# Connecticut Common Core Algebra 1 Curriculum 

## Professional Development Materials

## Unit 7 Introduction to Exponential Functions

## Contents

Activity 7.1.1b Is Population Growth Linear?
Activity 7.1.2 Is It a Good Deal?
Activity 7.2.3 Exploring the Meaning of Rational Exponents
Activity 7.3.3 Effects of Parameters
Activity 7.4.1 Tossing M\&Ms
Activity 7.5.3 Percent Change Situations
Activity 7.6.1 The Mathematics of Global Warming
Activity 7.6.2 Countering Global Warming
Unit 7 End-of-Unit Test*

* This item appears only on the password-protected web site.


## Is Population Growth Linear?

Since the 1960's, the world population has grown rapidly. During this same time period, world food production has also increased. There are many reasons why people have been able to produce more crops. Farmers rotate crops more efficiently causing crop yields to expand. New and improved fertilizers and chemical pesticides allow farmers to grow more crops on the same amount of land. In dry, arid regions, large irrigation systems convert land to farmland. Also, farmers use genetically altered crops that greatly increase crop yields.

Crop yields are increasing but so is the world population. Scientists need to know if one quantity is growing much faster than the other. What could happen if the world population is growing much faster than the amount of food produced?

Let's begin by exploring the growth in the world population. The table on the right gives year and population data for a sample of years from 1804 to 2010 .

## Question: Are the population vs. year data linear?

There are several ways to investigate this question. We will look at

| Year | Population <br> (in billions) |
| :---: | :---: |
| 1804 | 1 |
| 1850 | 1.2 |
| 1900 | 1.6 |
| 1927 | 2 |
| 1950 | 2.55 |
| 1955 | 2.78 |
| 1960 | 3.04 |
| 1965 | 3.35 |
| 1970 | 3.71 |
| 1975 | 4.09 |
| 1980 | 4.45 |
| 1985 | 4.85 |
| 1990 | 5.28 |
| 1995 | 5.7 |
| 2000 | 6.1 |
| 2005 | 6.48 |
| 2008 | 6.71 |
| 2010 | 6.87 | patterns in the graph, patterns in the table, the linear regression line, and its correlation coefficient. This will allow us to explain why we think the data are linear or not linear.

1. Make a graph of the data on the coordinate axes below.

2. Describe any patterns you see. What does your graph tell you about the linearity of the data?
3. Find average rates of change between the given pairs of points in the table. The first pair is done for you.

| Difference in <br> year $(\Delta \boldsymbol{x})$ | Year | Population in <br> billions | Difference in <br> population in <br> billions ( $\Delta \boldsymbol{y})$ | Average rate of <br> change $(\Delta \boldsymbol{y} / \Delta \boldsymbol{x})$ |
| :---: | :---: | :---: | :---: | :---: |
| $1850-1804=46$ | 1804 | 1 | $1.2-1=0.2$ | $0.2 / 46=0.00043$ |
|  | 1850 | 1.2 |  |  |
|  | 1900 | 1.6 |  |  |
|  | 1927 | 2 |  |  |
|  | 1950 | 2.55 |  |  |
|  | 1955 | 2.78 |  |  |
|  | 1960 | 3.04 |  |  |
|  | 1965 | 3.35 |  |  |
|  | 1970 | 3.71 |  |  |
|  | 1975 | 4.09 |  |  |
|  | 1980 | 4.45 |  |  |
|  | 1985 | 4.85 |  |  |
|  | 1990 | 5.28 |  |  |
|  | 1995 | 5.7 |  |  |
|  | 2000 | 6.1 |  |  |
|  | 2005 | 6.48 |  |  |
|  | 2008 | 6.71 |  |  |

4. Describe any patterns you see in the average rates of change column of the table. What do those patterns tell you about the linearity of the data?
5. Enter the data in your calculator and find the linear regression line and its correlation coefficient.

