Conic Sections

Unit 7

Unit Overview

In this unit you will investigate the curves formed when a plane intersects a cone. You will graph these curves known as the conic sections and you will identify conic sections by their equations.

Academic Vocabulary

Add these words and others you encounter in this unit to your vocabulary notebook.

- conic section
- ellipse
- hyperbola
- quadratic relation
- standard form

Essential Questions

How are the algebraic representations of the conic sections similar and how are they different?

How do the conic sections model real world phenomena?

EMBEDDED ASSESSMENTS

This unit has one embedded assessment, following Activity 7.5. It will give you the opportunity to demonstrate your ability to recognize and graph circles, ellipses, parabolas and hyperbolas.

Embedded Assessment 1	
Conic Sections	p. 409

UNIT 7

Getting Ready

Write your answers on notebook paper. Show your work.

1. Describe the geometric representation of each equation.

a. $5x + 3y = 30$

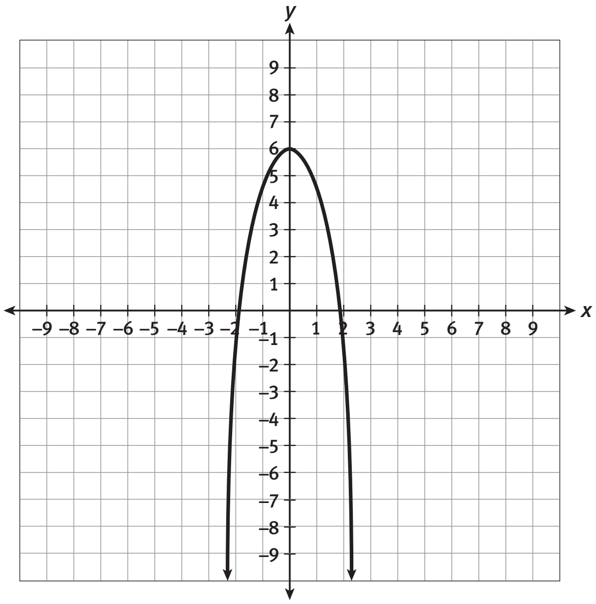
b. $x = 9$

c. $2(x - 2y) = 2x + y - 10$

2. Given the four quadratic equations below, explain how you could determine which equation is represented by the graph displayed.

$y = 2x^2 + 6$ $y = 2x^2 - 6$

$y = -6 - 2x^2$ $y = 6 - 2x^2$



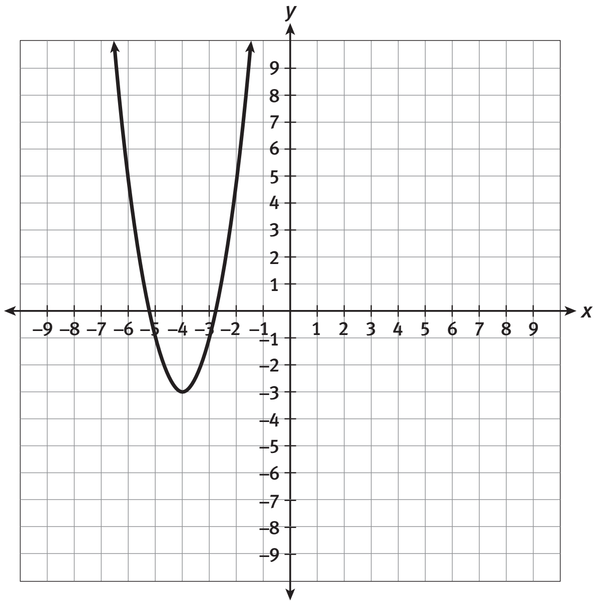
3. Model the process for completing the square using the equation $x^2 + 6x - 11 = 0$.

4. Write the equations of the diagonals of a rectangle that has vertices $(-2, 4)$, $(2, 4)$, $(-2, 2)$, and $(2, 2)$.

5. Find the distance between $(-5, 3)$ and $(2, 6)$.

6. Simplify $3(x - 5)^2$.

7. Identify 2 pairs of points that are symmetric about the line of symmetry in the parabola below.



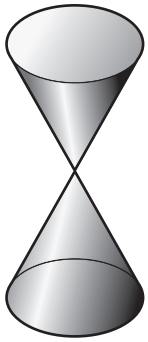
8. Simplify.

a. $\sqrt{200}$ **b.** $\sqrt{147}$

The Conic Sections
It's How You Slice It

ACTIVITY
7.1

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Use Manipulatives, Summarize/Paraphrase/Retell, Discussion Group, Group Presentation



In the 3rd century BCE the Greek mathematician Apollonius wrote an eight volume text, *Conic Sections*, detailing curves formed by the intersection of a plane and a double cone. Nearly two millennia later Johannes Kepler used one of these intersections to model the path planets follow when orbiting the sun. René Descartes also studied the work of Apollonius, discovering that the coordinate system he created, the Cartesian Plane, could be applied to the conic sections and each could be represented by a quadratic relation.

Follow the instructions for the figures your teacher has assigned.

Figure One

Materials:
Piece of plain paper
Index card
Scissors

Instructions:

1. In the center of a plain piece of paper, place a point and label it C.
2. Using one corner of an index card as a right angle cut the index card to form a right triangle.
3. Label the vertex of the right angle of the triangle Q and the vertices of the acute angles P_1 and P_2.
4. Place P_1 on C and mark the point on the paper where P_2 falls.
5. Repeat step four 25–30 times keeping P_1 on C and moving P_2 to different locations on the paper.
6. Join the points formed by P_2 with a smooth curve to form a closed geometric figure.
7. Using the definitions of the conic sections in the My Notes section, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.

1. a. How would the resulting figure change if P_2 were placed on C and the mark was made where P_1 falls?

b. Explain how the work you did to create your figure models the definition of the curve you created.

My Notes

ACADEMIC VOCABULARY

conic sections

MATH TERMS

A **circle** is the set of all points in a plane that are equidistant from a fixed point.

ACADEMIC VOCABULARY

An **ellipse** is the set of all points in a plane such that the sum of the distances from each point to two fixed points is a constant.

ACADEMIC VOCABULARY

A **hyperbola** is the set of all points in a plane such that the absolute value of the differences from each point to two fixed points is constant.

MATH TERMS

A **parabola** is the set of points in a plane that are equidistant from a fixed point and a fixed line.

My Notes

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Use Manipulatives, Summarize/Paraphrase/Retell, Discussion Group, Group Presentation

Figure Two

Materials:

Piece of plain paper

Piece of string between 3 and 8 inches long

Tape or tacks

Instructions:

1. Draw a line on the paper.
2. Place two points on the line and label them F_1 and F_2.
3. Using tape or tacks secure one end of the string to F_1 and the other end of the string to F_2.
4. Use a pencil to pull the string tight.
5. With the tip of the pencil on the paper and keeping the string tight, move the pencil until a closed geometric figure is formed.
6. Using the definitions of the conic sections in the My Notes section on page 379, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.

2. a. What would happen if F_1 and F_2 were closer to each other?

b. Explain how the work you did to create your figure models the definition of the curve you created.

Figure Three

Materials:

Piece of plain paper, waxed paper or patty paper

Instructions:

1. Label the top of one side of the paper *A*. Then turn the paper over as you would turn the page of a book and label the top of the other side of the paper *B*.
2. Place a point on side *A* about a third of the way down the page and in the middle. Label the point *F*.
3. On side B, place 25 points along the bottom edge of the page. The points should be evenly spaced out across the bottom of the page.
4. Fold the paper so that one point on the bottom falls on point *F* and crease the paper.
5. Repeat Step 4 for each point on the bottom of side *B*.
6. With a pencil trace the smooth curve formed by these folds.
7. Using the definitions of the conic sections in the My Notes section on page 379, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.

3. Explain how the work you did to create your figure models the definition of the curve you created.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Visualization, Use Manipulatives, Summarize/Paraphrase/Retell, Discussion Group, Group Presentation

My Notes

Figure Four

Materials:
Piece of plain paper
Compass and straight edge

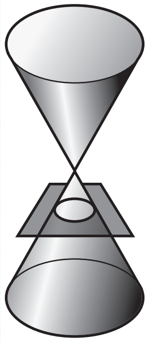
Instructions:

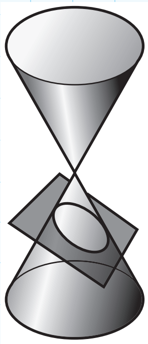
1. Draw a line, *l*, across the center of a piece of plain paper.
2. Place two points on the line and label them $F_1$and F_2.
3. Fold F_1 onto F_2 to find the midpoint of $\overline{F_1F_2}$ and mark the midpoint *C*.
4. Pick a length, *x*, that is less than the length of $\overline{F_1F_2}$ and greater than the length of $\overline{F_1C}$ or $\overline{CF_2}$.
5. Place the point of a compass on F_1 and using the compass, mark a point *x* units from F_1 on $\overline{F_1F_2}$.
6. Place the point of a compass on F_2 and using the compass, mark a point *x* units from F_2 on $\overline{F_1F_2}$.
7. Label the points identified in steps 5 and 6 V_1 and V_2.
8. Pick two numbers, *a* and *b*, so that $|a - b| = x$.
9. Assign a convenient unit of length for *a* and *b*. Set the pencil point and the compass point *a* units apart. Place the point of a compass on F_1 and draw an arc extending above and below line, *l*.
10. Move the point of the compass to F_2 and draw an arc of radius *a* extending above and below line, *l*.
11. Set the pencil point and the compass point *b* units apart. Place the point of a compass on F_1 and draw an arc of radius *b* extending above and below line, *l*.
12. Move the point of the compass to F_2 and draw an arc of radius *b* extending above and below line, *l*.
13. Place a point where the arcs of radius *a* intersect the arcs of radius *b*. You should have 4 points.
14. Repeat steps 8 through 13 with 3 additional values of *a* and *b*.
15. With a pencil connect the points to form two smooth curves.
16. Using the definitions of the conic sections in the My Notes section on page 379, identify the figure you created, sketch the figure in the space above these instructions, and, near the figure, write its name.

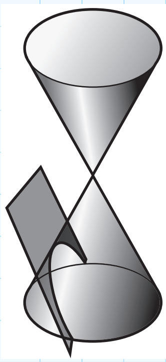
4. Explain the work you did to create your figure models the definition of the curve you created.

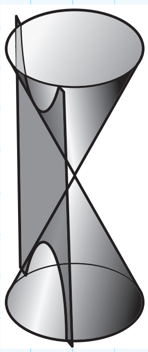
SUGGESTED LEARNING STRATEGIES: Visualization, Think/Pair/Share

My Notes

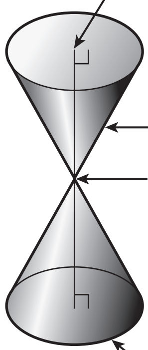








The four conic sections you have created are known as non-degenerate conic sections. A point, a line, and a pair of intersecting line are known as **degenerate conics**.



The figures to the left illustrate a plane intersecting a double cone. Label each conic section as an ellipse, circle, parabola or hyperbola.

