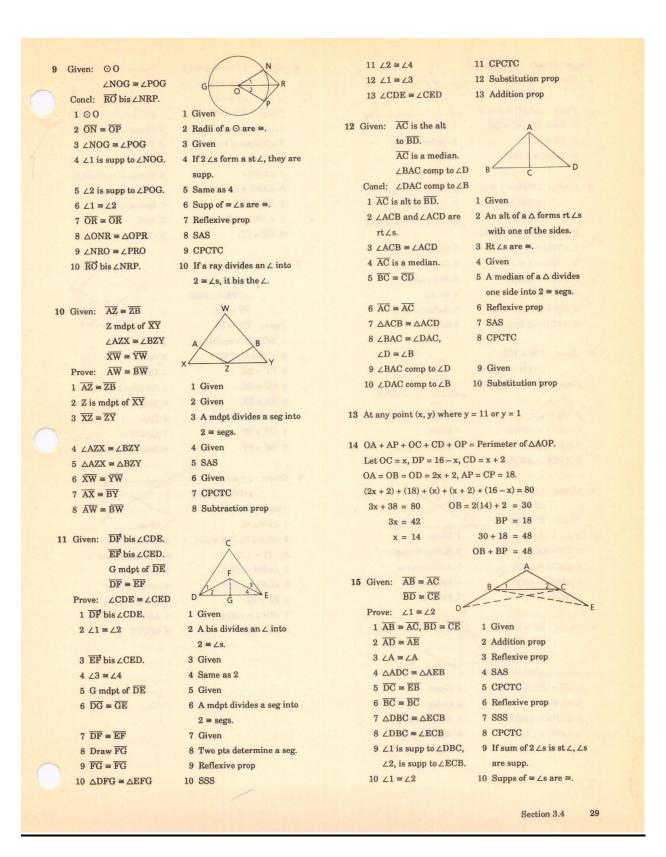
Reasons

- 1. $\overline{AE} \cong \overline{FC}$, $\overline{FB} \cong \overline{DE}$, (S) $\angle CFB \cong \angle AED$
- 2. $\overline{EF} \cong \overline{EF}$
- 3. $\overline{AF} \cong \overline{CE}$ (S)
- **4.** *△CFB Suppl. △BFA △AED Suppl. △DEC*
- 5. $\angle BFA \cong \angle DEC$ (A)
- 6. $\triangle BFA \cong \triangle DEC$
- 7. ≰1 ≅ ≰2

- 1. Given
- 2. Reflexive Property
- 3. Addition Postulate (1, 2)
- 4. Linear Pair Thm
- 5. Congruent Suppl. Thm
- 6. SAS (1, 5, 3)
- 7. CPCTC

Answers to Day 5





Answers to Isosceles Δ HW Day 6

20. Given:

∠A is the vertex of an isosceles △

The number of degrees in $\angle B$ is twice the number of centimeters in \overline{BC}

The number of degrees in ∠C is three times the number of centimeters is AB

$$m_{\angle}B = x + 6$$

 $m_{\angle}C = 2x - 54$

Find:

The perimeter of △ABC

$$x + 6 = 2x - 54$$

$$\Rightarrow x = 60$$

:. AB = AC =
$$\frac{2(60) - 54}{3}$$
 = 22
and
BC = $\frac{60 + 6}{3}$ = 33

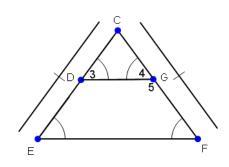
 $P_{\triangle ABC} = 2(22) + 33 = 77 \text{ cm}$

21.