Regression equation:
Correlation Coefficient: $r=$
6. What does the correlation coefficient tell you about the linearity of the data?
7. Compare your answers and ideas with those of the rest of the class. Do you agree? Did you or anyone else change their minds? Explain why or why not.

## Is It a Good Deal?

In a previous activity you saw that the world population vs. year data did not fit a linear model. Is there another kind of function that models the kind of growth we saw in the world population vs. year data? Yes! The exponential family of functions! To get familiar with this family of functions, let's explore another situation.

Situation: You are offered a job where you will earn $\$ 0.02$ on the first day of a job and then double your earnings each day.

Question: Should you take this job? Is it a good deal? Explain why or why not.

1. The table shows the daily wages for this job for the first nine days. Explain how you know that the data in the table are not linear.

| Day <br> $(\boldsymbol{x})$ | Amount Earned <br> $(\boldsymbol{y})$ |
| :---: | :---: |
| 1 | 0.02 |
| 2 | 0.04 |
| 3 | 0.08 |
| 4 | 0.16 |
| 5 | 0.32 |
| 6 | 0.64 |
| 7 | 1.28 |
| 8 | 2.56 |
| 9 | 5.12 |

2. Make a scatter plot of the nine data values from the table by hand or with your calculator. Label and scale the axes.

3. Describe all the patterns you see in the table and in your graph. Do the data look linear? Explain.
4. Do you think this job is a good deal? Explain why or why not.
5. Use the home screen of your graphing calculator to model the pattern and extend the table.

First, clear the home screen.
Next, enter 0.02, your earnings for the first day. Press enter.
Your screen should look like this: $\qquad$
Now, multiply this number by 2 to find the daily wage for the second day.
To do this, press the multiplication key followed by 2. Press enter.
You will see "ANS*2" to tell you that you have just multiplied the previous answer of .02 by 2 . Your screen should now look like this: $\qquad$


To repeat the previous command (multiply the previous answer by 2 ) just press enter again.
Now you see the daily wage for day 3 , which is $\$ 0.08$.
Your screen should now look like this: $\qquad$

| 02 | .02 |
| :--- | :--- |
| Ans*2 | .04 |
|  | $: 68$ |

Now you can continue the pattern by pressing enter again and again.


The next time you press enter, you will have the amount you earn on day 4.
Continue to press the enter key and keep track of the function values to fill in the missing values in the table. Begin filling in the amount earned on day 10 .

| Day <br> $(\boldsymbol{x})$ | Amount <br> Earned (y) | Day <br> $(\boldsymbol{x})$ | Amount <br> Earned (y) | Day <br> $(\boldsymbol{x})$ | Amount <br> Earned (y) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.02 | 11 |  | 21 |  |
| 2 | 0.04 | 12 |  | 22 |  |
| 3 | 0.08 | 13 |  | 23 |  |
| 4 | 0.16 | 14 |  | 24 |  |
| 5 | 0.32 | 15 |  | 25 |  |
| 6 | 0.64 | 16 |  | 26 |  |
| 7 | 1.28 | 17 |  | 27 |  |
| 8 | 2.56 | 18 |  | 28 |  |
| 9 | 5.12 | 19 |  | 29 |  |
| 10 |  | 20 |  | 30 |  |

6. How much money would your earn on day 30 ? Is this what you expected?
7. Do you think this job is a good deal? Explain why or why not.
8. Did your opinion of this job change? Explain.
9. Here is another way to represent the amount of money you will earn over time.
a. Enter the function $\mathrm{Y} 1=0.01^{*} 2^{\wedge} \mathrm{X}$ in the $\mathrm{Y}=$ menu on your calculator.
b. Then go to Table Set up and enter TblStart $=0$ and $\Delta \mathrm{Tbl}=1$
c. Press $2^{\text {nd }}$ Table to view the table.
d. How does the table on the calculator compare with the one you made in question 5 ?