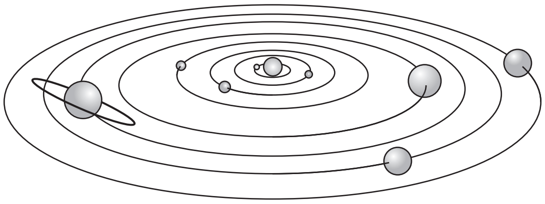
5. Describe the way in which a plane intersects the cone to form each of the conic sections.

6. How would a plane intersect the double cone to form a point?

7. How would a plane intersect the double cone to form a line?

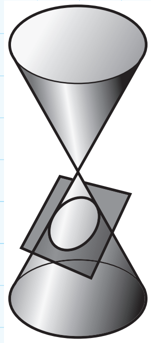
Ellipses and Circles
Round and Round We Go

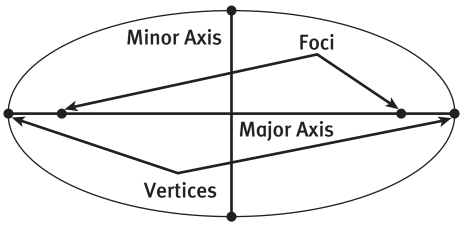
SUGGESTED LEARNING STRATEGIES: Shared Reading, Interactive Word Wall, Vocabulary Organizer, Marking the Text, Look for a pattern, Guess and Check



Prior to the 17th century, astronomers believed the orbit of the planets around the sun was circular. In the early 17th century, Johannes Kepler discovered that the orbital path was elliptical and the sun was not at the center of the orbit, but at one of the two foci.

An **ellipse** is the set of all points in a plane such that the sum of the distances from each point to two fixed points, called **foci**, is a constant. The **center** of an ellipse is the midpoint of the segment which has the foci as its endpoints. The **major** (longer) **axis** of an ellipse contains the foci and the center and has endpoints on the ellipse, the **vertices**. The **minor axis** of the ellipse is the line segment perpendicular to the major axis which passes through the center of the ellipse and has endpoints on the ellipse.





1. Match the graphs in the table on the following page with the corresponding equations from the list of equations given below by writing the equation in the appropriately headed column.

$$\frac{x^2}{16} + \frac{y^2}{81} = 1 \qquad \frac{(x-2)^2}{9} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{100} + \frac{y^2}{49} = 1 \qquad \frac{(x+3)^2}{4} + \frac{(y-1)^2}{36} = 1$$

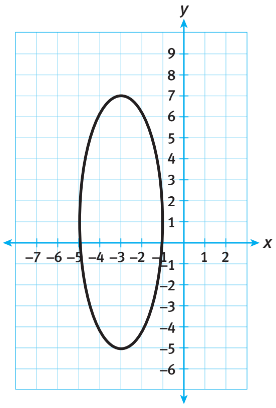
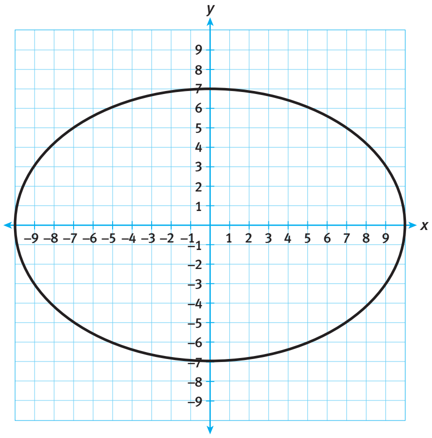
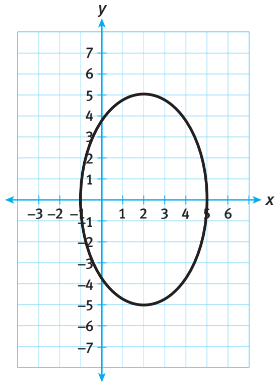
$$\frac{(x+1)^2}{64} + \frac{(y+4)^2}{9} = 1$$

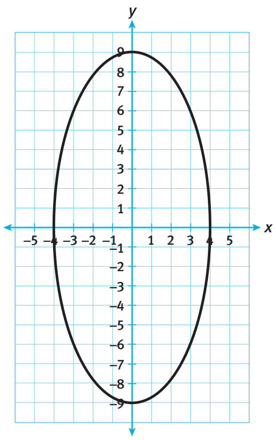
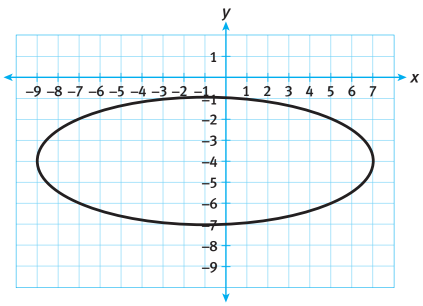
2. For each equation and graph, find the coordinates of the center point, the length of the major axis and the length of the minor axis to complete the chart.

Ellipses and Circles
Round and Round We Go

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Guess and Check

My Notes

Graph			
Equation			
Coordinates of Center of Ellipse			
Length of Major Axis			
Length of Minor Axis			

Graph		
Equation		
Coordinates of Center of Ellipse		
Length of major Axis		
Length of Minor Axis		

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Note taking, Vocabulary Organizer, Interactive Word Wall, Think/Pair/Share, Create Representations

My Notes

3. How are the denominators of the equations related to the major and minor axes of an ellipse?

4. How are numerators of the equations related to the center of the ellipse?

5. How can you determine the orientation of the major axis from the form of the equation of the ellipse?

6. If $a > b$, the **standard form of an ellipse** is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$.

a. Where is the center of the ellipse located?

b. How is the major axis oriented in the coordinate plane?

c. How long is the major axis and what are the coordinates of the endpoints?

d. How long is the minor axis?

ACADEMIC VOCABULARY

standard form of the equation of an ellipse

7. If $a > b$, the **standard form of an ellipse** is $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$.

a. Where is the center of the ellipse located?

b. What direction is the major axis?

c. How long is the major axis and what are the coordinates of the endpoints?

d. How long is the minor axis?

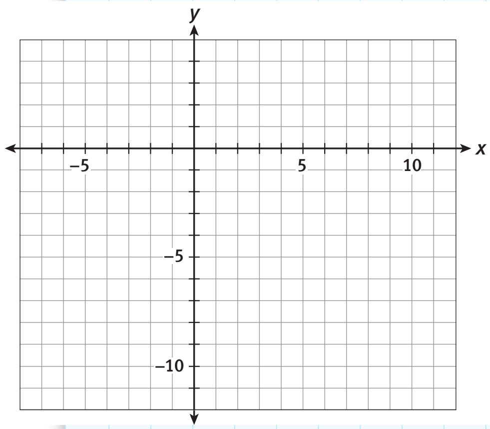
8. Using what you found in Items 6 and 7, find the following information for the ellipse $\frac{(x-2)^2}{64} + \frac{(y+5)^2}{25} = 1$.

a. the coordinates of the center

b. the length and coordinates of the endpoints of the major axis

c. the length and coordinates of the endpoints of the minor axis

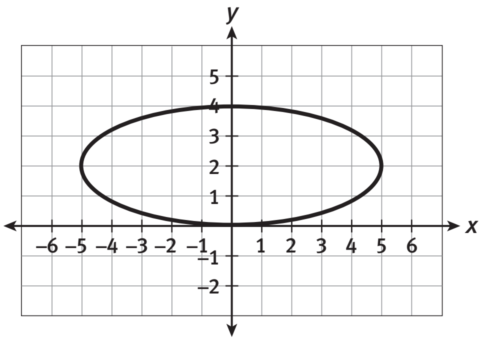
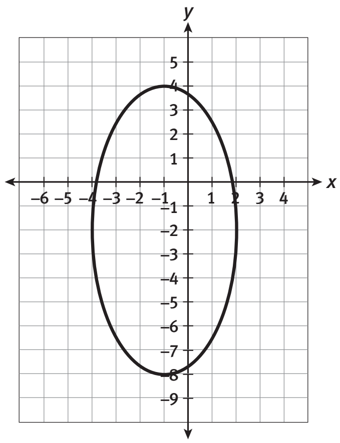
d. In the My Notes section, graph the ellipse and label the center and endpoints of the axes.

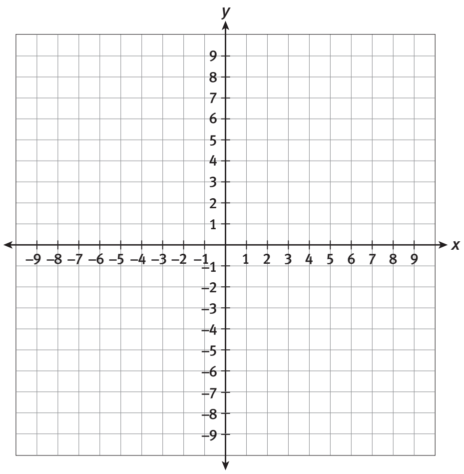
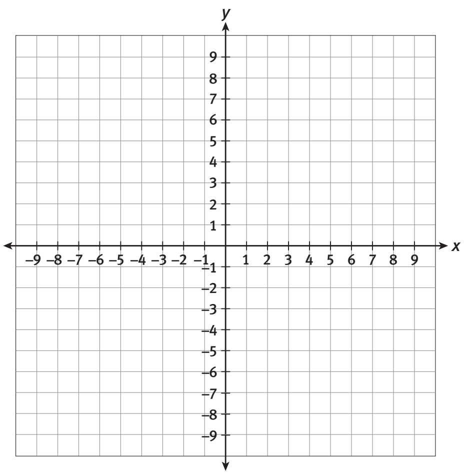


SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward/Think/Part/Share

My Notes

9. Complete the table below using the information given.

Ellipse		
Center		
Length and orientation of major axis		
Length and Orientation of Minor Axis		
Equation of Ellipse		

Graph		
Center	(0, 0)	(−1, 3)
Length and orientation of major axis	8 units vertical	12 units horizontal
Length and Orientation of Minor Axis	4 units horizontal	6 units vertical
Equation of Ellipse		

SUGGESTED LEARNING STRATEGIES: Create Representations, Work Backward, Vocabulary Organizer, Interactive Word Wall, Note Taking, Quickwrite

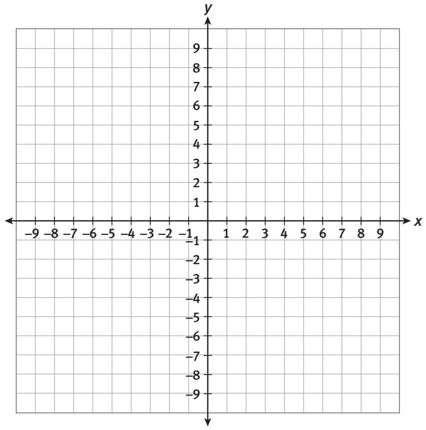
My Notes

CONNECT TO AP

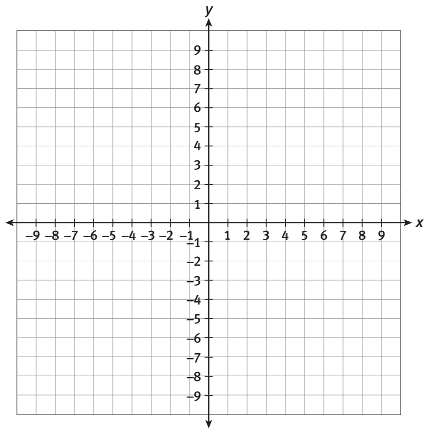
In calculus, you will have to quickly recognize a particular conic section from its equation and produce its sketch.