Given: $\overline{CE} \cong \overline{CF}$

∡E is supp.to ∡5

Prove: ΔCDG is isosceles



Statements

Reasons

1 $\overline{CE} \cong \overline{CF}$

∡E is supp.to ∡5

$$2. \angle E \cong \angle F$$

$$3. \not\Delta E \cong \not\Delta 3$$

5.
$$\not \Delta E \cong \not \Delta 4$$

7.
$$\overline{CD} \cong \overline{CG}$$

8. ΔCDG is Isosceles

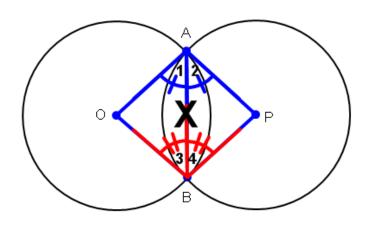
1. Given

2. If \triangle , then \triangle

- 3. Transitive Prop. (1, 2)
- 4. Linear Pair Thm
- 5. Congruent Suppl. Thm
- 6. Transitive Prop. (3, 5)
- 7. If \triangle , then \triangle
- 8. Definition of *Isosceles* Δ

23. Given: $\bigcirc O$, $\bigcirc P$; \overrightarrow{AB} bisects $\not AS$ OAP and OBP.

Prove: Figure AOBP is equilateral.



Statements

1. \bigcirc 0, \bigcirc P; \overrightarrow{AB} bisects \angle s OAP and OBP.

(A) 2.
$$\angle 1 \cong \angle 2$$

$$(A) \quad \cancel{4}3 \cong \cancel{4}4$$

(S) 3.
$$\overline{AB} \cong \overline{AB}$$

4.
$$\triangle AOB \cong \triangle APB$$

5.
$$\overline{AO} \cong \overline{AP}$$

$$\overline{BO} \cong \overline{BP}$$

6.
$$\overline{AO} \cong \overline{BO}$$

$$\overline{AP} \cong \overline{BP}$$

7.
$$\overline{AO} \cong \overline{BO} \cong \overline{AP} \cong \overline{BP}$$

8. Figure AOBP is equilateral

Reasons

- 1. Given
- 2. Definition of angle bisector
- 3. Reflexive Property
- 4. ASA (2, 3, 2)
- 5. CPCTC
- 6. All radii of a \odot are \cong
- 7. Transitive Prop. (5, 6)
- 8. If a figure has all sides $\cong \rightarrow equil$.

24 Given: Figure XSTOW is

equilateral and

equiangular.

Prove: AYTO is isos.

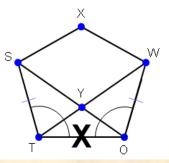
 XSTOW is equilateral and equiangular.

(S)
$$2\overline{ST} = \overline{WO}$$

(S)
$$3\overline{TO} \cong \overline{TO}$$

$$7 \overline{\text{TY}} \approx \overline{\text{YO}}$$

8 ΔΥΤΟ is isos.



- 1 Given
- 2 If a figure is equilateral, all sides are ≅.
- 3 Reflexive prop
- 4 If a figure is equiangular, all ∠s are ≅.
- 5 SAS (2, 4, 3)
- 6 CPCTC
- 7 If A then A
- 8 If a △ has at least 2 sides =, the △ is isos.

Given: △FED is equilateral

Find: x, y, and m∠F

$$6y + 12 = 3x - 6$$
 $(x + y) + (3x - 6) = 90$

$$\Rightarrow 6y = 3x - 18 \qquad \Rightarrow 4x + y = 96$$

$$\Rightarrow y = \frac{1}{2}x - 3 \longrightarrow 4x + \left(\frac{1}{2}x - 3\right) = 96$$