## Exploring the Meaning of Rational Exponents

1. What is a rational number? Why are they called rational numbers?

Now let's figure out what a rational exponent means. We start with a specific example. Look at the exponential pattern in the table below.
2. What do you think $9^{\frac{1}{2}}$ means? What do you think is its value?

| $\boldsymbol{x}$ | $\boldsymbol{9}^{\boldsymbol{x}}$ | Meaning | Value |
| :---: | :---: | :---: | :---: |
| -3 | $9^{-3}$ | $\frac{1}{9} \cdot \frac{1}{9} \cdot \frac{1}{9}$ | $\frac{1}{729}$ |
| -2 | $9^{-2}$ | $\frac{1}{9} \cdot \frac{1}{9}$ | $\frac{1}{81}$ |
| -1 | $9^{-1}$ | $\frac{1}{9}$ | $\frac{1}{9}$ |
| 0 | $9^{0}$ | 1 | 1 |
| $1 / 2$ | $9^{1 / 2}$ | 9 |  |
| 1 | $9^{1}$ | $9 \cdot 9$ | 9 |
| 2 | $9^{2}$ | $9 \cdot 9 \cdot 9$ | 81 |
| 3 | $9^{3}$ |  | 729 |

3. Does your estimate for $9^{\frac{1}{2}}$ in question 2 fit the pattern in the table?
4. Does your estimate for $9^{\frac{1}{2}}$ in question 2 fit with the rules for working with exponents? Review some of the exponent rules on the next page and then return to this question.

## Recall:

EX: $5^{2} \cdot 5^{4}=(5 \cdot 5) \cdot(5 \cdot 5 \cdot 5 \cdot 5)=5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5=5^{6}$
EX: $\left(4^{3}\right)^{2}=(4 \cdot 4 \cdot 4)^{2}=(4 \cdot 4 \cdot 4) \cdot(4 \cdot 4 \cdot 4)=4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4=4^{6}$
5. Simplify each expression by writing out what it means first. Leave your answer in exponential form.
a. $6^{4} \cdot 6^{3}$
b. $\left(9^{3}\right)^{2}$
c. $2^{3} \cdot 2^{5}$
d. $3^{4} \cdot 3^{7}$
e. $\left(8^{5}\right)^{3}$
f. $\left(12^{2}\right)^{4}$
6. Describe the two 'rules' for working with exponents that you see in the patterns above.
7. Based on the rules, what should $\left(9^{\frac{1}{2}}\right)^{2}$ mean? What should be its value?
8. Return to question 4. Does your estimate for $9^{\frac{1}{2}}$ in question 2 fit with the rules for working with exponents? Does that fit with the pattern in the table?
9. Discuss these ideas with your class and come up with your final estimate for the meaning of $9^{\frac{1}{2}}$.
10. What is the meaning of $25^{\frac{1}{2}}$ ? What is the meaning $49^{\frac{1}{2}}$ ? Check your answers on a calculator. (Hint: Because the calculator uses the order of operations you will need to enter $25^{\wedge}(1 / 2)$ for $25^{\frac{1}{2} .}$ )
11. You have seen that exponential growth may be modeled with the function $f(x)=a b^{x}$, where $a$ is the initial value and $b$ is the growth factor. Suppose the number of bacteria in a laboratory beaker after $x$ days is modeled by $f(x)=100 \cdot 4^{x}$. Find how many bacteria are in the beaker:
a. after 3 days.
b. after $\frac{3}{2}$ days.
c. $\operatorname{after} \frac{1}{2}$ day.

## Effects of Parameters

An exponential function is a function of the form $f(x)=a \cdot b^{x}$. In the previous activity we explored the roles of parameters $a$ and $b$. In this activity we will examine the $b$ parameter more closely. This parameter determines whether the function is increasing or decreasing and impacts the steepness of the curve.

1. Based on what we have already seen, what does the parameter $a$ indicate about the function and the graph?

Now let's look at functions for which $a=2$, that is, functions of the form $f(x)=2 \cdot b^{x}$. Let's see what happens when we change the value of $b$.