10. Use the information below. Write the equation and then graph the ellipse described.

a. length of vertical major axis: 14
length of minor axis: 8
center: (2, 3)



b. endpoints of major axis: (2, 2) and (−4, 2)
endpoints of minor axis: (−1, 0) and (−1, 4)



The foci of an ellipse are located on the major axis c units from the center. The values a, b, and c are related by the equation $c^2 = a^2 - b^2$.

The **eccentricity** of a conic section is $\frac{c}{a}$. The eccentricity of a conic section or an **orbit's eccentricity** indicates the roundness or flatness of the shape.

11. Give the coordinates of the foci of each ellipse.

a. $\frac{x^2}{81} + \frac{y^2}{25} = 1$

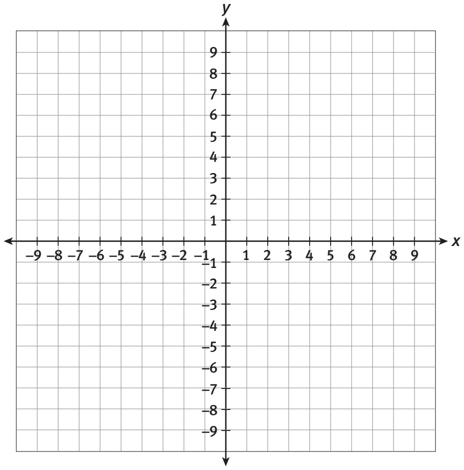
b. $\frac{(x+2)^2}{4} + \frac{y^2}{25} = 1$

SUGGESTED LEARNING STRATEGIES: Create Representations, Quickwrite, Think/Pair/Share

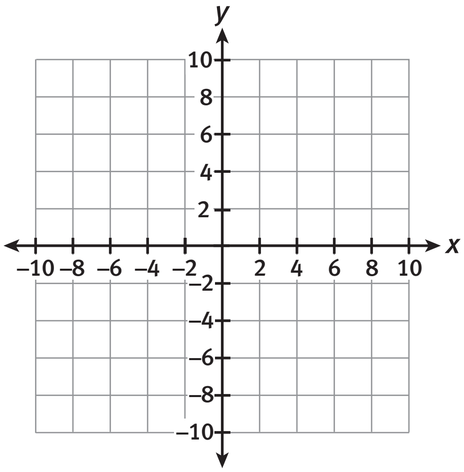
My Notes

12. Graph each ellipse. Determine the eccentricity.

a. $\frac{x^2}{100} + \frac{y^2}{1} = 1$



b. $\frac{x^2}{16} + \frac{y^2}{25} = 1$



13. What does the eccentricity tell you about the graph of an ellipse?

14. Consider the equation for an ellipse in which $a = b$.

a. Give a verbal, visual, and symbolic representation of the conic.

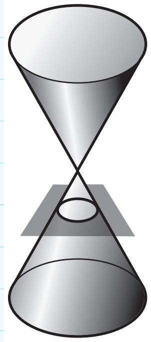
b. What is the eccentricity of the ellipse?

c. What does the eccentricity tell you about the major and minor axes of the ellipse?

d. What does the eccentricity tell you about foci of the ellipse?

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Interactive Word Wall, Marking the Text, Note taking, Activating Prior Knowledge, Think/Pair/Share, Create Representations

My Notes

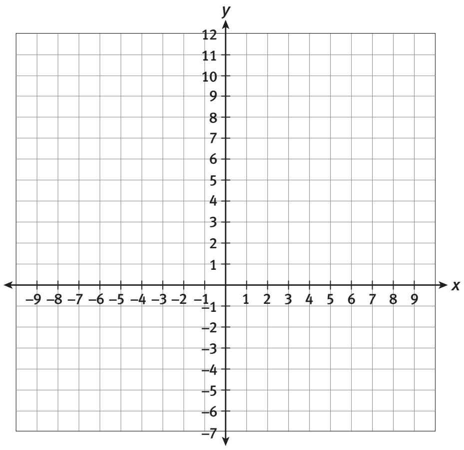


ACADEMIC VOCABULARY

standard form of the equation of a circle

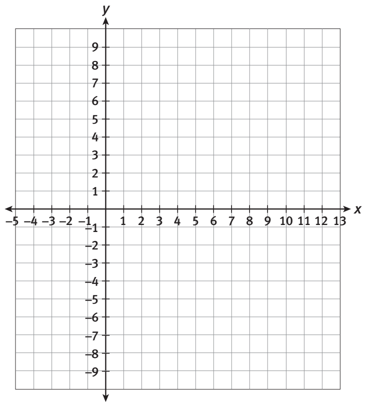
A **circle** is the set of all points in a plane that are equidistant from a fixed point the center. The **standard form of the equation of a circle** is $(x - h)^2 + (y - k)^2 = r^2$ where the center is (h, k) and the radius is r.

15. Write $\dfrac{(x + 2)^2}{4} + \dfrac{(y - 3)^2}{4} = 1$ in the standard form of a circle. Identify the center and radius and then graph the circle.

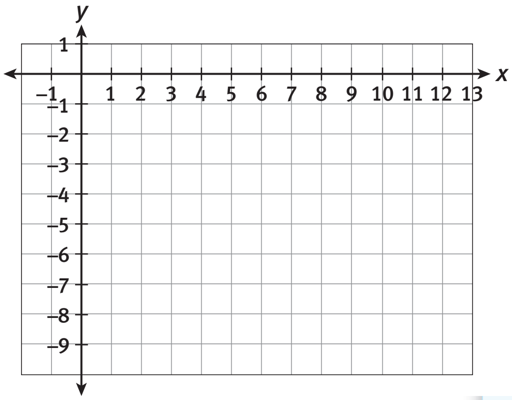


16. Graph each circle and label the center and radius.

a. $(x - 4)^2 + (y - 1)^2 = 49$



b. $(x - 3)^2 + (y + 5)^2 = 4$



17. Write the equation of each circle.

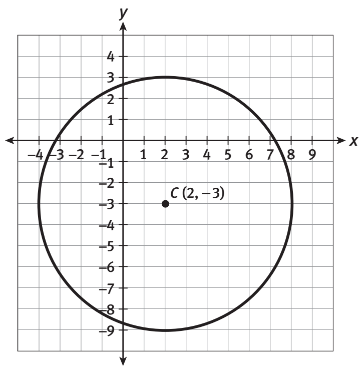
a. center $(-5, 2)$, radius 7

b. center $(1, 1)$ and passing through $(4, 5)$

SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

c.

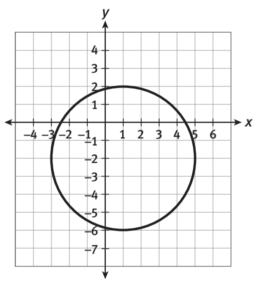


CHECK YOUR UNDERSTANDING

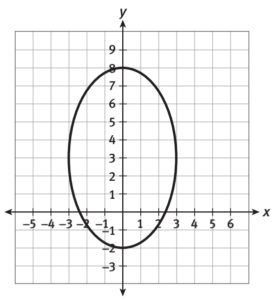
Write your answers on notebook paper or grid paper. Show your work.

1. Write the equation of each graph.

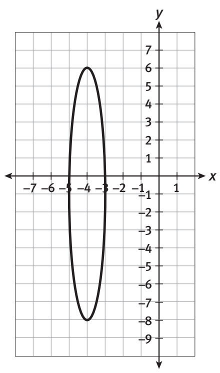
a.



b.



c.



2. Graph each equation. Label the center and endpoints of the major and minor axes.

a. $\frac{x^2}{81} + \frac{y^2}{16} = 1$

b. $\frac{(x + 5)^2}{121} + \frac{(y + 3)^2}{49} = 1$

3. Write the equation of an ellipse that has the endpoints of the major axis at (13, 0) and (−13, 0) and endpoints of the minor axis at (0, 5) and (0, −5).

4. Write the equation of a circle that has center (−6, 2) and a diameter of length 10.

5. If $a > b$, what are the endpoints of the major axis of the ellipse

$$\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1?$$

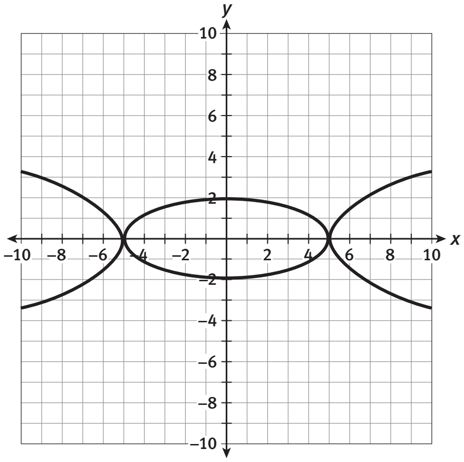
6. **MATHEMATICAL REFLECTION** How are circles and ellipses related?

Hyperbolas
What's the Difference?

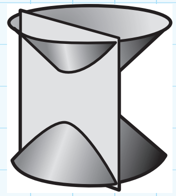
SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Vocabulary Organizer, Quickwrite, Close Reading, Graphic Organizer

Recall the definitions of ellipse and hyperbola:
An *ellipse* is the set of all points in a plane such that the sum of their distances to two fixed points is a constant. A *hyperbola* is the set of all points in a plane such that the absolute value of the difference of their distances to two fixed points, the *foci*, is a constant.

The ellipse $4x^2 + 25y^2 = 100$ and the hyperbola $4x^2 - 25y^2 = 100$ are graphed on the right.



My Notes



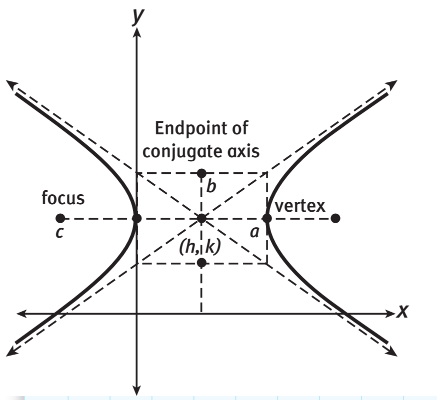
1. Tell the coordinates of the center and the endpoints of the major and minor axes of the ellipse.

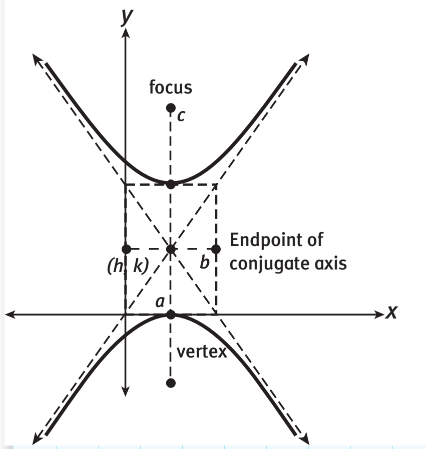
MATH TERMS
transverse axis
conjugate axis
asymptote

2. a. Using dashed line segments draw an auxiliary rectangle with vertices (5, 2), (5, −2), (−5, 2), and (−5, −2). Also using dashed lines, draw two diagonal lines that pass through the center and vertices of the rectangle and extend to the edges of the grid.

b. What relationships do the rectangle and lines have to the ellipse and hyperbola?

c. Why are dashed lines used when sketching the rectangle and diagonals of the rectangle?