$$\Rightarrow \frac{8}{2}X + \frac{1}{2}X = 99$$

$$\Rightarrow \frac{9}{2} x = 99$$

$$\Rightarrow$$
 y = $\frac{1}{2}(22) - 3 = 8$

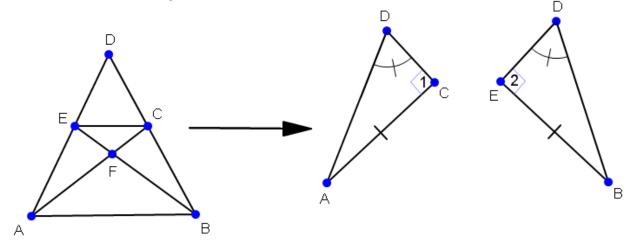
$$m_{2}F = 6(8) + 12 = 60^{\circ}$$

Page 160 #16

Given: $\overline{BE} \perp \overline{AD}$, $\overline{AC} \perp \overline{BD}$,

 $\overline{AC} \cong \overline{BE}, \ \overline{DE} \cong \overline{EC}$

Prove: ΔDEC is equilateral



Statements

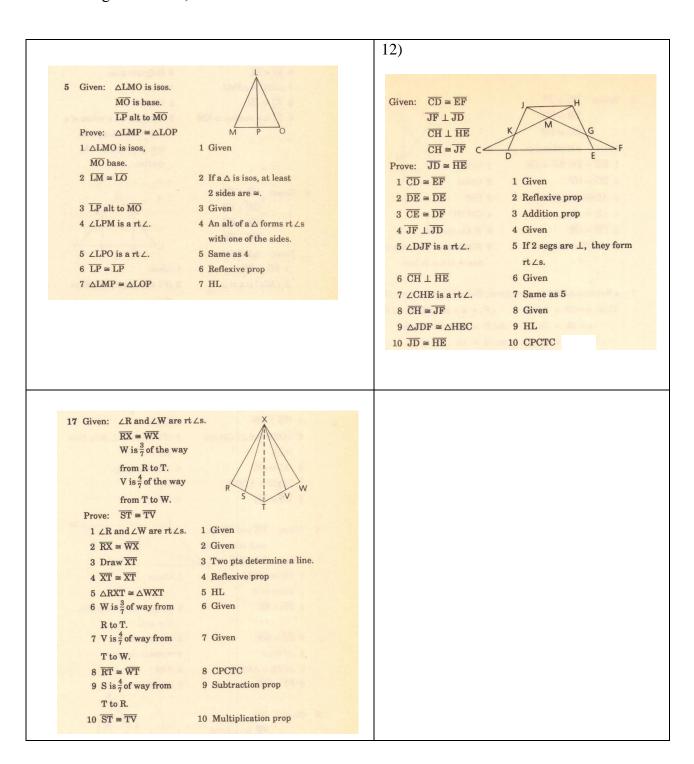
- (S) 1. $\overline{BE} \perp \overline{AD}$, $\overline{AC} \perp \overline{BD}$, $\overline{AC} \cong \overline{BE}$, $\overline{DE} \cong \overline{EC}$
 - 2. $\angle 1$ and $\angle 2$ are rt $\angle s$
- (A) 3. $\angle 1 \cong \angle 2$
- (A) 4. $\angle D \cong \angle D$
 - 5. $\triangle DAC \cong \triangle DEB$
 - 6. $\overline{DC} \cong \overline{DE}$
 - 7. $\overline{DC} \cong \overline{DE} \cong \overline{EC}$
 - 8. $\triangle DEC$ is equilateral

Reasons

- 1. Given
- 2. Def of \perp *lines*
- 3. all right $4s \cong$
- 4. Reflexive Property
- 5. AAS (3, 4, 1)
- 6. CPCTC
- 7. Transitive Prop. (1, 6)
- 8. If a figure has all sides $\cong \rightarrow equil$.

Proving Triangles Congruent with Hypotenuse Leg

Page 158 #'s 5, 12 and 17



Right Angle Theorem and Equidistance Theorems

Pages 182 – 183 #'s 4, 9, 14

4 Given: XR bis ∠YXZ.

 $\angle Y \cong \angle Z$

Concl: XR is an alt.

1 XR bis∠YXZ.

2 ∠YXR ≃ ∠ZXR

 $3 \angle Y \cong \angle Z$

 $4 \overline{XY} \cong \overline{XZ}$

 $5 \triangle YXR \cong \triangle ZXR$

6 ∠YRX ≃ ∠ZRX $7 \overline{XR} \perp \overline{YZ}$

8 XR is an alt.

1 Given

2 A bis divides an ∠ into

2 ≃ ∠s.

3 Given

4 If A then A

5 ASA

6 CPCTC

7 If 2 lines intersect to form

 \cong adj \angle s, the lines are \bot .

8 If a seg is drawn from a vertex of a △ and is ⊥ to

the opp side, it is an alt of

the \triangle .