Use your calculator to fill in the tables for questions 2-6. Then plot points by hand and sketch the graph. Answer the questions about each graph.
2. $y=2 \cdot 2^{x}$

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

a. Where is the $y$-intercept?

b. Is the function increasing or decreasing?
3. $y=2 \cdot 3^{x}$

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

a. Where is the $y$-intercept?

b. Is the function increasing or decreasing?
c. How does the steepness of the graph compare with $y=2 \cdot 2^{x}$ ?
4. $y=2 \cdot 4^{x}$

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

a. Where is the $y$-intercept?
b. Is the function increasing or decreasing?

c. How does the steepness of the graph compare with $y=2 \cdot 2^{x}$ ?
d. How does the steepness of the graph compare with $y=2 \cdot 3^{x}$ ?
5. $y=2 \cdot\left(\frac{1}{2}\right)^{x}$

| $x$ | $y$ |
| :---: | :---: |
| -4 |  |
| -3 |  |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

a. Where is the $y$-intercept?

b. Is the function increasing or decreasing?
c. How is this graph related to the graph in question $2, y=2 \cdot 2^{x}$ ?
6. $y=2 \cdot\left(\frac{1}{4}\right)^{x}$

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |

a. Where is the $y$-intercept?
b. Is the function increasing or decreasing?

c. How does the steepness of this graph compare with $y=2 \cdot\left(\frac{1}{2}\right)^{x}$ ?

For questions 7-12, first answer the question. Then check your answer on the calculator. You may have to adjust your window to get a good view of the graphs.
7. Which function, $f(x)=3(1.1)^{x}$ or $f(x)=3(1.2)^{x}$ is steeper? Explain how you know.
8. Which function, $f(x)=3(1.2)^{x}$ or $f(x)=3(1.25)^{x}$ is steeper? Explain how you know.
9. Which function, $f(x)=3(1.1)^{x}$ or $f(x)=3(0.11)^{x}$ is an increasing function? Explain how you know.
10. Which function, $f(x)=3\left(\frac{6}{7}\right)^{x}$ or $f(x)=3\left(\frac{7}{6}\right)^{x}$ is a decreasing function? Explain how you know.
11. Which function, $f(x)=3(1+.02)^{x}$ or $f(x)=3(1-.02)^{x}$, is an increasing function? Explain how you know.
12. Which function, $f(x)=300(.7+.4)^{x}$ or $f(x)=300(1.7+.2)^{x}$, is a decreasing function? Explain how you know.

## Domain and Range of Exponential Functions

13. Look at the table and graph for $y=2 \cdot 2^{x}$ in question 2 , and answer these questions.
a. Are there any restrictions on $x$ ? Can $x$ be any positive number? Can $x$ be zero? Can $x$ be any negative number?
b. Based on your answer to (a) what is the domain of this function?
c. Are there any restrictions on $y$ ? Can $y$ be any positive number? Can $y$ be zero? Can $y$ be any negative number?
d. Based on your answer to (c) what is the range of this function?
14. Look at the table and graph for $y=2 \cdot\left(\frac{1}{2}\right)^{x}$ in question 5 , and answer these questions.
a. Are there any restrictions on $x$ ? Can $x$ be any positive number? Can $x$ be zero? Can $x$ be any negative number?
b. Based on your answer to (a) what is the domain of this function?
c. Are there any restrictions on $y$ ? Can $y$ be any positive number? Can $y$ be zero? Can $y$ be any negative number?
d. Based on your answer to (c) what is the range of this function?