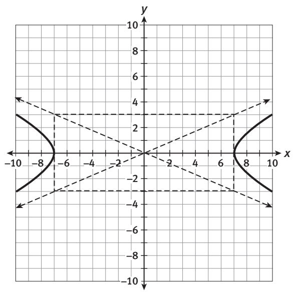
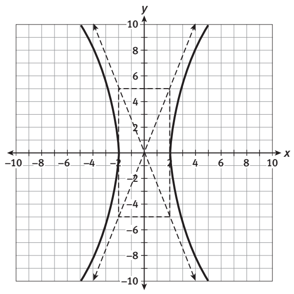
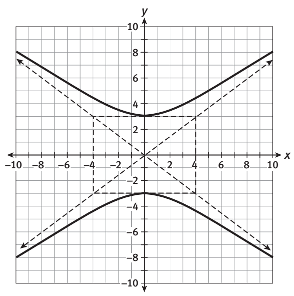
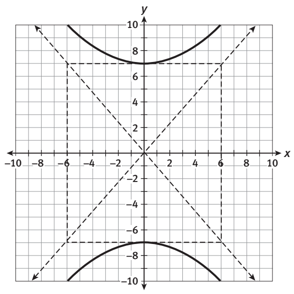


The **transverse axis** of a hyperbola has endpoints on the hyperbola. The *center* of a hyperbola is the midpoint of the transverse axis. The foci are on the line containing the transverse axis. The **conjugate axis** of the hyperbola is the line segment perpendicular to the transverse axis passing through the center of the hyperbola. The hyperbola has **asymptotes**, lines which the branches of the hyperbola approach. The asymptotes contain the center of the hyperbola and pass through the vertices of the auxiliary rectangle.

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Self/Peer Revision, Guess and Check

My Notes

3. Complete the table below. The first row has been done for you using the hyperbola $\frac{x^2}{49} - \frac{y^2}{9} = 1$ as an example.

Graph	Equation	Length, Endpoints, and Orientation of Transverse Axis	Length, Endpoints, and Orientation of Conjugate Axis	Equations of Asymptotes
	$\frac{x^2}{49} - \frac{y^2}{9} = 1$	14 units (7, 0), (−7, 0) horizontal	6 units (0, 3), (0, −3) vertical	$y^2 = \frac{9}{49}x^2$ $y = \pm\frac{3}{7}x$
	$\frac{x^2}{4} - \frac{y^2}{25} = 1$			
	$\frac{y^2}{9} - \frac{x^2}{16} = 1$			
	$\frac{y^2}{49} - \frac{x^2}{36} = 1$			

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Notetaking, Vocabulary Organizer, Create Representations

My Notes

4. How do the equations of the asymptotes relate to the equation of the hyperbola?

5. How can the direction in which the branches of the hyperbola open be determined by the equation?

The **standard form of a hyperbola** is $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, when the transverse axis is horizontal. The standard form of a hyperbola is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ when the transverse axis is vertical. The endpoints of the transverse axis are the **vertices** of the branches, and are located a units from the center of the hyperbola that is located at the point (h, k). The equations of the asymptotes are found by setting the quadratic terms equal to each other and solving for y.

ACADEMIC VOCABULARY

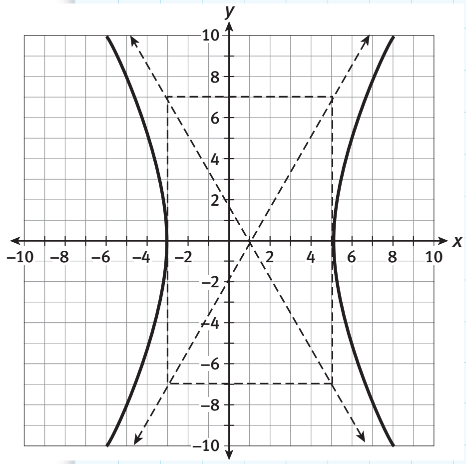
Standard form of the equation of a hyperbola

EXAMPLE 1

Sketch the hyperbola $\frac{(x-1)^2}{16} - \frac{y^2}{49} = 1$. Tell the coordinates of the center and the vertices, and give the equations of the asymptotes

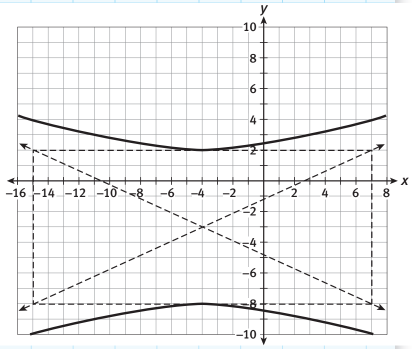
- The positive term is $\frac{(x-1)^2}{16}$, so the transverse axis is horizontal.
- Since a^2 is 16, then $a = 4$ and the transverse axis is 8 units long.
- The center is (1, 0).
- The vertices on the transverse axis are 4 units from the center: (−3, 0) and (5, 0).
- Setting $\frac{(x-1)^2}{16} = \frac{y^2}{49}$ and solving for y gives the equations of the asymptotes.

$$y^2 = \frac{49(x-1)^2}{16} \rightarrow y = \pm\frac{7(x-1)}{4}$$



SUGGESTED LEARNING STRATEGIES: Notetaking, Create Representations, Identify a Subtask

My Notes



EXAMPLE 2

Sketch the hyperbola $\frac{(y+3)^2}{25} - \frac{(x+4)^2}{121} = 1$. Tell the coordinates of the center and the vertices, and give the equations of the asymptotes.

- The positive term is $\frac{(y+3)^2}{25}$, so the transverse axis is vertical.
- Since a^2 is 25, then $a = 5$ and the transverse axis is 10 units long.
- The center is $(-4, -3)$.
- The vertices on the transverse axis are 5 units from the center: $(-4, 2)$ and $(-4, -8)$.
- Setting $\frac{(y+3)^2}{25} = \frac{(x+4)^2}{121}$ and solving for y gives the equations of the asymptotes.

$$(y+3)^2 = \frac{25(x+4)^2}{121} \rightarrow (y+3) = \pm\frac{5(x+4)}{11} \rightarrow y = -3 \pm \frac{5(x+4)}{11}$$

TRY THESE A

Write your answers on notebook or grid paper. Show your work. Sketch each hyperbola. Tell the coordinates of the center, label the vertices and give the equations of the asymptotes.

a. $\frac{x^2}{100} - \frac{y^2}{49} = 1$ **b.** $\frac{y^2}{9} - \frac{x^2}{64} = 1$ **c.** $\frac{x^2}{16} - \frac{(y+4)^2}{36} = 1$

d. $\frac{(x+2)^2}{25} - \frac{(y-3)^2}{9} = 1$

6. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is a hyperbola centered at the origin. Find each item.

a. the direction of the transverse axis

b. the length and endpoints of the transverse axis

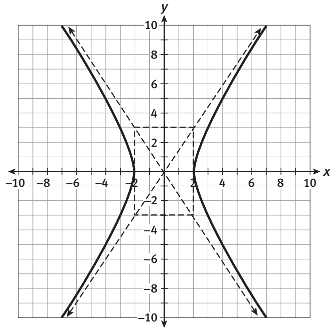
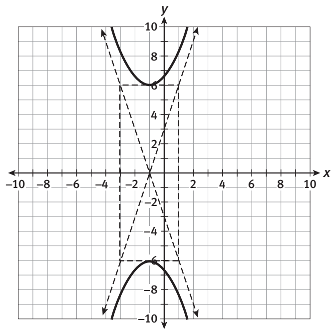
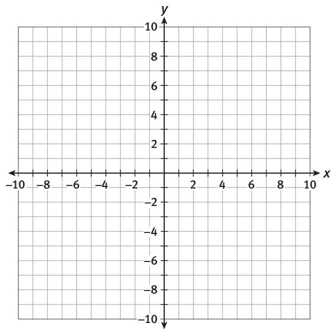
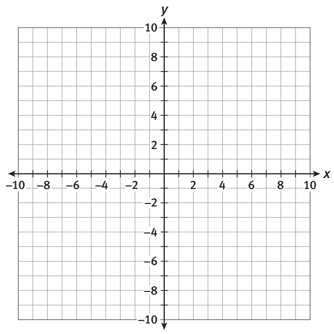
c. the length of the conjugate axis

d. the equation of the asymptotes

SUGGESTED LEARNING STRATEGIES: Think/Pair/Share, Self/Peer Revision, Create Representations

My Notes

7. Complete the table below using the information given.

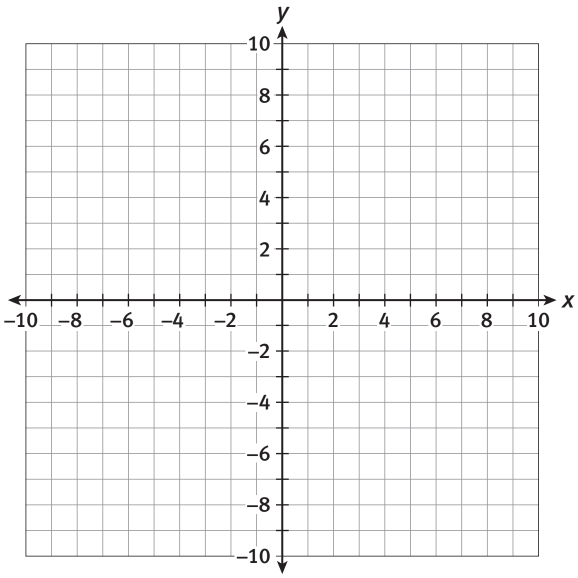
Hyperbola	Center	Length and Orientation of Transverse Axis	Length and Orientation of Conjugate Axis	Equation of Hyperbola
				
				
	(0, 0)	8 units vertical	4 units horizontal	
	(−1, 3)	12 units horizontal	6 units vertical	

SUGGESTED LEARNING STRATEGIES: Create Representations

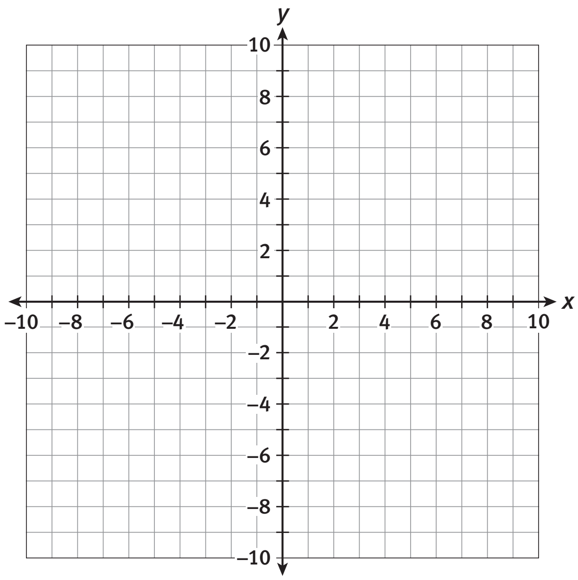
My Notes

8. Write the equation and graph the hyperbola described.

a. center (−1, 4), transverse axis 6 units, vertical conjugate axis 8 units



b. asymptotes $y = \pm\frac{3}{4}x$, vertices (4, 0), (−4, 0)