9 Given: 00

 $\angle B \cong \angle C$

Concl: AO ⊥ BC

100

 $2 \angle B \cong \angle C$

 $3 \overline{AC} \cong \overline{AB}$

4 Draw CO and BO

 $5 \overline{CO} \cong \overline{BO}$

 $6 \overline{AO} \cong \overline{AO}$

7 △AOC ≅ △AOB

8 41 = 42

 $9 \triangle AEC \cong \triangle AEB$

10 ∠AEC ≃ ∠AEB

11 AO ⊥ BC

1 Given

2 Given

3 If A then A

4 Two pts determine a seg.

5 Radii of a ⊙ are =.

6 Reflexive prop

7 SSS

8 CPCTC

9 SAS

10 CPCTC

11 If 2 lines intersect to form

 \cong adj \angle s, the lines are \bot .

14 If $b \perp a$, $(2x + 37)^{\circ} = 90^{\circ}$, $(2x + y)^{\circ} = 90^{\circ}$, and

 $(3y - 21)^{\circ} = 90^{\circ}$.

2x + 37 + 2x + y = 180, so 4x + y = 143.

Solving the 2 equations gives $x = 26\frac{1}{2}$ and y = 37.

 $2x + y = 2(26\frac{1}{2}) + 37 = 90$

So a 1 b.

2x + y + 3y - 21 = 180, so 2x + 4y = 201.

 $2x + 37 = 2(26\frac{1}{2}) + 37 = 90$

3y - 21 = 3(37) - 21 = 90

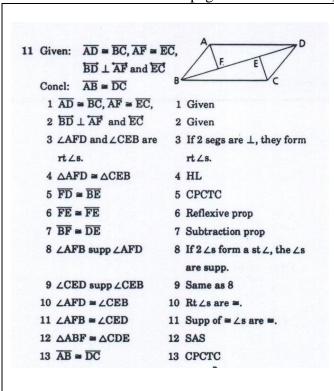
14 Given: WXYZ is a kite. 15 Given: ∠ADC and ∠ABC $\overline{WX} \cong \overline{WZ}, \overline{XY} \cong \overline{YZ}$ are rt∠s. Prove: △WPZ is a rt △. $\overline{AB} \cong \overline{AD}$ 1 WXYZ is a kite. 1 Given Concl: $\overrightarrow{AC} \perp \text{bis } \overline{BD}$. $\overline{WX} \cong \overline{WZ}, \overline{XY} \cong \overline{YZ}.$ 1 ∠ADC and ∠ABC are 1 Given $2 \overline{WY} \perp \overline{ZX}$ 2 Two pts =dist from rt∠s. endpts of a seg determine 2 ∠ADC ≅ ∠ABC 2 Rt∠s are ≅ $3 \overline{AB} \cong \overline{AD}$ ⊥ bis of that seg. 3 Given $4 \overline{AC} \cong \overline{AC}$ 4 Reflexive prop 5 △ADC ≃ △ABC 5 HL 3 ∠WPZ is a rt ∠. 3 ⊥ lines intersect to form $6 \ \overline{DC} \cong \overline{BC}$ 6 CPCTC 7 AC ⊥ bis BD. 7 Two pts =dist from 4 ΔWPZ is a rt Δ. 4 If a △ contains a rt ∠. endpts of a seg determine the \(\price \) bis of the seg. then it is a rt \triangle . 17 Given: F mdpt BC 16 Given: △ABC is isos, $\overline{DB} \cong \overline{EC}$ base BC. $\overline{DB} \perp \overline{DF}$ AD median BC $\overline{EC} \perp \overline{EF}$ Prove: AD is alt to BC. Concl: AF L BC 1 △ABC is isos, 1 Given base BC. 1 F mdpt BC 1 Given 2 AD median BC 2 Given $2 \overline{BF} \cong \overline{CF}$ 2 A mdpt divides a seg into $3 \overline{AB} \cong \overline{AC}$ 3 An isos △ has 2 sides ≥ 2 ≅ segs. $4 \ \overline{BD} \cong \overline{CD}$ 4 A median divides a seg $3 \overline{DB} \cong \overline{EC}$ 3 Given into 2 ≅ segs. 5 $\overline{AD} \perp \text{bis } \overline{BC}$. 4 $\overline{DB} \perp \overline{DF}$, $\overline{EC} \perp \overline{EF}$ 5 Two pts =dist from 4 Given endpts of a seg determine 5 ∠FDB is a rt ∠. 5 \(\precedit \) lines intersect to form rt. the \perp bis of that seg. 6 LFEC is a rt L. 6 Same as 5 6 AD is alt to BC. 6 A seg from a vertex of a \triangle ⊥ to opp side is an alt of 7 HL 7 △DBF = △ECF 8 CPCTC 8 ∠B ≃ ∠C 9 If A then A $9 \overline{AB} \cong \overline{AC}$ 10 Two pts =dist from the 10 AF L BC endpts of a seg determine the \perp bis of the seg (\overline{BC}). 20 Given: AB ≅ BC $\overline{AE} \cong \overline{EC}$ Concl: AD ≈ DC $1 \overline{AB} \cong \overline{BC}, \overline{AE} \cong \overline{EC}$ 1 Given 2 Two pts determine a line. 2 Draw AC 3 BE ⊥ bis AC. 3 Two pts =dist from the endpts of a seg determine the 1 bis of the seg. $4 \overline{AD} \cong \overline{DC}$ 4 A pt on the 1 bis of a seg