Special Cases: You have looked at functions for which $b>1$ and for which $0<b<1$. Now consider these special cases.
15. Let $b=1$. Make a table and sketch a graph of the function $y=2 \cdot 1^{x}$

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |



1. What is unusual about this function?
b. What are the domain and range for this function?
2. Let $b=0$. Make a table and sketch a graph of the function $y=2 \cdot 0^{x}$, for values of $x$ greater than zero.

| $x$ | $y$ |
| :---: | :---: |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

a. What is unusual about this function?

b. Why is -1 not in the domain of this function?
c. Enter $0^{\wedge} 0$ in your calculator. What is the result?
17. Let $b<0$. Make a table for the function $y=2 \cdot(-2)^{x}$.

| $x$ | $y$ |
| :---: | :---: |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |

a. Describe any patterns you see in the table.
b. Enter $\mathrm{Y} 1=2 *(-2)^{\wedge} \mathrm{X}$ in your calculator. Use the Zoom4 Decimal Window. Describe what the graph looks like.
18. When exponential functions are studied, usually we only consider cases where $b>1$ or $0<b<1$. Why do you think this is?

## Tossing M\&M's

Gather the following materials: $\quad \mathrm{M} \& \mathrm{Ms}$ (Don't eat them!)
Paper cup for M\&Ms
Paper plate or other surface for "tossing" M\&Ms
Directions: In your groups, start with $2 \mathrm{M} \& \mathrm{Ms}$ in your cup.
Toss the M\&Ms on your plate.
Count the number of M\&Ms that landed with the "mm" logo facing up.
Take that number of M\&Ms from the bag and add to your cup along with all the M\&Ms tossed on the plate. Now you are ready for the next roll.
Repeat this process for 10 trials or until you get past 90 M\&Ms.
Fill out the table as you go.

Example. Trial 0: Start with 2 M\&Ms, toss,
Trial 1: Now there are $3 \mathrm{M} \& \mathrm{Ms}$ to toss
Trial 2 Now there are $5 \mathrm{M} \& \mathrm{Ms}$ to toss
Trial 3 Now there are $10 \mathrm{M} \& \mathrm{Ms}$ to toss
$1 \mathrm{M} \& \mathrm{M}$ facing up, add $1 \mathrm{M} \& \mathrm{M}$
$2 \mathrm{M} \& \mathrm{Ms}$ facing up, add $2 \mathrm{M} \& \mathrm{Ms}$
$5 \mathrm{M} \& \mathrm{Ms}$ facing up, add $5 \mathrm{M} \& \mathrm{Ms}$

1. Now perform your group experiment and record your data in the table as you go. One student will toss, one will count, and one will record. When you are done, graph your results.

| Trial \# | \# M\&Ms <br> Tossed |
| :---: | :---: |
| 0 | 2 |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| 6 |  |
| 7 |  |
| 8 |  |
| 9 |  |
| 10 |  |


2. Are the data from the trials linear? Explain why or why not.
3. Is there a constant rate of change in the table? Explain why or why not.
4. Fill out the table below to see if we can find any pattern in the data.

- Find the difference between the outputs (\#M\&Ms) of the trials: (new - previous).
- Find the ratio between the outputs (\#M\&Ms) of the trials: (new)/(previous).

| Trials <br> (new to previous) | 4 to 3 | 5 to 4 | 6 to 5 | 7 to 6 | 8 to 7 | 9 to 8 | 10 to 9 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Difference |  |  |  |  |  |  |  |
| Ratio |  |  |  |  |  |  |  |

5. What pattern do you see in the differences on successive trials? Do the differences increase, decrease, or stay the about the same?
6. What pattern do you see in the ratios on successive trials? Do the ratios increase, decrease, or stay about the same?

## Exponential Functions

Your data may be modeled with an exponential function, which we can call the "M\&M function." Recall that an exponential function has the form $f(x)=a \cdot b^{x}$, where $a$ is the initial value and $b$ is the growth factor.
7. In your data, where do you find the initial value?
8. What is the initial value for the $M \& M$ function?
9. Estimate the growth factor for the $\mathrm{M} \& \mathrm{M}$ function.
10. Look at the table in question 4 . Where do you think the growth factor should appear?
11. Does the exact growth factor appear in the table?
12. Why do you think the ratios in the table do not exactly match the growth factor every single time?
13. Write an equation for the $M \& M$ function.
14. Fill out the following table using the $M \& M$ function. (Round the number of $M \& M$ s to the nearest integer.)