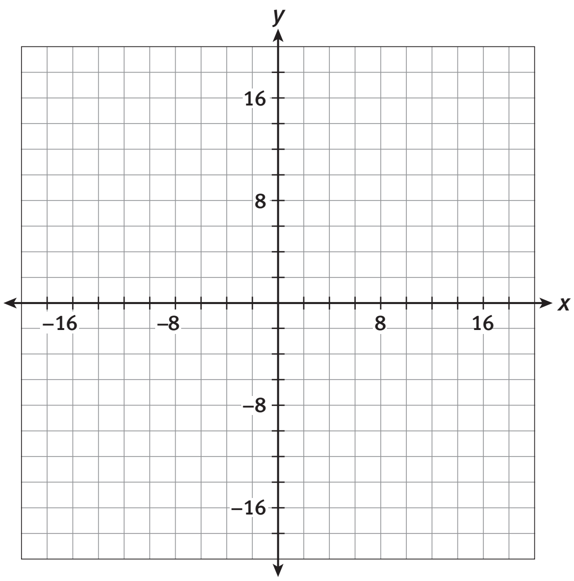
SUGGESTED LEARNING STRATEGIES: Create Representations

My Notes

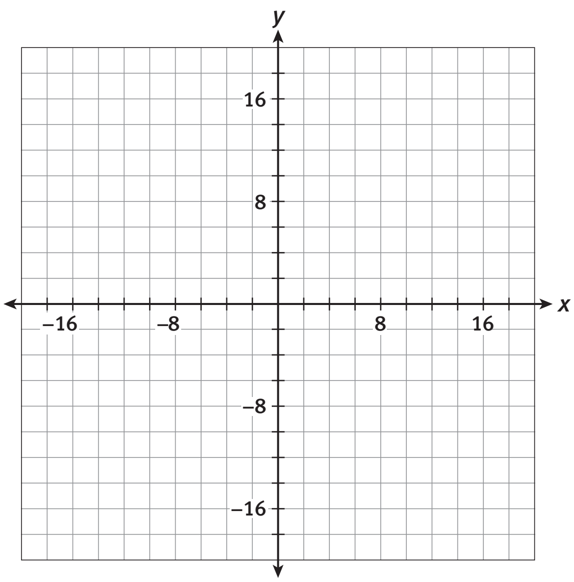
The **foci** of a hyperbola are located on the transverse axis c units from the center. The values a, b, and c are related by the equation $c^2 = a^2 + b^2$.

9. Graph each hyperbola and label the foci with their coordinates.

a. $\frac{x^2}{81} - \frac{y^2}{25} = 1$



b. $\frac{(y+2)^2}{4} - \frac{x^2}{25} = 1$



CHECK YOUR UNDERSTANDING

Write your answers on notebook or grid paper. Show your work.

For each hyperbola in Questions 1–5:

a. Give the coordinates of the center.

b. Tell the direction of the transverse axis.

c. Write the equations of the asymptotes.

d. Sketch the hyperbola and label the endpoints of the transverse axis.

1. $\frac{x^2}{16} - \frac{y^2}{49} = 1$

2. $\frac{y^2}{81} - \frac{x^2}{25} = 1$

3. $\frac{(x+5)^2}{9} - \frac{(y+2)^2}{4} = 1$

4. $\frac{(x-2)^2}{1} - \frac{(y+3)^2}{64} = 1$

5. $\frac{(y-2)^2}{100} - \frac{(x-5)^2}{4} = 1$

For each hyperbola in Questions 6–7:

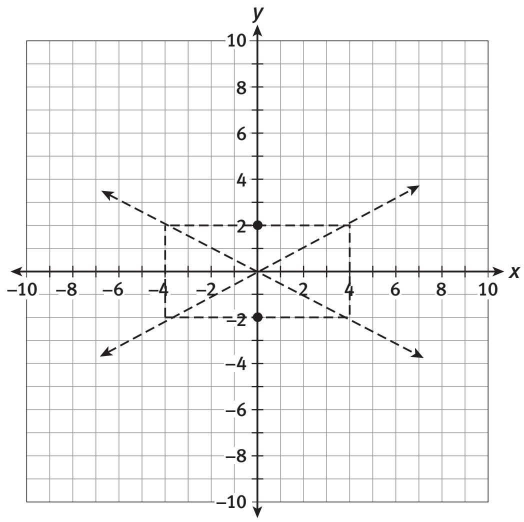
a. Give the coordinates of the center.

b. Tell the direction and length of the transverse axis.

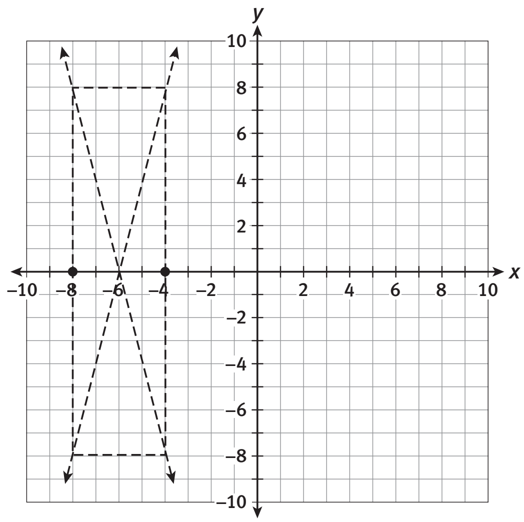
c. Write the equations of the asymptotes.

d. Write the equation of the hyperbola.

6.



7.

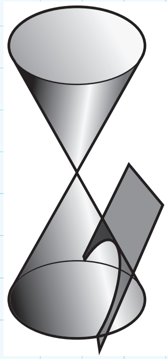


8. **MATHEMATICAL REFLECTION** How do the asymptotes of a hyperbola help you graph the hyperbola?

Parabolas
A Parabola on the Roof

ACTIVITY 7.4

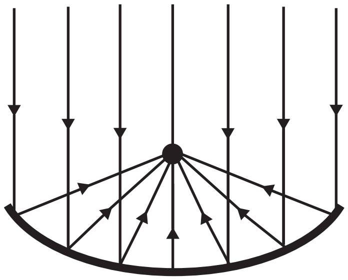
SUGGESTED LEARNING STRATEGIES: Shared Reading, Questioning the Text, Marking the Text, Vocabulary Organizer, Create Representations

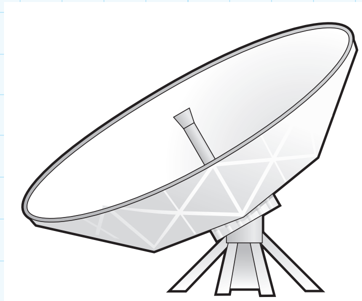


In previous units you learned about quadratic functions. The graph of a quadratic function is a parabola, one of the conic sections you have studied in this unit. In this activity, you will learn more about geometric properties of parabolas, their applications in real world settings, and how to recognize and graph them.

Many people have a parabola on the roof of their homes. The satellite television dishes used to detect television signals are parabolic reflectors. The reason these dishes are shaped like a parabola is due to the following geometric property of a parabola.

When any line parallel to the axis of a parabola hits its surface, the line is reflected through the focus.

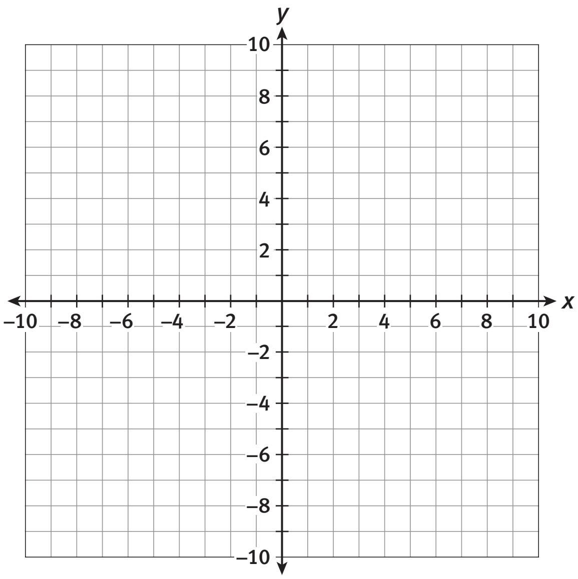




In a satellite dish, the device collects satellite signals over the surface area of the dish. The overall signal is amplified when the individual signals are all reflected to the focus point, where the actual antenna is located at 0.

A **parabola** is the set of points in a plane that are equidistant from a fixed point and a fixed line. The fixed point is called the **focus** and the fixed line is called the **directrix**.

1. Graph $y = x^2$.



2. Form the inverse relation by exchanging x and y and use your knowledge of the properties of inverses to graph this relation on the graph in Item 1.

SUGGESTED LEARNING STRATEGIES: Create Representations, Group Presentation, Quickwrite, Think/Pair/Share

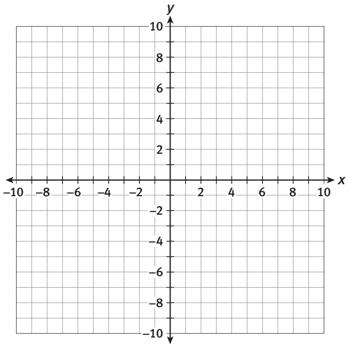
My Notes

MATH TIP

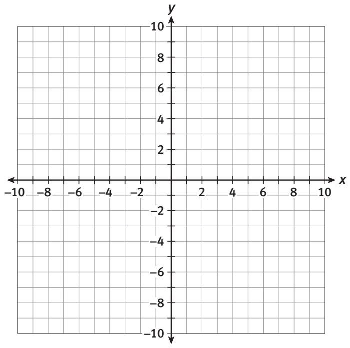
You can use key points and transformations when graphing vertical or horizontal parabolas.

3. For each parabola, write the inverse relation and then sketch the original parabola and its inverse.

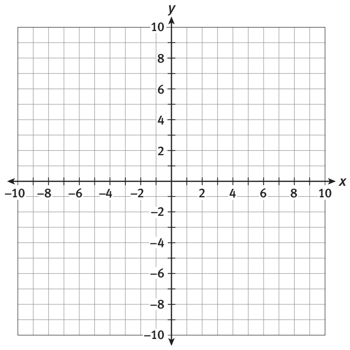
a. $y = x^2 + 2$



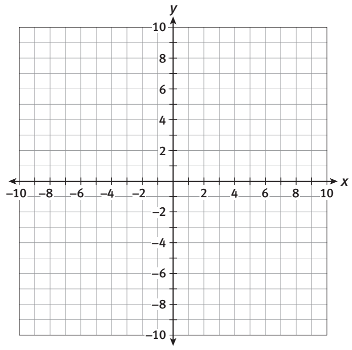
b. $y = (x + 1)^2$



c. $y = -2(x - 3)^2$



d. $y = \frac{1}{2}(x - 1)^2 + 3$



The inverse relations you graphed in Items 2 and 3 are parabolas with a horizontal axis of symmetry.