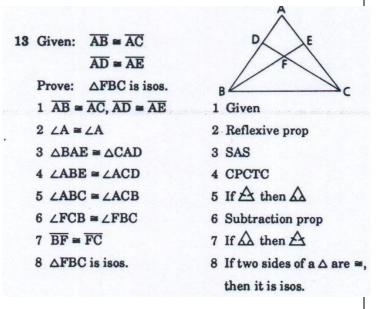
is =dist from the endpts

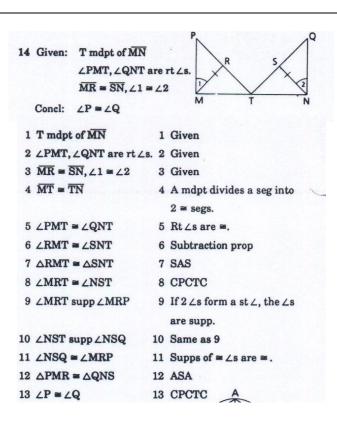
of the seg.

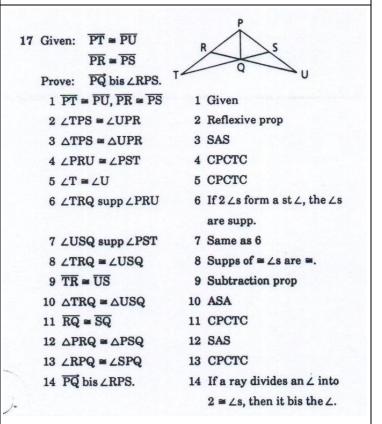
Answers to Detour Proofs

Detour Proofs pages 174- 175 #'s 11, 13, 14, 17

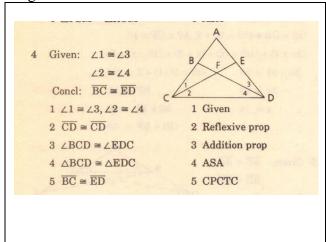












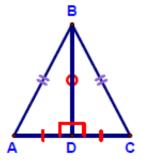
٥.

If the median to a side of a Δ is also an altitude to that side, then the Δ is isosceles.