| Trial \# | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| \# M\&Ms |  |  |  |  |  |  |  |  |


| Trial \# | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# M\&Ms |  |  |  |  |  |  |  |  |

15. According to the table, how many new M\&Ms are added between:

Trials 0 and 1 : $\qquad$ Trials 1 and 2: $\qquad$ Trials 2 and 3: $\qquad$
Trials 12 and 13 : $\qquad$ Trials 13 and 14 : $\qquad$ Trials 14 and 15 : $\qquad$
16. What do you notice about the change in the earlier trials compared to the change later on?
17. Summarize the differences between a linear function and an exponential function.

## Extending the M\&M Function

18. A 7 -ounce bag of M\&Ms has about 210 pieces of candy. How many trials would it take to use an entire bag of M\&Ms? Estimate an answer.
19. A 10 -pound bag of $M \& M s$ holds about 5,000 pieces of candy. How many trials would it take to use the whole bag? Estimate an answer.
20. Suppose we wanted to continue our experiment for 50 trials. How many M\&Ms do you think we would need? Estimate an answer.

## Using Technology to Explore the M\&M Function

Enter the data from your group's initial experiment into L1 and L2 on your graphing calculator. Find an exponential regression equation to model the data. (Press STAT $\rightarrow$ CALC, and scroll down to "0 ExpReg.")
21. Use your calculator to find the exponential equation that models your data. Write the equation below.
22. Is this equation exactly the same as the equation from question 13? Is the equation similar to the equation from question 13? Why do you think this is the case?
23. Use your calculator to answer question 18. Write the answer below. How close was your estimate?
24. Use your calculator to answer question 19. Write the answer below. How close was your estimate?
25. Use your calculator to answer question 20. Write the answer below. Is this surprising? Explain why or why not.

## Percent Change Situations

## For each problem:

A. Decide whether each situation is growth or decay and explain how you know.
B. Decide whether each situation is linear or exponential and explain how you know.
C. Identify your variables and write an equation for the function.
D. Answer the related question(s).

1. My pet iguana was 20 cm long when I got him. Then each month his length was $8 \%$ longer than the month before.
A.
B.
C.
D. How long was he after a year?
2. Tom's ATV (all-terrain vehicle) was worth $\$ 800$ when he purchased it. Each year it lost $16 \%$ of its value.
A.
B.
C.
D. How much was it worth after 4 years?
3. Sue's scarf was only 8 inches long. Her grandmother took it and each day she knitted $75 \%$ of its original length.
A.
B.
C.
D. How long was the scarf after 5 days?
4. Aunt Amy starts feeding four seagulls at the beach. Each minute the number of seagulls wanting to feed from her is about $50 \%$ more than the number there the minute before.
A.
B.
C.
D. How many seagulls is she feeding eight minutes later?
5. You lend $\$ 75$ to your brother, and each month for a year he will pay you only simple interest of $5 \%$. Then at the end of the year he will have to pay you the $\$ 75$ back.

First find out what simple interest means. Explain it here.
A.
B.
C.
D. How much total interest will your brother have paid you at the end of the year?
6. A local pond started with about 2400 fish. Due to contamination, the number of fish in the pond decreased. Each week the lake lost about $12 \%$ of the fish from the week before.
A.
B.
C.
D. How long until there is less than half of the original fish population left?

## The Mathematics of Global Warming

## Introduction

The term "global warming" refers to the increase in the average temperature of the Earth. In May of 2006, Al Gore, former senator, vice president, and presidential candidate, created a movie documentary, An Inconvenient Truth. The movie was designed to alert the public about the global warming crisis and to halt its progress. For more information about An Inconvenient Truth and global warming, visit http://www.takepart.com/an-inconvenient-truth to see the trailer for the movie.