4. Sketch and label the axis of symmetry for each graph in Item 3.

5. Label the coordinates of the vertex for each parabola in Item 3.

6. How can you determine whether or not a parabola has a vertical or horizontal axis of symmetry?

7. Sketch the graph of each parabola, labeling the vertex coordinates and the axis of symmetry. Use the My Notes section of your book.

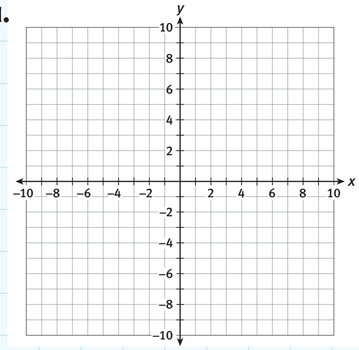
a. $y = x^2 - 5$

b. $x = 2y^2 + 3$

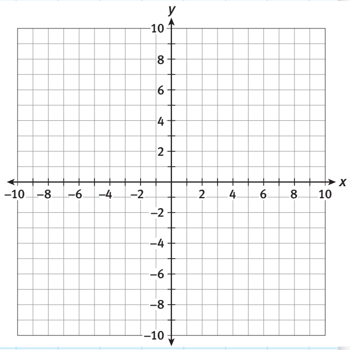
c. $y - 1 = 2(x + 1)^2$

d. $x + 4 = -(y - 3)^2$

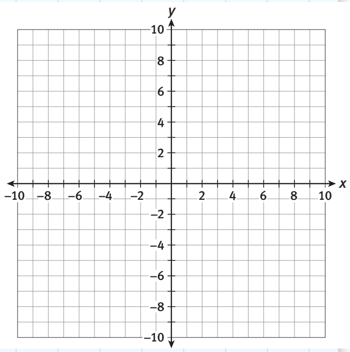
7a.



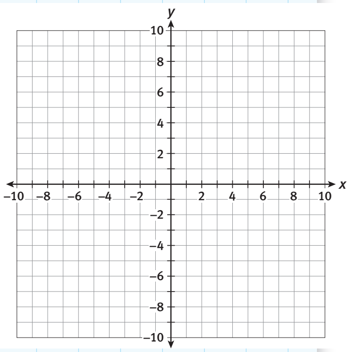
7b.



7c.



7d.



SUGGESTED LEARNING STRATEGIES: Note-taking, Visualization, Look for a Pattern, Create Representations, Identify a Subtask

My Notes

Standard Form of a Parabola

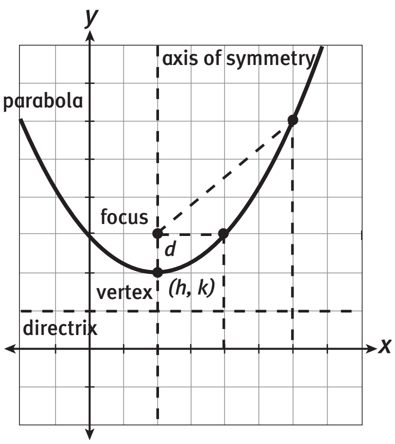
Vertical Axis of Symmetry

$$y - k = \frac{1}{4d}(x - h)^2$$

Horizontal Axis of Symmetry

$$x - h = \frac{1}{4d}(y - k)^2$$

where (h, k) is the vertex and d is the distance from the vertex to the focus.



To find the coordinates of the focus, you add or subtract d to either h or k depending on the orientation of the parabola.

8. For the vertical parabola, what are the coordinates of the focus?

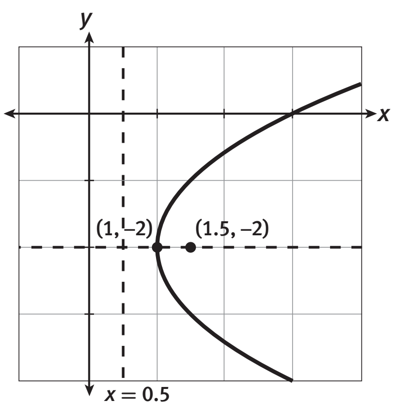
Recall that all points on a parabola are equidistant from the focus and the directrix, including the vertex. To find the equation of the directrix, you subtract d from h or k depending on the orientation of the parabola.

9. For the vertical parabola, what is the equation of the directrix?

EXAMPLE 1

Graph the parabola $x - 1 = \frac{1}{2}(y + 2)^2$. Find the equation of the axis of symmetry, the directrix and the coordinates of the vertex and focus.

- horizontal orientation
- vertex: $(1, -2)$
- axis of symmetry: $y = -2$
- Solve $\frac{1}{4d} = \frac{1}{2}$ to find d.
- $d = \frac{1}{2}$
- Add d to the x-coordinate of the vertex. Focus: $(1.5, -2)$
- Subtract d from the x-coordinate of the vertex.
- Directrix is $x = \frac{1}{2}$



My Notes

SUGGESTED LEARNING STRATEGIES: Identify a Subtask, Create Representations, Group Presentation, Marking the Text, Visualize, Debriefing

TRY THESE **A**

Graph the parabola. Find the equation of the axis of symmetry, the directrix, and the coordinates of the vertex and focus.

a. $y - 2 = \frac{1}{4}(x - 3)^2$

b. $x + 1 = \frac{1}{8}(y + 3)^2$

10. The curvature of a satellite dish is modeled by the parabola

$$y = \frac{1}{64}x^2 + 1$$

where x is measured in inches. If the tip of the antenna needs to be located at the focus of the parabola, then how long should the antenna be?

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

1. Graph the parabola. State the vertex and axis of symmetry.

a. $x = y^2 + 4$

b. $y = (x - 3)^2 + 1$

c. $y = 4(x + 2)^2$

d. $x + 3y^2 = 1$

2. State the coordinates of the focus and the directrix equation for each parabola.

a. $y = x^2$

b. $x = -\frac{1}{2}y^2$

c. $y - 2 = \frac{1}{12}(x + 1)^2$

d. $x = \frac{1}{16}(y - 2)^2$

3. Graph the parabola. State the vertex, axis of symmetry, focus and directrix.

a. $x - 3 = (y + 1)^2$

b. $y + 2 = \frac{1}{2}(x - 4)^2$

4. MATHEMATICAL REFLECTION How can you determine whether a parabola has a horizontal or vertical orientation? How does the equation of a parabola differ from the other conic sections you have studied?

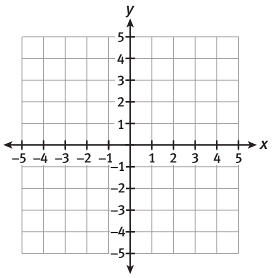
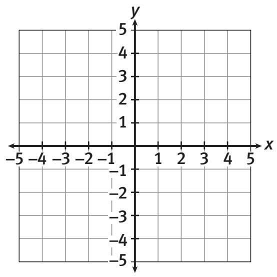
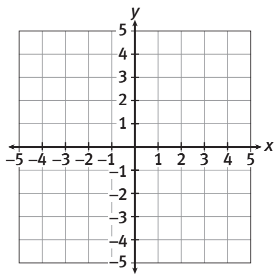
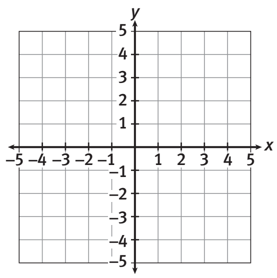
Identifying Conic Sections
How Can You Tell?

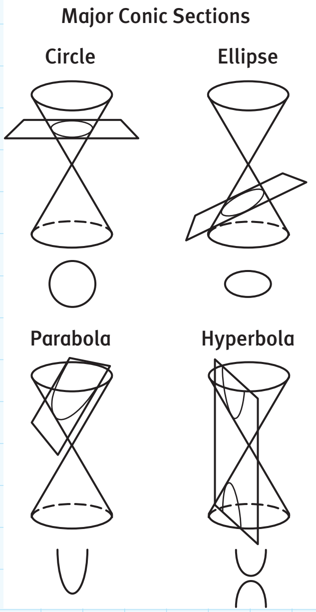
ACTIVITY
7.5

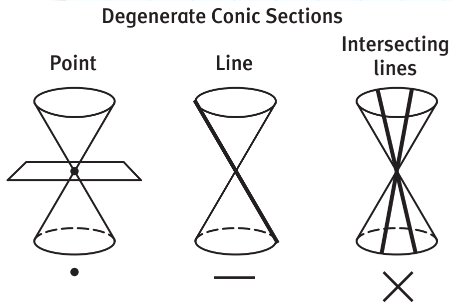
SUGGESTED LEARNING STRATEGIES: Shared Reading, Marking the Text, Activating Prior Knowledge, Create Representations, Think/Pair/Share

As you have been graphing and identifying geometric properties of the conic sections, you have generally been using the standard form of the relation. Each of the conic sections can also be represented by the general form $Ax^2 + Cy^2 + Dx + Ey + F = 0$, where A, C, D, E, and F are constants. The values of A, C, D, E, and F determine the conic and its properties.

1. Complete the chart below by sketching and identifying the conic section and stating the values of A and C.

Equation Conic Section Values of A and C	Graph
a. $x^2 + y^2 - 9 = 0$ Conic: $A =$ $C =$	
b. $x^2 + 9y^2 - 9 = 0$ Conic: $A =$ $C =$	
c. $9x^2 + y^2 - 9 = 0$ Conic: $A =$ $C =$	
d. $x^2 - 9y^2 - 9 = 0$ Conic: $A =$ $C =$	





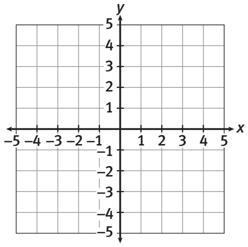
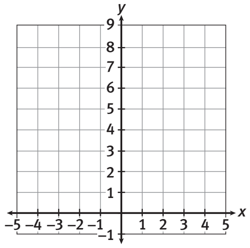
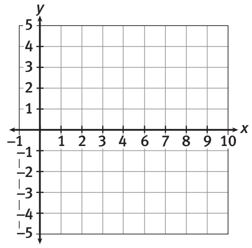
Identifying Conic Sections

How Can You Tell?

My Notes

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Create Representations, Think/Pair/Share, Look for a Pattern, Note-taking, Group Presentation, Vocabulary Organizer

1. *(continued)*

Equation Conic Section Values of A and C	Graph
e. $y^2 - 9x^2 - 9 = 0$ Conic: $A =$ $C =$	
f. $x^2 + y - 9 = 0$ Conic: $A =$ $C =$	
g. $y^2 + x - 9 = 0$ Conic: $A =$ $C =$	

ACADEMIC VOCABULARY

A **quadratic relation** has the general form $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$.

2. Compare and contrast the values of A and C. Make conjectures that complete the statement.

The graph of $Ax^2 + Cy^2 + Dx + Ey + F = 0$ is

a. a circle if ____________________

b. an ellipse if ____________________

c. a hyperbola if ____________________

d. a parabola if ____________________

MATH TERMS

A degenerate state is a limiting case in which an object changes its nature so that it belongs to another, usually simpler description. For example, the point is a degenerate case of the circle as the radius approaches 0, and the circle is a degenerate form of an ellipse as the eccentricity approaches 0. The **degenerate conic sections** are the point, the line, and two intersecting lines.

The **degenerate conic sections** are also represented by the equation $Ax^2 + Cy^2 + Dx + Ey + F = 0$.