Given: BD is a median

BD is an altitude

Prove: ABC is isosceles



Reasons

Statements

- 1. BD is a median
- 2. D is the midpoint of CA
- S 3. AD ≅ DC
 - 4. BD is an altitude
 - 5. BD ⊥ CA
 - 6. ∠BDA and ∠BDC are right ∠s
- A 7. ∠BDA ≅ ∠BDC
- S 8. BD ≅ BD
 - 9. △ABD ≅ △CBD
 - 10. BA ≅ BC
 - 11. AABC is isosceles

1. Given

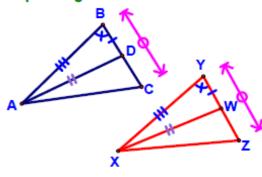
- 2. Definition of Median
- 3. Definition of Midpoint
- 4. Given
- 5. Defn. of Altitude
- 6. Defn. of ⊥ Segments
- 7. All Right Angles are Congruent
- 8. Reflexive Property
- 9. SAS (2, 5, 6)
- 10. CPCTC
- 11. Definition of Isosceles △

Prove that if 2 \triangle s are \cong , then any pair of corresponding medians are \cong .

Given: △ABC ≅ △XYZ

AD & XW are medians

Prove: AD ≅ XW



Reasons Reasons

Statements Statements

- 1. △ABC ≃ △XYZ
- S 2. AB ≅ XY
- A 3. ∠B ≅ ∠Y
 - 4. BC ≃ YZ
 - 5. AD & XW are medians
 - 6. D & W are midpoints
 - 7. $\overline{BD} \cong \frac{1}{3}\overline{BC}$, $\overline{YW} \cong \frac{1}{2}\overline{YZ}$
- S 8. BD ≈ YW
 - 9. △ABD ≅ △XYW
 - 10. AD ≃ XW

- 1. Given
- 2. CPCTC
- 3. CPCTC
- 4. CPCTC
- 5. Given
- 6. Definition of Median of △
- 7. Definition of Midpoint
- 8. Division Property of ≅ Segments
- 9. SAS (2, 3, 8)
- 10. CPCTC

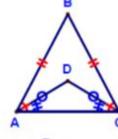
12.

Prove that if a \triangle is isosceles, then the \triangle formed by its base and the \angle bisectors of its base \angle s is also isosceles.

Given: △ABC is isosceles with vertex ∠ABC

AD & CD are _ bisectors

Prove: AADC is isosceles



Reasons

Statements

- 1. △ABC is isos, with vertex ∠ABC
- 2. AB = CB
- 3. ∠BAC = ∠BCA
- 4. AD & CD are _ bisectors
- 5. $\angle DAC \cong \frac{1}{2} \angle BAC$, $\angle DCA \cong \frac{1}{2} \angle BCA$
- 6. ZDAC = ZDCA
- 7. AD a CD
- 8. AADC is isosceles

- 1. Given
- 2. Definition of Legs of Isosceles △
- 3. If A, then A
- 4. Giver
- 5. Definition of Angle Bisector
- 6. Division Property of a Angles
- 7 If A. then A
- 8. Definition of Isosceles A

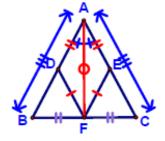
Prove that if a point on the base of an isos. \triangle is equidistant from the midpoints of the legs, then that point is the midpoint of the base.

Given: △ABC is isos, with vertex ∠BAC

D & E are midpoints

F is equidistant from D & E

Prove: F is the midpt. of BC



Statements

- 1. F is equidistant from D & E
- S 2. FD ≅ FE
 - 3. △ABC is isos, with vertex ∠BAC
 - S 4. AB ≅ AC
 - 5. D & E are midpoints
 - **6.** $\overline{AD} \cong \frac{1}{2}\overline{AB}$, $\overline{AE} \cong \frac{1}{2}\overline{AC}$
- S 7. AD ≅ AE
 - 8. Draw AF
- SS 9. AF ≅ AF
 - 10. △ADF ≅ △AEF
 - A 11.∠DAF ≅ ∠EAF
 - 12. △ABF ≅ △ACF
 - 13. BF ≃ FC
 - 14. F is the midpoint of BC

Reasons

- Given
- 2. Definition of Equidistant
- Giver
- Definition of Isosceles ∆
- 5. Given
- 6. Definition of Midpoint
- 7. Division Property of ≅ Segments
- 8. 2 points determine a line
- 9. Reflexive Property
- 10. SSS (2, 7, 9)
- 11. **CPCTC** (from 10)
- 12. SAS (4,11,9)
- 13. CPCTC (from 12)
- 14. Definition of Midpoint