## Modeling Carbon Dioxide Data

Global warming is related to amount of $\mathrm{CO}_{2}$ (carbon dioxide) in the atmosphere. To project what is ahead for future temperatures, we'll look at recent $\mathrm{CO}_{2}$ levels. Notice that these all are far above 300 ppmv (parts per million by volume).

The data below was gathered at Mauna Loa in Hawaii:


Source: http://en.wikipedia.org/wiki/File:Mauna_Loa_Carbon_Dioxide-en.svg
The lighter line that moves up and down accounts for the annual natural increase and decrease in the $\mathrm{CO}_{2}$ level in our atmosphere. The darker line represents the average $\mathrm{CO}_{2}$ level.

1. Fill in the with the average $\mathrm{CO}_{2}$ values. Take these values from the darker curve.

| Year | 1960 | 1965 | 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 | 2008 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| \# of Years <br> since 1960 |  |  |  |  |  |  |  |  |  |  |  |
| CO2 level <br> (ppmv) |  |  |  |  |  |  |  |  |  |  |  |

2. Looking at the graph or numerical data, do you think a linear model or an exponential model (or neither) would be best for modeling the $\mathrm{CO}_{2}$ data over time? Explain.
3. Enter the data into your graphing calculator and examine a plot of the data. Then, find a model that will fit the data as closely as possible. Write the equation for your model, in function notation, here and explain how you arrived at your equation.
4. What is the real world meaning for the value of each parameter in your equation? (What are the parameters?) Use complete sentences.

Parameter: $\qquad$
Meaning:

Parameter: $\qquad$
Meaning:
5. By what percent is the $\mathrm{CO}_{2}$ level changing each year?
6. Use your model to project the $\mathrm{CO}_{2}$ level in the year 2020. Show how you arrived at your projection.
7. Now, project ahead to the year 2050 (perhaps a good approximation for Gore's projection in "less than 50 years"). Write the predicted $\mathrm{CO}_{2}$ level here.

## Modeling Global Temperature Data

There are several ways to measure the average temperature of the entire earth. One method, used by the Goddard Institute for Space Study, collects data from stations all over the world. It shows that the earth's average surface temperature for the years 1951-1980 was $57.2^{\circ}$ Fahrenheit. Since there are fluctuations from year to year, a more accurate picture of how the global temperature changes over time can be found by looking at 10 -year periods (decades). The table below summarizes the data collected for the years 1910-2009.

| Decade <br> starting <br> with year | Years since <br> $\mathbf{1 9 0 0}$ | Average <br> Temperature <br> $\left({ }^{\circ} \mathbf{F}\right)$ |
| :---: | :---: | :---: |
| 1910 | 10 | 56.70 |
| 1920 | 20 | 56.88 |
| 1930 | 30 | 57.12 |
| 1940 | 40 | 57.26 |
| 1950 | 50 | 57.16 |
| 1960 | 60 | 57.13 |
| 1970 | 70 | 57.20 |
| 1980 | 80 | 57.52 |
| 1990 | 90 | 57.86 |
| 2000 | 100 | 58.12 |


|  |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |

8. Enter the data into your calculator and graph it. Include a sketch of the graph above. Clearly label your axes and choose appropriate scales. (Hint: Let $\mathrm{Ymin}=56.5$ and $\mathrm{Ymax}=58.5$ )
9. Looking at the graph above, do you think a linear model or an exponential model (or neither) would be best for modeling the surface temperature data over time? Explain.
10. Find a function that will fit the data as closely as possible. Explain how you arrived at your function.

Function:
Explanation:
11. Use your function to project the temperature of the earth for the decade starting in 2010. Show how you arrived at your projection.
12. Now, project ahead to the decade starting in 2050. Write the predicted temperature here. Do you think it is a good prediction? Explain why or why not.