3. What values of the coefficients would produce

a. a line?

b. a point?

SUGGESTED LEARNING STRATEGIES: Shared Reading, Vocabulary Organizer, Interactive Word Wall, Look for a Pattern, Quickwrite, Think/Pair/Share, Marking the Text, Activating Prior Knowledge, Note-taking

My Notes

TRY THESE A

Identify each equation as a *circle, ellipse, hyperbola, line*, or *parabola*.

a. $x^2 - 9y^2 + 10x + 54y - 47 = 0$

b. $x^2 + y^2 = 100$

c. $y^2 - 6y - x + 3 = 0$

d. $9x^2 + 4y^2 - 54x + 16y - 479 = 0$

e. $x^2 + 4y - 36 = 0$

f. $9y - 3x - 12 = 0$

g. $y^2 - 4x^2 + 32x + 4y - 96 = 0$

h. $9x^2 + 25y^2 = 225$

In Item 1(a), the values of *D* and *E* were zero. The **quadratic relations** below represent the graphs of four different circles, some of which have *C* and *D* coefficients.

Quadratic Relation	Center	Radius
$x^2 + y^2 = 16$	(0, 0)	4
$x^2 + y^2 + 6x = 7$	(−3, 0)	4
$x^2 + y^2 - 4y = 12$	(0, 2)	4
$x^2 + y^2 + 6x - 4y = 3$	(−3, 2)	4

4. Make several conjectures about the relationship between the coefficients of the terms of each quadratic relation and the center of the circle it represents.

Because graphing and identifying the geometric characteristics of a conic section is most easily done from the standard form of the relation, it is important to be able to write the general form in the standard form.

To find the center and radius of a circle given its general form, complete the square on each variable to write the equation in the form $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center of the circle and the radius is r.

EXAMPLE 1

Find the center and radius of $x^2 + y^2 + 8x - 10y - 8 = 0$.

- $(x^2 + 8x) + (y^2 - 10y) = 8$

  $\frac{1}{2}(8) = 4; (4)^2 = 16$ $\quad$ $\frac{1}{2}(-10) = (-5); (-5)^2 = 25$
- $(x^2 + 8x + 16) + (y^2 - 10y + 25) = 8 + 16 + 25$
- $(x + 4)^2 + (y - 5)^2 = 49$

Center: (−4, 5); radius: 7

MATH TIP

Since conic sections are second degree and are not always functions, they are also known as **quadratic relations**.

CONNECT TO AP

On AP Calculus exams, you may use a graphic calculator to graph a function, solve an equation, and perform other computations without having to show any additional work.

MATH TIP

To complete the square:

- Group like variables together and isolate the constant.
- Take one-half the coefficient on the linear term(s), square the result(s).
- Add the square(s) to both sides of the equation.
- Factor and simplify.

SUGGESTED LEARNING STRATEGIES: Create Representations, Note-taking, Discussion Group

My Notes

TRY THESE B

Write each circle in standard form. Find the center and radius. Then graph the circle in the My Notes space.

a. $x^2 + y^2 + 4x - 12y = 9$

b. $x^2 + y^2 + 2x + 6y - 15 = 0$

The general form of any conic can be written in standard form by completing the square.

EXAMPLE 2

Write $x^2 + 25y^2 + 6x - 100y + 9 = 0$ in standard form. Identify the conic and center.

- $x^2 + 6x + 25y^2 - 100y = -9$

  $x^2 + 6x + 25(y^2 - 4y) = -9$
- $\frac{1}{2}(6) = 3;\ (3)^2 = 9$ $\qquad \frac{1}{2}(-4) = -2;\ (-2)^2 = 4$
- $(x^2 + 6x + 9) + 25(y^2 - 4y + 4) = -9 + 9 + 100$
- $(x + 3)^2 + 25(y - 2)^2 = 100$

  $\frac{(x+3)^2}{100} + \frac{25(y-2)^2}{100} = 1$

  $\frac{(x+3)^2}{100} + \frac{(y-2)^2}{4} = 1$

  ellipse; center: $(-3, 2)$

EXAMPLE 3

Write $y^2 - 4x^2 - 8x - 18y + 13 = 0$ in standard form. Identify the conic and center.

- $y^2 - 18y - 4(x^2 + 2x) = -13$
- $\frac{1}{2}(-18) = -9;\ (-9)^2 = 81$ $\qquad \frac{1}{2}(2) = 1;\ (1)^2 = 1$
- $(y^2 - 18y + 81) - 4(x^2 + 2x + 1) = -13 + 81 - 4$
- $(y - 9)^2 - 4(x + 1)^2 = 64$

  $\frac{(y-9)^2}{64} - \frac{4(x+1)^2}{64} = 1$

  $\frac{(y-9)^2}{64} - \frac{(x+1)^2}{16} = 1$

  hyperbola; center: $(-1, 9)$

SUGGESTED LEARNING STRATEGIES: Note-taking, Discussion Group

My Notes

EXAMPLE 4

Write $9x^2 + 4y^2 + 36x - 8y + 4 = 0$ in standard form. Identify the conic and center.

- $9x^2 + 36x + 4y^2 - 8y = -4$

  $9(x^2 + 4x) + 4(y^2 - 2y) = -4$
- $\frac{1}{2}(4) = 2;\ 2^2 = 4$ $\qquad$ $\frac{1}{2}(-2) = -1;\ (-1)^2 = 1$
- $9(x^2 + 4x + 4) + 4(y^2 - 2y + 1) = -4 + 36 + 4$
- $9(x + 2)^2 + 4(y - 1)^2 = 36$

  $\frac{9(x+2)^2}{36} + \frac{4(y-1)^2}{36} = 1$

  $\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1$

  ellipse; center: $(-2, 1)$

EXAMPLE 5

Write $2y^2 - x - 8y = 0$ in standard form. Identify the conic and vertex.

- $2y^2 - 8y - x = 0$

  $2(y^2 - 4y) - x = 0$
- $\frac{1}{2}(4) = 2;\ 2^2 = 4$
- $2(y^2 - 4y + 4) - x = 0 + 8$
- $2(y - 2)^2 - x = 8$

  $x + 8 = 2(y - 2)^2$

  parabola; vertex: $(-8, 2)$

TRY THESE C

Write each equation in standard form. Identify the conic section and its geometric characteristics. Write your answers in the My Notes space.

a. $x^2 - y^2 + 8x + 6y - 18 = 0$

b. $4x^2 + y^2 + 16x - 6y + 9 = 0$

c. $x^2 - 6x - y + 4 = 0$

CHECK YOUR UNDERSTANDING

Write your answers on notebook paper or grid paper. Show your work.

1. For Parts (a)–(j) below, identify each equation as representing a *circle*, *ellipse*, *hyperbola*, or *parabola*. Write each equation in standard form. Graph each relation.

   a. $2x^2 - 8x - y + 5 = 0$

   b. $4y^2 - 25x^2 = 100$

   c. $4x^2 + y^2 - 40x + 6y = -93$

   d. $x^2 + y^2 - 8y - 20 = 0$

   e. $y^2 - 3x^2 + 6x + 6y - 394 = 0$

   f. $x^2 + 4y^2 + 2x - 24y + 33 = 0$

   g. $x^2 + y^2 + 2x - 6y - 15 = 0$

   h. $y^2 - x - 2y - 3 = 0$

   i. $6x^2 + 12x - y + 6 = 0$

   j. $4x^2 - 9y^2 - 8x - 32 = 0$

2. **MATHEMATICAL REFLECTION** Why is it useful to be able to change the form of the equation for a conic section?

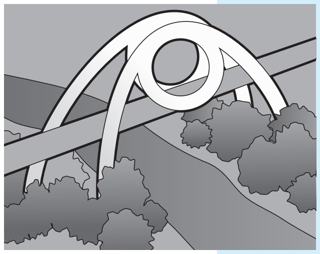
Conic Sections

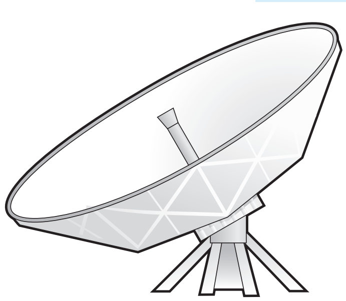
WORKING WITH US

Embedded Assessment 1

Use after Activity 7.5.

When studying astronomy we learn that stars, planets and comets have orbital paths that are circular, elliptical, parabolic and hyperbolic. Applications of the conic sections also occur in everyday life; such as machine gears, telescopes, headlights, radar, sound waves, navigation, roller coasters, hyperbolic cooling towers and suspension bridges.





State whether each equation represents a *circle*, *ellipse*, *hyperbola*, or *parabola*.

1. $x^2 + y^2 + 2x - 8 = 0$

2. $x^2 - 9y = 0$

3. $25y^2 - 9x^2 - 50y - 200 = 0$

4. $x^2 - 2x - y + 1 = 0$

5. $4x^2 + 3y^2 + 32x - 6y + 67 = 0$

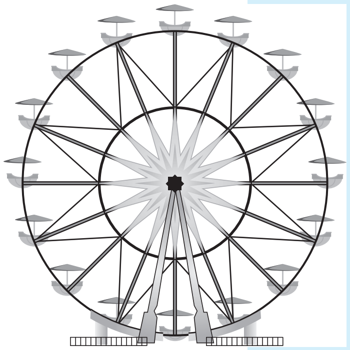
Sketch the graph of each equation.

6. $2y^2 + x - 12y + 10 = 0$

7. $x^2 + y^2 - 10x - 4y - 20 = 0$

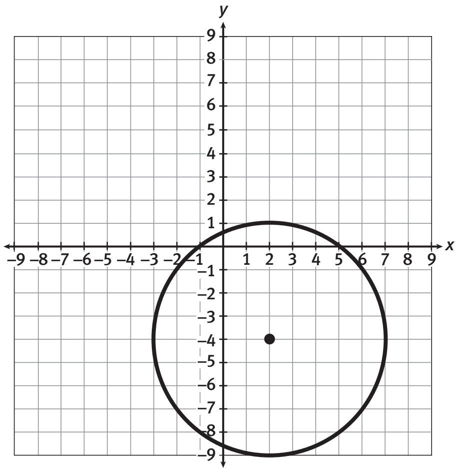
8. $9x^2 + 36y^2 - 216y = 0$

9. $16x^2 - 9y^2 - 144 = 0$



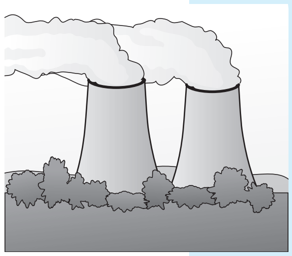
Give the standard equation of each graph.

10.