## The Ice Cap at Mount Kilimanjaro

Although near the equator, Mount Kilimanjaro in Tanzania, Africa, is covered with ice. For the past century, however, its ice has been melting. Some scientists believe this is due to global warming; others believe that there are other causes such as deforestation. Nevertheless the fact that the glacier (large body of ice) is melting is undisputable.
(http://dailymaverick.co.za/article/2010-09-28-revealed-the-real-cause-of-kilimanjaros-melting-ice-cap)

Between 1912 and 1989 the extent of ice covering Mount Kilimanjaro decreased 75\%. It then decreased further so that by 2000 its extent was only $19 \%$ of what it had been in 1912. Using an index number of 100 for 1912 we have the following data.
(Source: http://www.geo.umass.edu/climate/kibo.html)

| Year | Years since <br> $\mathbf{1 9 1 2}$ | Ice cover <br> $(\mathbf{1 9 1 2}=\mathbf{1 0 0})$ |
| :---: | :---: | :---: |
| 1912 |  | 100 |
| 1989 |  | 25 |
| 2000 |  | 19 |
| 2010 |  | 15 |

13. Find a linear model that fits these data.
14. Find an exponential model that fits these data.
15. Predict the ice cover in the year 2015 for both models.
16. By what percent is the ice cover changing each year?
17. Will the ice cover on Mount Kilimanjaro completely disappear some day? Defend your answer.

## Countering Global Warming

Use the equations we developed in Activity 7.6.1 to answer the following questions. Show all of your work!

1. Predict the $\mathrm{CO}_{2}$ level in the year 2025 .
2. Predict the temperature of the Earth in the year 2025.

## Green Energy

Individuals and governments have begun responding to global warming by investing in "green" energy sources. One green energy source is wind power. Below is a table of global wind power, in megawatts, from 1980-2009.
3. What kind of model should you create for these data? Explain.
4. Write a function to model the data. Use years since 1980 for the $x$ variable.
5. Explain the meaning of the " $a$ " and " $b$ " parameters in your equation.
6. According to you equation, by what percent is the world's wind power increasing each year?
7. Use your equation to predict the megawatts of wind power in 2025.

| Year | Years <br> since <br> $\mathbf{1 9 8 0}$ | World Wind <br> Power <br> (Megawatts) |
| :---: | :---: | :---: |
| 1980 |  | 10 |
| 1982 |  | 90 |
| 1984 |  | 600 |
| 1986 |  | 1,270 |
| 1988 |  | 1,580 |
| 1990 |  | 1,930 |
| 1992 |  | 2,510 |
| 1994 |  | 3,490 |
| 1996 |  | 6,100 |
| 1998 |  | 10,200 |
| 2000 |  | 17,400 |
| 2002 |  | 31,100 |
| 2004 |  | 47,620 |
| 2006 |  | 74,052 |
| 2008 |  | 120,550 |
| 2009 |  | 157,899 |

## Solar Power

The development of solar power may also help us combat global warming. The graph below shows the increase in solar power production in the years 1980-2009. Source: http://www.earthpolicy.org/datacenter/pdf/book wote energy solar.pdf

World Annual Solar Photovoltaics Production, 1975 - 2009

8. What kind of model do you think will best fit these data? Explain.
9. Write a function to model the data.
10. According to your equation, by what percent is the world's solar power production increasing each year?
11. Use your function to predict the megawatts of solar power in 2025.

## Geothermal Electricity

Geothermal electricity is another power source that may help us counter global warming. The function modeling global geothermal electricity-generating capacity in megawatts, from 19802010, is:

$$
f(x)=4000 \cdot(1.034)^{x}
$$

where $x$ is the number of years since 1980 .
12. From 1980 to 2010, by what percent is geothermal electricity-generating capacity increasing each year?
13. Use the equation given to fill out the table below.

| Year | 1980 | 1985 | 1990 | 1995 | 2000 | 2005 | 2010 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of years <br> since 1980 |  |  |  |  |  |  |  |
| Geothermal <br> power |  |  |  |  |  |  |  |

14. Graph the equation. Include title and labels and carefully scale axes.