11. a parabola with vertex (4, 1), axis of symmetry $y = 1$ and passing through the point (3, 3)

12. an ellipse with vertices of the major axis at (10, 2) and (−8, 2) and minor axis of length 6



Conic Sections

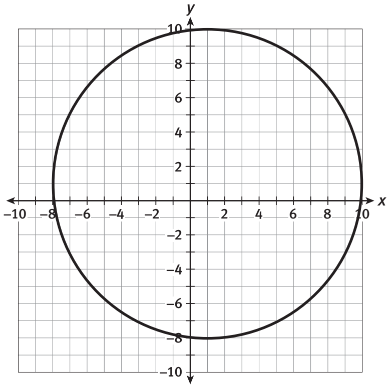
WORKING WITH US

	Exemplary	Proficient	Emerging
Math Knowledge	The student: • Correctly identifies the equations. (1–5) • Recognizes the conic section that the equation represents. (6–9)	The student: • Correctly identifies only three or four of the equations. • Recognizes only two or three of the equations.	The student: • Correctly identifies only one or two of the equations. • Recognizes only one of the equations.
Problem Solving	The student gives the correct standard equations. (10–12)	The student gives only two correct standard equations.	The student gives only one correct standard equation.
Representations	The student graphs the equations correctly. (6–9)	The student graphs only two or three of the equations correctly.	The student graphs only one of the equations correctly.

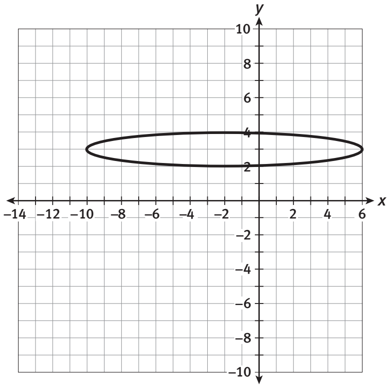
ACTIVITY 7.2

1. Write the equation of each ellipse and circle in standard form.

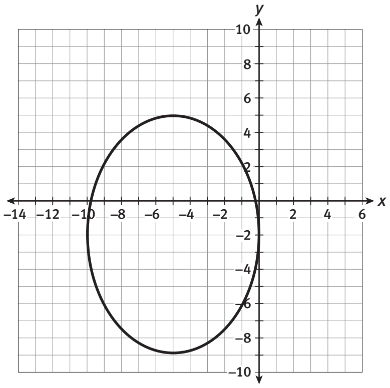
a.



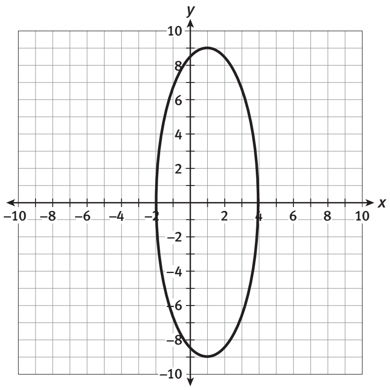
b.



c.



d.



2. Graph each equation. Label key points on each graph.

   a. $\frac{x^2}{25} + \frac{y^2}{81} = 1$

   b. $\frac{(x-2)^2}{16} + \frac{(y-3)^2}{49} = 1$

   c. $\frac{(x+3)^2}{16} + (y-7)^2 = 1$

3. Write the equation for each figure.

   a. a circle with center $(-3, 4)$ and radius 6

   b. a circle whose diameter has endpoints $(3, 5)$ and $(11, 5)$

   c. an ellipse centered at the origin and having a major axis of 10 units and a vertical minor axis of 6 units

   d. an ellipse with center $(3, 4)$, a horizontal minor axis 10 units long and a major axis 20 units long.

4. Given an ellipse with a major axis with endpoint $(-6, 0)$ and foci $(-4, 0)$ and $(4, 0)$.

   a. Explain how to find the center.

   b. Explain how to find the endpoints and the length of the minor axis.

   c. Write the equation of the ellipse.

ACTIVITY 7.3

For each hyperbola in Questions 5–9:

a. Give the coordinates of the center.
b. Tell the direction of the transverse axis.
c. Tell the equations of the asymptotes.
d. Sketch the hyperbola and label the endpoints of the transverse axis.

5. $\frac{x^2}{81} - \frac{y^2}{4} = 1$

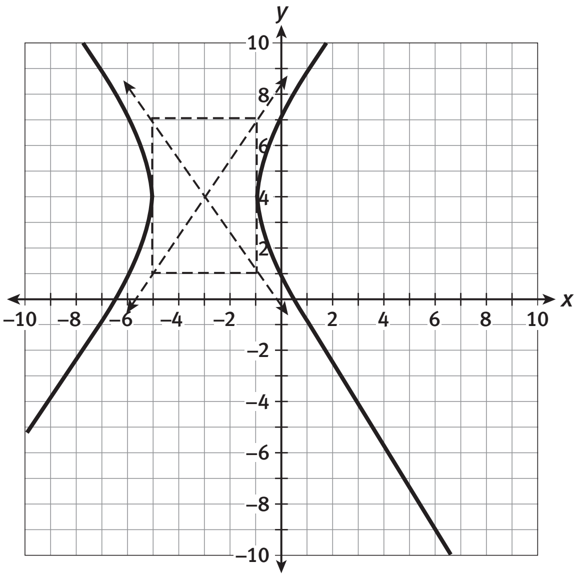
6. $\frac{y^2}{36} - \frac{x^2}{100} = 1$

7. $\frac{(x+7)^2}{4} - \frac{(y+4)^2}{64} = 1$

8. $\frac{(x-1)^2}{49} - \frac{(y-4)^2}{36} = 1$

9. $\frac{(y+3)^2}{121} - \frac{(x-3)^2}{9} = 1$

10. Label the coordinates of the center, the vertices and the foci of the hyperbola below.



Match each equation below with the correct graph.

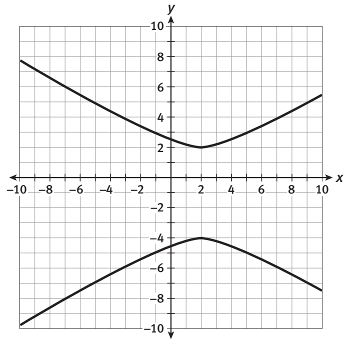
11. $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1$

12. $\frac{(x-2)^2}{16} - \frac{(y+1)^2}{9} = 1$

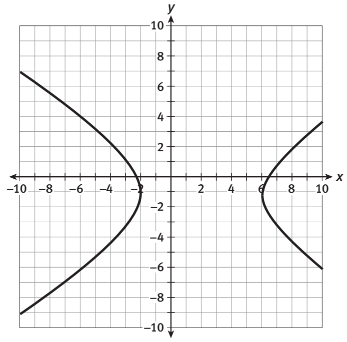
13. $\frac{(y+1)^2}{16} - \frac{(x-2)^2}{9} = 1$

14. $\frac{(y+1)^2}{9} - \frac{(x-2)^2}{16} = 1$

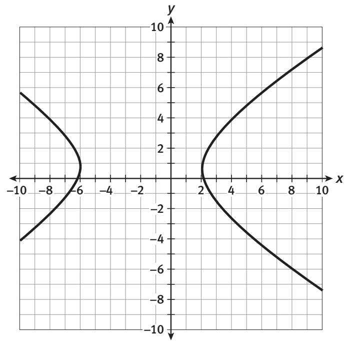
A.



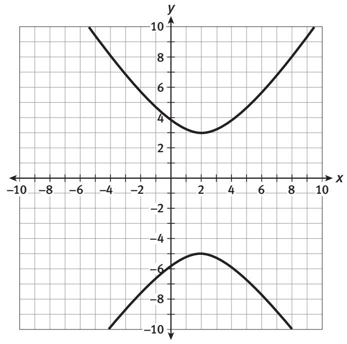
B.



C.



D.



ACTIVITY 7.4

15. Graph the parabola, state the vertex and axis of symmetry.

a. $y = -3x^2 + 5$

b. $x - 3 = -(y + 2)^2$

16. State the coordinates of the focus and the directrix equation for each parabola.

a. $y = \frac{1}{8}x^2 - 4$

b. $x + 4 = \frac{1}{20}(y - 1)^2$

17. Graph the parabola

$$x + 5 = -\frac{1}{4}(y - 2)^2$$

State the vertex, axis of symmetry, focus, and directrix.

ACTIVITY 7.5

For each equation in questions 18–25:

a. Identify each equation as representing a *circle*, *ellipse*, *hyperbola*, or *parabola*.

b. Write each equation in standard form.

c. Graph each relation.

18. $x^2 - 4y^2 - 8x + 16y - 36 = 0$

19. $3y^2 - x - 6y - 5 = 0$

20. $16y^2 + 25x^2 = 400$

21. $x^2 + y^2 - 16x + 6y - 27 = 0$

22. $25x^2 + 9y^2 - 50x - 54y - 119 = 0$

23. $x^2 - 2y - 6 = 0$

24. $9x^2 + 4y^2 + 8y - 140 = 0$

25. $x^2 - y^2 + 6y - 10 = 0$

26. $2x^2 - y + 16x + 28 = 0$

27. $x^2 - 4x + y + 3 = 0$

Reflection

An important aspect of growing as a learner is to take the time to reflect on your learning. It is important to think about where you started, what you have accomplished, what helped you learn, and how you will apply your new knowledge in the future. Use notebook paper to record your thinking about the following topics and to identify evidence of your learning.

Essential Questions

1. Review the mathematical concepts and your work in this unit before you write thoughtful responses to the questions below. Support your responses with specific examples from concepts and activities in the unit.

1 How are the algebraic representations of the conic sections similar and how are they different?

2 How do the conic sections model real world phenomena?

Academic Vocabulary

2. Look at the following academic vocabulary words:

- conic section
- ellipse
- hyperbola
- quadratic relation
- standard form

Choose three words and explain your understanding of each word and why each is important in your study of math.

Self-Evaluation

3. Look through the activities and Embedded Assessment in this unit. Use a table similar to the one below to list three major concepts in this unit and to rate your understanding of each.

Unit Concepts	Is Your Understanding Strong (S) or Weak (W)?
Concept 1	
Concept 2	
Concept 3	

a. What will you do to address each weakness?

b. What strategies or class activities were particularly helpful in learning the concepts you identified as strengths? Give examples to explain.

4. How do the concepts you learned in this unit relate to other math concepts and to the use of mathematics in the real world?

1. The hyperbola $\frac{y^2}{25} - \frac{x^2}{9} = 1$ has foci at the points

A. $(\sqrt{34}, 0), (-\sqrt{34}, 0)$

B. $(0, \sqrt{34}), (0, -\sqrt{34})$

C. $(5, 3), (5, -3)$

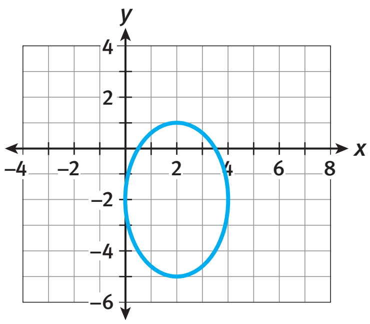
D. $(-5, 3), (-5, -3)$

1. Ⓐ Ⓑ Ⓒ Ⓓ

2. A circle has the equation $(x - 3)^2 + (y + 2)^2 = 16$. What is the radius of the circle?

2.

3. Given the equation and graph of the ellipse $\frac{(x-2)^2}{4} + \frac{(y+2)^2}{a^2} = 1$, what is the value of a?



3.

Read
Solve
Explain

4. Given the equation of the parabola $y = -\frac{1}{4}x^2 - \frac{1}{4}x + \frac{39}{16}$

Part A: Write the equation in standard form.

Answer and Explain

Part B: Find the vertex, focus, and directrix.

Answer and Explain

Part C: Sketch a graph.